



The optimal delayed retirement age in aging China: Determination and impact analysis[☆]

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ABSTRACT

Facing a deepening aging population, nations like China are calling for a gradually delayed retirement policy urgently. Based on the dynamic model of Miyazaki (2014), this paper first examines the difference between the agent's labor productivities in youth and old age (the age difference in labor productivity as defined below). We then establish an agent's optimal consumption model under the background of delayed retirement, and derive formulas to calculate the steady-state output per labor, wage rate, and rate of capital return. Based on the model, we propose a formula to determine the optimal delayed retirement age, which can optimize the government's delaying retirement policy. Moreover, we provide theoretical demonstrations and perform numerical simulations about how delayed retirement ages affect key variables such as the agent's optimal consumption and steady-state output per labor in the context of deepening population aging, and further analyze how the Chinese government determines the optimal delayed retirement age. Results are as follows: Firstly, extending the delayed retirement age will reduce the agent's optimal youth consumption, but increase the agent's optimal old-age consumption, life-time consumption, and the total consumption of economy in the current period. Secondly, the aging population problem is affected by both changes in the birth rate and the survival probability. From 2020 to 2100, the former is constantly declining in China, while the latter is gradually rising. Although these two drivers have different influences on the optimal delayed retirement age, the influence of a higher survival probability is greater, so aging postpones the optimal delayed retirement age in China. Thirdly, the degree of population aging and the agent's labor productivity of the elderly are two main factors affecting the optimal delayed retirement age. Although both factors present an increasing trend from 2020 to 2100, they have opposite implications: the former dominates the latter until 2075 and increases the optimal delayed retirement age. On the other hand, the inhibitory effect of the latter gradually increases and eventually dominates the former from 2075. Fourthly, under the cross influence of the above two factors, the optimal delayed retirement age in China shows an inverted U-shaped trend of rising initially and then decreasing after reaching the peak in 2075. Fifthly, we estimate that if the delayed retirement policy is adopted from 2025, it is reasonable to set the average annual delayed retirement time to two to three months. Finally, we propose measures and suggestions to mitigate the possible negative impacts of delayed retirement.

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1. Introduction

Population aging is now becoming increasingly severe in China. On the one hand, the birth rate has dropped to 8.52‰ in 2020, falling below 10‰ for the first time since 1978. On the other hand, the average life expectancy has reached 77.3 years old. The combined effect of the two factors leads to a constant increase in the old age dependency ratio, which has already reached 19.7%.¹ Meanwhile, the legal retirement age policy promulgated in 1978 is still being implemented, and the actual average retirement age is only 54.² With the intensified population aging and the prominent lag of the retirement age policy, a labor gap has generated and is getting increasingly worse in China. Fig. 1 shows that from 2016 to 2039, China's cumulative labor gap³ will reach 153 million, accounting for 11% of the current total population.⁴ Therefore, given the constant reduction in labor market supply caused by the decline of birth rate, it seems urgent to timely adjust the seriously outdated legal retirement age and gradually promote the delayed retirement policy to alleviate the labor supply shortage.

In fact, as an inevitable policy, delaying retirement age has already been considered by the Chinese government. The Ministry of Human Resources and Social Security (MOHRSS), the National Development and Reform Commission (NDRC) and other government agencies proposed to study the policy of flexibly delaying the pension age in 2012. Since then, the specific reform plan for delaying the legal retirement age has been under intensive investigation.⁵ In the 14th Five-year Plan for the Development of Human Resources and Social Security announced in June 2021, MOHRSS clearly claimed to “implement gradual postponement of the legal retirement age steadily”. Meanwhile, an extensive literature also holds a positive attitude towards the impact of delayed retirement. It is believed that delaying retirement can not only reduce human capital waste but also significantly ease the payment pressure of pension funds, improve the pension replacement rate and effectively reduce the gap of pension funds, thereby enhancing the sustainability of the pension system (Bazzana, 2020; Gruber, Milligan, & Wise, 2009). Thus, there is no doubt that implementation of a gradual retirement policy in China has become a foregone conclusion. However, a lot of questions remain unanswered. For example, what impact will an increase in the retirement age have on the agent's⁶ economic activity? Does there exist an optimal delayed retirement age for population aging, and how to determine it? This paper intends to analyze these issues.

Prior literature on the retirement age or the optimal delayed retirement age can be roughly divided into two branches. One branch of literature does not take social security system into account, but treats the retirement age as an endogenous variable in models and allows agents to individually determine when to retire. Therefore, the optimal retirement age is determined by maximizing the total life-time discounted utility of agent's consumption and leisure with the life cycle model. For example, Prettnier and Canning (2014) find that the optimal working hours are 44.22 years when the leisure preference parameter and life expectancy are set as 0.643 and 77.84 years, respectively. Bloom, Canning, and Moore (2014) construct an optimal life cycle model including the perfect capital market and calibrate the parameters of the model by using American data, and find that with the increase of average life expectancy, the optimal delayed retirement ages of agents born in 1901, 1951, and 1996 are 65.1, 68.1, and 70.3 years, respectively. Kuhn, Wrzaczek, Prskawetz, and Feichtinger (2015) add the trade-off choice between medical insurance and retirement to the life cycle model, and show that the optimal delayed retirement age is between 67 and 70 years. However, these conclusions are not practically relevant, due to a lack of social security system led by the government.

The other branch of literature includes the social security system in the models. For example, Tucker (2009) maximizes mortality weighted income from Social Security and private saving, and claims that the optimal retirement age is 62. However, such a model suffers some drawbacks, as its objective function is to maximize output or income. The fundamental purpose of social and economic development is not only to pursue the maximization of economic interests, which is just one part of the residents' good life, but should also be well allied with the goal proposed by the Chinese government.⁷ This is why policy-making, including the postponement of retirement, should focus on the fundamental goal to continuously meet people's ever-growing needs for a good life. There is no doubt that the notion of “a good life” is immensely rich, but according to the perspective of economics, the total life-time discounted utility of agent's youth and old-age consumption and leisure (hereinafter referred to as the agent's life-time discounted utility) could be a feasible and appropriate description. Some prior literature has already taken the agent's life-time discounted utility as the maximization goal and provided insight into delayed retirement within the framework of general equilibrium. Hansen & Lønstrup (2009) point out that the optimal working time is between 35 and 48 years when the life expectancy is 70 years by studying a pay-as-you-go (PAYG) pension system. Wang and Wang (2021) clarify the impact of delayed retirement in China through simultaneously deciding the

¹ The data on the birth rate and old age dependency ratio is from China Statistical Yearbook 2021 published by the National Bureau of Statistics of the People's Republic of China; the data on the average life expectancy is from The Statistical Bulletin on the Development of China's Health Sector in 2019, <http://www.nhc.gov.cn/guihuaxxs/s10748/202006/ebfe31f24cc145b198dd730603ec4442.shtml>.

² Source: http://www.mohrss.gov.cn/SYRlzyhshbzb/dongtaixinwen/buneyiawen/201607/t20160725_244160.html.

³ Labor gap is defined as the difference between the numbers of people exiting and entering the labor market in one year. Cumulative labor gap is the sum of the labor gap in each year. When more people exit the labor market than those enter it, a labor shortage occurs that year.

⁴ According to the data from China Statistical Yearbook in 2021, China's total population is 1.41 billion.

⁵ Source: http://www.gov.cn/zwqk/2012-06/27/content_2171218.htm; http://m.xinhuanet.com/2021-02/26/c_1127143915.htm.

⁶ Agents mentioned in this paper all stand for representative agents. According to the definition provided by Hartley (1996), a representative agent is generally used to refer to a certain type of typical decision-makers in a complete market whose features are homogeneous, while their heterogeneity is nonsignificant or hidden in the model. Furthermore, in the representative agent model, the decisions of multiple representative agents are linearly additive in mathematics.

⁷ The 19th National Congress proposed that the aspirations of the people to live a good life must always be the focus of our efforts. (Source: <http://theory.people.com.cn/n1/2018/0105/c40531-29747906.html>)

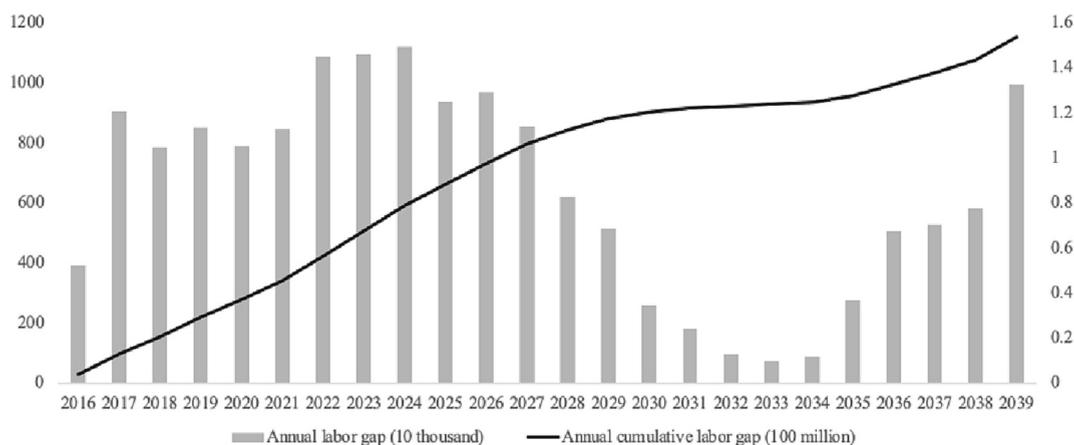


Fig. 1. Chinese annual labor gap and cumulative labor gap from 2016 to 2039.

Note: In Fig. 1, we assume that residents enter the labor market at 20 and retire at 54. The primary axis shows the annual labor gap and the secondary axis shows the cumulative labor gap since 2016; the annual number of newborns is calculated based on the total population and birth rate. The data is from the National Bureau of Statistics.

optimal delayed retirement age and the pension insurance contribution rate to maximize the agent's life-time discounted utility, and claim that the agent's optimal delayed retirement age is between 53 and 59 years old. However, there is still room for improvement. First, the existing literature ignores the change in the age structure of labor force caused by delayed retirement, while Aísa, Pueyo, and Sanso (2012) point out that different age structures of labor force have different labor productivity, which results in different labor supplies and wage incomes, and has a further series of effects on the agent's retirement benefits, consumption and saving decision. In addition, Bound, Stinebrickner, and Waidmann (2010), Tong and Liao (2017), and Attema, Brouwer, and Pinto (2022) also provide empirical evidence that the deterioration of health status could significantly reduce the labor productivity and labor participation rate of the elderly. Studies on developed countries such as Europe and the United States have also reached similar conclusions (Dostie, 2011; Van Ours & Stoeldraijer, 2011). While some prior studies have realized that labor productivity will decrease with the increase of workers' age, which has a significant impact on workers' retirement decisions, no prior research incorporates it into the model when determining the optimal retirement age. We have examined the difference between the agent's labor productivity in youth and old age, which is referred to simply as the agent's age difference in labor productivity.

Secondly, Kang (2012), Wang and Wang (2021) and other studies, when determining the leisure preference parameter in the agent's life-time discounted utility to study the problem of delayed retirement in China, mostly directly employ the value calibrated by studies on foreign countries, which may not be in line with Chinese reality. Yoshino and Hayashi (2002) point out that residents in different countries have their own ethnic characteristics that will be reflected in daily life and work behavior. Furthermore, trade-offs and preferences for work and leisure time vary widely with races (Kalleberg, 2011; Kaplan & Schulhofer-Wohl, 2018). Finally, although this class of the literature considers the adjustment and decision-making role of the social security system, the setting of the social security system is often simplified as much as possible due to the differences in domestic and foreign systems or model tractability. In fact, the endowment insurance system what China adopts is the combination of social account and individual accounts (Kang, 2012) which are respectively managed by the PAYG system and the fully-funded system. Despite the "empty account" problem in individual accounts at one time (Jia, 2017) resulting from the imbalance between revenue and expenditure and the pension gap, China has been gradually consolidating the individual accounts since 2005.⁸ In fact, there is a close relationship between the delayed retirement policy and the pension system. On the one hand, the formulation of delayed retirement policy will necessarily involve the collection mode and amount of endowment insurance to improve the quality of residents' life. On the other hand, the delayed retirement policy will lead to changes in the structure of the working and retired population, affecting revenue and expenditure of the pension system. Therefore, if all features above of Chinese current pension insurance system are not included in the model or are just considered in a simplified fashion, or even the impact of endowment insurance system is ignored, then the results are not practically relevant and provide little implications for the Chinese delayed retirement policy.

In this paper, we establish a model to determine the optimal delayed retirement age conditional on deepening population aging. In

⁸ In the Decision of the State Council on Improving the Basic Endowment Insurance System for Enterprise Employees issued by the State Council in December 2005, it is clearly stated that one of the main tasks of China's Endowment insurance system is to "gradually make real individual accounts and improve the basic system of the combination of social overall planning accounts and individual accounts" and "make real individual accounts and accumulating fundamental endowment insurance fund is an important solution to population aging". Source: http://www.gov.cn/zhuanti/2015-06/13/content_2878967.htm.

order to be consistent with the policy objectives of “comprehensively promoting consumption and enhancing the basic role of consumption in economic development” and “satisfying people’s good life”,⁹ our model maximizes the agent’s life-time discounted utility, incorporating the impacts of changing labor productivity caused by the changing population age structure, social security system, enterprise production, and other factors affected by delayed retirement. Through the model, we analyze the impact of different delayed retirement ages on agent’s youth and old-age consumption and savings. Then, we calibrate the model parameters such as agent’s leisure preference according to the requirements of the model and being closer to the reality in China. Based on numerical simulations, we estimate the optimal delayed retirement age adapted to China’s practical reality and then analyze its influence on the main economic indicators including agent’s youth and old-age consumption.

The contributions of this paper to prior literature are mainly as follows. First, this paper examines the effects of age difference in labor productivity on the optimal delayed retirement age. On the one hand, delaying retirement will increase the total labor supply. On the other hand, compared with youth, the agent’s labor productivity will decline in the elderly, resulting in a substantial reduction in effective labor supply. The first effect acts as an incentive for a delayed retirement, while the second works in the opposite direction. This is in a sharp contrast with most previous studies with the assumption that labor productivity is homogeneous in youth and old age (Hansen & Lønstrup, 2009; Wang & Wang, 2021).

Second, in the numerical analysis, this paper also examines the trend of Chinese population aging and labor productivity in the elderly from 2020 to 2100, and their influence on the optimal delayed retirement age. Our results first show that the optimal delayed retirement age in China presents a unique inverted U-shaped pattern under the foregoing influences. It will rise initially and then decline after reaching the peak in 2075.

Third, as noted, different ethnic characteristics will significantly affect residents’ preference for work-leisure. Our numerical results also show that Chinese agent is more diligent and hard-working, and also has a higher preference for work. Therefore, this paper utilizes the historical data of Chinese population structure and economic variables to calibrate the parameters of agent’s leisure preference, rather than imitating previous research nor directly applying the parameter values in literature. The optimal delayed retirement age obtained based on our model will be more applicable.

Fourth, different from the research of Miyazaki (2014), this paper discusses China’s delayed retirement policy and the impact of the following four factors will be considered comprehensively in determining the optimal delayed retirement age in China. The first is the full prediction and division of the aging degree of China’s population. According to the “World Population Prospects” issued by the United Nations, this paper calculates the degree of China’s population aging from 2025 to 2100, and divides it into moderate aging (from 2025 to 2040) and severe aging (from 2040 to 2100). During the period of moderate aging in China, the proportion of the population aged 60 and above will increase by 10%, which is considered an accelerated phase of population aging, and then the degree of population aging will gradually slow down. The second is the full characterization of Chinese basic endowment insurance system. The third is the full description and embodiment of Chinese development goals of “meeting people’s aspirations to live a good life” and “expanding domestic demand”. The fourth is to accurately estimate the labor productivity and work-leisure preference of Chinese people. Therefore, compared with prior research, this paper will be more reliable in the calculation of Chinese optimal delayed retirement age under the background of deepening population aging.

Fifth, according to the connotation of “gradual delayed retirement” put forward by the Chinese government, this paper further provides detailed plans and implementation steps for gradual delayed retirement under different aging degrees to support the design of a reasonable delayed retirement policy. For more specific innovations and features in other aspects of this paper, please refer to notes 1, 2 and 3.

The rest of the paper proceeds as follows: Section 2 develops the model to determine the agent’s optimal consumption conditional on delayed retirement and derives the formulas to calculate key variables such as steady-state output. Section 3 analyzes the influence of delayed retirement. Section 4 calibrates the model in the aspects of population aging, social security system, and agent’s production and consumption activities. Section 5 presents the calculation and numerical simulation results of the optimal delayed retirement age and the impacts under different degrees of population aging. Section 6 concludes the paper.

2. The model

Similar to Miyazaki (2014), we assume that the economy is a closed system that exists indefinitely and consists of an agent sector, a production sector, and a government sector. The production sector uses labor and capital for consumption goods production and capital accumulation. The agent sector supplies labor for consumption and private savings. The government sector undertakes pension payments and social capital investment. Under such assumptions, we extend the Miyazaki (2014) dynamic general equilibrium overlapping generation model (OLG model), and establish an agent’s optimal consumption model under delayed retirement, as well as derive formulas to calculate steady-state output per labor, wage rate, and rate of capital return. Based on the model, we propose a formula to determine the optimal delayed retirement age for the government’s delaying retirement policy. The specific methods and steps are as follows.

⁹ The data is from the report of the 19th National Congress and the Proposal of the CPC Central Committee on Formulating the 14th Five-year Plan and the Long-term Objectives for 2035 for National Economic and Social Development. http://www.gov.cn/zhengce/2020-11/03/content_5556991.htm

2.1. Agent's problem: Life-time discounted utility, optimal consumption and savings under the background of delayed retirement

2.1.1. Agent's existence status and age difference in productivity

First of all, we set and analyze the agent's existence status. It is assumed that an agent can live for two periods, referred to as young and old ages, and the length of each period is 1. There are two generations of young and old in each period, and following Cipriani (2014), everyone lives during the young ages and survives at the end with a probability of p . Therefore, if N_t is the population size of young agents in period t , the old-age agents in period t is pN_{t-1} , and the growth rate of youth population is $\frac{N_t}{N_{t-1}}$, denoted as η . Obviously, a decrease in the growth rate of the youth population η or an increase in the survival probability p increases the proportion of the elderly in the total population during the same period, which means that the degree of population aging is intensified. However, Miyazaki (2014) does not consider the impact of the growth rate of youth population η and survival probability p , and believes that the population of each period would remain unchanged, which is obviously unrealistic.

Secondly, we specify and analyze the difference between the agent's labor productivity in youth and old age. Most studies such as Hansen & Lønstrup (2009), Kang (2012), and Wang and Wang (2021) assume that the labor productivity of an agent is the same in youth and old age; that is, an agent supplies a unit of labor inelastically in youth and continues to work with the same labor productivity in old age before retirement. Apparently, such an assumption is not in line with reality because the labor productivity of an agent usually declines with age, which is supported by sufficient evidence from prior literature. For example, Aísa et al. (2012) and Attema et al. (2022) point out as an agent grows from youth to old age, his labor productivity will decrease due to his health condition and other reasons, leading to a decline in his wage income. Similarly, Ilmakunnas and Ilmakunnas (2018) show that compared to young age, there would be a drop in the agent's possibility of providing labor, work performance and labor productivity in old age. Dostie (2011) and Van Ours and Stoeldraijer (2011) conduct empirical tests on developed countries including Europe and the United States, and find that employees' wages and productivity decline in old age. In conclusion, it is reasonable and realistic to consider the difference between the agent's labor productivities in youth and old age and reflect the this difference in wage income.

Therefore, following Aísa et al. (2012), we specify the agent's age difference in productivity as follows: The agent at young can supply 1 unit of effective labor inelastically and obtain a wage income of w_t . If the agent remains alive in the old age period, the agent's labor productivity decreases with age (Ferreira & de A Pessôa, 2007). For this reason, following Aísa et al. (2012), a parameter $\delta = p^\xi$ is introduced,¹⁰ where $\xi \in [0, 1]$, $0 \leq \delta \leq 1$. δ represents the coefficient of the effective labor loss of the elderly. Obviously, the larger the value of ξ is, the lower the labor productivity of the elderly compared to the young will be. Therefore, δ reflects the level of labor productivity that the agent can achieve in old age (Aísa et al., 2012), which also means that the effective labor provided by the old agent in one unit time is only δ times of that provided by the young agent. The wage income of the elderly in period t is δw_{t+1} and δ is referred to as **the wage income loss of the elderly**.

Finally, we set and analyze the agent's total expected effective labor supply. We assume that the agent's working time of the elderly is d according to the government's policy of delaying retirement (hereinafter referred to as the delayed retirement age parameter),¹¹ and the leisure time is $(1 - d)$,¹² where $0 \leq d \leq 1$. L_t is the expected total effective labor supply in period t . Then,

$$L_t = N_t + dp\delta N_{t-1} = gN_{t-1} \quad (1)$$

where $g = \eta + dp\delta$. Apparently, the parameter $g = \frac{L_t}{N_{t-1}}$ represents the ratio of the expected total effective labor supply in period t to the youth population in the previous period $t - 1$, which indicates the increase ratio of effective labor supply attributed to the delayed retirement policy. Eq. (1) suggests that conditional on the deepening aging of population, the later the retirement is, the more sufficient the expected effective supply of labor would be.

2.1.2. Agent's life-time discounted utility function (agent's objective function of optimization problem) and budget constraints

Following Hansen & Lønstrup (2009) and Wang and Wang (2021), we assume that the agent's life-time discounted utility function is characterized by the following logarithmic form¹³:

¹⁰ According to Aísa et al. (2012), the parameter δ is positively related to the expected life expectancy p , which is not only consistent with the finding in Qiao and Wang (2019), but also supported by a lot of empirical evidence. On the one hand, the empirical studies by Singh and Miller (2004), Ranabhat, Atkinson, Park, Kim, and Jakovljevic (2018), and Muennig, Reynolds, Fink, Zafari, and Arline (2018) show that the expected life expectancy of agents is significantly positively correlated with their health level. Robine and Ritchie (1991), Lubitz, Cai, Kramarow, and Lentzner (2003) and Stiefel, Perla, and Zell (2010) even regard the expected life expectancy as a health indicator that reflects an individual's health level. On the other hand, Sengupta (2017), Kelani, Odunayo, Ozgebe, and Nwani (2019) and Siddique and Mohey-ud-din (2020) find that the health level is a decisive factor affecting the agent's labor supply and hence his wage income. Therefore, the specification of parameter δ is reasonable.

¹¹ The parameter d is entirely determined by the government, not the agent, and it is derived from the legal delayed retirement age and the life expectancy of the agent. Therefore, the parameter d is equivalent to the legal delayed retirement age in this model.

¹² Weagley and Huh (2004) claim that with the development of economy and the improvement of residents' income, residents are more inclined to increase their leisure time for relaxation or personal empowerment in the case that the original welfare level does not decline.

¹³ It is noteworthy that our model has an implicit assumption of total depreciation of physical capital, mainly for the following two reasons. First, from the perspective of theoretical modeling, total depreciation of physical capital is a common setting of OLG models such as Hansen & Lønstrup, 2009, Miyazaki (2014), and Wang and Wang (2021). Second, according to the data of China from 1952 to 2015, the depreciation rate of physical capital is between 5%–12.5% (Holz & Sun, 2018). Since the time span of each period of OLG model is >30 years, it is reasonable to assume total depreciation of physical capital.

$$U_t = \ln c_t^Y + \beta p [\ln c_{t+1}^O + \gamma \ln(1 - d)] \tag{2}$$

where c_t^Y and c_{t+1}^O stand for, respectively, the expected youth consumption and old-age consumption conditional on survival of the young agents in period t . $0 < \beta \leq 1$ is the time discount factor. γ is the agent's preference for leisure.

For the agent's budget constraints, the agent in youth engages in consumption and savings activities, and pays an extra payroll tax in proportion to wage income to the government for pension benefits of the elderly under the current social security system. To simplify model specifications, most previous literature regards China's pension insurance system as a PAYG system relying on social account management. In reality, the pension insurance system implemented in China is essentially a mixed system that is primarily based on the PAYG system with social account management and supplemented by the fully-funded system with individual account management.¹⁴ We also examine the empty account operation in individual accounts.¹⁵ In fact, as early as 2012, Hu Xiaoyi, deputy minister of MOHRSS, pointed out that "The empty account does exist and the amount is relatively high. However, the government is trying to make up for it."¹⁶ Since then, the government has introduced a series of policies and measures to compensate individual accounts. For example, the State Council in 2017 issued the *Implementation Plan for Transferring Part of State-owned Capital to Enrich Social Security Funds*. Thus, the budget constraint for youth is given by:

$$s_t + c_t^Y = (1 - \tau - \sigma)w_t \tag{3}$$

The right side of Eq. (3) is the net wage income of young agent. τ and σ represent the payroll tax rate of the social account and individual accounts, respectively. The left side shows the expenditure of young agent. c_t^Y and s_t denote the consumption and savings of the agent in youth, respectively. Following Cipriani (2014), the savings s_t and the taxes paid for individual accounts are lent to the insurance companies. Insurance companies carry out investment activities and obtain investment income in the next period for agents. The total return rate on investment of the savings and individual accounts is $\frac{R_{t+1}}{p}$.

Of the elderly, the agent works only during the period of delayed retirement d and spends all the income on consumption. So, the budget constraint of the elderly is given by:

$$c_{t+1}^O = \frac{R_{t+1}}{p} [s_t + \sigma(1 - e)w_t] + (1 - d)\theta_{t+1} + (1 - \tau)d\delta w_{t+1} \tag{4}$$

The left side of Eq. (4) is the agent's expected consumption c_{t+1}^O of the old age in $t + 1$, and the right side is the expected income of the old-age agent at the same time. δw_{t+1} denotes the expected wage income of the old-age agent in $t + 1$. The expected income includes three parts. The first part is the capital income $\frac{R_{t+1}}{p}s_t$ brought by the savings in youth. The second part is the expected net wage income $(1 - \tau)d\delta w_{t+1}$ ¹⁷ from delayed working for time d . And the third is the expected retirement benefits received after retirement, consisting of the benefits from the individual account $\frac{R_{t+1}}{p}\sigma(1 - e)w_t$ and from social account $(1 - d)\theta_{t+1}$, where e denotes the empty account ratio of the individual account, that is, the proportion of funds in the individual account transferred to the social account; θ_{t+1} is the retirement benefits from the social account received by the agent within a unit time, which is decided by the total tax paid during working period and the timing of delayed retirement d . Since the pension insurance tax paid to the individual account is recorded in person and managed based on the fully-funded accumulation system, the retirement benefits from the individual account include extra interest income, and this income has nothing to do with the delayed retirement time.¹⁸

2.1.3. The agent's optimal consumption and savings under the background of delayed retirement

We now can develop a model to determine the agent's optimal consumption and savings under the background of delayed retirement. Under the budget constraints (3) and (4), the agent chooses a certain consumption profile of c_t^Y and c_{t+1}^O to maximize the life-time discounted utility. The optimal consumption profile can be deduced by the Lagrange method to derive Eqs. (2)–(4) with respect to c_t^Y and c_{t+1}^O :

¹⁴ Different from the PAYG system that pays the pensions of current retirees directly through the contributions of active workers, the fully-funded system is essentially an institutional arrangement that compulsorily transfers part of the income of workers during their working period to the retirement period.

¹⁵ The so-called empty account means that there is no cash deposit in the individual account. Only the specific amount is recorded.

¹⁶ Source: http://www.mohrss.gov.cn/SYrlzyhshbzb/zcfg/SYzhengcejiedu/201303/t20130309_87374.html.

¹⁷ Since the pension insurance tax paid into the individual account during the period of delayed retirement is too late to enter capital market and obtain interest, the old agent will no longer pay the pension insurance tax in this model. Actually, such setting also fits in with China's current policy. The Social Insurance Law of the People's Republic of China stipulates that the individual accounts shall not be drawn ahead of schedule, and residents can draw their pension when they reach the legal retirement age and the accumulative payment is over 15 years.

¹⁸ The Decision of the State Council on Improving the Basic Pension Insurance System for Enterprise Employees promulgated in 2005 stipulates that the coverage period of pension from individual accounts is calculated according to the retirement age, and the coverage period depends on the average life expectancy. For example, the coverage period to retire at the age of 55 is 170 months, which is about 14 years. That is to say, the payment is completed at the age of 70. If the pension from the individual account is not fully drawn out after retirement, the family members can inherit it and draw out in one lump sum. Therefore, in the two-period OLG model, retirement benefits from individual accounts are independent of delayed retirement age.

$$\frac{c_{t+1}^{*O}}{c_t^{*Y}} = \beta R_{t+1} \tag{5}$$

Then, the optimal youth and old-age consumption and savings of the agent are given by:

$$c_t^{*Y} = \frac{(1 - \tau)w_t - \sigma e w_t}{1 + \beta p} + \frac{p\theta_{t+1}(1 - d) + (1 - \tau)dp\delta w_{t+1}}{(1 + \beta p)R_{t+1}} \tag{6}$$

$$c_{t+1}^{*O} = \frac{(1 - \tau)w_t - \sigma e w_t}{1 + \beta p} \beta R_{t+1} + \frac{\beta p\theta_{t+1}(1 - d) + (1 - \tau)\beta dp\delta w_{t+1}}{1 + \beta p} \tag{7}$$

$$s_t^* = \left[\frac{\beta p(1 - \tau) + \sigma e}{1 + \beta p} - \sigma \right] w_t - \frac{p\theta_{t+1}(1 - d) + (1 - \tau)dp\delta w_{t+1}}{(1 + \beta p)R_{t+1}} \tag{8}$$

Note 1: Compared with the existing studies, the optimal consumption and optimal savings obtained by Eqs. (5)–(8) have the following characteristics. First, this paper introduces the difference between the agent’s labor productivities in youth and old age, which is referred to as the age difference in labor productivity, which in turn leads to the difference in wage income received by the agent in young and old age: in period t , the wage income of the young agent is w_t , while that of the elderly agent is δw_t . The results of this paper are consistent with Hansen & Lønstrup, 2009 and Kang (2012), who assume that the agent’s labor productivity is homogeneous in age, and there is no difference in the wage income of agents between youth and old age. Second, this paper examines the social account and individual accounts, and the impact of the empty account on the optimal consumption and optimal savings. Eqs. (6) to (8) imply that the optimal consumption is negatively correlated with the tax rates of the social account and the individual accounts, as well as the empty account ratio of individual accounts, while the optimal saving is negatively and positively correlated with the tax rate of individual accounts and the empty account ratio, respectively. The effect of the social account tax rate on the optimal saving is relatively complicated. In contrast, Aísa et al. (2012), Miyazaki (2014), Wang and Wang (2021) and other prior studies either just consider the social account or neglect the issue of empty accounts, which serves as a special case of this paper. Third, this paper analyzes the change in population structure caused by changes in birth rate and survival probability, while Kang (2012) sets the survival probability as 1, which also serves as a special case of this paper. Fourth, when delayed retirement (at this time $d = 0$) and the age difference of agent’s labor productivity is not considered, Eqs. (6) to (8) are consistent with Abel (2003) and Geng, Sun, and Zheng (2016), among others. Thus, our model extends the models considered in prior studies, and our results are more general.

2.2. Formulas of steady-state output per labor, wage rate, and rate of capital return under delayed retirement

2.2.1. Production sector problem

We assume that the production sector uses labor and capital to produce consumption goods in a perfectly competitive market, and the production function takes a standard Cobb-Douglas production form as follows:

$$Y_t = AK_t^\alpha L_t^{1-\alpha} \tag{9}$$

where Y_t is the total output of consumption goods considering delayed retirement. A is the total factor productivity (TFP), which indicates the productivity of technology. K_t is the total capital in period t , and α is capital-output elasticity with $0 < \alpha < 1$. Therefore, the effective wage rate of the current working population in period t is the marginal product of the effective labor force L_t in Eq. (9):

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) \frac{Y_t}{L_t} = A(1 - \alpha)k_t^\alpha \tag{10}$$

with $k_t = \frac{K_t}{L_t}$. Similarly, the rate of return to capital R_t and the output per effective labor y_t are given by:

$$R_t = \frac{\partial Y_t}{\partial K_t} = \alpha \frac{Y_t}{K_t} = A\alpha k_t^{\alpha-1}, \quad y_t = \frac{Y_t}{L_t} = Ak_t^\alpha \tag{11}$$

2.2.2. Government sector problem

We assume that the fiscal revenue and expenditure of the government sector is balanced, and the expenses of taxation in the social account collected by the government are paid for pensions of the retired agent in the current period. Therefore, the budget constraint of the government sector in the period t satisfies:

$$w_t L_t + e\sigma w_t N_t = N_{t-1} p\theta_t (1 - d) \tag{12}$$

where the left side is source of endowment insurance collected by the government from the working labor during period t , which includes the endowment insurance tax $\tau w_t L_t$, collected from all working labor and transferred into the social account, and pension funds from individual accounts $e\sigma w_t N_t$, transferred to cover the gap in social account by government. The right side shows the pension expenditure in the current period. Solving for θ_t from Eq. (12), we can obtain old-age agent’s retirement benefits θ_t from the social account for 1 unit time as follows:

$$\theta_t = \frac{(\tau g + e\sigma\eta)}{(1-d)p} w_t \tag{13}$$

Comparing Eq. (13) with the retirement benefits from the individual accounts within a unit time $\frac{R_{t+1}}{p} \sigma(1-e)w_t$, we can find the similarities and differences between the retirement benefits from the social account and the individual accounts. First, the pension funds in the social account and the individual accounts are respectively managed by the PAYG system and the fully-funded system, which leads to differences in the composition of retirement benefits from the two accounts. Eq. (13) shows that the retirement benefits from the social account within one unit of time under the PAYG system θ_t are positively related to the birth rate η , the growth rate of the effective working labor g , the tax rate τ , and the legal delayed retirement age d , and are negatively related to the population of dependents N_{t-1} and the survival probability p . Different from the social account, the retirement benefits from the individual accounts with the fully-funded system is $\frac{R_{t+1}}{p} \sigma(1-e)w_t$, which only relates to the individuals' contribution ratio σ , rate of return on investment R_{t+1} and survival probability p , and has nothing to do with the birth rate and the working population. Second, according to Eqs. (12) and (13), the retirement benefits from the social account are mainly affected by two aspects in terms of the relationship with delayed retirement. On the one hand, the later the retirement is, the greater the labor supply in the economy, so the more the total retirement benefits θ_t transferred to the dependents within a unit of time between generations. On the other hand, given θ_t , delaying the retirement will lead to a shortening of pension coverage period, causing retirement benefits $N_{t-1}p\theta_t(1-d)$ from social account to fall. For individual accounts, retirement benefits have nothing to do with the delayed retirement age. Third, the mechanisms through which pension funds in the two accounts affect the economy are also different. The pension funds in the social account are essentially to keep the balance of revenue and expenditure in the current year or future years, not for the purpose of capital accumulation and investment. However, pension funds in individual accounts enter the capital market, which influences capital accumulation and has a direct impact on productive activities.

2.2.3. Market clearing conditions and general equilibrium

The agent's savings and taxes paid into individual accounts enter the production sector through insurance companies to produce consumption goods, and realize the maximization of profits and agent's optimal consumption and savings. Therefore, the capital market clearing condition can be established as:

$$K_{t+1} = N_t [s_t^* + (1-e)\sigma w_t] \tag{14}$$

The left side of Eq. (14) is the capital stock in period $t + 1$, and the right side indicates the agent's optimal savings and the retirement benefits from the individual accounts, excluding the empty account in period t .

Combined with Eqs. (8) and (13), the market clearing condition (14) can be rewritten as:

$$K_{t+1} = N_t \left[\frac{\beta p(1-\tau-\sigma e)}{1+\beta p} w_t - \frac{(\tau g + e\sigma\eta)w_{t+1} + (1-\tau)pd\delta w_{t+1}}{(1+\beta p)R_{t+1}} \right] \tag{15}$$

Note that the capital stock per labor $k_{t+1} = \frac{K_{t+1}}{N_{t+1}}$. Eq. (15) can be deformed into:

$$k_{t+1} = \frac{\beta p(1-\tau-\sigma e)}{(1+\beta p)} w_t - \frac{(\tau g + e\sigma\eta) + (1-\tau)dp\delta}{(1+\beta p)R_{t+1}} w_{t+1} \tag{16}$$

Substituting Eqs. (10) and (11) into Eq. (16), we have

$$gk_{t+1} = \frac{\beta p(1-\tau-\sigma e)A(1-\alpha)}{(1+\beta p)} k_t^\alpha - \frac{(1-\alpha)[(\tau g + e\sigma\eta) + (1-\tau)dp\delta]}{\alpha(1+\beta p)} k_{t+1} \tag{17}$$

Setting $k = k_t = k_{t+1}$ in Eq. (17), we obtain the unique steady-state equilibrium solution of capital stock per labor k^* ¹⁹ under the condition of market clearing:

$$k^* = \left\{ \frac{\alpha g(1+\beta p) + (1-\alpha)[(\tau g + e\sigma\eta) + (1-\tau)dp\delta]}{A\alpha\beta p(1-\alpha)(1-\tau-\sigma e)} \right\}^{\frac{1}{\alpha-1}} = f(d, p)^{\frac{1}{\alpha-1}} \tag{18}$$

where $f(d, p) = \frac{\alpha g(1+\beta p) + (1-\alpha)[(\tau g + e\sigma\eta) + (1-\tau)dp\delta]}{A\alpha\beta p(1-\alpha)(1-\tau-\sigma e)}$. According to Eqs. (10) and (11), as well as the market clearing condition, the formulas of output per labor, wage rate, and rate of capital return are given by:

$$y^* = A(k^*)^\alpha, w^* = A(1-\alpha)(k^*)^\alpha, R^* = A\alpha(k^*)^{\alpha-1} \tag{19}$$

Note 2: As mentioned in Section 2.1.3, since our model considers delayed retirement, the age difference of agent's labor productivity, the aging of the population and the reality of Chinese social security system, the steady-state solution k^* , and then w^* , R^* , and y^* are all affected by four types of parameters: delayed retirement parameter d , the wage income loss parameter of the elderly δ , population aging parameters p and η and social security system parameters τ , σ , and e . Compared with our results, [Geng et al. \(2016\)](#) neglect delayed retirement (which means $d = 0$), [Hansen & Lønstrup \(2009\)](#) leave out of consideration the age difference of agent's

¹⁹ The economic state at this time is referred to as steady state equilibrium, or steady state for short.

labor productivity (which means $\delta = 1, \xi = 0$), [Aísa et al. \(2012\)](#) ignore the individual accounts and empty accounts problem, and [Kang \(2012\)](#) does not consider the impact of life expectancy changes on population structure. So, studies above are all special cases of this paper.

2.3. Optimal delayed retirement age

Eqs. (6) to (8) show the optimal choices made by the agent of savings and two periods of consumption for any given legal delayed retirement age d . The Chinese government, who claims that “the aspirations of the people to live a good life must always be the focus of our efforts”, should take the agent’s optimal choices into consideration when formulating the delayed retirement policy, and determine the optimal legal delayed retirement age under the government’s budget constraint. Therefore, this section develops a method to determine the optimal delayed retirement age for the government’s policy, based on the agent’s optimal consumption and savings and the resource constraint of the government sector.

Following [Hansen & Lønstrup \(2009\)](#), we assume that when formulating delayed retirement policies, the government takes into account the agent’s total consumption and leisure preferences, namely the agent’s life-time discounted utility given by Eq. (2). We denote $U_t^*(d)$ as the agent’s life-time discounted utility in the steady state obtained by summing up the two-period optimal consumption and leisure utility, hereinafter also referred to as the agent’s life-time discounted utility under the two-period optimal consumption. Therefore, the goal to achieve when the government formulates the delayed retirement policy is to maximize the agent’s life-time discounted utility under the two-period optimal consumption, that is,

$$\max_d U_t^*(d) = \max_d \{ \ln c_t^{*Y} + \beta p [\ln c_{t+1}^{*O} + \gamma \ln(1-d)] \} \tag{20}$$

In addition, the resource constraint faced by the government is the balance of payments, that is,

$$Y_t^* = N_t c_t^{*Y} + p N_{t-1} c_{t-1}^{*O} + L_{t+1} k_{t+1}^* \tag{21}$$

where $Y_t^* = L_t y^*$ can be derived from Eqs. (11) and (19), which represents the steady-state output in period t ; the right side of the above equation stands for the steady-state expenditure of period t . Therefore, Eqs. (20) and (21) are the optimal delayed retirement age determination model in the context of population aging. We take the partial derivative of the agent’s life-time discounted utility function $U_t^*(d)$ in Eq. (20) with respect to the delayed retirement parameter d and set it equal to zero $\frac{\partial U_t^*}{\partial d} = 0$. We have the following:

$$\frac{1}{c^{*Y}} \frac{\partial c^{*Y}}{\partial d} + \frac{\beta p}{c^{*O}} \frac{\partial c^{*O}}{\partial d} - \beta \gamma p \frac{1}{1-d} = 0 \tag{22}$$

By substituting Eq. (15), and the wage rate and total return rate of capital in a steady state in Eq. (19) into Eqs. (6) to (8), we can obtain the optimal youth consumption c_t^{*Y} , the optimal old-age consumption c_t^{*O} and the optimal savings s_t^* in the steady state. Then, we substitute them into Eq. (22) and rearrange. Finally, we solve Eq. (22) to obtain the optimal delayed retirement age, denoted as d^* . Eq. (22) shows that the parameters and functional forms in the equation are so complicated that it is difficult to obtain a closed-form solution for the optimal delayed retirement age d^* , as in the case of [Hansen & Lønstrup \(2009\)](#). However, Section 5 shows that the solution to Eq. (22), the optimal delayed retirement age d^* , does exist and can be yielded by numerical simulations. Appendix C also proves the existence of d^* and Eq. (23). Furthermore, the existence of d^* demonstrates that if $d^* \neq 0$, delaying retirement will improve the agent’s utility, and d^* must be an interior point of (0,1). The numerical simulations in the following part also show that the solution to Eq. (22), the optimal delayed retirement age d^* , does exist and can be obtained through numerical simulations. See Section 5 for details. In addition, according to Eqs. (20), (19), (6), and (7), it is easy to know that d^* depends on the survival probability p , the birth rate η and the agent’s expected labor productivity of the elderly ρ . Therefore, without loss of generality, when d^* is the interior point of (0,1), we have

$$d^* = \operatorname{argmax}_d U_t^*(d) = d^*(p, \eta, \delta) \tag{23}$$

That is, d^* satisfies Eq. (23) and the following equation is established:

$$\frac{\partial U_t^*}{\partial d} = \frac{1}{c^{*Y}} \frac{\partial c^{*Y}}{\partial d} + \frac{\beta p}{c^{*O}} \frac{\partial c^{*O}}{\partial d} - \beta \gamma p \frac{1}{1-d} \begin{cases} \geq 0, d \in [0, d^*] \\ < 0, d \in (d^*, 1] \end{cases} \tag{24}$$

Note 3: Compared with the existing research, Eq. (23) for determining the optimal delayed retirement age obtained in this paper has the following different characteristics. First, from the agent’s life-time discounted utility function given by Eq. (2), it can be seen that the optimal delayed retirement age determined in this paper is decided exogenously by the government, not endogenously by the agent sector. Obviously, the setting of this paper is more in line with the reality in China, which is also the most prominent difference between this paper and [Prettner and Canning \(2014\)](#). Second, in Eqs. (22) and (23), setting $\delta = 1, \xi = \sigma = e = 0$ can yield the same results of [Hansen & Lønstrup \(2009\)](#), [Miyazaki \(2014\)](#). Apparently, the findings of this paper generalize the conclusion of [Hansen & Lønstrup \(2009\)](#), [Miyazaki \(2014\)](#) because the age difference of agent’s labor productivity and factors such as individual accounts and empty accounts problem in the social security system are included.

3. The effect of delayed retirement

According to Eqs. (6) to (8), (18), (19), (22), and (23), the optimal delayed retirement age d^* is mainly a function of the following two types of parameters: ① Population aging parameters, namely birth rate η and survival probability p ; ② Wage income loss parameter of the elderly δ . Since the explicit solution of the optimal delayed retirement age d^* is not available, we numerically analyze the influence of the above two types of parameters on the optimal delayed retirement age d^* . This section will focus on the effects of delayed retirement age on other economic variables.

3.1. The effect of delayed retirement on capital stock per labor, wage rate, output per labor, and rate of return on capital

From Eq. (18), it is easy to infer the relationship between the delayed retirement age and capital stock per labor k^* under the steady-state equilibrium. The delayed retirement parameter $d \in [0, 1]$ satisfies:

$$\frac{\partial k^*}{\partial d} = -\frac{1}{1-\alpha} \frac{\partial f}{\partial d} f^{\frac{\alpha-1}{\alpha}} < 0 \tag{25}$$

Using facts from Eq. (19), we can show that for any $d \in [0, 1]$, the following is true:

$$\frac{\partial y^*}{\partial d} < 0, \frac{\partial w^*}{\partial d} < 0, \frac{\partial R^*}{\partial d} > 0 \tag{26}$$

Therefore, the following propositions can be obtained from Eqs. (25) and (26):

Proposition 1. Delaying retirement will decrease the capital stock per labor, the output per labor, and the wage rate in the steady state equilibrium, but will increase the rate of return on capital.²⁰

3.2. The effect of delayed retirement on youth and old-age consumption

According to Eqs. (3) and (26), the following relationship can be obtained in the steady state:

$$\frac{\partial c^{*Y}}{\partial d} + \frac{\partial s^*}{\partial d} = \frac{1-\tau-\sigma}{2} \frac{\partial w^*}{\partial d} < 0 \tag{27}$$

which means that $\frac{\partial c^{*Y}}{\partial d} < 0$ and $\frac{\partial s^*}{\partial d} < 0$ at least one holds. From Proposition 1, it can be seen that delayed retirement will lead to a decrease in the wage rate and an increase in the rate of return on capital. According to Kaganovich and Zilcha (2012), a rational agent at this time will increase savings by reducing consumption in youth to increase the income of savings and ensure that old-age consumption does not decline, thereby $\frac{\partial c^{*Y}}{\partial d} < 0$ (See Appendix B for derivation). If U_t^* is quasi-concave in d ,²¹ according to Eq. (24), it can be further inferred that when $d \in [0, d^*]$, the parameter d containing the optimal delayed retirement age d^* has the following relationship with the optimal consumption of the elderly:

$$\frac{\partial c^{*O}}{\partial d} = \frac{c^{*O}}{\beta p} \left[\beta \gamma p \frac{1}{1-d} - \frac{1}{c^{*Y}} \frac{\partial c^{*Y}}{\partial d} \right] > 0 \tag{28}$$

When $d \in (d^*, 1]$, the derivation process of Eqs. (24) and (28) indicates that Eq. (28) does not necessarily hold.

Then, we examine the impact of $d \in [0, d^*]$ on the agent's life-time total consumption ($c_t^{*Y} + \beta p c_{t+1}^{*O}$). If U_t^* is quasi-concave in d , from Eqs. (5) and (24), we can derive:

$$\frac{\partial c^{*Y}}{\partial d} \geq \frac{\beta \gamma p}{1-d} c^{*Y} - \frac{\partial c^{*O}}{\partial d} \frac{p}{R^*} \tag{29}$$

Based on Eq. (29), it can be found that the relationship between the agent's total life-time consumption ($c_t^{*Y} + \beta p c_{t+1}^{*O}$) and the parameters d is as follows:

$$\frac{\partial (c_t^{*Y} + \beta p c_{t+1}^{*O})}{\partial d} \geq \frac{\beta \gamma p}{1-d} c^{*Y} - \frac{\partial c^{*O}}{\partial d} \frac{p}{R^*} + \beta p \frac{\partial c^{*O}}{\partial d} = \frac{\beta \gamma p}{1-d} c^{*Y} + p \left(\frac{\beta R^* - 1}{R^*} \right) \frac{\partial c^{*O}}{\partial d} \tag{30}$$

²⁰ Studies like Cipriani and Pascucci (2020) also find that delayed retirement time has a negative relationship with the capital stock per labor and wage rate in steady-state equilibrium.

²¹ The adoption of the quasi-concave assumption here is mainly for the following reasons. According to Equation (20), as well as Equations (6) and (7) for optimal consumption, it is easy to know that the function of U_t^* in d is extremely complex, and related to several parameters including β , p , and η . So, it is quite difficult to prove the quasi-concavity property of U_t^* in d . However, in Appendix C we prove that when $d^* \neq 0$, the maximum of function U_t^* can be obtained at an inner point d^* within (0,1). Based on the numerical simulation, the function U_t^* is indeed quasi-concave in d , so the maximum point in d^* is unique. In addition, the hypothesis of quasi-concavity is also a very commonly used hypothesis with clear meaning in economics (Arrow & Enthoven, 1961; Mas-Colell, Whinston, & Green, 1995). Therefore, it is natural and reasonable to assume that U_t^* is quasi-concave in d .

Eq. (30) suggests that as long as $\beta R^* \geq 1, \frac{\partial(c_t^Y + \beta p c_{t+1}^O)}{\partial d} > 0$ must hold. βR^* depicts the discounted present value of the expected total future return on the capital. In the case of normal operation of the economy, $\beta R^* = \beta(1 + r^*) \geq 1$ usually holds, confirmed by the following numerical simulation as well, where r^* is the rental rate of the capital. In particular, when agent's discounted factor $\beta = 1, R^* = 1 + r^* > 1$.

Therefore, based on Eqs. (27), (28), and (30), the following propositions can be obtained:

Proposition 2. If U_t^* is quasi-concave in d , when the delayed retirement age d is not higher than the optimal delayed retirement age d^* , namely $d \leq d^*$, extending delayed retirement age will reduce the agent's optimal youth consumption c^{*Y} , but increase the optimal old-age consumption c^{*O} . Meanwhile, if the condition $\beta R^* \geq 1$ holds, the agent's total life-time consumption $c_t^{*Y} + \beta p c_{t+1}^{*O}$ will also increase.

3.3. The effect of delayed retirement on aggregate output, wages, consumption, utility and retirement benefits from the social account

From the production function in Eq. (9), delaying retirement will increase the labor supply, so $\frac{\partial L_t}{\partial d} > 0$; and delaying retirement does not affect the total capital in the steady state K^* , that is $\frac{\partial K^*}{\partial d} = 0$, then

$$\frac{\partial Y_t^*}{\partial d} = A(1 - \alpha)(K^*)^\alpha L_t^{-\alpha} \frac{\partial L_t}{\partial d} > 0 \tag{31}$$

Eqs. (19) and (21) imply that the total wage W_t^* and total consumption C_t^* in period t are:

$$W_t^* = w^* L_t = A(1 - \alpha) Y_t^*, C_t^* = C_t^{Y*} + C_t^{O*} = Y_t^* - N_t k^* \tag{32}$$

so

$$\frac{\partial W_t^*}{\partial d} > 0, \frac{\partial C_t^*}{\partial d} = \frac{\partial Y_t^*}{\partial d} - N_t \frac{\partial k^*}{\partial d} > 0 \tag{33}$$

Finally, according to Eqs. (31) to (33), the following propositions can be obtained:

Proposition 3. In the steady state equilibrium, delaying retirement will currently increase the total output, total wage, and total consumption.

Finally, we examine the relationship between the optimal delayed retirement age d^* and social welfare. Eqs. (23) and (24) indicate that:

Proposition 4. In the steady state equilibrium, if U_t^* is quasi-concave in d , there exists an optimal delayed retirement age d^* that maximizes the agent's life-time discounted utility U_t^* under the two-period optimal consumption. Furthermore, when the delayed retirement age d is not higher than the optimal delayed retirement age d^* , delaying retirement will increase the agent's life-time discounted utility; when the delayed retirement age d is higher than the optimal delayed retirement age d^* , delaying retirement will reduce the agent's life-time discounted utility.

According to Eqs. (12), (26), and (32), it is easy to infer:

$$\frac{\partial(\tau w^* L_t)}{\partial d} = \tau \frac{\partial W_t^*}{\partial d} > 0, \frac{\partial(e \sigma N_t w^*)}{\partial d} = e \sigma N_t \frac{\partial w^*}{\partial d} < 0 \tag{34}$$

Since the proportion of empty accounts e will gradually approach 0 under the institutional arrangement of the government to consolidate the individual accounts, $e \sigma N_t \frac{\partial w^*}{\partial d}$ will also gradually tend to 0. Therefore, from Eq. (34), the relationship between the income from social account in the steady state equilibrium ($\tau w^* L_t + e \sigma N_t w^*$) and the delayed retirement age satisfies:

$$\frac{\partial(\tau w^* L_t + e \sigma N_t w^*)}{\partial d} > 0 \tag{35}$$

And the relationship between the individual account income $(1 - e) \sigma w^* N_t$ and the delayed retirement age is:

$$\frac{\partial[(1 - e) \sigma w^* N_t]}{\partial d} = (1 - e) \sigma N_t \frac{\partial w^*}{\partial d} < 0 \tag{36}$$

Proposition 5 can be obtained from Eqs. (35) and (36):

Proposition 5. Under the institutional arrangement that the government implements to make real individual accounts, an increase in the delayed retirement age will increase the income of the social account in the steady state equilibrium, but decrease the income of the individual accounts.

Propositions 1-5 are also verified in the numerical simulation part of **Section 5** below.

4. Calibration

Based on the predictions of Chinese future population structure in existing research and reports, Chinese current pension insurance system and retirement system, this section will set the parameters of population aging, social security system and production and consumption activities in the model.

4.1. Prediction and setting of China's population aging parameters

For setting of population aging, two issues are mainly involved. The first is how to classify the degree of population aging. The second is how to set population aging parameters, namely birth rate and survival probability, under the corresponding population aging degree.

First, the international standard for classifying the degree of population aging is usually as follows. If a country's population aged 60 and above accounts for >10% of the total population, the country has entered an aging society. Between 10% and 20%, it belongs to the stage of mild aging; between 20% and 30% is the stage of moderate aging. >30% is the stage of severe aging.²² At present, according to the data of the seventh census of the National Bureau of Statistics, the proportion of the elderly over 60 years old in the total population is 18.7%, which suggests that China is currently in a state of mild aging.

Second, we set population aging parameters, namely the birth rate and survival probability. We will use the World Population Prospects 2019 released by the United Nations Department of Economic and Social Affairs in June 2019 for population structure measurement and forecasting. The main reasons are as follows: First, the World Population Prospects report released by the United Nations Department of Economic and Social Affairs provides the most authoritative data of the world's population; Second, World Population Prospects 2019 not only estimates the population structure of countries in the world every five years from 1950 to 2020, but also predicts the population structure of countries in the next 80 years. The time period is long enough, and a unified measurement caliber is employed to facilitate comparative analysis. Third, the population structure predicted by the World Population Prospects 2019 is relatively close to the actual situation. Taking China's situation in 2020 as an example, the World Population Prospects 2019 predicted that the proportion of the elderly over 60 years old in China's total population in 2020 would be 17.35%, which is extremely close to the actual figure of 18.73% released in 2021 by National Bureau of Statistics of China.

The following will specifically explain the method of setting survival probability and birth rate, which is divided into four steps. First, we divide the degree of population aging, and select time nodes with important policy significance to investigate. Table 1 shows the calculation and prediction of the aging degree of China's population. According to the China Statistical Yearbook released in 2021 and World Population Prospects 2019, the proportion of the elderly over 60 years old in the total population in China from 2020 to 2100 can be obtained respectively. Then, we refer to the above classification method of population aging degree, make a specific judgment on the degree of population aging in each year, and select the aging situation in 2020, 2025, 2030, 2040, 2050, and 2100 as the study years. This choice is mainly for the following reasons: One is the availability of data. The World Population Prospects 2019 was released in 2019, with 2020 as the starting year and five-year intervals to provide the forecast data of the global population structure in 2020, 2025, 2030 and other years in turn. The forecast period is 2020–2100. The second is to consider the changing trend of population aging. Table 1 shows the process of deepening aging of China's population during the period from 2020 to 2100. Taking 2020 as the starting point for the investigation, 2025 is the year when China transitions from mild to moderate population aging. In 2040, the proportion of the elderly aged over 60 years in the total population will be 29.91%, which is an important time node for China to transition from moderate aging to severe aging. It is noteworthy that the proportion of the population aged 60 and above will increase by 10% within only 15 years from 2025 to 2040 in China, which is considered an accelerated period of population aging. However, after 2040, the pace of population aging in China will slow down. Third, we consider the five-year planning cycle of China's national economy. The end of 2020 and the end of 2025 are the completing points in time for China's 13th Five-Year Plan and 14th Five-Year Plan, respectively, and the starting points in time for the 14th Five-Year Plan and 15th Five-Year Plan, respectively, and the years 2030, 2040, 2050 and so on.²³

Second, we set and measure the population structure. The model in this paper is a two-period model, assuming that agent starts working from the age of 20,²⁴ the time span of youth and old age is 35 years old, and the age spans of young people and old people are 20–54 years old and 55–90 years old respectively. Based on this, according to the China Statistical Yearbook and World Population Prospects 2019, the ratio of the population of youth to the elderly in 2020–2100 can be calculated, that is, the reciprocal of the old-age dependency ratio, which can reflect how many young people support each elderly person. It can be estimated that about 2 young people support 1 elderly person in 2020, and by 2050 this ratio will drop to 0.99.

²² The data comes from the press conference on the operation of the national economy in April held by the State Council Information Office on May 17, 2021, see <http://www.scio.gov.cn/xwfbh/xwfbh/wqfbh/44687/45564/wz45566/Document/1704041/1704041.htm>.

²³ Source: http://www.xinhuanet.com/2021-03/13/c_1127205564.htm.

²⁴ Kang (2012), Yan (2018), Geng and Sun (2020) and other studies generally set the starting working age of Chinese agent as 20 years old. The possible reasons are as follows: The average working age of Chinese residents before entering university is about 17–20 years old, which also conforms to the Labor Law of the People's Republic of China and other relevant laws. That is, the legal working age of Chinese residents is over 16 years old. While the average age of employment for college graduates is about 22, and higher for graduate students, it is still a small fraction of the aforementioned large employment population. Therefore, it is reasonable to set the starting working age of Chinese residents at 20 years old, which is also convenient for research.

Table 1
Calculation and prediction of the aging degree of Chinese population.

Year	60+ year population ratio	Degree of population aging	Ratio of youth to the elderly	Average life expectancy	Survival probability p	Birth rate η
2020	18.73%	mild aging	1.9657	77.30	63.71%	1.2524
2025	20.55%	moderate aging	1.6978	78.30	66.57%	1.1302
2030	24.83%	moderate aging	1.4078	79.30	69.43%	0.9774
2040	29.91%	moderate aging	1.2246	81.30	75.14%	0.9202
2050	34.62%	severe aging	0.9920	83.30	80.86%	0.8021
2100	37.82%	severe aging	0.9496	84.30	83.71%	0.7949

Note 1: The data in Table 1 is based on World Population Prospects 2019, and the data in Table 2-3 and Fig. 2-6 is calculated based on Table 1.

Note 2: The data in Table 1 is calculated based on the World Population Prospects 2019. The data in Table 2-3 and Fig. 2-6 below are all calculated based on Table 1, so no further explanation will be given below. The 60+ year population ratio refers to the ratio of the elderly population over the age of 60 to the total population of the year; the ratio of youth to the elderly refers to the ratio of the population of youth aged 20–54 to the population of 55–90 based on the model set in this paper. The data for 2020 is from the China Statistical Yearbook 2021, and the data for 2025–2100 is from the World Population Prospects 2019.

Third, we determine the average life expectancy and survival probability. As mentioned above, the National Health Commission issued a bulletin in 2019, which claimed that the average life expectancy of residents increased to 77.3 years. We assume that the agent starts working at the age of 20, and the time span of youth and old age is 35 years. Therefore, we set the agent's survival probability of the elderly $p = (77.3 - 55)/35 \approx 63.71\%$. Further, we assume that the average life expectancy increases at a rate of one year every five years in China, and evidence is as follow. First, an important goal of China's 14th Five-Year Plan period in terms of people's livelihood and well-being is to increase the average life expectancy by one year.²⁵ Second, from the historical data, the average life expectancy in China has been increasing by 1 year every 4 to 5 years,²⁶ and then it is easy to calculate the survival probability in other years.

Fourth, based on the settings of this model, we calculate the birth rate. As mentioned above, it can be obtained:

$$\frac{\text{population of young people aged } 20-54}{\text{population of old people aged } 55-90} = \frac{\eta N_{t-1}}{p N_{t-1}} = \frac{\eta}{p}, \text{ from which the birth rate } \eta \text{ can be calculated.}$$

4.2. Setting social security system parameters

As mentioned above, China's current pension insurance system is a mixed system; that is, the pension taxes paid by individuals and enterprises go into individual accounts and social account, respectively. Before Chinese overall consideration of reducing social insurance tax rates, according to the Decision on Improving the Basic Endowment Insurance System for Enterprise Employees promulgated by the State Council in 2005, the proportion of individual contributions was 8%, while the proportion of corporate contributions was 20%.²⁷ Therefore, the existing literature usually sets the payroll tax rate corresponding to the social account as $\tau = 0.2$, and the payroll tax rate corresponding to the individual account as $\sigma = 0.08$ (Wang & Wang, 2021). In addition, studies show that due to the different levels of contribution rates in different provinces and cities, when conducting research on China's social security system, it should be assumed that a unified social planning contribution rate used across the country is $\tau = 0.1$ (Kang, 2012).

Afterwards, with the emergence of new economic development situations such as the deepening of the aging population, to ease the payment pressure of enterprises and ensure that social insurance benefits are paid in full and on time, the government has gradually reduced social insurance rates since 2015. In the Notice of Comprehensive Plan for Reducing Social Insurance Rates issued in 2019, it is clearly pointed out: "If the contribution rate of pension insurance is higher than 16%, it can be reduced to 16%. If it is currently lower than 16%, it is necessary to study and propose transitional measures."²⁸ Therefore, considering comprehensively, we set the

²⁵ For details, please refer to the Government Work Report delivered by Premier Li Keqiang on March 5, 2021, <http://www.gov.cn/guowuyuan/2021/zfgzbg.htm>.

²⁶ Yu Xuejun, deputy director of the National Health Commission, pointed out at a press conference held by the State Council Information Office on October 28, 2020: "From 2015 to the end of 2019, the average life expectancy of Chinese residents increased from 76.3 years to 77.3 years, that is to say, increased by 1 year every 4 years" (see http://www.gov.cn/xinwen/2020-10/28/content_5555672.htm). However, this paper assumes that the average life expectancy of Chinese residents will no longer increase when it reaches a peak of 84.3 years in 2055, because the growth of average life expectancy will slow down significantly at this time. Furthermore, according to the World Health Statistics Report 2021 released by the World Health Organization in May 2021, Japan has the longest average life expectancy in the world, with 84.3 years. Therefore, this paper takes the current highest average life expectancy in the world as the ultimate goal.

²⁷ Source: http://www.gov.cn/zhuanti/2015-06/13/content_2878967.htm.

²⁸ Source: http://www.gov.cn/zhengce/content/2019-04/04/content_5379629.htm; http://www.gov.cn/fuwu/2019-04/10/content_5381120.htm

contribution rate of the social account as $\tau = 0.16$, and the contribution rate of the individual account as $\sigma = 0.08$.

In addition, China's endowment insurance system still has the empty account problem, which has the following characteristics: First, the problem of empty account does exist, but since 2005, Chinese government is trying to make up for it. Second, China has not realized the national planning of pension insurance system so that pension funds in each province has a surplus or loss, and the extent of empty account varies from province to province.²⁹ In view of the lack of direct data, we refer to the practice of [Geng et al. \(2016\)](#), and compare three cases of individual accounts without misappropriation, misappropriation of half and complete misappropriation, namely $e = 0, 0.5$, and 1.

4.3. Setting production and consumption activities parameters

The parameters related to production and consumption activities involved in our model mainly include the following four categories. The first one is the elasticity of capital output α . [Lin & Gong \(2007\)](#) set Chinese elasticity of capital output equal to 0.65, [Wang and Wang \(2021\)](#) used a capital output elasticity of 0.5, [Yan \(2018\)](#), [Wang and Qiao \(2018\)](#) and many other studies believe that the elasticity of capital output is 0.3. In general, with the continuous development of China's economy, the contribution of capital to output has declined, and the estimated value of the capital output elasticity has also continued to decrease, so this paper adopts $\alpha = 0.3$.

The second one is the time discount factor β . Overall, the previous literatures have relatively uniform values for the time discount factor, assuming that the annual time discount factor is 0.99 ([Guo, Yu, & Gong, 2021](#)). Since the time span of each period in our model is 35 years, the discount factor for each period is $\beta = 0.99^{35} \approx 0.7$.

The third one is wage income loss parameter of the elderly, δ . According to [Section 2.1](#), δ is mainly affected by the survival probability p and coefficient of agent's effective labor loss of the elderly ξ . The parameter $1 - \xi$ reflects the ability of the agent to offer effective labor of the elderly, rather than the willingness to work. In fact, a large number of studies point out that there exists a strong correlation between health and labor ability. Health is an important factor affecting human capital, and healthy human capital is also a driving force for a country's economic growth ([Bloom, Canning, Kotschy, & Prettnner, 2019](#)). Thus, health is an important factor affecting workforce retirement decisions ([McGarry, 2004](#); [Stabridis & van Gameren, 2018](#)). [Hou, Wang, Wang, and Zhao \(2021\)](#) further emphasize that the effective use of the old-age labor force is the solution to population aging, the feasibility of which depends on the health of the elderly. Therefore, taking 2020 as an example here, we describe the calculation process of parameter ξ as follows. According to the hypothesis of "the age span of the elderly is 55-90 years old and the span period is 35 years" mentioned above and the data of healthy life expectancy in [Table 2](#), it can be calculated that over a span of 35 years from the cutoff of 55 years of age, the proportion of time when the elderly can provide effective labor in 2020 is $\frac{68.20-55}{35}$. The proportion is the parameter $1 - \xi$, which reflects old people's abilities to offer effective labor, and also $\xi = \frac{35-(68.20-55)}{35}$. Then, according to [Section 2.1](#), the expected labor productivity of the elderly $E(\delta)$ in 2020 is obtained. The expected labor productivity in other years can be obtained in the same fashion (see [Table 2](#)). The healthy life expectancy in [Table 2](#) represents the life expectancy when the agent can offer effective labor depending on health condition. This paper follows [Li, Liu, Liang, Wang, and Li \(2018\)](#) to use the annual average increase method. By setting the annual average increase value of Chinese agent's healthy life expectancy equal to 0.016,³⁰ we can estimate healthy life expectancy in each year, which is reported in [Table 2](#).

The final parameter is the agent's leisure parameters γ . For the agent's leisure preference parameters, there are few related measurements in China, and most of the studies directly cite the settings of studies on foreign countries, and the selected parameter values vary greatly. For example, [Kang \(2012\)](#) refers to the practice of [Hansen & Lønstrup \(2009\)](#) and sets the agent's leisure parameter to 1.5; [Wang and Wang \(2021\)](#) set the leisure preference parameter γ to 0.588. This paper draws on the method of [Wang and Qiao \(2018\)](#) to recalibrate the leisure preference parameters γ based on our model. Since China has not yet implemented a delayed retirement policy in 2023, this paper sets this year as the base year. According to the reality of China's population aging in 2023 and [Eq. \(22\)](#), we propose that when the agent retire normally at the age of 55, the value of the leisure preference parameter γ is 0.26.

5. Numerical simulation of the optimal delayed retirement age estimation and its impact under different population aging degrees

In this section, with the optimal delayed retirement age determination models (20) and (21) and [Eq. \(23\)](#) established in [Section 2](#), and the relevant parameter values determined in [Section 4](#) to fit in with the reality of China, we will first examine the impact of the survival probability, birth rate and expected labor productivity on the determination of the optimal delayed retirement age. Based on

²⁹ It can be inferred from the Comprehensive Plan of Lowering Social Insurance Premium Rate that the reform of social insurance premium collection system in China includes "accelerating the provincial planning of pension insurance" and "increasing the proportion of central allocation of pension insurance funds". Hence, the pension funds burden among provinces is not the same. As the Table on the Difference between Contributions and Allocations from the Central Regulation Fund in 2020 released by the Ministry of Finance presents, some provinces and cities like Guangdong, Beijing and Shanghai have pension fund balances, while other provinces and cities in northeast China need funds from the central government to make up the shortfall. As a result, the empty account problems in different provinces and cities are not consistent.

³⁰ According to [Wu, Dong, Wang, and Zhang \(2019\)](#), the data of China's sixth population census in 2010 and the national 1% population sample survey in 2015 showed that the healthy life expectancy of residents aged 60-69 in China increased by 0.016 years on average within five years. Depending on the calculation of [Li et al. \(2018\)](#), the healthy life expectancy of China in 2013 was 68.03 years, from which the healthy life expectancy of 2020-2100 can be inferred.

Table 2
Estimation of healthy life expectancy and expected labor productivity in China.

Year	Healthy life expectancy	Coefficient of effective labor loss of the elderly ξ	Wage income loss parameter of the elderly δ
2020	68.20	0.6229	0.7552
2025	68.28	0.6206	0.7769
2030	68.36	0.6183	0.7981
2040	68.52	0.6137	0.8391
2050	68.68	0.6091	0.8786
2100	69.48	0.5863	0.9010

Note: The data in Table 2 is calculated based on Table 1.

this, numerical simulations are carried out to calculate the optimal delayed retirement age in China from 2025 to 2100 and the length of delayed retirement months under the progressive delayed retirement policy. Then, numerical simulation method is used to discuss the impact of delayed retirement on production, consumption and other economic activities, and the five propositions deduced in Section 4 are verified through numerical simulation.³¹

5.1. Analysis of influencing factors of optimal delayed retirement age under different population aging degrees

According to Eqs. (20) to (23), the optimal delayed retirement age d^* is mainly affected by the survival probability p , birth rate η and wage income loss parameter of the elderly δ . This section will estimate and analyze the relationship between the optimal delayed retirement age d^* and the three parameters conditional on their value ranges.

5.1.1. Analysis of the influence of high survival probability and low birth rate on the optimal delayed retirement age

As mentioned in Section 2.1.1, the survival probability and birth rate are the fundamental factors affecting the degree of population aging, so this paper also calls these two parameters as population aging parameters. The high survival probability and low birth rate are the primary reasons leading to the aging of population and the implementation of delayed retirement policy, which is also the basic status quo of China. Fig. 2 is the numerical simulation diagram obtained according to Eqs. (20) and (21) and Eq. (23) and using the relevant parameter values determined in Section 4. Fig. 2(a) and (b) present that the corresponding high survival probability and low birth rate in 2025, 2040, 2050, and 2100 have opposite effects on the optimal delayed retirement age. They play a driving role and a restraining role, respectively. In addition, looking at different years, Fig. 2(a) indicates that, under the same survival probability, the birth rate in the above four years decreased in turn, the corresponding degree of population aging deepened in turn, and the corresponding optimal retirement age gradually decreased. This is consistent with the aforementioned conclusion that low birth rate has an inhibitory effect on the optimal delayed retirement age; similarly, Fig. 2(b) can also confirm the conclusion that high survival probability has a push-up effect on the optimal delayed retirement age. Mechanisms are investigated separately as follows.

First, the survival probability has an impact on the optimal delayed retirement age from two aspects: A higher survival probability means a continuous increase in the life expectancy of residents and a deepening of the population aging, so that a high survival probability pushes up the optimal delayed retirement age. But from Eq. (1), it can be seen that the survival probability is positively correlated with labor productivity. The increase in the total supply of effective labor suggests that an agent provides more effective labor during one unit time of delayed retirement, which has the inhibitory effect on delayed retirement time. However, after the pushing up and restraining effects of the above two aspects are balanced, the promotion effect of the former dominates, resulting in a positive correlation between the survival probability and optimal delayed retirement age, which is also consistent with Wang and Wang (2021).

Second, we examine the impact of low birth rates. Table 1 shows that from 2020 to 2100, Chinese birth rate is in the range of 0.7–1.3 and presents a declining trend. At the same time, the optimal delayed retirement age will decrease with the decline of the birth rate, so a low birth rate has a restraining effect on delayed retirement,³² which is shown in Fig. 2(b). The specific analysis is as follows. Fig. 2(d) of the numerical simulation obtained by applying Eq. (18) shows that a low birth rate will lead to an increase in capital per labor, and the positive correlation between the wage rate and the capital stock per labor shown by the second formula in Eq. (19) shows that the low birth rate will increase the wage rate, which will have two opposite effects on delayed retirement. On the one hand, an increase in the wage rate means that the opportunity cost of retirement leisure time will increase, weakening agent's demand for retirement leisure, so a low birth rate can contribute to delayed retirement. At the same time, according to the proof in Appendix A, $\frac{\partial c^{y^*}}{\partial w} > 0$ and when $\frac{\partial c^{y^*}}{\partial w} < (1 - \tau - \sigma)$, $\frac{\partial d^*}{\partial w} > 0$, this suggests that rising wage rates could prompt the rational agent to increase consumption and savings in youth, to ensure the current level of consumption and increase consumption of the elderly appropriately. This

³¹ The population aging degree in 2025, 2030, and 2040 are moderate in China. Sections 5.2 and 5.3 examine the influencing factors and propositions on the optimal delayed retirement age under different degrees of population aging. The results in 2030 are similar to 2025 and 2040, so Fig. 2-5 only show the results for moderate population aging (2025 and 2040) and severe population aging (2050 and 2100).

³² The research of Feng, Ai, and Liu (2020) shows that intergenerational rearing has a significant impact on fertility rate: the probability of offspring bearing is significantly increased by about 6–9 percentage points after the father exceeds the retirement age compared with before, indicating that delayed retirement will reduce the birth rate. This also confirms the restraining relationship between delayed retirement and low birth rate from the other side.

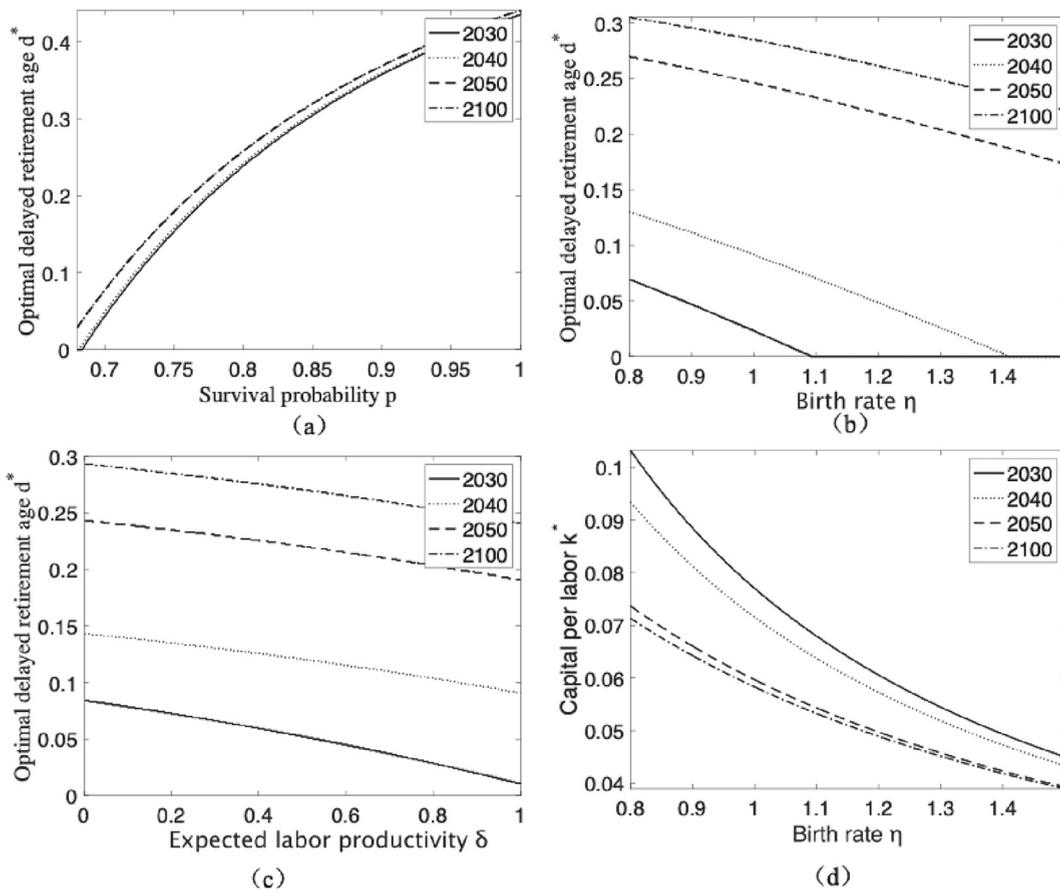


Fig. 2. The relationship between optimal delayed retirement age and its impacting factors.

will lead to a decrease in the marginal utility of consumption of the elderly, and a corresponding increase in the marginal utility of retirement leisure, which in turn will increase agent’s demand for retirement leisure. Therefore, a low birth rate has a restraining effect on delayed retirement. After the opposite effects of the above two aspects are cancelled out, the restraining effect of the second aspect is dominant, so that a low birth rate, which promotes population aging, has a restraining effect on the optimal delayed retirement age.

Finally, the influence of the two population aging parameters on the optimal delayed retirement age is compared: Fig. 2(a) and (b) show in turn that the slope of the relationship between the survival probability and the optimal delayed retirement age is higher than 0.4, while the slope of the relationship between the birth rate and the optimal delayed retirement age is lower than 0.1, indicating that the survival probability has a much greater impact on the optimal delayed retirement age than the birth rate. Therefore, the combined effect of high survival probability and low birth rate, which promote the deepening of population aging, will still push up the optimal delayed retirement age.

5.1.2. Impact of expected labor productivity on the optimal delayed retirement age

Fig. 2(c) shows that the wage income loss parameter of the elderly δ is negatively correlated with the optimal delayed retirement age. According to Eq. (1), a higher expected labor productivity ρ indicates that an agent can provide more effective labor supply per unit time in the old age, which has a restraining effect on delayed retirement. Tables 1 and 2 show that the aging of the population will continue to deepen from 2020 to 2100, and labor productivity is expected to gradually increase. According to the previous analysis, the increasing degree of population aging and expected labor productivity will have an opposite effect on the optimal delayed retirement age, so there will be a counteracting effect.

Note: Fig. 2(a)-(c) shows the relationship between the optimal delayed retirement age d^* and the survival probability p , birth rate η and the wage income loss parameter of the elderly δ in 2030, 2040, 2050, and 2100 corresponding to different degrees of population aging. Fig. 2(d) shows the relationship between birth rate and capital per labor.

5.2. Estimation and analysis of the optimal retirement policy under population aging

From the prediction of China’s future population aging degree in Section 4.1, it can be seen that in the next 80 years, with the decline in birth rate and the increase in life expectancy, the degree of population aging in China will continue to deepen. Since it is

Table 3

Analysis of optimal delayed retirement age and delayed retirement plans in China under different degrees of population aging.

Year	Optimal delayed retirement age d^*	Average months of retirement	Progressive average months of retirement	Actual optimal delayed retirement age (years)
2025	0.03	–	–	–
2030	1.05	2.52	2.52	56.05
2040	5.78	4.62	5.67	60.78
2050	9.45	4.54	4.41	64.45
2060	10.60	3.63	1.38	65.60
2075	10.75	2.58	0.12	65.75

Note: The data in Table 3 is calculated based on Table 1. The average months of retirement refer to the average months that require to be delayed each year starting from 2025, in order to achieve the optimal number of delayed retirement years in that year.

difficult to estimate the future trends of technological development and capital output elasticity, this section first refers to the existing literature such as Geng and Sun (2020). We assume that only birth rates and average life expectancy change in the economy and the total factor productivity is standardized to 1. As for the parameters such as capital output elasticity α , empty account rate e , leisure preference γ , which have been discussed and determined in Section 4 above, and they are $\alpha = 0.3$, $e = 0$, $\gamma = 0.12$.

According to Models (21) and (22) and Eq. (24), the optimal delayed retirement ages in China in the years of 2025, 2030, 2040, 2050, 2075, and 2100 are calculated through numerical simulations, as shown in Table 3. Based on the results in Tables 1, 2, and 3, as well as in Fig. 3 and discussion in Section 5.1, we find that with the deepening of the aging of the population and the improvement of the elderly's expected labor productivity, the optimal delayed retirement age presents nonlinear changes. First, during the accelerated population aging period from 2025 to 2040, the promotion of population aging on the optimal delayed retirement age will always be dominant, leading to a rapid increase in the optimal delayed retirement age. But after 2040, the aging rate of China's population will slow down, while the expected labor productivity of the elderly will continue to increase, and its inhibitory effect on the optimal deferred retirement age will gradually prevail. The optimal deferred retirement age will start to decline after reaching its peak in 2075, and the optimal delayed retirement age will also present an inverted U-shape. In summary, China should gradually increase the delayed retirement age, and the final optimal delayed retirement age is about 65 years old, which is close to the actual average retirement age among OECD countries (Yang & Xie, 2014).

In fact, in November 2013, the Chinese government has put forward the concept of "progressive extension of retirement age", in which "progressive" includes multiple meanings. Specifically in terms of delayed retirement age and time arrangement, first it means to provide relevant groups with several preparation years in advance. Second, it means delaying a few months each year to achieve a smooth transition for a longer delayed retirement time.³³ Since the optimal delayed retirement age in 2025 is 0.03, this paper assumes that 2025 is the year from which the delayed retirement policy is implemented, and proposes a progressive delayed retirement plan. Based on the optimal delayed retirement age from 2025 to 2100 calculated above, we can set up multiple delayed retirement goals, and achieve them one by one. As shown in Table 3, the optimal delayed retirement age in 2030 is 1.05 years old, then the average annual number of retirement months from 2025 to 2030 is 2.52 months. On this basis, in order to achieve the sub-target of 5.78 years of delayed retirement in 2040, the progressive average annual number of retirement months from 2030 to 2040 should be 5.67 months ($= \frac{5.78 - 1.05}{10} \times 12$ months). On the whole, in the context of the increasing aging of the population and the gradual improvement of the labor productivity of residents of the elderly, it is reasonable and feasible to set the average number of retirement months per year at 2–3 months.

5.3. Simulation analysis of the impact of delayed retirement on various economic activity variables under different degrees of population aging

5.3.1. Simulation analysis on the impact of delayed retirement on capital return rate, capital per labor, wage rate and output per labor

Fig. 4 shows numerical simulation results of the relationship between delayed retirement and the rate of return on capital, capital stock per labor, wage rate, and output per labor. It can be found that delayed retirement will increase the rate of return on capital factors while reducing the capital stock per labor, the wage rate and the output per labor. It shows that Proposition 1 holds. In addition, by comparing results of 2030, 2040, 2050, and 2100, in the order of severity of aging from moderate to severe, it is found that the impact of deepening population aging and delayed retirement on economic variables are different. The deepening of population aging will reduce the return rate of capital, and at the same time, it will increase the capital stock, wage rate and output per labor.³⁴ The numerical simulation results in the following also find that population aging and delayed retirement have mutually offsetting effects.

5.3.2. Analysis on the impact of delayed retirement on optimal consumption and total consumption, total output and total age in the current period

Fig. 5(a)–(c) show results of retirement age's influence on the optimal consumption under different degree of population aging. We

³³ Source: <http://cpc.people.com.cn/n/2013/1212/c83083-23820444.html>.

³⁴ Since Propositions 1 to 3 mainly focus on the impacts of delayed retirement age on other economic activities under different levels of population aging, the empty account rate is set to 0 here. The issue of empty accounts will be discussed in detail in Proposition 5.

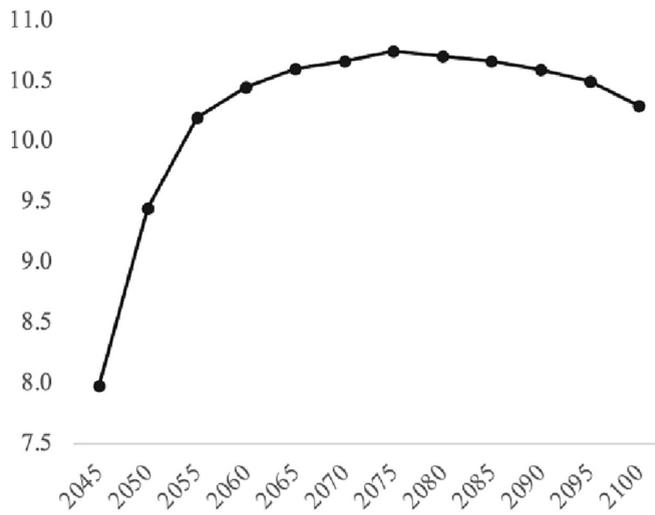


Fig. 3. Trend of optimal delayed retirement age.
Note: The vertical axis is the number of years of delayed retirement.

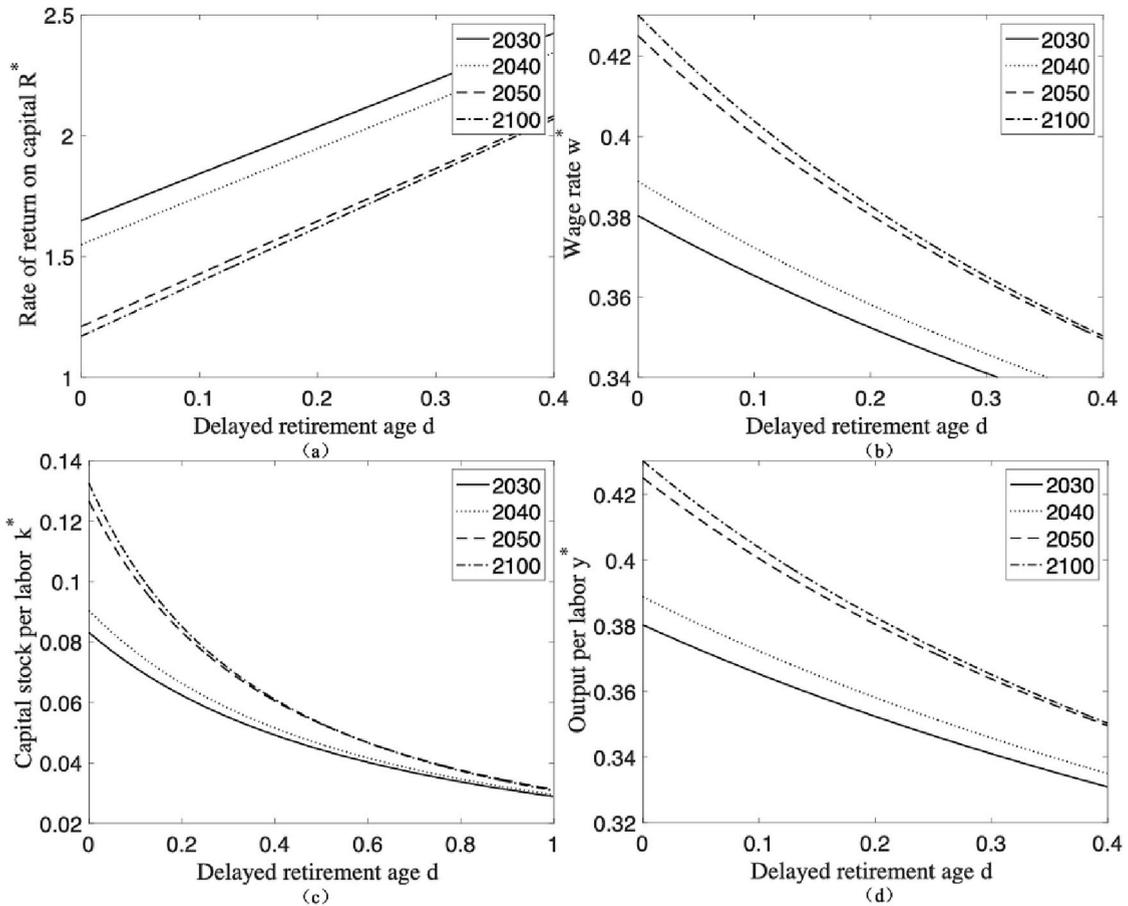


Fig. 4. The impacts of delayed retirement on R^* , w^* , k^* , and y^* .
Note: Fig. 4 (a) - (d), respectively, show the impacts of the optimal delayed retirement age d^* on the return rate of capital, capital stock per labor, wage rate and output per labor in four years with different degrees of aging: 2030, 2040, 2050, and 2100.

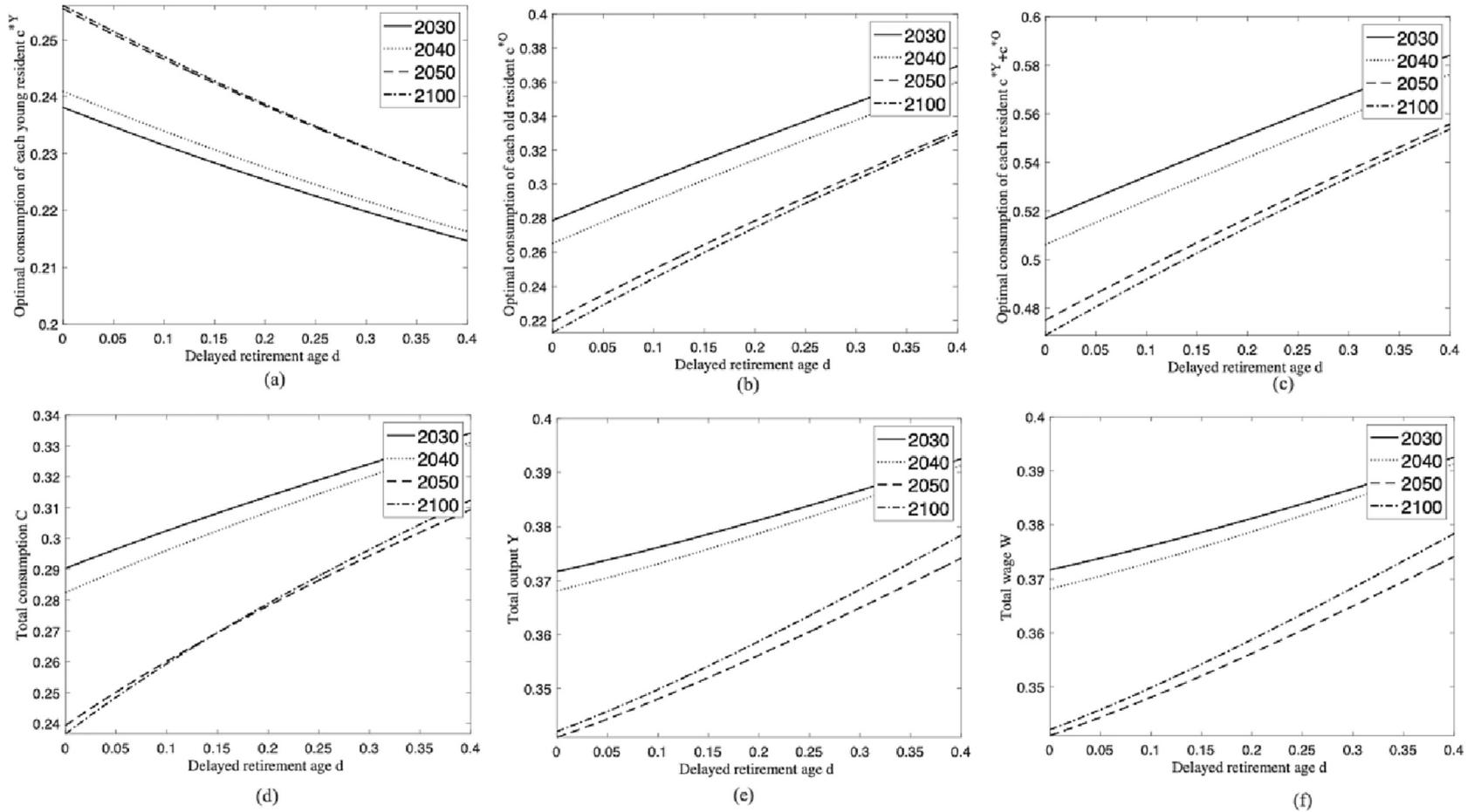


Fig. 5. The impacts of delayed retirement on optimal consumption, total consumption, total output and total wage.

find that delaying retirement will reduce the optimal consumption of the young agent, but increase the optimal consumption of the old agent and the total optimal consumption in the whole lifetime. As shown in the previous numerical simulation results, delayed retirement will lead to a decrease in the wage rate per unit time, thereby reducing the optimal consumption of the young agent. And delayed retirement will increase the wage income of the elderly, thereby increasing the optimal consumption of the elderly. The optimal consumption level will eventually lead to a simultaneous increase in the optimal consumption of the agent of the elderly and the optimal total consumption in their lifetime. Thus, the numerical simulation here shows that [Proposition 2](#) holds.³⁵

Further, we examine the impact of delayed retirement policy on the economy's current total output, total wages, and total consumption. In order to eliminate the influence of the exogenous variable of initial population size, we normalize the initial size of the population to 1. [Fig. 5\(d\)-\(f\)](#) shows that since delayed retirement will greatly increase the labor supply, the output per labor y^* will decline, but the total output of the economy Y^* will increase in the current period, which will lead to an increase in total wage W^* and total consumption C^* in the current period. Thus, the numerical simulations show that [Proposition 3](#) holds.

5.3.3. Analysis on the impact of delayed retirement on agent's life-time discounted utility and retirement benefits

Since the degree of population aging only affects the change in the degree of retirement benefits, not its direction, we only show the situation in 2030 with a moderate degree of population aging for brevity.

[Fig. 6\(a\)](#) shows that there is an inverted U-shaped relationship between the agent's life-time discounted utility and the delayed retirement age, that is, with the increase of the delayed retirement age d , the agent's life-time discounted utility increases first, and then decreases after reaching the maximum value at the optimal delayed retirement age d^* . Thus, [Proposition 4](#) is also confirmed.

Then, we examine the relationship between delayed retirement age and retirement benefits. For the social account, delaying retirement will increase the pension tax levied at contribution rate τ (that is, the pension tax from the non-empty account) and reduce the pension tax transferred from individual accounts, shown in [Fig. 6\(b\)](#), which indicates that delaying retirement will have both positive and negative impacts on the total income of social account (the sum income of the non-empty portion of the social account and the empty accounts). [Fig. 6](#) shows that the higher the empty account rate e , the greater the negative impact, so that the income of the social account in the case of empty accounts decreases with an increase in delayed retirement age. When the individual accounts are gradually compensated, that is, the empty account rate e gradually tends to zero, the increase in delayed retirement will steadily increase the income of the social account, as shown in [Fig. 6\(c\)](#) when $e = 0$. For individual accounts, total retirement benefits are negatively correlated with delayed retirement age d and empty account rate e , as shown in [Fig. 6\(d\)](#). Numerical simulations above prove [proposition 5](#).

6. Conclusions

Given the deepening aging of China's population from 2020 to 2100, we develop a model to determine the optimal delayed retirement age by maximizing the agent's life-time discounted utility, from the perspective of government policy-making. In fact, maximizing the agent's life-time discounted utility also reflects China's claim that the aspirations of the people to live a good life must always be the focus of Chinese government. More specifically, we extend the dynamic general equilibrium generation overlapping model of [Miyazaki \(2014\)](#) to establish the agent's optimal consumption model conditional on delayed retirement, as well as derive formulas to calculate steady-state output per labor, wage rate, and rate of capital return. Based on the theoretical model, we propose a formula to determine the optimal delayed retirement age, which aims to provide references for the government's optimizing delayed retirement policy. The most prominent innovation and feature of our model is that we take the age difference in agent's productivity into account, which is generally ignored by prior research, but does exist in reality. Based on data of the Chinese population structure during 2020–2100, forecasted by the United Nations, we simulate the Chinese optimal delayed retirement age and its impacting factors. We also analyze how delayed retirement affects the agent's wage rate, total consumption, retirement benefits, total output, and other related economic variables, and further estimate the optimal delayed retirement age in China and its impact through numerical simulations. The main conclusions are as follows:

First, we discuss how delayed retirement affects relevant economic variables. For the production sector, delaying retirement leads to a decline in steady-state capital stock per labor, wage rate, and output per labor, as well as an increase in steady-state capital return rate, and total output, total wage, and total consumption in the current period. For the agent sector, when the delayed retirement age is not higher than the optimal delayed retirement age, delaying retirement will decrease the youth consumption, but increase the old-age consumption and life-time consumption. There is an inverted U-shaped relationship between agent's life-time discounted utility and the delayed retirement age, and it reaches the maximum at the optimal delayed retirement age. For the government sector, in the context of making real the empty accounts, delaying retirement will increase the income of the social account and decrease the income of the individual accounts.

Second, from 2020 to 2100, both the degree of population aging and the labor productivity of the elderly keep growing, but they have positive and negative effects on the optimal delayed retirement age, respectively. In addition, the two parameters that drive the population aging, namely increasing survival probability and decreasing birth rate, have complex effects on the optimal delayed retirement age. An increase in the survival probability p raises the optimal delayed retirement age. However, a decrease in the birth rate η decreases the optimal delayed retirement age. Overall, the influence of the survival probability p on the optimal delayed

³⁵ Although the [proposition 2](#) is conditional on $d \in [0, d^*]$, according to the results of the numerical simulation, the proposition also holds when $d \in [d^*, 1]$.

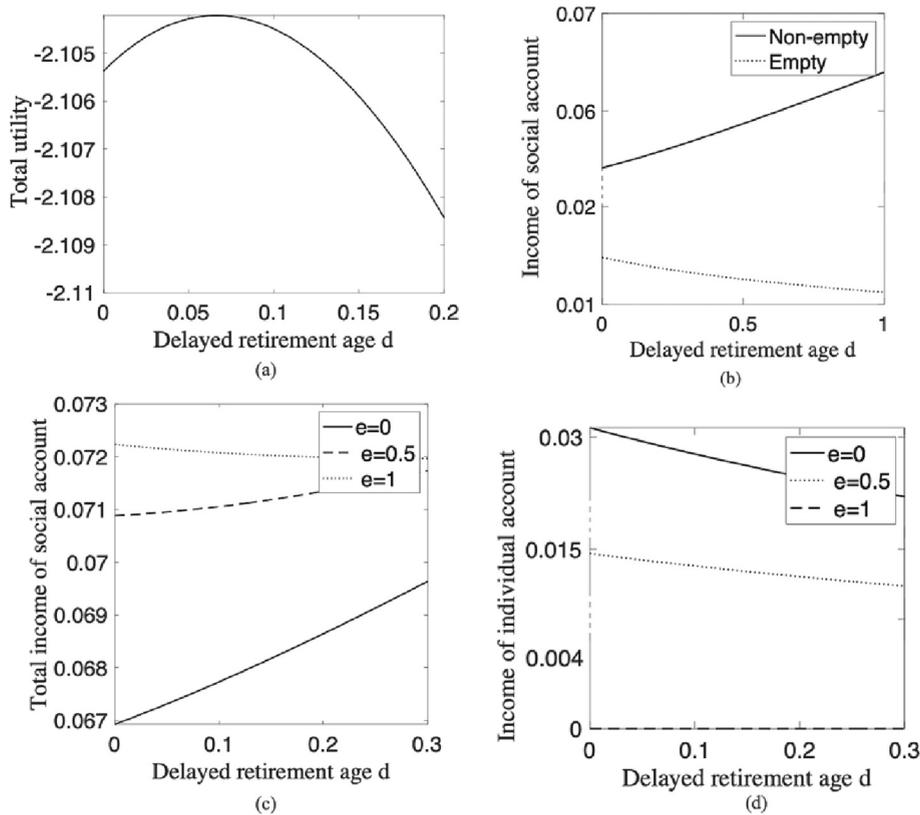


Fig. 6. The impacts of delayed retirement on agent's life-time discounted utility and retirement benefits.

retirement age is dominant, so the deepening population aging shows a promoting effect on the optimal delayed retirement age.

Third, under the combined effect of the aging of the Chinese population and the continuous improvement of old-age labor productivity, the optimal delayed retirement age presents an inverted U-shaped trend of rising initially and then falling after reaching its peak. Through simulation and calculation, the optimal delayed retirement years in 2030, 2040, 2050, and 2075 are 1.05 years, 5.78 years, 9.45 years, and 10.75 years, respectively. However, after 2075, the effect of increases in effective labor supply caused by the continuous improvement of labor productivity will gradually exceed that deepening population aging, and become the dominant factor. Correspondingly, the optimal delayed retirement age gradually reduces from 10.75 years in 2075 to 10.30 years in 2100. It is reasonable to set gradual delayed retirement plan as a delay of two to three months for each year if the policy begins from 2025.

Based on the above conclusions, the following countermeasures and suggestions can be drawn. First, based on our model, we can calculate the optimal delayed retirement age from 2025 to 2100, and design a progressive delayed retirement plan that matches synchronously with Chinese 5-year plan. Second, it is necessary to fully examine the impact factors of the delayed retirement age. In the future, the deepening of population aging and the continuous improvement of labor productivity will appear simultaneously, and these two factors will affect the optimal delayed retirement in opposite directions. Therefore, the offsetting effect between population aging and labor productivity should be fully considered when determining the delayed retirement policy. In addition, residents' preference for work-leisure should also be fully considered, so people can obtain both happiness and satisfaction in work and life. Third, a policy mechanism should be established to help increase residents' consumption in their youth. Delayed retirement has different impacts on residents' youth and old-age consumption. According to the numerical simulation results, delayed retirement will reduce residents' consumption in youth. In this regard, we consider establishing an income distribution coordination mechanism between the young and old, or set a minimum consumption guarantee, so that the huge negative impact of delayed retirement on young residents' consumption can be alleviated or avoided. Fourth, the delayed retirement policy has two impacts on the pension insurance system. On one hand, it slightly increases the income of the social account. On the other hand, it greatly reduces the income of the individual accounts. This implies that the delayed retirement policy will be more beneficial for residents to enjoy the economic development results brought about by the intergenerational transfer of pensions. But at the same time, it is not conducive to individual capital accumulation. Therefore, when implementing the delayed retirement policy, we should vigorously promote the development of commercial endowment insurance to ensure the steady improvement of residents' endowment benefits. Fifth, the numerical simulation results show that to a certain extent, the empty account problem hinders the positive effect of delaying retirement on the social account income, and it is difficult to alleviate the pension gap through the transfer of pensions from individual accounts to the social account (Xie, 2005). Therefore, while implementing the delayed retirement policy, individual accounts should be compensated

completely. Otherwise, it may have a greater negative impact on the pension insurance system.

Data availability

No data was used for the research described in the article.

Appendix A. This appendix proves that in steady state

$$\frac{\partial c^{*Y}}{\partial w^*} > 0, \text{ and if } \frac{\partial c^{*Y}}{\partial w^*} < \frac{1}{2}(1 - \tau - \sigma), \text{ then } \frac{\partial s^*}{\partial w^*} > 0 \tag{A1}$$

Substitute Eqs. (14) and (19) into Eq. (7) and recall the second formula in Eq. (20) to rearrange and obtain the following representation of the optimal youth consumption in the steady state:

$$\begin{aligned} c^{*Y} &= \frac{1}{\beta p} \left[g + \frac{1-\alpha}{\alpha} (\tau g + e\sigma\eta + (1-\tau)dp^{\xi+1}) \right] k^* \\ &= \frac{1}{\beta p} \left[g + \frac{1-\alpha}{\alpha} (\tau g + e\sigma\eta + (1-\tau)dp^{\xi+1}) \right] \left(\frac{w^*}{A(1-\alpha)} \right)^{\frac{1}{\alpha}} \end{aligned} \tag{A2}$$

in which $g = \eta + dp\delta$ from Eq. (1). Apparently, $\frac{1}{\beta p} [g + \frac{1-\alpha}{\alpha} (\tau g + e\sigma\eta + (1-\tau)dp\delta)] > 0$, so that $\frac{\partial c^{*Y}}{\partial w^*} > 0$. Then inspection of Eq. (3) yields that the optimal savings in steady state satisfies:

$$\frac{\partial s^*}{\partial w^*} = (1 - \tau - \sigma) - \frac{\partial c^{*Y}}{\partial w^*} \tag{A3}$$

Therefore, if $\frac{\partial c^{*Y}}{\partial w^*} < (1 - \tau - \sigma)$, then $\frac{\partial s^*}{\partial w^*} > 0$.

Appendix B. Proof of $\frac{\partial c^{*Y}}{\partial d} < 0$ is as follows

Simplifying (A2) gives

$$c^{*Y} = \frac{(g + \frac{1-\alpha}{\alpha}v)}{\beta p} k^* \tag{B1}$$

where $v = \tau g + e\sigma\eta + (1 - \tau)dp\delta$.

Substituting k^* into Eq. (18) and rearranging yields:

$$\begin{aligned} \frac{\partial \ln c^{*Y}}{\partial d} &= \frac{\partial \ln(g + \frac{1-\alpha}{\alpha}v)}{\partial d} + \frac{\partial \ln k^*}{\partial d} \\ &= p\delta \frac{\alpha}{1-\alpha} \frac{(1-\alpha)(1+\beta p)\eta(1-\tau-e\sigma) - (\alpha\beta p + 1)g}{[\alpha g + (1-\alpha)v][\alpha g(1+\beta p) + (1-\alpha)v]} \end{aligned} \tag{B2}$$

By sorting out Eq. (B2), it can be inferred that the condition that $\frac{\partial c^{*Y}}{\partial d} < 0$ always holds is $\tau + e\sigma - \left(1 - \frac{1-\alpha}{\alpha} - \frac{1}{1+\beta p}\right) > 0$. Under the parameter setting in Section 5, the condition holds.

We provide an explanation for the existence of interior point solutions to Eq. (22). According to Eq. (20), since the utility function $U_t^*(d)$ is a continuous function on the closed interval $[0,1]$, it must reach the maximum at the boundary points 0,1 or an interior point in $(0,1)$. Supposing that $U_t^*(d)$ reaches the maximum at point d^* , the following three cases are discussed

- (i) If $d^* = 0$, it means that the optimal utility $U_t^*(d)$ can be realized without implementing the delayed retirement policy.
- (ii) $d^* = 1$ cannot be true. It is easy to deduce from Eq. (22) and expressions of c^{*Y} and c^{*O} in Eqs. (6) and (7) that

$$\lim_{d \rightarrow 1^-} \frac{\partial U_t^*(d)}{\partial d} = \lim_{d \rightarrow 1^-} \left(\frac{1}{c^{*Y}} \frac{\partial c^{*Y}}{\partial d} + \frac{\beta p}{c^{*O}} \frac{\partial c^{*O}}{\partial d} - \beta \gamma p \frac{1}{1-d} \right) \rightarrow -\infty$$

It shows that there exists at least one left neighborhood of 1 in $[0,1]$ in which $\frac{\partial U_t^*}{\partial d} < 0$ holds. So, $U_t^*(d)$ cannot reach the maximum at $d = 1$.

- (iii) According to the discussion of (i) and (ii), if delayed retirement policy can make the utility $U_i^*(d)$ achieve the optimal, $d^* \in (0, 1)$ is established, which also means the interior point of $(0, 1)$. d^* is both the global maximum point and the local maximum point, and $\frac{\partial U_i^*(d)}{\partial d} \Big|_{d=d^*} = 0$ holds, which means that Eq. (22) in the revised version has an interior point solution in $(0, 1)$. Therefore, Eq. (24) and Proposition 4 also hold.

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