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Multi-criteria Large-Scale Group Decision-Making in Linguistic Contexts: A Perspective of Conflict Analysis and Resolution

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Abstract

As a universal phenomenon, conflicts exist widely in various fields such as politics, economic life, military, and culture. Group decision-making techniques that effectively identify and resolve conflict during the decision-making process will result in stronger group consensus, while existing studies rarely discuss the multi-criteria large-scale group decision-making (LSGDM) from the perspective of conflict analysis and resolution. This paper systematically studies conflict analysis and resolution approach to obtain consensus decision results. Conflicts among decision makers (DMs) in LSGDM are divided into two kinds: goal conflicts and cognitive conflicts. Based on Pawlak conflict analysis, we introduce three relations among DMs, i.e., conflict, neutrality, alliance into multi-criteria LSGDM in linguistic contexts. Based on linguistic assessment, an improved Pawlak conflict analysis is used to analyze goal conflicts, and the alliance of DMs and the weight of criteria are obtained. According to three cognitive conflict relations, a conflict coordination and feedback mechanism is designed to resolve cognitive conflicts between alliance pairs. Finally, an illustrative example is used to verify the effectiveness and applicability of the proposed model.

Keywords Large-scale group decision-making (LSGDM) · Conflict analysis and resolution · Cognitive conflicts · Goal conflicts

1 Introduction

Group decision-making (GDM) is widespread in our daily life. As an extension of GDM, large-scale group decision-making (LSGDM) refers to finding a solution or alternative that is generally accepted by a large number of decision makers (DMs) with different backgrounds and preferences under several criteria/attributes in a

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decision-making event (Palomares et al. 2013; Labella et al. 2018; Quesada et al. 2015; Ding et al. 2020). The multi-criteria LSGDM problems have the following characteristics: (1) The number of participating members is greater than or equal to 20 and the group has high complexity; (2) Decision attributes present multidimensionality, complexity and randomness; (3) It needs to satisfy the higher consistency requirement of large group preference. The applications of multi-criteria LSGDM are also very wide, such as data-driven circular economy (Kamble et al. 2021), sustainable building material selection (Chen et al. 2021b), transportation management (Chen et al. 2021a), construction project evaluation (Xiao et al. 2020), etc.

Currently, the consensus reaching process (CRP) is a key issue in multi-criteria LSGDM, and there are three research trends developed in the CRP of LSGDM: (1) The expression of opinion. Initially, the opinions of DMs are expressed in numerical forms, e.g., [0, 1]. Due to time pressure and uncertainty of people's cognition, DMs cannot provide accurate quantitative values, but tend to adopt linguistic terms to express their opinions on alternatives (Rodríguez et al. 2021; García-Zamora et al. 2022). In the more complex and uncertain decision-making environment, linguistic terms may be an information representation model that is closer to natural language and people's cognitive habits than exact numerical models. The fuzzy linguistic approach has been applied to solve many real-world LSGDM problems. Even though, in computing linguistic variables, this method may lead to the loss of information. Thus, Herrera and Martínez (2000) proposed a 2-tuple fuzzy linguistic representation model, which allows a continuous representation of the linguistic information. Combining linguistic terms with 2-tuple linguistic representation method, we can model linguistic LSGDM problems and find consensus solutions. In this sense, there are many types of linguistic expression studied in LSGDM, such as hesitant fuzzy linguistic information (Yu et al. 2021), multigranular unbalanced hesitant fuzzy linguistic information (Zhang et al. 2019), etc. (2) Clustering Methods in LSGDM. In LSGDM, we need to cluster large-scale group into several subgroups. Clustering methods of LSGDM can be implemented based on opinion similarities/distances, preference information, and DMs' relations. In the literature, we can find well-known clustering methods such as k-means (Liu and Li 2021), fuzzy c-means (Palomares et al. 2013), grey clustering (Dong et al. 2018), but also other developed clustering techniques (Xu et al. 2015; Li et al. 2018). (3) Behavior management and feedback. Behavior management is about coordinating DMs who refuse to make changes. According to behavior management and predetermined consensus level, we can design a feedback mechanism to achieve the consensus. Non-cooperative behavior detection process in LSGDM is proposed by Palomares et al. (2013), some scholars have systematically studied various non-cooperative behaviors and its feedback mechanism, such as heterogeneous preferences (Chao et al. 2021), fairness concern (Du et al. 2022a), biased DMs (Rabiee et al. 2021), etc. It can be seen that there is a number of advanced linguistic assessment models to solve CRP of LSGDM, but their clustering algorithms, behavior management and feedback mechanism only take into account the similarity of opinions, ignoring the existence of various conflicts that seriously affect the decision-making process. As a universal phenomenon, conflicts exist widely in various fields such as politics, economic life, military, and culture, it arises from the opposition of the goals or cognition pursued by all agents in the system. Therefore, it is necessary to propose a new linguistic assessment model from the perspective of conflict to solve CRP of LSGDM.

Pawlak (1984, 1998, 2005) believed that agents have their own opinions, beliefs, views, votes with respect to some disputed issues, criteria, or solutions. In general the agents may be individuals, groups, companies, states, political parties etc. In order to better analyze and resolve conflicts, Prof. Pawlak first used rough set theory to analyze problems of conflict. In his model, he defines the conflict information system, and provides the concept of conflict function and the conflict degree with the help of auxiliary functions and distinguishable matrices. By employing the method of rough-set attribute reduction and the voting functions to extract decision rules, Pawlak conflict analysis can determine the alliances among agents (Przybyła-Kasperek 2020; Du et al. 2022b). The applications of Pawlak conflict analysis mainly include labor management negotiations (Ali et al. 2019), public engagement programme (Tam and Tong 2011). Inspired by Pawlak conflict analysis, due to the inconsistency of goals or cognition, there are also some conflicts among DMs in LSGDM. Priem et al. (1995) point out that group decision-making techniques that effectively identify and resolve conflict during the decision-making process will result in stronger group consensus. Therefore, conflict seriously affects the CRP of LSGDM.

Existing research only treats conflict as inconsistent opinions and trust relationships between DMs (Cai et al. 2017; Ding et al. 2019; Liu et al. 2019; Wan et al. 2020; Liao et al. 2021). Nevertheless, conflicts in group can be divided into goal conflicts and cognitive conflicts (Cosier and Rose 1977). Goal conflict is an interpersonal relationship involving competition regarding decision outcomes and payoffs; cognitive conflict between DMs is the awareness of inconsistent inferences drawn from identical information (Cosier and Rose 1977). Let's begin with an example, the selection of the civil airport location is a complex LSGDM problem involving citizens, environmental protection department, enterprises, development planning department and other stakeholders. In this problem, citizens and environmental protection department are obviously concerned about noise, pollution criteria; development planning department and enterprises pay attention to criteria with economic benefits. In addition, due to differences in knowledge, background, and resources, these DMs' opinions on airport alternatives are inconsistent. Therefore, goal conflicts appear because different DMs represent different interest groups, and make different preferences on criteria from their own interest. Cognitive conflict refers to the inconsistency in the evaluation of alternatives by DMs with various knowledge levels or experiences.

It can be seen that goal conflict and cognitive conflict have their own characteristics, so we need to deal with the two kinds of conflicts separately. In LSGDM, this conflict of goals involving interests is indirectly reflected in DMs' preferences for criteria and the inconsistency of the importance for each criterion (Tang et al. 2020). Cosier and Rose (1977) pointed out that goal conflict involves competition for payoffs and is a zero-sum game, then goal conflicts are difficult to resolve in a short period of time. So, we focus on analyzing the conflict of goals and mining the conflict relations between DMs. In addition, Piaget (1977) believed that adjustment is an effective method to resolve cognitive conflicts, i.e., when the original cognitive preference cannot adapt to the requirements of the environment, the individual can only change their cognition to meet that. So, we design a feedback mechanism to resolve cognitive conflicts. In short, based on Pawlak conflict analysis, this paper proposes a multi-criteria LSGDM model in linguistic contexts from the perspective of conflict analysis and resolution, the main contributions are summarized as follows:

- For goal conflict analysis, we propose an improved Pawlak conflict model in linguistic contexts. Pawlak conflict relations of goal provide a detailed description for the relationship between pair of DMs. Through improved Pawlak conflict analysis, we can obtain the alliances of DMs and the weights of criteria. In addition, the concept of the strength of alliance is introduced.
- For cognitive conflict resolution, we design a conflict coordination and feedback mechanism. From the perspective of conflict analysis, Power-Average operator is used to determine the opinions of alliances. Based on Pawlak conflict relations of cognition and the strength of alliance, we design an interactive conflict coordination and feedback mechanism to resolve cognitive conflicts between alliances.

The remaining part of the paper proceeds as follows. Section 2 introduces the basic theory of 2-tuple linguistic model and Pawlak's conflict analysis. Section 3 defines the basic concepts of multi-criteria LSGDM problem and discusses how to analyze goal conflicts and resolve cognitive conflicts in LSGDM problems, respectively. In Sect. 4, we summarize the framework and decision-making steps of the proposed method. An illustrative example and comparisons study are provided in Sect. 5. The conclusions and future study are drawn in Sect. 6.

2 Preliminaries

In this section, we review the related concepts of 2-tuple linguistic model and Pawlak conflict analysis.

2.1 2-Tuple Linguistic Model

Suppose $S = \{s_i \mid i = -g, \dots, -1, 0, 1, \dots, g\}$ is a finite and completely ordered set of linguistic terms, the potential of the set is an odd number. The term s_i represents a possible value for a linguistic variable. Generally, linguistic variables need to satisfy some characteristics: 1) Linguistic term set is ordered, $s_i \geq s_j$ iff $i \geq j$; 2) There is a negation operator, $neg(s_i) = s_{-i}$, in particular, $neg(s_0) = s_0$. According to Herrera and Martínez (2000), we have the following definitions.

Definition 1 Let $S = \{s_i \mid i = -g, \dots, -1, 0, 1, \dots, g\}$ be a linguistic term set, β be a number in the granularity interval of the linguistic term set S. For $\beta \in [-g, g]$, let $i = round(\beta)$ and $\alpha = \beta - i$ be two values such that $i \in [-g, g]$ and $\alpha \in [-0.5, 0.5)$. Thus, α is called a symbolic translation with rounding operation *round*.

Definition 2 Let S be a linguistic term set and $\beta \in [-0.5, 0.5)$ a granularity interval. Then the 2-tuple that expresses the equivalent information to β can be defined with the following function:

$$\Delta : [-g, g] \longrightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), with \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases}$$

where Δ is a one to one mapping function, the inverse function of Δ is denoted as Δ^{-1}: $S \times [-0.5, 0.5) \longrightarrow [-g, g]$ with $\Delta^{-1}((s_i, \alpha)) = i + \alpha$.

The comparison and negation operator of 2-tuple linguistic information has been studied in Herrera and Martínez (2000). Details are shown in below.

- *Comparison of 2-Tuples:* Let (s_k, α_1) and (s_l, α_2) be two 2-tuples. Then, a) if $k < l$ then $(s_k, \alpha_1) < (s_l, \alpha_2)$; b) if $k = l$, then i) if $\alpha_1 = \alpha_2$, then $(s_k, \alpha_1) = (s_l, \alpha_2)$; ii) if $\alpha_1 < \alpha_2$, then $(s_k, \alpha_1) < (s_l, \alpha_2)$.
- *Negation Operator of 2-Tuple:* the negation operator over 2-tuples is shown as $Neg((s_i, \alpha)) = \Delta(-\Delta^{-1}(s_i, \alpha))$.

2.2 Pawlak Conflict Analysis

Pawlak (1984, 1998, 2005) proposed a rough set-based conflict analysis method to describe the complicated structure of conflict. Let $IS = (U, C)$ be an information system, the elements of the universe U are called agents, C is a set of disputes. The set of values of $c \in C$ is denoted as $V^c = \{-1, 0, 1\}$, $c(x)$ is the opinion of agent x about dispute c. -1, 0, 1, respectively represent that agent holds an attitude of opposition, neutrality and support towards dispute. Pawlak provided a simple example of the Middle East conflict to demonstrate the validity of the concept of conflict information system. In his example, there are six agents: Israel (1), Egypt (2), Palestinians (3), Jordan (4), Syria (5), Saudi Arabia (6), and five disputes: autonomous Palestinian state on the West Bank and Gaza (a), Israeli military outpost along the Jordan River (b), Israeli retains East Jerusalem (c), Israeli military outposts on the Golan Heights (d), Arab countries grant citizenship to Palestinians who choose to remain within their borders (e). So, this Middle East conflict can be clearly depicted in the form of a matrix in Table 1.

Definition 3 Let $IS = (U, C)$ be an information system, for $\forall x, y \in U$ and $\forall c \in C$, auxiliary function $\phi_c : U \times U \longrightarrow \{-1, 0, 1\}$ can be defined:

$$\phi_c(x, y) = \begin{cases} 1 & \text{if } c(x) \cdot c(y) = 1 \vee x = y, \\ 0 & \text{if } c(x) \cdot c(y) = 0 \wedge x \neq y, \\ -1 & \text{if } c(x) \cdot c(y) = -1. \end{cases}$$

Table 1 Information system for the middle east conflict

	a	b	c	d	e
1	−1	+1	+1	+1	+1
2	+1	0	−1	−1	−1
3	+1	−1	−1	−1	0
4	0	−1	−1	0	−1
5	+1	−1	−1	−1	−1
6	0	+1	−1	0	+1

$\phi_c(x, y) = 1$ means that agent x and y hold the same attitude toward dispute c, $\phi_c(x, y) = 0$ means that at least one agent holds a neutral attitude toward dispute c, $\phi_c(x, y) = -1$ means that agent x and y hold the different attitude (support or opposition) toward dispute a.

In order to evaluate opinions between agents x and y with respect to the set of disputes $B \subseteq C$, Pawlak put forward the concept of distance function.

Definition 4 Let $IS = (U, C)$ be an information system, for $\forall x, y \in U$ and $B \subseteq C$, distance function $\rho_C : U \times U \longrightarrow [0, 1]$ can be defined:

$$\rho_C(x, y) = \frac{\sum_{c\in B} \phi^*_c(x, y)}{|B|},$$

where

$$\begin{aligned}\phi^*_c(x, y) &= \frac{1 - \phi_c(x, y)}{2}\\ &= \begin{cases} 0 & \text{if } c(x) \cdot c(y) = 1 \vee x = y,\\ 0.5 & \text{if } c(x) \cdot c(y) = 0 \wedge x \neq y,\\ 1 & \text{if } c(x) \cdot c(y) = -1.\end{cases}\end{aligned}$$

By the Definition 4, we can obtain three relations between a pair of agents about dispute set. For $B \subseteq C$, a pair $x, y \in U$ is said to be

- In conflict $\mathbb{R}^-_B(x, y)$ iff $\rho_B(x, y) > 0.5$,
- Neutral $\mathbb{R}^0_B(x, y)$ iff $\rho_B(x, y) = 0.5$,
- Allied $\mathbb{R}^+_B(x, y)$ iff $\rho_B(x, y) < 0.5$.

3 Conflict Analysis and Resolution for Multi-criteria LSGDM

The multi-criteria LSGDM problems studied in this article have the following main elements.

- $A = \{a_1, a_2, \dots, a_n\}$, $n \geq 2$, a finite non-empty set of alternatives, which are regarded as possible solutions to LSGDM problem.
- $C = \{c_1, c_2, \dots, c_m\}$, $m \geq 2$, a finite non-empty set of evaluation criteria and its weight vector is $\boldsymbol{w} = (w_1, w_2, \dots, w_m)^T$, with $w_j \geq 0$ and $\sum_{j=1}^{m} w_j = 1$.
- $E = \{e_1, e_2, \dots, e_q\}$, $20 \leq q \leq 50$, a finite non-empty set of experts from different professional fields, which are invited to evaluate alternatives. In addition, the number of experts in an LSGDM problem should be no less than 20 (Liu et al. 2015; Ding et al. 2020). In the real-world LSGDM problem, it does not seem to be reasonable to ask too many people to be an expert in a concrete area (García-Zamora et al. 2022). Therefore, considering the actual decision-making situation and the characteristics of the CRP, this paper defines the number of experts participating in the decision process as $q \in [20, 50]$.
- Let $S^{\diamond} = \{s^{\diamond}_{-3}, s^{\diamond}_{-2}, s^{\diamond}_{-1}, s^{\diamond}_{0}, s^{\diamond}_{1}, s^{\diamond}_{2}, s^{\diamond}_{3}\}$ be a linguistic term set with respect to attitude on criteria, where $s^{\diamond}_{-3}$ = extremely opposed, $s^{\diamond}_{-2}$ = opposed, $s^{\diamond}_{-1}$ = somewhat opposed, $s^{\diamond}_{0}$ = neutral, $s^{\diamond}_{1}$ = somewhat supportive, $s^{\diamond}_{2}$ = supportive, $s^{\diamond}_{3}$ = extremely supportive. Each expert can use linguistic terms in $S^{\diamond}$ to express her/his attitude on criteria. Let $S^{\circ} = \{s^{\circ}_{-4}, s^{\circ}_{-3}, s^{\circ}_{-2}, s^{\circ}_{-1}, s^{\circ}_{0}, s^{\circ}_{1}, s^{\circ}_{2}, s^{\circ}_{3}, s^{\circ}_{4}\}$ be a linguistic term set, where s°_{-4} = extremely poor,s°_{-3} = very poor, s°_{-2} = poor, s°_{-1} = somewhat poor, s°_{0} = medium, s°_{1} = somewhat good, s°_{2} = good, s°_{3} = very good, s°_{4} = extremely good. Each expert can use linguistic term set S° to express her/his opinions on alternatives under the set of criteria, forming an evaluation matrix $V_p = (v^p_{ij})_{n \times m}$, v^p_{ij} denotes the linguistic evaluation of alternative x_i with respect to criterion c_j.

From the perspective of conflict analysis and resolution, the multi-criteria LSGDM problem in this paper mainly includes the following contents.

- *Goal Conflict Analysis* (Sect. 3.1). In Sect. 3.1.1, we first introduce the definition of auxiliary function, which is the basis of goal conflict analysis. According to auxiliary function, we propose the three relations with respect to goal as: conflict, neutrality, alliance and the corresponding conflict set, neutrality set, alliance set. Through three goal conflict relations, we can determine the maximal alliance of experts and the strength of alliance, shown in Sect. 3.1.2.
- *Cognitive Conflict Resolution* (Sect. 3.2). In Subsection 3.2.1, we use Power Average operator to aggregate opinions of individual expert into a collective opinion from the perspective of conflict resolution. According to opinions of alliances, we will design an interactive conflict coordination and feedback mechanism to resolve cognitive conflicts between alliances in Sect. 3.2.2.
- *Selection Process of Alternatives* (Sect. 3.3). After conflict analysis and resolution, this subsection first determine the weights of alliances and criteria, respectively. Then the final evaluation value of each alternative is obtained by using the WA operator. So, the order ranking of each alternative is determined.

In the following subsections, we will present more modeling details.

3.1 Goal Conflict Analysis

3.1.1 Goal Conflict Relations

In order to express relations between agents we define three basic binary relations on the universe: conflict, neutrality and alliance. To this end Pawlak defined the auxiliary function. So, we need the following auxiliary function in linguistic contexts.

Definition 5 Let $c_j \in C$ be a criterion in LSGDM, for $\forall x, y \in E$, $c(x,j) \in S^{\diamond}$ and $c(y,j) \in S^{\diamond}$ denote conflict attitudes of agent x and y towards dispute c_j, respectively. Then, auxiliary function$: E \times E \longrightarrow [0,1]$ can be defined:

$$d(c(x,j), c(y,j)) = \frac{\left|\Delta^{-1}(s^{\diamond}_{xj}, \alpha_{xj}) - \Delta^{-1}(s^{\diamond}_{yj}, \alpha_{yj})\right|}{2g}. \tag{1}$$

According to Pawlak conflict analysis, In what follows we will respectively define three novel basic relations $\mathbb{R}^{\asymp}_{\diamond}$, $\mathbb{R}^{\approxeq}_{\diamond}$ and $\mathbb{R}^{=}_{\diamond}$ over E^2 called alliance, neutrality and conflict relations by using goal conflict distance function.

Definition 6 Let $IS^{\diamond} = (E, C)$ be an information system in LSGDM. For $\forall x, y \in E$, the goal conflict distance function $\rho^{\diamond} : E \times E \longrightarrow [0,1]$ can be defined:

$$\rho^{\diamond}(x,y) = \frac{1}{m} \sum_{j=1}^{m} d(c(x,j), c(y,j)). \tag{2}$$

Then, let $GCS = (E, \rho^{\diamond})$ be a goal conflict space, given a pair of thresholds (ν, μ) with $0 \le \nu \le \mu \le 1$, we can obtain three goal conflict relations $\left\langle \mathbb{R}^{\asymp}_{\diamond}, \mathbb{R}^{\approxeq}_{\diamond}, \mathbb{R}^{=}_{\diamond} \right\rangle$ between any agent pair with respect to the dispute set. For any pair of expert x and y, is said to be

- in conflict $\mathbb{R}^{\asymp}_{\diamond}(x,y)$ iff $\rho^{\diamond}(x,y) > \mu$,
- neutral $\mathbb{R}^{\approxeq}_{\diamond}(x,y)$ iff $\nu \le \rho^{\diamond}(x,y) \le \mu$,
- allied $\mathbb{R}^{=}_{\diamond}(x,y)$ iff $\rho^{\diamond}(x,y) < \nu$.

The three goal conflict relations are more general extension of Pawlak's conflict, neutral, and allied relations, which can avoid some inconsistencies. This provides a clear description for the relation of any expert pair. To identify conflict objects, alliance objects, and neutral objects for each expert, the conflict set, neutrality set, alliance set of each expert can be defined as follows.

Definition 7 Let $GCS = (E, \rho^{\diamond})$ be a goal conflict space, given a pair of thresholds (ν, μ) with $0 \le \nu \le \mu \le 1$. Then, for any $x \in E$, the conflict set $CO^{(\nu,\mu)}_{\diamond}(x)$, neutrality set $NE^{(\nu,\mu)}_{\diamond}(x)$, and alliance set $AL^{(\nu,\mu)}_{\diamond}(x)$ are defined as:

- $CO^{(\nu,\mu)}_{\diamond}(x) = \{y \in E \mid \rho^{\diamond}(x,y) > \mu\}$;

- $NE_{\diamond}^{(\nu,\mu)}(x) = \{y \in E \mid \nu \le \rho^{\diamond}(x,y) \le \mu\}$;
- $AL_{\diamond}^{(\nu,\mu)}(x) = \{y \in E \mid \rho^{\diamond}(x,y) < \nu\}$.

It can be seen that thresholds are important for determining conflict relations, so we use decision with decision-theoretic rough set, proposed by Yao (2010), to compute thresholds. The value of $\rho^{\diamond}(x,y)$ can indicate the probability to which x is in conflict with y, Lang et al. (2017) provided a detailed explanation. Thus, for any $x, y \in E$, let $\Omega = \left\{ y \in CO_{\diamond}^{(\nu,\mu)}(x), y \in AL_{\diamond}^{(\nu,\mu)}(x) \right\}$ be two states, which denotes that DM x and y are in conflict and allied, respectively. a_C, a_N and a_A represents three actions of classifying y into $CO_{\diamond}^{(\nu,\mu)}(x)$, $NE_{\diamond}^{(\nu,\mu)}(x)$ and $AL_{\diamond}^{(\nu,\mu)}(x)$. As shown in Table 2, λ_{CC}, λ_{NC} and λ_{AC} are the cost loss functions of taking actions a_C, a_N and a_A, respectively, when y belongs to $CO_{\diamond}^{(\nu,\mu)}(x)$; λ_{CA}, λ_{NA} and λ_{AA} are the cost loss functions of taking actions a_C, a_N and a_A, respectively, when y belongs to $AL_{\diamond}^{(\nu,\mu)}(x)$.

Theorem 1 *Let $GCS = (E, \rho^{\diamond})$ be a conflict space, for any $x, y \in E$, if the loss functions satisfy that $0 \le \lambda_{CC} \le \lambda_{NC} \le \lambda_{AC}$, $0 \le \lambda_{AA} \le \lambda_{NA} \le \lambda_{CA}$, then the following holds true.*

- If $\rho^{\diamond}(x,y) > \mu$, then $y \in CO_{\diamond}^{(\nu,\mu)}(x)$;
- If $\nu \le \rho^{\diamond}(x,y) \le \mu$, then $y \in NE_{\diamond}^{(\nu,\mu)}(x)$;
- If $\rho^{\diamond}(x,y) < \nu$, then $y \in AL_{\diamond}^{(\nu,\mu)}(x)$.

Where,

$$\mu = \frac{\lambda_{CA} - \lambda_{NA}}{(\lambda_{CA} - \lambda_{NA}) + (\lambda_{NC} - \lambda_{CC})},$$
$$\nu = \frac{\lambda_{NA} - \lambda_{AA}}{(\lambda_{NA} - \lambda_{AA}) + (\lambda_{AC} - \lambda_{NC})},$$
$$\gamma = \frac{\lambda_{CA} - \lambda_{AA}}{(\lambda_{CA} - \lambda_{AA}) + (\lambda_{AC} - \lambda_{CC})}.$$

Proof The proof is given in the Appendix 1. □

Theorem 1 shows that the loss function of different actions in different states will affect the identification of the conflict relation between experts. Yao (2010)

Table 2 Goal Conflict Loss Function of x and y on C

	$y \in CO_{\diamond}^{(\nu,\mu)}(x)$	$y \in AL_{\diamond}^{(\nu,\mu)}(x)$
a_C	λ_{CC}	λ_{CA}
a_N	λ_{NC}	λ_{NA}
a_A	λ_{AC}	λ_{AA}

put forward that losses may be estimated through techniques such as cost-effective analysis and cost-benefit analysis, the estimation of losses is much domain dependent and needs careful investigation based on domain knowledge when applying the decision-theoretic rough set model. In this paper, users can determine the loss of dividing conflict, neutrality, and alliance relations in different states based on the degree of mutual trust between experts, such as social networks, beliefs, values, etc. Then, we normalize this qualitative or quantitative loss information to a dimensionless loss value.

3.1.2 The Alliances of DMs

Formation of coalitions is a very import issue in conflict analysis and many results have been obtained in this area, the idea of coalition is a consequence of the assumed alliance relation (Pawlak 2005). Pawlak used conflict graph to represent the above defined relations, which clearly expresses the relationship between any two experts. From the conflict graph we can also observe alliances among experts.

Through goal conflict relations $\left\langle \mathbb{R}^{\asymp}_{\diamond}, \mathbb{R}^{\approx}_{\diamond}, \mathbb{R}^{=}_{\diamond} \right\rangle$, we also have the goal conflict graph, where experts are represented by circles. In order to find maximal alliances, all cliques should be identified in the graph. So the subset of vertices such that every two vertices are connected by dashed line is determined, that is, maximal connected subgraph connected by dashed line. For example, in Fig. 1, there are two alliances: $\{e_1, e_6\}$ and $\{e_2, e_3, e_4, e_5\}$. Graph is easy to understand but not easy to compute. In what follows, we introduce the definition of maximal alliance.

Definition 8 Let $GCS = (E, \rho^{\diamond})$ be a goal conflict space, for $X \subseteq E$, if $X \times X$ are said to be allied $\mathbb{R}^{=}_{\diamond}$, then X is called an alliance. If there is no alliance Y that satisfies $X \subset Y$, then X is called a maximal alliance in LSGDM.

According to the Definitions 7 and 8, if X is a maximal alliance, then $\bigcap_{x \in X} AL^{(\nu,\mu)}_{\diamond}(x) = X$, which provides an effective aid for us to find alliances in LSGDM. Therefore, we design an incremental algorithm for computing maximal

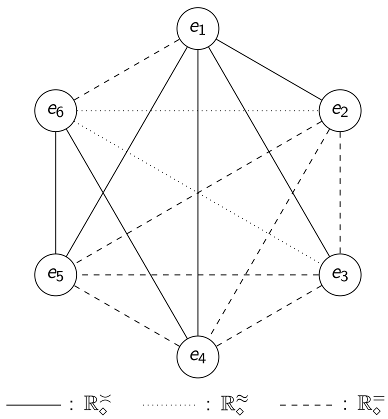

Fig. 1 Goal conflict graph of a multi-criteria LSGDM

alliances, shown in Algorithm 1. The AL is the set of all maximal alliances, it includes the alliance with only one expert and the alliance with multiple experts. If a maximal alliance has only one expert, then the expert is in conflict with others, he/she is only allied with himself/herself.

Algorithm 1 The Algorithm for Computing Alliances

Require: alliance set $AL_{\diamond}^{(\nu,\mu)}(x)$ and E
Ensure: maximal alliance AL
1: $AL \leftarrow \{\}$
2: **for** $\forall x \in E$ **do**
3:     **if** $AL_{\diamond}^{(\nu,\mu)}(x) = \{x\}$ **then**
4:         $AL \leftarrow AL \cup \{x\}$
5:     **end if**
6: **end for**
7: **return** AL
8: $E \leftarrow E - AL$
9: **for** construct X satisfying $\bigcap_{x \in X} AL_{\diamond}^{(\nu,\mu)}(x) = X$ **do**
10:     **repeat**
11:         find Y where $X \subset Y = \bigcap_{x \in Y} AL_{\diamond}^{(\nu,\mu)}(x)$
12:         $X \leftarrow Y$
13:     **until** $Y = X$
14:     **return** $AL \leftarrow X \cup AL$
15: **end for**
16: **repeat** operations 9–15

Different from subgroups in conventional LSGDM, an expert may belong to multiple alliances at the same time in this paper. For example, there are three alliances: $\{e_1, e_6\}$, $\{e_2, e_3, e_4\}$ and $\{e_2, e_5\}$. Obviously, $\{e_2, e_3, e_4\} \cap \{e_2, e_5\} = e_2 \neq \emptyset$, and we call this overlapping subgroups. Driven by interests, and resources, stakeholders expect different stakeholder groups interact, one individual may belong to several stakeholder groups (Rowley and Moldoveanu 2003; Jansson 2005). Therefore, there may be some overlapping (similar) memberships across multiple stakeholder groups. In view of this, we have the following definition.

Definition 9 For any maximal alliance $X \in AL$, if there is a maximal alliance $Z \in AL$ satisfying $X \cap Z \neq \emptyset$, then X is called weak alliance; otherwise the strong alliance. The strength of alliance X can be calculated as

$$\delta = 1 - \frac{\left|\bigcup_{Z \in AL, Z \neq X} X \cap Z\right|}{|X|} \tag{3}$$

Obviously, $\delta \in [0, 1]$, if $\delta = 1$, then X is a strong alliance; otherwise weak alliance. Strong alliance and weak alliance reflect the strength and stability of an alliance, which can affect the resolution of cognitive conflict between alliances in the next section. Thus, in the above example, the strength of alliance $\{e_2, e_5\}$ is 0.5, and that of alliance $\{e_2, e_3, e_4\}$ is 0.67.

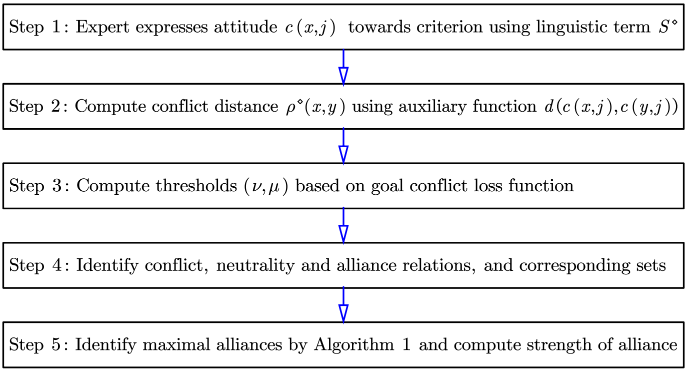

Fig. 2 The flow chart of goal conflict analysis

In order to facilitate the understanding of goal conflict analysis proposal, we provide a flow chart to describe all the steps, shown in Fig. 2.

3.2 Cognitive Conflict Resolution

Cognitive conflicts mainly include conflicts within alliances and conflicts between alliances. To obtain the cognition of alliances, we use Power Average operator to aggregate opinions of individual expert into a collective opinion from the perspective of conflict resolution. Then, we will design an interactive conflict coordination and feedback mechanism to resolve cognitive conflicts between alliances.

3.2.1 Cognitive Conflicts within Alliances

In an alliance, due to the pressure of conformity or the group with strong cohesion, reaching internal consensus without considering more alternatives may cause decision-making errors. If conflict is stimulated in a way that presents multiple different perspectives, it is possible to come up with more ideas and improve the accuracy and effectiveness of decision-making. So, the opinion with less conflict is more likely to be accepted by others. Power average (PA), proposed by Yager (2001), is a useful aggregation operator that can naturally reflect the interrelationships among aggregated arguments by permitting them to support and reinforce each other (Chen et al. 2021c; Xiong et al. 2019). Through this mechanism, smaller weights can be automatically assigned to the opinion with unduly high conflict. Thus, the PA operator is suitable for integrating opinions within a coalition. Let $v_{ij}^{p} \in S^{\circ}$ and $AL = \left\{X_1, X_2, \dots, X_Q\right\}$ be the set of maximal alliance in LSGDM, for cognitive conflict within alliances, we use PA operator to aggregate experts' opinions in alliance, i.e., an alliance evaluation matrix $V_{X_k} = (v_{ij}^{X_k})_{n\times m}$, $X_k \in AL$, such that

$$v_{ij}^{X_k} = \text{P-A}\left(v_{ij}^1, v_{ij}^2, \dots, v_{ij}^{|X_k|}\right) = \bigoplus_{p=1}^{|X_k|} \frac{(1 + T(e_p))v_{ij}^p}{\sum_{p=1}^{|X_k|}(1 + T(e_p))} \tag{4}$$

where

$$T(e_p) = \sum_{r=1, r\neq p}^{|X_k|} \text{Sup}(e_p, e_r).$$

$\text{Sup}(e_p, e_r)$ denotes the support for DM e_p from DM e_r. Using the conflict distance, we let $\text{Sup}(e_p, e_r) = 1 - (1/nm)\sum_{i=1}^{n}\sum_{j=1}^{m} d\left(v_{ij}^p, v_{ij}^r\right)$, cognitive conflict distance $d\left(v_{ij}^p, v_{ij}^r\right)$ is same to the definition of goal auxiliary function in Eq. (1). Thus, we see the more similar, the closer two experts, the more they support each other, the fewer conflicts they have. From the perspective of conflict resolution, experts with higher support should be assigned greater weights.

3.2.2 Cognitive Conflicts Between Alliances

Similar to goal conflict analysis, we first need to analyze conflict, neutrality and alliance relations between alliances in cognitive conflict, which can help us identify alliance pairs in conflict. For these alliance pairs, we can design an effective feedback mechanism to resolve cognitive conflicts.

Definition 10 Let $IS^\circ = (AL, C, A)$ be an information system of alliances in LSGDM, for any alliance $X_k, X_l \in AL$, $V_{X_k} = (v_{ij}^{X_k})_{n\times m}$ and $V_{X_l} = (v_{ij}^{X_l})_{n\times m}$ denotes the decision matrix of alliance X_k and X_l, respectively. Then, the cognitive conflict distance function $\rho^\circ : AL \times AL \longrightarrow [0, 1]$ between any pair of alliances can be defined as

$$\rho^\circ\left(X_k, X_l\right) = \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m} d\left(v_{ij}^{X_k}, v_{ij}^{X_l}\right). \tag{5}$$

Obviously, $\rho^\circ\left(X_k, X_l\right) = \rho^\circ\left(X_l, X_k\right)$. By the Eq. (1), $d\left(v_{ij}^{X_k}, v_{ij}^{X_l}\right)$ is called cognitive auxiliary function, where

$$d\left(v_{ij}^{X_k}, v_{ij}^{X_l}\right) = \frac{\left|\Delta^{-1}(s_{kij}^\circ, \alpha_{kij}) - \Delta^{-1}(s_{lij}^\circ, \alpha_{lij})\right|}{2g}.$$

Similar to goal conflict analysis, In what follows we can respectively have three basic relations $\mathbb{R}_\circ^{\asymp}$, $\mathbb{R}_\circ^{\approx}$ and $\mathbb{R}_\circ^{=}$ over AL^2 called alliance, neutrality and conflict relations by using cognitive conflict distance function. Let $CCS = (AL, \rho^\circ)$ be a cognitive conflict space, given a pair of thresholds (φ, ψ) with $0 \leq \varphi \leq \psi \leq 1$. Then, for any pair of alliance X_k and X_l, is said to be

- In conflict $\mathbb{R}_{\circ}^{\asymp}(X_k, X_l)$ iff $\rho^{\circ}(X_k, X_l) > \psi$,
- Neutral $\mathbb{R}_{\circ}^{\approx}(X_k, X_l)$ iff $\varphi \le \rho^{\circ}(X_k, X_l) \le \psi$,
- Allied $\mathbb{R}_{\circ}^{=}(X_k, X_l)$ iff $\rho^{\circ}(X_k, X_l) < \varphi$.

Thus, $\left\langle \mathbb{R}_{\circ}^{\asymp}, \mathbb{R}_{\circ}^{\approx}, \mathbb{R}_{\circ}^{=} \right\rangle$ is called cognitive conflict relations, the determination of (φ, ψ) is similar to that of (ν, μ).

Obviously, we always hope that the cognitive conflict between any pair of alliances is as small as possible. For the alliance pair (X_k, X_l) satisfying $\mathbb{R}_{\circ}^{\asymp}(X_k, X_l)$, we need gradually shift their relation to $\mathbb{R}_{\circ}^{\approx}$ or $\mathbb{R}_{\circ}^{=}$ by adjusting their opinions. Suppose the alliance pair (X_k, X_l) in conflict $\mathbb{R}_{\circ t}^{\asymp}$ is selected to make adjustment in the t-th iteration, let $\rho^{\circ}\left(X_k, X_l\right)$ be abbreviated as ρ_{kl}°, the evaluation matrix $V_{X_k}^{t+1} = (v_{ij(t+1)}^{X_k})_{n\times m}$ and $V_{X_l}^{t+1} = (v_{ij(t+1)}^{X_l})_{n\times m}$ of the alliance pair (X_k, X_l) in $t+1$-th iteration can be modified by the following strategies:

$$v_{ij(t+1)}^{X_k} = \xi_t^k v_{ij(t)}^{X_k} \oplus (1 - \xi_t^k) v_{ij(t)}^{X_l} \tag{6}$$

$$v_{ij(t+1)}^{X_l} = \xi_t^l v_{ij(t)}^{X_l} \oplus (1 - \xi_t^l) v_{ij(t)}^{X_k} \tag{7}$$

where ξ_t^k is the adjustment coefficient, it reflects the retention ratio of the original opinion of the alliance X_k in the adjusted opinion. Then, $1 - \xi_t^k$ denotes the proportion of alliance X_k adopting the opinion of alliance X_l. The same is true for alliance X_l. Through the coordination, the cognitive conflict between alliance X_k and X_l will become less in the next interaction. In addition, due to $\rho^{\circ}\left(X_k, X_l\right) = \rho^{\circ}\left(X_l, X_k\right)$, then (X_k, X_l) and (X_l, X_k) are disordered.

Since adjustment coefficient plays a key role in cognitive conflict resolution, we need to design a method for determining adjustment coefficient. Considering the resolution of cognitive conflict, an alliance that has more conflicts with other alliances needs to make more adjustments. At the same time, strong alliances are more stable than weak alliances, while weak alliances are more likely to accept others' opinions or suggestions. In each round of modification, weak alliances can make larger adjustments, while strong alliances should avoid excessive adjustments. Inspired by Tang et al. (2020), we calculate the adjustment coefficient as follows.

$$\xi_t^k = 1 - \frac{\sum_{z=1, z\neq l}^{Q} \rho_{kz}^{\circ t}}{(1+\delta_k)\left(\sum_{z=1, z\neq l}^{Q} \rho_{kz}^{\circ t} + \sum_{z=1, z\neq k}^{Q} \rho_{lz}^{\circ t}\right)}, \tag{8}$$

$$\xi_t^l = 1 - \frac{\sum_{z=1, z\neq k}^{Q} \rho_{lz}^{\circ t}}{(1+\delta_l)\left(\sum_{z=1, z\neq k}^{Q} \rho_{lz}^{\circ t} + \sum_{k=1, z\neq l}^{Q} \rho_{kz}^{\circ t}\right)}, \tag{9}$$

where

$$\delta_k = 1 - \frac{\left|\bigcup_{u=1,u\neq k}^{Q} X_k \cap X_u\right|}{\left|X_k\right|}, \delta_l = 1 - \frac{\left|\bigcup_{u=1,u\neq l}^{Q} X_l \cap X_u\right|}{\left|X_l\right|}.$$

It can be seen that $\xi_t^k, \xi_t^l \in (0,1)$. $\sum_{z=1,z\neq l}^{Q} \rho_{kz}^{\circ t}$ and $\sum_{z=1,z\neq k}^{Q} \rho_{lz}^{\circ t}$ are total cognitive conflicts for X_k and X_l with other alliances, respectively. δ_k and δ_l are the strength of alliance X_l and X_l. Compared with Tang et al. (2020), we further consider the impact of strength of alliance on conflict resolution, it is more in line with the actual decision-making situation. For $\xi_t^k = \xi_t^l$, only when the support degrees and the strength of alliances X_k and X_l are equal respectively. Thus, the smaller the strength of an alliance, the smaller its adjustment coefficient.

Suppose the alliance pair with maximal cognitive conflict is (X_y, X_z). In actual LSGDM problems, the alliance may accept or reject the modification strategy. Therefore, we have the following three situations to update the opinions of alliances.

- If both alliances X_y and X_z accept modification strategy, then for any alliance $X_k \in AL$, its opinion in $(t+1)$-th iteration can be updated as

$$v_{ij(t+1)}^{X_k} = \begin{cases} \xi_t^k v_{ij(t)}^{X_k} \oplus (1-\xi_t^k) v_{ij(t)}^{X_z}, & k = y \\ \xi_t^k v_{ij(t)}^{X_k} \oplus (1-\xi_t^k) v_{ij(t)}^{X_y}, & k = z \\ v_{ij(t)}^{X_k}, & k \neq y, z \end{cases} \tag{10}$$

- If alliance X_y accepts and X_z rejects modification strategy, then for any alliance $X_k \in AL$, its opinion in $(t+1)$-th iteration can be updated as

$$v_{ij(t+1)}^{X_k} = \begin{cases} \xi_t^k v_{ij(t)}^{X_k} \oplus (1-\xi_t^k) v_{ij(t)}^{X_z}, & k = y \\ v_{ij(t)}^{X_k}, & k \neq y \end{cases} \tag{11}$$

- If both alliances X_y and X_z reject modification strategy, then for any alliance $X_k \in AL$, its opinion in $(t+1)$-th iteration can be updated as

$$v_{ij(t+1)}^{X_k} = v_{ij(t)}^{X_k}. \tag{12}$$

Theorem 2 *In the t-th round of cognitive conflict resolution, $t \geq 0$, for any $X_k, X_l \in AL$, if $\rho_{kl}^{\circ t} > \psi$, then*

$$\max_{X_k, X_l \in AL} \left\{\rho_{kl}^{\circ t+1}\right\} \leq \max_{X_k, X_l \in AL} \left\{\rho_{kl}^{\circ t}\right\}. \tag{13}$$

Proof The proof is given in the Appendix 1. □

Theorem 2 shows that the adjustment strategy satisfies the convergence, i.e., $\lim_{t\to+\infty} \rho_{kl}^{\circ t+1} = 0$. In each round of modification, the conflicts in alliance pairs are smaller than that in previous round. After several rounds of modifications, the cognitive conflicts between alliances have been continuously resolved, and the consistency of opinions has been continuously improved. Algorithm 2 gives a detailed description of the resolution of cognitive conflict.

Algorithm 2 The Resolution Algorithm for Cognitive Conflict between Alliances

Require: evaluation matrix V_{X_k}, $k = 1, 2, \ldots, Q$; cognitive conflict thresholds (φ, ψ); the maximum number of iterations $T > 0$.

Ensure: evaluation matrix $V^*_{X_k}$.

1: Let P^t be the set of alliances which reject adjustment strategy, and set $P^0 = \emptyset$. For $t = 0$, $v^{X_k}_{ij(0)} = v^{X_k}_{ij}$.

2: For any (X_k, X_l) in all pairs of alliances, if $X_k \in P^t \wedge X_l \in P^t$, skip (X_k, X_l); else use Eq. (5) to obtain $\rho^{\circ t}_{kl}$. Then, obtain three cognitive conflict relations $\langle \mathbb{R}^{\asymp}_{\circ}, \mathbb{R}^{\approx}_{\circ}, \mathbb{R}^{=}_{\circ} \rangle$ in t-th iteration.

3: If $t \geq T \vee P^t = AL \vee \mathbb{R}^{\asymp}_{\circ t} = \emptyset$, go to operation 5; else, continue.

4: Find the alliance pair with maximal cognitive conflict degree, $\rho^{\circ t}_{yz} = \max_{X_k, X_l \in \mathbb{R}^{\asymp}_{\circ t}} \{\rho^{\circ t}_{kl}\}$. Then, we have the following three situations.

- Both X_y and X_z accept modification strategy, the adjusted opinions of all alliances are calculated as Eq. (10). Then, let $t = t + 1$, $P^{t+1} = P^t$; and return to operation 2.
- X_y accepts and X_z rejects modification strategy, the adjusted opinions of all alliances are calculated as Eq. (11). Then, let $t = t + 1$, $P^{t+1} = P^t \cup X_z$; and return to operation 2.
- Both X_y and X_z reject modification strategy, the adjusted opinions of all alliances are calculated as Eq. (12). Then, let $t = t + 1$, $P^{t+1} = P^t \cup X_y \cup X_z$; and return to operation 2.

5: Set $\left(v^{X_k}_{ij(*)}\right)_{n \times m} = \left(v^{X_k}_{ij(t)}\right)_{n \times m}$, that is, $V^*_{X_k} = V^t_{X_k}$.

In order to facilitate the understanding of cognitive conflict resolution proposal, we provide a flow chart to describe all the steps, shown in Fig. 3.

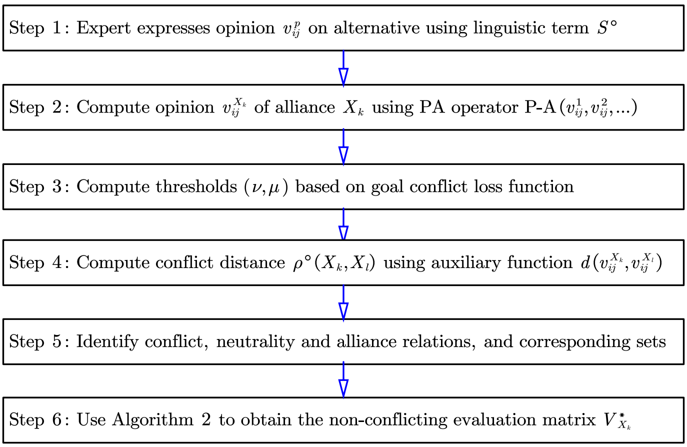

Fig. 3 The flow chart of cognitive conflict resolution

3.3 Selection Process of Alternatives

Suppose Algorithm 2 iteration stops at t^* rounds, we have $V^*_{X_k} = V^{t^*}_{X_k}$, $\rho^{\circ *}_{kl} = \rho^{\circ t^*}_{kl}$. By the cognitive conflict function, the $1 - \rho^{\circ *}_{kl}$ can be regarded as the support of X_k from X_l. An alliance with more support from other alliances should be assigned bigger weight. Then, the weight of alliance X_k is

$$u_k = \frac{\sum_{l=1,l\neq k}^{Q}(1 - \rho^{\circ *}_{kl})}{\sum_{k=1}^{Q}\sum_{l=1,l\neq k}^{Q}(1 - \rho^{\circ *}_{kl})} \tag{14}$$

According to goal auxiliary function, we can obtain the goal conflict degree $GCD(c_j)$ of each criterion and goal conflict degree GCD of information system as follows.

$$GCD(c_j) = \frac{1}{q(q-1)} \sum_{(x,y)\in E\times E} d(c(x,j), c(y,j)) \tag{15}$$

$$GCD = \frac{1}{m}\sum_{j=1}^{m} GCD(c_j) \tag{16}$$

$GCD(c_j)$ holds $GCD(c_j) \in [0, 1]$. As mentioned in the introduction, the importance for each criterion indirectly reflects the interests of the expert. Criteria with a high degree of conflict imply a strong conflict of interest between alliances. In order to reach a consensus result accepted by the majority of the alliances, we should assign smaller weights to criteria with higher conflict degrees. Therefore, we develop a method to obtain the weight vector of criteria based on goal conflict degree $GCD(c_j)$. The weight of a criterion is

$$w_j = \frac{1 - GCD(c_j)}{\sum_{j=1}^{m}(1 - GCD(c_j))} \tag{17}$$

By the weight vector of criteria $\boldsymbol{w} = (w_1, w_2, \dots, w_m)$ and weight vector of alliances $\boldsymbol{u} = (u_1, u_2, \dots, u_Q)$, we can obtain the consensus evaluation v_i of each alternative as

$$v_i = \bigoplus_{j=1}^{m}\bigoplus_{k=1}^{Q} w_j u_k v^{X_k}_{ij(t^*)} \tag{18}$$

Thus, the ranking of alternatives can be determined by v_i.

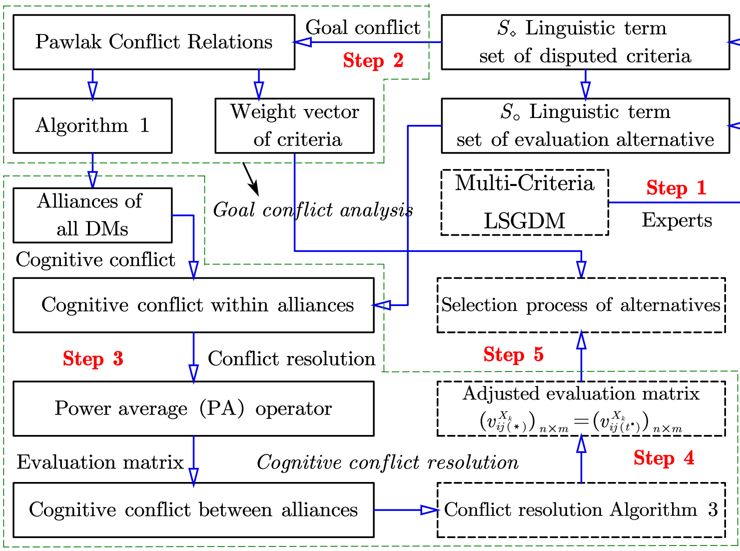

Fig. 4 The framework of conflict analysis and resolution for multi-criteria LSGDM

4 Framework of Conflict Analysis and Resolution for Multi-Criteria LSGDM

Figure 4 clearly shows the framework of conflict analysis and resolution for multi-criteria LSGDM problems. The detailed decision-making steps can be summarized as follows.

Step 1: According to specific LSGDM problems, we collect the linguistic information of experts' conflict attitudes towards disputed criteria and the experts' linguistic evaluation information with respect to alternatives.

Step 2: For goal conflict, we first compute conflict distance function based on Eq. (2), then Theorem 1 is used to determine conflict thresholds, the Algorithm 1 is used to determine the interest alliance formed by experts.

Step 3: For cognitive opinions of alliances, we use the proposed P-A operator to aggregate DMs' opinions in alliance into an alliance evaluation matrix V_{X_k}, shown in Eq. (4).

Step 4: For cognitive conflict between alliances, Algorithm 2 is used to obtain adjusted alliance evaluation matrix $V^*_{X_k}$, $k = 1, 2, \dots, Q$.

Step 5: After conflict analysis and resolution, Eqs. (14) and (17) are used to determine the weight of criterion and alliance, respectively, then we can use Eq. (18) to obtain the consensus evaluation value of each alternative and the ranking of alternatives.

5 Illustrative Example

Since the signing of the "Kyoto Protocol", climate change has become an important issue worldwide, and reducing greenhouse gas emissions in response to climate warming has become a worldwide consensus. Major economies have put forward the goal of "carbon reduction", such as The United States, Japan, and the European Union propose to achieve carbon neutrality by 2050. China has made a commitment to strive to reach the peak of carbon dioxide emissions by 2030, and strive to achieve carbon neutrality by 2060.

Suppose a local government intends to select an industrial enterprise to provide incentives and subsidies for its industrial green technology R &D. The three candidate industrial enterprises are denoted as $A = \{a_1, a_2, a_3\}$. The local government invited 20 experts denoted as $E = \{e_1, e_2, \dots, e_{20}\}$ from various fields such as environmental protection, market, economy, finance, and public management and considered four criteria for each alternative: emission reduction benefit (c_1), technology economic benefit (c_2), R &D cost industrial enterprise (c_3), and the set of criteria is $C = \{c_1, c_2, c_3\}$. Based on experience and historical data, the local government gives the goal conflict loss function in Table 3 and the cognitive

Table 3 Goal conflict loss function of the example

	$y \in CO_\diamond^{(\nu,\mu)}(x)$	$y \in AL_\diamond^{(\nu,\mu)}(x)$
a_C	$\lambda_{CC} = 0.071$	$\lambda_{CA} = 0.235$
a_N	$\lambda_{NC} = 0.221$	$\lambda_{NA} = 0.17$
a_A	$\lambda_{AC} = 0.39$	$\lambda_{AA} = 0.14$

Table 4 Cognitive conflict loss function of the example

	$y \in CO_\circ^{(\varphi,\psi)}(x)$	$y \in AL_\circ^{(\varphi,\psi)}(x)$
a_C	$\lambda_{CC} = 0.04$	$\lambda_{CA} = 0.455$
a_N	$\lambda_{NC} = 0.14$	$\lambda_{NA} = 0.43$
a_A	$\lambda_{AC} = 0.81$	$\lambda_{AA} = 0.353$

Table 5 Goal conflict information system

	c_1	c_2	c_3		c_1	c_2	c_3
e_1	$s_0^\diamond$	$s_1^\diamond$	$s_{-1}^\diamond$	e_{11}	$s_1^\diamond$	$s_2^\diamond$	$s_{-1}^\diamond$
e_2	$s_2^\diamond$	$s_0^\diamond$	$s_0^\diamond$	e_{12}	$s_1^\diamond$	$s_2^\diamond$	$s_{-3}^\diamond$
e_3	$s_2^\diamond$	$s_0^\diamond$	$s_0^\diamond$	e_{13}	$s_2^\diamond$	$s_{-1}^\diamond$	$s_1^\diamond$
e_4	$s_0^\diamond$	$s_1^\diamond$	$s_{-1}^\diamond$	e_{14}	$s_2^\diamond$	$s_2^\diamond$	$s_3^\diamond$
e_5	$s_0^\diamond$	$s_1^\diamond$	$s_{-1}^\diamond$	e_{15}	$s_1^\diamond$	$s_2^\diamond$	$s_{-1}^\diamond$
e_6	$s_2^\diamond$	$s_0^\diamond$	$s_0^\diamond$	e_{16}	$s_0^\diamond$	$s_1^\diamond$	$s_{-1}^\diamond$
e_7	$s_1^\diamond$	$s_2^\diamond$	$s_{-3}^\diamond$	e_{17}	$s_{-1}^\diamond$	$s_1^\diamond$	$s_2^\diamond$
e_8	$s_1^\diamond$	$s_2^\diamond$	$s_{-3}^\diamond$	e_{18}	$s_2^\diamond$	$s_3^\diamond$	$s_2^\diamond$
e_9	$s_2^\diamond$	$s_2^\diamond$	$s_2^\diamond$	e_{19}	$s_2^\diamond$	$s_2^\diamond$	$s_2^\diamond$
e_{10}	$s_0^\diamond$	$s_1^\diamond$	$s_{-1}^\diamond$	e_{20}	$s_0^\diamond$	$s_1^\diamond$	$s_{-1}^\diamond$

Table 6 Cognitive conflict information system

	c_1	c_2	c_3		c_1	c_2	c_3
e_1				e_{11}			
a_1	s°_{-2}	s°_{2}	s°_{2}	a_1	s°_{-2}	s°_{1}	s°_{-3}
a_2	s°_{-3}	s°_{-2}	s°_{-1}	a_2	s°_{-1}	s°_{-2}	s°_{-3}
a_3	s°_{3}	s°_{-1}	s°_{0}	a_3	s°_{3}	s°_{4}	s°_{-1}
e_2				e_{12}			
a_1	s°_{1}	s°_{4}	s°_{-3}	a_1	s°_{-1}	s°_{3}	s°_{-2}
a_2	s°_{0}	s°_{3}	s°_{3}	a_2	s°_{1}	s°_{2}	s°_{-2}
a_3	s°_{-2}	s°_{-4}	s°_{1}	a_3	s°_{2}	s°_{1}	s°_{-1}
e_3				e_{13}			
a_1	s°_{0}	s°_{0}	s°_{2}	a_1	s°_{0}	s°_{4}	s°_{-4}
a_2	s°_{1}	s°_{3}	s°_{-3}	a_2	s°_{-2}	s°_{-2}	s°_{-2}
a_3	s°_{0}	s°_{3}	s°_{-4}	a_3	s°_{-1}	s°_{1}	s°_{0}
e_4				e_{14}			
a_1	s°_{1}	s°_{1}	s°_{-2}	a_1	s°_{1}	s°_{2}	s°_{-4}
a_2	s°_{1}	s°_{-2}	s°_{3}	a_2	s°_{3}	s°_{2}	s°_{1}
a_3	s°_{-2}	s°_{3}	s°_{2}	a_3	s°_{1}	s°_{-1}	s°_{-2}
e_5				e_{15}			
a_1	s°_{-1}	s°_{-3}	s°_{-2}	a_1	s°_{2}	s°_{4}	s°_{-3}
a_2	s°_{1}	s°_{-4}	s°_{3}	a_2	s°_{0}	s°_{-1}	s°_{-3}
a_3	s°_{-2}	s°_{2}	s°_{2}	a_3	s°_{4}	s°_{-1}	s°_{4}
e_6				e_{16}			
a_1	s°_{-1}	s°_{-2}	s°_{-3}	a_1	s°_{1}	s°_{-1}	s°_{1}
a_2	s°_{2}	s°_{-1}	s°_{4}	a_2	s°_{-3}	s°_{-3}	s°_{-1}
a_3	s°_{3}	s°_{0}	s°_{-2}	a_3	s°_{-2}	s°_{0}	s°_{4}
e_7				e_{17}			
a_1	s°_{0}	s°_{-2}	s°_{2}	a_1	s°_{2}	s°_{-2}	s°_{1}
a_2	s°_{-2}	s°_{-2}	s°_{2}	a_2	s°_{-2}	s°_{3}	s°_{4}
a_3	s°_{-1}	s°_{1}	s°_{0}	a_3	s°_{-1}	s°_{-2}	s°_{-4}
e_8				e_{18}			
a_1	s°_{-2}	s°_{3}	s°_{4}	a_1	s°_{0}	s°_{1}	s°_{0}
a_2	s°_{-2}	s°_{-2}	s°_{-4}	a_2	s°_{2}	s°_{-1}	s°_{-4}
a_3	s°_{2}	s°_{2}	s°_{4}	a_3	s°_{3}	s°_{4}	s°_{-3}
e_9				e_{19}			
a_1	s°_{-1}	s°_{-1}	s°_{-4}	a_1	s°_{-2}	s°_{-2}	s°_{0}
a_2	s°_{-2}	s°_{4}	s°_{1}	a_2	s°_{0}	s°_{0}	s°_{-2}
a_3	s°_{4}	s°_{1}	s°_{1}	a_3	s°_{-3}	s°_{4}	s°_{-2}
e_{10}				e_{20}			
a_1	s°_{-2}	s°_{0}	s°_{2}	a_1	s°_{1}	s°_{-3}	s°_{-4}
a_2	s°_{-1}	s°_{-3}	s°_{0}	a_2	s°_{-1}	s°_{-1}	s°_{3}
a_3	s°_{1}	s°_{0}	s°_{0}	a_3	s°_{0}	s°_{-4}	s°_{0}

conflict loss function in Table 4, respectively. The linguistic information of experts' conflict attitudes on criteria and the linguistic information of experts' evaluation on alternatives are presented in Tables 5 and 6, respectively.

Step 1: As shown in Tables 5 and 6, DMs' conflict attitudes towards criteria are linguistic terms with granularity interval $[-3, 3]$, DMs' evaluation on alternatives are linguistic terms with granularity interval $[-4, 4]$.

Step 2: Based on Table 3, the thresholds are $\mu = 0.30$ and $\nu = 0.15$. According to Eq. (2) and Algorithm 1, the alliances of experts are presented in Table 7.

Step 3: For cognitive conflicts within 5 alliances, by the PA operator, we can obtain alliance evaluation matrix as follows.

$$V^0_{X_1} = \begin{pmatrix} (s^\circ_0, -0.273) & (s^\circ_0, 0.118) & (s^\circ_{-1}, -0.083) \\ (s^\circ_{-1}, 0.118) & (s^\circ_{-2}, -0.264) & (s^\circ_0, 0.137) \\ (s^\circ_1, -0.397) & (s^\circ_0, 0.398) & (s^\circ_1, 0.36) \end{pmatrix}$$

$$V^0_{X_2} = \begin{pmatrix} (s^\circ_0, 0) & (s^\circ_2, -0.473) & (s^\circ_{-2}, -0.03) \\ (s^\circ_0, 0.231) & (s^\circ_1, -0.274) & (s^\circ_0, 0.499) \\ (s^\circ_0, -0.011) & (s^\circ_0, -0.011) & (s^\circ_{-1}, -0.227) \end{pmatrix}$$

$$V^0_{X_3} = \begin{pmatrix} (s^\circ_{-1}, 0.376) & (s^\circ_2, -0.167) & (s^\circ_0, -0.43) \\ (s^\circ_{-1}, 0.213) & (s^\circ_{-1}, 0.014) & (s^\circ_{-2}, -0.044) \\ (s^\circ_2, 0.029) & (s^\circ_1, 0.424) & (s^\circ_1, 0.188) \end{pmatrix}$$

$$V^0_{X_4} = \begin{pmatrix} (s^\circ_0, -0.494) & (s^\circ_0, 0.01) & (s^\circ_{-2}, 0.01) \\ (s^\circ_1, -0.232) & (s^\circ_1, 0.234) & (s^\circ_{-1}, -0.014) \\ (s^\circ_1, 0.246) & (s^\circ_2, 0.005) & (s^\circ_{-2}, 0.485) \end{pmatrix}$$

$$V^0_{X_5} = \begin{pmatrix} (s^\circ_2, 0) & (s^\circ_{-2}, 0) & (s^\circ_1, 0) \\ (s^\circ_{-2}, 0) & (s^\circ_3, 0) & (s^\circ_4, 0) \\ (s^\circ_{-1}, 0) & (s^\circ_{-2}, 0) & (s^\circ_{-4}, 0) \end{pmatrix}$$

Step 4: According to Table 4, we have $\varphi = 0.1$ and $\psi = 0.2$. Then, we use Algorithm 2 to resolve cognitive conflict between 5 alliances. By the Eq. (5), then the cognitive conflict matrix of paired alliances is

Table 7 Alliances of the example

Alliance X_k	DMs	Strength δ_k
X_1	$e_1, e_4, e_5, e_{10}, e_{11}, e_{15}, e_{16}, e_{20}$	$\delta_1 = 0.75$
X_2	e_2, e_3, e_6, e_{13}	$\delta_2 = 1$
X_3	$e_7, e_8, e_{11}, e_{12}, e_{15}$	$\delta_3 = 0.6$
X_4	$e_9, e_{14}, e_{18}, e_{19}$	$\delta_4 = 1$
X_5	e_{17}	$\delta_5 = 1$

$$\left(\rho_{kl}^{\circ 0}\right)_{Q\times Q}=\begin{pmatrix} 0 & 0.149 & 0.124 & 0.176 & 0.362\\ 0.149 & 0 & 0.19 & 0.113 & 0.31\\ 0.124 & 0.19 & 0 & 0.172 & 0.427\\ 0.176 & 0.113 & 0.172 & 0 & 0.358\\ 0.362 & 0.31 & 0.427 & 0.358 & 0\end{pmatrix}.$$

Thus, $\mathbb{R}_{\circ 0}^{\asymp}=\left\{(X_1,X_5),(X_2,X_5),(X_3,X_5),(X_4,X_5)\right\}$ and $t<T$. At this time, $\mathbb{R}_{\circ 0}^{\asymp}(X_3,X_5)=0.427$ has the maximal cognitive conflict. Based on Eqs. (8) and (9), we have $\xi_0^3=0.8$ and $\xi_0^5=0.66$. Both X_3 and X_5 accept adjustment strategy, let $t=1$ and $P^1=P^0=\emptyset$, then we have

$$V_{X_3}^1=\begin{pmatrix} (s_0^\circ,-0.099) & (s_1^\circ,0.066) & (s_0^\circ,-0.144)\\ (s_{-1}^\circ,-0.029) & (s_0^\circ,-0.189) & (s_{-1}^\circ,0.165)\\ (s_1^\circ,0.423) & (s_1^\circ,-0.261) & (s_0^\circ,0.15)\end{pmatrix},$$

$$V_{X_5}^1=\begin{pmatrix} (s_1^\circ,0.108) & (s_{-1}^\circ,0.303) & (s_1^\circ,-0.486)\\ (s_{-2}^\circ,0.413) & (s_2^\circ,-0.355) & (s_2^\circ,-0.055)\\ (s_0^\circ,0.03) & (s_{-1}^\circ,0.164) & (s_{-2}^\circ,-0.236)\end{pmatrix}.$$

For X_1, X_2, X_4, then $V_{X_1}^1=V_{X_1}^0$, $V_{X_2}^1=V_{X_2}^0$, $V_{X_4}^1=V_{X_4}^0$, then we have

$$\left(\rho_{kl}^{\circ 1}\right)_{Q\times Q}=\begin{pmatrix} 0 & 0.149 & 0.106 & 0.176 & 0.217\\ 0.149 & 0 & 0.132 & 0.113 & 0.166\\ 0.106 & 0.132 & 0 & 0.136 & 0.197\\ 0.176 & 0.113 & 0.136 & 0 & 0.213\\ 0.217 & 0.166 & 0.197 & 0.213 & 0\end{pmatrix}.$$

Thus, $\mathbb{R}_{\circ 1}^{\asymp}=\left\{(X_1,X_5),(X_4,X_5)\right\}$ and $t<T$. At this time, $\mathbb{R}_{\circ 1}^{\asymp}(X_1,X_5)=0.217$ has the maximal cognitive conflict. Thus, $\xi_1^3=0.755$ and $\xi_1^5=0.714$. In this round, X_1 accepts and X_5 rejects adjustment strategy, let $t=2$ and $P^2=P^1\cup X_5=\left\{X_5\right\}$, then we have

$$V_{X_1}^2=\begin{pmatrix} (s_0^\circ,0.066) & (s_0^\circ,-0.082) & (s_{-1}^\circ,0.308)\\ (s_{-1}^\circ,-0.055) & (s_{-1}^\circ,-0.306) & (s_1^\circ,-0.42)\\ (s_0^\circ,0.463) & (s_0^\circ,0.096) & (s_0^\circ,0.479)\end{pmatrix}.$$

For X_2, X_3, X_4,X_5, then $V_{X_2}^2=V_{X_2}^1$, $V_{X_3}^2=V_{X_3}^1$, $V_{X_4}^2=V_{X_4}^1$, $V_{X_5}^2=V_{X_5}^1$, then we have

$$\left(\rho_{kl}^{\circ 2}\right)_{Q\times Q}=\begin{pmatrix} 0 & 0.121 & 0.088 & 0.175 & 0.164\\ 0.121 & 0 & 0.132 & 0.113 & 0.166\\ 0.088 & 0.132 & 0 & 0.136 & 0.197\\ 0.175 & 0.113 & 0.136 & 0 & 0.213\\ 0.164 & 0.166 & 0.197 & 0.213 & 0\end{pmatrix}.$$

Thus, $\mathbb{R}_{\circ 2}^{\asymp}=\left\{(X_4,X_5)\right\}$ and $t<T$. At this time, $\mathbb{R}_{\circ 2}^{\asymp}(X_4,X_5)=0.213$, X_4 and X_5 are selected to adjust their opinions. Then, $\xi_2^4=0.777$ and $\xi_2^5=0.723$. X_4 acceptsadjustment strategy, let $t=3$ and $P^3=P^2$, we have

$$V_{X_4}^3 = \begin{pmatrix} (s_0^\circ, -0.137) & (s_0^\circ, -0.147) & (s_{-1}^\circ, -0.431) \\ (s_0^\circ, 0.243) & (s_1^\circ, 0.326) & (s_0^\circ, -0.354) \\ (s_1^\circ, -0.025) & (s_1^\circ, 0.372) & (s_{-2}^\circ, 0.324) \end{pmatrix}.$$

For X_1, X_2, X_3, X_5, then $V_{X_1}^3 = V_{X_1}^2, V_{X_2}^3 = V_{X_2}^2, V_{X_3}^3 = V_{X_3}^2, V_{X_5}^3 = V_{X_5}^2$, we can obtain

$$\left(\rho_{kl}^{\circ 3}\right)_{Q\times Q} = \begin{pmatrix} 0 & 0.121 & 0.088 & 0.136 & 0.164 \\ 0.121 & 0 & 0.132 & 0.093 & 0.166 \\ 0.088 & 0.132 & 0 & 0.121 & 0.197 \\ 0.136 & 0.093 & 0.121 & 0 & 0.165 \\ 0.164 & 0.166 & 0.197 & 0.165 & 0 \end{pmatrix}.$$

As shown in Fig. 5, it can be seen that as the round of iterations increases, cognitive conflict of alliance pairs tends to decrease, alliance pairs in conflict gradually become neutral or non-conflicting. In 3-th round of modification, we have $\mathbb{R}_{\circ 3}^{\asymp} = \emptyset$, which denotes that all alliance pairs have been classified into $\mathbb{R}_{\circ 3}^{=}$ or $\mathbb{R}_{\circ 3}^{\approx}$. We can move on to the next step.

Step 5: According to *Step 4*, we have $V_{X_1}^* = V_{X_1}^3, V_{X_2}^* = V_{X_2}^3, V_{X_3}^* = V_{X_3}^3, V_{X_4}^* = V_{X_4}^3$, $V_{X_5}^* = V_{X_5}^3$, $\left(\rho_{kl}^{\circ *}\right)_{Q\times Q} = \left(\rho_{kl}^{\circ 3}\right)_{Q\times Q}$. By the Eq. (14), the weight vector of alliances is $\boldsymbol{u} = (0.204, 0.203, 0.201, 0.2, 0.192)^T$. By the Eqs. (15) and (16), the goal conflict degrees of criteria are $GCD(c_1) = 0.181$, $GCD(c_2) = 0.176$, $GCD(c_3) = 0.332$; the goal conflict degree of information system is $GCD = 0.230$. According to Eq.(17) the weight vector of criteria is $\boldsymbol{w} = (0.354, 0.357, 0.289)^T$. Thus, by the Eq. (18), we can obtain

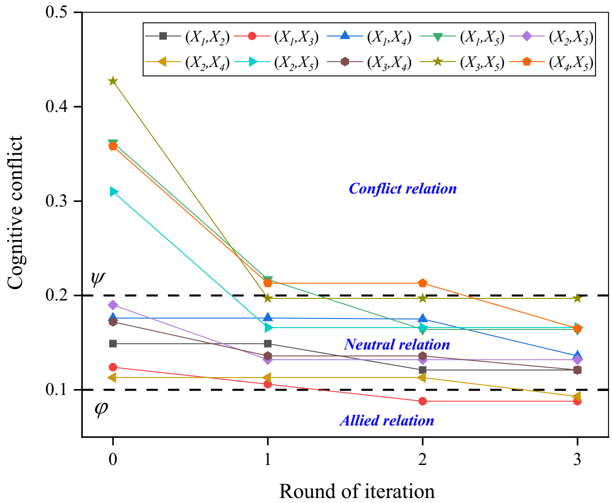

Fig. 5 Evolution of cognitive conflict of paired alliance in each round

$$(v_i)_{Q\times 1} = \left((s_0^\circ, -0.036), (s_0^\circ, 0.03), (s_0^\circ, 0.049)\right)^T.$$

Then, consensus ranking is $a_3 \succ a_2 \succ a_1$.

5.1 Comparison and Discussion

In order to clearly show the difference between the conventional LSGDM models and the proposed model, in this section, we provide a comparative analysis with several other classic studies. It should be noted that in order to facilitate the comparison between different methods, we only select some conventional LSGDM methods (Tang et al. 2020; Liu and Li 2021; Dong et al. 2018; Wang et al. 2018) similar to the modeling condition of this paper. Meanwhile, since the mechanisms of the models are not exactly the same, the results in this paper are only used to illustrate the differences in the contributions of these models.

Clustering a large-scale group into multiple subgroups is an important step in LSGDM. The clusters of Tang et al. (2020) are $\{e_1, e_{15}, e_{16}\}$, $\{e_2, e_3, e_6, e_{13}\}$, $\{e_4, e_5, e_{10}, e_{11}, e_{20}\}$, $\{e_7, e_8, e_{12}\}$, $\{e_9, e_{14}, e_{17}, e_{18}, e_{19}\}$; that of Liu and Li (2021) are $\{e_1, e_{10}, e_{15}, e_{16}\}$, $\{e_2, e_3, e_6, e_{13}\}$, $\{e_4, e_5, e_{11}, e_{20}\}$, $\{e_7, e_8, e_{12}\}$, $\{e_9, e_{14}, e_{17}, e_{18}, e_{19}\}$; that of Dong et al. (2018) is E, that of Wang et al. (2018) are $\{e_1, e_{16}\}$, $\{e_2, e_3, e_6, e_{13}, e\}$, $\{e_4, e_5, e_{10}, e_{11}, e_{20}\}$, $\{e_7, e_8, e_{12}, e_{15}\}$ and $\{e_9, e_{14}, e_{17}, e_{18}, e_{19}\}$. The existing studies mainly obtain subgroups based on some clustering algorithms, such as fuzzy c-means, k-means, grey clustering, hierarchical clustering, and their subgroups are non-overlapping. However, driven by interests, and resources, stakeholders expect different stakeholder groups interact, so one individual may belong to several stakeholder groups (Rowley and Moldoveanu 2003; Jansson 2005). Based on Pawlak conflict analysis, this paper treats subgroup as an alliance and can obtain the overlapping subgroups. Thus, the clusters of this paper are $\{e_1, e_4, e_5, e_{10}, e_{11}, e_{15}, e_{16}, e_{20}\}$, $\{e_2, e_3, e_6, e_{13}\}$, $\{e_7, e_8, e_{11}, e_{12}, e_{15}\}$, $\{e_9, e_{14}, e_{18}, e_{19}\}$ and $\{e_{17}\}$. In addition, we divide the alliance into strong alliances and weak alliances, and the above methods all ignore the stability of the cluster. Therefore, the classification of large-scale groups from the perspective of interest alliance is more practical and has more management significance.

As shown in Table 8, the ranking of Tang et al. (2020) and Liu and Li (2021) is the same as that of this paper, which is $a_3 \succ a_2 \succ a_1$. Even though, Tang et al. (2020) use simple weighting operator to aggregate interest conflict and cognitive conflict, while

Table 8 Comparison with other LSGDM models

Models	Main contribution	Ranking
Tang et al. (2020)	Conflict resolution	$a_3 \succ a_2 \succ a_1$
Liu and Li (2021)	RT-PROMETHEE II	$a_3 \succ a_2 \succ a_1$
Dong et al. (2018)	Non-cooperative behaviors	$a_2 \succ a_3 \succ a_1$
Wang et al. (2018)	Cloud model-based consensus	$a_2 \succ a_3 \succ a_1$
This paper	Goal conflict and cognitive conflict	$a_3 \succ a_2 \succ a_1$

these are two different types of conflict, and weighted aggregation may not be appropriate. In addition, they also ignore the stability and overlap of subgroups, which may make the designed conflict resolution algorithm unable to fully satisfy the interests of the subgroup. The method of Liu and Li (2021) does not allow experts to adequately express their opinions, and its ranking may not be accepted by most experts. Compared with the above two methods, this paper design a conflict resolution algorithm considering the strength of alliances, so that experts can fully express their opinions. Compared with the method of Dong et al. (2018) and Wang et al. (2018), there are some differences in ranking, the ranking of these two methods is $a_2 \succ a_3 \succ a_1$, that of this paper is $a_3 \succ a_2 \succ a_1$. The above two methods need the reference point, i.e., consensus opinion, the centricity-oriented method may lead to biases because different aggregation functions may cause different results. Our paper is constructed based on pair comparisons, its adjustment strategy is dynamic in each iterative round.

In the following, we will compare the differences between our method and some key progresses on CRP from the perspective of modeling mechanism. It mainly involves (1) weight determination, (2) consensus measure and (3) feedback mechanism. In terms of weight determination, some CRP studies mainly used the majority principle, i.e., weight relies on the size of subgroup (Xu et al. 2015; Zhang et al. 2017). Even though, different subgroups with different inner characteristics but the same number of DMs will have the same weight. To address this concern, size and cohesion (Rodríguez et al. 2018), cooperation behavior (Palomares et al. 2013) are proposed. Our method determines the weight by the conflict degree. An alliance/criterion with a low degree of conflict should be assigned bigger weight, which better meets the requirements of conflict resolution. In terms of consensus measure, the distances to the collective opinion (Herrera et al. 1996; Ben-Arieh and Chen 2006) and distances between DMs (Kacprzyk and Fedrizzi 1988; Bordogna et al. 1997) are widely used to measure consensus level. However, these methods require a consensus reference point. Using different aggregation functions, the centricity-oriented method may lead to some biases. This paper directly measures the conflict degree of the alliance pair to reflect the consensus level without any consensus reference point. Therefore, the computational process is simplified. In addition, we divide consensus measures into three relations: conflict, neutrality, and alliance, which provides a detailed description of the relations between DMs. In terms of feedback mechanism, many existing classical models (Herrera-Viedma et al. 2002; Mata et al. 2009) incorporate feedback mechanisms based on this process, where moderator advises DM being farthest from consensus to modify his/her opinion. In this paper, we identify the alliance pair with maximal conflict to modify their assessments, and the feedback mechanism relies on the specific coalition strength and conflict degree of the subgroup. The purpose of the feedback mechanism is to eliminate conflict relations and properly retain neutral relations, which can avoid excessive consensus.

6 Conclusion

This paper proposes a multi-criteria LSGDM model in linguistic contexts from the perspective of conflict analysis and resolution. The main conclusions of this model can be summarized as follows.

- We consider two kinds of conflicts that exist in LSGDM: goal conflicts and cognitive conflicts. Goal conflict refers to the attitude of experts towards the disputed criteria; Cognitive conflict refers to the inconsistent linguistic evaluation provided by experts and their formed alliances.
- In the goal conflict analysis, we propose the concept of three relations among experts and cluster numerous experts into several interest alliances. Considering the real-world situation, there may be overlap between alliances. Furthermore, we can define the strength of alliances.
- In cognitive conflict resolution, we use PA operator to determine the opinions of alliances. Based on three relations with respect to cognitive conflict and the strength of alliances, we design a conflict coordination and feedback mechanism to resolve cognitive conflict between alliance pairs, which does not need the aggregated group opinion as a reference point.
- The two kinds of conflicts are studied separately, the analysis of goal conflict can be regarded as a clustering process, and the resolution of cognitive conflict can be regarded as a CRP for LSGDM.

In the proposed three relations, we focus on conflict relation and allied relation. For neutral relation, we have not systematically studied. In LSGDM, the behavior of experts with neutral relations may seriously affect the decision-making result. Therefore, reaching a consensus considering experts with neutral relations is the focus of our next study. In addition, this paper assumes that the conflict attitude and evaluation information are complete. However, in real life, we may not be able to obtain all the information. How to reach a consensus in an incomplete information environment is a future research challenge.

Appendix A: Proof of Theorem 1

By the Table 2, the expected loss functions produced by three actions for DM y are shown below.

$$\begin{aligned}
C^x(a_C \mid y) =& \rho^\diamond(x,y)\cdot\lambda_{CC} + (1-\rho^\diamond(x,y))\cdot\lambda_{CA};\\
N^x(a_N \mid y) =& \rho^\diamond(x,y)\cdot\lambda_{NC} + (1-\rho^\diamond(x,y))\cdot\lambda_{NA};\\
A^x(a_A \mid y) =& \rho^\diamond(x,y)\cdot\lambda_{AC} + (1-\rho^\diamond(x,y))\cdot\lambda_{AA}.
\end{aligned}$$

According to minimum-risk principle of Bayesian decision, we have the following rules:

(C): If $S^x(a_C \mid y) \le W^x(a_N \mid y)$ and $S^x(a_C \mid y) \le N^x(a_A \mid y)$, then $y \in CO_\diamond^{(\nu,\mu)}(x)$;

(N): If $W^x(a_N \mid y) \le S^x(a_C \mid y)$ and $W^x(a_N \mid y) \le N^x(a_A \mid y)$, then $y \in NE_\diamond^{(\nu,\mu)}(x)$;

(A): If $N^x(a_A \mid y) \le S^x(a_C \mid y)$ and $N^x(a_A \mid y) \le W^x(a_N \mid y)$, then $y \in AL_\diamond^{(\nu,\mu)}(x)$.

We know $0 \le \lambda_{CC} \le \lambda_{NC} \le \lambda_{AC}$ and $0 \le \lambda_{AA} \le \lambda_{NA} \le \lambda_{CA}$, then rules ($S$), ($W$), and ($N$) can be simplified as:

(C): If $\rho^\diamond(x,y) > \mu$ and $\rho^\diamond(x,y) > \gamma$, then $y \in CO_\diamond^{(\nu,\mu)}(x)$;

(N): If $\rho^\diamond(x,y) \le \mu$ and $\rho^\diamond(x,y) \ge \nu$, then $y \in NE_\diamond^{(\nu,\mu)}(x)$;

(A): If $\rho^{\diamond}(x,y) < \nu$ and $\rho^{\diamond}(x,y) < \gamma$, then $y \in AL_{\diamond}^{(\nu,\mu)}(x)$, where

$$\mu = \frac{\lambda_{CA} - \lambda_{NA}}{(\lambda_{CA} - \lambda_{NA}) + (\lambda_{NC} - \lambda_{CC})},$$
$$\nu = \frac{\lambda_{NA} - \lambda_{AA}}{(\lambda_{NA} - \lambda_{AA}) + (\lambda_{AC} - \lambda_{NC})},$$
$$\gamma = \frac{\lambda_{CA} - \lambda_{AA}}{(\lambda_{CA} - \lambda_{AA}) + (\lambda_{AC} - \lambda_{CC})}.$$

Appendix B: Proof of Theorem 2

Without loss of generality, suppose the alliance pair (X_y, X_z) has a maximal cognitive conflict satisfying $\rho_{yz}^{\circ t} > \psi$ in the t-th iteration. In the $(t+1)$-th iteration, we discuss in three situations.

For $\xi_t^k, \xi_t^l \in (0,1)$, if both alliance X_k and X_l accept adjustment strategies, then

$$\begin{aligned}
\rho_{yz}^{\circ t+1} &= \frac{1}{2gnm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|\Delta^{-1}v_{ij(t+1)}^{X_y} - \Delta^{-1}v_{ij(t+1)}^{X_z}\right| \\
&= \frac{1}{2gnm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|\left(\xi_t^y\Delta^{-1}v_{ij(t)}^{X_y} + (1-\xi_t^y)\Delta^{-1}v_{ij(t)}^{X_z}\right)\right. \\
&\qquad \left. - \left(\xi_t^z\Delta^{-1}v_{ij(t)}^{X_z} + (1-\xi_t^z)\Delta^{-1}v_{ij(t)}^{X_y}\right)\right| \\
&= \frac{\left|\xi_t^y + \xi_t^z - 1\right|}{2gnm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|\Delta^{-1}v_{ij(t)}^{X_y} - \Delta^{-1}v_{ij(t)}^{X_z}\right| \\
&\le \frac{1}{2gnm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|\Delta^{-1}v_{ij(t)}^{X_y} - \Delta^{-1}v_{ij(t)}^{X_z}\right| = \rho_{yz}^{\circ t}.
\end{aligned}$$

For any $X_k \in AL, k \neq y,z$, we can obtain $\rho_{yk}^{\circ t+1}, \rho_{zk}^{\circ t+1} < \rho_{yz}^{\circ t}$. Then,

$$\begin{aligned}
\max_{X_k,X_l \in AL}\left\{\rho_{kl}^{\circ t+1}\right\} &= \max\left\{\rho_{yz}^{\circ t+1}, \max_{\substack{X_k,X_l \in AL,\\ k,l \neq y,z}}\left\{\rho_{kl}^{\circ t+1}\right\}, \max_{\substack{X_k \in AL,\\ k \neq y,z}}\left\{\rho_{yk}^{\circ t+1}\right\}, \max_{\substack{X_k \in AL,\\ k \neq y,z}}\left\{\rho_{zk}^{\circ t+1}\right\}\right\} \\
&< \max\left\{\max_{\substack{X_k,X_l \in AL,\\ k,l \neq y,z}}\left\{\rho_{kl}^{\circ t}\right\}, \rho_{yz}^{\circ t}\right\} = \rho_{yz}^{\circ t} = \max_{X_k,X_l \in AL}\left\{\rho_{kl}^{\circ t}\right\}.
\end{aligned}$$

If one alliance accepts while the other rejects. Suppose alliance X_k accepts and X_l rejects modification strategy, then we have

$$\max_{X_k, X_l \in AL} \left\{ \rho_{kl}^{ot+1} \right\} = \max \left\{ \max_{\substack{X_k, X_l \in AL, \\ k, l \neq y, z}} \left\{ \rho_{kl}^{ot+1} \right\}, \max_{X_k \in AL} \left\{ \rho_{yk}^{ot+1} \right\} \right\}$$

$$< \max \left\{ \max_{\substack{X_k, X_l \in AL, \\ k, l \neq y, z}} \left\{ \rho_{kl}^{ot} \right\}, \rho_{yz}^{ot} \right\} = \rho_{yz}^{ot} = \max_{X_k, X_l \in AL} \left\{ \rho_{kl}^{ot} \right\}.$$

If both alliance X_y and X_z reject adjustment strategies, then $v_{ij(t+1)}^{X_k} = v_{ij(t)}^{X_k}$. So, $\max_{X_k, X_l \in AL} \left\{ \rho_{kl}^{ot+1} \right\} \leq \max_{X_k, X_l \in AL} \left\{ \rho_{kl}^{ot} \right\}$.

In summary, it can be proved that $\max_{X_k, X_l \in AL} \left\{ \rho_{kl}^{ot+1} \right\} \leq \max_{X_k, X_l \in AL} \left\{ \rho_{kl}^{ot} \right\}$ is established.

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Declarations

Conflict of interest The authors declare that there are no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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