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Research paper

The sustainable hybrid truck-drone delivery model with stochastic customer existence

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ABSTRACT

Drone delivery is a fast delivery mode that has gained tremendous attention from academia and various companies in recent years. However, due to limited battery and payload capacities which may reduce the system's efficiency, it is better to coordinate ground vehicles and drones to take advantage of both trucks' large capacity and the drone's high speed. As a restriction for realistic parcel delivery systems, customer presence is only sometimes deterministic. For instance, a customer makes an order from an e-retailer, but due to various probable reasons, he cannot be present at home to get service. In this paper, we have introduced a sustainable hybrid truck-drone delivery model with stochastic customer presence. We have modeled the system with the Markov chain and proposed a linear mathematical model. This work processes with a heuristic and a Branch-and-Bound algorithm. Also, we have carried out numerous computational experiments to evaluate the proposed solution methods' performance, where the results show the efficiency of the proposed algorithms. Finally, we performed a detailed sensitivity analysis on a case study and studied various aspects of the problem. The results highlight that truck and drone coordination reduces completion time, operational costs, truck emissions, and social penalties.

1. Introduction

In recent years, with the increase in E-commerce and door-to-door popularity, customers' expectations for fast and high service quality, and various urban restrictions, including narrow streets and dense traffic, have urged city logistics to implement innovative delivery approaches. As an innovative approach, logistics providers proposed using unmanned aerial vehicles (UAVs), also called drones, for delivery tasks. For example, Amazon started a drone delivery system in 2016, Walmart announced its experimental drone delivery service, and delivery companies FedEx and UPS have begun drone delivery (Liu et al., 2022).

UAVs produce less air pollution than conventional road vehicles (Asadi et al., 2022). It does not need road infrastructures, and dense traffic does not reduce its efficiency. UAVs travel direct distances and provide high travel speeds (Moshref-Javadi & Winkenbach, 2021). Although this mode of transportation can increase service quality (Luo et al., 2021), UAVs have some restrictions, including limited battery and payload capacities. For that matter, it is better to use hybrid truck-drone delivery systems (Wang et al., 2019).

Truck and drone coordination and developing decision-making models in a real-world environment face many challenges (Ghelichi et al., 2022). For example, Chen et al. (Chen et al., 2021) mentioned security and safety as the most significant issues in drone delivery. It is also contrary to city logistics ideals to park a truck at the customer location for operating the drone, which may result in double park. Taking mentioned issues into account, we worked on the hybrid truck-drone routing problem with sight radius and rendezvous locations. In this paper, we have two major questions. First, what are the impacts of truck and drone coordination on sustainability dimensions? Second, as customer presence uncertainty is unavoidable in city logistics, is considering stochastic customer existence in routing problem improves the systems' efficiency? To answer these questions, this paper proposes a sustainable hybrid truck-drone delivery model with stochastic customer existence and aims to analyze the sustainability dimensions of truck and drone coordination and the effects of customer presence uncertainty on the truck and drone delivery system.

Fig. 1 depicts our proposed delivery system. In this study, parcels are loaded on a truck equipped with a drone and transferred to customer or rendezvous locations. The truck acts as a mobile depot and carries the drone while transporting between each pair of locations, and the drone cannot travel directly from/to the depot. In this delivery system, the first-class customers only get service with a truck, the second-class

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customers can get service by truck or drone, and the third-class customers get service only with a drone. For servicing second-class and third-class customers with the drone, the truck stops at a rendezvous location, and from there, the drone makes various deliveries while the truck is stationary. In this paper, we considered a recourse strategy: if a scheduled customer has no order, the system ignores that and services the next scheduled customer.

In so many delivery systems, when the decision maker is deciding on customers' schedules, customer presence status is not revealed, but he may be informed of the customer presence status before servicing that. As shown in Fig. 1, when the delivery system is not informed about the customer presence status, the hybrid truck-drone system must transport to all of the customer locations which increases the delivery time, operational costs, emissions, and social penalty. On the contrary, when a customer is not existing and the system is informed before the truck or drone transport to the customer's location, the system ignores the non-existed customer and services the next customer.

The main contributions of this study can be summarized as follows.

- We have considered stochastic customer presence and proposed a linear mathematical model for the sustainable hybrid truck-drone routing problem.
- We have proposed an effective Branch-and-bound and a heuristic Branch-and-Bound algorithm and analyzed the proposed delivery system with real case data.

The remainder of this paper is as follows. Section *2* represents an overview of the literature. Section *3* prepares a formal system description and a linear mathematical model for the problem. Also, we have described our proposed heuristic Branch-and-Bound algorithm and the Branch-and-Bound algorithm in Section *4*. Section *5* illustrates computational results, and Section *6* concludes our study.

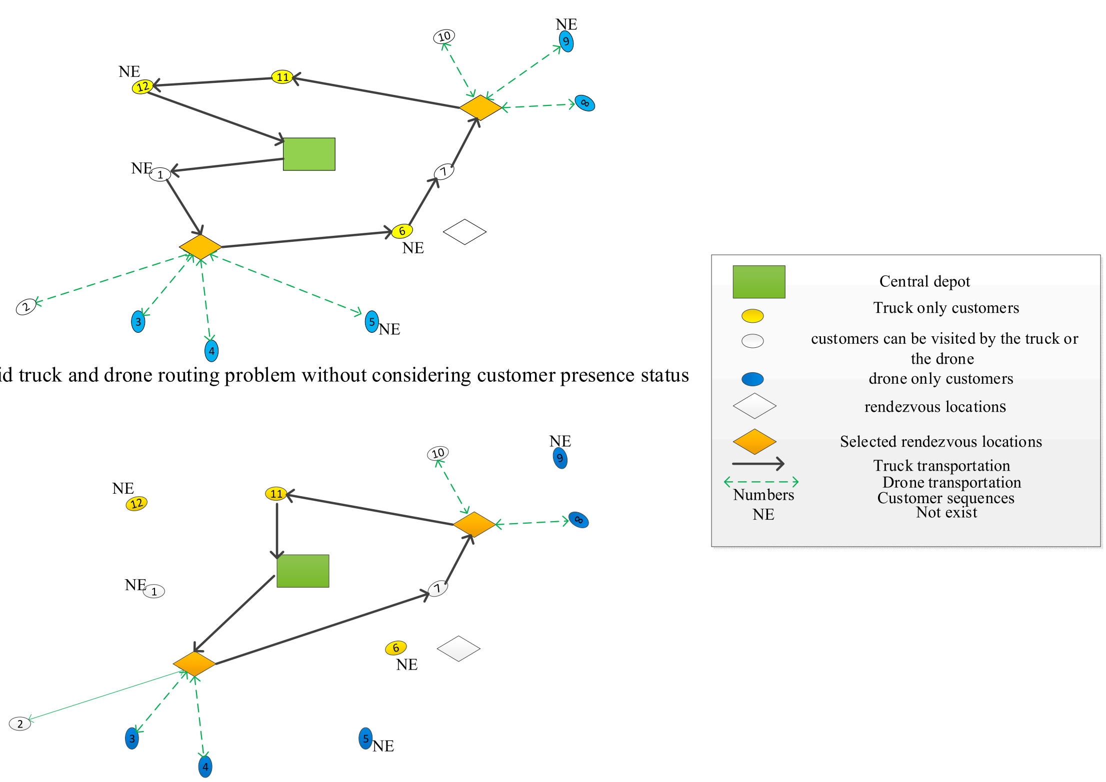
2. Related works

2.1. Truck-drone routing problem

Murray and Chu (Murray & Chu, 2015) were the first researchers that suggested deploying a truck and a drone and proposed the flying sidekick traveling salesman problem (*FSTSP*). In this model, the drone can deliver only one parcel per delivery, and a maximum traveling distance restriction exists for each drone delivery route. Both delivery vehicles depart from and return to the depot only once, where the drone can be transported by truck or independently. Freitas and Penna (Freitas & Penna, 2019) considered the same problem and proposed an effective general variable neighborhood search algorithm to solve the *FTSP*.

Murray and Raj (Murray & Raj, 2020) coordinated a truck with a fleet of heterogeneous drones and suggested the multiple flying sidekicks traveling salesman problem (*mFSTSP*). In this study, the authors considered the completion time objective and proposed a three-phased iterative heuristic. Raj and Murray (Raj & Murray, 2020) considered the tradeoff between the speed and flying range of the drone and assumed that higher speeds of the drone reduce the flying range and consequently limit the efficiency of the delivery system. They considered a decision variable for the drone's speed and extended the multiple flying sidekicks traveling salesman problems with variable drone speeds (*mFSTSP-VDS*). The authors focused on the completion time objective and suggested a three-phased iterative heuristic, and indicated that optimizing drone speed reduces truck travel distances, drone power consumption, and drone loitering in rendezvous locations.

Jeong et al. (Jeong et al., 2019) assumed that due to weather conditions or the sensitivity of various facilities, the drone is restricted to flying in specific areas. They also considered the impact of parcel weights on the drone energy consumption and proposed the flying sidekick traveling salesman problem with energy consumption and no-fly zones (*FTSP-ECNZ*). In this study, the authors focused on the completion time and proposed a two-phased heuristic to solve the problem.



a- Hybrid truck and drone routing problem without considering customer presence status

b- Hybrid truck and drone routing problem when the carrier is informed about customer presence status before servicing that

Fig. 1. The proposed delivery system.

Bouman et al. (Bouman et al., 2018) extended the traveling salesman problem with drones (*TSP-D*). In this problem, the drone cannot travel directly from/to the depot in this delivery system. The authors suggested a dynamic programming approach to solve the problem. Tu et al. (Tu et al., 2018) considered a delivery system where the truck is equipped with m drones and proposed the traveling salesman problem with multiple drones (*TSP-mD*). They focused on the operational costs objective and proposed an adaptive large neighborhood search algorithm to solve the problem. Poikonen et al. (Poikonen et al., 2019) also worked on the *TSP-D*. They suggested a dynamic programming approach to find a lower bound and proposed a new branch-and-bound algorithm to solve the problem.

Moshref-Javadi et al. (Moshref-Javadi et al., 2020) assumed a delivery system with a truck and multiple drones, where the truck stops at customer locations and drones can be launched multiple times at each truck stop. They focused on minimizing customer waiting time and extended the traveling repairman problem (*TRP*). The authors suggested a hybrid metaheuristic based on Simulated Annealing and Tabu Search. Lue et al. (Luo et al., 2021) considered multiple homogenous drones and the multi-visit capability of drones and proposed the multi-visit traveling salesman problem with multi-drones (*MTSP-MD*), They focused on the time required for the system to service all of the customers and proposed a multi-start tabu search algorithm to solve the problem.

Wang et al. (Wang et al., 2019) suggested a delivery system that employs independent trucks, trucks that are equipped with a drone, and independent drones. The authors proposed a three-stage-heuristic algorithm consisting of problem preprocessing, flight segment construction, and global routes construction.

Another extension of the hybrid truck-drone routing problem is the vehicle routing problem with drones which considers a delivery system with multiple trucks equipped with one or multiple drones. Sacramento et al. (Sacramento et al., 2019) considered a fleet of homogenous vehicles where each of them collaborated with a single drone. The authors focused on the operational costs objective function and proposed an adaptive large neighborhood search to solve the problem. Schermer et al. (Schermer et al., 2019) assumed a fleet of homogenous trucks where each of them is equipped with the same number of homogenous drones. They focused on the completion time objective function and proposed a hierarchical heuristic that allocates customers to the truck routes, then assigns them to the drone, and finally defines customers' sequences. Chen et al. (Chen et al., 2021) assumed a fleet of homogenous trucks where each of them collaborated with several self-driving drones. They assumed the dispatch-wait-collect system. In this system, all of the drones are collected at the same location where they are departing. They focused on the delivery time objective function and considered a time window for delivery nodes. The authors proposed an adaptive large neighborhood search to solve the problem.

As another extension for the coordination of ground vehicles and UAVs, Luo et al. (Luo et al., 2017) considered a delivery system with a truck and a drone where the drone can be recycled and launched only at rendezvous nodes. The authors assumed that the rendezvous locations have enough space for parking a truck and operating a drone. They considered the total routing time and proposed two constructive heuristics. Karak and Abdelghany (Karak & Abdelghany, 2019) proposed a two-echelon truck and drone routing problem, where the truck acts as a mobile depot and drones service the customers. The authors assumed that the truck can stop only at specific stations which may be different from the customer locations. They considered the operational costs objective function and proposed three heuristics based on Clarke and Wright algorithm.

Since rendezvous locations are not limited to customer locations, models that incorporate rendezvous locations may prepare better solutions than models that allow the truck to stop only at customer locations. Only a few studies have considered rendezvous locations in the literature. Considering only one drone departure per truck stop is an unrealistic restriction. Nevertheless, only a limited number of studies have addressed the possibility of multiple drone departures per truck stop. In addition, most studies have focused on either completion time or total delivery time dimensions, and sustainable models are scarce. No study considers all sustainability factors (economic, environmental, and social) as one decision criterion. Thus, this study considers rendezvous locations and multiple drone departures per truck stop and presents a weighted multi-objective model for the sustainable hybrid truck-drone routing problem.

2.2. Stochastic vehicle routing problem

In the stochastic *VRP* (*SVRP*), one or some of the parameters are uncertain, and the decision maker must decide on the solution before all values of stochastic parameters are determined (Oyola et al., 2018). For modeling *SVRP*, some studies considered chance-constrained programming (Basso et al., 2021; Karimi & Setak, 2018), and some focused on the stochastic program with recourse (Noorizadegan & Chen, 2018).

Goodarzi et al. (Goodarzi et al., 2022) assumed stochastic truck arrival time and proposed an $M/M/c$ queue model for VRP in a cross-dock environment. Latorre-Biel et al. (Latorre-Biel et al., 2021) considered stochastic and correlated demand and proposed a semi-heuristic approach combined with a demand predictor to solve the problem. Florio et al. (Florio et al., 2021) focused on the *VRP* with stochastic demand and stochastic route duration constraint and proposed a branch and price algorithm. Zhang et al. (Zhang et al., 2021) focused on the *VRP* with uncertain deadlines, where the probability of serving customers before their related deadlines must be greater than a pre-defined value. They proposed a robust optimization framework to study the worst bounds of the problem. Ji et al. (Ji et al., 2020) considered a two-echelon inventory routing problem with uncertain demand and proposed a robust mathematical model.

Recently, some studies paid attention to stochastic drone delivery. For example, Asadi et al. (Asadi et al., 2022) worked on the drone delivery of medical inventories. They assumed that drones dispatch from a drone hub and deliver medical inventories to hospitals. The authors assumed stochastic demand for each hospital and proposed a Markov decision process model. Liu et al. (Liu et al., 2022) considered stochastic travel time and extended the *FSTSP* with stochastic travel time. They formulated the problem into a Markov decision process and used reinforcement learning algorithms to solve the problem. Ghelichi et al. (Ghelichi et al., 2022) considered demand location uncertainty during disasters and developed a chance-constrained programming formulation to find the optimal location of drone take-off platforms; using a decomposition method, they proposed a three-stage heuristic algorithm. Sawadsitang et al. (Sawadsitang et al., 2019) Considered uncertain conditions for the drone take-off and breakdown during the delivery process and proposed a three-stage planning framework for truck and drone routing problems.

The use of stochastic models has been limited in the literature on the truck-drone routing problem. To the extent of our knowledge, no study has addressed stochastic customer presence. For this reason, this paper considers stochastic customer existence and proposes a linear formulation for the truck and drone routing problem with stochastic customer existence.

2.3. Exact solution methods

The truck and drone routing problem is known as an NP-hard problem (Jeong et al., 2019). For this reason, exact solution methods still need to be discovered, and most studies focus on heuristic and metaheuristic algorithms. We are aware of some studies on this topic. Tamke and Buscher (Tamke & Buscher, 2021) considered the vehicle routing problem with drones and proposed a branch-and-cut algorithm. Cavani et al. (Cavani et al., 2021) focused on the traveling salesman problem with multiple drones and proposed a compact Mixed-Integer Linear Programming. The authors considered the completion time objective function and proposed a decomposition method based on compact Mixed-Integer Linear Programming and a branch-and-cut algorithm. Dell'Amico et al. (Dell'Amico et al., 2021) considered the *FSTSP* and proposed a branch-and-bound algorithm for small-sized instances and an iterative heuristic algorithm for large-sized instances. For the parallel drone scheduling multiple traveling salesman problem, Saleu et al. (Saleu et al., 2022) proposed a branch-and-cut algorithm. In this problem, parcel deliveries divide among trucks and drones, where each truck completes the service in a classical tour. In contrast, each drone prepares service in multiple back-and-forth trips. Najy et al. (Najy et al., 2022) considered a coloration of a truck and a drone and proposed the inventory routing problem with drone. The authors considered holding costs and operational costs and proposed a branch-and-cut algorithm to solve the problem.

Although there are a few studies that paid attention to the exact solution methods there is no exact algorithm that fits our problem. This paper considers non-linear formulations of stochastic customer existence, multiple drone departures per truck stop, and sustainable dimensions. So, to prepare high-quality solutions, this paper proposes a new branch-and-bound algorithm and a new heuristic branch-and-bound algorithm.

2.4. Research gap

Table 1 briefly compares studies focused on coordinating trucks and drones. In the truck-drone routing problem literature, sustainable models are scarce, and no study considers all sustainability factors (economic, environmental, and social) as one decision criterion. Although some studies considered stochastic environments, no study addressed stochastic customer presence in truck and drone routing problems. As a result, this paper proposes a weighted multi-objective model for the sustainable hybrid truck-drone routing problem with stochastic.

3. Formal problem description and formulations

This section defines a sustainable hybrid truck-drone delivery model with stochastic customer existence. In this system, we equipped the truck with a drone and loaded parcels on the truck at the depot location. It transfers parcels to the customer or some selected rendezvous locations. The drone delivers parcels to the customers when the truck stops at a rendezvous location. After servicing all customers, the truck turns back to the depot. Assumptions about the proposed delivery system are as follows.

- Only one truck that is equipped with a drone is considered.
- Only delivery operations are considered.
- All of the customers can only be visited once.
- There is a sight radius for the drone operation.
- There are three kinds of customers. Since we cannot deliver heavy parcels with a drone, first-class customers must be serviced only with a truck. Also, we can service Second-class customers with a truck or a drone. Because of narrow streets, third-class customers must be serviced only with the drone.
- The drone can be operated only from rendezvous locations. Each rendezvous point represents a parking location with sufficient space for truck parking and drone operation.
- Due to city logistics ideals, customer locations differ from rendezvous locations.
- The drone only delivers one parcel per trip. But by replacing and charging its batteries it can serve all second-class and third-class customers.
- The truck has enough capacity to serve all the demands.
- Each customer exists with the probability of p.
- This delivery system uses a recourse strategy. If a customer does not exist at a defined delivery point in a scheduled plan, the system ignores that and services the next scheduled customer.
- Parking the truck in customer locations may result in double park and increase traffic, reducing city livability. For this reason, a social penalty is considered for parking the truck in the customer locations.
- As an environmental factor, we considered truck emissions and assumed that the overall emission of using a drone is negligible.
- After a trip, whenever a drone returns to the truck, its battery will be replaced with a fully-charged battery. Battery replacement time is assumed to be negligible.
- The truck and the drone travel at constant speeds.
- The impacts of parcel weights on drone speed and energy conservation are not considered.
- The truck driver operates the drone.
- This paper considers depreciation costs, taxes, and fuel costs for the truck and drone operational costs.

The problem defines by $G = (N, A)$ as a directed graph, where A is the set of arcs and N is the set of various locations, including depot locations, customer locations, and rendezvous locations.

3.1. Stochastic routing formulations

As mentioned before, the system will discard customer locations with no demand from servicing plan, and the delivery system services the next scheduled customer. In this situation, if another customer is assigned to the current truck stop, the drone services that; otherwise, the truck travels to the following scheduled truck stop. In this graph, the expected value of the truck traveling distance equals $\sum_{k,f} \overline{p_{kf}}.d_{kf}$, where d_{kf} is the distance and $\overline{p_{kf}}$ is the probability for the truck to travel from k to f.

In the next step, we should calculate $\overline{p_{kf}}$. In this directed graph, it is clear that the probability for the truck to travel an arc from k to f is not dependent on how the truck arrives at node k, and we can calculate it as follows:

$$\overline{p_{kf}} = \left(1-(1-p)^{N_k}\right).\left(1-(1-p)^{N_f}\right).\left(\prod_{\forall h \in SR_{kf}} (1-p)^{N_h}\right) \tag{1}$$

Where N_k is the number of customers assigned to truck stop k and SR_{kf} is the set of all truck stops selected for servicing between nodes k and f. As N_k and the sequence of truck stops are variables, Formula *(1)* is nonlinear.

Lemma 1. *we can redefine Formula (1) based on customer sequences in Formula (2).*

$$\overline{p_{kf}} = \sum_{j>i} p^2 (1-p)^{j-i-1}.U_{k,i}.U_{f,j} \tag{2}$$

If a customer is assigned to the truck stop k at sequence i, $U_{k,i}$ will be a binary variable that gets value 1. The probability for the truck to visit the depot at the sequence *(1)* and N is equal to *1*, so similar to Formula *2*, the probability for the truck to travel between depot and truck stop f is equal to Formulas (3 and 4):

$$\overline{p_{depot,f}} = \sum_{j>1} p(1-p)^{j-1-1}.U_{f,j} \tag{3}$$

$$\overline{p_{f,depot}} = \sum_{j>1} p(1-p)^{N-j-1}.U_{f,j} \tag{4}$$

See Appendix *A* for the proof of Lemma 1.

3.2. Mathematical model formulation

In this section, we propose a *MILP* model for the problem. The objective components are completion time, overall costs, emissions, and social penalty for parking the truck in second-class customer locations.

Table 1
Literature analysis of truck and drone routing problem.

Reference	Vehicle characteristics			Environment		Objective function	Solution method
	Drone departure location		Multiple drone departure per truck stop	Deterministic	Stochastic		
	Only customer locations	Rendezvous locations					
Luo et al. (Luo et al., 2021)	✓	–	–	✓	–	Minimizing the completion time	A route-first-drone-second construction algorithm and a TS algorithm
Wang et al. (Wang et al., 2019)	✓	–	–	✓	–	Minimizing the completion time	A three stage-heuristic named Hybrid Truck-Drone Delivery Algorithm: stage 1- Problem Preprocessing, 2- Flight Segment Construction and Global Routes Construction
Freitas and Penna (Freitas & Penna, 2019)	✓	–	–	✓	–	Minimizing total travel time	A hybrid general variable neighborhood search
Chen et al. (Chen et al., 2021)	✓	–	✓	✓	–	Minimizing total travel time	An adaptive large-neighborhood search algorithm
Murray and Chu (Murray & Chu, 2015)	✓	–	–	✓	–	Minimizing the completion time	A route and re-assign heuristic for the flying sidekick traveling salesman problem and a heuristic algorithm for the parallel drone scheduling TSP
Jeong et al. (Jeong et al., 2019)	✓	–	–	✓	–	Minimizing the completion time	A two-phased heuristic contains a construction phase and a search phase
Murray and Raj (Murray & Raj, 2020)	✓	–	–	✓	–	minimizing total travel time	A three-phased iterative heuristic
Raj and Murray (Raj & Murray, 2020)	✓	–	–	✓	–	Minimizing total travel time	A three-phased heuristic; 1- partition customers and create a TSP tour, 2- create UAV sorties, and 3-schedule activities and obtain timings
Bouman et al. (Bouman et al., 2018)	✓	–	–	✓	–	Minimizing total travel time	Dynamic programming
Poikonen and Golden (Poikonen & Golden, 2019)	✓	–	–	✓	–	Minimizing total travel time	A branch and bound algorithm and three heuristics which are as follows:1- Greedy Sequence, 2- Greedy Sequence with Local Search, and 3-Partial Solve with Greedy Insert
Tu et al. (Tu et al., 2018)	✓	–	–	✓	–	Minimizing total traveling costs	1-A Large-neighborhood Search algorithm and 2- greedy randomized adaptive search procedure
Schermer et al. (Schermer et al., 2019)	✓	–	–	✓	–	Minimizing the completion time	A hierarchical heuristic; 1- allocation and sequencing, and 2- drone assignment and scheduling
Poikonen et al. (Poikonen et al., 2019)	✓	–	–	✓	–	Minimizing total travel time	Branch and bound and a constructive algorithm named divide-and-conquer algorithm
Karak and Abdelghany (Karak & Abdelghany, 2019)	–	✓	✓	✓	–	Minimizing total operation cost	Three heuristics were proposed: 1- a hybrid Clarke and Wright, 2- vehicle-driven heuristic (routing vehicle then improving drone route), and 3- vehicle-driven heuristic (constructs the drone routes before the vehicle route)
Liu et al. (Liu et al., 2022)	✓	–		–	✓	Minimizing total travel time	Reinforcement learning algorithms including the deep Q-network and the Advantage Actor-Critic algorithm
Sawadsitang et al. (Sawadsitang et al., 2019)	✓	–		–	✓	Minimizing total operation cost	A Decomposition Algorithm
Kitjacharoenchai et al. (Kitjacharoenchai et al., 2020)	✓	–	–	✓	–	Minimizing total travel time	1- Drone Truck Route Construction heuristic and 2- A Large-neighborhood Search algorithm
Sacramento et al. (Sacramento et al., 2019)	✓	–	–	✓	–	Minimizing total costs	An Adaptive Large-neighborhood Search
Pina-Pardo (Pina-Pardo et al., 2021)	✓	–	–	✓	–	Minimizing total travel time	A decomposition approach
This work	–	✓	✓	–	✓	Minimizing weighted multi-objective dimensions included completion time, operational costs, emissions, and social penalties	A branch-and-bound algorithm and a heuristic branch-and-bound algorithm

The relevant notation used in the model is as follows.

Sets	
S	Set of all locations containing customer locations and rendezvous locations.
S_1	Set of first-class customer locations, we must serve with a truck.
S_2	Set of second-class customer locations, we must serve with a truck or drone.
S_3	Set of third-class customer locations, we must serve only with a drone.
S_4	All rendezvous locations where the truck can stop and operate the drone.
De	Depot node $De = \{1, N\}$.
ST	The truck can stop in all of these locations. $ST = S_1 \cup S_2 \cup S_4 \cup De$.
SD	Set of all customer locations where can get service with a drone, $SD = S_2 \cup S_3$.
SND	All locations where the truck can stop, but the drone cannot operate. $SND = S_1 \cup S_2 \cup DE$.
SN	Sequence of customers $i = \{1, \ldots, N\}$.
DR	Drone sequence number for each truck stop. $DR = \{1, \ldots, R\}$.
Parameters:	
d_{kf}	Distance between nodes k and f.
p	Customer existence probability.
SR	Sight radius for operating the drone.
V_t	Truck speed.
V_{dr}	Drone speed.
EM	Truck Emission rate per meter.
Co_{tr}	Truck travel cost per meter.
Co_{dr}	Drone travel cost per meter.
SP_k	Social penalty considered for parking the truck in the customer k's location.
BE	Battery endurance of the drone.
Indexes:	
i, j	Customer sequences.
k, f	Rendezvous nodes, customer locations and depot.
r	Sub-sequence in rendezvous nodes.
Variables:	
z_k^r	A binary variable gets the value of *1* in case of assigning a customer to sub-sequence r of truck stop location k.
h_{kf}^r	A binary variable gets the value of *1* in case of assigning customer k to sub-sequence r of truck stop location f.
$U_{k,i}^r$	A binary variable will be *1* if the overall sequence customer assigned to sub-sequence r in truck station k is i.
$\overline{d_{ij}}$	Truck traveling distance between sequences i and j, if the delivery system services the customer assigned to sequence j exactly after the customer assigned to sequence i.
Ed_{ij}	The distance value that we expect the truck to travel between two sequences of i and j.

3.2.1. Objective functions

$$Min\ obj_1 = \sum_{i,j \in SN} \frac{Ed_{ij}}{V_t} + 2\sum_{r \in DR}\sum_{f \in S_4}\sum_{k \in SD} \frac{h_{kf}^r p d_{kf}}{V_{dr}} \tag{5}$$

$$Min\ obj_2 = \sum_{i,j \in SN} Co_{tr} Ed_{ij} + 2\sum_{r \in DR}\sum_{f \in S_4}\sum_{k \in SD} Co_{dr} h_{kf}^r p d_{kf} \tag{6}$$

$$Min\ obj_3 = \sum_{i,j \in SN} EMEd_{ij} \tag{7}$$

$$Min\ obj_4 = \sum_{\forall k \in S_2} SP_k p z_k^1 \tag{8}$$

3.2.2. Constraints

$$z_k^1 = 1 \forall k \in S_1 \cup DE \tag{9}$$

$$z_k^1 + \sum_{r \in DR, f \in S_4} h_{kf}^r = 1 \forall k \in S_2 \tag{10}$$

$$z_k^r = 0 \forall k \in SND, and\ (r \geq 2) \in R \tag{11}$$

$$\sum_{r \in DR, f \in S_4} h_{kf}^r = 1 \forall k \in S_3 \tag{12}$$

$$\sum_{f \in S_4} h_{kf}^r d_{kf} \leq SR \forall k \in SD, and\ r \in DR \tag{13}$$

$$\frac{2\left(\sum_{f \in S_4} h_{kf}^r d_{kf}\right)}{V_{dr}} \leq BE \forall k \in SD, and\ r \in DR \tag{14}$$

$$\sum_{k \in SD} h_{kf}^r = z_f^r \forall f \in S_4 \quad and\ r \in DR \tag{15}$$

$$\sum_{i \in SN} U_{k,i}^r = z_k^r \forall k \in ST and\ r \in DR \tag{16}$$

$$U_{1,1}^1 = U_{N,N}^1 = 1 \tag{17}$$

$$\sum_{r \in DR, k \in ST} U_{k,i}^r = 1 \forall i \in SN \tag{18}$$

$$U_{k,i}^r \leq U_{k,i-1}^{r-1} \forall k \in S_4\ and\ (r \geq 2) \in DR\ \ and\ (i > 1) \in SN \tag{19}$$

$$\overline{d_{ij}} \geq d_{kf} - M\left(1 - \sum_{r \in DR} U_{k,i}^r\right) - M\left(1 - \sum_{r \in DR} U_{f,j}^r\right) \forall k \text{ and } f \in ST, and\ i, j \in SN \tag{20}$$

$$Ed_{ij} = p^2(1-p)^{j-i-1}\overline{d_{ij}} \forall j > i > 1, \text{and} \neq N \in SN \tag{21}$$

$$Ed_{ij} = p(1-p)^{j-i-1}\overline{d_{ij}} \forall (j > i = 1 \text{or } (i > 1 \text{ and } j = N)) \in SN \tag{22}$$

$z_k^r, h_{kf}^r, U_{k,i}^r$: *Binary variables* $\overline{d_{ij}}, Ed_{ij}$: *Positive variables*

The first objective function is minimizing the completion time. The second is related to minimizing operational costs. The third is reducing truck emissions, and the fourth is diminishing the social penalty for parking a truck in second-class customer locations. We cannot ignore parking the truck in first-class customer locations, so we only considered second-class customers in the social penalty dimension.

Constraint 9 ensures that the truck will visit first-class customers. Constraint 10 guarantees that the truck serves second-class customers or that a rendezvous location will assign to them. Because of assuming we only can control the drone operation from the rendezvous location, constraint 11 ensures that each first-class and second-class customer location has just one customer. Constraint 12 makes sure that all third-class customers designate to rendezvous locations. Constraint 13 considers sight radius for the assignment of customers to the rendezvous locations. Constraint 14 considers the battery endurance of the drone for departing from the rendezvous location, servicing the customer, and then returning to the truck. When SR is smaller than $\frac{V_{dr}BE}{2}$ constraint 13 will be activated, otherwise, constraint 14 will be activated.

Constraints (15 and 16) link between customers' overall sequence and their related truck stop. Constraint 17 ensures that 1 and N sequences designate to the depot. Constraint *18* ensures that for each customer (each sub-sequence at each truck stop), only one overall sequence must be assigned. Constraint 19 forces that all sub-sequences of a rendezvous location must be given to overall sequences one after one. In other words, before the truck departs from a rendezvous location, the system should serve all assigned customers. Constraints (20, 21, and 22) calculate the expected value of truck travel distance between each pair of overall sequences.

3.3. Weighted objective model

Logistic service providers mostly focus on completion time or operational costs. However, delivery systems affect the environment, and city livability and sustainability dimensions should be considered. In order to model a real-life problem and properly analyze the effects of drone specifications, and environmental and social penalties on the performance of the hybrid truck-drone delivery system, we propose using a weighted objective approach that combines several objectives to achieve a single preferred solution. In this way, we can analyze the performance of the system in various conditions including considering only completion time, focusing only on operational cost, or considering all four objective functions together.

This approach combines the completion time dimension, the transportation cost dimension, the emission dimension, and the social penalty dimension with priority weights into equation (23).

$$\overline{OBj} = w_1 obj_1 + w_2 obj_2 + w_3 obj_3 + w_4 obj_4 \tag{23}$$

w_s demonstrates the importance of dimension $s \in \{1, \ldots, 4\}$. For this function, $0 \le w_s \le 1$, and $\sum_{s=1,2,3,4} w_s = 1$.

4. Solution method

In this section, we proposed a heuristic Branch-and-Bound algorithm (*HB&B*) and a Branch-and-bound algorithm for the stochastic hybrid truck-drone routing problem.

4.1. Heuristic branch-and-bound

This algorithm defines the first sequence for the depot and constructs new branches based on the proposed five customer selection rules. At a new branch,*HB&B*schedules a non-selected customer and its related truck stop. If the customer is from first or second class and gets service from the truck, its location is considered as its truck stop location. For scheduling a second or third-class customer that gets service with the drone, a rendezvous location must be defined for its truck stop location.

After defining a new branch,*HB&B* checks for three boundary conditions, and if one of them is not satisfied, it closes that new branch. Then *HB&B* gets back to the previous stage and constructs a new branch. Based on the customer selection rules, if at any stage, *HB&B* cannot construct a new branch, it gets back to the previous stage. Whenever in a branch all customers are scheduled and the objective value is smaller than the upper bound, *HB&B replaces* the upper bound with the new solution. This process will be finished when *HB&B* cannot construct any new branches. Algorithm 1 depicts the overall framework of the *HB&B*procedure.

Algorithm 1

. depicts the overall framework of the HB&B procedure.

Where *i* is the branching stage. Suppose the truck stop location of stage *i-1* is the *f*, and $k^i(f)$ defines the number of different truck stop locations analyzed at stage *i* after truck stop *f*. In that case, $\overline{set_a}$ is the set of all parking stations that can cover customer *a*. For a scheduled customer *a*, $\overline{set_a}$ contains only one truck stop location. *u(a)* is the sequence number of customer *a*. $sel_t(i)$ is the set of truck stop locations that are not selected until stage *i* and $sel_c(i)$ is the set of customer locations that are not selected until stage *i*, *LB* is the lower bound, and *UB* is the upper bound.

In this algorithm, we have defined five rules for branching.

Rule 1- We define a small neighborhood zone as follows: If the truck stop location of stage *i-1* is *f*, the maximum number of branches from *f* at stage *i* with different truck stop locations are limited to $\bar{k}$. It says that at most $\bar{k}$ neighborhood truck stops among non-selected truck stops can be selected for the following sequence. If we consider a big value for $\bar{k}$, we can find the optimal solution, but we propose to restrict that to achieve a near-optimal solution in a reasonable run time. We defined this rule in Algorithm 1 (lines 7, 10, 18, 27, and 38). Fig. 2 demonstrates how this rule defines a small neighborhood zone for a selected truck stop at stage *i*.

Rule 2- Consider constructing a large neighborhood zone with the $\bar{L}$ nearest truck stops for each truck stop. If *j* is the last selected truck stop and $k^i(j)$ is bigger than one, the next truck stop shall select among the large neighborhood zone of *j*. We have considered this rule in Algorithm 2 (lines 5, 6, 7, 8, and 9). This rule ensures that by selecting all truck stops located in the large neighborhood zone of *j* in

```
Require: Distance matrix
1: For all i= {1, …, N}, and all f∈ ST, k^i(f) ← 0
2:      i← 1
3:      Select the depot for the first sequence and the first truck stop
4:      i← i + 1
5:      while i> 1
6:        For stage i;
7:         If k^i(f) ≤ k̄
8:              If the last truck stop (f) ∈ S_4, and there is a non-selected customer (a), that (d_af = min d_ak |∀k ∈ {f} ∪ (S_4 ∩ sel_t(i)))
9:                 Select the customer a for the stage i
10:                k^i(f) ← k̄
11:                u(a)← i
12:                sel_c(i + 1) ← sel_c(i) − {a}
13:                i← i + 1
14:             Else;
15:                  Based on customer selection algorithm, select a customer{a} and related truck stop {j}
16:                  set̄_p ← {j}, and u(a)← i
17:                 If f is the truck stop location of stage i-1, and j≠ f,
18:                     k^i(f) ← k^i(f) + 1
19:                 Else;
20:                     Continue with pervious k^i(f)
21:                 End if
22:                Calculate the LB
23:                If boundary conditions are satisfied
24:                     If i=N-1,
25:                          Update UB, and i← i − 1
26:                     Else;
27:                          k^{i+1}(j) ← 0
28:                          sel_t(i + 1) ← sel_t(i) − {j}
29:                          sel_c(i + 1) ← sel_c(i) − {a}
30:                          i← i + 1
31:                     End if
32:                Else;
33:                     Discard this branch, i← i, and continue by selecting a new customer and truck stop
34:                End if
35:             End if
36:       Else;
37:          i← i − 1
38:       End if
39:     End while
40: Demonstrate the UB
```

Fig. 2. Large and small neighborhood zones for a selected truck stop at stage i in the proposed *HB&B*, when $\overline{L} = 4$ and $\overline{k} = 3$.

previous stages, we can check only one truck stop for the following sequence. In Fig. 2, $\overline{L}$ equals 4.

In this algorithm, the large neighborhood zone of a truck stop location is always constant, but the small neighborhood zone is related to the non-selected truck stops, thus, at various branches starting from the same truck stop location it may be different. Fig. 2 compares the small and large neighborhood zones.

Algorithm 2. Customer selection

Require: travel time matrix, last selected truck stop $\{j\}$, $sel_c(i)$, and $sel_t(i)$

If the last selected truck stop location is ($j \in S_4$), and there is a non-selected customer ($a \in sel_c(i) \cap S_2 \cap S_3$) in sight radius of j, which is not selected in a same customer combination assigned to $\{j\}$ at the same stage in previous branches after selecting truck stop $\{j\}$
    select customer a for the next customer sequence
else;
    Based on mco_j, find the customer with smallest change in objective values and related truck stop for the next sequence
        If $k^i(j) > 1$, and related truck stop is not in large neighborhood zone of j
            a cannot be selected and $k^i(j) \leftarrow k^i(j) + 1$
        Else;
            Select a for the next customer
        End if,
End if

Rule 3- If rendezvous location f has the minimum distance to customer $a \in S_3$, and this rendezvous location is selected, we should assign customer a to that. It says that even if the truck visits the other rendezvous locations that can cover customer a, location f has the minimum distance and drone travel time to the customer a. So, customer a must be assigned to rendezvous location f in the optimal solution. We have considered this rule in Algorithm 1 (lines 8, 9, 10, 11, 12, and 13).

Rule 4- If the last truck stop location is a rendezvous location, we must check all customers it can cover for the next branching stage.

Rule 5- The next customer always is selected based on the minimum changes in objective values after the last selected truck stop. The second, third, and fourth rules construct the customer selection algorithm as follows:

Where mco_{ja} is the change of objective values if customer a is selected for the following sequence. We can calculate mco_j as follows:

$$mco_j = \begin{cases} \left(w_2.Co_{tr} + w_3EM + w_1\dfrac{1}{V_t}\right)d_{ja} \forall p \in S_1 \\ \min\left\{\left(\left(w_2.Co_{tr} + w_3.EM + \dfrac{w_1}{V_t}\right)d_{ja} + w_4.SP_k\right), \min\left\{\begin{matrix}\left(\left(w_2.Co_{tr} + w_3.EM + w_1\dfrac{1}{V_t}\right)d_{jl}\right) + \\ \left(w_2.Co_{dr} + \dfrac{w_1}{V_{dr}}\right) \ d_{la}, \forall (l \in S_4) \neq j\end{matrix}\right\}\right. \forall p \in S_2 \\ \min\left\{\left(\left(w_2.Co_{tr} + w_3.EM + \dfrac{w_1}{V_t}\right)d_{jl}\right) + \left(w_2Co_{dr} + \dfrac{w_1}{V_{dr}}\right) \ d_{la}, \forall (l \in S_4) \neq j\right\} \forall p \in S_3 \end{cases} \tag{24}$$

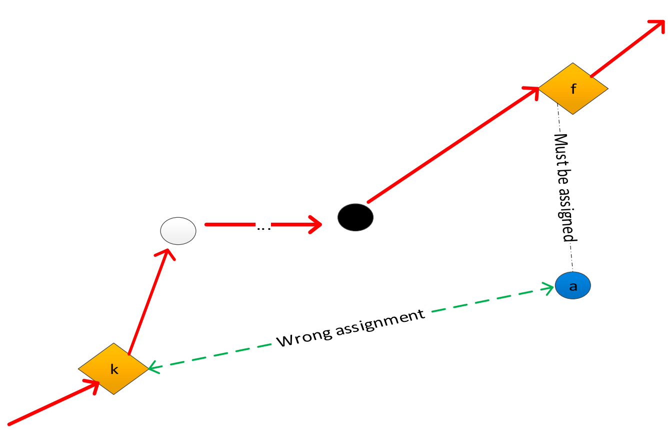


Fig. 3. Description of the delivery system.

In algorithm *2*, line *2* ensures that after rendezvous location *j*, the system will check only one branch for the same combination of customers assigned to *j* with different customer sequences.

In this algorithm, we have defined three boundary conditions.

Lemma 2. Algorithm 3 *presents a lower bound for the system when we are at stage i* (*depot and i-1 customers are scheduled*).

Algorithm 3. *LB calculation*

Require: Distance matrix, $sel_c(i)$, $sel_t(i)$, $\overline{set_a}$

For all j and $a \in SC \cup De$

Calculate $E\acute{C}C_{ja}$ and ECC_{ja}

If $j \in S_1 or De$

For $a \in S_1 or\ depot$

$E\acute{C}C_{ja} \leftarrow (w_2.Co_{tr} + w_3.EM + \frac{w_1}{V_t})d_{ja};$

For $a \in S_2\ or\ S_3$

$E\acute{C}C_{ja} \leftarrow \min\{ \left(w_2.Co_{tr} + w_3.EM + \frac{w_1}{V_t}\right) d_{jl} + (w_2.Co_{dr} + \frac{w_1}{V_{dr}})\ d_{la}\ \ ,\forall\ l \in\ \overline{set_a}\}$, for l=a, $d_{la} = 0$

If j $\in S_2$

For a $\in S_1 or\ depot$

$E\acute{C}C_{ja} \leftarrow\ \min\{(w_2.Co_{dr} + \frac{w_1}{V_{dr}})d_{jl} + \left(w_2.Co_{tr} + w_3.EM + \frac{w_1}{V_t}\right) d_{la}\ \ ,\forall\ l \in\ \overline{set_j}\}$, for l=j, $d_{lj} = 0$

For a $\in S_2\ or\ S_3$

$E\acute{C}C_{ja} \leftarrow\ \min\{\left(w_2.Co_{dr} + \frac{w_1}{V_{dr}}\right)(d_{jl} + d_{kp}) + \left(w_2.Co_{tr} + w_3.EM + \frac{w_1}{V_t}\right) d_{lk}, \forall\ l \in\ \overline{set_j}\ and\ k \in\ \overline{set_a}\}$

,

Else (j $\in S_3$);

For a $\in S_1 or\ depot$

$E\acute{C}C_{ja} \leftarrow\ \min\{(w_2.Co_{dr} + \frac{w_1}{V_{dr}})d_{jl} + \left(w_2.Co_{tr} + w_3.EM + \frac{w_1}{V_t}\right) d_{la}\ \ ,\forall\ l \in\ \overline{set_j}\}$

For p $\in S_2\ or\ S_3$

$E\acute{C}C_{ja} \leftarrow \min\{\left(w_2.Co_{dr} + \frac{w_1}{V_{dr}}\right)(d_{jl} + d_{kp}) + \left(w_2.Co_{tr} + w_3.EM + \frac{w_1}{V_t}\right) d_{lk}, \forall\ l \in\ \overline{set_j}\ and\ k \in\ \overline{set_a}\}$

End if

End if

For all scheduled customers (*j* and $a \in SC$),

$ECC_{ja} \leftarrow p^2(1-p)^{u(p)-u(j)-1}.E\acute{C}C_{ja}$

For scheduled (*j*= depot) and scheduled $a \in SC.\ u(j) = 1$

$ECC_{ja} \leftarrow p(1-p)^{u(p)-1-1}.E\acute{C}C_{ja}$

For scheduled ($j \in SC$) and $a = depot.\ u(a) = N$

$ECC_{ja} \leftarrow p(1-p)^{N-u(j)-1}.E\acute{C}C_{ja}$

For all scheduled ($j\in SC \cup De$) and non-scheduled a until stage i:

In related to *j*, give an order from *1* to *N-i-1*, from smallest to biggest value of $\{E\acute{C}C_{ja}, \forall\ a \in sel_c(i)\}$, and put it in ord_{ja}

For scheduled customer *j* and non-scheduled customer *a*:

$ECC_{ja} \leftarrow p^2(1-p)^{i+ord_{ja}-u(j)-1}.E\acute{C}C_{ja}$

For (*j*=depot) and non-scheduled customer *a* until stage *i*:

$ECC_{ja} \leftarrow p(1-p)^{i+ord_{ja}-2}.E\acute{C}C_{ja}$

For non-scheduled customer *j*, and (*a*=depot)

In related to *a*, give an order from *1* to *N-i-1*, from smallest to biggest value of $\{E\acute{C}C_{ja}, \forall\ j \in sel_c(i)\}$, and put it in ord_{ja}

$ECC_{ja} \leftarrow p(1-p)^{ord_{ja}-1}.E\acute{C}C_{ja}$

For all pair of non-scheduled customers (*j* and *a*) until stage *i*:

In related to *j*, give an order from *1* to *N-i-2*, from smallest to biggest value of $\{E\acute{C}C_{ja}, \forall\ a \in sel_c(i)\}$, and put it in ord_{ja}

$ECC_{ja} \leftarrow p^2(1-p)^{ord_{ja}-1}.E\acute{C}C_{ja}/2$

For all selected truck stops from second class customers

$z_k^1 \leftarrow 1$

EC4$\leftarrow \sum_{\forall k \in S_2} SP_k.p.z_k^1$

calculate LB:

LB$\leftarrow \sum_{\forall (j \neq p)} ET_{ja}$ + EC4

Boundary condition 1- Lower bound must be smaller than the upper bound.

Boundary condition 2- Considering the system cannot cover a third-class customer with remaining rendezvous locations. In that case, this branch cannot present a feasible solution.

Boundary condition 3- For the deterministic state, if the last selected truck stop f is a rendezvous location, there cannot be a selected customer (a) assigned to another rendezvous location in previous stages, that ($d_{af} = \min d_{al} | \forall l \in S_4$). As depicted in Fig. 3, in the customer selection algorithm, customer a can be assigned to a rendezvous location k that $d_{ak} > d_{af}$. Suppose in the next branching stages rendezvous location *f* with the minimum distance to customer a is the choice in the following branching stages. In that case, this branch cannot present the optimal solution for the deterministic delivery system. For this reason, we used this rule as a heuristic boundary condition for the stochastic model.

SC is the set of all customers. If customer a is selected for the following sequence after customer *j*, $E\acute{C}C_{ja}$ is an estimation for the minimum change of the objective value. ECC_{ja} is a variable to calculate a lower bound for the expected value of the objective value. *EC4* calculates the fourth objective value (social penalty) for the scheduled second-class customers. For the scheduled customers, ECC_{ja} equals the expected value of objective value change for servicing customer a exactly after customer *j*. ord_{ja} is a ranking number for estimating the minimum objective value changes for servicing node a after node *j*. In this algorithm, we assign the depot to both sequences of *1* and *N*.

In this paper, the objective expected value is calculated based on customers' sequences and relation between each pair of sequences. At any stage part of customers are scheduled and part of them are not scheduled. Thus, this algorithm calculates the objective expected value between scheduled customers and estimates that between for two non-scheduled customers or a non-scheduled customer and a scheduled customer. See Appendix *B* for the proof of Lemma *2*.

4.2. Branch-and-bound algorithm

Some restrictions and procedures in the proposed *HB&B* may result in non-optimal solutions. For the *B&B* algorithm, we do not regard the first, the second, and the third rules of customer selection and the third boundary condition. It is clear that for the deterministic systems, the second customer selection rule or third boundary condition aims to reduce drone travel time while truck travel time is not changed. Nevertheless, in a stochastic system, the number of customers assigned to rendezvous locations impacts the probability of visiting them and changes the expected value of truck traveling distance. Consequently, these rules may obstruct optimal solution achievement.

5. Computational study

We implemented our proposed algorithm in Python 3.9.12 and used CPLEX to solve the proposed MIP model. All computations are performed on a LENOVO Laptop with Intel(R) Core (TM) i7-9750HF CPU @ 2.60 GHz, 2.59 GHz, and 16 GB installed RAM running Windows 10.

5.1. Test instances

To evaluate the performance of *HB&B* and *B&B*, we generated *180* small-sized and *180* medium-sized, and *60* large-sized instances. We generated customer locations in a $1000*1000m^2$ square. Then randomly assigned to three customer classes. Considering the necessity of covering the rendezvous locations of the second-class and the third-class customers, we created them so that at least one rendezvous node is in sight radius of each second-class or third-class customer. Also, we consider the sight radius for this data generation *100* m. Two locations for the depot are the vertex of the square *(0,0)* and the center of the square *(500,500)*. We assumed *10* m per second for truck speed and *20* m per second for drone speed. In this way, we created *42* various instance types and generated *10* instances for each type.

5.2. Performance of the proposed solution methods

In this section, we evaluated the proposed *B&B* and the *HB&B* performance. For this analysis, we only focused on the first objective value. The computational time is limited to 3600 s, and p is assumed to be *0.5*. For the *MILP*, we considered the best result for $r = 2, 3, 4$, and 5. As demonstrated in Table 2, among *60* instances with *16* customers and *4* rendezvous locations, *MILP* solved only one of them, and the problem characteristics significantly influenced it. After seeing the results, we can conclude that the proposed *B&B* is more consistent, reaching the optimal solutions in shorter CPU times. Compared to the exact methods, *HB&B* is more effective and prepares faster solutions, and among *180* small-sized instances, it solved *177* instances optimally.

In this section, we analyzed the performance of the *HB&B* for medium-sized instances due to various values of $\overline{L}$ and $\overline{k}$. For this section, the limit of computational time is *4800* s, and we considered p equal to (*0.5*). For the medium-sized instances, *B&B* could not solve the instances with *33* nodes (*28* customers and *5* rendezvous locations), so the average Gap is calculated based on the best solution found. As shown in Table 3, the average CPU time of *HB&B* for all medium-sized instances is less than *1.38* s. Even with $\overline{L} = 3$ and $\overline{k} = 2$, it can reach the optimal solution for most of the test instances, demonstrating the high performance of *HB&B* for medium-sized instances (see Table 4).

In the *FSTSP, and TSP-D,* rendezvous locations are not considered, and in the two-echelon truck and drone routing problem, servicing with the truck is not considered. However, this paper proposes a hybrid truck-drone routing problem that considers both truck servicing and

Table 2
Performance of the *MILP*, *B&B* and, *HB&B*.

Number of customers				Number of rendezvous locations	Depot Position	Number of instances	*MILPs*			*B&B*			*HB&B* $\overline{L} = 4, \overline{k} = 3$			
First-class customers	Second-class customers	Third-class customers	Total				Solved instances (%)	CPU time (sec) Best	CPU time (sec) Average	Solved instances (%)	CPU time (sec) Best	CPU time (sec) Average	Solved instances (%)	CPU time (sec) Best	CPU time (sec) Average	Gap (%)
2	2	2	6	2	(0,0)	10	100	0.01	0.02	100	0	0.01	100	0	0	0
3	3	3	9	3	(0,0)	10	100	1.33	5.02	100	0.03	0.24	100	0	0	0
3	6	3	12	3	(0,0)	10	100	90.54	131.30	100	7.89	14.12	100	0	0	0
3	3	6	12	3	(0,0)	10	100	28.71	63.10	100	3.28	8.33	100	0	0	0
6	3	3	12	3	(0,0)	10	100	77.80	112.05	100	7.51	17.06	100	0	0	0
4	4	4	12	4	(0,0)	10	100	56.44	84.16	100	8.22	14.69	100	0	0	0
4	8	4	16	4	(0,0)	10	0	–	–	100	78.14	220.36	100	0	0	0
4	4	8	16	4	(0,0)	10	10	3054.50	–	100	66.95	126.60	100	0	0	0
8	4	4	16	4	(0,0)	10	0	–	–	100	90.01	182.58	100	0	0	0
2	2	2	6	2	(500,500)	10	100	0.03	0.07	100	0.01	0.01	100	0	0	0
3	3	3	9	3	(500,500)	10	100	7.39	20.80	100	0.07	0.28	100	0	0	0
3	6	3	12	3	(500,500)	10	100	154.50	628.12	100	9.50	21.92	100	0	0	0
3	3	6	12	3	(500,500)	10	100	112.41	300.05	100	5.46	13.22	100	0	0	0
6	3	3	12	3	(500,500)	10	100	225.34	692.90	100	10.47	20.33	100	0	0	0
4	4	4	12	4	(500,500)	10	100	166.70	545.00	100	6.93	15.02	100	0	0	0
4	8	4	16	4	(500,500)	10	0	–	–	100	112.64	289.20	100	0	0.01	0.01
4	4	8	16	4	(500,500)	10	0	–	–	100	88.75	210.33	100	0	0.01	0.03
8	4	4	16	4	(500,500)	10	0	–	–	100	128.22	310.65	100	0	0.01	0.02

Table 3
Performance of *HB&B* due to various values of $\overline{k}$ and $\overline{L}$.

Number of customers				Number of rendezvous locations	Number of instances	Depot position	*B&B*	*HB&B*							
								$\overline{L}=3$				$\overline{L}=4$			
								$\overline{k}=2$		$\overline{k}=3$		$\overline{k}=2$		$\overline{k}=3$	
First-class customers	Second-class customers	Third-class customers	Total				CPU time (sec)	CPU time (sec)	Gap (%)	CPU time (sec)	Gap (%)	CPU time (sec)	Gap (%)	CPU time (sec)	Gap (%)
5	5	10	20	4	10	(0,0)	514.04	0.01	0	0.01	0	0.01	0	0.01	0
5	10	5	20	4	10	(0,0)	1032.30	0.01	0	0.02	0	0.02	0	0.03	0
10	5	5	20	4	10	(0,0)	708.77	0.02	0	0.02	0	0.02	0	0.02	0
6	6	12	24	4	10	(0,0)	2280.19	0.03	0	0.03	0	0.03	0	0.03	0
6	12	6	24	4	10	(0,0)	4091.33	0.03	0	0.03	0	0.03	0	0.03	0
12	6	6	24	4	10	(0,0)	3273.64	0.03	0.37	0.04	0.30	0.04	0.37	0.05	0
7	7	14	28	5	10	(0,0)	–	0.03	0	0.03	0	0.03	0	0.06	0
7	14	7	28	5	10	(0,0)	–	0.04	0	0.04	0	0.06	0	0.06	0
14	7	7	28	5	10	(0,0)	–	0.04	1.27	0.40	1.27	0.42	1.27	0.87	0
5	5	10	20	4	10	(500,500)	639.52	0.01	0	0.02	0	0.02	0	0.02	0
5	10	5	20	4	10	(500,500)	1360.40	0.02	0	0.02	0	0.02	0	0.02	0
10	5	5	20	4	10	(500,500)	914.40	0.01	0	0.02	0	0.02	0	0.02	0
6	6	12	24	4	10	(500,500)	2819.53	0.05	0	0.04	0	0.04	0	0.08	0
6	12	6	24	4	10	(500,500)	–	0.04	0	0.06	0	0.09	0	1.00	0
12	6	6	24	4	10	(500,500)	3744.60	0.04	0.51	0.04	0	0.04	0.51	0.04	0
7	7	14	28	5	10	(500,500)	–	0.30	1.09	0.44	1.09	0.46	0	1.38	0
7	14	7	28	5	10	(500,500)	–	0.13	0.3	0.14	0	0.18	0.3	0.30	0
14	7	7	28	5	10	(500,500)	–	1.23	0	2.47	0	0.66	0	0.81	0

Table 4
Comparison of *VDH* and *HB&B* for large-sized instances.

Number of customers				Number of rendezvous locations	Number of instances	Depot position	*VDH*	*HB&B* $\overline{k}=3$, $\overline{L}=3$	
First-class customers	Second-class customers	Third-class customers	Total				Average CPU time (sec)	Average CPU time (sec)	Average Gap (%)
10	10	10	30	5	10	(0,0)	0.54	0.93	−1.84
15	15	15	45	10	10	(0,0)	3.76	24.14	−2.44
20	20	20	60	15	10	(0,0)	7.46	1092.60	−4.22
10	10	10	30	5	10	(500,500)	0.87	1.71	−1.07
15	15	15	45	10	10	(500,500)	4.01	116.18	−2.80
20	20	20	60	15	10	(500,500)	6.80	1360.88	−3.22

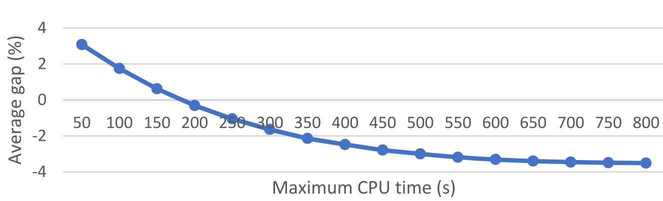


Fig. 4. The average gap between *VDH* and *HB&B* for various maximum CPU times for *20* instances with *60* customers.

rendezvous locations. To the best of our knowledge, there is no solving algorithm in the literature fully matches the problem. For this reason, we considered the vehicle-driven heuristic (*VDH*) proposed by (Karimi & Setak, 2018). This algorithm uses a Clarke and Wright heuristic for truck and drone routing. Then it starts to discard rendezvous locations with the most considerable cost-saving value. Assume all customers get service from the remaining rendezvous locations; reconstructing vehicle and drone routes will happen. Considering problem assumptions, we adapted *VDH* to our problem. To prepare a better comparison, we considered the deterministic model, where the demand existence probability for all customers is assumed to be (*1*). As shown in Table *4*, *VDH* solves most of the instances in less CPU time, but *HB&B* outperforms the *VDH*in terms of objective value, demonstrating a good performance for a newly implemented heuristic Branch-&-Bound.

To profile the performance of the solutions of the *HB&B* algorithm, we considered *20* instances with the size of *60* customers and *15* rendezvous locations and present the average gap between *VDH* and *HB&B* for various maximum CPU times, in Fig. 4. As it is shown, for CPU time of more than 150 s, *HB&B* performs better solutions.

5.3. Case study analysis

Because of proposing the stochastic hybrid truck-drone routing problem for city logistics, test instances should consider its restrictions. For example, the truck travels in road distances and the drone in Euclidean distances. We used data from the National Post Company of the Islamic Republic of Iran in Boujnord city as real data. This logistic provider prepares delivery services for different supply chain agents. As a brief description of Boujnord city, it is the capital of the north Khorasan province of Iran, and its population is about *320,000*.

This logistic service provider has *30* carriers for Boujnord city, distributing annually around 1800 parcels per day on average. As a rule, delivery service carriers must visit customer locations two times on different days. If a customer cannot receive his parcel at the defined location, the parcel will return to the delivery stations. According to historical data, around (*28.64* percent) of the deliveries at customer locations have failed. For this reason, we considered demand existence probability equal to (*0.72*). To prepare a realistic data instance, we randomly selected a random working day on August 2022. Then implementing a municipal map on the *GIS* software and using *UTM* of customer locations, we calculated road distances and Euclidean distances between each pair of customer locations assigned to each carrier.

In this delivery system, around *5* percent of the parcels are heavier

Table 5
Comparing the performance of *HTDS* and *TOS*.

priority weights		$w_s = 0.25$	$w_1 = 1$	$w_2 = 1$	$w_3 = 1$	$w_4 = 1$
Objective value	*HTDS*	25.27	9.3	62.4	7.2	0
	TOS	101.75	12.01	172.8	109.2	112.9

To prepare a better comparison between *HTDS* and *TOS*, we have defined *GCT* for the completion time gap, *GOC* for the operational costs gap, *GE* for the truck emission gap, and *GS* for the social penalty gap.

than 4 lbs. Therefore, their delivery was impossible with drones. So, we considered customers with parcels heavier than 4 lbs to be truck-only customers. We also considered customer locations in narrow and dead-end alleys as drone-only customers.

Currently, the National Post Company of the Islamic Republic of Iran is not using drones for delivery services. It has a defined specific city area for each carrier, and the responsibility of all parcels in each area belongs to the related carrier. Traffic congestion and average truck speed in variously defined areas are different. However, we estimated the average truck speed at *8* m per second to analyze the system. Like Pina-Pardo et al. (Pina-Pardo et al., 2021), we consider the drone speed equal to 60 km/h, which is equal to (*16*.6 m/s). We have defined *5*rendezvous locations for this data for each delivery area.

Drone traveling cost per kilometer presumably equals *0.01$*, mainly for maintenance and depreciation costs. Also, we assumed the truck traveling cost per kilometer is *0.5$*, primarily for fuel consumption, maintenance, and depreciation costs.

Our fuel consumption estimation of the van was *0.12* L of diesel per kilometer. As a liter of diesel has *835* g of carbon and to combust this carbon to *CO2*, we will need *1920* g of oxygen; we assumed consuming a liter of diesel emits *2640* g of *CO2*. So, we estimated the emission of a van equal to *0.316* kg per kilometer. In this case, we supposed that the average value of the social penalty is equal to *0.1*. For this case, sight radius is equals *1000* m.

In this section, we have considered *p* equal to *0.5* and compared the truck-only delivery system (*TOS*), and the hybrid truck-drone delivery system (*HTDS*) in Table 5, which highlights that truck and drone coordination can reduce the objective value up to *75* percent.

In the sustainable model with $w_s = 0.25$, the system's sensitivity due to drone speed is not considerable. For the case study data, even if the drone speed will reduce to *8* m per second, the optimal solution is still the same. As depicted in Fig. 5, reducing the drone speed significantly impacts three sustainable dimensions (economic, environmental, and social) if the decision maker focuses only on the completion time. See the gap between *HTDS* and *TOS* performance in Fig. 6. When drone speed reduces to *10* m per second, *TOS* outperforms *HTDS* in the completion time criteria.

In this section, we have assumed that the truck traveling cost per kilometer is constant and equal to *0.5 $* and analyzed the system's performance for various values of drone operational costs. As displayed in Figs. 7 and 9, in this case study, when $w_2 = 1$, the model shows higher sensitivity due to various values of drone operational costs than the model with $w_s = 0.25$, but both models demonstrate similar behavior for changes of drone operational costs. In both models, an increase in drone operational costs increases other dimensions. Figs. 8 and 10 show the gap between the performance of *TOS* and *HTDS*, which highlights the importance of drone operational costs for hybrid truck-drone systems. In this case, when drone operational costs increases to *0.2 $/m*, *TOS* outperforms the *HTDS* in the completion time, and the operation cost criteria.

This section defines *ep* as the environmental penalty for each kilogram emission. In this case study, when $w_1 = w_2 = 0$, the decision model uses less truck transportation and truck stop locations to reduce emissions and social penalties. When $w_s = 0.25$, Fig. 11 demonstrates the number of truck stop locations for various values of *ep* and drone operational costs, and Fig. 12 demonstrates the number of truck stop locations for various values of social penalty (*sp*) and drone operational costs. As shown before, for the same values of drone operational costs, an increase in the social or environmental penalty reduces the number of truck stops.

5.3.1. Comparing the stochastic model and deterministic model

To analyze the effects of customer presence probability on the performance of the proposed model, we have depicted the expected value of the gap between the deterministic model and the stochastic model for various values of drone operational costs and customer presence probabilities in Fig. 13. As it is demonstrated, for smaller values of drone operational costs, the stochastic model shows better performance in our case (see Fig. 14).

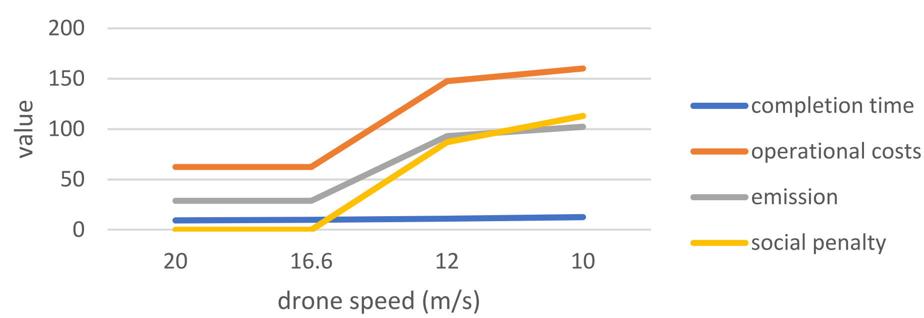


Fig. 5. Performance of the truck and drone delivery system for various values of drone speed, when $w_1 = 1$.

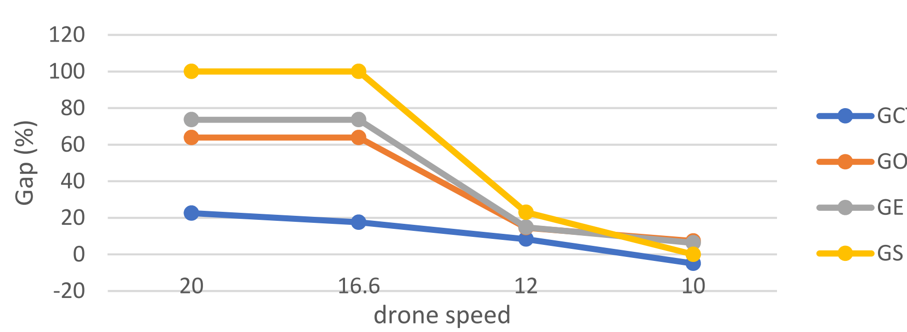


Fig. 6. The Gap between *HTDS* and *TOS* performance for various values of drone speed, when $w_1 = 1$.

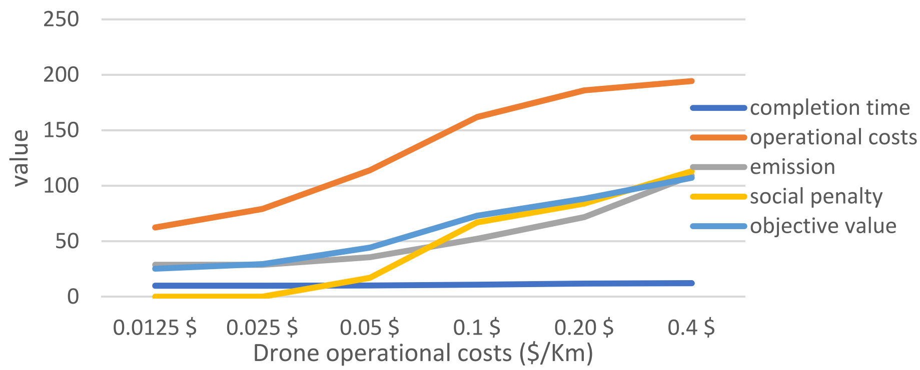


Fig. 7. System's performance for various values of drone operational costs, when $w_s = 0.25$.

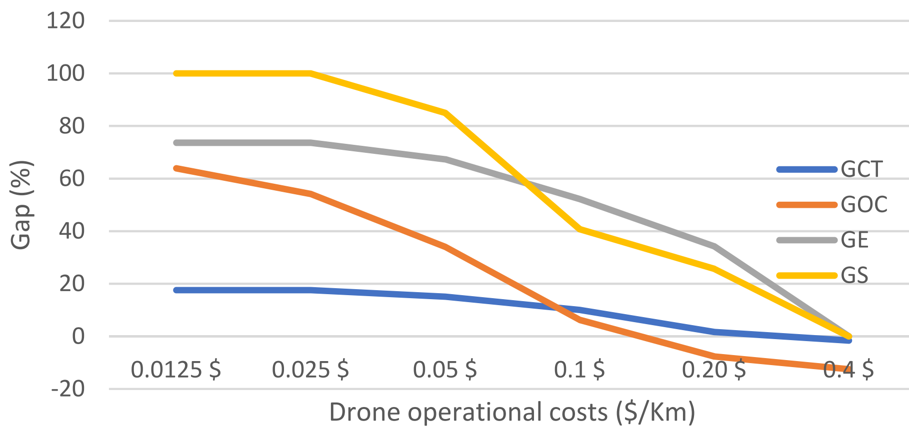


Fig. 8. The gap between the performance of *HTDS* and *TOS* for various values of drone operational costs, when $w_s = 0.25$.

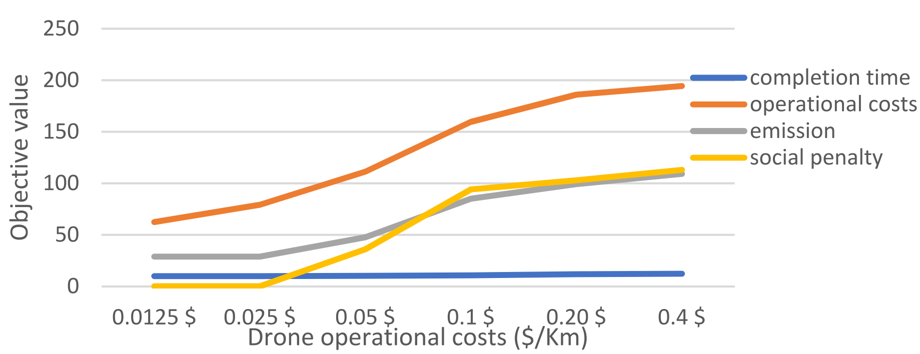


Fig. 9. System's performance for various values of drone operational costs, when $w_2 = 1$.

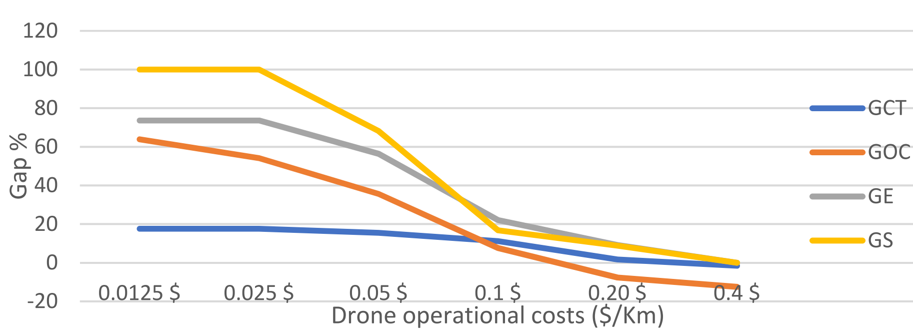


Fig. 10. The gap between the performance of *HTDS* and *TOS* for various values of drone operational costs, when $w_2 = 1$.

To analyze the effectiveness of our proposed stochastic model, we should compare the results of the stochastic and deterministic models for customer presence data over a long period of time. For this reason, we considered the delivery of all tasks of August*2022* and compared the objective value of stochastic and deterministic solutions for them. Based on the historical data, we assumed *0.72* for customer presence probability and presented the gap between the deterministic model and the stochastic model for real customers' existence status in Fig. 13. As it is

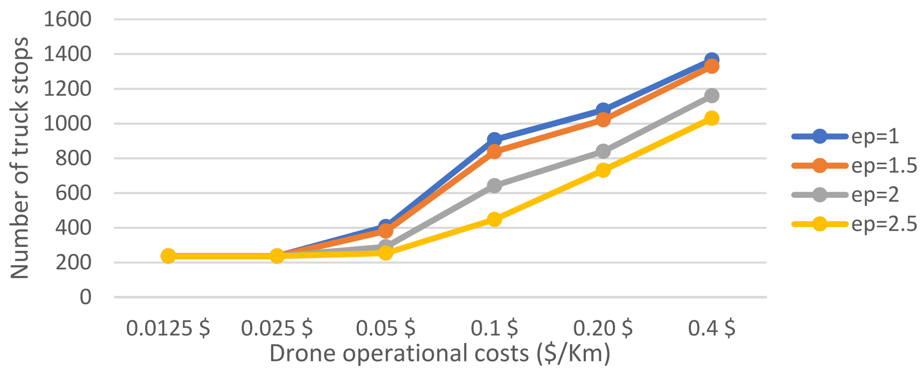


Fig. 11. The number of truck stop locations for various values of *ep* and drone operational costs, when $w_s = 0.25$.

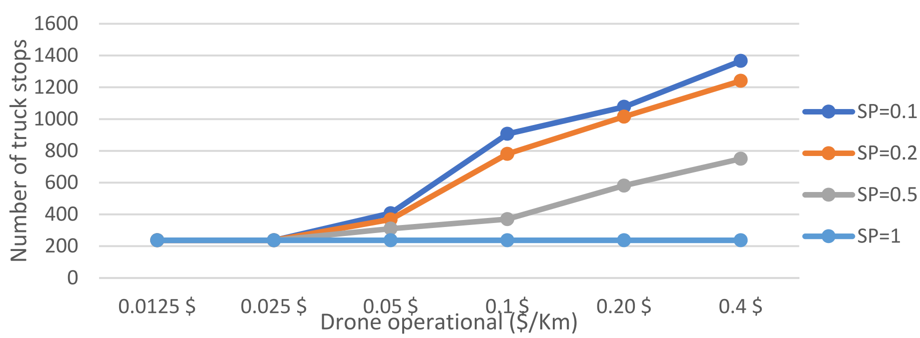


Fig. 12. The number of truck stop locations for various values of *sp* and drone operational costs, when $w_s = 0.25$.

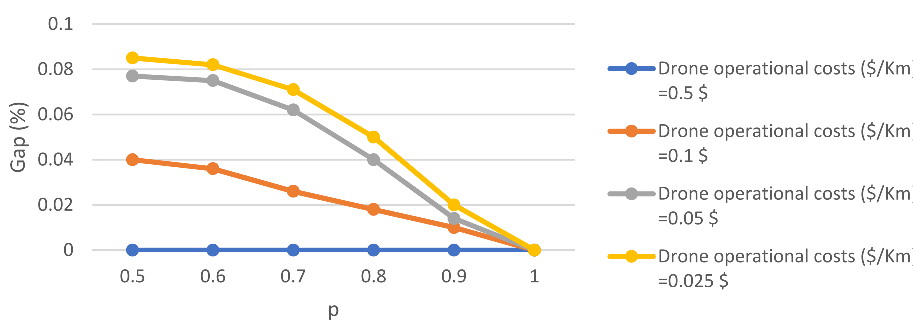


Fig. 13. The gap between the deterministic model and the stochastic model for various *p* and drone operational costs for a random data at august 2022.

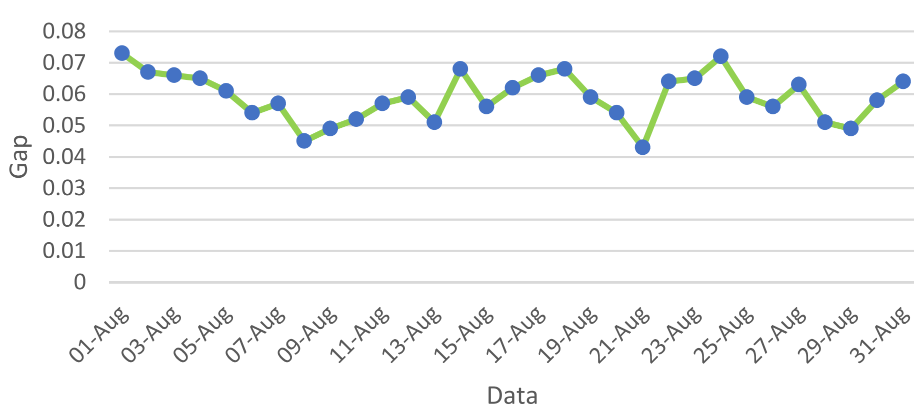


Fig. 14. The gap between the deterministic model and the stochastic model for delivery tasks of August 2022 and their real presence status.

demonstrated, the minimum gap between solutions of the stochastic and deterministic models is *0.043*, and the maximum gap is *0.073*. It highlights the efficiency of the proposed stochastic model.

Using locker boxes in last-mile delivery is mentioned as an effective and flexible delivery mode (Grabenschweiger et al., 2021). In a delivery system with stochastic customer presence, one may suggest delivery systems with locker boxes. Thus, this paper compares the drone delivery system and locker box delivery system for the case study data. In delivery systems with locker boxes, we have a location routing problem, where locker box locations must be defined, customers must be assigned to selected locations and vehicle routes must be scheduled. This problem is similar to our problem where there are only third-class customers and drone delivery time is not considered. In this section, we adapted our *HB&B*to the delivery system with locker boxes.

In the locker box delivery mode, even if customers are not present at delivery points, their parcels must be delivered to locker boxes. For this analysis, the customer compensation fee is assumed to becf, and the first, second and third objective functions can be formulated as follows:

$$Min\ obj_1 = \sum_{k,f \in \{S_4, De\}} \frac{d_{kf} TT_{kf}}{V_t} \tag{25}$$

$$Min\ obj_2 = \sum_{k,f \in \{S_4, De\}} Co_{tr} d_{kf} TT_{kf} + \sum_{k \in S_3, f \in S_4, r \in DR} h^r_{kf}.d_{kf}.cf \tag{26}$$

$$Min\ obj_3 = \sum_{k,f \in \{S_4, De\}} EMd_{kf} TT_{kf} \tag{27}$$

For this problem, TT_{kf} is a binary variable, which gets value of *1* if the truck travels from rendezvous node k to f.

In this section, we only considered third-class customers, so the fourth objective value will be equal to zero. We assumed that the covering radius equals *1000* m, and cf is equal to $\frac{Co_{tr}}{2}$. In our case, carriers must visit customer locations two times on different days. If a customer cannot receive his parcel at the defined location, the parcel will return to the delivery station. However, in the locker box delivery mode, trucks transport each parcel once. Thus, for better analysis, we considered delivery tasks of August 2022 and considered all of them to be from third-class customers. Then we corrected the data for locker box delivery mode and reduced the duplicated customer nodes on two consecutive days. For this case, Fig. 15 presents the gap between the average objective value in the stochastic hybrid truck-drone delivery model and the locker box delivery model for various drone operational costs. As it is shown, for drone operational costs smaller than *0.07 $* per kilometer, the hybrid truck-drone delivery system outperforms the locker box delivery system. However, when drone operational costs increase, it is suggested to use a locker box delivery system.

6. Conclusion

This study considers the completion time and three sustainability dimensions (economic, environmental, and social) as one decision criterion and a weighted multi-objective model for the sustainable hybrid truck-drone routing problem with stochastic customer existence developed. We have defined a recourse strategy for this delivery system: if a customer has no order in a scheduled plan, the system ignores that and services the next scheduled customer. We proposed a *MILP* model for the suggested delivery system and developed a Branch-and-bound and a heuristic Branch-and-Bound algorithm.

We have conducted a series of numerical experiments where our proposed solution methods provided encouraging results. *B&B* could solve instances with up to 28 nodes, and *HB&B* provides high-quality solutions in short CPU times for large-size instances.

Based on a realistic case evaluation, we obtain some insights. Generally, the stochastic model consistently outperforms the deterministic model, and it can reduce the objective value up to *8.5* percent for small values of customer presence probability. Truck and drone coordination reduces the weighted sustainable objective value by up to *75* percent. If only the completion time dimension is considered, for drone speeds faster than *10* m per second, *HTDS* outperforms *TOS*. If the logistic service provider focuses only on the operational costs, for drone operational costs larger than *0.2 $* per kilometer, *TOS* outperforms *HTDS* in the completion time and the operation cost criteria. For the sustainable weighted multi-objective model, increasing environmental and social penalties reduces the number of truck stop locations. It forces the system to service second class-customers with the drone. This paper also compares the locker box delivery system and drone delivery system, where the results highlight that for drone operational costs smaller than *0.07 $* per kilometer, the drone delivery system outperforms the locker box delivery system.

In urban delivery systems with congested areas, stochastic truck travel time is unavoidable. Future research could investigate that in hybrid truck-drone delivery systems. In real cases, logistic service providers implement a fleet of trucks and drones. Thus, it will be more practical to investigate hybrid truck-drone models with multi-trucks and multi-drones. As a limitation of drones, drone energy conservation is dependent on their speed and parcel weights, and future research could study parcel weights and drone speed interactions on truck-drone scheduling. Although delivery systems must work in most kinds of weather, drones are vulnerable to weather, and their performance is reduced in bad weather. So, it should be considered in future studies. The battery of drones has a limited life cycle, so its effects on economic and environmental criteria should be studied. Because of safety issues, some customers cannot trust the drone delivery system, so it is suggested to study truck-drone delivery systems with an alternative delivery. Finally, there are also opportunities to present more efficient solution methods.

CRediT authorship contribution statement

Ebrahim Teimoury: Conceptualization, Formal analysis, Funding acquisition, Investigation, Validation, Project administration, Resources, Supervision, Writing – original draft, Writing – review & editing, Resources, Supervision, Project administration. **Reza Rashid:** Conceptualization, Formal analysis, Data curation, Methodology, Investigation, Resources, Software, Writing – original draft,

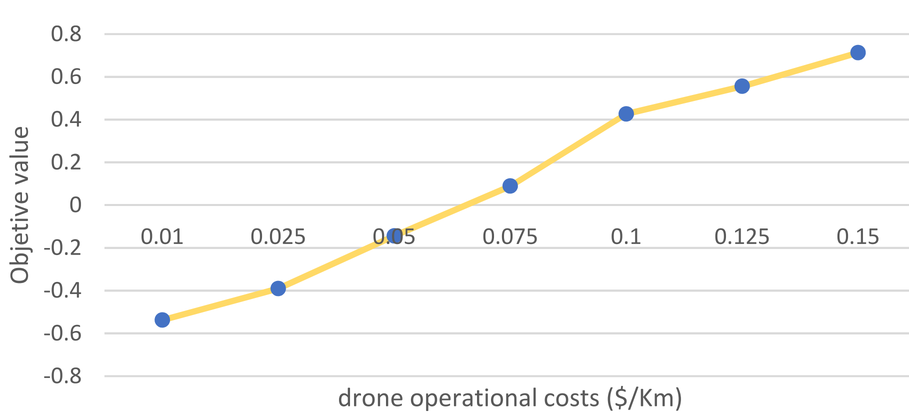


Fig. 15. The gap between the average objective value in the stochastic hybrid truck-drone delivery model and locker box delivery model for delivery tasks of August *2022*and various values of drone operational costs.

Visualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A. (Proof of lemma 1)

In the proposed directed graph, if the system services customer x, the probability for the system to service customer y exactly after x is not dependent on how the truck or drone arrived at location x. Considering this, we can model the proposed delivery system with a Markov chain, and the states could be assumed to be customer sequences. In this stochastic model, the probability of visiting each customer is equals p, so the probability of state $(1 < i < N)$ equals p. The system services j exactly after i, when there is no demand for all customers assigned between sequences of i and j, while customer j has an order and the system must consider visiting him. There are $(j - i - 1)$ customers between sequences i and j. *So,* if we are in state i, the probability of servicing j exactly after i would be equal to $p(1-p)^{j-i-1}$.

If there is a customer assigned to the truck stop k at sequence i and a customer assigned to the truck stop f at sequence j, the probability for the system to service j exactly after i and force the truck to travel between k and f equals $p^2(1-p)^{j-i-1}$. As we may assign more than one customer to a truck stop with different sequences, Formula *2* considers all related probabilities for each pair of sequences. For a better description, see Fig. 16. The red line demonstrates truck and drone transportation direction when the customer at sequence j is serviced exactly after the customer at sequence i.

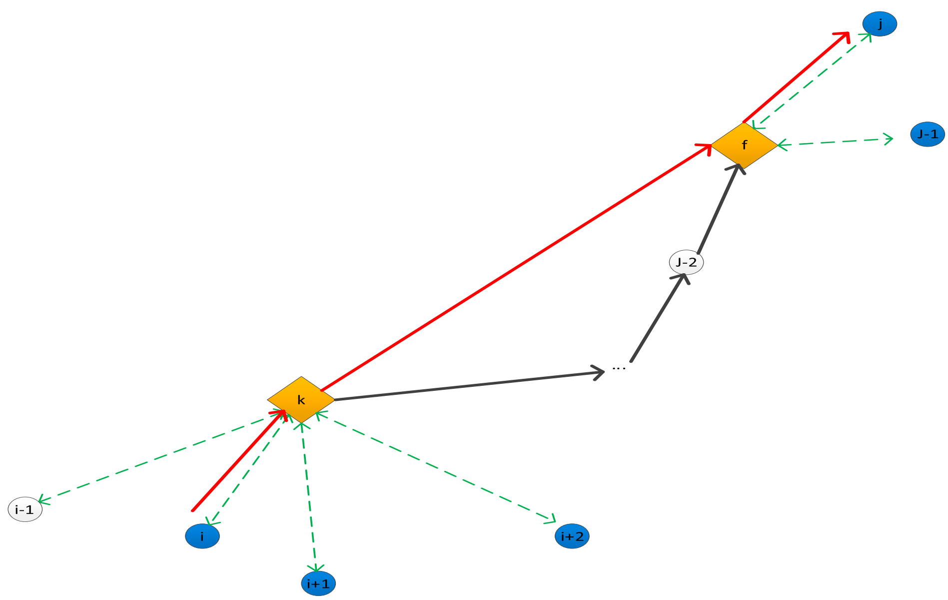


Fig. 16. The delivery system description is like this when customers assigned to sequences between i and j have no order.

Appendix B. (Proof of lemma 2)

In algorithm *3*, lines (*3* to *19*) calculate the minimum objective value changes required for servicing between each pair of customers or between the depot and customers.

In algorithm 1, for the scheduled customer ($a \in S_2$ *or* S_3), $\overline{set_a}$ is updated to the assigned truck stop, and for each pair of scheduled customers .$E\acute{C}C_{ja}$ calculates the exact objective values between related nodes. Because of knowing sequences of scheduled customers, ECC_{ja} (calculated in lines (20–25)) is the exact expected objective value for servicing customer a after j.

If a, j, or both of them are not scheduled, $E\acute{C}C_{ja}$ is equal to or less than the exact objective value for servicing node a exactly after node j. It is clear that if A is bigger than B, $(1-p)^A < (1-p)^B$. So, for calculating ECC_{ja}, bigger values of $E\acute{C}C_{ja}$ are reduced with smaller coefficient probabilities (because of larger values of ord_{ja} and smaller values of $(1-p)^{ord_{ja}}$). On the other hand, *EC4* always calculates the exact values for the fourth objective value. Consequently, $\sum_{\forall(j \neq p)} ECC_{ja}$ constantly is smaller than the expected objective values.

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