China Economic Review 80 (2023) 102014

Contents lists available at ScienceDirect

China Economic Review

journal homepage: www.elsevier.com/locate/chieco

Multi-province comparisons of digital financial inclusion performance in China: A group ranking method with preference analysis

Yelin Fu, Zelong Yi *

College of Economics, Shenzhen University, Shenzhen, China

ARTICLE INFO

Keywords:
Digital financial inclusion
Group ranking
Half-quadratic theory
Preference analysis

ABSTRACT

The Peking University Digital Financial Inclusion Index of China (PKU-DFIIC) measures the digital financial inclusion (DFI) performance in China from three dimensions: breadth of coverage, depth of use and level of digitalization. This paper extends the PKU-DFIIC to measure and compare the provincial DFI performance in terms of proposing a group ranking method with preference analysis. Specifically, we identify a panel of experts by comprehensively considering all possible importance orders of three dimensions, and formulate a preliminary group decision matrix. Then the preference analysis is carried out with respect to preferential difference and preferential priority. A revised group decision matrix is derived with the expert-specific ranking as the inputs. Finally, a group ranking method is given based on the half-quadratic theory, in conjunction with the consensus index and trust level denoting the level of agreement and reliability of the group ranking. An empirical study using the PKU-DFIIC data from 2016 to 2020 is conducted to demonstrate our method, along with the spatial-temporal analysis of the results and a set of policy implications to promote the DFI development in China.

1. Introduction

As a key enabler to implement the Sustainable Development Goals (SDGs)[1] adopted by the United Nations in 2015 (Gutiérrez-Romero & Ahamed, 2021), financial inclusion (interchangeably known as "inclusive finance") means that individuals and businesses have access to useful and affordable financial products and services that satisfy their needs - transactions, payments, savings, credit and insurance - delivered in a responsible and sustainable manner.[2] In addition to the efforts exerted by traditional financial institutions, the digital and innovative technologies including artificial intelligence, big data, cloud computing and blockchain, has significantly expanded the coverage and accessibility of financial inclusion. Furthermore, the ongoing COVID-19 pandemic has reinforced the demand for increased digital financial inclusion (DFI), which involves the deployment of the cost-saving digital instruments to reach currently financially excluded and underserved populations, micro, small and medium enterprises (MSEMs), with a series of digital financial services suited to their needs that are responsibly delivered at a cost affordable to customers and sustainable for providers. Significant progress has been made during the last decade to advance DFI around the globe. Starting from 2010, over 55 economies

* Corresponding author.
E-mail addresses: msylfu@gmail.com (Y. Fu), yizl@szu.edu.cn (Z. Yi).

[1] https://www.undp.org/sustainable-development-goals
[2] https://www.worldbank.org/en/topic/financialinclusion/overview

https://doi.org/10.1016/j.chieco.2023.102014
Received 24 November 2022; Received in revised form 25 May 2023; Accepted 22 June 2023
Available online 25 June 2023
1043-951X/

have made commitments to financial inclusion, and more than 60 countries either have launched or are developing a national strategy. Specifically, the G20 committed to advance financial inclusion worldwide and reaffirmed its commitment to implement the “G20 High-Level Principles for Digital Financial Inclusion”. Moreover, Chinese authorities adopted DFI as a national strategy. In 2016, the State Council issued “Development plan for promoting financial inclusion (2016–2020)”. This plan elevates financial inclusion to the level of national strategy and emphasizes the promotion of financial inclusion with innovative financial products and services. Within this context, DFI - a recent trend and business model in financial inclusion - has been flourishing in China. China is the first economy that eliminates extreme poverty and promotes DFI through financial technology (FinTech) (Wang & Fu, 2022). For instance, Tencent and Ant Financial have been improving the accessibility of financial inclusion in remote and rural areas in China relying on the data accessibility of Wechat and Taobao, respectively (Huang, Ito, Iwata, McKenzie, & Urata, 2022).

Along with the implementation of financial inclusion strategy at different levels, the measurement of financial inclusion has received considerable attentions of academic researchers, governments, and organizations (Cámara & Tuesta, 2014; Demirgüç-Kunt & Klapper, 2012; Demirgüç-Kunt, Klapper, Singer, & Van Oudheusden, 2015; Tram, Lai, & Nguyen, 2021), with the purpose of collecting comprehensive information of financial inclusion, unearthing the impact of stackholders' initiatives, and guiding the policy roadmap of financial inclusion implementation. We refer the readers to Pesqué-Cela, Tian, Luo, Tobin, and Kling (2021) for a systematic review of the framework and methods related to measuring financial inclusion. However, traditional financial inclusion is primarily measured using the indicators corresponding to the usage of and/or access to traditional financial services, such as the number of branches and/or ATMs per capita and bank accounts per capita (Beck, Demirguc-Kunt, & Peria, 2007; Honohan, 2008), and do not fully capture the contribution of the growing significant role of FinTech. DFI performance measurement has received scant but increasing attention in recent years. Khera, Ng, Ogawa, and Sahay (2022) join the adventure to incorporate both indicators related to the usage of and access to digital financial services into the measurement of financial inclusion, and then propose a novel DFI index covering 52 emerging market and developing economies, by means of a three-stage principal component analysis (PCA) and a widely utilized objective weighting scheme. In addition, the Institute of Digital Finance at Peking University and Ant Financial Services Group form a joint research team to develop a unique index series - “The Peking University Digital Financial Inclusion Index of China (PKU-DFIIC)[3]”, the framework of which is given in Fig. 1 below. It is observed that the DFI in China is typically measured in three dimensions, namely, breadth of coverage, depth of use and level of digitalization. Specifically, the breadth of coverage is revealed by the number of e-accounts and others; the depth of usage is quantified from the perspective of payment, money fund, credit, insurance, investment and credit investigation, while the concept of usage is broken down into the number of actual users, the number of transactions per capita and the average transaction amount per capita; the level of digitalization is evaluated in mobility, affordability, credit and convenience (Guo et al., 2019). Regarding to the framework of PKU-DFIIC, the breadth of coverage is a prerequisite, the depth of usage denotes the practice use status, and the level of digitalization reflects the potential condition. The data associated with these dimensions is collected from the Ant Financial's massive dataset about DFI.

The main purpose of this paper is to measure and compare the provincial DFI performance in China, and carry out a spatial-temporal analysis using the panel data of PKU-DFIIC between 2016 and 2020. The Analytic Hierarchy Process (AHP) method is originally employed to subjectively determine the weights associated with breadth of coverage, depth of use and level of digitalization as: 0.540, 0.297 and 0.163, respectively. This set of weights is resulted from the subjective judgements of experts, and therefore reveals the personal knowledge and collective wisdom of experts. Albeit many merits of the AHP weighting scheme, the debate remains. It is not uncommon that the comparison decision matrix of AHP may exhibit a substantial degree of variability, which inevitably give rise to conflicting weights. Instead of obtaining the precise weights as AHP, this paper intends to consider all possible importance orders among three dimensions of PKU-DFIIC. In view of the fact that different ranked dimension weights may generate different evaluation results, all possible ranked dimension weights are extensively explored to facilitate the collective commitment, in which an importance order among the three dimensions of PKU-DFIIC is reasonably defined as an expert (Dyer & Sarin, 1979; Fu, Lai, & Yu, 2021; Fu, Lu, Yu, & Lai, 2022; Song, Fu, Zhou, & Lai, 2017). In doing so, a group decision matrix is formulated to comprehensively evaluate the digital financial inclusion performance using the data of PKU-DFIIC. It is evident that the experts' preference structures about alternatives could be diversified (Huang, Chang, Li, & Lin, 2013; Huang & Li, 2012). Some experts may have extreme preference about certain alternatives, while others may have the similar preferences. We are therefore concerned with the preference analysis about the group decision matrix, with the goal of posing more satisfactory commitments for the experts and achieving a more robust consensus. Specifically, we unveil two preferential aspects elaborated by Huang and Li (2012): preferential differences and preferential priorities. Expert's preferential differences show the preference degrees among various alternatives, while preferential priorities indicate the most favorable ranking of the alternatives. The preference analysis gives rise to a revised group decision matrix, the elements of which are the expert-specific preference rankings of all alternatives according to the performance results with preference analysis. A group ranking method is proposed to consolidate and aggregate the experts' rankings based on the half-quadratic theory (Hochbaum & Levin, 2006; Mohammadi & Rezaei, 2020). A half-quadratic programming approach is devised by Mohammadi and Rezaei (2020) to compute the weights with respect to the experts' rankings. A ranking deviating from most of other rankings is straightforwardly labeled as an outlier, and is thereby assigned with a small weight. This implies that such a ranking has a smaller effect on the group ranking.

This study substantively contributes to several strands of the DFI measurement literature. First, we propose a well-crafted mechanism to measure and compare the provincial DFI performance in China based on the framework of PKU-DFIIC. Instead of eliciting weights associated with various dimensions (Guo et al., 2019; Khera et al., 2022), all possible importance orders of the dimensions of

[3] https://en.idf.pku.edu.cn/

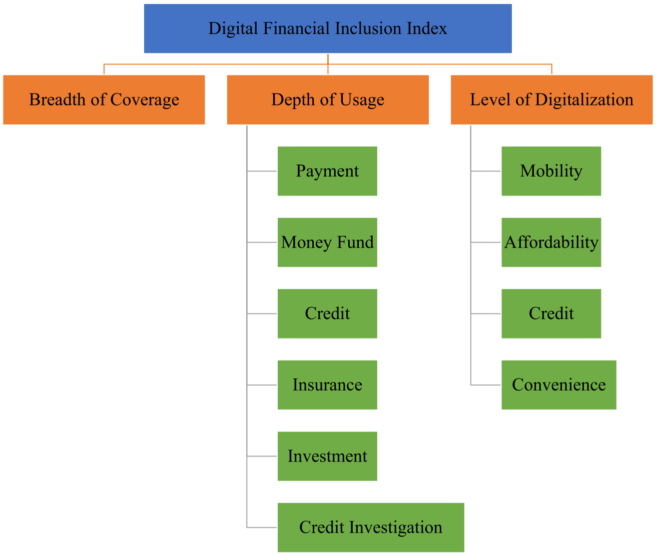

Fig. 1. The framework of PKU-DFIIC.

PKU-DFIIC are explored and then identified as a panel of experts. Second, a group ranking method based upon the half-quadratic theory is developed to aggregate the expert-specific rankings with preference analysis. In addition, a consensus index and trust level for the group ranking are provided to validate the effectiveness of our method. Third, the provincial DFI in China has been measured and compared in a spatial-temporal manner, in conjunction with meaningful managerial insights and policy roadmap to support the solid development of DFI in China.

The remainder of this paper is organized as follows. In Section 2, we review the related literature, and in Section 3, we propose the group ranking method. An empirical study that measures and compares the province-level DFI performance using the panel data from 2016 to 2020 is performed in Section 4. We conclude in Section 5 by elaborating our method and providing suggestions for future research.

2. Literature review

An extensive literature review related to both impact and measurement of DFI and group ranking is presented to highlight the significance of this study.

2.1. Impact and measurement of digital financial inclusion

The development of DFI has material impacts on both economic growth and environmental sustainability. Qualitative studies including case studies and expert survey and quantitative analysis indicate that the DFI can build a viable channel to serve the bottom of the pyramid (BOP) segment (Gupta & Kanungo, 2022). Lin, Peng, and Wu (2022) claim that digital finance significantly increases both Micro and Small Enterprises' (MSEs') probability of applying for new investment projects and the number of projects applied, based on the China MSE survey data. Similarly, Liu, Luan, Wu, Zhang, and Hsu (2021) show that DFI can effectively promote the China's economic growth, using the provincial panel data from 2011 to 2019 in China. Apart from these economic effects, Wang, Wang, Ren, and Wen (2022) find that DFI increases CO_2 emissions of local cities, but reduces those of neighboring cites. Furthermore, the breadth of coverage and depth of use are significantly correlated with CO_2 emissions. In addition, Ozturk and Ullah (2022) conclude that DFI increases economic growth but decreases environmental quality through the surge of CO_2 emissions, based upon an empirical study in 42 one belt and road initiative (OBRI) countries from 2007 to 2019. Kanungo and Gupta (2021) notice that DFI can improve the social-economic well-being in India. Lin and Ma (2022) state that breadth of coverage, depth of use, and degree of digitization are conducive in improving the quantity and quality of green technological innovation in China.

In view of the significant impacts of DFI, up to our knowledge, only two methods have been developed to measure the DFI performance in recent studies. The PKU-DFIIC is proposed by the Institute of Digital Finance at Peking University and Ant Financial Services Group (Guo et al., 2019), which considers three dimensions of financial services: breadth of coverage, depth of use and level of digitization, and uses the AHP and Coefficient of Variation to determine subjective-objective integrated weights for the indicators. In parallel, Khera et al. (2022) construct a new and comprehensive DFI index by employing a three-stage principal component analysis (PCA) and a widely used objective weighting scheme. Sawada (2022) comments on this novel index and suggests five directions for further improvement.

2.2. Group ranking

Our paper also contributes to the growing literature stream of group ranking, in which a plenty of appropriate mathematical models and effective algorithms have been proposed. Cook and Kress (1990) devise a Data Envelopment Analysis (DEA) model with assurance region side constraints to aggregate votes from a preferential ballot. González-Pachón and Romero (2001) propose an interval goal programming approach to aggregating incomplete individual patterns of preference in a group decision-making (GDM) problem. Hochbaum and Levin (2006) present a new paradigm using an optimization framework providing a specific performance measure for the quality of the aggregate ranking. Ma (2016) determines a complete ranking list meeting group consensus preferences for group ranking problem, in which a new group consensus mining approach and an optimization model involving maximum consensus sequences are given. Moreno-Centeno and Escobedo (2016) develop an axiomatic method to aggregate a set of incomplete rankings into a consensus ranking. Bustince et al. (2022) introduce the concepts of a ranking fusion function and a score function as general means to aggregate individual rankings. Aledo, Gámez, and Molina (2016); Aledo, Gámez, and Rosete (2018, 2021) employ the extension sets, evolution strategies and greedy algorithms based on different sort-first and cluster-second strategies to aggregation ranking, respectively. Yoo, Escobedo, and Skolfield (2020) design a generalization of the seminal Kendall tau correlation coefficient to handle various ranking formats including non-strict and incomplete preferences. Yoo and Escobedo (2021) also introduce a binary programming formulation for the generalized Kemeny ranking aggregation problem, in which the rankings maybe complete and incomplete, with and without ties.

3. The group ranking method

The proposed group ranking method originates from the social choice theory developed by Sen (1977, 1999), according to which a mechanism is designed to summarize a collective or social preference order over alternatives from a set of individual preference orders over those same alternatives available to a society of individuals. We begin with formulating a group decision matrix, in which the experts are identified using different importance orders of three dimensions of PKU-DFIIC, and the expert-specific evaluation results could be obtained by means of a sophisticated mathematical transformation; we then implement the preference analysis about the above group decision matrix, to obtain a revised group decision matrix, the elements of which are the expert-specific preference rankings of all alternatives according to the performance results with preference analysis; the group ranking of all provinces is ultimately aggregated based upon a half-quadratic programming approach, in conjunction with the development of consensus index and trust level to indicate the level of agreement and reliability of the group ranking.

3.1. Group decision matrix

Consider that the PKU-DFIIC measures the provincial DFI performance in accordance with three core dimensions: breadth of coverage, depth of use and level of digitalization, and unlike the PKU-DFIIC assigns subjective weights to these dimensions using AHP, this study investigates all possible importance orders among these dimensions to identify six experts (Fu et al., 2021, 2022; Song et al., 2017): E1: $w_{\text{breadth of coverage}} \geq w_{\text{depth of use}} \geq w_{\text{level of digitalization}}$; E2: $w_{\text{breadth of coverage}} \geq w_{\text{level of digitalization}} \geq w_{\text{depth of use}}$; E3: $w_{\text{depth of use}} \geq w_{\text{breadth of coverage}} \geq w_{\text{level of digitalization}}$; E4: $w_{\text{depth of use}} \geq w_{\text{level of digitalization}} \geq w_{\text{breadth of coverage}}$; E5: $w_{\text{level of digitalization}} \geq w_{\text{breadth of coverage}} \geq w_{\text{depth of use}}$; E6: $w_{\text{level of digitalization}} \geq w_{\text{depth of use}} \geq w_{\text{breadth of coverage}}$. Without loss of generality, we explore provincial DFI performance assessed by E1: $w_{\text{breadth of coverage}} \geq w_{\text{depth of use}} \geq w_{\text{level of digitalization}}$, the results of which could be easily migrated to other experts. Additionally, the DFI performance of province $i, i = 1, 2, \ldots, m$ corresponding to three dimensions could be denoted as $x_{it}, t = 1, 2, 3$, and the weights with respect to breadth of coverage, depth of use and level of digitalization are indicated as w_1, w_2, w_3. Therefore, the DFI performance of province i evaluated by E1, v_{i1}, is described as:

$$\begin{cases} v_{i1} = max \sum_{t=1}^{3} y_{it} w_t \\ s.t. w_1 \geq w_2 \geq w_3 \\ \sum_{t=1}^{3} w_t = 1, w_t \geq 0 \end{cases} \tag{1}$$

in which the min-max normalization scheme is employed to normalize x_{it} as: $y_{it} = \frac{x_{it} - \min_i \{x_{it}\}}{\max_i \{x_{it}\} - \min_i \{x_{it}\}}$.

Theorem 1. (Song et al., 2017) The DFI performance of province $i, i = 1, 2, \ldots, m$ measured by E1 could be analytically derived as $v_{i1} = max\left\{y_{i1}, \frac{y_{i1}+y_{i2}}{2}, \frac{y_{i1}+y_{i2}+y_{i3}}{3}\right\}$.

Theorem 1 demonstrates that the provincial DFI performance results are analytically obtained as a closed-form expression, without the determination of precise weights. Such a scheme differs from the conventional wisdom that assigns exact weights to each dimension, and makes the DFI performance measurement easier and simpler. This study comprehensively takes the assessment results of all experts into account, and proposes the following group decision matrix for DFI performance evaluation:

$$V_{m\times 6} = \begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ v_{21} & v_{22} & v_{23} & v_{24} & v_{25} & v_{26} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{m1} & v_{m2} & v_{m3} & v_{m4} & v_{m5} & v_{m6} \end{bmatrix},$$

where $v_{i2} = max\left\{y_{i1}, \frac{y_{i1}+y_{i3}}{2}, \frac{y_{i1}+y_{i3}+y_{i2}}{3}\right\}$, $v_{i3} = max\left\{y_{i2}, \frac{y_{i2}+y_{i1}}{2}, \frac{y_{i2}+y_{i1}+y_{i3}}{3}\right\}$, $v_{i4} = max\left\{y_{i2}, \frac{y_{i2}+y_{i3}}{2}, \frac{y_{i2}+y_{i3}+y_{i1}}{3}\right\}$, $v_{i5} = max\left\{y_{i3}, \frac{y_{i3}+y_{i1}}{2}, \frac{y_{i3}+y_{i1}+y_{i2}}{3}\right\}$, $v_{i6} = max\left\{y_{i3}, \frac{y_{i3}+y_{i2}}{2}, \frac{y_{i3}+y_{i2}+y_{i1}}{3}\right\}$.

In regard to such a panel-of-experts paradigm, the decision maker pragmatically tends to average the opinions of experts but err in the determination of weights (Mannes, 2009). Extant studies have pointed out that the average aggregation scheme may lack of commitment among experts especially when some experts have extreme preferential conditions for certain alternatives (Huang et al., 2013). This paper is therefore accompanied by an investigation of the endogenous preference structure of the above group decision matrix $V_{m\times 6}$ for better evaluating the provincial DFI performance in China.

3.2. Preference analysis

The preference analysis of the aforementioned group decision matrix $V_{m\times 6}$ is carried out from two aspects: preferential differences describing the preference degrees among provinces for each expert, and preferential priorities describing the favorite ranking of the provinces for each expert. Consistent with Matsatsinis, Grigoroudis, and Samaras (2005) and Huang and Li (2012), this paper defines the preferential difference between province i and k by expert j as below:

$$\alpha_{ikj} = \left|v_{ij} - v_{kj}\right|, 0 \leq \alpha_{ikj} \leq 1, i \neq k, \tag{2}$$

in which v_{ij} is the DFI performance of province i measured by expert j. In view of that α_{ikj} only measures the preferential difference between province i and k by expert j, the relative preferential difference of expert j in the group could therefore be calculated as (Huang & Li, 2012):

$$\alpha_j = \frac{\sum_{i<k}\alpha_{ikj}}{\sum_{j=1}^{6}\sum_{i<k}\alpha_{ikj}}, 0 < \alpha_j < 1, i \neq k, 1 \leq i < k \leq m, \tag{3}$$

in which α_j is computed in terms of summing up α_{ikj} for C_m^2 times to consider all possible combinations of province pairs. A large α_j implies that the preferential difference for expert j is large, which indicates that expert j is able or willing to decisively discriminate the provinces with respect to DFI performance, and thus may have significantly different preferences among these provinces. Therefore, α_j is a promising metric for the expert j to denote her/his preferential difference in the group.

In addition, it is also noted that experts would be particularly aware of whether their most favorable provinces is adopted in a group decision making problem, and naturally give the most importance to these provinces. In this regard, the preferential priority of province i by expert j is defined as $\varphi_{ij} = \frac{m}{\psi_{ij}}$, in which ψ_{ij} is the preferential ranking of province i by expert j. For the alternatives with the same ranking position, they will be assigned with the rankings position that is the average of their original ones. The preferential priority of province i in the group could therefore be obtained as $\varphi_i = \frac{\sum_{j=1}^{6}\varphi_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{6}\varphi_{ij}}$.

The investigation of preferential differences and preferential priorities give rise to a revised group decision matrix below:

$$V'_{m\times 6} = \begin{bmatrix} v_{11}\times\alpha_1\times\varphi_1 & v_{12}\times\alpha_2\times\varphi_1 & v_{13}\times\alpha_3\times\varphi_1 & v_{14}\times\alpha_4\times\varphi_1 & v_{15}\times\alpha_5\times\varphi_1 & v_{16}\times\alpha_6\times\varphi_1 \\ v_{21}\times\alpha_1\times\varphi_2 & v_{22}\times\alpha_2\times\varphi_2 & v_{23}\times\alpha_3\times\varphi_2 & v_{24}\times\alpha_4\times\varphi_2 & v_{25}\times\alpha_5\times\varphi_2 & v_{26}\times\alpha_6\times\varphi_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{m1}\times\alpha_1\times\varphi_m & v_{m2}\times\alpha_2\times\varphi_m & v_{m3}\times\alpha_3\times\varphi_m & v_{m4}\times\alpha_4\times\varphi_m & v_{m5}\times\alpha_5\times\varphi_m & v_{m6}\times\alpha_6\times\varphi_m \end{bmatrix}.$$

Such a revised decision matrix with preference analysis makes the consensus- and commitment-achieving in a decision group more realistic and reasonable (Fu et al., 2021). In accordance with the DFI performance matrix $V'_{m\times 6}$, the ranking associated with expert j is denoted as $R^j, j = 1, 2, \ldots, 6$, which could be different among experts because of the different weight constraints. Furthermore, the ranking of province i obtained by expert j is described as $R_i^j, j = 1, 2, \ldots, 6$.

3.3. A half-quadratic programming approach

We propose a half-quadratic programming approach to implement the group ranking aggregation based upon different expert-specific rankings, in which the weights with respect to each experts are estimated in an objective manner to alleviate the decision bias (Mohammadi & Rezaei, 2020). A general situation involving n experts is explored in this section, in which $j = 1, 2, \ldots, n$.

3.3.1. Half-quadratic minimization

The Euclidean norm is known as the most-widely accepted loss function that is utilized in a variety of environments, while least square fitting is commonly recognized as the most famous regression instrument that employs the Euclidean norm as the loss function (Mohammadi & Rezaei, 2020). Albeit the Euclidean norm is simple to use and generates a closed-form solution, it is extremely sensitive to outliers and performs poorly in uncertain circumstances (Demšar, 2006). The robust estimators are capable of handling this sensitivity issue in an effective way (Mohammadi & Rezaei, 2020). The half-quadratic functions are stimulated by M-estimator, which is a group of robust estimators in the field of robust statistics. The optimum of the half-quadratic functions could be derived by means of half-quadratic minimization with guaranteed convergence. The half-quadratic functions and the corresponding minimizer functions are reported in Table 1 below, in which β is a positive constant, the j^{th} element in a vector like s is denoted as s_j, and σ and γ are the parameters of the half-quadratic functions.

Consider the general half-quadratic minimization problem as below,

$$\min_{s} \sum_{j} g(s_j), \tag{4}$$

in which $g(\blacksquare)$ represents a specific the half-quadratic function listed in Table 1. The multiplicative (Geman & Reynolds, 1992) and additive (Geman & Yang, 1995) forms of the half-quadratic programming can effectively find a local optimal solution to this minimization problem. According to a systematic analysis of the convergence rate obtained by the multiplicative and additive forms, both the bound and the number of iterations required for convergence for the additive form are always larger than those of the multiplicative form (Nikolova & Ng, 2005). The multiplicative form is therefore preferred when the computation efficiency is considered. In view of the multiple criteria decision making (MCDM) feature of DFI performance evaluation, this paper employs the multiplicative form of the half-quadratic programming to solve the above minimization problem (Mohammadi & Rezaei, 2020). In doing so, the half-quadratic minimization problem (4) could be restated as

$$\min_{s,\lambda} \left\{ \sum_{j} \lambda_j s_j^2 + \psi(\lambda_j) \right\}, \tag{5}$$

in which $\lambda_j, \lambda_j > 0$ is the half-quadratic auxiliary variable, and $\psi(\blacksquare)$ is the convex conjugate of $g(\blacksquare)$ denoted as $\psi(\lambda_j) = \max_{e} \{ e\lambda_j - g(e) \}$.

The half-quadratic multiplicative theory proposes the following iteration process to achieve convergence when solving the half-quadratic minimization problem (5) (Geman & Reynolds, 1992):

$$\begin{cases} \lambda_j^{l+1} = \delta\left(s_j^l\right), \\ s^{l+1} = \underset{s}{argmin} \sum_{j} \lambda_j^{l+1} s_j^w, \end{cases} \tag{6}$$

in which l and $l+1$ denote the iteration counter, $\delta(\blacksquare)$ is the minimizer function associated with $g(\blacksquare)$ in Table 1. Obviously, different half-quadratic functions could give rise to different λ_j. In line with Mohammadi and Rezaei (2020), we in particular investigate the Welsch M-estimator since it is recognized as the most viable and outlier-resistant estimator among the half-quadratic functions (He et al., 2014).

3.3.2. A half-quadratic-based compromise ranking

Different expert-specific rankings are typically aggregated in terms of minimizing the Euclidean distance to individual obtained ranking. Then the relevant minimization problem is

Table 1
Different M-estimators and their corresponding minimizer function (He, Zheng, Tan, & Sun, 2014).

Estimators	$l_1 - l_2$	Fair	log-cosh	Welsch	Huber
Half-quadratic function $g(s_j)$	$\sqrt{\beta + s_j^2} - 1$	$\frac{\lvert s_j\rvert}{\beta} - log\left(1 + \frac{\lvert s_j\rvert}{\beta}\right)$	$log(cosh(\beta s_j))$	$1 - exp\left(-\frac{s_j^2}{\sigma^2}\right)$	$\begin{cases} \frac{s_j^2}{2}, & \lvert s_j\rvert \le \gamma \\ \gamma\lvert s_j\rvert - \frac{\gamma^2}{2}, & \lvert s_j\rvert \rangle \gamma \end{cases}$
Minimizer function relevant to the multiplicative form $\delta(s_j)$	$\frac{1}{\sqrt{\beta + s_j^2}}$	$\frac{1}{\beta(\beta + \lvert s_j\rvert)}$	$\frac{\beta}{s_j} tanh(\beta s_j)$	$exp\left(-\frac{s_j^2}{\sigma^2}\right)$	$\begin{cases} 1, & \lvert s_j\rvert \le \gamma \\ \frac{\gamma}{\lvert s_j\rvert}, & \lvert s_j\rvert \rangle \gamma \end{cases}$
Minimizer function relevant to the additive form $\delta(s_j)$	$s_j - \frac{s_j}{\sqrt{\beta + s_j^2}}$	$s_j - \frac{s_j}{\beta\sqrt{\beta + \lvert s_j\rvert}}$	$s_j - \beta tanh(\beta s_j)$	$s_j - s_j exp\left(-\frac{s_j^2}{\sigma^2}\right)$	$\begin{cases} 0, & \lvert s_j\rvert \le \gamma \\ s_j - \gamma sign(s_j), & \lvert s_j\rvert \rangle \gamma \end{cases}$

$$\min_{R^*} \frac{1}{2}\sum_{j=1}^{n}\left\|R^j - R^*\right\|_2^2, \tag{7}$$

in which n is the number of experts, R^j is the ranking of the j^{th} expert, and R^* is the group ranking. It is not difficult to obtain the closed-form solution to the minimization problem (7) as:

$$R^* = \frac{1}{n}\sum_{j=1}^{n} R^j, \tag{8}$$

which in essence is the average of rankings generated by experts. However, this scheme is not a reliable estimator since it is extremely sensitive to outliers (Demšar, 2006). Specifically, an expert who provides different rankings from others, would significantly affect the group ranking. Therefore, we employ the half-quadratic functions to aggregate the rankings, which are immune to outliers (Huber, 2004). In this regard, the proposed minimization problem to estimate R^* is

$$\min_{R^*} \frac{1}{2}\sum_{j=1}^{n} g\left(\left\|R^j - R^*\right\|_2\right), \tag{9}$$

in which $g(\blacksquare)$ is an half-quadratic function. Geman and Reynolds (1992) claim that half-quadratic programming could effectively solve this minimization problem, even it is not convex.

Based upon the multiplicative form of the half-quadratic programming, the half-quadratic minimization problem (9) can be reformulated as

$$\min_{R^*,\lambda} J(R^*,\lambda) = \sum_{j=1}^{n} \lambda_j \left\|R^j - R^*\right\|_2^2 + \psi(\lambda_j), \tag{10}$$

in which $\lambda \in R^n$ is the half-quadratic auxiliary variable. In accordance with the half-quadratic multiplicative theory, the following procedures must be iterated until the two variables have achieved convergence (Geman & Reynolds, 1992).

$$\begin{cases} \lambda_j = \delta\left(\left\|R^j - R^*\right\|_2\right), j = 1, 2, \ldots, n, \\ R^* = \underset{R^*}{argmin} \sum_{j=1}^{n} \lambda_j \left\|R^j - R^*\right\|_2^2. \end{cases} \tag{11}$$

Theorem 2. (Mohammadi & Rezaei, 2020).

(i) The solution to the first stage in (11) is obtained by the minimizer function presented in Table 1.

(ii) The optimum for the second stage in (11) is derived as $R^* = \sum_{j=1}^{n} \zeta_j R^j$, *in which* $\zeta_j = \frac{\lambda_j}{\sum_{j=1}^{n}\lambda_j}$.

Theorem 2 indicates that the group ranking is aggregated by computing the weighted sum of the rankings given by all experts, in which the expert-wise weights are calculated using the minimizer function. Apparently, the weights with respect to the expert-specific rankings conform to the non-zero and unit-sum properties.

In summary, the working procedure of the group ranking method based upon the half-quadratic theory is presented as **Algorithm 1** below.

Algorithm 1. Group ranking.

1: **Input:** Expert-specific rankings $R^j, j = 1, 2, ..., n$.
2: **while** Not Congverged **do**
3: $\lambda_j = \delta\left(\left\|R^j - R^*\right\|_2\right), j = 1, 2, ..., n,$
4: $\zeta_j = \frac{\lambda_j}{\sum_{j=1}^{n} \lambda_j}$
5: $R^* = \sum_{j=1}^{n} \zeta_j R^j$
6: **end while**
7: **Output:** Group ranking R^*.

Table 2
The PKU-DFIIC data ranging from 2016 to 2020.

Province	Breadth of coverage					Depth of use					Level of digitization				
	2016	2017	2018	2019	2020	2016	2017	2018	2019	2020	2016	2017	2018	2019	2020
Anhui	194.89	234.70	273.41	301.15	323.75	229.95	309.55	309.62	341.53	366.15	338.54	324.48	393.79	401.16	408.38
Beijing	285.65	316.12	353.87	384.66	397.00	263.74	357.24	366.78	402.07	445.83	329.90	326.02	420.19	406.11	436.02
Chongqing	214.03	249.50	285.11	311.03	329.39	211.54	301.21	285.60	310.36	343.91	340.10	319.57	384.74	396.36	397.12
Fujian	240.47	275.40	312.31	340.65	359.21	245.12	334.33	334.30	363.73	401.80	306.70	314.47	407.76	414.89	409.82
Gansu	189.28	227.38	261.29	287.31	308.87	172.66	240.39	227.52	251.74	265.35	310.24	304.10	356.54	369.65	367.36
Guangdong	240.07	275.91	312.44	339.98	356.94	236.50	328.17	329.93	364.50	404.35	295.07	304.92	399.86	387.77	409.06
Guangxi	193.51	232.73	270.41	295.26	311.98	202.21	279.52	272.49	292.48	313.24	360.15	326.44	381.93	362.35	390.41
Guizhou	180.70	227.77	267.39	292.66	313.24	182.70	258.44	241.33	245.66	258.20	353.03	316.99	373.01	462.23	380.81
Hainan	210.09	253.39	294.40	319.41	335.87	220.35	297.53	300.23	314.46	337.24	322.83	309.34	377.54	369.19	383.46
Hebei	191.55	232.89	264.06	284.39	304.10	196.87	273.45	267.92	297.33	318.42	321.46	313.87	371.55	400.62	391.92
Heilongjiang	191.24	226.00	256.12	275.79	290.48	206.54	275.86	254.88	274.64	293.69	350.97	323.77	372.28	383.30	380.09
Henan	200.65	241.45	278.46	309.34	331.16	199.22	279.56	275.74	301.85	321.21	340.80	328.09	389.27	422.92	408.32
Hubei	215.55	253.63	292.56	320.79	336.54	233.41	317.58	322.44	348.53	369.58	331.83	331.10	402.99	400.97	411.73
Hunan	186.13	223.47	258.07	282.28	302.28	219.80	297.70	286.55	311.81	347.44	318.07	318.96	382.19	379.62	402.30
Inner Mongolia	202.00	238.92	269.49	291.45	310.40	184.89	249.20	232.31	260.31	275.66	404.00	340.10	349.76	363.16	367.40
Jiangsu	233.22	272.32	311.95	341.50	362.11	253.08	328.93	333.09	365.50	395.01	322.80	324.69	408.62	387.38	421.70
Jiangxi	188.79	228.52	266.46	294.32	316.14	222.74	305.92	296.52	319.18	353.23	341.08	324.38	394.00	403.46	398.52
Jilin	191.94	227.45	256.55	275.75	290.78	204.14	273.62	255.23	275.93	297.63	323.59	310.72	378.46	390.01	385.29
Liaoning	207.74	239.87	271.81	292.44	307.11	220.06	291.27	279.48	302.52	328.12	330.21	313.57	375.01	421.66	386.33
Ningxia	205.92	242.42	274.25	299.04	320.45	179.62	252.21	225.27	241.55	262.72	293.12	305.24	355.14	388.74	361.52
Qinghai	177.73	215.67	251.69	272.90	292.06	182.26	251.09	235.31	252.75	264.67	308.11	301.42	351.43	366.30	379.58
Shaanxi	211.17	246.48	281.05	308.21	329.53	202.87	276.00	277.15	309.14	331.73	337.60	317.47	379.31	402.11	402.11
Shandong	209.80	247.19	281.99	309.97	331.66	217.81	290.92	287.85	318.54	343.49	334.58	319.92	388.48	362.98	409.00
Shanghai	274.25	305.89	346.33	378.25	395.20	281.48	396.05	400.40	439.91	488.68	309.94	330.31	440.26	385.58	450.08
Shanxi	205.51	243.02	277.03	305.61	327.29	189.38	254.98	249.73	277.89	291.37	352.96	324.92	367.19	420.25	383.04
Sichuan	197.00	231.87	266.15	291.22	310.76	216.54	301.54	295.83	319.53	344.86	335.38	325.14	384.51	400.84	396.05
Tianjin	225.41	257.90	295.35	323.86	340.29	231.61	310.13	317.94	349.01	373.91	339.15	322.91	386.10	440.83	408.74
Tibet	167.21	209.29	249.82	271.14	290.18	202.53	273.79	267.16	293.21	319.38	332.66	314.10	368.33	398.23	361.67
Xinjiang	190.32	228.82	267.35	293.48	310.22	190.11	249.10	232.94	256.31	273.85	303.31	313.56	357.37	439.16	364.88
Yunnan	185.37	223.54	262.29	284.43	302.46	203.17	282.85	278.84	291.12	309.45	348.65	316.08	376.06	375.07	387.78
Zhejiang	254.44	290.06	330.17	362.40	382.07	270.62	366.40	372.01	404.65	439.25	308.66	322.66	421.07	382.41	429.98

3.3.3. Consensus index and trust level

Among the half-quadratic functions, the Welsch estimator has been extensively applied with an impressive performance in a variety of fields (He et al., 2014; He, Zhang, Sun, & Yin, 2015). He et al. (2014) stated that the parameter σ in the Welsch estimator could be orchestrated recursively in each iteration as,

$$\sigma = \frac{\sum_{j=1}^{n} \|R^j - R^*\|_2^2}{2m^2}. \tag{12}$$

In the presence of computed σ in the optimization process, the consensus index and trust level of the group ranking derived by **Algorithm 1** are elaborated below to manifest the validity of proposed group ranking method.

Definition 1. (Consensus Index.) (Mohammadi & Rezaei, 2020) A consensus index C shows the extent to which all expert-specific rankings being used agree upon the group ranking.

The consensus index calculates the similarity of the group ranking with each expert-specific ranking, when the Welsch estimator is employed as the half-quadratic function. In this sense, the consensus index C of a group ranking R^* associated with rankings $R^j, j = 1, 2, \ldots, n$ could be calculated as

$$C(R^*) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{N_\sigma(R_i^* - R_i^j)}{N_\sigma(0)}, \tag{13}$$

in which $N_\sigma(\blacksquare)$ is the probability density function of the Gaussian distribution with the mean as zero and the standard deviation as σ, and $N_\sigma(0)$ is utilized to normalize the similarity computation, to make sure the $C(R^*)$ distributes in $[0, 1]$. It is observed that when there a complete agrement as $R_i^* = R_i^j$, the consensus index would be $C(R^*) = 1$. Furthermore, the consensus index decreases as rankings diverge from one another. In other words, if one ranking differs from the others, it can have a negative impact on the consensus index. In the meanwhile, this particular ranking is considered as an outlier in the half-quadratic functions being utilized. Therefore, the group ranking will be less affected by this characteristic ranking, which can significantly affect the consensus index.

Definition 2. (Trust level.) (Mohammadi & Rezaei, 2020) A trust level T is the degree to which one can accredit the group ranking.

The trust level reflects the reliability of the group ranking. For example, a ranking deviating profoundly from the majority of

Table 3
Group decision matrix with preference analysis.

Province	E1	E2	E3	E4	E5	E6	φ_i
Anhui	0.4373	0.4373	0.4683	0.4988	0.5292	0.5292	0.0258
Beijing	1.0000	1.0000	0.9070	0.8851	0.9206	0.8851	0.1655
Chongqing	0.3803	0.3846	0.3803	0.3870	0.4021	0.4021	0.0168
Fujian	0.6462	0.6462	0.6346	0.6231	0.6049	0.6049	0.0483
Gansu	0.1750	0.1750	0.1030	0.0906	0.1205	0.0906	0.0085
Guangdong	0.6296	0.6250	0.6341	0.6341	0.5987	0.5987	0.0428
Guangxi	0.2564	0.2652	0.2564	0.2825	0.3262	0.3262	0.0125
Guizhou	0.2159	0.2169	0.1446	0.1446	0.2178	0.2178	0.0102
Hainan	0.4277	0.4277	0.3853	0.3429	0.3395	0.3395	0.0172
Hebei	0.2449	0.2449	0.2613	0.3023	0.3433	0.3433	0.0129
Heilongjiang	0.1222	0.1222	0.1540	0.1819	0.2097	0.2097	0.0094
Henan	0.3952	0.4561	0.3952	0.4009	0.5284	0.5284	0.0217
Hubei	0.4947	0.5005	0.4947	0.5251	0.5670	0.5670	0.0333
Hunan	0.3203	0.3203	0.3872	0.4239	0.4605	0.4605	0.0187
Inner Mongolia	0.1893	0.1893	0.1325	0.1105	0.1279	0.1105	0.0092
Jiangsu	0.6734	0.6765	0.6488	0.6488	0.6795	0.6795	0.0621
Jiangxi	0.3577	0.3577	0.4123	0.4150	0.4178	0.4178	0.0186
Jilin	0.1484	0.1484	0.1711	0.2197	0.2684	0.2684	0.0101
Liaoning	0.2474	0.2474	0.3034	0.3034	0.2802	0.2918	0.0124
Ningxia	0.2834	0.2834	0.1515	0.1010	0.1417	0.1010	0.0103
Qinghai	0.0832	0.1108	0.0832	0.1160	0.2040	0.2040	0.0088
Shaanxi	0.3819	0.4134	0.3819	0.3887	0.4584	0.4584	0.0184
Shandong	0.4315	0.4622	0.4315	0.4531	0.5362	0.5362	0.0274
Shanghai	0.9944	0.9944	1.0000	1.0000	1.0000	1.0000	**0.2069**
Shanxi	0.3474	0.3474	0.2456	0.2448	0.2952	0.2448	0.0126
Sichuan	0.3195	0.3195	0.3760	0.3829	0.3899	0.3899	0.0148
Tianjin	0.5014	0.5014	0.5020	0.5176	0.5332	0.5332	0.0321
Tibet	0.1327	0.0891	0.2654	0.2654	0.0891	0.1336	0.0098
Xinjiang	0.1876	0.1876	0.1278	0.0978	0.1128	0.0978	0.0087
Yunnan	0.2113	0.2113	0.2224	0.2595	0.2966	0.2966	0.0113
Zhejiang	0.8602	0.8602	0.8229	0.8063	0.8166	0.8063	0.0828
α_j	0.1660	**0.1681**	0.1657	0.1661	0.1665	0.1675	

Table 4
Revised group decision matrix.

Province	Result by E1	Results by E2	Result by E3	Result by E4	Result by E5	Result by E6	Ranking by E1	Ranking by E2	Ranking by E3	Ranking by E4	Ranking by E5	Ranking by E6
Anhui	0.0019	0.0019	0.0020	0.0021	0.0023	0.0023	10	10	9	9	10	10
Beijing	0.0275	0.0278	0.0249	0.0243	0.0254	0.0245	2	2	2	2	2	2
Chongqing	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	15	15	16	15	15	15
Fujian	0.0052	0.0052	0.0051	0.0050	0.0049	0.0049	5	5	5	5	5	5
Gansu	0.0002	0.0003	0.0001	0.0001	0.0002	0.0001	28	28	30	31	29	31
Guangdong	0.0045	0.0045	0.0045	0.0045	0.0043	0.0043	6	6	6	6	6	6
Guangxi	0.0005	0.0006	0.0005	0.0006	0.0007	0.0007	19	19	20	20	19	19
Guizhou	0.0004	0.0004	0.0002	0.0002	0.0004	0.0004	24	24	26	26	24	24
Hainan	0.0012	0.0012	0.0011	0.0010	0.0010	0.0010	12	13	15	16	16	16
Hebei	0.0005	0.0005	0.0006	0.0006	0.0007	0.0007	20	20	19	18	18	18
Heilongjiang	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	30	29	27	25	25	25
Henan	0.0014	0.0017	0.0014	0.0014	0.0019	0.0019	11	11	11	11	11	11
Hubei	0.0027	0.0028	0.0027	0.0029	0.0031	0.0032	7	7	7	7	7	7
Hunan	0.0010	0.0010	0.0012	0.0013	0.0014	0.0014	16	16	13	12	12	12
Inner Mongolia	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	25	25	28	28	28	29
Jiangsu	0.0069	0.0071	0.0067	0.0067	0.0070	0.0071	4	4	4	4	4	4
Jiangxi	0.0011	0.0011	0.0013	0.0013	0.0013	0.0013	14	14	12	13	14	14
Jilin	0.0002	0.0003	0.0003	0.0004	0.0005	0.0005	27	27	24	24	23	23
Liaoning	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	21	21	18	19	21	20
Ningxia	0.0005	0.0005	0.0003	0.0002	0.0002	0.0002	22	22	25	27	27	28
Qinghai	0.0001	0.0002	0.0001	0.0002	0.0003	0.0003	31	30	31	29	26	26
Shaanxi	0.0012	0.0013	0.0012	0.0012	0.0014	0.0014	13	12	14	14	13	13
Shandong	0.0020	0.0021	0.0020	0.0021	0.0024	0.0025	9	9	10	10	9	9
Shanghai	0.0341	0.0346	0.0343	0.0344	0.0345	0.0347	1	1	1	1	1	1
Shanxi	0.0007	0.0007	0.0005	0.0005	0.0006	0.0005	18	18	21	21	20	22
Sichuan	0.0008	0.0008	0.0009	0.0009	0.0010	0.0010	17	17	17	17	17	17
Tianjin	0.0027	0.0027	0.0027	0.0028	0.0029	0.0029	8	8	8	8	8	8
Tibet	0.0002	0.0001	0.0004	0.0004	0.0001	0.0002	29	31	22	23	31	27
Xinjiang	0.0003	0.0003	0.0002	0.0001	0.0002	0.0001	26	26	29	30	30	30
Yunnan	0.0004	0.0004	0.0004	0.0005	0.0006	0.0006	23	23	23	22	22	21
Zhejiang	0.0118	0.0120	0.0113	0.0111	0.0113	0.0112	3	3	3	3	3	3

rankings would be assigned a smaller weight in **Algorithm 1**, and thus, has a smaller influence on the group ranking. When the weight associated with such a ranking is smaller than those of rest rankings, the trust level would be less affected as well. Therefore, the trust level could be calculated as

$$T(R^*) = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n}\zeta_j\frac{N_\sigma\left(R_j^* - R_j^i\right)}{N_\sigma(0)}, \tag{14}$$

where $\zeta_j, j = 1, 2, \ldots, n$ is obtained in **Algorithm 1**. In this regard, the trust level is distorted by the rankings that are deviated from the majority of rankings, and therefore is capable of measuring the reliability of the group rankings. Interestingly, the trust level equals to the consensus index when each ranking has the identical importance level, that is, $\zeta_j = \frac{1}{n}, j = 1, 2, \ldots, n$.

4. An empirical study

The proposed group ranking method is applied to measure and compare the DFI performance of 31 provinces (municipalities, autonomous regions, collectively referred to as "provinces") in China using the PKU-DFIIC data ranging from 2016 to 2020 (See Table 2). 31 provinces are alphabetically listed in Table 2, and the related dataset is available from the joint research team formed by the Institute of Digital Finance at Peking University and Ant Financial Services Group. As a leader of DFI, China advances the development of online investment, mobile payment, BigTech credit and central bank digital currency.

In what follows, we employ the proposed group ranking method to compare the provincial DFI performance in China using the PUK-DFIIC 2020 data, the results of which are easily to migrated to other years. Based upon the normalized PUK-DFIIC 2020 data for 31 provinces, we obtain the expert-specific performance evaluation results in accordance with **Theorem 1**, and then derive the group decision matrix as the Table 3 below. Specially, the normalized values associated with breadth of coverage, depth of use and level of digitalization are 0.3142, 0.4683 and 0.5292, respectively. Then the score determined by E1 is calculated as $max\left\{0.3142, \frac{0.3142+0.4863}{2}, \frac{0.3142+0.4863+0.5292}{3}\right\} = 0.4373$. There are significant diversities about the evaluation results and corresponding rankings among different experts. These observations necessitate the analysis of their preference structures and aggregation of

Table 5
Group ranking results.

Province	Ranking by E1	Ranking by E2	Ranking by E3	Ranking by E4	Ranking by E5	Ranking by E6	R^*	Group ranking
Anhui	10	10	9	9	10	10	9.5208	10
Beijing	2	2	2	2	2	2	2.0000	2
Chongqing	15	15	16	15	15	15	15.1902	15
Fujian	5	5	5	5	5	5	5.0000	5
Gansu	28	28	30	31	29	31	30.2606	31
Guangdong	6	6	6	6	6	6	6.0000	6
Guangxi	19	19	20	20	19	19	19.4792	19
Guizhou	24	24	26	26	24	24	24.9583	24
Hainan	12	13	15	16	16	16	15.5894	16
Hebei	20	20	19	18	18	18	18.3162	18
Heilongjiang	30	29	27	25	25	25	25.6638	25
Henan	11	11	11	11	11	11	11.0000	11
Hubei	7	7	7	7	7	7	7.0000	7
Hunan	16	16	13	12	12	12	12.4421	12
Inner Mongolia	25	25	28	28	28	29	28.0888	28
Jiangsu	4	4	4	4	4	4	4.0000	4
Jiangxi	14	14	12	13	14	14	13.3306	13
Jilin	27	27	24	24	23	23	23.7311	23
Liaoning	21	21	18	19	21	20	19.5737	20
Ningxia	22	22	25	27	27	28	26.5824	27
Qinghai	31	30	31	29	26	26	28.1013	29
Shaanxi	13	12	14	14	13	13	13.4477	14
Shandong	9	9	10	10	9	9	9.4792	9
Shanghai	1	1	1	1	1	1	1.0000	1
Shanxi	18	18	21	21	20	22	20.9087	21
Sichuan	17	17	17	17	17	17	17.0000	17
Tianjin	8	8	8	8	8	8	8.0000	8
Tibet	29	31	22	23	31	27	25.8025	26
Xinjiang	26	26	29	30	30	30	29.5579	30
Yunnan	23	23	23	22	22	21	21.9754	22
Zhejiang	3	3	3	3	3	3	3.0000	3
Weights	0.0315	0.0315	0.1902	0.2889	0.1801	0.2777	Confidence Index	0.9673
							Trust Level	0.9838

individual rankings. We thereby analyze the preferential differences and preferential priorities of this group decision matrix and report them in Table 3 as well. It is observed that the preference degrees among different provinces for six experts change mildly. Furthermore, the preferential priority of Shanghai is largest, followed by Beijing. This implies that Shanghai is most preferred in the decision group in terms of ranking, reflecting the compromise solution among different experts.

On the strength of the preferential differences α_j and preferential priorities φ_i, a revised group decision matrix is obtained in Table 4 below, along with the rankings of provinces determined by six experts.

According to the expert-specific rankings of provincial DFI performance listed in Table 4, Table 5 tabulates the group ranking results, in which the expert-specific rankings are used as the inputs of the proposed half-quadratic programming approach to derive the group ranking, as well as the weights with respect to each expert, confidence index and trust level. The proposed group ranking method assigns the largest and smallest weights to expert 4 and experts 1&2, respectively. This in a sense implies that the preference relationship among breadth of coverage, depth of use and level of digitalization is depth of use$\succ$level of digitalization$\succ$breadth of coverage, which is completely different from the importance order of PKU-DFIIC. Furthermore, the confidence index and trust level of this ranking is 0.9673 and 0.9838, respectively. This manifests that the agreement among all expert-specific ranking and the reliability with respect to the group ranking are sufficiently applicable.

We also compare the ranking results between the group ranking and PKU-DFIIC 2020, which is shown in the Fig. 2 below. The PKU-DFIIC 2020 ranks 31 provinces according to the report released by the Institute of Digital Finance, Peking University (Guo et al., 2019), in which the weights with respect to breadth of coverage, depth of use and level of digitalization as: 0.540, 0.297 and 0.163, respectively. It is found that 24 out of 31 provinces are ranked differently, which is mainly attributed to different evaluation methods. In addition, the ranking positions of Bejing (2), Fujian (5), Guangdong (6) and Jiangsu (4), Shanghai (1), Yunnan (22) and Zhejiang (3) are unchanged. This means that the DFI performance of these provinces are sufficiently robust. In this regard, the results demonstrate that the ranking positions of top performers are immune to the choice of ranking methods.

Similarly, we derive the group rankings for each province in China between 2016 and 2020 in Table 6 and Fig. 3 below. It is temporally observed that the rankings of Inner Mongolia exhibit the largest variation from 2016 to 2020, followed by those of Xinjiang and Guizhou, while Zhejiang has the most robust ranking.

Additionally, the weights aggregating the expert-specific rankings into the group ranking, confidence index, and trust level are reported in the Table 7 and Fig. 4 below. We find that the optimal weights associated with different experts change across different years, and the agreement and reliability about the group ranking are remarkably applicable during these years. Moreover, Fig. 4 indicates that 2017 and 2019 have the similar pattern about the distribution of weights, 2017, 2019 and 2020 simultaneously assign

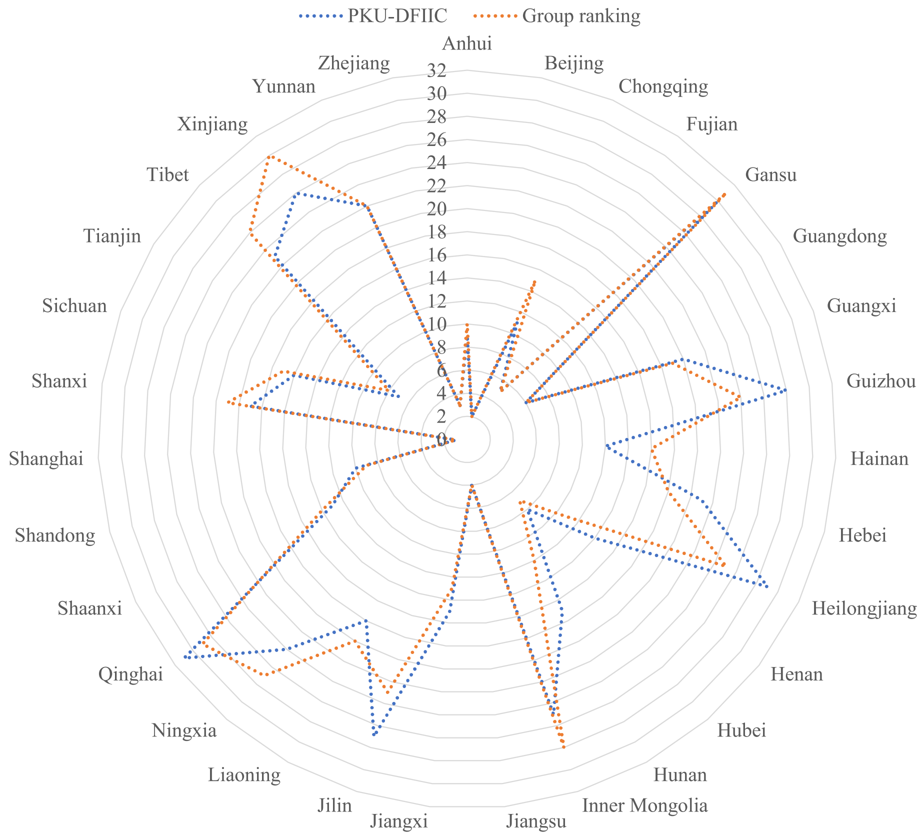

Fig. 2. Ranking comparisons.

Table 6
Group rankings for each province in China between 2016 and 2020.

Province	2016	2017	2018	2019	2020	Average group ranking
Anhui	11	11	9	12	10	10.6
Beijing	1	2	2	2	2	1.8
Chongqing	16	15	15	20	15	16.2
Fujian	6	6	5	7	5	5.8
Gansu	30	30	30	30	31	30.2
Guangdong	7	7	6	8	6	6.8
Guangxi	10	13	17	24	19	16.6
Guizhou	18	24	23	5	24	18.8
Hainan	15	19	11	15	16	15.2
Hebei	28	25	22	21	18	22.8
Heilongjiang	13	18	26	28	25	22
Henan	24	9	13	11	11	13.6
Hubei	9	5	7	10	7	7.6
Hunan	23	20	16	22	12	18.6
Inner Mongolia	4	4	29	29	28	18.8
Jiangsu	5	8	4	6	4	5.4
Jiangxi	14	14	10	16	13	13.4
Jilin	25	27	21	27	23	24.6
Liaoning	17	22	19	13	20	18.2
Ningxia	27	28	27	25	27	26.8
Qinghai	31	31	31	31	29	30.6
Shaanxi	21	21	18	17	14	18.2
Shandong	19	17	12	19	9	15.2
Shanghai	2	1	1	1	1	1.2
Shanxi	12	16	24	14	21	17.4
Sichuan	22	12	14	18	17	16.6
Tianjin	8	10	8	4	8	7.6
Tibet	26	26	25	23	26	25.2
Xinjiang	29	29	28	9	30	25
Yunnan	20	23	20	26	22	22.2
Zhejiang	3	3	3	3	3	3

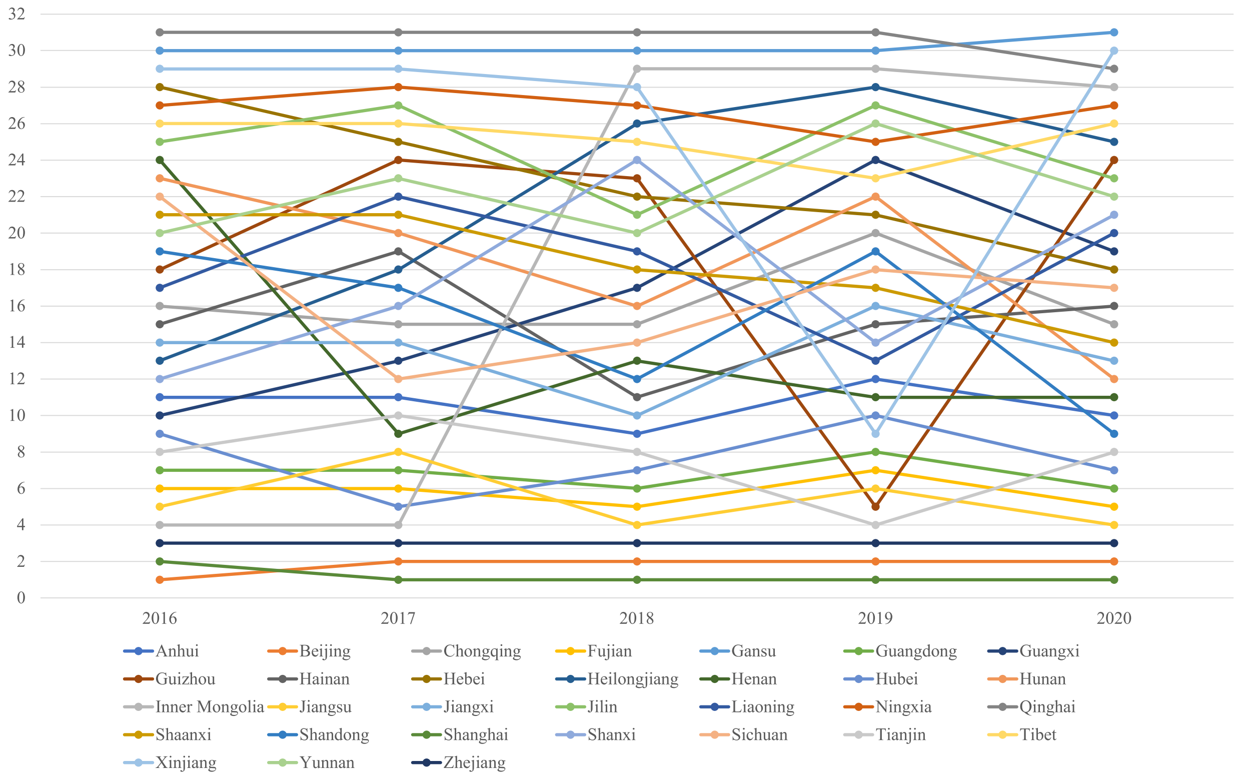

Fig. 3. Group rankings for each province in China between 2016 and 2020.

Table 7
The weights, confidence index, and trust level in the group ranking.

Year	E1	E2	E3	E4	E5	E6	confidence index	trust level
2016	0.2777	0.2583	0.1197	0.2354	0.0517	0.0573	0.9674	0.9780
2017	0.0537	0.0401	0.4187	0.4336	0.0269	0.0269	0.9471	0.9815
2018	0.0087	0.0079	0.1001	0.2528	0.3019	0.3285	0.9554	0.9846
2019	0.2077	0.1490	0.3054	0.3081	0.0108	0.0189	0.9549	0.9811
2020	0.0315	0.0315	0.1902	0.2889	0.1801	0.2777	0.9673	0.9838

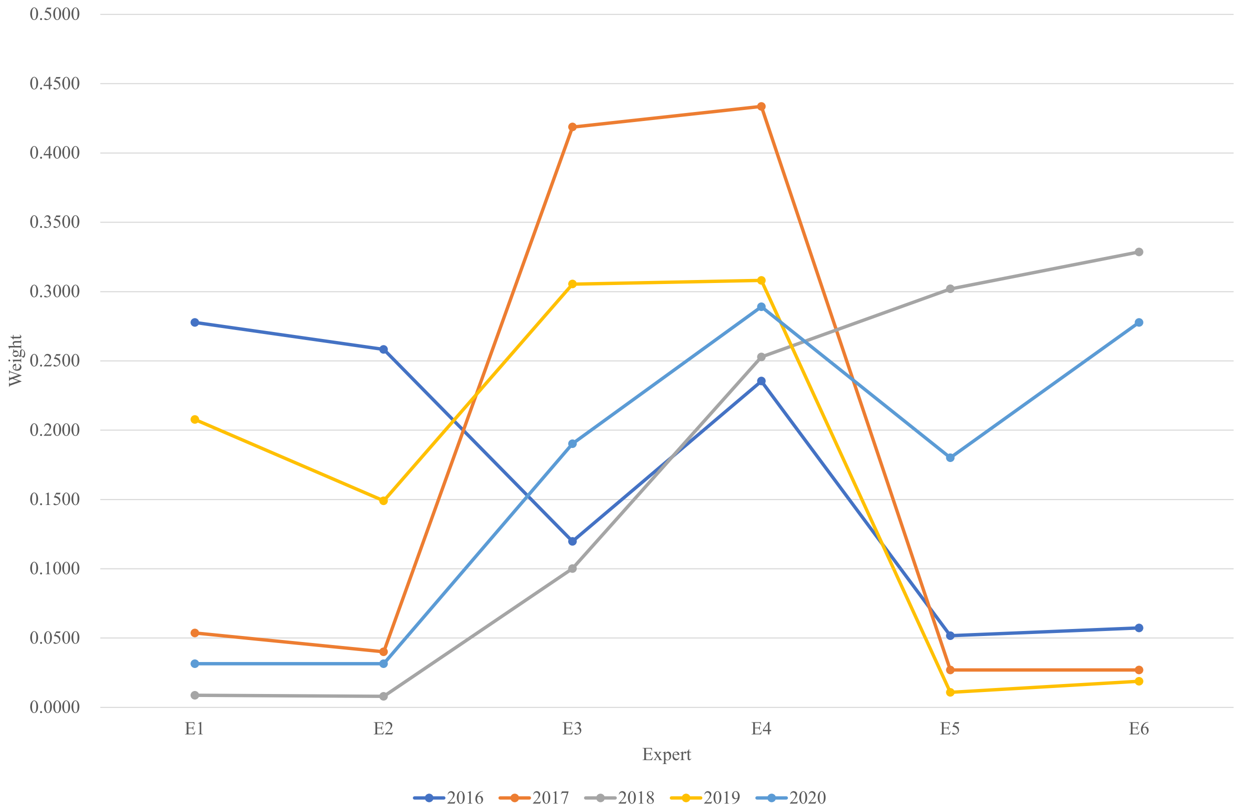

Fig. 4. The weights aggregating the expert-specific rankings.

the largest weight to E4, while 2016 and 2018 believe E1 and E5 are the most important, respectively. This is mainly attributed to the data-driven weighting nature of the proposed group ranking method, and thereby makes the group ranking more flexible.

We also compute the average group rankings for each province in China between 2016 and 2020, which are shown in Fig. 5 below. Smaller average group ranking value in particular represent better DFI performance. By setting the interval of these values as 9.8, 31 provinces are categorized into 3 grades. The first-class performers with average group ranking interval $[1.2, 11]$ include 9 provinces: Anhui, Beijing, Fujian, Guangdong, Hubei, Jiangsu, Shanghai, Tianjin, Zhejiang; the second-class performers with group ranking interval $[11, 20.8]$ include 13 provinces: Chongqing, Guangxi, Guizhou, Hainan, Henan, Hunan, Inner Mongolia, Jiangxi, Liaoning, Shaanxi, Shandong, Shanxi, Sichuan; the third-class performers with group ranking interval $[20.8, 30.6]$ include 9 provinces: Gansu, Hebei, Helongjiang, Jilin, Ningxia, Tibet, Xinjiang, Yunnan. Such a classification is consistent with the provincial digitalization levels and economic development status in China. In other words, the developed provinces usually have better DFI performance.

To facilitate the spatial comparison and analysis with respect to DFI performance in China, 31 provinces are grouped into 7 geographic regions (see Table 8). We calculate the regional average group rankings for each year, which are reported in Table 9 and Fig. 6. The DFI performance exhibit remarkable regional differences. We find that although the rankings among these regions change across years, East China always takes the leading position with respect to the DFI performance in China, while Northwest China and Southwest China typically have the inferior performance. These regional differences are mainly resulted from the differences of provincial DFI performance.

The results pose two-fold important policy implications for advancing the DFI in China. On the one hand, the DFI development in China remains unbalanced, in accordance with both the provincial and geographically regional comparisons. The leading provinces are mainly distributed in the Yangtze River delta region, Pearl River Delta region, and Beijing-Tianjin-Hebei region, which are the

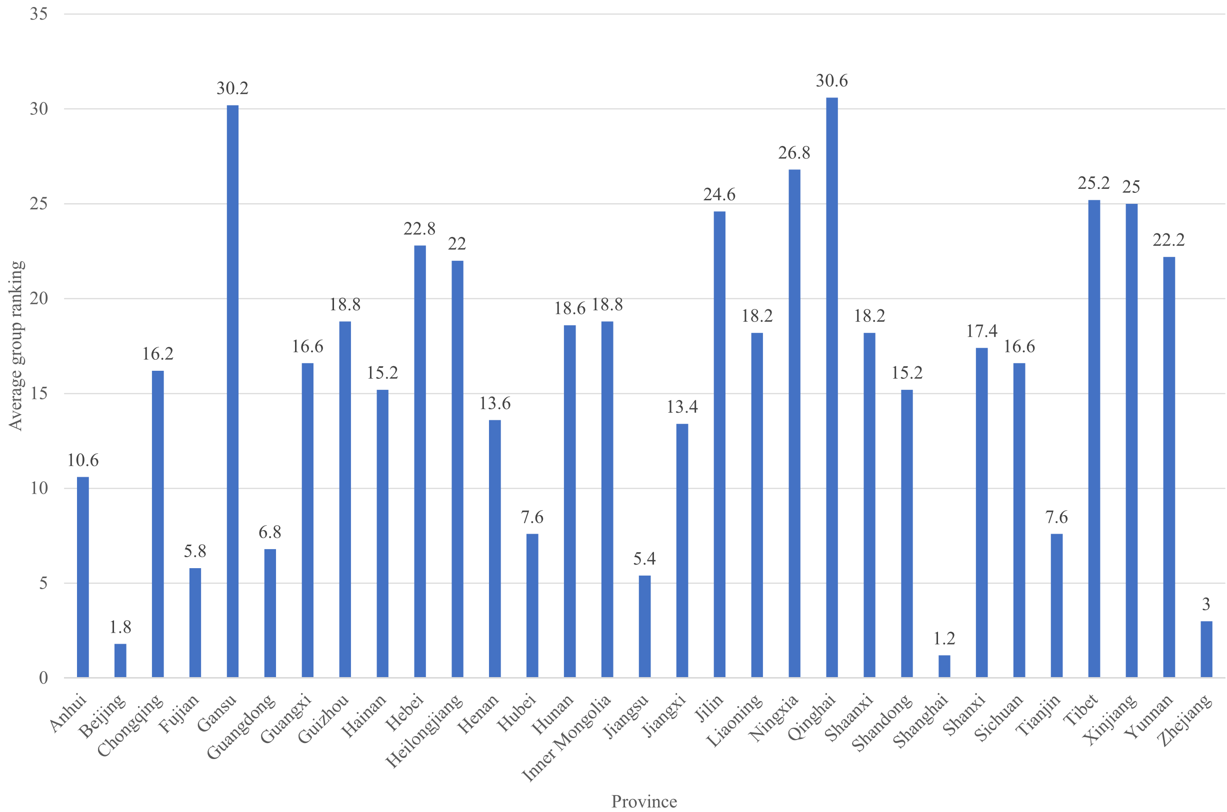

Fig. 5. Average group rankings for each province in China between 2016 and 2020.

Table 8
Seven regions in China.

Region	Province
North China	Beijing, Hebei, Inner Mongolia, Shanxi, Tianjin
Northeast China	Heilongjiang, Jilin, Liaoning
East China	Anhui, Fujian, Jiangsu, Jiangxi, Shandong, Shanghai, Zhejiang
Central China	Henan, Hubei, Hunan
South China	Guangdong, Guangxi, Hainan
Southwest China	Chongqing, Guizhou, Sichuan, Yunnan
Northwest China	Gansu, Ningxia, Qinghai, Shaanxi, Xinjiang

Table 9
Regional average group rankings in China.

Region	2016	2017	2018	2019	2020
North China	10.60	11.40	17.00	14.00	15.40
Northeast China	18.33	22.33	22.00	22.67	22.67
East China	8.57	8.57	6.29	9.14	6.43
Central China	18.67	11.33	12.00	14.33	10.00
South China	10.67	13.00	11.33	15.67	13.67
Southwest China	20.40	20.00	19.40	18.40	20.80
Northwest China	27.60	27.80	26.80	22.40	26.20

economically developed areas in China. This reflects that the DFI performance is in a sense coupled with the economic development level. Therefore, policymakers should strength the implementations of DFI initiatives to promote the DFI performance and alleviate the uneven status in China, with the ultimate purpose of boosting the economic development and enhancing the social well-beings. On the other hand, the DFI performance captures not only the values of breadth of coverage, depth of use and level of digitalization, but also the endogenous preferences among them. Policymakers should better trade-off the performance among the three dimensions of DFI, especially for the laggards with the goal of catching up with the leaders while coping with the preference diversities among different

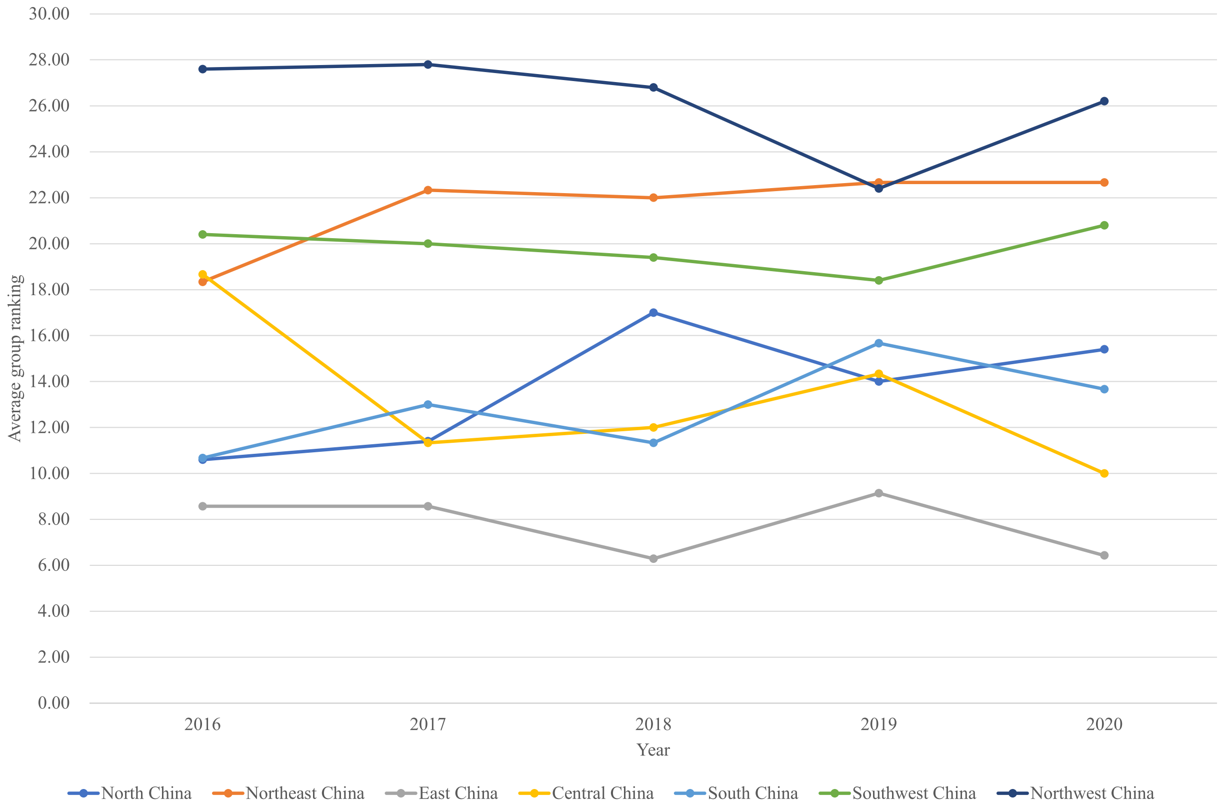

Fig. 6. Regional average group rankings in China.

experts.

5. Concluding remarks and future research

This study measures and compares the provincial DFI performance in China under the framework of the PKU-DFIIC developed by the Institute of Digital Finance at Peking University and Ant Financial Services Group. A group ranking method with preference analysis is developed. Specifically, the importance orders among breadth of coverage, depth of use and level of digitalization are comprehensively considered to identify the experts for formulating a preliminary group decision matrix. The preference structure of this group decision matrix is analyzed from two aspects, namely, preferential differences and preferential priorities. We thereby obtain a revised group decision matrix, the elements of which are the expert-specific rankings about the alternatives. A group ranking method is lastly proposed based on the half-quadratic theory, where the weights with respect to each expert are estimated in an objective manner for alleviating the decision bias. In addition, consensus index and trust level are constructed to manifest the merits of the proposed group ranking method. An empirical study using the provincial DFI data between 2016 and 2020 is carried out to validate our method, in conjunction with spatial-temporal analysis and meaningful insights. Two-fold policy implications are suggested to promote the DFI development in China.

Our DFI measurement method has several remarkable advantages over the previous measures. First, instead of relying on a single expert like the PKU-DFIIC and Khera et al. (2022)'s comprehensive DFI index, consolidating the opinions from a panel of experts allows us to capture the collective wisdom from a group decision-making perspective. Second, the data-driven rationale of our method reduces the decision bias and enhances the result objectiveness and flexibility, while the existing measures requires the subjective information to elicit the weights associated with the dimensions of DFI. Third, the preference analysis and group ranking views allow for more detailed understanding of the DFI performance. This would help inform policymakers in promoting the DFI in an appropriate manner. To the best of our knowledge, this work is among the first to measure the DFI performance under a group-decision making framework.

This research can be further extended in several directions to address other open questions regarding DFI performance evaluation. First, this paper demonstrates the research opportunities to investigate different M-estimators and their corresponding minimizer functions in aggregating the group ranking, which could be possible future research avenues. Second, our work considers the deterministic scenario, in which the expert-specific evaluation results are precisely obtained. It will be meaningful to explore the uncertain assessment results when the experts have unknown or imprecise preferences. Thirdly, the time-consuming in conducting the calculation of our method increases exponentially with respect to the number of dimensions, future research should be aware of the performance evaluation with more dimensions. Finally, it will be interesting to integrate the empirical tests with our computational results.

Data availability

The authors do not have permission to share data.

Acknowledgments

We would like to express our gratitude to the Editor-in-Chief, Executive Editor and the anonymous review team for helpful comments on earlier drafts of this paper. This research is financially supported by the National Natural Science Foundation of China (Nos. 72101098, 72171154 & 72222019), Guangdong Basic and Applied Basic Research Foundation (No. 2022A1515010232).

References

Aledo, J. A., Gámez, J. A., & Molina, D. (2016). Using extension sets to aggregate partial rankings in a flexible setting. *Applied Mathematics and Computation, 290*, 208–223.
Aledo, J. A., Gámez, J. A., & Rosete, A. (2018). Approaching rank aggregation problems by using evolution strategies: The case of the optimal bucket order problem. *European Journal of Operational Research, 270*, 982–998.
Aledo, J. A., Gámez, J. A., & Rosete, A. (2021). A highly scalable algorithm for weak rankings aggregation. *Information Sciences, 570*, 144–171.
Beck, T., Demirguc-Kunt, A., & Peria, M. S. M. (2007). Reaching out: Access to and use of banking services across countries. *Journal of Financial Economics, 85*, 234–266.
Bustince, H., Bedregal, B., Campion, M., Da Silva, I., Fernandez, J., Indurain, E., … Santiago, R. (2022). Aggregation of individual rankings through fusion functions: Criticism and optimality analysis. *IEEE Transactions on Fuzzy Systems, 30*, 638–648.
Cámara, N., & Tuesta, D. (2014). *Measuring financial inclusion: A muldimensional index*. BBVA Research Paper.
Cook, W. D., & Kress, M. (1990). A data envelopment model for aggregating preference rankings. *Management Science, 36*, 1302–1310.
Demirgüç-Kunt, A., & Klapper, L. F. (2012). *Measuring financial inclusion: The global findex database*. World Bank Policy Research Working Paper.
Demirgüç-Kunt, A., Klapper, L. F., Singer, D., & Van Oudheusden, P. (2015). *The global findex database 2014: Measuring financial inclusion around the world*. World Bank Policy Research Working Paper.
Demšar, J. (2006). Statistical comparisons of classifiers over multiple data sets. *The Journal of Machine Learning Research, 7*, 1–30.
Dyer, J. S., & Sarin, R. K. (1979). Group preference aggregation rules based on strength of preference. *Management Science, 25*, 822–832.
Fu, Y., Lai, K. K., & Yu, L. (2021). Multi-nation comparisons of energy architecture performance: A group decision-making method with preference structure and acceptability analysis. *Energy Economics, 96*, Article 105139.
Fu, Y., Lu, Y., Yu, C., & Lai, K. K. (2022). Inter-country comparisons of energy system performance with the energy trilemma index: An ensemble ranking methodology based on the half-quadratic theory. *Energy, 261*(Part A), 125048.
Geman, D., & Reynolds, G. (1992). Constrained restoration and the recovery of discontinuities. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 14*, 367–383.
Geman, D., & Yang, C. (1995). Nonlinear image recovery with half-quadratic regularization. *IEEE Transactions on Image Processing, 4*, 932–946.
González-Pachón, J., & Romero, C. (2001). Aggregation of partial ordinal rankings: An interval goal programming approach. *Computers & Operations Research, 28*, 827–834.
Guo, F., Wang, J., Wang, F., Kong, T., Zhang, X., & Cheng, Z. (2019). *The peking university digital financial inclusion index of China (2011-2018)* (pp. 1–70). Working paper, Institute of Digital Finance, Peking University.
Gupta, S., & Kanungo, R. P. (2022). Financial inclusion through digitalisation: Economic viability for the bottom of the pyramid (BOP) segment. *Journal of Business Research, 148*, 262–276.
Gutiérrez-Romero, R., & Ahamed, M. (2021). COVID-19 response needs to broaden financial inclusion to curb the rise in poverty. *World Development, 138*, Article 105229.
He, R., Zhang, Y., Sun, Z., & Yin, Q. (2015). Robust subspace clustering with complex noise. *IEEE Transactions on Image Processing, 24*, 4001–4013.
He, R., Zheng, W.-S., Tan, T., & Sun, Z. (2014). Half-quadratic-based iterative minimization for robust sparse representation. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 36*, 261–275.
Hochbaum, D. S., & Levin, A. (2006). Methodologies and algorithms for group-rankings decision. *Management Science, 52*, 1394–1408.
Honohan, P. (2008). Cross-country variation in household access to financial services. *Journal of Banking & Finance, 32*, 2493–2500.
Huang, Y., Ito, T., Iwata, K., McKenzie, C., & Urata, S. (2022). Digital finance in Asia: Editors' overview. *Asian Economic Policy Review, 17*, 163–182.
Huang, Y.-S., Chang, W.-C., Li, W.-H., & Lin, Z.-L. (2013). Aggregation of utility-based individual preferences for group decision-making. *European Journal of Operational Research, 229*, 462–469.
Huang, Y.-S., & Li, W.-H. (2012). A study on aggregation of TOPSIS ideal solutions for group decision-making. *Group Decision and Negotiation, 21*, 461–473.
Huber, P. J. (2004). *Robust statistics. 523*. John Wiley & Sons.
Kanungo, R. P., & Gupta, S. (2021). Financial inclusion through digitalisation of services for well-being. *Technological Forecasting and Social Change, 167*, Article 120721.
Khera, P., Ng, S., Ogawa, S., & Sahay, R. (2022). Measuring digital financial inclusion in emerging market and developing economies: A new index. *Asian Economic Policy Review, 17*, 213–230.
Lin, A., Peng, Y., & Wu, X. (2022). Digital finance and investment of micro and small enterprises: Evidence from China. *China Economic Review, 75*, Article 101846.
Lin, B., & Ma, R. (2022). How does digital finance influence green technology innovation in China? Evidence from the financing constraints perspective. *Journal of Environmental Management, 320*, Article 115833.
Liu, Y., Luan, L., Wu, W., Zhang, Z., & Hsu, Y. (2021). Can digital financial inclusion promote china's economic growth? *International Review of Financial Analysis, 78*, Article 101889.
Ma, L.-C. (2016). A new group ranking approach for ordinal preferences based on group maximum consensus sequences. *European Journal of Operational Research, 251*, 171–181.
Mannes, A. E. (2009). Are we wise about the wisdom of crowds? The use of group judgments in belief revision. *Management Science, 55*, 1267–1279.
Matsatsinis, N., Grigoroudis, E., & Samaras, A. (2005). Aggregation and disaggregation of preferences for collective decision-making. *Group Decision and Negotiation, 14*, 217–232.
Mohammadi, M., & Rezaei, J. (2020). Ensemble ranking: Aggregation of rankings produced by different multi-criteria decision-making methods. *Omega, 96*, Article 102254.
Moreno-Centeno, E., & Escobedo, A. R. (2016). Axiomatic aggregation of incomplete rankings. *IIE Transactions, 48*, 475–488.
Nikolova, M., & Ng, M. K. (2005). Analysis of half-quadratic minimization methods for signal and image recovery. *SIAM Journal on Scientific Computing, 27*, 937–966.

Ozturk, I., & Ullah, S. (2022). Does digital financial inclusion matter for economic growth and environmental sustainability in OBRI economies? An empirical analysis. *Resources, Conservation and Recycling, 185*, Article 106489.
Pesqué-Cela, V., Tian, L., Luo, D., Tobin, D., & Kling, G. (2021). Defining and measuring financial inclusion: A systematic review and confirmatory factor analysis. *Journal of International Development, 33*, 316–341.
Sawada, Y. (2022). Comment on "measuring digital financial inclusion in emerging market and developing economies: A new index". *Asian Economic Policy Review, 17*, 233–234.
Sen, A. (1977). Social choice theory: A re-examination. *Econometrica, 45*, 53–88.
Sen, A. (1999). The possibility of social choice. *American Economic Review, 89*, 349–378.
Song, L., Fu, Y., Zhou, P., & Lai, K. K. (2017). Measuring national energy performance via energy trilemma index: A stochastic multicriteria acceptability analysis. *Energy Economics, 66*, 313–319.
Tram, T. X. H., Lai, T. D., & Nguyen, T. T. H. (2021). Constructing a composite financial inclusion index for developing economies. *The Quarterly Review of Economics and Finance*. https://doi.org/10.1016/j.qref.2021.01.003
Wang, X., & Fu, Y. (2022). Digital financial inclusion and vulnerability to poverty: Evidence from Chinese rural households. *China Agricultural Economic Review, 14*, 64–83.
Wang, X., Wang, X., Ren, X., & Wen, F. (2022). Can digital financial inclusion affect CO2 emissions of China at the prefecture level? Evidence from a spatial econometric approach. *Energy Economics, 109*, Article 105966.
Yoo, Y., & Escobedo, A. R. (2021). A new binary programming formulation and social choice property for kemeny rank aggregation. *Decision Analysis, 18*, 296–320.
Yoo, Y., Escobedo, A. R., & Skolfield, J. K. (2020). A new correlation coefficient for comparing and aggregating non-strict and incomplete rankings. *European Journal of Operational Research, 285*, 1025–1041.