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# Pricing shared vehicles

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ABSTRACT

This paper analyzes profit-maximizing pricing in a model of shared vehicle (SV) market, with particular emphasis on spatial inequality of demand. I show that the best policy assigns a score to every location, and rewards (penalizes) customers for relocating the vehicle to a place with higher (lower) score. Such spatially explicit pricing enables providers to expand the vehicle dropoff “home” area into otherwise unprofitable low-density suburban areas and into for-fee parking zones. A greater geographic coverage has positive spillovers on operations within the initial home area. The empirical part of the paper uses novel microdata on SV trips to develop a strategy to estimate demand parameters, extrapolate them into larger counterfactual home area, evaluate optimal location scores, and predict profit gains from the expansion.

## 1. Introduction

### 1.1. Shared vehicles

Shared vehicle (SV) service is a rapidly growing industry, with dozens of operators serving multiple countries around the world. Estimates of the industry size vary, but are generally measured in hundreds of thousands of vehicles and tens of millions of customers.[1] While several modes of SV service are available, this paper focuses on vehicle sharing characterized by the following features. First, vehicles are owned or leased by dedicated *operators*. Second, customers of such service typically book a vehicle of choice via an operator-specific mobile app, walk to the vehicle, drive it while paying per minute, and drop it off at a permitted location. Vehicles are typically not serviced between customers and are available for the next booking immediately after drop-off. This paper is primarily focused on *free-floating* vehicle sharing, which allows drop-off at any legal parking location within a predetermined *home area*. The results of the paper can also be extended to station-based service that requires vehicles to be returned to one of several addresses.

SV technology has a potential to become a major method of passenger ground transportation around the world, as it has clear advantages over traditional transportation modes. Compared to public transit, shared vehicles have all advantages of private vehicles, as they relieve passengers from the need to (i) adjust to operator schedules, (ii) make connections, (iii) make unnecessary enroute stops, and (iv) share space with strangers. Free-floating SV also solve the last-mile problem as they can be dropped off at the destination.

In comparison with taxi or ridehailing, SV have a clear cost advantage, as the driver’s labor is done by the customer and therefore not included into the rental price.

In comparison with private vehicles, SV also have a cost advantage due to higher utilization and, consequently, shorter parking durations. According to a widely accepted estimate, private automobiles are utilized for only 5% of all time and are parked otherwise (e.g. Appendix B in Shoup, 2005); at the same time, the data described in Section 4.3 indicates that shared automobiles are utilized about 30% of all time. Therefore, SV technology allows to meet the same transportation demand with $\frac{.3}{.05} = 6$ times fewer vehicles and requires $6\frac{1-.05}{1-.3} \approx 8$ times less parking space, both of which dramatically reduce capital costs of transportation industry.

In addition, a private car must always be picked up where it has been previously dropped off, which may sometimes turn such car into a liability, rather than asset, for travelers using multiple modes of transportation. An SV user is not obliged to return to previously dropped off vehicle and can pick another one, located more conveniently, for the next ride. This means shared vehicles offer more mobility than private vehicles in congested cities.

A key drawback of SV business model is high cost of relocation of vacant vehicles, which makes this service viable only if a vehicle dropped off by a previous customer is expected to be booked by a next customer at the same location within reasonable period of time. This feature leads existing SV operators to limit their home areas to high-density urban districts. For example ShareNow, the world’s biggest provider of free-floating shared automobiles, serves only 16

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[1] “The Carsharing Telematics Market” by Berg Insight, accessed on Feb.1, 2021.

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large European cities; even in those cities, the home area is typically limited to high-density urban core. Low-density districts are typically excluded from home areas due to expectation of low vehicle turnover rate.

### 1.2. The contribution

The goal of this paper is to develop a spatially-explicit theoretic model of SV market, and to specify the profit-maximizing pricing policy. The model features arbitrary geography (i.e. the set of origins/destinations and distances between them) and spatial inequality of travel demand (i.e. heterogeneous rate of customer arrival, ranging from zero to infinity), as well as arbitrary home area and pricing policy by competing operators. In addition, the model features arbitrary location-specific per-minute parking fees that SV operators must pay, which enables to consider private garages and parking lots as part of home area.

In practice, SV operators typically charge a per-minute rate, regardless of pickup and dropoff locations within the home area, and do not allow drop-off at private for-fee parking lots. There are several reasons to think that such pricing policies are suboptimal, and that this paper can contribute to more efficient SV service, in terms of both operator profits and consumer surplus. First, the industry is new, less than 8 years old in most cities, and may have not yet converged to its steady state where competition causes extinction of suboptimal policies. Second, sharp discontinuity of dropoff policy at the home area boundary can hardly be considered profit-maximizing. Finally, lack of cooperation between SV operators and private parking garages also indicates a potential for welfare gains. Theoretically, shared vehicles have the most comparative advantage precisely in areas of expensive parking, because they spend much less time being parked. The existing SV operators did not yet learn to exploit this comparative advantage.

The above-mentioned feature of SV service, that relocation of vacant vehicles is economically impractical, implies that they are almost always located where previous customers dropped them off. Accounting for this feature, this paper uses recursive economics methods to formulate profit-maximizing pricing schedule.

The key theoretical finding is that every pick-up/drop-off location should be associated with a location *score*, with higher score typically corresponding to higher-travel-demand and/or lower-parking-fee locations. The location score reflects the expected profit to the operator from a vacant vehicle at that location, relative to other locations. Each customer, in addition to conventional per-minute rental rate, is charged the difference between the pick-up and drop-off location scores. Thus, customers traveling from city centers to suburban areas are typically charged an extra fee, while customers traveling in opposite direction earn a bonus of the same amount. In practice, some operators divide their home areas into two or three zones, with dropoff fees in more remote zones. Rewards for taking the vehicle out of the periphery are much less common, and address-specific fees (which enable dropoff at various parking lots with varying rates) are non-existent. For example GIG CarShare, primarily serving San Francisco and being the largest operator in the United States, does not allow dropoff at San Francisco curbside parking (i.e. does not use most of parking supply), presumably because of block-specific municipal parking rates.

While this paper does not explicitly analyze the consumer surplus, I show that profit-maximizing pricing policy allows SV operators to serve much larger areas, leading to unambiguous welfare gains not only for the operators but also for customers traveling to/from these newly added areas. Also, the density of vacant vehicles in the initial home area is shown to increase, which benefits customers traveling within the initial area.

In addition to theoretical contribution, this paper fits the model to novel data on carsharing trip demand in Moscow, Russia, featuring over 300 K trip observations. The primary objective of the empirical exercise is to develop a computational strategy to analyze the spatial predictors of trip demand, extrapolate estimates to predict trip demand to/from territories not served at the time of data collection, compute optimal counterfactual location scores, and predict profit gains from serving a larger area with pricing based on such scores. This computational methodology, after some adjustments, can be used by SV operators for real-world pricing decisions.

While this paper highlights only instantaneous effects of better pricing policy, dynamic effects may exist, too. Unlike private vehicles that cause only negative congestion externalities, shared vehicles also create positive agglomeration externalities: more vacant vehicles in an area means the nearest vehicle is more proximate to the customer, meaning more customers may wish to join the service. Thus, the demand observed in the data may be far from its steady-state level, and earlier entry of SV operators into new territory, facilitated by pricing policy of this paper, may have long-term consequences for industry growth.

### 1.3. Related literature

The idea of measuring location scores for suppliers of transportation services dates back to Koopmans (1949). Brendel et al. (2020) propose a very similar idea of lump-sum fees/bonuses, depending on pick-up and drop-off location, as part of SV fares. Their pricing was implemented by a real-world SV operator in Gottingen, Germany, and was shown to have a positive effect on vehicle demand. The magnitude of fees/bonuses however is not theoretically informed and is chosen experimentally. Such approach is practical with two or three pricing areas, but not with block- or address-specific pricing as proposed in the current paper.

Bimpikis et al. (2019), a theoretical analysis of optimal pricing in ride-hailing, in their appendix C derive results similar to location scores of the current paper. In a related paper, Zhang et al. (2019) propose introduction of negative prices to encourage optimal rebalancing of shared bicycles. The current paper goes beyond these theoretical results by offering an intuitive interpretation of location scores, an empirical computation strategy, and highlighting their role in the expansion of area of service.

Another strand of transportation literature proposes to maintain a desirable spatial distribution of vacant vehicles by offering drop-off-location incentives to individual customers. Wagner et al. (2015), in a counterfactual analysis of a free-floating SV service, study the effects of offering customers a small fixed bonus for altering their destination location by few hundred meters, towards a spot more desirable for the operator. Lippoldt et al. (2019) empirically analyze the effects of such policy implemented in practice. However, there is no theoretical proof of optimality of such policy; in particular, it is not clear why incentives for a dropping-off customer are better than incentives for the next (picking-up) customer.

The industrial-organization literature on vehicle sharing is still emerging. Cao et al. (2021) discover that entry of a second bike-sharing platform in a Chinese city *increases* the number of bikes supplied by the first platform. The suggested channel of causality is via network effects: an additional platform increases the overall number of consumers; the new consumers use the incumbent platform as well. Yan (2021) studies the spatial mismatch between customers and shared bikes, and ways to overcome it.

Beginning from Lagos (2000), a stream of economics literature models search frictions between passengers and vehicles in the taxi/ridehailing market; examples include Lagos (2003), Frechette et al. (2019), and Castillo (2020). Buchholz (2021), another example of this literature, offers spatially-explicit pricing for the New York taxi market. However, the latter has no concept of location scores; instead, price depends on location via district-specific multipliers, i.e. specific to origin but not to destination. Besbes et al. (2021) study ride-hailing surge pricing of similar type, and point out that, in spatially heterogeneous environment, it can lead to suboptimal relocation decisions by the drivers.

All above mentioned models beginning from Lagos (2000) present search frictions in the form of classic one-to-one matching of vehicles to customers. The matching process is typically a black-box "matching function"; Castillo (2020) details a matching procedure that mimics that of Uber, a private market mediator. The division of welfare surplus from a match is typically determined by the social planner, efficiently or not; in Castillo (2020) the social planner is replaced by Uber; in Brancaccio et al. (2020), Nash bargaining is assumed.

The model of this paper can also be viewed as a matching model, as it specifies (i) the probability of being matched to a vehicle for every customer, and (ii) the Poisson rate of customer arrival for every vacant vehicle. At the same time, SV operators, unlike ridehailing platforms, do not match their customers to specific vehicles but rather offer them the entire menu of vacant vehicles to choose from. In addition, full automation of the supply side excludes any possibility of bargaining; operators post take-or-leave price quotes instead. Given these features, the model of this paper is best viewed as a model of oligopoly or monopolistic competition between SV operators, where vehicle demand is imperfectly elastic due to idiosyncratic heterogeneity of vacant vehicle locations. A "toy" model of Section 3.3 also determines the equilibrium number of vehicles by the operator free-entry condition.

There is empirical literature analyzing various aspects of free-floating SV service. Becker et al. (2017) study the determinants of SV choice, as opposed to other transportation options, using data from Basel, Switzerland. Ampudia-Renuncio et al. (2020) analyze the spatial distribution of SV trips in Madrid. Balac et al. (2017) argue that increased parking rates will induce travelers to switch from personal to shared vehicles, as the latter typically have shorter parking durations.

Another strand of literature studies social impacts of SV service, dating back to McCarthy (1984). Mounce and Nelson (2019) is a recent example of this line of research, discussing the potential role of free-floating carsharing in future urban transportation systems, and highlighting its complementarity with public transit. In operations research, Banerjee et al. (2021) consider a model of station-based SV provision, that explicitly accounts for the number of vacant vehicles at each station. This paper focuses on computational issues of finding optimal state-dependent prices given high dimensionality of the problem. Jorge et al. (2015) pursues similar objectives.

No existing study, to the best of my knowledge, questioned optimality of existing SV home area or analyzed possible expansion of such area.

## 2. The model

### 2.1. Time and space

Define by *zone* a geographic area such that any two points within the area are of walking distance from each other, e.g. a residential neighborhood or a couple of blocks in the central business district of a city. Within a zone, all events occur in a single-dimensional space in which all locations are ex-ante identical to each other; one can think of each zone as being a circular street.

The world consists of a set $\mathcal{I}$ of heterogeneous zones[2] that are sufficiently distant from each other, so that vehicles are demanded for travel between them. One of transportation options is an SV.

Time in the model is continuous. For transparency of the main results, we will assume that all parameters in the model are time-invariant, thus only steady states will be analyzed.

[2] All mathematical notation is cataloged in Appendix A.

### 2.2. Travel demand

Emergence of potential customers in each zone $i$ willing to travel to another zone $j$ by SV is a Poisson process, such that the expected number of potential customers emerging during a time interval $\mathrm{d}t$ within a space segment of walking time $\mathrm{d}l$ is equal to $\lambda_{ij}\mathrm{d}t\mathrm{d}l$. We will refer to $\lambda_{ij}$ as (bilateral) *demand density*. The exact destination location within the destination zone is random, uniformly distributed across space, and is independent from the origin location. The ride between the two zones takes $h_{ij}$ minutes, regardless of the exact locations of origin and destination.

I assume every zone $i$ has strictly positive outbound $\sum_j \lambda_{ij} > 0$ and inbound $\sum_j \lambda_{ji} > 0$ travel demand. Furthermore, I assume that travel demand satisfies the *connectedness* condition: the set $\mathcal{I}$ of all zones cannot be partitioned into subsets $\{\mathcal{I}_1, \mathcal{I}_2\}$ such that $\lambda_{ij} = 0$ for every $i \in \mathcal{I}_1$ and every $j \in \mathcal{I}_2$. In other words, the SV service area cannot be split into two independent areas that can be analyzed separately.

It is quite plausible that customers' value of travel $v$ is proportional to $h_{ij}$, as the cost of their outside transportation options (e.g. of taxi) likely increases with travel time. I assume that customers have a heterogeneous $v$, distributed exponentially with mean $\theta h_{ij}$, where $\theta > 0$ is mean value per minute of travel: $\Pr(\text{value} > v) = \exp(-\frac{v}{\theta h_{ij}})$. The assumption of exponential value distribution contributes to analytical tractability of the model, as I show in Section 2.4 that other elements of the model are exponentially distributed, too.

Vacant vehicles are scattered along space within each zone. As the space within a zone is continuous while vacant vehicles are discrete, customers will almost surely need to walk to the SV of their choice. We assume that walking incurs a dollar-valued disutility $w$ per minute of walk.[3] During the walking time $x$, the vehicle must be reserved by the customer and unavailable for alternative uses.

As the space at the destination zone is homogeneous, all customers are always better-off traveling directly to the destination point, and no walking costs are incurred upon arrival.[4] The vehicle becomes available to other customers immediately after the end of the trip. Given random and uniform distribution of trip destination locations within each zone, vehicles available to the next customer are also distributed randomly and uniformly across space. Fig. 1 illustrates a typical trip by an SV. All customers have a reservation utility of zero: if the value of travel $v$ is below monetary and walking costs detailed below, the customer will not use an SV and take the outside opportunity (e.g. another transportation method).

The total price $p_{ij}$ for renting a vehicle may generally depend on the walking time $x$ between the customer's origin location and the vehicle, due to the need to reserve the vehicle during this time. Then, the customer selects the vehicle $k$ that minimizes the total cost of the trip $p_{ij}(x_k) + wx_k$. As elaborated in the previous paragraph, no vehicle is chosen if $v < \min_k\{p_{ij}(x_k) + wx_k\}$.

### 2.3. Travel supply

The supply side of the market is presented from the perspective of a typical SV company, *operator 0* henceforth, facing competition with other similar operators. Operator 0 has a fleet of SV such that, in the steady state, the density of its vacant vehicles in zone $i$ is $\mu_i$, i.e. on average there are $\mu_i\mathrm{d}l$ vacant vehicles in the space of walking

[3] Another potential cost, that of obtaining information about vehicles via an operator's mobile app, is omitted from analysis for mathematical tractability. Such cost creates an advantage for operators with more vehicles and therefore explains existence of operators with large market share.

[4] In congested cities, drivers may have to park some distance away from destination and walk. Because such cost of travel is not specific to shared mobility, it is not modeled explicitly. One could assume this time cost is part of travel time $h_{ij}$.

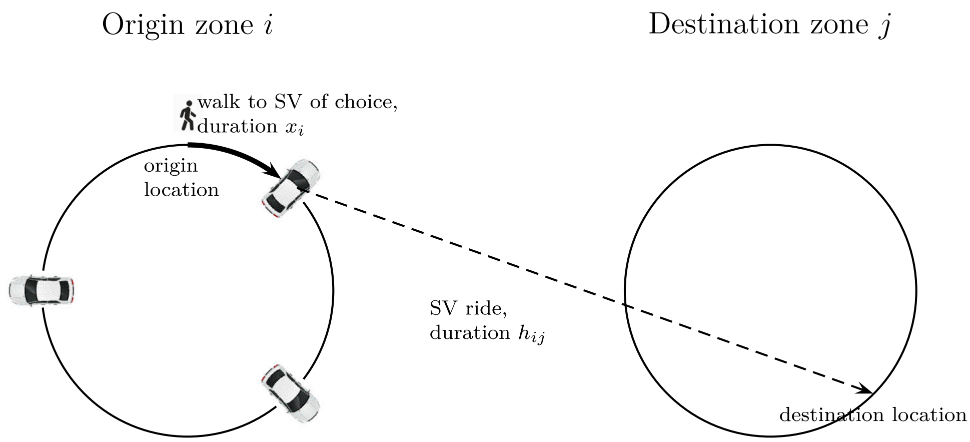


**Fig. 1.** Illustration of a typical trip. Vehicle image courtesy of Macrovector/Freepik.

time d$l$. Operator 0's vacant vehicles can be taken to any destination $j \in \mathcal{I}, j \neq i$. Because other operators may have different geographic coverage of service, the density of vacant vehicles by other operators is destination-specific and is denoted $\tilde{\mu}_{ij}$. For example, if some other operator allows its vehicles in zone $i$ to travel to zone $j$ but not to $k$, then such vehicles are included into the calculation of $\tilde{\mu}_{ij}$ but excluded from $\tilde{\mu}_{ik}$. In particular, $\tilde{\mu}_{ij} = 0$ if no other operator is serving zone $j$, meaning that operator 0 is a monopolist for that destination.

The price per trip chosen by operator 0 is assumed to be a linear function of walking time $x$ (but not necessarily linear in travel time $h_{ij}$) $p_{ij}(x) = p^a_{ij} + p'_i x$. Here $p'_i$ is the rate per minute of SV reservation, while $p^a_{ij}$ is the fare for the vehicle use. Likewise, other operators' price is $\tilde{p}_{ij}(x) = \tilde{p}^a_{ij} + \tilde{p}'_i x$. For notational simplicity, $\tilde{p}_{ij}(\cdot)$ is identical across all other operators.

Operator costs associated with each vehicle include:

- Movement costs (fuel, cleaning, maintenance, depreciation, etc.): $c$ per minute, $ch_{ij}$ per $i \to j$ trip;
- Parking costs of $g_i$ per minute in zone $i$. The parking cost is paid when the vehicle is vacant or reserved;
- Fixed costs $\phi_c$ per minute: vehicle lease or loan payments, insurance, and any other expenses unrelated to how the vehicle is used.

For each of its vehicles, operator 0 maximizes its stream of *gross profit*, equal to the stream of revenue minus movement and parking costs. As rental periods tend to be very short, measured in minutes or hours, there is no discounting of future periods: gross profit earned at 1pm has the same value to the operator as gross profit earned at 5pm of the same day. Instead of *discounted* stream of profit commonly used in economics literature, I will assume that operators maximize the *expected* gross profit earned per minute per vehicle, denoted $\phi$ and introduced formally in Section 2.5.

Section 3.3 considers equilibrium vehicle density determined by free entry of new operators, meaning that the average flow of gross profit $\phi$ equals the flow of fixed costs $\phi_c$.

### 2.4. Vehicle demand

Given spatial density of vacant vehicles $\mu_i$, the probability that a customer walking in a certain direction will reach a vacant vehicle within time $x$ is (dividing time interval $[0, x]$ into segments of infinitesimal length $\Delta$) $\lim_{\Delta\to 0} 1 - (1 - \mu_i \Delta)^{\frac{x}{\Delta}} = 1 - \exp(-\mu_i x)$.

Denote by $\mathbf{p} = \{p_{ij}(\cdot), \tilde{p}_{ij}(\cdot), \forall i, j\}$ the vector of price schedules by all operators. We now calculate the probability $D_{ij}(x, \mathbf{p})$ that a customer originating her trip in zone $i$ at distance of $x$ minute-walk from a specific vehicle $k$ of operator 0, and destined to zone $j$, will indeed book this vehicle. Such outcome will take place if all of the following three independent events occur:

1. The customer's value exceeds the travel cost, $v > p_{ij}(x) + wx$, which occurs with probability $\exp(-\frac{p_{ij}(x)+wx}{\theta h_{ij}})$.[5]
2. No other vehicle of operator 0 is more proximate to the customer. As the customer can walk in two directions, there should be no other vehicle within walking time $[-x, x]$ from the customer's position, which occurs with probability $\exp(-2\mu_i x)$.
3. Vehicles of other operators are distant enough. As trip costs of other operators may differ, denote by $z_{ij}(x, \mathbf{p})$ the maximum walking time between the customer's origin and some alternative vehicle more attractive than $k$: $z = 0$ if $\tilde{p}_{ij}(0) \geq p_{ij}(x) + wx$, and $\tilde{p}_{ij}(z) + wz \equiv p_{ij}(x) + wx$ otherwise. Linearity of $\tilde{p}_{ij}(\cdot)$ allows us to express $z$ explicitly:

$$z_{ij}(x, \mathbf{p}) \equiv \max\left\{\frac{p_{ij}(x) + wx - \tilde{p}^a_{ij}}{\tilde{p}'_i + w}, 0\right\}. \tag{1}$$

Vehicle $k$ will be taken if there are no competing vehicles by other operators within walking time $[-z_{ij}(x, \mathbf{p}), z_{ij}(x, \mathbf{p})]$ from the customer's position, which occurs with probability $\exp(-2\tilde{\mu}_{ij} z_{ij}(x, \mathbf{p}))$.

Given the above calculations, the probability $D_{ij}(\cdot)$ is given by

$$D_{ij}(x, \mathbf{p}) = \exp\left(-\frac{p_{ij}(x) + wx}{\theta h_{ij}} - 2\mu_i x - 2\tilde{\mu}_{ij} z_{ij}(x, \mathbf{p})\right). \tag{2}$$

Note that $z_{ij} = 0$ in (1) only if $p_{ij}(0)$ is lower than $\tilde{p}_{ij}(0)$ and only for sufficiently short walking time $x$. To streamline the exposition of the results that follow, we will ignore the zero lower bound on $z$. This will cause low-price operators to incorrectly estimate demand for their vehicles from customers within short walking time. This assumption does not bias the results that follow due to symmetry of operators in equilibrium.

[5] Note that if $p_{ij}(x)$ is negative, i.e. operator 0 is paying a bonus to its customer for the trip (which is relevant in some applications below), this probability may exceed unity. While mathematically incorrect, this feature proxies for the fact that negative prices may attract individuals who have no intrinsic value of an $i \to j$ trip, ride the vehicle only to collect the bonus, and thus were not included into the calculation of demand density $\lambda_{ij}$.

Given this assumption, $z_{ij}(x,\mathbf{p}) = \frac{p^a_{ij} - \tilde{p}^a_{ij} + (p'_i + w)x}{\tilde{p}'_i + w}$. The mean walking time before an $i \to j$ trip is equal to[6]

$$\bar{x}_{ij}(\mathbf{p}) = \frac{\int_{x=0}^{\infty} x D_{ij}(x,\mathbf{p})\mathrm{d}x}{\int_{x=0}^{\infty} D_{ij}(x,\mathbf{p})\mathrm{d}x} = \frac{\theta h_{ij}}{p'_i + w + 2\theta h_{ij}\left(\mu_i + \tilde{\mu}_{ij}\frac{p'_i + w}{\tilde{p}'_i + w}\right)}. \quad (3)$$

The mean walking time increases with travel time $h_{ij}$, as people traveling further will search for an SV within a larger area around trip origin. The relationship is linear if there are no alternative options ($\mu_i = 0, \tilde{\mu}_{ij} = 0$) but is concave otherwise, as presence of other vehicles imposes an upper bound on $\bar{x}_{ij}$. As one might expect, walking disutility $w$ and reservation cost $p'_i$ reduce the walking time, while competitors' reservation cost $\tilde{p}'_i$ increases it.

Define the *departure rate* $q_{ij}$ as the Poisson rate of vacant vehicle booking in zone $i$ for a trip to $j$:

$$q_{ij}(\mathbf{p}) \equiv 2\lambda_{ij}\int_{x=0}^{\infty} D_{ij}(x,\mathbf{p})\mathrm{d}x = 2\lambda_{ij}\exp\left(-\frac{p^a_{ij}}{\theta h_{ij}} - 2\tilde{\mu}_{ij}\frac{p^a_{ij} - \tilde{p}^a_{ij}}{\tilde{p}'_i + w}\right)\bar{x}_{ij}(\mathbf{p}). \quad (4)$$

### 2.5. Operator profit: formal definition

Define by the vehicle *cycle* the time between (i) the moment it becomes vacant in some zone and (ii) the moment it is dropped off by a customer in another zone. Each cycle then includes three stages: (i) vacancy, (ii) walking-time reservation, and (iii) travel-time vehicle use. Denote by $Q_i(\mathbf{p}) = \sum_{j\neq i} q_{ij}(\mathbf{p})$ the overall departure rate out of zone $i$ to any destination, so that $\frac{1}{Q_i}$ is the expected duration of vacancy at $i$.

Relocation of a vehicle between zones can be viewed as a Markov chain. Denote by $s_{ij}(\mathbf{p}) \equiv \frac{q_{ij}(\mathbf{p})}{Q_i(\mathbf{p})}$ the Markov transition probability that a vehicle cycle beginning at $i$ will end at $j$. Denote by $\tau_i$ the expected duration of a cycle originating at $i$, $\tau_i(\mathbf{p}) = \frac{1}{Q_i(\mathbf{p})} + \sum_{j\neq i} s_{ij}(\mathbf{p})(\bar{x}_{ij}(\mathbf{p}) + h_{ij})$. Denote by $\Psi_i$ the expected gross profit per cycle originating at $i$,

$$\Psi_i(\mathbf{p}) = -\frac{g_i}{Q_i(\mathbf{p})} + (p'_i - g_i)\sum_{j\neq i} s_{ij}(\mathbf{p})\bar{x}_{ij}(\mathbf{p}) + \sum_{j\neq i} s_{ij}(\mathbf{p})(p^a_{ij} - ch_{ij}). \quad (5)$$

Finally, the vector $\{f_1, \ldots, f_{|\mathcal{I}|}\}$ of steady-state distribution of cycle origins satisfies the standard Markovian condition

$$\sum_{j\neq i} s_{ji}(\mathbf{p}) f_j(\mathbf{p}) = f_i(\mathbf{p}), \forall i. \quad (6)$$

Given the above notation, the expected gross profit flow $\phi$ is the maximal ratio of expected gross profit per cycle to the expected cycle duration:

$$\phi = \max_{\forall p^a_{ij}, p'_i} \frac{\sum_i f_i(\mathbf{p})\Psi_i(\mathbf{p})}{\sum_i f_i(\mathbf{p})\tau_i(\mathbf{p})}. \quad (7)$$

Maximization of (7) is equivalent to maximization of the expected gross profit *net* of foregone opportunities during the cycle:

$$\max_{\forall p^a_{ij}, p'_i} \sum_i f_i(\mathbf{p})(\Psi_i(\mathbf{p}) - \phi\tau_i(\mathbf{p})). \quad (8)$$

Under the profit-maximizing (highest) value of $\phi$, the maximal value of (8) is zero.

[6] In (3) we assume distance $x$ between a vacant vehicle and potential customer ranges from zero to infinity, while Section 2.1 has assumed that both customer and vehicle are on the circle of finite size. While it is straightforward to impose a finite upper bound on $x$ in (3) and thereafter, unbounded $x$ proxies for the possibility of walk across two or more neighboring zones to reach a vacant SV.

## 3. Analysis

### 3.1. Optimal price

To find the optimal price menu that maximizes (8), it is convenient to employ a recursive method of analysis. Given heterogeneity of origin/destination zones, presence of a vacant vehicle in different zones may be of different value to the operator. For example, presence of a vehicle in a low-demand-density zone may reduce the operator's profit due to long time of waiting for the next customer. To quantify such heterogeneity, denote by $R_i$ the value of a vacant vehicle being in zone $i$, relative to the value of being in the "numeraire" zone 1 with $R_1 = 0$. Intuitively, $R_i$ is the expected profit gain from the fact that the vehicle is in $i$ rather than in zone 1. Alternatively, $R_i$ can be interpreted as the expected cumulative profit, on top of "normal" profit flow $\phi$, for a vacant vehicle in zone $i$, until the moment it ends up in the "numeraire" zone 1. We will refer to $R_i$ as the *score* of zone $i$.

For optimal price, consider a vacant SV parked in some zone $i$. A customer who appeared $x$-min walk away from the SV and destined to zone $j$ will book the vehicle with probability $D_{ij}(x,\mathbf{p})$. In case of booking, the operator's profit net of foregone opportunities is $p_{ij}(x) - g_i x - ch_{ij} - \phi(x + h_{ij})$; the value of the vehicle dropped off in $j$ is $R_j$.

The probability that such customer appears within time period $\mathrm{d}t$ and within walking-time interval $[x, x+\mathrm{d}x]$ is $2\lambda_{ij}\mathrm{d}t\mathrm{d}x$; the coefficient 2 here is because the customer may appear on both sides of the vehicle.

If the vehicle was not reserved during the time interval $\mathrm{d}t$, the operator incurs a parking cost $g_i\mathrm{d}t$ and a foregone profit of $\phi\mathrm{d}t$. The value of the vehicle remaining in $i$ is $R_i$.

Given this analysis, the score $R_i$ can be defined as follows:

$$\begin{aligned} R_i = \lim_{\mathrm{d}t\to 0}\mathrm{d}t \max_{p_{ij}(\cdot),\forall j} \sum_{j\neq i} 2\lambda_{ij}\int_{x=0}^{\infty} D_{ij}(x, p_{ij}(x))\left[p_{ij}(x) - ch_{ij}\right.\\ \left. -\phi(h_{ij} + x) - g_i x + R_j\right]\mathrm{d}x\\ + \left[1 - \mathrm{d}t\sum_{j\neq i} 2\lambda_{ij}\int_{x=0}^{\infty} D_{ij}(x, p_{ij}(x))\mathrm{d}x\right]\left[-(g_i + \phi)\mathrm{d}t + R_i\right]. \end{aligned} \quad (9)$$

In (9), the profit-maximizing price $p_{ij}(x)$ for a particular value of $x$ can be found independently from other values of $x$, by maximizing the integrands. The first-order condition reads (dividing by $\mathrm{d}x\mathrm{d}t$, dropping the higher-order terms)

$$\frac{\mathrm{d}D_{ij}(x,\mathbf{p})}{\mathrm{d}p_{ij}(x)}\left[p_{ij}(x) - (c+\phi)h_{ij} - (g_i + \phi)x + R_j - R_i\right] + D_{ij}(x,\mathbf{p}) = 0. \quad (10)$$

From (2), we have that $\frac{\mathrm{d}D_{ij}(x,\mathbf{p})}{\mathrm{d}p_{ij}(x)} = -\frac{D_{ij}(x,\mathbf{p})}{m_{ij}h_{ij}}$, where

$$m_{ij} = \frac{\theta(\tilde{p}'_i + w)}{\tilde{p}'_i + w + 2\theta h_{ij}\tilde{\mu}_{ij}}. \quad (11)$$

Then, the profit-maximizing price $p^{OPT}_{ij}(x)$ can be found from (10) as

$$p^{OPT}_{ij}(x) = (m_{ij} + c + \phi)h_{ij} + (g_i + \phi)x + R_i - R_j, \quad (12)$$

consistently with our linearity-in-$x$ assumption. Intuitively, the price consists of three elements. The optimal per-minute reservation cost $p'^{OPT}_i = g_i + \phi$ simply covers parking fees and foregone opportunities during the reservation time. (12) implies that the operator should not attempt to make more profit during that time, as a higher rate would reduce the walking range and thus the number of customers.

The remaining component $p^{a\,OPT}_{ij} = (m_{ij} + c + \phi)h_{ij} + R_i - R_j$ is further divided into vehicle use fare and the relocation fee. The per-minute vehicle use fare includes movement cost $c$, opportunity cost $\phi$, and the per-minute *markup* $m_{ij}$. The latter is used to cover operator's losses during vehicle vacancy. From (11), the per-minute markup is

- constant and equal to $\theta$ when there is no competition with other operators $\tilde{\mu}_{ij} = 0$,
- decreasing with $h_{ij}$ from $\theta$ to zero when competition exists, $\tilde{\mu}_{ij} > 0$.

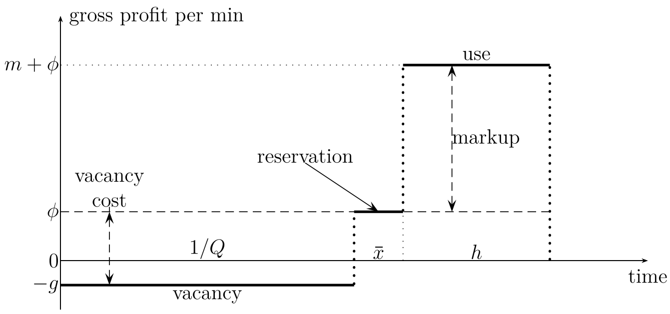


**Fig. 2.** Flow of gross profit during a typical cycle.

Thus, the customer always pays $c+\phi$ per minute of vehicle use, i.e. the operating cost plus the foregone gross profit per minute; in addition, she should be charged a markup up to $\theta$ per minute, with highest markup corresponding to (i) a monopolized market or (ii) short trips. The markup increasing with monopolization is a standard result in the industrial organization literature. The per-minute markup decreasing with trip duration is because longer trip duration makes customers search for vacant vehicles in larger area, and to walk longer distances towards a cheap option, essentially increasing the price elasticity of demand.

Finally, the relocation fee $R_i - R_j$ is a lump-sum fee (bonus) for taking the vehicle to a location that is less (more) attractive to the operator.

Given (11), the term $\frac{p^a_{ij}}{\theta h_{ij}} + \frac{2\tilde{\mu}_{ij} p^a_{ij}}{\tilde{p}'_i + w}$ in (4) is equal to $\frac{p^a_{ij}}{m_{ij} h_{ij}}$. Denote $\mathbf{p}^{OPT} = \{p^{OPT}_{ij}(\cdot), \tilde{p}_{ij}(\cdot), \forall i, j\}$ the price schedule optimal for operator 0, given price schedules of others. Then, the optimal departure rate (4) is

$$q_{ij}(\mathbf{p}^{OPT}) = 2\lambda_{ij} \exp\left(-1 - \frac{c+\phi}{m_{ij}} + \frac{R_j - R_i}{m_{ij} h_{ij}} + \frac{2\tilde{\mu}_{ij}\tilde{p}^a_{ij}}{\tilde{p}'_i + w}\right) \bar{x}_{ij}(\mathbf{p}^{OPT}). \quad (13)$$

By plugging (12) back into (9), dropping the second-order terms, and rearranging, we end up with the following system of equations (cf. (13)):

$$g_i + \phi = \sum_{j \neq i} q_{ij}(\mathbf{p}^{OPT}) m_{ij} h_{ij}, \forall i. \quad (14)$$

In the above equation, the left-hand side is the expected per-minute cost of a vacant vehicle in zone $i$, equal to the sum of foregone gross profit and parking cost. The right-hand side is how much profit (above the normal flow $\phi$) a vacant vehicle is expected to generate in the next minute; it is equal to the product of the departure rate, price markup per minute of next-trip use, and duration of the next trip.

This is a system of $|\mathcal{I}|$ equations with an equal number of unknowns: gross profit flow $\phi$ and $|\mathcal{I}| - 1$ zone scores $R_i, i = 2 \ldots |\mathcal{I}|$.

### *3.2. Properties of optimal price*

To develop intuition of the optimal markup and profit calculations, Fig. 2 illustrates the flow of gross profit during a typical vehicle cycle, dropping zone subscripts. To account for vehicle opportunity costs, gross profit should be compared to its expectation $\phi$ rather than to zero. Profit-maximizing pricing equates the expected loss during the vacancy phase $\frac{\phi+g}{Q}$ to the gain during vehicle use, $mh$, which is captured by optimality condition (14), for every zone. Note that $m+\phi$ in Fig. 2 is the gross profit during the vehicle-use phase *without* relocation fees; the latter do not affect profit directly because their expected flow is zero, and their primary role is to regulate vacancy durations.

To understand properties of zone scores $R_i$, observe that both $m_{ij}$ and $h_{ij}$ in (14) are bounded from above. Then, (13) implies that a low demand density $\lambda_{ij}, \forall j \neq i$ in some origin zone $i$ should be balanced by a reduced zone score $R_i$, to make this zone profitable for the operator. The relationship here is logarithmic, so when demand density approaches zero, the zone score approaches negative infinity. At the same time, $R_i$ has a finite upper bound because (i) the demand density is finite, (ii) $R_1 = 0$ and the difference $R_i - R_1$ must be finite due to connectedness of zones 1 and $i$.

A lower zone score, in turn, means that the trip cost is increased for the customer taking the vehicle to that zone, and is reduced by the same amount for the next customer who will take the vehicle out of the zone. Intuitively, a price discount for outbound customers helps to boost departures from low-demand zones; the discount is funded by the previous customer who left the vehicle in such zone.

A higher parking rate $g_i$ in (14) should be balanced by higher departure rates $q_{ij}$, which is achieved by lower zone score $R_i$. For example, if an SV operator expands its home area to a private garage with parking rates higher than those of nearby municipal parking, the garage should be assigned a lower score. This means that a customer who drops off a vehicle at the garage will be charged an extra lump-sum fee, while the next customer who takes the vehicle out of the garage will earn a lump-sum bonus of the same amount; the bonus accelerates the departure out of the garage. On Fig. 2, higher per-minute cost of vacancy at the garage $\phi + g_i$ is offset by a shorter vacancy duration $\frac{1}{Q_i}$; the total cost of vacancy (area of rectangle) should remain constant if the garage location score is right.

### *3.3. Demand density and equilibrium*

This section studies how SV market equilibrium properties depend on overall demand density. For that purpose, we consider the simplest possible spatial structure with only two identical zones and with symmetric travel demand between them. Due to symmetry, all location subscripts are dropped throughout this section. Because for any origin there is only one possible destination, we have $s_{ij} = 1$; symmetry of the two zones also implies $f_i = \frac{1}{2}$, hence the two components of the sum in (8) are identical. Zone symmetry also means $R_i = 0$. We also assume zero parking rates $g$.

In equilibrium, all operators charge identical rates for reserved vehicles $p' = \tilde{p}' = \phi$. The profit-maximizing markup (11) is then $m(\tilde{\mu}) = \frac{\theta(\phi+w)}{\phi+w+2\theta h\tilde{\mu}}$, and the trip fare is

$$p^a(\tilde{\mu}) = (m(\tilde{\mu}) + c + \phi)h. \quad (15)$$

Furthermore, assume all operators have small fleets so there are no other vehicles by the same operator in a given area, $\mu = 0$. Then the mean walking time (3) is (dropping the price argument throughout this section) $\bar{x}(\tilde{\mu}) = \frac{\theta h}{\phi+w+2\tilde{\mu}\theta h} = \frac{m(\tilde{\mu})h}{\phi+w}$. The departure rate (4) is

$$q(\lambda, \tilde{\mu}) = 2\lambda \exp\left(-\frac{p^a(\tilde{\mu})}{\theta h}\right) \bar{x}(\tilde{\mu}). \quad (16)$$

Because $m(\tilde{\mu}) < \theta$ for $\tilde{\mu} > 0$, the departure rate is increasing in $m(\tilde{\mu})$ and thereby decreasing in $\tilde{\mu}$.

Then, optimality condition (14) is simplified to

$$\phi = q(\lambda, \tilde{\mu}) m(\tilde{\mu}) h. \quad (17)$$

In free-entry equilibrium, all operators are breaking even, meaning that the equilibrium gross profit $\phi$ is equal to fixed cost $\phi_c$. Then, Eq. (17) becomes a condition on equilibrium density of vacant vehicles $\tilde{\mu}$. We can study the effects of exogenous parameters on such density. For example, because both $m(\tilde{\mu})$ and $q(\lambda, \tilde{\mu})$ are decreasing with $\tilde{\mu}$, while $q$ is proportional to $\lambda$, we can conclude that a higher demand density $\lambda$ is increasing $\tilde{\mu}$. The relationship however is not linear. For $\lambda$ smaller than $\lambda_0 = \frac{1}{2}\frac{\phi_c(\phi_c+w)}{(\theta h)^2} \exp\left(1 + \frac{c+\phi_c}{\theta}\right)$ such that $\phi_c \equiv q(\lambda_0, 0)m(0)h$, SV service is not sustainable and the equilibrium vehicle density is zero. This is because, even in the absence of competition, demand for a vacant vehicle is limited by the walking costs. But as the demand density $\lambda$ rises to infinity, vehicle density is asymptotically a square root of demand density, $\tilde{\mu} \approx \left(\frac{\lambda}{2} \exp\left(-\frac{c+\phi_c}{\theta}\right) \frac{\phi_c+w}{\phi_c}\right)^{\frac{1}{2}}$. The relationship

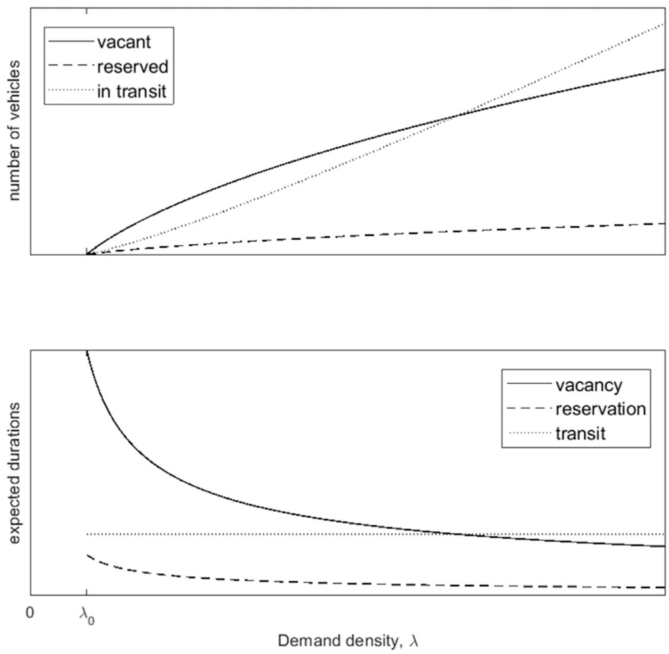


**Fig. 3.** Number of vehicles and expected durations by phase.

is concave because higher vehicle density reduces both price markup and the customer walking range, thus affects expected revenue more strongly than demand density does.

Besides the density of vacant vehicles $\tilde{\mu}$, it is useful to calculate the density of vehicles reserved by walking customers. It is equal to the density of vacant vehicles $\tilde{\mu}$, times the rate at which vacant vehicles become reserved, $q(\lambda, \tilde{\mu})$, times the expected duration of reservation $\bar{x}(\tilde{\mu})$. By comparing the resultant expression against the equilibrium condition (17), we conclude that the equilibrium density of reserved vehicles is $\frac{\phi_c}{\phi_c+w}\tilde{\mu}$, i.e. is a constant fraction of the density of vacant vehicles, and therefore is also asymptotically proportional to $\lambda^{\frac{1}{2}}$.

Similarly, we can find the equilibrium number of vehicles in transit, per unit of zone (parking) space: it is equal to the product of vacant vehicle density $\tilde{\mu}$, departure rate $q(\lambda, \tilde{\mu})$, and trip duration $h$. Asymptotically, as both $\lambda$ and $\tilde{\mu}$ grow large, this quantity approaches $\lambda h \exp\left(-\frac{(c+\phi_c)}{\theta}\right)$, i.e. is asymptotically proportional to the demand density $\lambda$. We may conclude that in low-density municipalities, the ratio of vacancy time to revenue time is higher, which justifies the lower bound on demand density for the SV service to become viable. Fig. 3 visualizes these findings.

### *3.4. Expansion to periphery*

This section extends the stylized example of the previous section to theoretically investigate SV operation in low-density areas that are adjacent to higher-density areas with existing service. We label the two symmetric zones of Section 3.3 as "central", and suppose there is also a third "peripheral" zone. For mathematical tractability, we will assume the travel time between any two zones is still $h$, and parking costs $g$ in all zones are still assumed zero. The density of demand for travel between the peripheral and a central zone is denoted $\lambda_{01}$, with $\lambda_{01} < \lambda$.

First, we consider the case when SV operators use location-invariant pricing, and investigate conditions that make SV operators unwilling to serve the peripheral zone under such pricing. Suppose the initial state coincides with that of Section 3.3: there are multiple small operators who serve only the central zones, the density of vacant vehicles $\tilde{\mu}$ in central zones is defined by (17), there are no vacant SV in the peripheral zone, and $\phi = \phi_c$. The pricing policy of all operators is given by $p' = \phi_c$ and $p^a = p^a(\tilde{\mu})$ given by (15).

Suppose one of operators considers expansion of service into the peripheral zone with the same pricing policy. What is the impact of such expansion on the net profit per cycle (8)? The latter consists of three elements corresponding to three phases of each cycle: vacancy, reservation, and use. Given location-invariant tariff structure, the revenue from vehicle use $p^a$ does not depend on origin and destination; so does the cost of vehicle use $(c + \phi_c)h$. Therefore, home zone expansion does not affect this component of (8). Further, the revenue from reservation time $p'x$ under optimal $p'$ always equals the opportunity cost of such time $\phi_c x$. Therefore, although the mean walking/reservation time $\bar{x}$ is higher for trips to/from the peripheral zone due to lack of competition with other operators, the net profit from this phase of the cycle remains unchanged and equal to zero.

Therefore, net profit (8) of the expanding operator changes only via changing vacancy times. The latter is reduced in the central zones, due to addition of the new destination, but can be high in the peripheral zone if demand density $\lambda_{01}$ is low. Appendix B shows that, when $\lambda_{01} \to 0$, the unconditional expected vacancy duration of the expanding operator is doubled, dropping its net profit below zero. As $\lambda_{01}$ increases, expected vacancy duration drops, eventually making the operator's net profit positive. The cutoff demand density is $\hat{\lambda} = \frac{\lambda(\phi_c+w)}{2(\phi_c+w+2\tilde{\mu}\theta h)}$; no operator will want to expand under location-invariant pricing unless $\lambda_{01} \geq \hat{\lambda}$.

In contrast, Section 3.1 has shown that any zones can be served under optimal pricing, as long as the system-wide gross profit $\phi$ is sufficient to cover the fixed cost $\phi_c$. Under optimal pricing, expansion of home area cannot reduce $\phi$;[7] therefore, if operators were able to survive serving central zones (if $\lambda > \lambda_0$ in the notation of Section 3.3), they can always profitably expand to the peripheral zone.

What are the welfare effects of such expansion? Under free entry, SV operators continue to earn zero net profit. New customers, traveling between center and periphery, are clearly better off, as they get access to previously non-existent service. To see the effect on initial customers, traveling between the two central zones, consider the change in central-zone vacant SV density $\tilde{\mu}$. Besides departures to the other central zone at rate $q(\lambda, \tilde{\mu})$, these vehicles now also depart to peripheral zones with some rate $q_{01} > 0$ and price markup $m_{01} > 0$. Therefore, the equilibrium condition (17) becomes $\phi_c = \phi = q(\lambda, \tilde{\mu})m(\tilde{\mu})h + q_{01}m_{01}h$. Because $\phi_c$ is constant, the term $q(\lambda, \tilde{\mu})m(\tilde{\mu})$ must decrease for the equation to remain true, which means (as discussed under Eq. (16)) vacant vehicle density $\tilde{\mu}$ must increase. Such increased density benefits consumers via two channels: reduced walking times $\bar{x}$, and reduced price markups $m$.

Thus, we may conclude, expansion of SV home area under optimal pricing is a Pareto improvement. If any home area is found where SV service may survive, it can be expanded indefinitely if complemented by optimal pricing, as derived in Section 3.1.

## 4. An empirical application: overview and data

The rest of the paper conducts various empirical tests of the above theoretical model, using novel SV trip microdata from Russia. The main focus is the carsharing market of Moscow, for which parameters of the model were calibrated or estimated. Then, I conduct a counterfactual experiment that allows a small share of the fleet to serve a larger counterfactual area, using location-specific pricing based on Section 3.1. The proposed methodology can be used for real-world pricing decisions of SV operators, after adjustments to include data available to SV operators but not available within the present study.

[7] For proof, observe that exclusion of zone $i$ from home area is equivalent to $R_i = -\infty$. Optimization over $R_i$ in (7) can only increase $\phi$.

### 4.1. Cross-market comparisons

The empirical model is based on real-time data on vacant shared vehicles around Russia published at an aggregator website carsharing.gde-luchshe.ru in December 2019. The website collected data from the majority of Russian SV operators, those that were willing to share data, and published it in HTML format. The data includes: vehicle location, model, operator, fuel left. Importantly, a unique vehicle ID is also provided, which allows us to track vehicles over time. Data quality was verified by manually comparing real-time data from gde-luchshe.ru to that from mobile apps of the respective operators. The data collection method was pioneered by Kortum et al. (2016) and Wielinski et al. (2018), for SV trips in Western Europe and in North America.

The data used in this paper was downloaded with 5-minute intervals from 10:43am on December 17, 2019, until 8:48am on December 24, 2019, with occasional lapses due to website unavailability. The total number of time observations is 1976.

The observed operators served a total of 15 Russian cities; several other cities were covered by unobserved operators. These cities are sufficiently distant from each other and can be treated as independent markets, which allows us to conduct back-of-envelope tests of theoretical predictions of Section 3.3. Because larger cities tend to be more densely populated, demand density $\lambda$ can be roughly proxied by population size.[8] Section 3.3 predicts that SV service can exist only in dense enough cities; in the data, 10 out of 15 cities with SV service have population over 1 million people. Moreover, within this sample of cities, the observed number of SV units per capita is positively correlated with population size (Fig. 4, top panel). Lack of data from some operators means the true number of SV units is greater; however, because most vehicle supply is by operators serving multiple cities, it is unlikely that an operator decision to share data is correlated with size of individual cities. Hence, the correlation between the two variables should not be affected by the measurement error.

Section 3.3 also predicts that lower demand density should be associated with longer vacancy times. Fig. 4, bottom panel, corroborates this prediction.

### 4.2. The market of Moscow

According to truesharing.ru, a shared mobility news outlet that catalogs SV operators, carsharing service emerged in Moscow in 2015 and had been growing rapidly until the Covid pandemic. In late 2019, Moscow was served by 14 free-floating carsharing operators; additional 4 operators specialized in certain suburban areas of the city. The fleet size has doubled in 2019, reaching approximately 30000 vehicles by the end of the year. Of these, over 95% were operated by the "Big Four" operators, Yandex Drive (about 50% of the market), Delimobil (30%), BelkaCar and YouDrive. Vehicle models, rates, and home areas were quite similar across operators. The primary tariff was a location-invariant per-minute rate, which could depend on the time of the day; daily and sometimes 3-hour tariffs were also available. Operators also charged a much lower reservation rate, but universally offered 20–25 min of free time to walk to the vehicle. The home areas of all operators included nearly all territory inside the MKAD (a loop highway around the city with 15–20 km radius), as well as certain hand-picked high-density areas outside of MKAD. The outside-MKAD home area was quite limited: approximately 280 km$^2$ for Delimobil, 180 km$^2$ for Yandex Drive, and only 95 km$^2$ for BelkaCar,[9] i.e. a small fraction of the 30000 km$^2$ Moscow region, or even of 900 km$^2$ of the inside-MKAD territory. Fig. 5 illustrates the home area of Delimobil (i.e. the largest among existing operators), labeled as the *D-area* henceforth, in early 2020.

[8] Official city limits in Russia are often redrawn at will of individual politicians, making official city areas and population densities an unreliable measure of real urban density.

[9] Author's estimate based on published home area maps

**Table 1**
Percentiles of rental duration distribution.

| Percentile | 10 | 25 | 50 | 75 | 90 | 99 |
|---|---|---|---|---|---|---|
| Rental duration, min | 10 | 20 | 35 | 60 | 95 | 245 |

As of 2022, the market is still dominated by the same Big Four. The competitive fringe shrank from 10 to 6 firms; all 4 suburban operators shut down.

### 4.3. SV trip data

Among the Big Four operators mentioned in Section 4.2, data from Delimobil and YouDrive is available; seven small operators are also observed. The observed operators represent about 40% of the Moscow region SV market. Similar home areas and pricing policies of all operators, observed and unobserved, along with limited ability of operators to choose strategically the location of vacant vehicles, imply that observed operators are representative of the whole market.

Define *the Moscow region* as the rectangle of territory between $[55.3750N, 56.0458N]$ of latitude and $[37.1125E, 38.1167E]$ of longitude, as illustrated on Fig. 5, totaling $75 \times 62 = 4650km^2$. In total, 10507 vehicles, belonging to 9 operators, were observed vacant within the Moscow region during at least one of 1976 time moments. On average, each of these vehicles was observed vacant 63.85% of the time. Of all observed vehicles, 8748 (83.3%) belonged to Delimobil, 1283 (12.2%) to YouDrive, and another 476 (4.5%) to remaining operators. Almost all vehicles had gasoline engines; over 75% of them were economy class such as VW Polo and Hyundai Solaris.

Because the actual revenue trips are not directly observed, they have to be inferred from the data. For a given vehicle, by a *movement event* I define a combination of start and end locations and times such that (i) the vehicle was not available between start and end times and (ii) either latitude or longitude of the vehicle have changed by more than 100 m. A total of 497 K such movement events were identified. For every such event, define the *movement speed* as the ratio of (i) the direct distance between start and end locations and (ii) difference between end and start times. I discard movement events with (i) speed exceeding 60 kmh (4.9% of all observations), as these are often errors in location recording, and (ii) speed below 2 kmh (31.9% of observations), which are likely to include non-revenue trips, or rental trips with long stopovers, or rental round-trips. The remaining 314 K movement events are labeled as (one-way) *rental events*; the duration of each rental event includes the walking time $x$ and the time of vehicle use, $h$.

The mean duration of a rental event is 49.5 min; some distribution percentiles are shown in Table 1.

Because data was recorded with 5-minute intervals, two or more consecutive rental events of the same vehicle could have been recorded as one, if the vacancy period between the trips fell within a 5-minute interval between data recordings.

Not surprisingly, there is a clear daily cycle in travel demand. The peak demand is observed during rush hours of 6–8 am and 7−9 pm, with 3000+ rental events per hour initiated during this time. The lowest demand is between 3am and 4am, with less than 400 rental events per hour initiated. While time-varying demand can make optimal pricing vary over time, as well, this paper considers only time-invariant prices for transparency of other results, leaving the time dimension for future research.

### 4.4. Zones and predictors of demand

To predict travel demand to/from areas currently not served by SV operators, we associate such demand econometrically with its potential spatial predictors. The data I use includes brightness of night lights, as well as various information from OpenStreetMap (OSM).

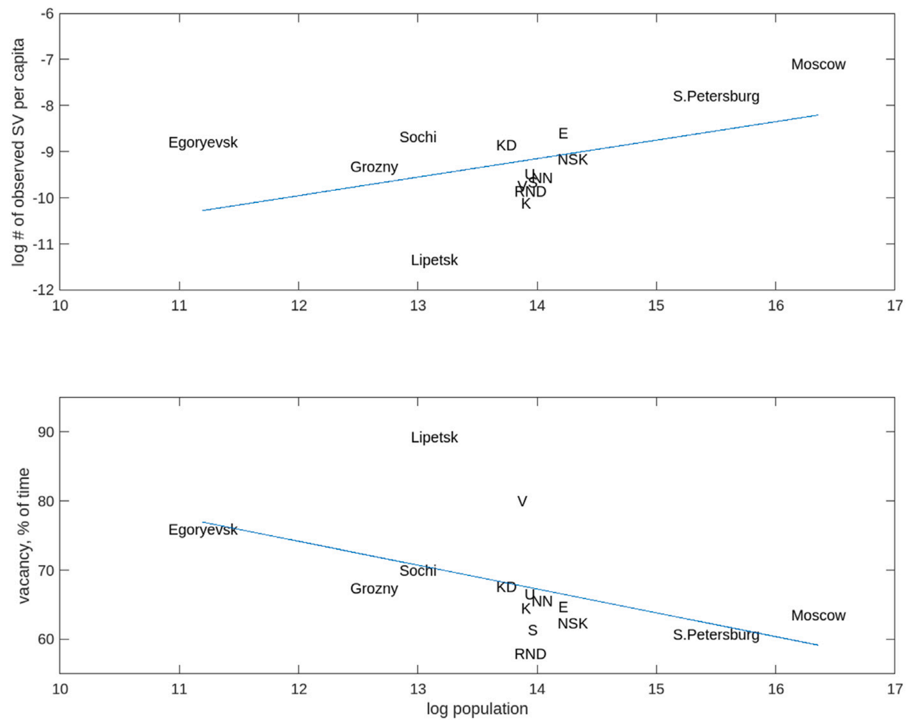


**Fig. 4.** City size and use of shared cars. Abbreviation: E — Ekaterinburg, K — Krasnoyarsk, KD — Krasnodar, NN — Nizhny Novgorod, NSK — Novosibirsk, RND — Rostov na Donu, S — Samara, U — Ufa, V — Voronezh.

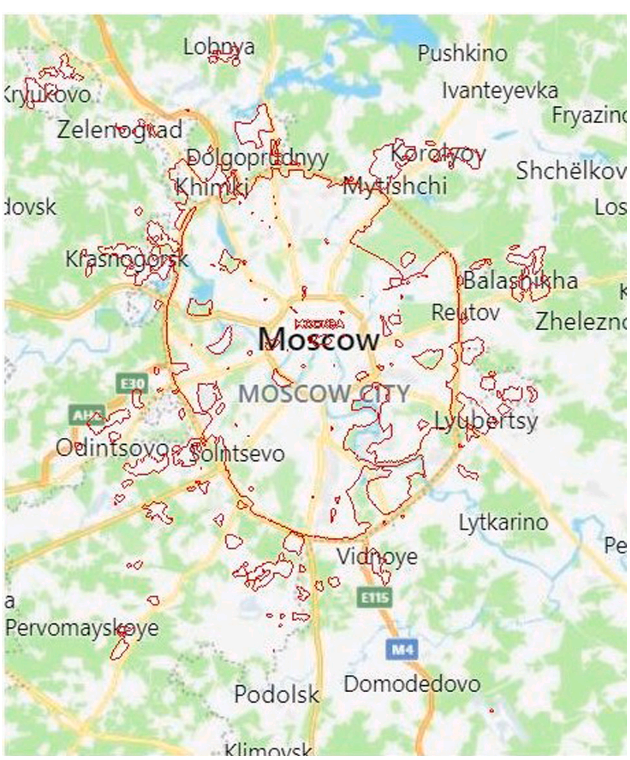


**Fig. 5.** Home area of *Delimobil* (the *D-area*) in the Moscow region, January 2020. Sources: Bing maps, Delimobil.

Night light radiance is commonly used in economics to predict local economic activity. I use the standard source of such data, published by the Earth Observations Group at NOAA/NCEI. The data is provided monthly for every rectangle of the Earth surface measuring 15 arc-seconds of latitude (about 463 m) and 15-arc seconds of longitude (259 m at the latitude of Moscow). The spatial units of the theoretical model, the zones, are set equal to these night-light rectangles in the empirical analysis below. Thus, the Moscow region spans 161 zones North–South and 241 zones East–West, each zone occupying 12 hectares of land. Variable $lnlight_i$ is defined as the log of night light radiance in zone $i$. To eliminate short-term lights caused by construction or social events, I take the minimum of two 2019 months, November (latest available at the time of download) and March.

The OSM data used, all for December 2019, includes: building floor space, points of interest, accessibility by automobile, bus, and rail. Variable $fls_i$ is defined as total floor space of all buildings located in zone $i$. Floor space of each building, in turn, is calculated as product of building footprint and the number of levels. OSM provides each building coordinates in polygon format, which allows to calculate the footprint. The number of levels is provided for about 40% of buildings; for others, I estimate the number of levels as non-parametric function of footprint and building type, using buildings with observed levels.

The variable $poi_i$ is defined as the total number of points of interest in zone $i$; these include categories like “playground”, “restaurant”, “pharmacy”, “government”, “supermarket”, or “hotel”.

The dummy $access_i$ indicates accessibility of zone $i$ by automobile, as evidenced by presence of the following objects: presence of certain categories of “highway lines” such as “living street”, “primary” or ”road” (but not “footway” or “steps”); presence of parking (excluding “private”, “permit”, etc.); presence of “highway points” (meaning road intersections). Although SV service is theoretically impossible in zones with $access_i = 0$, empirically we do observe vehicles in such zones; this can be explained by errors in OSM categorization of infrastructure.

The dummy $bus_i$ indicates the presence of a bus stop in zone $i$. Likewise, $MT_i$ indicates presence of a rail station or a subway entrance in zone $i$. Covariance of mass transit infrastructure with SV demand is ambiguous: on the one hand, presence of mass transit indicates high population density, and also allows connections between SV and public transportation; on the other hand, it may imply better alternatives to SV trips.

**Table 2**
Zone-level mean values of demand predictors: Delimobil home area (D-area) and elsewhere.

| Demand predictor | D-area | Other Moscow region |
|---|---|---|
| Night light radiance | 55.4161 | 16.9350 |
| Floor space (1000 $m^2$) | 87.3903 | 11.5368 |
| Points of interest | 12.2056 | 0.7721 |
| Accessible | 0.8678 | 0.4387 |
| Bus | 0.5257 | 0.0821 |
| Mass transit | 0.0559 | 0.0038 |

### 4.5. Home areas of SV operators

As mentioned above, most destinations outside of MKAD are currently not served by SV operators. To correctly assess the model parameters, it is therefore essential to know which zones in our model are included into the home area of which operator. This paper uses the data on the boundaries of home areas of the Big four operators in Moscow. Even though we do not have trip data for two of these operators, Yandex Drive and BelkaCar, information on their home areas is used to assess the density $\tilde{\mu}$ defined in Section 2.3.

Moscow SV operators do not publish their home areas in a machine-readable format; the data for each operator was available only in the image format via the operator's mobile app. To digitize the data, screenshots from the apps were made, geolocated, and processed.

Delimobil, the largest provider by observed fleet and by home area, served 8783 zones out of 38801 in the Moscow region; the set of these zones constitutes the D-area and is denoted $\mathcal{I}_D$. Table 2 provides the mean values of demand predictors outlined in Section 4.4, in the D-area and elsewhere in the Moscow region. It is evident that SV operators hand-pick territory with high economic activity for their home areas.

## 5. Fitting the model to data

### 5.1. Supply-side parameters

A number of techniques were used to fit parameters of the theoretical model of Section 2 to Moscow SV market. Parameters of the supply side, such as operating costs $c$, transit times $h_{ij}$, and supply of competing vehicles $\mu_i, \tilde{\mu}_{ij}$, $i, j \in \mathcal{I}_D$, were calibrated using the data of Section 4.3, SV operator price quotes, and other publicly available data about the industry. The details of calibration are provided in Appendix D.

To calibrate the per minute rate $r$, used to infer other parameters below, we use the price schedule of Delimobil, described in Appendix D.3. Averaging the price across all rental events yields approximately $r = 6.75$ RUB/min (0.11 USD/min in Dec. 2019).

The gross profit $\phi$ is calibrated by assuming it is equal to the fixed cost of operation $\phi_c$, which in turn consists mainly of vehicle purchase costs. Assuming new economy-class vehicles were bought for 800 K RUB,[10] funded via 3-year loans with 12% interest,[11] and sold for 480 K RUB after three years,[12] the associated annuity cost is 184.6 K RUB/year. The other major fixed costs are insurance, estimated at 35 K RUB/year, and that of unlimited parking permit, 27.6 K RUB/year in 2019.[13] Given these estimates, we infer $\phi_c = (184.6K + 35K + 27.6K)/365 = 677$ RUB/day.

[10] A typical list price of VW Polo and Hyundai Solaris, the most common economy SV, in Dec. 2019

[11] A typical car loan interest rate in Dec. 2019

[12] YouDrive, one of major operators, leases new vehicles from any investor, and buys them out for 60% of initial value after 3 years of service, presumably only to auction them off for the same expected price. Source: https://youdrive.partners/, accessed on Jan 21, 2021.

[13] Source: https://truesharing.ru/news/15684/, accessed 21.01.2021.

The annual parking permit used by SV operator exempts them from paying per-minute parking fees, hence we assume $g_i = 0$ throughout the rest of the paper.

### 5.2. Demand-side parameters

The demand-side parameters are estimated from fitting the observed trip data to predicted trip demand. To remove potential effects of the operator or vehicle type, in the estimation procedure we will focus on the economy-class vehicles by Delimobil, which include various versions of Hyundai Solaris, Kia Rio, Renault Kaptur and Sandero, and VW Polo. One observation is then a cycle by an economy-class Delimobil vehicle, that begins with some period of continuous vacancy and ends with a rental event defined in Section 4.3, and with both origin and destination inside the D-area. Such cycles are denoted *sample cycles*; their total number is 256 K, or 81.6% of all rental events in the Moscow region. Such choice of observation sample not only includes most available data, but is also most relevant for a study of a counterfactual home area expansion, because (i) Delimobil already served more territory than other operators, and (ii) residents in lower-density areas tend to have lower incomes and are thus more likely to demand economy-class vehicles.

The bilateral demand density $\lambda_{ij}$ is assumed to follow the gravity law

$$\lambda_{ij} = A_i A_j H(h_{ij}). \tag{18}$$

Here $A_i$ is *spatial demand density* in zone $i$. For 22 zones with 200+ observed originating cycles, $A_i$ is estimated individually (i.e. is a zone fixed effect), to capture possible correlation between trip observations with origin or destination at $i$. For other zones, $\ln A_i$ is assumed to be a linear combination of observed zone characteristics:

$$\begin{aligned}\ln A_i(\beta) = \beta_0 + \beta_l lnlight_i + \beta_{acc} access_i + \beta_{bus} bus_i + \beta_{MT} MT_i \\ + \beta_{fls} fls_i + \beta_{poi} poi_i + \beta_{R2} R2_i,\end{aligned} \tag{19}$$

where $R2_i$ is an indicator of zone $i$ being inside the Garden ring,[14] and other variables are defined in Section 4.4. The vector of unknowns $\beta$ then includes the 22 zone fixed effects and all betas in (19).

Function $H(\cdot)$ in (18) measures the effect of distance on trip demand, and is assumed piecewise linear in logs:

$$\ln H(h, \gamma) = \begin{cases} \gamma_1 \ln h, h \le \hat{h} \\ \gamma_2 \ln h + (\gamma_1 - \gamma_2) \ln \hat{h}, h > \hat{h}, \end{cases}$$

where cutoff $\hat{h}$ is assumed to equal 15 min. We have $\gamma = \{\gamma_1, \gamma_2\}$; we can also consider $\lambda_{ij}$ as function of unknowns $\beta, \gamma$.

The cost of walking $w$ is identified from sensitivity of vehicle demand to presence of other vehicles in the same zone, varying both over time and across space.

Due to lack of data on actual fares for every rental event, the parameter $\theta$ is not identifiable from demand data alone. For example, in (4), an increase in $\theta$, which proportionately increases all $\theta h_{ij}$, will result in the same departure rate $q_{ij}$ if complemented by a proportionate increase in $w$ and exponential increase in all $\lambda_{ij}$. This paper derives $\theta$ from the supply side of the SV market, from the identifying assumption that SV operators choose their per-minute rate $r$ optimally within the class of flat location-invariant free-reservation rates, as discussed further in Appendix C is thus identified from (C.3).

[14] A loop avenue around the city center with 3 km radius.

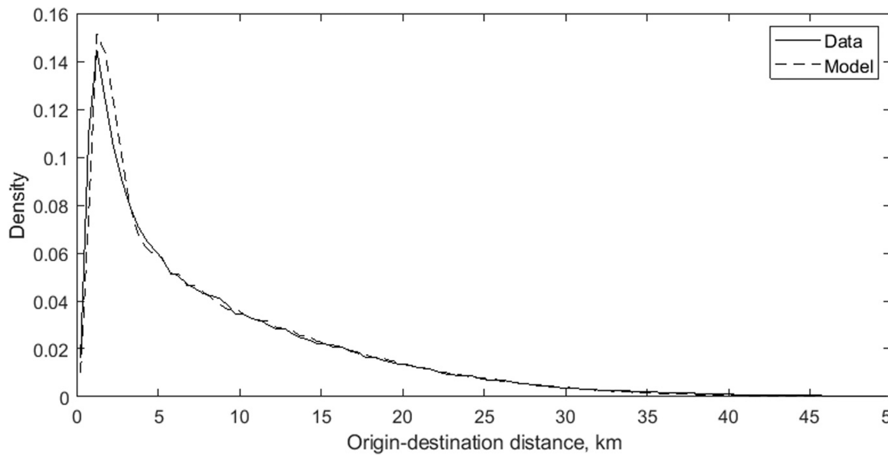


**Fig. 6.** Distribution of direct distance between origin and destination, model vs. data.

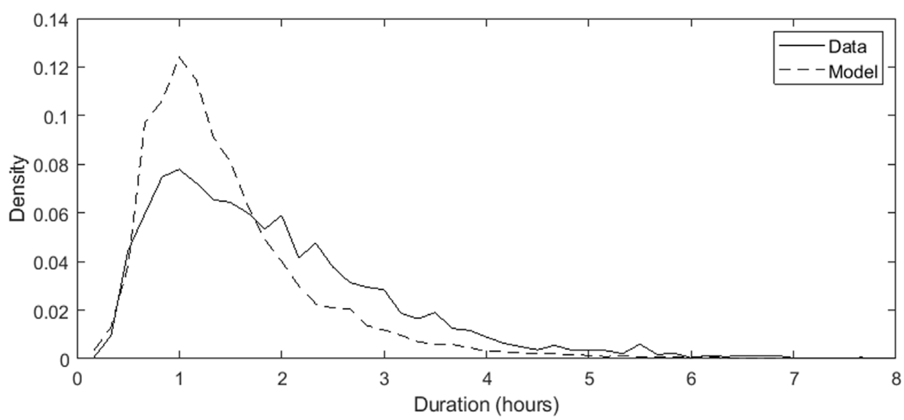


**Fig. 7.** Distribution of mean vacancy durations across zones, model vs. data.

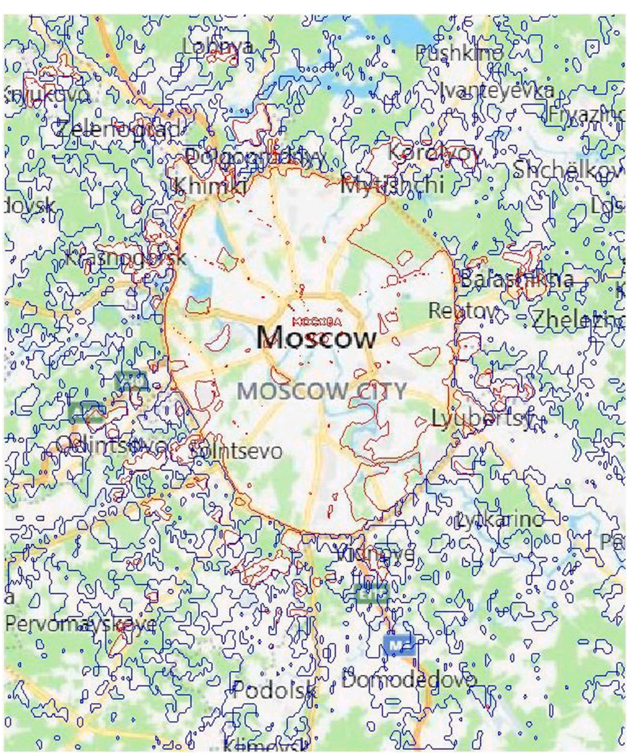


**Fig. 8.** Red boundary: D-area, blue boundary: new home area. Map source: Bing.

### 5.3. *The econometric model*

The unknown demand-side parameters $\Theta = \{\beta, \gamma, \theta, w\}$ are estimated via maximum likelihood using sample cycles defined in Section 5.2. For every sample cycle $k$, we make use of the following data: location of origin and vacancy $i(k) \in \mathcal{I}_D$, beginning of vacancy time $t_0(k)$, end of vacancy time $t_1(k)$, and terminal location (the trip destination) $j(k) \in \mathcal{I}_D$.

At any given moment $t$, the probability of a vacant vehicle in zone $i$ to be booked for zone $j$ within infinitesimal period $\mathrm{d}t$, given flat pricing outlined in Appendix C, is (cf. (4))

$$q_{ij}(\Theta, t)\mathrm{d}t = 2\lambda_{ij}(\beta, \gamma) \exp\left(-\frac{r}{\theta}\right) \frac{\theta h_{ij}}{w + 2\theta h_{ij}(\mu_i(t) + \tilde{\mu}_{ij}(t))}\mathrm{d}t. \tag{20}$$

The probability that a vehicle remains vacant in zone $i$ from time $t_0$ until $t_1$ is $\exp\left(-\int_{t_0}^{t_1} Q_i(\Theta, t)\mathrm{d}t\right)$, where $Q_i(\Theta, t) \equiv \sum_{j \neq i} q_{ij}(\Theta, t)$. Therefore, the likelihood of an observed sample cycle $k$ is given by $\exp\left(-\int_{t_0(k)}^{t_1(k)} Q_{i(k)}(\Theta, t)\mathrm{d}t\right) q_{i(k)j(k)}(\Theta, t_1(k))\mathrm{d}t$, and the unknowns $\Theta$ can be found by solving

$$\max_{\Theta} \sum_k \left[-\int_{t_0(k)}^{t_1(k)} Q_{i(k)}(\Theta, t)\mathrm{d}t + \ln q_{i(k)j(k)}(\Theta, t_1(k))\right] \tag{21}$$

subject to constraint (C.3).

To reduce dimensionality of the problem and computer memory requirements, 8783 destinations (but not origins) were aggregated into 2683 clusters of up to 2 × 2 zones. Because vehicle demand depends on the number of competing vehicles in the origin, but not destination, zone, it is desirable for parameter estimation to preserve the most granular definition of the origin zone.

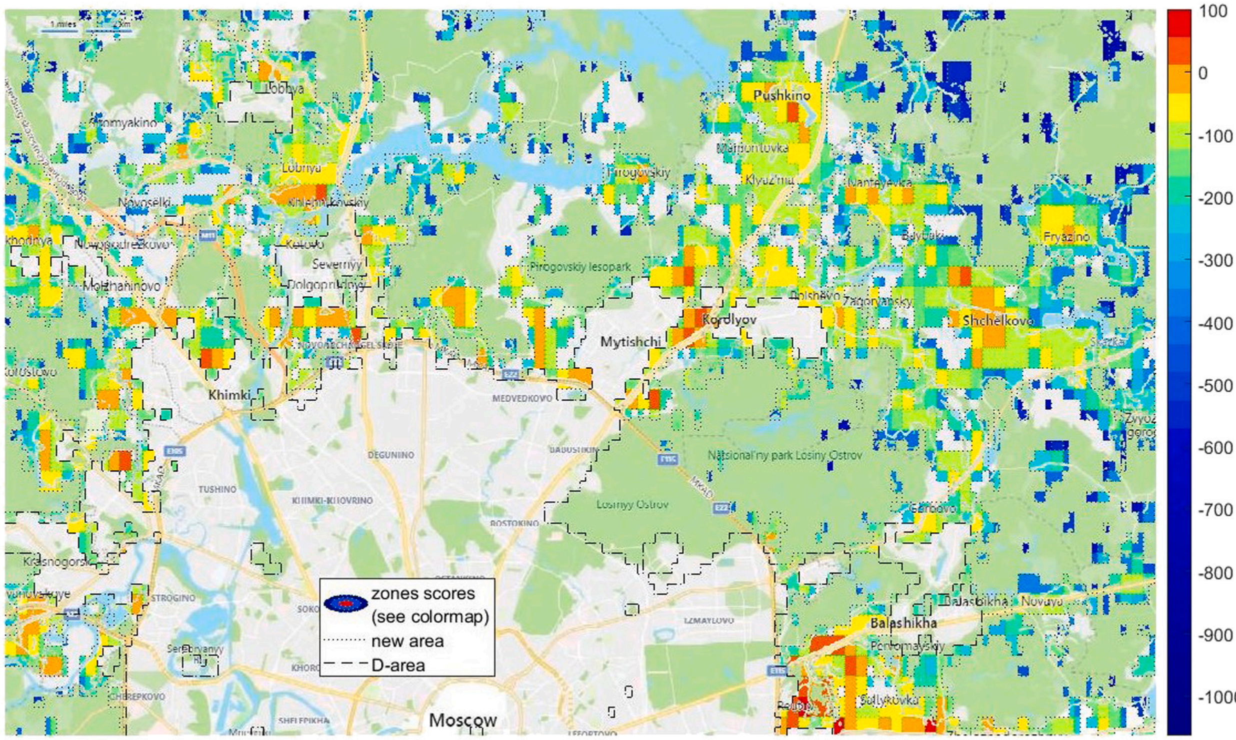


**Fig. 9.** Zone scores, North-East of Moscow. Map source: Bing.

**Table 3**
Estimated model parameters and $t$-statistics.

| Parameter | Notation | Value ($t$-stat) |
|---|---|---|
| Mean value of travel, RUB/min | $\theta$ | 3.690(7.73) |
| Walking cost, RUB/min | $w$ | 27.58(17.54) |
| Effect of $h$ under 15 min | $\gamma_1$ | 0.1795(0.64) |
| Effect of $h$ over 15 min | $\gamma_2$ | −3.394(−35.79) |
| Constant | $\beta_0$ | 1.256(1.66) |
| Log night lights | $\beta_l$ | 0.03821(0.51) |
| Road access | $\beta_{acc}$ | 1.228(4.00) |
| Bus stop | $\beta_{bus}$ | 0.1946(2.21) |
| Train or subway | $\beta_{MT}$ | 0.2196(1.59) |
| Floor space (1000 m$^2$) | $\beta_{fls}$ | 0.003144(5.88) |
| Points of interest | $\beta_{poi}$ | 0.003377(2.11) |
| Inside Garden ring | $\beta_{R2}$ | −0.5877(−3.06) |
| Fixed effects for 22 zones | | yes |

Table 3 presents the estimation results. The value of $w$ implies that a typical customer is willing to pay 27.6 RUB (i.e. the cost of $\approx$ 4 min rental period) for an SV located 1 min-walk closer. The value of $\theta$ implies that the price elasticity of total demand for vehicles by all operators is $-\frac{r}{\theta} = -1.83$;[15] it also implies that, in the absence of competition with other operators, price markup could have been 3.96 RUB/min rather than currently estimated $m_0 = 1.13$. The values of $\gamma$ mean that, for trips up to 15 min, demand is insensitive to travel time; a 1% increase in travel time beyond 15 min reduces the number of customers considering travel by 3.4%.

### 5.4. Goodness of fit

To verify goodness of fit, we replace $\mu_i(t)$ and $\tilde{\mu}_{ij}(t)$ in (20) by their averages over time $\mu_i, \tilde{\mu}_{ij}$, and calculate all model parameters in the resultant time-invariant environment. Thus, the model parameters are calculated without accounting for the daily cycle of vehicle use that tends to concentrate vacant vehicles in the center during working hours and in the periphery at night; the goodness-of-fit would be even higher if this cycle was accounted for.

Table 4 compares the empirical distribution of sample cycle origins against the distribution $f_i$ predicted by (6). The results are compared across four areas of Moscow, arranged from most central to most peripheral.

The density of vacancies in the city center is underestimated, perhaps because the model ignores the hub-and-spoke road pattern, reducing the travel costs around the city center. The density in the periphery is underestimated too, meaning that the counterfactual exercise of Section 6 offers a conservative estimate of SV demand in areas currently not served.

Fig. 6 illustrates how the model fits the observed distribution of trip distances. Despite only two degrees of freedom in the corresponding parameter $\gamma$, the match is almost perfect. The mean geodesic trip distance is 8.95 km empirically vs. 8.71 km predicted by the model.

Fig. 7 compares the distribution of empirical and predicted mean vacancy durations across zones. The empirical duration for zone $i \in \mathcal{I}_D$ was calculated as the mean value of $t_1(k) - t_0(k)$ across all sample cycles $k$ originating from $i$. Zone observations were weighted by the number of sample cycles originating from respective zones. The model-predicted vacancy duration for zone $i$ is $\frac{1}{Q_i}$; the observation weights are $f_i$. The mean vacancy duration is 1.87 h empirically vs. 1.43 h predicted by the model. Multiple factors can contribute to the discrepancy between the empirical and model-predicted distribution. The most obvious is lack of daily cycle in the theoretical model: vehicles have long vacancy durations at night; such vacancy usually occurs in specific (residential) zones. This inflates the right tail of the empirical, but not model-predicted, distribution.

The gross profit flow calculated according to (7) is $\phi_0 = 745$ RUB/day, which is 10% above the fixed cost $\phi_c$ calibrated in Section 5.1. The mismatch may be due to the fact that vehicles are sometimes taken out of service and do not generate revenue, which our model does not account for, or due to some omitted fixed costs. $\phi_0$

[15] Jorge et al. (2015), in another calibrated model of carsharing demand, consider price elasticity of −1.5 in the baseline scenario.

**Table 4**
Observed vs. predicted distribution of vacant sample vehicles over space.

| Area of Moscow region | Inside Garden ring | Between Garden and 3rd ring | Between 3rd ring and MKAD | Outside MKAD |
|---|---|---|---|---|
| % of Delimobil home area | 1.83 | 5.99 | 65.23 | 26.94 |
| Observed % of cycle origins | 4.00 | 11.06 | 60.18 | 24.75 |
| Predicted % of cycle origins | 3.12 | 12.42 | 69.21 | 15.25 |

is used as benchmark gross profit in the counterfactual exercise of the next section.

## 6. Counterfactual expansion of home area

This section studies an hypothetical expansion of the home area of Delimobil, the largest observed operator. The expansion will be marginal in terms of fleet size: we will assume that only a small fraction of Delimobil vehicles are allowed to serve larger area, so that, for any such vehicle, the density of competing vehicles (both by Delimobil $\mu_i$ and by others $\tilde{\mu}_{ij}$) remains unchanged. In particular, in the newly added area we have $\mu_i = \tilde{\mu}_{ij} = 0$. To isolate the effect of location-specific pricing in the newly added home area, we will assume that pricing of Delimobil within the initial D-area is unchanged. In the new zones, optimal zone scores are used, but other elements of the tariff (free reservation, flat per-minute rate) remain unchanged too.

Introducing location-specific pricing allows to expand home area to all zones where any travel demand exists. We will assume that the *new area* of Delimobil includes all previously unserved zones in the Moscow region with a positive amount of building floor space, 12442 zones total. Fig. 8 illustrates the counterfactual home area.

To reduce the computational burden, I aggregate the new zones into 4789 clusters of up to 2-by-2 zones. The set of new-area clusters is denoted $\mathcal{I}_{new}$.

The fact that Delimobil and other operators did not serve the new area in Dec. 2019 provides additional inference about travel demand parameters in that area: the gross profit from serving the new area under location-invariant pricing should not exceed $\phi_0$ estimated in Section 5.3 (see Section 3.4 for theoretical discussion), which we label *non-expansion constraint*.

I assume that the bilateral demand density $\lambda_{ij}$ is unchanged from (18), and spatial demand density $A_i$ for cluster $i \in \mathcal{I}_{new}$ is the sum of $A_j$ across zones $j$ that constitute cluster $i$. $A_j$, in turn, is defined by (19); the parameter values $\beta$ are generally extrapolated from estimates of Table 3, but $\beta_0$ is capped from above by the non-expansion constraint. Given estimated model parameters, the latter is indeed binding: we reduce $\beta_0$ to $\beta_1 = -0.6807$ to meet the constraint.

Since pricing is assumed unchanged in the D-area, all its zones are used as numeraire with $R_i = 0$. The vector $\mathbf{R}$ of unknown scores $R_i$ in the new clusters is defined by equality (14) for these clusters. In that formula, given free-reservation flat-rate pricing and zero competition with other vehicles, mean walking time is (cf. (3)) $\bar{x}_{ij} = \frac{\theta h_{ij}}{w}$, markup is (cf. (B.2)) $m_{ij} = r - c - \phi\left(1 + \frac{\theta}{w}\right)$, thus (14) takes form

$$C_i(\mathbf{R}) \equiv \sum_{j \neq i, j \in \mathcal{I}_D \cup \mathcal{I}_{new}} 2\lambda_{ij} \exp\left(-\frac{r}{\theta} + \frac{R_j - R_i}{\theta h_{ij}}\right) \bar{x}_{ij} m_{ij} h_{ij} - \phi = 0, \forall i \in \mathcal{I}_{new}. \tag{22}$$

and gross profit $\phi$ is still found from (7).

The system (22,7) is solved iteratively, exploiting the fact that the dependence of $C_i$ on every particular $R_j, j \neq i$ is small. This allows us to treat the latter as given when calculating the next-iteration value of $R_i$. Specifically, we start with initial values $R_i^{[0]} = 0, \forall i \in \mathcal{I}_{new}$ and $\phi^{[0]} = \phi_0$. Every iteration $a$ consists of the following steps:

1. For every cluster $i \in \mathcal{I}_{new}$, treat previous-iteration $\phi^{[a-1]}$ and $R_j^{[a-1]}, j \neq i$, as given, and make one step in the Newton's method to update $R_i$ for solving $C_i = 0$: $R_i^{[a]} = R_i^{[a-1]} - C_i(\mathbf{R}^{[a-1]})\left(\frac{\partial C_i(\mathbf{R}^{[a-1]})}{\partial R_i^{[a-1]}}\right)^{-1}$. Certain bounds on maximum change of $R_i$ were imposed.
2. Given new vector $\mathbf{R}^{[a]}$, find new $\phi^{[a]}$ using (7).

The algorithm shows excellent convergence properties; after 10 iterations, the change in all $R_i$ does not exceed 1 RUB, and the change in $\phi$ is under 1 RUB/day.

The resultant gross profit is 792 RUB/day, which is 6.18% higher than the benchmark profit $\phi_0$. Given very narrow margins between gross profit and fixed cost, such increase may become a lifeline for operator survival and growth.

The resultant location scores range from 148 RUB (2.38 USD in Dec.2019) to −1063 RUB (−17 USD), with 8.81% of new area having positive score, and with median score of −132 RUB (−2.12 USD). Positive location scores imply that SV operations there would be profitable even without location-specific pricing. The highest location scores are in a satellite town of Reutov, a high-density area just outside of MKAD; as of Dec.2021 (two years after data collection), Delimobil has indeed expanded its home area to this territory, without location-specific pricing.

Fig. 9 illustrates the zone scores for North-Eastern suburbs of Moscow. E.g. the above-mentioned Reutov is the red "hottest" area east of the city center.

Table 5 illustrates various descriptive statistics of counterfactual change. The new area is split into zone with positive scores, negative but above median (−132 RUB), below median. Most trips still originate from the D-area; among the remaining trips, the majority originate in zones with above-median score.

As predicted by Section 3.4, adding new destinations reduces vacancy durations in the D-area. Vacancy durations in the new area are greater, which is compensated by greater distances and rental periods of trips originating from there. Not surprisingly, larger home area means that trips originating in the D-area have increased distances (and thus rental periods), too.

One peculiar feature of the most remote zones (last column of Table 5) is a significant gap in the distance/duration between arriving and departing trips. This is because customers dropping off a vehicle in such a zone have to pay a significant lump-sum relocation fee, in addition to per-minute fees; customers traveling from further away are more likely to accept the deal because for them, the relocation fee is a smaller fraction of the total fare. Conversely, customers picking up the vehicle from a remote zone earn a significant lump-sum relocation bonus, which is especially attractive for customers traveling to proximate destinations. Therefore, vehicles dropped off in remote places are likely to be taken to a nearby town before being returned to Moscow.

## 7. Conclusion

This paper introduces a theory of profit-maximizing pricing of shared vehicles based on a novel model of SV market. The paper shows that introduction of location-specific pricing for shared vehicles may be instrumental in geographic expansion of the service, connecting many more communities. Availability of shared vehicles scattered around larger territory may reduce individual vehicle ownership, making it primarily a rural phenomenon. Reduced vehicle ownership, in turn, would have a positive effect on household welfare, as purchasing a car is typically the second-largest expense of a household. Fewer vehicles would also reduce parking demand, making cities less congested.

**Table 5**
Parameters of counterfactual equilibrium.

| Area | D-area (before/after) | New area by zone score | | |
|---|---|---|---|---|
| | | $> 0$ | $[-132, 0)$ | $< -132$ |
| % of home area | 100/41.4 | 5.2 | 24.1 | 29.3 |
| Percent of cycle origins | 100/94.2 | 2.0 | 2.9 | 0.9 |
| Vacancy duration (h) | 1.43/1.35 | 1.93 | 1.95 | 1.66 |
| Distance, arriving trips (km) | 8.71/9.36 | 13.43 | 20.75 | 26.64 |
| Distance, departing trips (km) | 8.71/9.38 | 18.32 | 19.69 | 16.94 |

In low-density areas, vacant shared vehicles would be too sparse to become a viable transportation option for local everyday trips. But they still could be used for longer-distance trips, especially for trips to large cities with high cost of parking a private vehicle.

Since the data for this paper was collected and the first draft was written, several SV operators in Moscow made their pricing more aligned with the proposed theory; in particular, Delimobil introduced drop-off fees (and, under some conditions, pick-up bonuses) in certain areas of low demand. The theory of this paper allows to make further steps in the right direction, by developing an explicit methodology to calculate an individual location score at every address. In particular, this methodology allows SV operators to make a partnership offer to owners of any parking garage under any schedule of parking rates. Adding such garages to the home area has a quadratic effect on the number of origin–destination pairs that can be served.

Further theoretical contributions could be as follows. Demand density $\lambda$ could be made endogenous and dependent on the density of vacant vehicles in the area, because new customers are more likely to join the service when more vehicles are available. A daily cycle of travel demand and parking rates could be accounted for. This would open the possibility of time-varying zone scores to encourage trips to locations where the number of bookings is expected to increase in the near future, or where parking is expected to become cheaper. Optimal pricing in presence of other parking regulations, such as time limits or number-of-vehicles limits (as in Toronto) could be analyzed.

On the empirical side, actual street patterns could be accounted for when calculating travel times. Partnership with SV operators could result in access to additional data on actual trip fares, customer origin locations, and reservation times, leading to better models of SV demand.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

## Appendix A. Notational glossary

Notation used within a single section is omitted.

| Notation | Description | Section of 1st use |
|---|---|---|
| $\beta$ | Coefficients of spatial demand density | 5.2 |
| $\gamma$ | Coefficients of distance effect | 5.2 |
| $\theta$ | Mean value of travel, RUB/min | 2.2 |
| $\lambda$ | Bilateral demand density | 2.2 |
| $\mu(\tilde{\mu})$ | Density of vacant vehicles by same (other) operator(s) | 2.3 |
| $\tau$ | Expected cycle duration, min | 2.5 |
| $\phi(\phi_c)$ | Vehicle gross profit (fixed cost), RUB/day | 2.3 |
| $\Psi$ | Gross profit per vehicle cycle, RUB | 2.5 |
| $A$ | Spatial demand density | 5.2 |
| $c$ | Cost of vehicle movement (RUB/min) | 2.3 |
| $D$ | Prob. of vehicle booking by customer | 2.4 |
| $f$ | Probability of specific cycle origin | 2.5 |
| $g$ | Cost of parking (RUB/min) | 2.3 |
| $h$ | Time of travel (vehicle use), min | 2.2 |
| $\mathcal{I}$ | Set of zones of home area | 2.1 |
| $m$ | Price markup, RUB/min | 3.1 |
| $p, p^a, p'$ | Trip fare: total (RUB), vehicle use (RUB), reservation rate (RUB/min) | 2.2 |
| $q(Q)$ | Rate of vacant vehicle departure to specific (any) destination | 2.4 |
| $R$ | Location (zone) score, RUB | 3.1 |
| $r$ | Flat per-minute trip fare, RUB/min | Appendix C |
| $s$ | Probability of specific destination | 2.5 |
| $v$ | Idiosyncratic value of travel, RUB | 2.2 |
| $w$ | Disutility of walk, RUB/min | 2.2 |
| $x$ | Walking time, min | 2.2 |

## Appendices B, C, D. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ecotra.2022.100296.

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