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Research paper

Incentive contracts with demand guarantee in BOT toll road projects

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ABSTRACT

Demand uncertainty is a significant issue faced by both the government and private firms in build-operate-transfer (BOT) toll road projects. To encourage private firms' participation, the government usually provides them with demand guarantee. This paper investigates and compares the optimal BOT contracts with the minimum demand guarantee (MDG) and the flexible demand guarantee (FDG), respectively, in environments of symmetric and asymmetric information. It is determined that under the MDG with asymmetric information, the private firm's optimal effort decreases with respect to the guarantee level, which indicates that the MDG provides the private firm with a disincentive to exert effort to increase traffic demand. Under the FDG with asymmetric information, the private firm's optimal effort decreases with respect to the guarantee level and threshold coefficient. Through a comparison of the environment with asymmetric information, the optimal toll price, optimal guarantee level and resulting social welfare are higher under the FDG for a small threshold coefficient. Therefore, it is concluded that the government may choose the FDG instead of the MDG for a small threshold coefficient, which means that the FDG is suitable for toll roads with relatively low demand. Conversely, the government prefers to choose the MDG instead of the FDG when the threshold coefficient is large enough, which means that the MDG is suitable for toll roads with relatively high demand. We further demonstrate that our model results still qualitatively hold when traffic congestion is considered.

1. Introduction

The build-operate-transfer (BOT) scheme has been frequently employed in toll road projects. In a typical BOT road project, the private firm is responsible for project financing, construction, and operation for many years, and then the road is transferred to the government. In return, the private firm is granted the right to charge users a reasonable toll price within the concession period. It has been recognized that the BOT model enables efficient project delivery, improved service quality and reduced financial burden for the government (Brandao & Saraiva, 2008; Li et al., 2005).

BOT toll road projects typically have a long operating period and face high demand uncertainty. Flyvbjerg et al. (2005) examined 210 transport infrastructure projects in 14 countries and pointed out that demand forecasts are overestimated in 90% of rail projects and that the actual demand deviates from the forecasted demand by more than 20% in 50% of road projects. Demand uncertainty may lead to the failure of BOT transport projects and become the main obstacle of private firms' participation. Albalate and Bel (2009) suggested that a private firm would suffer financial difficulties if the actual traffic is significantly lower than the expected traffic. To mitigate demand risk and encourage private firms' participation in toll road projects, the government usually offers the fixed minimum demand guarantee (MDG) in a BOT contract. Under the MDG, the government guarantees a minimum demand level and compensates the private firm for unrealized traffic demand when the actual demand is below the guarantee level. In practice, the MDG exists in many BOT transport projects, such as the Hong Kong Cross-Harbour Tunnel (Tam, 1999), Metro Line 4 of the São Paulo Subway System (Brandão et al., 2012) and Beijing Metro Line 4 (Chang, 2013), to reduce the demand risk faced by private firms.

Although the MDG is an effective way to attract private firms' participation, it has also caused controversy for many years. Wibowo (2004) indicated that the MDG can reduce risks but is not free of costs. Feng et al. (2015) investigated the impacts of the MDG on toll charges as well as road quality and capacity and suggested that imposing the MDG could increase toll charges while decreasing road quality. Song et al. (2018) proposed that a higher MDG level could significantly reduce social welfare. Under the MDG, the government needs to guarantee all

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unrealized demand and assume full demand risk. In particular, a high MDG level makes the private firm receive a high revenue without risks and, as a result, reduces the private firm's effort to improve traffic demand.

To overcome the pitfalls of the MDG and fairly allocate demand risk between the government and private firms, the flexible demand guarantee (FDG) has been proposed, under which the government assumes limited liability and compensates only the private firm for a portion of the unrealized traffic demand. Different from the MDG, which is a rigid guarantee contract, the advantage of the FDG is that the government can transfer partial demand risks to private firms by setting a critical guarantee level, which means that the government will subsidize the private firm based on the realized demand states. Specifically, if the actual traffic demand is lower than the critical guarantee level, then the private firm assumes the loss of operating revenue that is incurred by the insufficient demand. Therefore, to avoid this loss, the private firm has incentives to exert more efforts to improve traffic demand beyond the critical guarantee level. As a result, the government only needs to undertake a smaller guarantee expenditure. The FDG has already been analyzed in the supply chain literature (Lariviere, 2002; Tsay, 1999; Tsay & Lovejoy, 1999). However, its application in a BOT toll road has not been noted by scholars, and how to choose between the MDG and the FDG remains unclear. Although the FDG helps to achieve a relatively fairer risk allocation between the government and private firms, it is not certain whether such a fairer risk allocation will enable a higher social welfare by encouraging the private firm to exert additional effort.

Some studies on demand guarantee in BOT projects exist (Feng et al., 2015; Song et al., 2018; Wibowo, 2004). However, these studies tend to consider the guarantee level exogenous and are unable to evaluate the rationality of the demand guarantee level and the effectiveness of its incentive effect on the private firm's effort. In addition, it is very common in the literature to implicitly assume that the government and the private firm have symmetric information in forecasting future demand (Feng et al., 2016; Niu & Zhang, 2013; Tan et al., 2010; Tan & Yang, 2012a). This finding may be true when the accumulated demand data from previous projects cannot be applied to the current project or both the government and the private firm lack demand forecast experience. However, in practice, the government and the private firm may possess different information about future demand.

Different from the aforementioned studies, this paper makes the demand guarantee level endogenous and aims to compare the role of the MDG and FDG under both symmetric information and asymmetric information to determine which guarantee type should be preferred by the government. In particular, we investigate how to simultaneously determine the guarantee level and other important contract variables from the perspective of maximizing social welfare under the two guarantee types. Moreover, to further explore the incentive effect of demand guarantee, we take into account the impact of the private firm's effort on traffic demand instead of assuming that the demand is only price-dependent. With this assumption, we investigate how the optimal guarantee level, toll price, and subsidy vary with the private firm's effort.

This paper is organized as follows: Section 2 presents the literature review. Section 3 presents some basic assumptions and notations. Section 4 investigates the optimal incentive contract with the MDG. Section 5 investigates the optimal incentive contract with the FDG. Section 6 compares and discusses the two demand guarantee types in environments with symmetric and asymmetric information. Section 7 further discusses the optimal decision-making under the two types of demand guarantees by taking into account travelers' value of time (VOT). Section 8 presents policy implications and conclusions. Appendix A presents a table of the frequently applied notations. The proofs are provided in Appendix B.

2. Literature review

Some studies investigate the optimal BOT contract design without considering the impact of traffic demand guarantee in both the environment with symmetric information and that with asymmetric information.

Regarding contract design under symmetric information Verhoef et al. (1996) examined the congestion pricing problem of two alternative routes in a road network and indicated that whether two-route tolling is more efficient than one-route tolling depends on congestion costs. By taking into account the users' heterogeneous values of time, Yang et al. (2002) examined the profitability and welfare gain of a private toll road under different combinations of road capacity and toll charge in a given road network. Subprasom and Chen (2007) focused on the choice of toll price and road capacity for a BOT toll road to trade off between private investors' profits and social welfare. Verhoef (2007) investigated the second-best road pricing strategy and showed that a competitive auction with the level of road use as the decision criterion can produce the socially optimal road capacity and toll price. Light (2009) developed an analytical framework to investigate the optimal toll price and road capacity as well as their fiscal impacts. Li and Cai (2017) elaborated the impacts of government incentives on the choices of investment timing, capacity, and price under demand uncertainty and showed that revenue guarantee, concession period extension, lump-sum subsidy and unit subsidy can induce timely investment. Lu and Meng (2017) proposed a two-stage stochastic programming model with uncertain traffic demand and presented the optimal highway capacity and toll adjustment strategy. By investigating the single-period concession structure and two-period concession structure in BOT road projects, Zhang, Feng, and Zhang (2018) investigated and compared the optimal BOT contract design, including the toll price, road capacity, road quality and concession period under the single-concession and two-concession period structure. Rouhani (2018) compared the social welfare inside and outside the congestion zone and indicated that the optimal toll rate is significantly overestimated without considering the spillover effects.

In addition, Engel et al. (2001) characterized an optimal risk-sharing contract and showed that it can be implemented with a fairly straightforward mechanism of "least-present-value-of-revenue auction". Guo and Yang (2009) considered a BOT contract as a combination of a concession period, road capacity, and toll charge and examined optimal BOT contracts that maximize social welfare and simultaneously allow the private sector an acceptable profit. Tan and Yang (2012a) investigated the full and partial flexibility of BOT road contracts under demand uncertainty and indicated that different concession periods and toll prices should be adopted under different levels of demand. By varying road users' VOT, Tan and Yang (2012b) analyzed Pareto-efficient BOT toll road contracts and showed that whether service quality is better, identical to, or lower than the socially optimal level depends on the curvature of the mean residual VOT function. Niu and Zhang (2013) considered the negotiation power of the private firm and investigated the impact of demand uncertainty on Pareto-efficient BOT contracts. Wang, Xiong, et al. (2018) constructed a stylized model to investigate how to design an optimal BOT contract with a government subsidy when there is a constraint on capacity use. Zhang, Feng, Zhang, et al. (2018) investigated the effects of the service level on BOT transport project contracts and revealed that the presence of service level requirements increases traffic demand and consumer surplus while reducing private firms' profits.

Regarding contract design under asymmetric information Feng et al. (2016) investigated how the government encouraged private firms to improve noncontractible service quality in toll road projects. They indicated that a government subsidy is helpful in achieving this aim when both parties sufficiently value future transactions and when the marginal cost of public funds is sufficiently low. Shi et al. (2016) examined the optimal road BOT contracts when the private firm's cost information is asymmetric. They showed that carefully designed government guarantees can induce the private firm to reveal its true cost information. Li et al. (2019) characterized the optimal contracts under both the regulation regime and the deregulation regime and concluded that whether deregulation dominates regulation under information asymmetry depends on the interplay of the shadow cost of public funds, demand volatility, the government's imperfect information on the firm's cost structure and the franchise fee.

Since government guarantees are prevalent in BOT infrastructure projects (Engel et al., 2013; Irwin, 2007; Sun & Zhang, 2014; Vassallo & Soliño, 2006; Wang & Liu, 2015; Wibowo & Kochendoerfer, 2010), other studies examine the value of government guarantees or their impacts.

The value of government guarantees Chiara et al. (2007) determined the fair value of minimum revenue guarantee based on Bermudan real options, which provided a foundation for renegotiation between the government and private firms. Alonso-Conde et al. (2007) assessed the real option value of government guarantee in the case of Melbourne City-Link. Jun (2010) demonstrated the combined impacts of minimum revenue guarantee and revenue cap agreements on the project based on option pricing theory. By applying the real option method to an operating BOT highway project, Galera and Soliño (2010) obtained the estimated value of a given MDG. Similarly, Ashuri et al. (2011) applied the real option method to determine the effects of the minimum revenue guarantee option on a concessionaire's economic profile. Wibowo et al. (2012) formulated real option models to assess the value of three types of government guarantees in BOT toll road projects, including land-capping instruments, full toll adjustment guarantee and fair compensation guarantee in the event of nationalization. Liu et al. (2014) applied real option theory to quantify the actual value of restrictive competition guarantee in public-private partnership (PPP) projects.

The impact of government guarantees There are a handful of papers that focus on the impact of government guarantees. Kokkaew and Chiara (2013) presented a revenue guarantee model in privately built transport projects, where minimum revenue guarantee can promote a fairer risk allocation between the host government and the concessionaire. Based on a revised net present value (NPV) financial evaluation model and Monte Carlo simulation, Sun and Zhang (2014) showed that the combination of a minimum revenue guarantee and royalty collection can balance the risks and benefits between the government and private investors in BOT concessions. Based on reciprocal preference theory, Wang, Cui, and Liu (2018) established an optimal incentive mechanism to improve the project revenue. They showed that by designing different guarantee strategies for different participants, the government can apply reciprocal preferences to incentivize investors to exert more effort.

The above literature review indicates that our paper is generally related to two branches of literature—literature on PPP contract design in both environments of symmetric information and asymmetric information and literature on government guarantees in PPP projects. The first branch of literature typically investigates the optimal PPP contract decision variables such as toll price, capacity, and subsidy. Noting the prevalence of government guarantees in sharing demand risks with the private firm, some scholars begin to focus on PPP contract design in the presence of government guarantees. However, previous literature typically treats government guarantees exogenously given and investigates their impact on the contract parameters. In practice, the PPP contract and the guarantee level are interrelated and different contract designs would in return affect the government's decision on the guarantee level. Therefore, different from previous literature, we take the guarantee level as endogenous and investigate the joint design of contract parameters and government guarantee in both environments of symmetric information and asymmetric information. This constitutes one contribution of our paper. Most studies in the second branch of literature focus on MDG and either consider their values or their impacts. By revealing the backfiring result of MDG in providing disincentives for the private firm to improve traffic demand, we have a different focus to compare MDG and FDG. The discussions on FDG are rarely considered by previous literature even though it is also widely adopted in practice. This constitutes another contribution of our paper.

3. Basic assumptions and notations

Consider a BOT toll road project in which a private firm builds the road and charges road users a toll to recover upfront investment and gain profit throughout the predefined concession period. Following some of the existing literature (Wang, Xiong, et al., 2018; Xiao et al., 2016), it is assumed that traffic demand q linearly decreases as the toll price p increases, namely, $q = a - p$, where a represents the market size (Parker & Van Alstyne, 2005; Zhang, Feng, Zhang, et al., 2018). Decreasing demand functions have been widely applied in BOT toll road projects to investigate the optimal contract design problem (Guo & Yang, 2009; Tan et al., 2010; Tan & Yang, 2012a). According to Feng et al. (2016) and Qiu and Wang (2011), in addition to the toll price, traffic demand is also affected by the private firm's effort. Moreover, traffic demand also faces great uncertainty, which has been stressed by many previous studies (Erhun et al., 2008; Feng et al., 2016; Lu & Meng, 2017; Niu & Zhang, 2013; Tan & Yang, 2012a). Therefore, we also take into account the impacts of the private firm's effort e and demand uncertainty ε in determining traffic demand.

It is assumed that the private firm's effort e and demand uncertainty ε have an impact on traffic demand. Therefore, the price-sensitive demand function can be rearranged as $q = a - p + e + \varepsilon$. Let I denote the private firm's initial construction cost, which is dependent on the road capacity and is assumed to be exogenously given. In this paper, we do not consider fixed operating & maintenance (O&M) costs but consider only the operating cost from the effort exerted by the private firm to increase traffic demand; this is because considering a fixed operating cost only complicates the analysis without affecting the main model results. Actually, if the fixed operating cost is much lower than the construction cost, then incorporating a fixed operating cost would only be a trivial extension (Guo & Yang, 2009; Tan et al., 2010).

Assume that the cost incurred by the private firm's effort is e^2, that is, the marginal cost of effort increases as the private firm's effort increases. The quadratic function of the firm's effort cost is common in the literature (Wang, Cui, & Liu, 2018). For ease of analysis, we assume that the random variable ε in this demand function follows a uniform distribution in the interval $[-\varepsilon_0, \varepsilon_0]$, which gives rise to $E(\varepsilon) = 0$ and $\mathrm{var}(\varepsilon) = (\varepsilon_0)^2/3$. In our following analysis, we refer to ε_0 as the demand spread. Following Tan and Yang (2012a) and Li et al. (2019), it is assumed that both the government and the private firm are risk neutral. In this paper, we assume that the concession period is equal to the lifetime of the project life, which is consistent with the results in many previous reports (Niu & Zhang, 2013; Tan et al., 2010; Tan & Yang, 2012b).

4. Incentive contract with the MDG

In this section, we investigate the optimal contract in the presence of the MDG. The sequence of events is described as follows: First, the government determines the toll price and MDG level under demand uncertainty at the contracting stage to maximize social welfare. Second, the private firm determines its effort level to increase traffic demand. Third, demand uncertainty realizes, and the MDG takes effect. If the actual demand is lower than the guarantee level, then the government will subsidize the private firm for the demand shortfall. Otherwise, if the actual demand is higher than the guarantee level, the MDG has no use, and the government does not need to compensate the private firm.

Let v denote the unit subsidy that is transferred to the private firm, and let y denote the MDG level when it takes effect. The total expected subsidy would then be $vE(y-q)^+$, where $(y-q)^+ = y - q$ if $y > q$ and E is an expectation operator. As a result, the private firm's expected profit is modeled as follows:

$$\Pi_m(p, y, e) = E(pq) + vE[(y-q)^+] - e^2 - I \tag{1}$$

Here, the subscript m denotes the MDG in the subsequent analysis. The first term of equation (1) represents the operating revenue of private firm. To guarantee the concavity of the profit function $\Pi_m(p, y, e)$ with respect to e, it is assumed that $v < 4\varepsilon_0$.

Consumer surplus can be modeled as the area below the demand curve and above the toll price. Therefore, the expected consumer surplus can be calculated as follows:

$$CS\left(p,e\right)=E\left[\int_0^q(a-\omega+e+\varepsilon)d\omega-q(a-q+e+\varepsilon)\right]=\frac{1}{2}\left((a-p+e)^2+\frac{\varepsilon_0{}^2}{3}\right) \tag{2}$$

where we define $\mathrm{var}(\varepsilon)$ such that $\mathrm{var}(\varepsilon) = E(\varepsilon^2) - \mu_\varepsilon{}^2$. As can be seen from equation (2), we observe that the private firm's high effort leads to high consumer surplus because more travelers would be attracted to use the road when the private firm exerts high effort.

Different from the private firm, the government's objective is to maximize social welfare, which can be modeled as the sum of the private firm's profit and consumer surplus minus the government expenditure. Here, the government subsidy that is transferred to the private firm is derived from public funds (Baron & Myerson, 1982; Gagnepain et al., 2013), which will generate social costs. Studies have measured the marginal social cost of the government subsidy and concluded that it is approximately 0.9 in developing countries and approximately 0.3 in developed countries (Snow & Warren, 1996; World Bank, 1998).

Let λ ($\lambda > 1$) denote the government expenditure that is incurred by each unit of subsidy, which includes both the subsidy and the associated social cost. The total government expenditure is $\lambda v E[(y-q)^+]$. Therefore, the expected social welfare can be written as

$$W_m(p,y,e)=\Pi_m(p,y,e)+CS(p,e)-\lambda vE[(y-q)^+] \tag{3}$$

4.1. Contract design for the MDG with symmetric information

In this section, assume that the government can observe the private firm's effort level; so the first-best results can be obtained by assuming that the private firm's effort is contractible. Following Qiu and Wang (2011), assume that the government simultaneously determines (p, y, e) to maximize the expected social welfare, which is an optimization problem without any constraints. In particular, Proposition 1 presents the first-best result.

Proposition 1. *The first-best contract offered by the government can be expressed as* $(p_{ms}, y_{ms}, e_{ms}) = (0, 2a - \varepsilon_0, a)$.

Proposition 1 shows that the socially optimal toll price is equal to zero because, mathematically, social welfare decreases with the toll price. Even though a low toll price leads to a low private firm's profit, it generates a high consumer surplus. Moreover, since a low toll price leads to high traffic demand, it makes the MDG less likely to take effect and reduces the government subsidy that is transferred to the private firm. Therefore, the impacts of the toll price on consumer surplus and government expenditure are stronger than those on the private firm's profit. Proposition 1 further shows that the socially optimal guarantee level is equal to the lowest guarantee level that makes the total government subsidy equal zero.

The absence of the government guarantee helps to avoid the social cost that is incurred by government subsidy. Proposition 1 also shows that the socially optimal effort is equal to the market size a; although exerting high effort incurs a considerable effort cost for the private firm, it also results in high consumer surplus. In particular, when the private firm's effort is equal to a, i.e., $e = a$, the increase in consumer surplus would be equal to the increase in the effort cost.

According to Proposition 1, the following corollary regarding the socially optimal guarantee level y_{ms} and private firm's effort e_{ms} can be obtained.

Corollary 1. *The socially optimal guarantee level y_{ms} and private firm's effort e_{ms} are independent of both v and λ.*

Corollary 1 shows that the socially optimal guarantee level and private firm's effort are independent of both the unit subsidy and marginal social cost. The logic behind the corollary is detailed here. To maximize social welfare, the government should reduce the social cost incurred by the government subsidy that is transferred to the private firm, which means that the smallest guarantee level should be exactly equal to the actual traffic demand. Based on the expression $\lambda v E[(y-q)^+] = 0$, it can be easily determined that the social cost is equal to zero, and the socially optimal guarantee level is independent of both v and λ. Moreover, since the socially optimal toll price is equal to zero, the government needs only to maximize consumer surplus by requiring the private firm to exert more effort. As a result, $e_{ms} = a$, which means that the socially optimal effort is also independent of both v and λ.

4.2. Contract design for the MDG with asymmetric information

In this section, we consider the case in which the private firm's effort is noncontractible, which means that the private firm's effort is its private information. Therefore, the government determines the optimal contract (p, y) to incentivize the private firm to select the optimal effort level e, which will lead to the second-best results. To solve for the optimal decisions of both the government and the private firm, a principal-agent model is adopted, that is, the government faces the following mathematical program model:

$$\begin{aligned} &\max_{(p,y)} \quad W_m(p,y,e) \\ &s.t. \quad \Pi_m(p,y,e) \geq \overline{\Pi} \\ &\Pi_m(p,y,e) \geq \Pi_m(p,y,e'), e \neq e' \\ &p > 0, y > 0, e > 0 \end{aligned} \tag{4}$$

where the first constraint and the second constraint represent the private firm's participation constraint and incentive compatibility constraint, respectively; and $\overline{\Pi}$ represents the private firm's reservation utility.

The following lemma examines the properties of the private firm's optimal effort given the contract (p, y).

Lemma 1. *Given (p, y), the private firm's optimal effort $e_m(p, y)$ satisfies $e_m(p,y) = \frac{2\varepsilon_0 p - v(y-a+p+\varepsilon_0)}{4\varepsilon_0 - v}$, which is (i) decreasing in v if and only if $y > a - \varepsilon_0 - \frac{p}{2}$; (ii) decreasing in y; (iii) increasing in p if and only if $v < 2\varepsilon_0$; and (iv) increasing in ε_0 if and only if $y > a - \frac{p}{2} - \frac{v}{4}$.*

The impact of the unit subsidy on the private firm's effort is straightforward. Lemma 1 (i) indicates that a high unit subsidy induces the private firm to exert less effort to increase traffic demand so that more subsidies can be collected from the government. This is because, in the presence of a sufficiently low demand guarantee, it becomes difficult for the government to share demand risks with the private firm. As a result, the private firm has to exert more effort to improve demand and thus obtain more subsidies as the unit subsidy increases. Lemma 1 (ii) shows that the government's high guarantee level also discourages the private firm from exerting effort to increase traffic demand. In fact, a higher guarantee level would lead the private firm to more subsidies and thus reduces the private firm's incentive to exert effort to improve the demand.

Furthermore, one might conjecture that a high toll price can incentivize the private firm to exert more effort to increase traffic demand. Interestingly, Lemma 1 (iii) shows that it is true only if the unit subsidy is sufficiently low. Mathematically, the private firm's optimal effort can be rewritten as $e_m(p, y) = \frac{2\varepsilon_0 p - v(y-a+p+\varepsilon_0)}{4\varepsilon_0 - v} = \frac{(2\varepsilon_0 - v)p - v(y-a+\varepsilon_0)}{4\varepsilon_0 - v}$. As a result, $e_m(p, y)$ is increasing in p if and only if $v < 2\varepsilon_0$. To understand this, we should note that the private firm's revenue comes from two sources- project revenue and government subsidies. Therefore, if the unit subsidy is sufficiently low, then the private firm would care more about project revenue and thus exert more effort to increase demand as the toll price increases. On the contrary, if the unit subsidy is sufficiently high, then the private firm would place emphasis on the government subsidies and thus exert less effort so that more subsidies can be collected. Finally, Lemma 1 (iv) shows that the private firm effort is increasing in demand spread if and only if the guarantee level is sufficiently high. In fact, in the presence of a sufficiently low guarantee level, a larger demand spread would make the demand more likely to exceed the guarantee level, which consequently reduces the private firm's incentive to exert effort to improve demand so that more government subsidies can be collected. However, when the guarantee level is sufficiently high, then the private firm would like to exert more effort in the presence of a larger demand spread to reduce the impact of the uncertainty on the traffic demand.

By solving model (4), Proposition 2 characterizes the government's optimal decision regarding contract (p, y).

Proposition 2. *The optimal toll price p_m^* is determined by the following equation, and the optimal guarantee level y_m^* satisfies $y_m^* = a - \varepsilon_0 - \frac{p_m^*}{2} + \sqrt{\frac{4\varepsilon_0 - v}{v}\Lambda(p_m^*)}$.*

$$((12\varepsilon_0\lambda - 2\varepsilon_0 - v)p + 4\varepsilon_0 a(1-2\lambda))\sqrt{\frac{\Lambda(p)}{v(4\varepsilon_0 - v)}} - \frac{3}{2}p^2 + 3ap - a^2 - \overline{\Pi} - I = 0$$

where $\Lambda(p) = \frac{3}{4}p^2 - ap + \overline{\Pi} + I$.

Plugging p_m^* and y_m^* into $e_m(p,y)$, the optimal effort of the private firm is $e_m^* = \frac{p_m^*}{2} - \sqrt{\frac{v}{4\varepsilon_0 - v}\Lambda(p_m^*)}$. Since the closed-form solutions with respect to p and y cannot be obtained, we present the following numerical study to characterize the impact of the key parameters on the government's optimal decisions.

4.2.1. Sensitivity analysis

In this subsection, the sensitivity analysis will be carried out numerically. Following Wang, Xiong, et al. (2018) and Sun and Zhang (2014), the main objective of the numerical analysis is to be illustrative of our previous model results. The parameter values are not based on a real case due to data availability. However, the parameter values will be consistent with some previous empirical results as well as our model assumptions. For example, we take the shadow cost of government subsidy $\lambda - 1$ as $\lambda - 1 = 0.5$ to be consistent with the previous empirical results that the shadow cost should reside in $[0.3, 0.9]$ (Snow & Warren, 1996; World Bank, 1998). The rationale for setting other parameter values are as follows. The market size a captures the potential maximum traffic demand when the toll price equals to zero, which is set as $a = 30$. Since the private firm typically needs to undertake a relatively large upfront investment, which is set as $I = 245$. The private firm's reservation utility is positive, which can be set as $\overline{\Pi} = 66$. Table 1 presents a summary on the specific parameter values mentioned above.

By fixing $\varepsilon_0 = 9$, Fig. 1 presents the impacts of the unit subsidy on the optimal guarantee level, optimal toll price and resulting social welfare. Fig. 1 (a) shows that a high unit subsidy leads to a low guarantee level, which is intuitive since the government would prefer to reduce the total government subsidy that is transferred to the private firm. There exists a balance between the guarantee level and the unit subsidy. Fig. 1 (a) also shows that a high unit subsidy leads to a high toll price. Given a reduction in the total government subsidy, the main aim of a high toll price is to guarantee the private firm's participation. Therefore, the reduction in government expenditure will also sacrifice some consumer surplus. Fig. 1 (b) further shows that an increase in the unit subsidy reduces social welfare, which is mainly derived from the reduction in consumer surplus due to the increase in the toll price.

By fixing $v = 8$, Fig. 2 presents the impacts of the demand spread. Fig. 2 (a) shows that a larger demand spread also leads to a lower guarantee level. Note that a higher demand spread implies higher total demand risk. Thus, the government tends to reduce the guarantee level to reduce the demand risk she undertakes. On the part of the private firm, in order to reduce the influence of demand uncertainty, the private firm will exert more effort to improve traffic demand and consequently to obtain more operating revenue. When there exists a high operating revenue, the government only needs to set a low toll price to satisfy the private firm's participation. In summary, an increase in the private firm's effort level and a decrease in the toll price can improve consumer surplus, and a decrease in the guarantee level can reduce government expenditure. Thus, social welfare is increasing as the demand spread increases, as depicted in Fig. 2 (b).

Table 1
Values of selected parameters.

a	λ	I	$\overline{\Pi}$
30	1.50	245	66

5. Incentive contract with the FDG

Although the MDG is widely applied in practice, it has attracted some criticism. Some researchers suggest that the demand risk is not fairly shared between the government and the private firm under the MDG. As suggested by Lemma 1, a MDG provides private firms with a disincentive to exert effort to increase traffic demand. This type of guarantee makes all unrealized demand the responsibility of the government. Moreover, the MDG requires the government to set a high toll price to increase the private firm's revenue when traffic demand is sufficiently low, which reduces consumer surplus and social welfare and may lead to contract renegotiation (Xiong & Zhang, 2016).

Different from the MDG, in addition to the guarantee level y, the government also needs to set the threshold level $(1-k)y$, which determines whether the government takes full responsibility for the unrealized demand under the FDG, where parameter k is referred to as the threshold coefficient to measure the flexibility with a higher value that represents greater flexibility. In particular, the parameter k is exogenously given and satisfies $0 < k < \min\left\{1, \frac{2\varepsilon_0}{\varepsilon_0 + v(\lambda - 1)}\right\}$ in the subsequent analysis. After the actual traffic demand is observed during the operating period, if the demand q is higher than the threshold level $(1-k)y$, then the government takes full responsibility for the demand shortfall $y - q$, and the subsidy received by the private firm will be $v(y-q)$. Otherwise, if q is lower than the threshold level $(1-k)y$, then the government only takes partial responsibility for the demand shortfall, that is, the government compensates the private firm for the demand shortfall ky, and the subsidy received by the private firm will be vky. In this case, the government needs to assume the demand risk between the guarantee level and the threshold level, and the private firm needs to assume the demand risk below the threshold level. Fig. 3 depicts the FDG in detail: if traffic demand is q_1, then the guaranteed demand that the private firm receives is ky; otherwise, if traffic demand is q_2, then the guaranteed demand that the private firm receives is $y - q_2$.

Based on the previous description of the FDG, the total government subsidy transferred to the private firm can be calculated. If traffic demand q resides in the interval $(0, (1-k)y)$, that is, $\varepsilon < (1-k)y - a + p - e$, the government subsidy can be expressed as $v\int_{-\varepsilon_0}^{(1-k)y-a+p-e}\frac{ky}{2\varepsilon_0}d\varepsilon$. However, if traffic demand q resides in the interval $((1-k)y, y)$, that is, $(1-k)y - a + p - e < \varepsilon < y - a + p - e$, the government subsidy can be expressed as $v\int_{(1-k)y-a+p-e}^{y-a+p-e}\frac{y-q}{2\varepsilon_0}d\varepsilon$. Therefore, the expected total government subsidy can be expressed as $v\int_{-\varepsilon_0}^{(1-k)y-a+p-e}\frac{ky}{2\varepsilon_0}d\varepsilon + v\int_{(1-k)y-a+p-e}^{y-a+p-e}\frac{y-q}{2\varepsilon_0}d\varepsilon$. Therefore, under the FDG, the private firm's expected profit can be expressed as follows:

$$\Pi_f(p,y,e) = E(pq) + v\int_{-\varepsilon_0}^{(1-k)y-a+p-e}\frac{ky}{2\varepsilon_0}d\varepsilon + v\int_{(1-k)y-a+p-e}^{y-a+p-e}\frac{y-q}{2\varepsilon_0}d\varepsilon - e^2 - I \quad (5)$$

Fig. 1. Impacts of the unit subsidy v when $\varepsilon_0 = 9$.

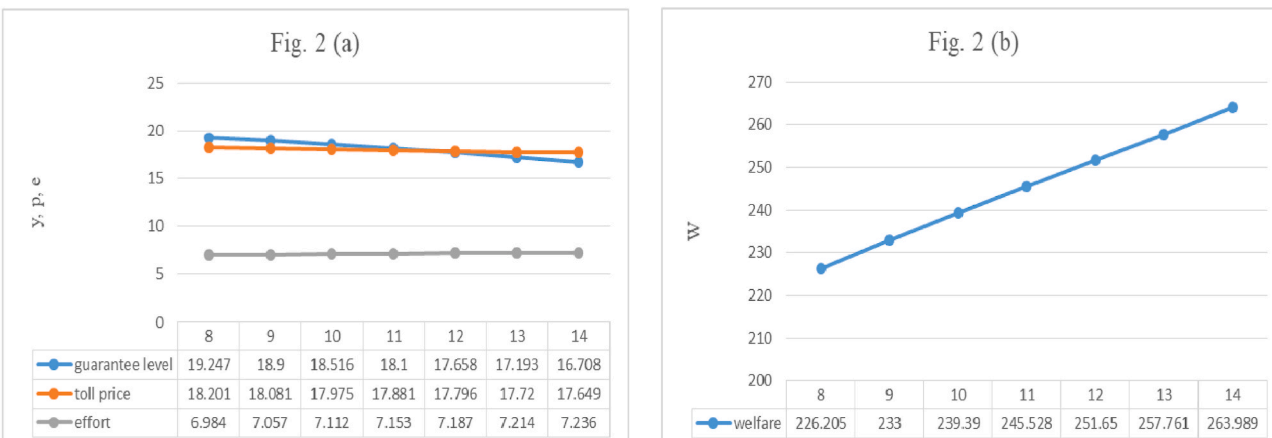

Fig. 2. Impacts of the demand spread ε_0 when $v = 8$.

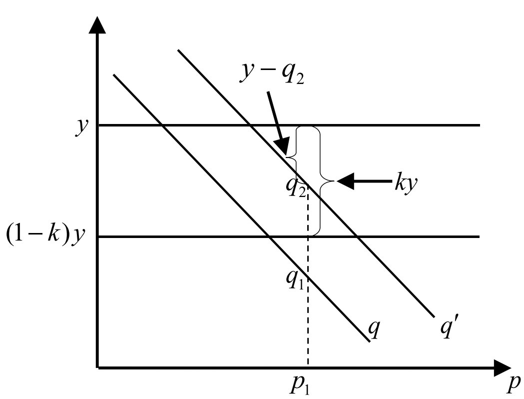

Fig. 3. Description of government subsidies under the FDG.

where the first term is the private firm's operating revenue, and the second term and third term are the subsidies that the private firm receives when the demand q is lower and higher, respectively, than the threshold level $(1-k)y$. Especially, the subscript f denotes the FDG in the subsequent analysis.

Note that the FDG does not change the consumer surplus, which is still $CS\left(p,e\right) = \frac{1}{2}\left((a-p+e)^2 + \frac{\varepsilon_0^2}{3}\right)$. Combining the private firm's profit and consumer surplus, the expected social welfare will be

$$W_f(p,y,e) = \Pi_f(p,y,e) + CS(p,e) - \lambda\left(v\int_{-\varepsilon_0}^{(1-k)y-a+p-e}\frac{ky}{2\varepsilon_0}d\varepsilon + v \times \int_{(1-k)y-a+p-e}^{y-a+p-e}\frac{y-q}{2\varepsilon_0}d\varepsilon\right) \tag{6}$$

where the third term represents the total government expenditure incurred under the FDG. Similar to the case of the MDG, we investigate the first-best results under symmetric information and then further investigate the government's optimal contract design under asymmetric information.

5.1. Contract design for the FDG with symmetric information

We consider the case in which the private firm's effort is contractible, that is, we investigate the first-best contract (p,y,e) to maximize social welfare. Based on equation (6), Proposition 3 presents the results.

Proposition 3. *Under the FDG, the first-best contract can be expressed as* $(p_{fs},y_{fs},e_{fs}) = \left(0,\frac{2\varepsilon_0(2a-\varepsilon_0)}{2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)}, a+\frac{vk(\lambda-1)(2a-\varepsilon_0)}{2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)}\right)$.

With similar intuitions to Proposition 1, where a low toll price generates high consumer surplus, Proposition 3 shows that the socially optimal toll price under the FDG is also equal to zero. Compared to a road that has a high toll price, a road that has a low toll price can attract more travelers and increase the consumer surplus. Moreover, Proposition 3 gives the socially optimal guarantee level and private firm's effort. We can determine that both the guarantee level and the private firm's effort are dependent on the unit subsidy, marginal social cost, and threshold coefficient and parameter ε_0.

Furthermore, according to Proposition 3, the following corollary can be obtained.

Corollary 2. *Both the socially optimal guarantee level y_{fs} and the socially optimal effort e_{fs} are increasing in v, λ and k.*

Corollary 2 presents the impacts of some important parameters on the socially optimal guarantee level and socially optimal effort. Note that the social cost incurred by the total government subsidy is increasing in the unit subsidy v, marginal social cost λ and threshold coefficient k. To offset the increase in social cost and consequently increase the social welfare, the socially optimal effort level should be increased to improve traffic demand. As traffic demand increases, the government can increase the guarantee level to reduce the probability that the FDG will take effect.

5.2. Contract design with the FDG with asymmetric information

In this section, we consider the case in which the private firm's effort is its private information, which cannot be observed by the government. Therefore, the government determines the optimal contract (p, y) and the private firm determines the optimal effort level e, which will lead to the second-best results. To investigate the optimal decisions of the government and the private firm, an optimization model needs to be formulated, that is, the government faces the following mathematical program:

$$\begin{aligned} &\max_{(p,y)} \quad W_f(p,y,e) \\ &s.t. \quad \Pi_f(p,y,e) \geq \overline{\Pi} \\ &\Pi_f(p,y,e) \geq \Pi_f(p,y,e'), e \neq e' \\ &p > 0, y > 0, e > 0 \end{aligned} \tag{7}$$

where the first constraint and second constraint are the private firm's participation constraint and incentive compatibility constraint, respectively. The following lemma examines the properties of the private firm's optimal effort given the contract (p, y).

Lemma 2. *Given (p, y), the private firm's optimal effort $e_f(p, y)$ satisfies $e_f(p,y) = \frac{p}{2} - \frac{vky}{4\varepsilon_0}$, which (i) decreases in v, (ii) decreases in y, (iii) decreases in k, (iv) increases in p, and (v) increase in ε_0.*

The intuition regarding the impact of the guarantee level on the private firm's effort under the FDG is similar to that under the MDG in Lemma 1 (ii). In fact, as the guarantee level increases, the government will assume more demand risk. As a result, the private firm can obtain more guaranteed revenue, which leads the private firm to exert less effort to reduce effort cost. Lemma 2 further shows that a high threshold level (small k) leads to high effort by the private firm. This is because a high threshold level makes the private firm assume more demand risk. As a result, the private firm can only obtain less guaranteed revenue when the actual traffic demand is lower than the threshold level. To achieve the reservation utility, the private firm is more willing to exert effort to increase the operating revenue. These intuitions can also be applied to the impact of the toll price on the private firm's effort. Specifically, if the threshold level is sufficiently high ($ky < 2\varepsilon_0$), the private firm is more willing to exert effort to increase the demand under a high toll price. In this case, the private firm can obtain more operating revenue to offset the loss of revenue from government guarantee. Otherwise, if the threshold level is sufficiently low ($ky > 2\varepsilon_0$), the government will assume all demand risk. As a result, the private firm's effort to increase traffic demand is correspondingly reduced. In addition, the private firm has an incentive to exert more effort to increase traffic demand when there exists a larger demand spread. This is because the private firm needs to assume more demand risk under a larger demand spread. Therefore, the private firm is willing to exert more effort to reduce the impact of demand uncertainty.

By solving model (7), Proposition 4 characterizes the government's optimal decision regarding (p, y).

Proposition 4. *The optimal toll price p_f^* can be determined by the following equation, and the optimal guarantee level y_f^* satisfies $y = \frac{2\varepsilon_0}{\rho}\Gamma(p_f^*)$.*

$$\left(a - \frac{p}{2} - \frac{vk}{2\rho}\Gamma(p) + \lambda(3p - 2a)\right)\sqrt{\left(a - \frac{p}{2} - \varepsilon_0\right)^2 + \frac{\rho}{vk}\Lambda(p)} - \left(a - \frac{3p}{2} + \frac{vk}{2\rho}\Gamma(p)\right)\left(a - \frac{p}{2} + \left(\lambda - \frac{1}{2}\right)\frac{vk}{\rho}\Gamma(p)\right) = 0$$

where $\rho = 8\varepsilon_0 + vk - 4\varepsilon_0 k$ and $\Gamma(p) = 2a - p - 2\varepsilon_0 + \sqrt{(2a - p - 2\varepsilon_0)^2 + \frac{4\rho}{vk}\Lambda(p)}$.

Plugging p_f^* and y_f^* into $e_f(p, y)$, the private firm's optimal effort is $e_f^* = \frac{\rho p_f^* - vk\Gamma(p_f^*)}{2\rho}$. Since the closed-form solutions of p_f^* and y_f^* cannot be obtained, we present a numerical study to characterize the impacts of the unit subsidy, demand spread and threshold coefficient on the government's optimal decisions.

5.2.1. Sensitivity analysis

Similar with Section 4.2.1, we numerically investigate the impact of some key parameters on the optimal guarantee level, optimal toll price, and resulting social welfare. The values of some important parameters remain equivalent to those in Table 1. In addition, we let $k = 0.13$ and $\varepsilon_0 = 9$, respectively. Fig. 4 presents the impacts of the unit subsidy on the optimal guarantee level, optimal toll price, and resulting social welfare.

Following similar intuition to Figs. 1 and 4 (a) shows that the optimal guarantee level decreases, and the optimal toll price increases as the unit subsidy increases. Fig. 4 (b) shows that the resulting social welfare decreases as the unit subsidy increases, because as the unit subsidy increases, the government would prefer to set a low guarantee level to reduce the social cost incurred by the total subsidy. In addition, to guarantee the private firm's participation, the government must set a high toll price to increase the firm's operating revenue as the guarantee level decreases. As the unit subsidy increases, both an increase in the toll price and a decrease in the private firm's effort can cause a reduction in consumer surplus, which further reduces social welfare. These results also imply that under the FDG, the government can set a low unit subsidy to incentivize the private firm to increase effort.

We continue to fix $k = 0.13$ and $v = 8$, then Fig. 5 presents the impact of demand spread ε_0 on the optimal guarantee level, optimal toll price, and resulting social welfare.

Fig. 5 (a) shows that the optimal guarantee level increases, and the optimal toll price decreases as the demand spread ε_0 increases. Fig. 5 (b) shows that the resulting social welfare increases as the demand spread increases. Different from the MDG, the government will offer a higher guarantee level in the face of a higher demand risk level under the FDG. This is because the government does not need to undertake all demand risks when the demand is below the guarantee level. Rather, the improvement of the guarantee level will leave the private firm with more demand risks. The improved guarantee level also leads to a low toll price. Again, the private firm will be inclined to exert more effort to reduce the impact of demand uncertainty. The joint impact of the low toll price and the private firm's high effort level improves social welfare.

By fixing $v = 5$ and $\varepsilon_0 = 9$, Fig. 6 presents the impacts of the threshold coefficient k on the optimal guarantee level, optimal toll price, and resulting social welfare.

Fig. 6 shows that the optimal guarantee level decreases, the optimal toll price increases and resulting social welfare decreases as the threshold coefficient k increases. The logic behind these results is expressed as follows: As the threshold coefficient k increases, the total government subsidy also increases, which produces a high social cost. To reduce the social cost incurred by the total subsidy, the government

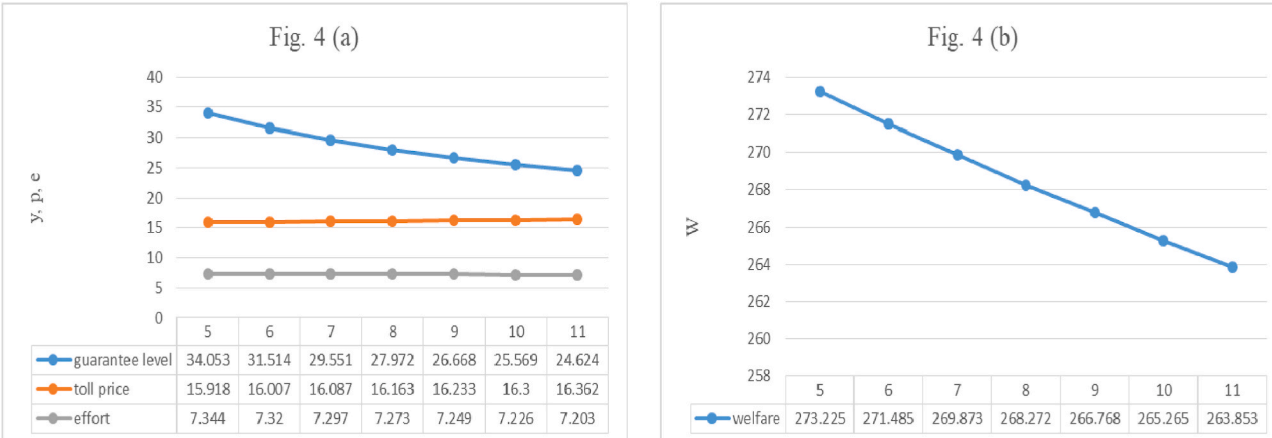

Fig. 4. Impacts of the unit subsidy v if $k = 0.13$ and $\varepsilon_0 = 9$.

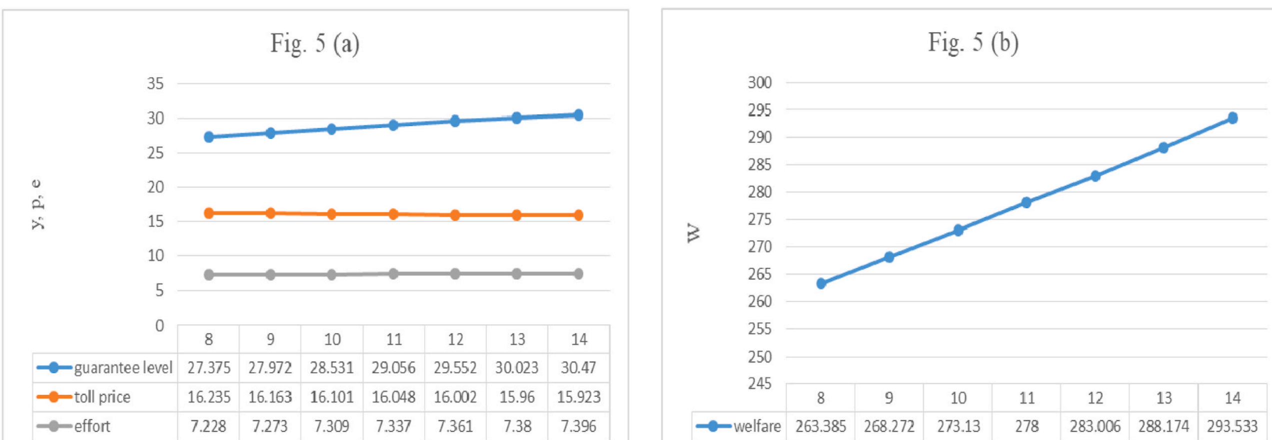

Fig. 5. Impacts of the demand spread ε_0 if $k = 0.13$ and $v = 8$.

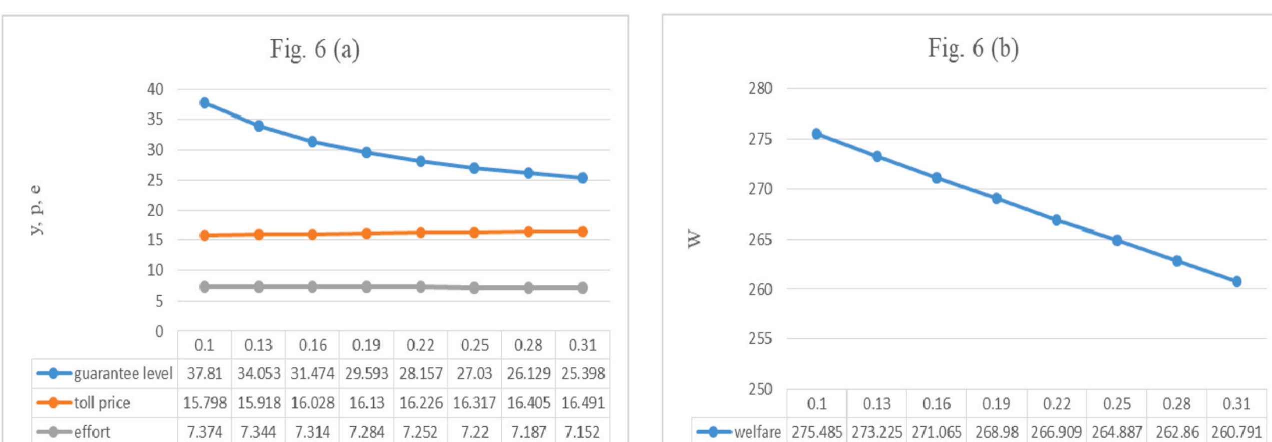

Fig. 6. Impact of the threshold coefficient k if $v = 5$ and $\varepsilon_0 = 9$.

would reduce the guarantee level and raise the toll price to guarantee that the private firm can obtain a reasonable profit. Therefore, it is concluded that the optimal guarantee level decreases and the optimal toll price increases as the threshold coefficient k increases. As the guarantee level decreases, the private firm tends to reduce its effort to obtain more government subsidies. In this case, both a high toll price and low effort by the private firm cause a decrease in consumer surplus, which will further reduce social welfare, as shown in Fig. 6 (b). Thus, when the threshold coefficient k increases, the government should set a high toll price and low guarantee level to incentivize the private firm to exert more effort. Similarly, the results also indicate that given the smaller threshold coefficient k, the government should set a low toll price and high guarantee level to incentivize the private firm to exert more effort. In this case, both a low toll price and the high effort of private firm generate high social welfare.

To further test for the robustness of the results described above, we will continue to change the parameter values. By fixing $v = 7$, $v = 9$, $v = 11$, $\varepsilon_0 = 9$ and relaxing the threshold coefficient k, Fig. 7, Figs. 8 and 9, respectively, present the impacts of adjusting the threshold coefficient k on the optimal guarantee level, optimal toll price, and resulting social welfare.

Figs. 7, Figs. 8 and 9 show that the optimal guarantee level decreases, optimal toll price increases and resulting social welfare decreases as the threshold coefficient k increases. These results follow intuitions that are similar to those in Fig. 6.

6. Comparison between the MDG and FDG

6.1. Comparison under symmetric information

By comparing the socially optimal toll price, private firm's effort, guarantee level, private firm's profit and social welfare, the following proposition can be obtained.

Proposition 5. *In the presence of symmetric information,*

(i) the socially optimal toll prices are equal, that is, $p_{ms} = p_{fs} = 0$;
(ii) the socially optimal effort is greater under the FDG than that under the MDG, i.e., $e_{ms} < e_{fs}$;
(iii) the socially optimal guarantee level is higher under the FDG than under the MDG, i.e., $y_{ms} < y_{fs}$;
(iv) the socially optimal private firm's profit is higher under the MDG than under the FDG, i.e., $\Pi_{ms}(p_{ms}, y_{ms}, e_{ms}) > \Pi_{fs}(p_{fs}, y_{fs}, e_{fs})$; and
(v) the socially optimal social welfare is higher under the MDG than under the FDG, i.e., $W_{ms}(p_{ms}, y_{ms}, e_{ms}) > W_{fs}(p_{fs}, y_{fs}, e_{fs})$.

Proposition 5 (i) shows that the socially optimal toll prices are both equal to zero under the two guarantee types in the environment with symmetric information because a low toll price leads to high traffic demand, which increases consumer surplus and social welfare. Proposition 5 (ii) shows that the government would require the private firm to exert more effort to increase traffic demand under the FDG. Proposition 5 (iii) shows that the government needs to set a high guarantee level under the FDG because the socially optimal guarantee level is increasing with respect to the socially optimal effort. Since the FDG results in high effort by the private firm, it also leads to a high guarantee level to reduce the social cost incurred by the total subsidy.

Since both the socially optimal toll price and the socially optimal guarantee level generate zero operating revenue or government subsidy for the private firm, the private firm would receive a low profit under the FDG due to the cost of high effort, as shown in Proposition 5 (iv). Even though the private firm's high effort increases consumer surplus, it also incurs a high cost of effort. Proposition 5 (v) implies that compared to the MDG, the increased consumer surplus under the FDG is insufficient for compensating for the private firm's effort cost. As a result, social welfare is lower under the FDG.

6.2. Comparison under asymmetric information

In this section, we will numerically compare MDG and FDG under asymmetric information. In doing so, we use parameter values given in section 4.2.1 and section 5.2.1. This is because the comparisons should be conducted under the government's and the private firm's optimal decisions which have been numerically given in previous sections. This implies that if we use a different data set in this section from previous sections, then the comparisons would be independent of the previous analysis. However, it is worth mentioning that if we replace the data set with another one simultaneously in these sections, then our following numerical comparisons continue to hold.

Figs. 10 and 11 respectively present the comparisons of the optimal guarantee levels under the two types of demand guarantees as the unit subsidy and the demand spread varies. The comparisons show that for the small threshold coefficient k, the government would prefer to offer a higher guarantee level under the FDG than that under the MDG. The reason is that the small threshold coefficient k makes it more difficult for the FDG to take effect and leads to a small total government subsidy even if the demand exceeds the threshold level $(1 - k)y$. As a result, the private firm's reservation utility would be difficult to achieve. To guarantee the private firm's participation, the government needs to set a high guarantee level under the FDG.

Figs. 12 and 13 respectively show the comparisons of the optimal toll prices under the two types of demand guarantees as the unit subsidy and the demand spread varies. The comparisons show that for the small threshold coefficient k, the government would prefer to set a higher toll price under the MDG than that under the FDG. Since the guarantee level is higher under the FDG, the government would set a low toll price to balance the private firm's operating revenue and the total subsidy to increase consumer surplus.

Figs. 14 and 15 respectively show the comparisons of the private firm's effort under the two types of demand guarantees as the unit subsidy and the demand spread varies. The comparisons show that when the threshold coefficient k is sufficiently small, the private firm exerts more effort under the FDG than that under the MDG. Furthermore, it can be determined that as the unit subsidy v increases, the optimal private firm's effort under the MDG is more likely to be equal to that under the FDG with a high threshold coefficient k. This is mainly because as the unit subsidy v increases, the private firm's effort decreases more under

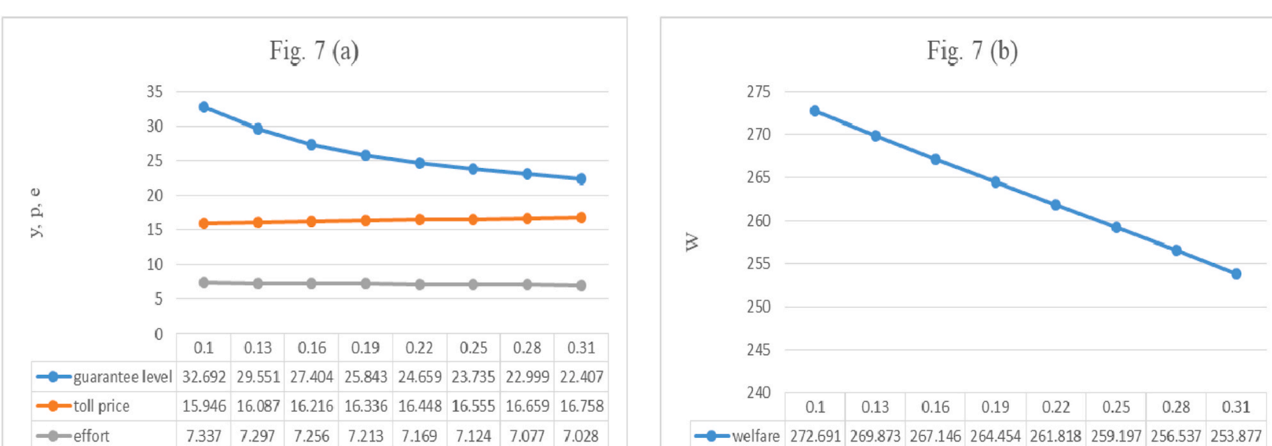

Fig. 7. Impact of the threshold coefficient k if $v = 7$ and $\varepsilon_0 = 9$.

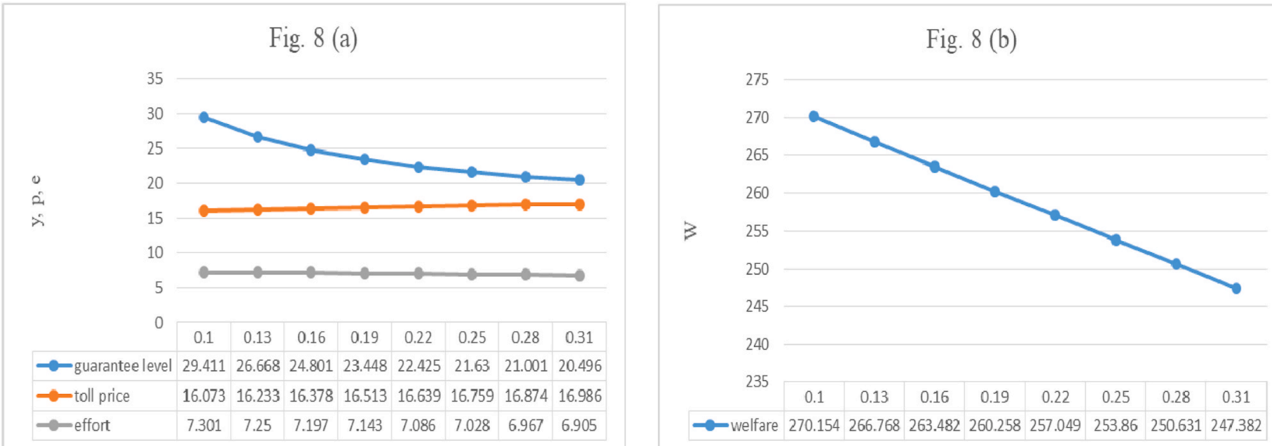

Fig. 8. Impact of the threshold coefficient k if $v = 9$ and $\varepsilon_0 = 9$.

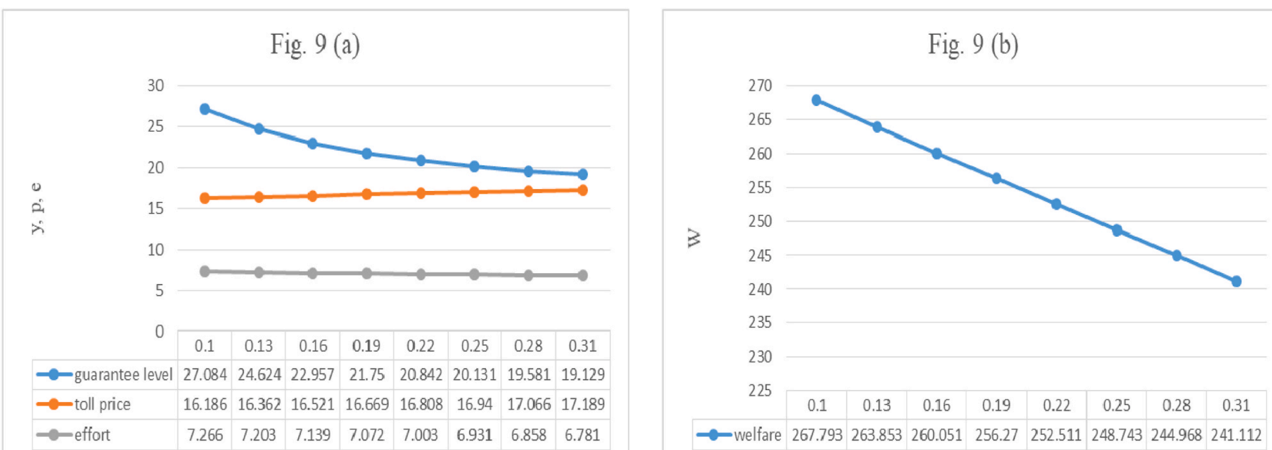

Fig. 9. Impact of the threshold coefficient k if $v = 11$ and $\varepsilon_0 = 9$.

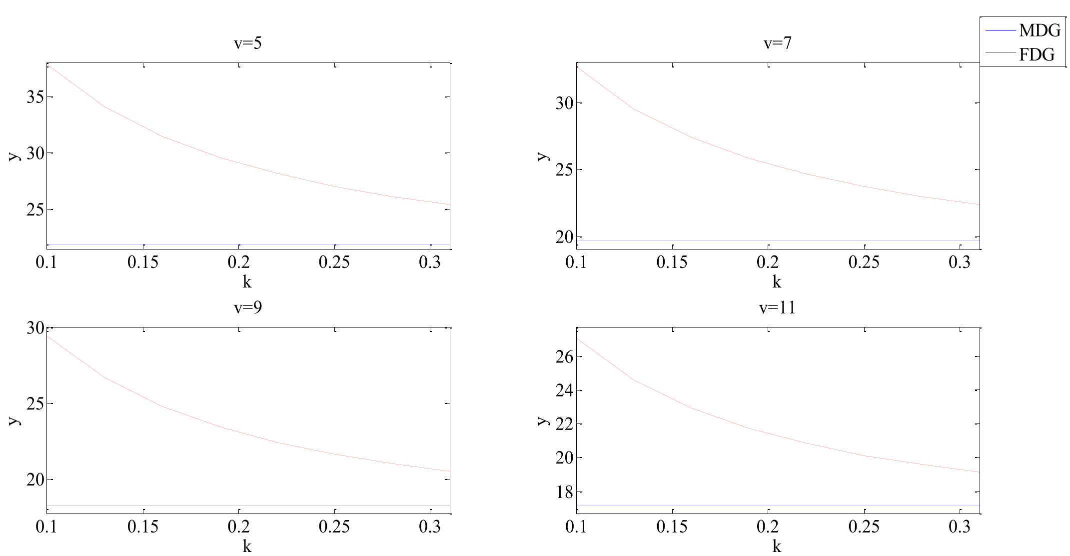

Fig. 10. Comparison of guarantee levels under two types of demand guarantees when v takes different values for $\varepsilon_0 = 9$.

the MDG. To see this numerically, let us take $k = 0.13$ as an example. Fig. 14 shows that the private firm's effort levels under FDG are 7.344, 7.297, 7.249, 7.203, respectively, as v takes 5, 7, 9, 11. The private firm's effort levels under MDG are 7.244, 7.125, 6.984, 6.823, respectively. Therefore, the variation intervals are larger under the MDG than that under the FDG, implying more reduction of private firm's effort

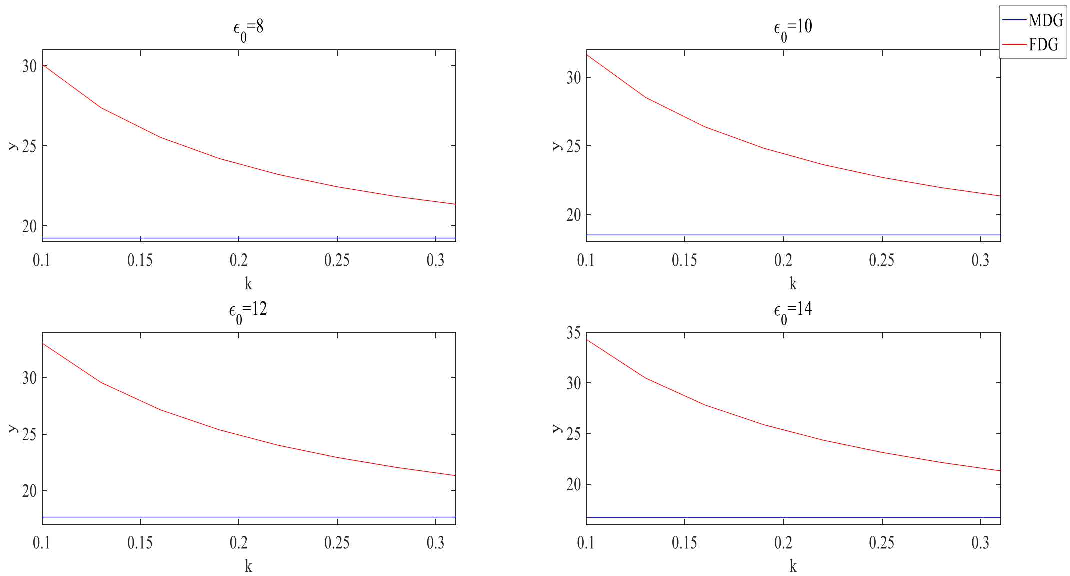

Fig. 11. Comparison of guarantee levels under two types of demand guarantees when ε_0 takes different values for $v = 8$.

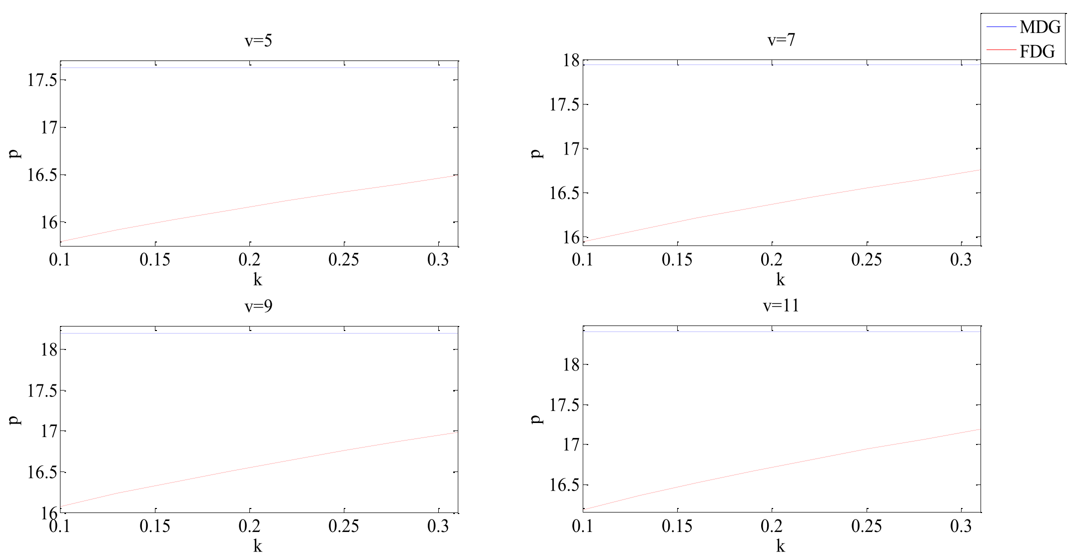

Fig. 12. Comparison of toll prices under two types of demand guarantees when v takes different values for $\varepsilon_0 = 9$.

under the MDG as v increases. It can be also found that as demand spread ε_0 increases, the private firm's optimal effort under the FDG is more likely larger than that under the MDG. This is because as ε_0 increases, the private firm's effort is reduced less as the threshold coefficient k increases under the FDG.

Figs. 16 and 17 respectively show the comparisons of social welfare under the two types of demand guarantees as the unit subsidy and the demand spread varies. This comparisons show that for a small threshold coefficient k, social welfare is higher under the FDG than that under the MDG. Even though a high guarantee level entails a considerable social cost, a low toll price and the private firm's high effort can also increase consumer surplus. Therefore, this finding implies that the social cost incurred by a high guarantee level under the FDG is lower than the increase in consumer surplus incurred by a low toll price and the private firm's high effort.

By comparing the two types of demand guarantees under asymmetric information, the following managerial implications can be obtained. First, since the MDG provides the private firm with a disincentive to exert effort to increase traffic demand, the government should set a low guarantee level to prevent the private firm's ex post opportunistic behavior while achieving the private firm's reservation utility by offering a high toll price. Second, recall that the private firm's effort in improving traffic demand is decreasing in the guarantee level under MDG. Thus, if a high guarantee level is required under the MDG, then the government should carefully design performance evaluation standards such as road maintenance and rehabilitation requirements to deter the private firm's disincentive to exert more effort. This is because the private firm tends to exert less effort to improve traffic demand in the presence of a high guarantee level. Note further that, in practice, one important way for the private firm to exert effort to improve traffic demand is by improving road quality. Therefore, if the performance evaluation standards such as road maintenance and rehabilitation requirements can be carefully designed, then the private firm's effort can be monitored and thus the unintended disincentive resulting from a high guarantee level can be avoided to some extent. Third, since the government could transfer some demand risk to the private firm under the FDG, the private firm exerts more effort to increase traffic demand under the FDG than under the MDG. Therefore, it can be concluded that the

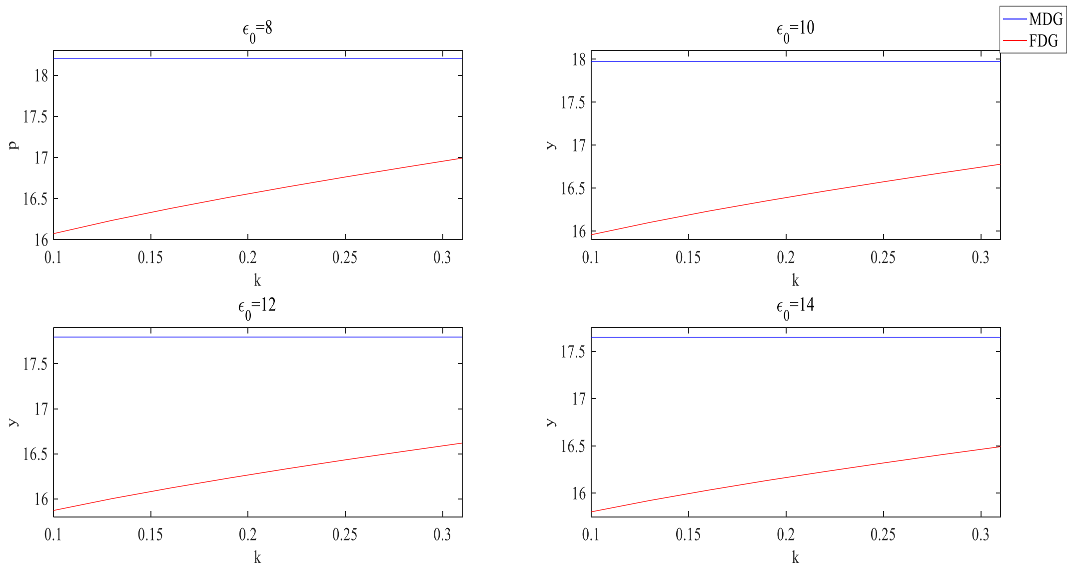

Fig. 13. Comparison of toll prices under two types of demand guarantees when ε_0 takes different values for $v = 8$.

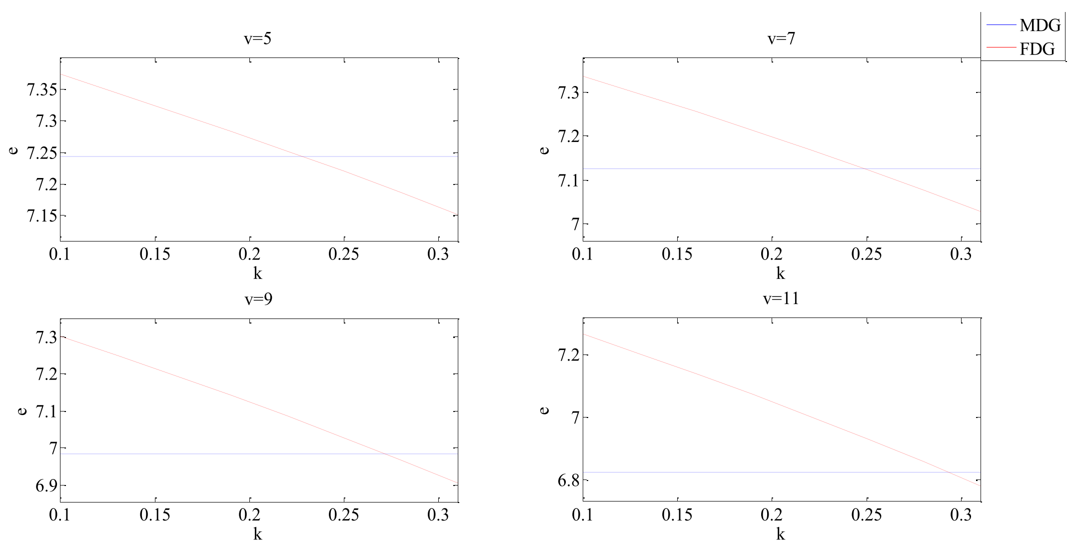

Fig. 14. Comparison of effort level under two types of demand guarantees when v takes different values for $\varepsilon_0 = 9$.

FDG is suitable for toll roads with high demand risk, while the MDG is suitable for toll roads with low demand risk.

7. Extension: The impact of traffic congestion

In our previous analysis, we assumed a negligible impact of traffic congestion on traffic demand. In this section, we will show that our main model results still qualitatively hold even if the congestion effect is considered.

Before carrying out the theoretical analysis, we need to introduce two additional key parameters, that is, τ denotes the generalized travel toll price of a trip for travelers through the road and c denotes road capacity. Therefore, the toll-dependent demand function can be rewritten as $q^\tau = a - \tau + e + \varepsilon$ (Feng et al., 2018; Tan & Yang, 2012b; Zhang, Feng, Zhang, et al., 2018). The total toll price τ includes the toll price p charged by the private firm and the time cost $\beta \frac{q}{c}$ of travelers (Guo & Yang, 2009; Tan et al., 2010). Therefore, τ can be further expressed as $\tau = p + \beta \frac{q}{c}$. Note that the parameter $\beta (\beta > 0)$ denotes the travelers' unit VOT, which converts travel time into an equivalent monetary cost. By plugging the total toll price τ into the linear demand function, traffic demand can be determined by the following demand-supply equilibrium (Tan et al., 2010), that is, $q^\tau = \frac{c}{\beta + c}(a - p + e + \varepsilon)$. With a slight abuse of notation, let $\frac{1}{\alpha}$ denote $\frac{c}{\beta+c}$ for the remainder of this paper.

7.1. Contract design with the MDG in the presence of traffic congestion

According to the previously discussed parameters and demand function q^τ, the private firm's profit function can be rewritten as

$$\Pi_m^\tau(p, y, e) = E(pq^\tau) + vE[(y - q^\tau)^+] - e^2 - I \tag{8}$$

For any given toll price, the actual traffic demand q^τ will be lower than q. Therefore, different from equation (2) presented in section 4, consumer surplus in this case can be calculated as follows:

$$\begin{aligned} CS^\tau\left(p, e\right) &= E\left[\int_0^{q^\tau} (a - \omega + e + \varepsilon) d\omega - q^\tau (a - q^\tau + e + \varepsilon)\right] \\ &= \frac{1}{2\alpha^2}\left((a - p + e)^2 + \frac{{\varepsilon_0}^2}{3}\right) \end{aligned} \tag{9}$$

Social welfare consists of the private firm's profit, consumer surplus

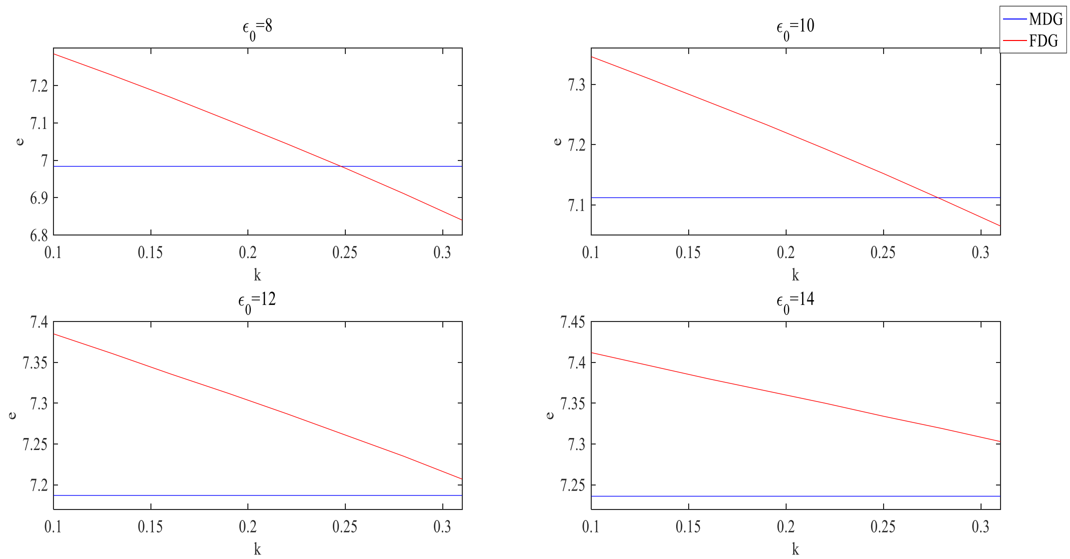

Fig. 15. Comparison of effort level under two types of demand guarantees when ε_0 takes different values for $v = 8$.

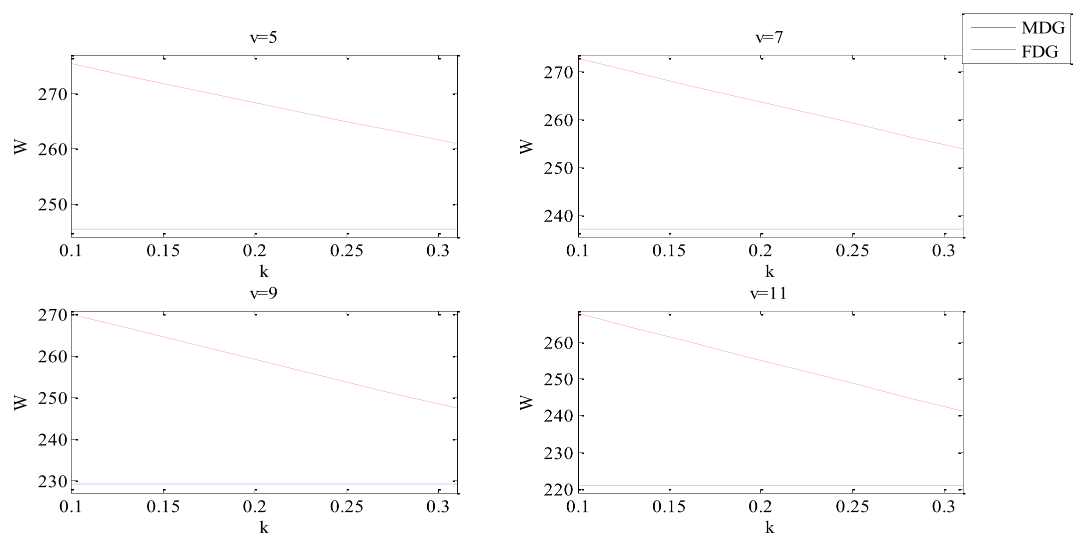

Fig. 16. Comparison of social welfare under two types of demand guarantees when v takes different values for $\varepsilon_0 = 9$.

and government expenditure caused by the MDG, which can be reformulated as follows:

$$W_m^\tau(p,y,e) = \Pi_m^\tau(p,y,e) + CS^\tau(p,e) - \lambda v E[(y-q^\tau)^+] \tag{10}$$

(1) Contract design under symmetric information

Since the information is symmetric, the government can observe the private firm's effort and make it choose the optimal effort that maximizes social welfare. According to the social welfare function given in equation (10), Proposition 1 presents the first-best results.

Lemma 3. *In the presence of traffic congestion, the first-best contract offered by the government can be expressed as* $(p_{ms}^\tau, y_{ms}^\tau, e_{ms}^\tau) = \left(\frac{2a(\alpha-1)}{4\alpha-3}, \frac{2a}{4\alpha-3} - \frac{\varepsilon_0}{\alpha}, \frac{a}{4\alpha-3}\right)$.

Similar to Proposition 1, Lemma 3 presents the socially optimal toll price, guarantee level and private firm's effort. Different from Proposition 1, we find that the socially optimal toll price is greater than 0, because in addition to the toll price and private firm's effort, consumer surplus is also affected by traffic congestion: traffic congestion reduces the consumer surplus. To offset the decrease in social welfare caused by the decrease in consumer surplus, the government should raise the toll price to reduce traffic demand and impact of traffic congestion. Lemma 3 also illustrates that the private firm's effort level required by the government is lower in the presence of traffic congestion.

(2) Contract design under asymmetric information

In the presence of traffic congestion, the second-best problem that the government faces is expressed as follows. Similarly, the first constraint and second constraint are still the private firm's participation constraint and incentive compatibility constraint, respectively. By solving for model (11), Lemma 4 presents the optimal contract (p, y) and the private firm's optimal effort e.

$$\begin{aligned}
&\max_{(p,y)} \quad W_m^\tau(p,y,e) \\
&s.t. \quad \Pi_m^\tau(p,y,e) \geq \overline{\Pi} \\
&\Pi_m^\tau(p,y,e) \geq \Pi_m^\tau(p,y,e'), e \neq e' \\
&p > 0, y > 0, e > 0
\end{aligned} \tag{11}$$

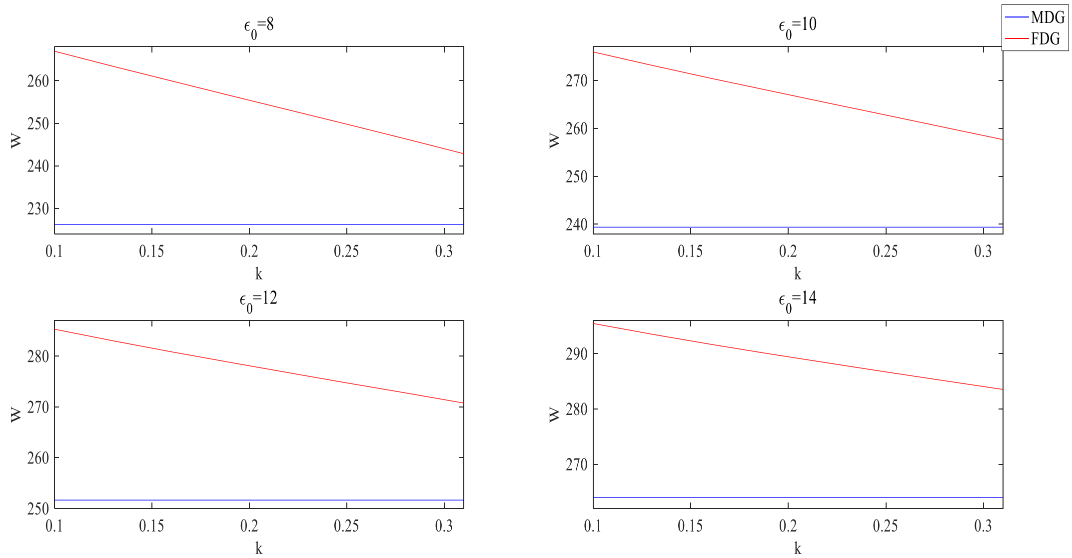

Fig. 17. Comparison of social welfare under two types of demand guarantees when ε_0 takes different values for $v = 8$.

Lemma 4. *In the presence of traffic congestion, the optimal toll price $p_m^{\tau*}$ can be determined by the following equation, and the optimal guarantee level $y_m^{\tau*}$ can be expressed as $y_m^{\tau*} = \frac{1}{\alpha}\left(a - \varepsilon_0 - \tilde{\alpha}p_m^{\tau*} + \sqrt{\gamma\overline{\Lambda}(p_m^{\tau*})}\right)$.*

$$\left(a-\tilde{\alpha}p-\frac{1}{\alpha}\sqrt{\frac{\alpha v\overline{\Lambda}(p)}{4\varepsilon_0\alpha-v}}\right)\left(\sqrt{\frac{(4\varepsilon_0\alpha-v)\overline{\Lambda}(p)}{\alpha v}}-\frac{a}{\alpha'}+\overline{\alpha}p\right)-\frac{2\varepsilon_0\lambda(2\alpha a-\alpha''p)}{\tilde{\alpha}v}\sqrt{\frac{\alpha v\overline{\Lambda}(p)}{4\varepsilon_0\alpha-v}}=0$$

where $\alpha' = 2\alpha-1$, $\alpha'' = 4\alpha-1$, $\tilde{\alpha} = \frac{\alpha'}{2\alpha}$, $\overline{\alpha} = \frac{\alpha''}{2\alpha\alpha'}$ and $\overline{\Lambda}(p) = \frac{\alpha''}{4\alpha}p^2 - ap + \alpha(I+\overline{\Pi})$. The private firm's optimal effort can be expressed as $e_m^{\tau*} = \frac{(2\varepsilon_0-v)p_m^{\tau*}-v(\alpha y_m^{\tau*}-a+\varepsilon_0)}{4\varepsilon_0\alpha-v}$.

Since the analytical solutions are more difficult to calculate in Lemma 4, we still numerically investigate the impact of the unit subsidy on the government's optimal decisions. The parameter values remain the same as those in Table 1. In addition, we introduce $\beta = 1$ and $c = 50$. By fixing $\varepsilon_0 = 9$, Fig. 18 depicts the impacts of the unit subsidy on the optimal guarantee level, optimal toll price, and resulting social welfare.

Fig. 18 illustrates that the optimal guarantee level decreases, the optimal toll price increases, and the optimal social welfare decreases as the unit subsidy increases. Based on the data listed in Fig. 18, it is concluded that for any given unit subsidy, the government should set a higher guarantee level and a lower toll price in the presence of traffic congestion. For any given toll price and private firm's effort, the presence of traffic congestion leads to a low traffic demand and further generates a low operating revenue. Hence, to satisfy the private firm's reservation utility, the government should provide a higher guarantee level. A higher guarantee level can raise the private firm's guaranteed revenue; therefore, the government can set a lower toll price. Since a lower traffic demand decreases consumer surplus and a higher guarantee level increases guaranteed negative externalities, there is lower social welfare in the presence of traffic congestion.

It should be emphasized that even the presence of traffic congestion leads to different sizes of the optimal guarantee level, toll price and social welfare. The direction of the impacts of unit subsidy on these contract variables would not change.

7.2. Contract design with the FDG in the presence of traffic congestion

The private firm's guaranteed revenue under the FDG can be expressed as $v\left(\int_{(1-k)\alpha y-a+p-e}^{\alpha y-a+p-e}\frac{y-q^{\tau}}{2\varepsilon_0}d\varepsilon+\int_{-\varepsilon_0}^{(1-k)\alpha y-a+p-e}\frac{ky}{2\varepsilon_0}d\varepsilon\right)$. Thus, the private firm's profit function and social welfare function can be

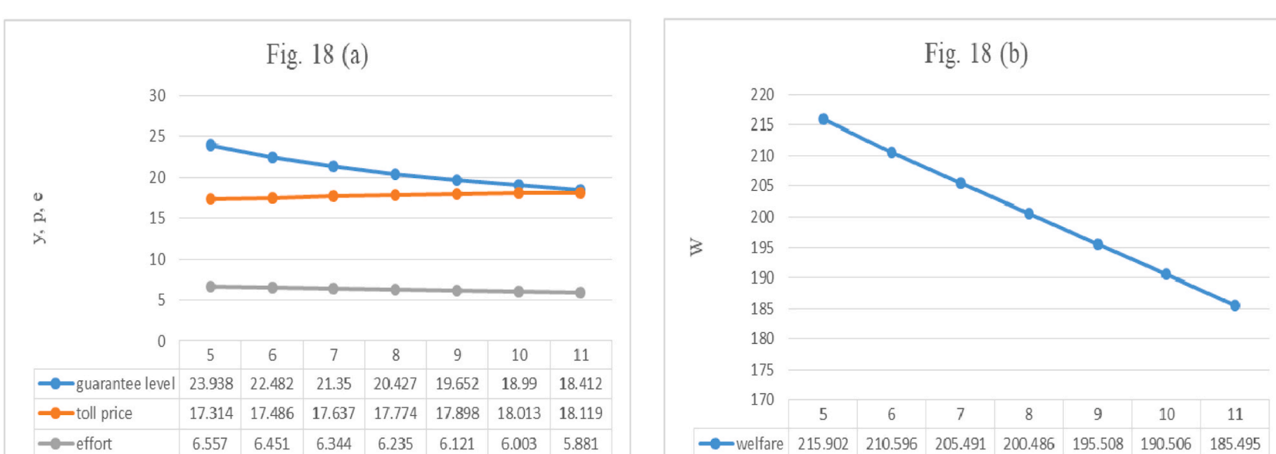

Fig. 18. Impacts of the unit subsidy v when accounting for travelers' VOT.

rewritten as follows:

$$\Pi_f^\tau(p,y,e)=E(pq^\tau)+v\int_{(1-k)\alpha y-a+p-e}^{\alpha y-a+p-e}\frac{y-q^\tau}{2\varepsilon_0}d\varepsilon+v\int_{-\varepsilon_0}^{(1-k)\alpha y-a+p-e}\frac{ky}{2\varepsilon_0}d\varepsilon-e^2-I \tag{12}$$

$$W_f^\tau(p,y,e)=\Pi_f^\tau(p,y,e)+CS^\tau(p,e)-\lambda\left(v\int_{(1-k)\alpha y-a+p-e}^{\alpha y-a+p-e}\frac{y-q^\tau}{2\varepsilon_0}d\varepsilon+v\times\int_{-\varepsilon_0}^{(1-k)\alpha y-a+p-e}\frac{ky}{2\varepsilon_0}d\varepsilon\right) \tag{13}$$

(1) Contract design under symmetric information

We consider the case in which the private firm's effort is contractible to derive the first-best contract (p,y,e).

Lemma 5. *In the presence of traffic congestion, the first-best contract can be described as* $p_{fs}^\tau=a-\frac{\varepsilon_0\alpha'(a(2-k)-vk(\lambda-1))}{\varepsilon_0(2-k)(\alpha''-2)-2\alpha vk(\lambda-1)}$, $y_{fs}^\tau=\frac{2\varepsilon_0(2\alpha a-\varepsilon_0(\alpha''-2))}{\alpha(\varepsilon_0(2-k)(\alpha''-2)-2\alpha vk(\lambda-1))}$ *and* $e_{fs}^\tau=\frac{(a(2-k)-vk(\lambda-1))\varepsilon_0}{\varepsilon_0(2-k)(\alpha''-2)-2\alpha vk(\lambda-1)}$.

Lemma 5 presents the socially optimal toll price, guarantee level and private firm's effort under the FDG. It is obvious that the socially optimal toll price is not equal to 0; the reason is similar to Lemma 3. The socially optimal guarantee level y_{Qs}^τ makes the guaranteed revenue equal to zero, and thus, fails to incur the social cost of public funds.

(2) Contract design under asymmetric information

In the presence of traffic congestion, the second-best problem faced by the government is expressed as follows. The government chooses the optimal contract (p,y) to maximize social welfare and the private firm chooses the optimal effort e to maximize profit. By solving for model (14), Lemma 6 presents the optimal toll price and guarantee level.

$$\begin{aligned}&\max_{(p,y)}\quad W_f^\tau(p,y,e)\\&s.t.\quad \Pi_f^\tau(p,y,e)\geq\overline{\Pi}\\&\Pi_f^\tau(p,y,e)\geq\Pi_f^\tau(p,y,e'),e\neq e'\\&p>0,y>0,e>0\end{aligned} \tag{14}$$

Lemma 6. *In the presence of traffic congestion, the optimal toll price* $p_f^{\tau*}$ *can be determined by the following equation, and the optimal guarantee level* $y_f^{\tau*}$ *is* $y_f^{\tau*}=\frac{2\varepsilon_0}{\alpha\rho'}\Gamma(p_f^{\tau*})$.

$$\left(a-\widetilde{\alpha}p-\frac{vk\overline{\Gamma}(p)}{2\alpha\rho'}+\frac{\lambda}{2\widetilde{\alpha}}(\alpha''p-2\alpha a)\right)\left(\frac{2\varepsilon_0K\overline{\Gamma}(p)}{\alpha\rho'}+\widetilde{\alpha}p-a+\varepsilon_0\right)-\left(\frac{a}{\alpha'}-\overline{\alpha}p-\frac{vk\overline{\Gamma}(p)}{2\alpha\rho'}\right)\left(a-\widetilde{\alpha}p-\frac{2\varepsilon_0\vartheta\overline{\Gamma}(p)}{\alpha\rho'}\right)=0$$

where the parameters K and ϑ are given by $K=(2-k)\alpha+\frac{vk}{4\varepsilon_0}$, $\vartheta=\frac{vk}{4\varepsilon_0}-\frac{\lambda\alpha^2vk}{2\varepsilon_0}$. In addition, $\overline{\Gamma}(p)=2\alpha(a-\varepsilon_0)-\alpha'p+\sqrt{(2\alpha(a-\varepsilon_0)-\alpha'p)^2+\frac{4\alpha\rho'}{vk}\overline{\Lambda}(p)}$. Plugging $p_f^{\tau*}$ and $y_f^{\tau*}$ into the e, the optimal private firm's effort can be expressed as $e_f^{\tau*}=\frac{p_f^{\tau*}}{2\alpha}-\frac{vky_f^{\tau*}}{4\varepsilon_0}$.

Let us numerically perform a sensitivity analysis to investigate the impacts of the parameter k on the government's optimal decisions and the resulting social welfare. In particular, the unit subsidy and the demand spread are first fixed at $v=8$ and $\varepsilon_0=9$ in the numerical analysis.

In the presence of traffic congestion, the social welfare is lower, and the guarantee level is higher. However, similar to the case in which traffic congestion is not considered, Fig. 19 illustrates that the optimal guarantee level decreases, the optimal toll price increases, and the optimal social welfare decreases as the threshold coefficient k increases. Moreover, regardless of whether we account for traffic congestion, the FDG always results in higher social welfare.

8. Conclusions

Traffic demand guarantees offered by the government are common in BOT toll roads with the aim to share demand risks with the private firm and encourage its participation. In this paper, we investigate and compare the optimal contract design with two guarantee types—the MDG and FDG—in the environment of both symmetric information and asymmetric information. We take into account the private firm's effort in improving traffic demand. To investigate the impact of traffic congestion on the comparison results, we have further made an extension to reexamine the results by incorporating travelers' VOT into the model.

We discover that under the MDG with symmetric information, the socially optimal toll price is equal to zero, and the socially optimal guarantee level and private firm's effort are independent of both the unit subsidy and the marginal social cost. For the case with asymmetric information, the private firm's optimal effort is decreasing with respect to the guarantee level, which indicates that the MDG provides the private firm with a disincentive to exert effort to increase traffic demand.

Under the FDG with symmetric information, the socially optimal toll price is also equal to zero. Different from the MDG case with symmetric information, the socially optimal guarantee level and private firm's

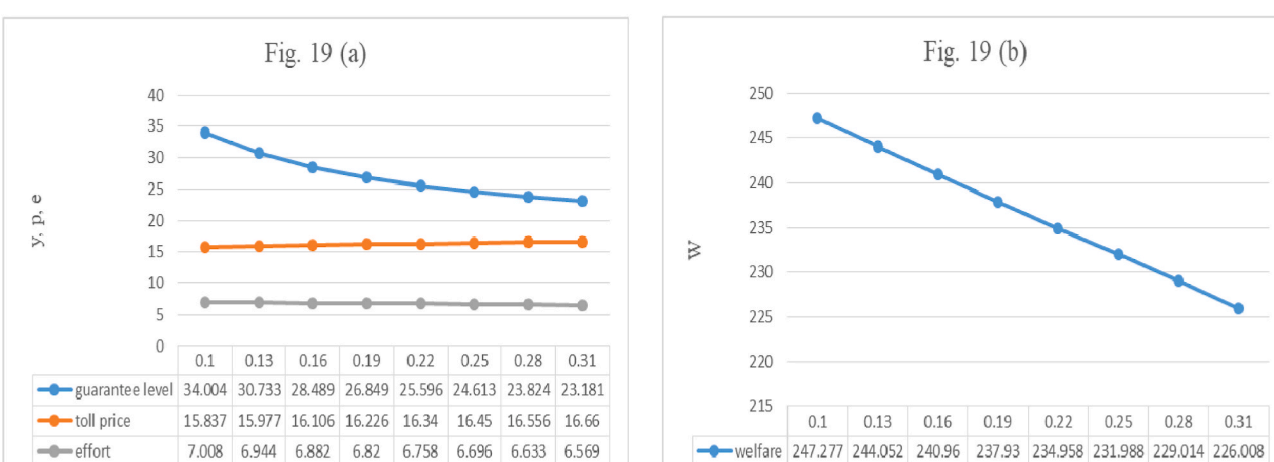

Fig. 19. Impacts of the threshold coefficient k when accounting for travelers' VOT.

effort are increasing in the unit subsidy, marginal social cost and threshold coefficient. For the case with asymmetric information, the private firm's optimal effort is decreasing with respect to the guarantee level and threshold coefficient.

For the comparison, we determine that in the environment with symmetric information, the socially optimal toll prices are equal under the two guarantee types. The socially optimal guarantee level and private firm's effort are higher under the FDG than those under the MDG. However, in the environment with asymmetric information, we discover that for a small threshold coefficient, the optimal toll price, optimal guarantee level and resulting social welfare are higher under the FDG. Moreover, we have also demonstrated that the model results still qualitatively hold when the traffic congestion is considered.

It is worth pointing out that we consider the government's simultaneous use of toll price and government subsidy to improve social welfare. The toll price alone cannot fulfill the role. The underlying logic is as follows. Note that social welfare is dependent on the private firm's effort level. Thus, if the government only takes advantage of the toll price to increase the private firm's effort or to satisfy the private firm's participation constraint, then the toll price has to be increased. However, the improvement of the toll price would damage the consumer surplus, which subsequently leads to a lower social welfare level. However, when government subsidies are present, a lower toll price will be required to attract private firm's participation. The improvement in consumer surplus due to low toll price is able to cover the loss in consumer surplus incurred by subsidy provision. Therefore, the government's simultaneous use of toll price and government subsidy instruments can generate higher social welfare. Our consideration of both toll price and subsidies is also consistent with previous literature that has investigated how subsidies can aid the toll price to achieve the government's aim (Feng et al., 2016; Shi et al., 2016; Tan & Yang, 2012b).

The following managerial implications can be obtained when the information between the two parties is asymmetric. First, the government can set a feasible guarantee level and toll price to incentivize the private firm to select a self-enforced effort to maximize social welfare, regardless of whether travelers' VOT is taken into account. Moreover, the government may choose the FDG instead of the MDG when there is a small threshold coefficient, which means that the FDG is suitable for toll roads with relatively low demand. Conversely, the government prefers to choose the MDG instead of the FDG when the threshold coefficient is large enough, which means that the MDG is suitable for toll roads with relatively high demand. In addition, since the social welfare under the FDG is decreasing with respect to the threshold coefficient, it is possible to make social welfare under the two types of demand guarantees equal if an appropriate threshold coefficient is set, which means that there is no difference for the government to choose which demand guarantee in different demand states.

There are avenues for future research. First, this paper does not consider renegotiation between the government and the private firm. When demand uncertainty is realized during the operating period, either the government or the private firm may request renegotiation to adjust the guarantee level according to the realized demand. This renegotiation may affect the impacts of the two types of demand guarantees on the private firm's effort and their comparison. Second, both the government and the private firm may possess different demand forecasting techniques; as a result, they may have different future demand information. Asymmetric information about future traffic demand may also affect the comparison between the MDG and the FDG.

Data availability statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request, including the simulated data in the sensitivity analysis.

CRediT authorship contribution statement

Yunpeng Zhao: Formal analysis, Writing – original draft. **Jinbo Song:** Project administration. **Zhuo Feng:** Formal analysis, Writing – review & editing. **Lulu Jin:** Formal analysis.

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Appendix A

This appendix provides a table of some frequently employed variables and parameters.

q	traffic demand
p	toll price
a	market size
e	private firm's effort
ε	random variable that captures demand uncertainty
I	private firm's initial construction cost
v	unit subsidy
y	guarantee level
λ	marginal social cost
$\overline{\Pi}$	private firm's reservation utility
$\Pi_m(p, y, e)$	private firm's profit under the MDG without considering congestion externalities
$CS(p, e)$	consumer surplus without considering congestion externalities
$W_m(p, y, e)$	social welfare under the MDG without considering congestion externalities
k	threshold coefficient
$\Pi_f(p, y, e)$	private firm's profit under the FDG without considering congestion externalities
$W_f(p, y, e)$	social welfare under the FDG without considering congestion externalities
β	travelers' TOV
c	road capacity
τ	generalized travel toll price of a trip

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(*continued*)

$\Pi_m^\tau(p,y,e)$	private firm's profit under the MDG when considering congestion externalities
$CS^\tau(p,e)$	consumer surplus when considering congestion externalities
$W_m^\tau(p,y,e)$	social welfare under the MDG when considering congestion externalities
$\Pi_f^\tau(p,y,e)$	private firm's profit under the FDG when considering congestion externalities
$W_f^\tau(p,y,e)$	social welfare under the FDG when considering congestion externalities

Appendix B. . Proofs

Proof of Proposition 1.

Taking the derivative of $W_m(p,y,e)$ with respect to p, we can obtain that $\frac{\partial W_m(p,y,e)}{\partial p} = -p-(\lambda-1)\frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0}$. Note that if $y < a-p+e-\varepsilon_0$, then the guarantee level set by the government is lower than the smallest value of the actual traffic demand. Therefore, the MDG would fail. Consequently, we only need to consider the scenario $y \geq a-p+e-\varepsilon_0$, which immediately yields $\frac{\partial W_m(p,y,e)}{\partial p} < 0$. This finding means that social welfare is decreasing in p. Thus, the socially optimal toll price is equal to zero, namely, $p_{ms} = 0$.

Taking the derivative of $W_m(p,y,e)$ with respect to y gives $\frac{\partial W_m(p,y,e)}{\partial y} = -(\lambda-1)\frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0}$. Since we only need to consider the scenario $y \geq a-p+e-\varepsilon_0$, $\frac{\partial W_m(p,y,e)}{\partial y} < 0$. This result means that social welfare decreases with respect to y. Further, the socially optimal guarantee level is $y = a-p+e-\varepsilon_0$. In addition, taking the derivative of $W_m(p,y,e)$ with respect to e, we have $\frac{\partial W_m(p,y,e)}{\partial e} = a-e+(\lambda-1)\frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} = 0$, which means $e = \frac{2\varepsilon_0 a+v(\lambda-1)(y-a+p+\varepsilon_0)}{2\varepsilon_0+v(\lambda-1)}$. Plugging p_{ms} and y into e, the socially optimal private firm's effort is $e_{ms} = a$. Similarly, plugging p_{ms} and e_{ms} into $y = a-p+e-\varepsilon_0$, the socially optimal guarantee level is $y_{ms} = 2a-\varepsilon_0$.

Proof of Corollary 1.

It immediately follows that the socially optimal guarantee level y_{ms} and private firm's effort e_{ms} are independent of both v and λ based on the expressions y_{ms} and e_{ms}.

Proof of Lemma 1

Taking the derivative of $\Pi_m(p,y,e)$ with respect to e, the first-order condition can be expressed as

$$\frac{\partial \Pi_m(p,y,e)}{\partial e} = p - \frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} - 2e = 0$$

By rearranging this first-order condition, the optimal private firm's effort $e_m(p,y)$ satisfies $e_m(p,y) = \frac{2\varepsilon_0 p - v(y-a+p+\varepsilon_0)}{4\varepsilon_0 - v}$.

(i) Taking the derivative of $e_m(p,y)$ with respect to v gives $\frac{de_m(p,y)}{dv} = \frac{2\varepsilon_0(-2y+2a-p-2\varepsilon_0)}{(4\varepsilon_0-v)^2}$. Therefore, if $y > a-\frac{p}{2}-\varepsilon_0$, we have $\frac{de_m(p,y)}{dv} < 0$, that is, the optimal private firm's effort is decreasing in v.

(ii) Taking the derivative of $e_m(p,y)$ with respect to y gives $\frac{de_m(p,y)}{dy} = -\frac{v}{4\varepsilon_0-v}$. Due to $v < 4\varepsilon_0$, we can obtain that $\frac{de_m(p,y)}{dy} < 0$, that is, the optimal private firm's effort is decreasing in y.

(iii) Taking the derivative of $e_m(p,y)$ with respect to p gives $\frac{de_m(p,y)}{dp} = \frac{2\varepsilon_0-v}{4\varepsilon_0-v}$. Since $v < 4\varepsilon_0$, the denominator is greater than 0. Thus, the optimal private firm's effort is increasing in p if $v < 2\varepsilon_0$ while decreasing in p if $2\varepsilon_0 < v < 4\varepsilon_0$.

(iv) Taking the derivative of $e_m(p,y)$ with respect to ε_0 gives $\frac{de_m(p,y)}{d\varepsilon_0} = \frac{v(4y-4a+2p+v)}{4\varepsilon_0-v}$. Thus, if $y > a-\frac{p}{2}-\frac{v}{4}$, we have $\frac{de_m(p,y)}{d\varepsilon_0} > 0$, which means that the optimal private firm's effort is increasing in ε_0.

Proof of Proposition 2.

Proposition 1 has shown that social welfare is decreasing in y. Thus, the optimal guarantee level should bind the private firm's participation constraint; then, we have $\Pi_m(p,y,e) = \overline{\Pi}$. In addition, the private firm aims to maximize its profit by selecting the optimal effort e, which means that the incentive compatibility constraint can be replaced by $p - \frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} - 2e = 0$.

To solve the optimization model, we need to formulate a Lagrange function as follows:

$$L_m(p,y,e,\mu_1,\mu_2) = W_m(p,y,e) + \mu_1(\Pi_m(p,y,e)-\overline{\Pi}) + \mu_2\left(p - \frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} - 2e\right)$$

where μ_1 and μ_2 represent the Lagrange multipliers. Taking the derivatives of $L_m(p,y,e,\mu_1,\mu_2)$ with respect to p, y, e, μ_1 and μ_2, respectively, gives

$$\begin{cases} \frac{\partial L_m(p,y,e,\mu_1,\mu_2)}{\partial p} = \mu_1(a-2p+e) - p - (\lambda-1-\mu_1)\frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} + \mu_2\left(1-\frac{v}{2\varepsilon_0}\right) = 0 \\ \frac{\partial L_m(p,y,e,\mu_1,\mu_2)}{\partial y} = (1+\mu_1-\lambda)\frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} - \mu_2\frac{v}{2\varepsilon_0} = 0 \\ \frac{\partial L_m(p,y,e,\mu_1,\mu_2)}{\partial e} = a-e+\mu_1(p-2e) + (\lambda-1-\mu_1)\frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} + \mu_2\left(\frac{v}{2\varepsilon_0}-2\right) = 0 \\ \frac{\partial L_m(p,y,e,\mu_1,\mu_2)}{\partial \mu_1} = p(a-p+e) + \frac{v(y-a+p-e+\varepsilon_0)^2}{4\varepsilon_0} - e^2 - I - \overline{\Pi} = 0 \\ \frac{\partial L_m(p,y,e,\mu_1,\mu_2)}{\partial \mu_2} = p - \frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} - 2e = 0 \end{cases}$$

Based on the last equation in these first-order conditions, it readily follows that $\frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0} = p - 2e$. Based on the second equation, we have $(1 + \mu_1 - \lambda)(p - 2e) = \mu_2 \frac{v}{2\varepsilon_0}$. Plugging $\frac{v(y-a+p-e+\varepsilon_0)}{2\varepsilon_0}$ and $\mu_2 \frac{v}{2\varepsilon_0}$ into the first equation and the third equation gives $\mu_1 = \frac{2p-a+e}{2a-3p}$ and $\mu_2 = \frac{(a-p+e)(a-p-e)}{2a-3p}$. Plugging μ_1 and μ_2 into the second equation, the optimal toll price p and the optimal guarantee level y can be determined by the following equation set:

$$\begin{cases} 2\varepsilon_0\left(\frac{p+2(y-a+\varepsilon_0)}{4\varepsilon_0 - v}\right)\left(a+\frac{v(a-\varepsilon_0-y)-2\varepsilon_0 p}{4\varepsilon_0 - v} - \lambda(2a-3p)\right) - (a-p)^2 + \left(\frac{(2\varepsilon_0 - v)p + v(a-\varepsilon_0-y)}{4\varepsilon_0 - v}\right)^2 = 0 \\ p\left(a+\frac{v(a-\varepsilon_0-y)-2\varepsilon_0 p}{4\varepsilon_0 - v}\right) + \varepsilon_0 v\left(\frac{p+2(y-a+\varepsilon_0)}{4\varepsilon_0 - v}\right)^2 - \left(\frac{(2\varepsilon_0 - v)p + v(a-\varepsilon_0-y)}{4\varepsilon_0 - v}\right)^2 - I - \overline{\Pi} = 0 \end{cases}$$

Solving the second equation in this equation set gives $y = a - \varepsilon_0 - \frac{p}{2} \pm \sqrt{\frac{4\varepsilon_0 - v}{v}\Lambda(p)}$, where $\Lambda(p) = \frac{3}{4}p^2 - ap + I + \overline{\Pi}$. Since $y > a - p + e - \varepsilon_0$, we have $a - \varepsilon_0 - \frac{p}{2} < y$. Consequently, the solution $y = a - \varepsilon_0 - \frac{p}{2} - \sqrt{\frac{4\varepsilon_0 - v}{v}\Lambda(p)}$ should be removed. Plugging $y = a - \varepsilon_0 - \frac{p}{2} + \sqrt{\frac{4\varepsilon_0 - v}{v}\Lambda(p)}$ into the first equation in this equation set gives the following equation that determines the optimal toll price p_m^*

$$((12\varepsilon_0\lambda - 2\varepsilon_0 - v)p + 4\varepsilon_0 a(1-2\lambda))\sqrt{\frac{\Lambda(p)}{v(4\varepsilon_0 - v)}} - \frac{3}{2}p^2 + 3ap - a^2 - \overline{\Pi} - I = 0$$

The optimal guarantee level can be expressed as $y_m^* = a - \varepsilon_0 - \frac{p_m^*}{2} + \sqrt{\frac{4\varepsilon_0 - v}{v}\Lambda(p_m^*)}$.

Proof of Proposition 3.

Taking the derivative of $W_f(p,y,e)$ with respect to p gives $\frac{\partial W_f(p,y,e)}{\partial p} = -p - (\lambda - 1)\frac{vky}{2\varepsilon_0} < 0$. Consequently, the socially optimal toll price p_{fs} is equal to 0. Taking the derivative of $W_f(p,y,e)$ with respect to y gives $\frac{\partial W_f(p,y,e)}{\partial y} = -\frac{vk}{2\varepsilon_0}(\lambda - 1)((2-k)y - a + p - e + \varepsilon_0)$. Therefore, social welfare is decreasing in y if and only if $y \geq \frac{a-p+e-\varepsilon_0}{2-k}$. To make $v\int_{-\varepsilon_0}^{(1-k)y-a+p-e}\frac{ky}{2\varepsilon_0}d\varepsilon + v\int_{(1-k)y-a+p-e}^{y-a+p-e}\frac{y-q}{2\varepsilon_0}d\varepsilon = 0$, the optimal guarantee level of maximizing social welfare should satisfy $y = \frac{2(a-p+e-\varepsilon_0)}{2-k}$. Taking the derivative of $W_f(p,y,e)$ with respect to e gives $\frac{\partial W_f(p,y,e)}{\partial e} = a - e + (\lambda - 1)\frac{vky}{2\varepsilon_0} = 0$; then we have $e = a + (\lambda - 1)\frac{vky}{2\varepsilon_0}$. Plugging p_{fs} and y into e yields $e_{fs} = a + \frac{vk(\lambda-1)(2a-\varepsilon_0)}{2\varepsilon_0 - k\varepsilon_0 - vk(\lambda-1)}$. Plugging p_{fs} and e_{fs} into y, the socially optimal guarantee level becomes $y_{fs} = \frac{2\varepsilon_0(2a-\varepsilon_0)}{2\varepsilon_0 - k\varepsilon_0 - vk(\lambda-1)}$.

Proof of Corollary 2.

Taking the derivative of y_{fs} with respect to v gives $\frac{dy_{fs}}{dv} = \frac{2\varepsilon_0 k(\lambda-1)(2a-\varepsilon_0)}{(2\varepsilon_0 - k\varepsilon_0 - vk(\lambda-1))^2}$ Since $2a - \varepsilon_0 > 0$ and $(\lambda - 1)$, we can obtain that $\frac{dy_{fs}}{dv} > 0$.

Taking the derivative of y_{fs} with respect to λ gives. $\frac{dy_{fs}}{d\lambda} = \frac{2\varepsilon_0 vk(2a-\varepsilon_0)}{(2\varepsilon_0 - k\varepsilon_0 - vk(\lambda-1))^2} > 0$

Taking the derivative of y_{fs} with respect to k gives. $\frac{dy_{fs}}{dk} = \frac{2\varepsilon_0(\varepsilon_0 + v(\lambda-1))(2a-\varepsilon_0)}{(2\varepsilon_0 - k\varepsilon_0 - vk(\lambda-1))^2}$ Since $\lambda - 1 > 0$ and $2a - \varepsilon_0 > 0$, we have $\frac{dy_{fs}}{dk} > 0$.

Taking the derivative of e_{fs} with respect to v gives $\frac{de_{fs}}{dv} = \frac{\varepsilon_0 k(2-k)(\lambda-1)(2a-\varepsilon_0)}{(2\varepsilon_0 - k\varepsilon_0 - vk(\lambda-1))^2}$ Since $2 - k > 0$ and $2a - \varepsilon_0 > 0$, we have $\frac{de_{fs}}{dv} > 0$.

Taking the derivative of e_{fs} with respect to λ gives. $\frac{de_{fs}}{d\lambda} = \frac{\varepsilon_0 vk(2-k)(2a-\varepsilon_0)}{(2\varepsilon_0 - k\varepsilon_0 - vk(\lambda-1))^2} > 0$

Taking the derivative of e_{fs} with respect to k gives $\frac{de_{fs}}{dk} = \frac{2\varepsilon_0 v(\lambda-1)(2a-\varepsilon_0)}{(2\varepsilon_0 - k\varepsilon_0 - vk(\lambda-1))^2} > 0$

Proof of Lemma 2.

The first-order condition of $\Pi_f(p,y,e)$ with respect to e is $\frac{\partial \Pi_f(p,y,e)}{\partial e} = p - \frac{vky}{2\varepsilon_0} - 2e = 0$. Thus, the optimal private firm's effort $e_f(p,y)$ should satisfy $p - \frac{pky}{2\varepsilon_0} - 2e_f(p,y) = 0$, which gives $e_f(p,y) = \frac{p}{2} - \frac{vky}{4\varepsilon_0}$.

(i) Taking the derivative of $e_f(p,y)$ with respect to v gives $\frac{de_f(p,y)}{dv} = -\frac{ky}{4\varepsilon_0}$. Since $\frac{ky}{4\varepsilon_0} > 0$, it readily follows that $\frac{de_f(p,y)}{dv} < 0$.
(ii) Taking the derivative of $e_f(p,y)$ with respect to y gives $\frac{de_f(p,y)}{dy} = -\frac{vk}{4\varepsilon_0}$. Since $\frac{vk}{4\varepsilon_0} > 0$, we can obtain that $\frac{de_f(p,y)}{dy} < 0$.
(iii) Taking the derivative of $e_f(p,y)$ with respect to k gives $\frac{de_f(p,y)}{dk} = -\frac{vy}{4\varepsilon_0} < 0$.
(iv) Taking the derivative of $e_f(p,y)$ with respect to p gives $\frac{de_f(p,y)}{dp} = \frac{1}{2} > 0$.
(v) Taking the derivative of $e_f(p,y)$ with respect to ε_0 gives $\frac{de_f(p,y)}{d\varepsilon_0} = \frac{vky}{4\varepsilon_0^2} > 0$.

Proof of Proposition 4.

Similarly, the private firm's participation constraint should be binding, that is, $\Pi_f(p,y,e) = \overline{\Pi}$. Moreover, the incentive compatibility constraint can be replaced by $p - \frac{vky}{2\varepsilon_0} - 2e = 0$. The Lagrange function associated with the optimization model (7) is

$$L_f(p,y,e,\mu_1,\mu_2) = W_f(p,y,e) + \mu_1(\Pi_f(p,y,e) - \overline{\Pi}) + \mu_2\left(p - \frac{vky}{2\varepsilon_0} - 2e\right)$$

where μ_1 and μ_2 are the Lagrange multipliers. Taking the derivative of $L_f(p,y,e,\mu_1,\mu_2)$ with respect to p, y, e, μ_1 and μ_2, respectively, gives

$$\begin{cases}\dfrac{\partial L_f(p,y,e,\mu_1,\mu_2)}{\partial p}=-p-(\lambda-1)\dfrac{vky}{2\varepsilon_0}+\mu_1\left(a-2p+e+\dfrac{vky}{2\varepsilon_0}\right)+\mu_2=0\\ \dfrac{\partial L_f(p,y,e,\mu_1,\mu_2)}{\partial y}=(\mu_1-\lambda+1)\dfrac{vk((2-k)y-a+p-e+\varepsilon_0)}{2\varepsilon_0}-\mu_2\dfrac{vk}{2\varepsilon_0}=0\\ \dfrac{\partial L_f(p,y,e,\mu_1,\mu_2)}{\partial e}=a-e+(\lambda-1)\dfrac{vky}{2\varepsilon_0}+\mu_1\left(p-\dfrac{vky}{2\varepsilon_0}-2e\right)-2\mu_2=0\\ \dfrac{\partial L_f(p,y,e,\mu_1,\mu_2)}{\partial \mu_1}=p(a-p+e)+\dfrac{vky}{2\varepsilon_0}((1-k)y-a+p-e+\varepsilon_0)+\dfrac{vk^2y^2}{4\varepsilon_0}-e^2-I-\overline{\Pi}=0\\ \dfrac{\partial L_f(p,y,e,\mu_1,\mu_2)}{\partial \mu_2}=p-\dfrac{vky}{2\varepsilon_0}-2e=0\end{cases}$$

Based on the last equation in these first-order conditions, it readily follows that $\frac{vky}{2\varepsilon_0}=p-2e$. Plugging $\frac{vky}{2\varepsilon_0}=p-2e$ into the third equation, we can obtain that $\mu_2=\frac{a-p+e+\lambda(p-2e)}{2}$. In addition, plugging μ_2 and $\frac{vky}{2\varepsilon_0}$ into the first equation, we have $\mu_1=\frac{\lambda(p-2e)-a+p+3e}{2(a-p-e)}$. Plugging μ_1 and μ_2 into the second equation, we can obtain that $(a-p+e+\lambda(3p-2a))((2-k)y-a+p-e+\varepsilon_0)-(a-p-e)(a-p+e+\lambda(p-2e))=0$. Consequently, the optimal toll price p and the optimal guarantee level y can be determined by the following equation set

$$\begin{cases}\left(a-\dfrac{p}{2}-\dfrac{vky}{4\varepsilon_0}+\lambda(3p-2a)\right)\left(\left(2-k+\dfrac{vk}{4\varepsilon_0}\right)y-a+\dfrac{p}{2}+\varepsilon_0\right)-\left(a-\dfrac{3p}{2}+\dfrac{vky}{4\varepsilon_0}\right)\left(a-\dfrac{p}{2}+\left(\lambda-\dfrac{1}{2}\right)\dfrac{vky}{2\varepsilon_0}\right)=0\\ \dfrac{vk(8\varepsilon_0+vk-4\varepsilon_0k)}{16{\varepsilon_0}^2}y^2-\dfrac{vk(2a-p-2\varepsilon_0)}{4\varepsilon_0}y-\left(\dfrac{3}{4}p^2-ap+I+\overline{\Pi}\right)=0\end{cases}$$

Based on the second equation in this equation set, we can obtain that $y=\frac{2\varepsilon_0}{\rho}\Gamma(p)$, where $\rho=8\varepsilon_0+vk-4\varepsilon_0k$ and $\Gamma(p)=2a-p-2\varepsilon_0+\sqrt{(2a-p-2\varepsilon_0)^2+\frac{4\rho}{vk}\Lambda(p)}$. Plugging y into the first equation this equation set gives the following equation, which determines the optimal toll price p_f^*.

$$\left(a-\frac{p}{2}-\frac{vk}{2\rho}\Gamma(p)+\lambda(3p-2a)\right)\sqrt{\left(a-\frac{p}{2}-\varepsilon_0\right)^2+\frac{\rho}{vk}\Lambda(p)}-\left(a-\frac{3p}{2}+\frac{vk}{2\rho}\Gamma(p)\right)\left(a-\frac{p}{2}+\left(\lambda-\frac{1}{2}\right)\frac{vk}{\rho}\Gamma(p)\right)=0$$

In particular, the optimal guarantee level can be expressed as $y_f^*=\frac{2\varepsilon_0}{\rho}\Gamma(p_f^*)$.

Proof of Proposition 5.

(i) Since $p_{ms}=0$ and $p_{fs}=0$, we can conclude that $p_{ms}=p_{fs}=0$.

(ii) Based on the expressions y_{ms} and y_{fs}, we can obtain that $y_{ms}-y_{fs}=-\frac{(k\varepsilon_0+vk(\lambda-1))(2a-\varepsilon_0)}{2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)}$. Since $2a-\varepsilon_0>0$ and $2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)>0$, we have $y_{ms}-y_{fs}<0$, which means that $y_{ms}<y_{fs}$.

(iii) Based on the expressions e_{ms} and e_{fs}, we have $e_{ms}-e_{fs}=-\frac{vk(\lambda-1)(2a-\varepsilon_0)}{2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)}<0$, which means that $e_{ms}<e_{fs}$.

(iv) Based on the expressions $\Pi_{ms}(p_{ms},y_{ms},e_{ms})$ and $\Pi_{fs}(p_{fs},y_{fs},e_{fs})$, we can obtain that

$$\Pi_{ms}(p_{ms},y_{ms},e_{ms})-\Pi_{fs}(p_{fs},y_{fs},e_{fs})=\frac{2avk(\lambda-1)(2a-\varepsilon_0)}{2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)}+\left(\frac{vk(\lambda-1)(2a-\varepsilon_0)}{2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)}\right)^2$$

Since $2a-\varepsilon_0>0$ and $2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)>0$, we can infer that $\Pi_{ms}(p_{ms},y_{ms},e_{ms})-\Pi_{fs}(p_{fs},y_{fs},e_{fs})>0$, which means that $\Pi_{ms}(p_{ms},y_{ms},e_{ms})>\Pi_{fs}(p_{fs},y_{fs},e_{fs})$.

(v) Based on the expressions $W_{ms}(p_{ms},y_{ms},e_{ms})$ and $W_{fs}(p_{fs},y_{fs},e_{fs})$, we can obtain that

$$W_{ms}(p_{ms},y_{ms},e_{ms})-W_{fs}(p_{fs},y_{fs},e_{fs})=\frac{1}{2}\left(\frac{vk(\lambda-1)(2a-\varepsilon_0)}{2\varepsilon_0-k\varepsilon_0-vk(\lambda-1)}\right)^2>0$$

which means that $W_{ms}(p_{ms},y_{ms},e_{ms})>W_{fs}(p_{fs},y_{fs},e_{fs})$.

Proof of Lemma 3. This proposition can be proved with a similar approach to Proposition 1.

Proof of Lemma 4. This proposition can be proved with a similar approach to Proposition 2.

Proof of Lemma 5. This proposition can be proved with a similar approach to Proposition 3.

Proof of Lemma 6. This proposition can be proved with a similar approach to Proposition 4.

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