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Research paper

Crowding externalities and optimal subsidies in public transport: Revisiting the Parry–Small model

Tamás Strommer [a,b,*], Daniel Hörcher [c,a], András Munkácsy [b]

[a] Budapest University of Technology and Economics, 3 Műegyetem rkp., Budapest H-1111, Hungary
[b] KTI Hungarian Institute for Transport Sciences and Logistics, 3-5 Than Károly Street, Budapest H-1119, Hungary
[c] Imperial College London, Exhibition Road, London SW7 2AZ, United Kingdom

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ABSTRACT

Subsidising public transport is a widely used policy, normally justified by redistributional objectives. Frequent use of subsidies, however, does not mean uniformity, as the actual subsidy rate varies between countries and cities. Excessive subsidisation of public transport can also be criticised by the presence of externalities, most importantly on-board crowding. This paper revisits Parry & Small (2009), one of the most influential contributions in the recent history of public transport economics. Despite the popularity of the Parry–Small model, it neglects existing empirical evidence on the effects of crowding on passengers' wellbeing. The purpose of the present study is to redesign and recalibrate their model by expressing crowding disutility as a multiplier of the value of in-vehicle travel time, and also to extend the analysis to the context of the Hungarian interurban transport system. Our results show that the crowding externality is a critical component of optimal pricing, as it decreases the optimal subsidy rate by 4 to 29%, even at a moderate level of capacity utilisation. This means that optimal fares are higher than they were previously found. We extend the original analysis to the case of Hungarian interurban transport and highlight the model's sensitivity to the assumed substitution pattern.

1. Introduction

The efficient operation of the transport system is a cornerstone of well-functioning cities. Since the recognition of this idea, greater attention has been focused on resource-efficient policies on the strategic, tactical as well as operational levels of decision making in urban governance. Public transport supply is one of the most frequently debated subjects in the policy arena. Transport pricing is a well-established and frequently used method of influencing traveller decisions and regulating demand, but existing transport economic findings are far from being consensual among decision makers and the public.

This study examines the optimal pricing and subsidy decisions of public transport operators. Although the daily operation of mobility service providers are often supported by public funds, the volume of external funding is mostly determined by ad-hoc political decisions or easily applicable but theoretically unjustified subsidy formulae instead of an accurate definition of the efficient subsidy. The appropriate rate of subsidies and the suitable funding method of public transport is important due to their impact on service attractiveness, efficiency and reliability; and also due to income distributional reasons (Gwilliam, 2008). Economic theory suggests that the presence of scale economies and substitution with under-priced car use may justify unprofitable service provision to certain extent; see our detailed literature review below. However, recent studies show that significant differences can be observed between major cities around the world in terms of the subsidies granted to transport agencies (see, for example, Doll & van Essen, 2008; Parry & Small, 2009; Tscharaktschiew & Hirte, 2012). Underfunding is a typical phenomenon in car dependent societies, including the United States, while recent trends indicate that unjustified policy decisions may happen in the opposite extreme as well: in the aftermath of the Covid-19 pandemic, several European countries introduced practically fare-free access to public transport (Cantner et al., 2022).

One of the critical source of externalities in public transport is crowding, i.e. the cost of inconvenience when the occupancy rate of vehicles is high. This form of congestion received increased attention in the literature since advanced empirical methods (mainly discrete choice models) made it possible to quantify the user cost of crowding (Hörcher & Tirachini, 2021; Li & Hensher, 2011; Wardman & Whelan, 2011). According to empirical estimates, the user cost of crowding might double the value of in-vehicle crowding in peak periods. Given that crowding is a cost imposed by one traveller on another, its externality

* Corresponding author at: KTI Hungarian Institute for Transport Sciences and Logistics, 3-5 Than Károly Street, Budapest H-1119, Hungary.
E-mail addresses: strommer.tamas@kti.hu (T. Strommer), d.horcher@imperial.ac.uk (D. Hörcher), munkacsy.andras@kti.hu (A. Munkácsy).

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nature makes it a crucial factor to consider in welfare-oriented pricing exercises.

This paper revisits one of the most influential papers on optimal pricing and subsidy decisions in public transport. Parry and Small (2009) developed and quantified a microeconomic model to determine the welfare maximising subsidy rate and the corresponding capacity decisions, including the optimal vehicle size, service frequency and line density. Our study is motivated by the observation that Parry and Small designed their model in absence of empirical evidence on the user cost of crowding, which is only partly circumvented in their paper with intuitively calibrated elasticities. This paper extends the Parry–Small model by replacing their crowding cost modules with a crowding-dependent travel time multiplier approach, which is widely used in the rapidly evolving empirical literature. This theoretical contribution makes the Parry–Small model consistent with recent findings in the literature and suitable for precise calibration.

The second contribution of the paper is that we apply the model in a new policy context, the Hungarian interurban public transport system. This application of the model might have clear policy relevance, given that, to the best of our knowledge and according to a recent review by Matyas, Hörcher, and Pawlak (2022), optimal fares and subsidies have never been estimated for any Eastern (post-communist) member states of the European Union.

The paper is structured as follows: the rest of this section reviews the relevant literature of public transport economics, and explains the policy context of our new application in more depth. Section 2 covers the paper's methodology. We revise the essential parts of Parry and Small (2009) and introduce the methodological contributions of our paper. Section 3 describes new data sources. Section 4 presents the results of a numerical application of the improved model, and Section 5 completes the paper with concluding remarks.

1.1. The literature: Optimal pricing of public transport

The optimisation of pricing and subsidy policies in public transport is among the most popular subjects of transport economics. A recent review by Hörcher and Tirachini (2021) cover around 350 influential studies of this literature since the seminal contribution of Herbert Mohring (1972), while Jara-Díaz, Gschwender, and Hörcher (2023) focus on the link between optimal pricing and capacity provision specifically.

The methodological foundations of this literature are rooted in the microeconomic concept of *marginal cost pricing*. This theory suggests that resource efficiency is maximised when each traveller pays a monetary sum equal to the net marginal social cost of the trip, including a range of possible welfare effects within the transport market (in various modes of transport) and beyond. The financial incentive ensures that each trip creates more value for the traveller than net cost for society; wasteful trips are avoided but no useful trip is priced off the service. The optimal subsidy is a derived quantity. When the optimal fare (per person) is greater than the average operating cost, the service is profitable—otherwise the operator should rely on public subsidies.

Naturally, the derivation of the marginal social cost is complicated and thus involved a learning curve as scholars and practitioners gradually identified new ways in which travel generates costs and benefits for the operator, other users, and external parties within society. Therefore, it is instructive to review this literature chronologically.

Under the traditional economic school of thought, pricing decisions are determined by the cost structure of the producer and potential market interactions with other suppliers. The estimation of operating cost functions dates back to the early 20th century as well, usually confirming the presence of scale and density economies in public transport. As most of the public transport literature as well as this paper use standard assumptions on operating cost functions, we refer to Small and Verhoef (2007) and Jara-Díaz (2007) who provide reviews that apply across modes.

Researchers recognised very soon that several social costs of public transport emerge in the form of a marginal user externality, i.e. various ways in which the marginal traveller affects the travel experience of other users. The crucial insight of Mohring (1972) has been that the average waiting time is endogenous with respect to service frequency, and frequency increases with demand, so that the marginal trip may, on average, contribute to wait time reduction for fellow travellers. This positive externality is often called the Mohring Effect (see a specific review in Silva, 2021). A key consequence of external waiting time savings under endogenous frequency is that the net marginal cost of public transport use is less than the average operating cost, thus justifying a financial deficit and positive subsidies when pricing and capacity provision are both optimal. The same way as waiting times depend on service frequency, access and egress times depend on stop spacing (Mohring, 1972) and line density (Chang & Schonfeld, 1991; Kocur & Hendrickson, 1982), and directness of public transport trips depends on network structure (Fielbaum, Jara-Díaz, & Gschwender, 2020), reinforcing the case for subsidised operations.

The marginal trip may affect the average user cost even if capacity (that is, the line layout, service frequencies and vehicle size) are independent of demand. First, it has been recognised early that boarding and alighting takes time, and therefore the average journal time of everyone on board depends on boarding and alighting volumes (Jansson, 1980). Second, public transport is by no means less sensitive to capacity scarcity, the same way as roads are congestible. The phenomenon of in-vehicle crowding has been identified in pioneering theoretical studies such as Kraus (1991) and Jara-Díaz and Gschwender (2003), and later on became a leading theme in public transport research since advances in discrete choice modelling made crowding cost measurement reliable (Li & Hensher, 2011; Wardman & Whelan, 2011). See a thematic review in Hörcher (2021). Both boarding and alighting delay and the inconvenience of crowding are negative externalities, having (i) a positive impact on the marginal social cost of travelling, and (ii) a negative impact on the optimal subsidy (Tirachini, Hensher, & Rose, 2013).

As crowding forms the basis of some of this paper's main contributions, let us summarise the core features of the related literature:

- An analytical expression has been created for the modelling of discomfort on public transport services with a calculation of disutility from travelling early or late; an important indicator is the number of passengers standing compared to the capacity of the vehicle, and also a conclusion that passengers with a seat and those who standing must be handled differently (de Palma, Kilani, & Proost, 2015).
- According to the modelling of rail transport (Paris RER) decisions with no fare or a uniform fare, ridership is too concentrated on timely trains. Marginal-cost-pricing calls for time-dependent fares that smooth train loads and generate more revenue than an optimal uniform fare (de Palma, Lindsey, & Monchambert, 2017).
- Jin, Schmöcker, and Maadi (2019) analyse the interaction between fares and public transport service quality. They found that with higher fares the operator has more resources to provide a better service, while demand depends on both service quality and fare. Their model includes the optimisation of the transportation network, stop spacing, the size of the dense service area in a city, as well as optimal fare and demand elasticity. Jin et al. (2019) also illustrate the possible existence of critical fare points, at which the demand and service quality of public transport suddenly drop if an attractive alternative mode appears.
- Tirachini et al. (2013) examine the multiple dimensions of passenger crowding related to public transport demand, supply and operations, including effects on operating speed, waiting time, travel time reliability, passengers' wellbeing, valuation of waiting and in-vehicle time savings, route and bus choice, and optimal levels of frequency, vehicle size and fare. In addition, crowding

externalities are estimated for rail and bus services in Sydney, in order to show the impact of crowding on the estimated value of in-vehicle time savings and demand prediction.

– Haywood, Koning, and Monchambert (2017) assess discomfort with PT crowding at various density levels across heterogeneous individuals using a different methodology. They found negative relationship of in-vehicle density on reported satisfaction across most individual characteristics. They also found an increase in crowding costs over users' income. Furthermore, they identified three key drivers: dissatisfaction with standing and not being seated; less opportunities to make use of the time during the journey; the physical closeness of other travellers.
– The significant change brought by the proliferation of information and communication technologies (ICTs) also has to be highlighted, as it has made multitasking a realistic option during travel (through learning, working, various communication forms etc., see Keseru, Bulckaen, Macharis, Minnen, Glorieux et al., 2015; Keseru & Macharis, 2018; Munkácsy, Keserű, & Siska, 2022). This change made travel time almost as valuable as other activities—in an appropriate travel environment the value of crowding cost savings can be significant (International Transport Forum, 2018).

The literature reviewed so far is concerned with first-best (unconstrained) partial equilibrium models. Further ingredients of the model presented below include substitution between public transport modes and car use. If sub-optimal pricing in one of the substitutes of public transport is given as an exogenous constraint, then reducing the fares may be a second-best response. This argument is often raised in today's policy debates on sustainability. Small and Verhoef (2007) solve the two-mode problem generically, Tirachini and Hensher (2012) extend the analysis to three modes, one of which is an active mode that public transport should not substitute ideally, while Gómez-Lobo, Tirachini, and Gutierrez (2022) solved for the optimal second-best tariff with an arbitrary number of modes. Theory confirms that the extent to which public transport fares should be lower than their first-best level is determined by the quantitative (i) magnitude of under-pricing in car use, (ii) substitution between public transport and the under-priced mode, and (iii) substitution with active modes.

Subsidisation of transport services is a frequently used second-best policy in developed countries, although the usually applied subsidy volume widely varies by cities and countries (Basso & Silva, 2014; Parry & Small, 2009). The optimal subsidy rate is a key research topic of transport management, since it has strong connections with sustainability, social equity, traffic management and congestion pricing (Börjesson, Fung, & Proost, 2017; Holmgren, 2010; Kębłowski, 2020). Several studies examined and modelled subsidy policies at different city sizes, transport service frequencies, vehicle capacities, bus lane policies, and bus stop designs (Basso & Silva, 2014; Börjesson et al., 2017; Börjesson, Fung, Proost, & Yan, 2019).

In a literature hallmarked by the contributions above, Parry and Small (2009) were the first to publish a comprehensive model of public transport pricing that they put into an empirically relevant context by calibrating optimal subsidy levels for Washington, DC and Los Angeles in the US, and London in the UK. The study, published in the American Economic Review, has attracted considerable attention, partly because they developed an iterative methodology which enabled them to solve the model in a regular spreadsheet. Since 2009, the Parry–Small model has become a core reference point in the literature. However, it has become outdated in terms of the appropriate representation of crowding inconvenience: in 2009, Parry and Small could not rely on the vast empirical literature developed in the 2010s in which crowding costs are expressed as a multiplier of the value of time. We devote the present paper to revisit their seminal model and update their results with a more consensual representation of in-vehicle crowding.

1.2. Policy context: Interurban transport in Hungary

The uniqueness of the Hungarian municipality system is its strongly centralised pattern, the effect of which can be observed both in the structure of the transport network and also in the main connections of transport services. Due to the country's radially structured transport infrastructure, the lack of transverse connections, and the poor permeability of the Danube River—which divides the country in halves—the vast majority of road and rail traffic flows directly through, or in the immediate vicinity of the capital city, Budapest. This effect puts a huge weight and traffic on the road and rail network of Central Hungary. One quarter of the country's population lives in Budapest and its metropolitan area (Hungary had a population of 9.77 million in 2020). Beside the complex and overcongested transportation system of Budapest and its agglomeration, the biggest traffic appears on the roads and rails between the central region of Hungary and the larger cities of the country. Although of lesser importance in terms of traffic volume, railway lines and bus routes serving rural settlements also play an important role in the country's transport system (Lieszkovszky, 2018).

An important feature of public transport in Hungary is that state-owned service providers still dominate the market. Almost exclusively, the Hungarian state-owned company group, the MÁV–Volán Group's subsidiaries are responsible for the passenger transport on rail and for the interurban bus services—and in some cities for the local transport service as well, on the basis of a contract between the municipality and the company. The MÁV–Volán Group has an important role to play in providing public transport services between different regions of the country, in cities and metropolitan areas, as well as in villages, towns and cities connecting them to the core of the transportation system.

The examination of the subsidisation issue is complicated by the fact that in the European Union, hence in Hungary also, transport service providers often operate on a regulated market within the framework of public service. In a regulated market like this, there is competition when one comes to entering the market, but companies already in the market are providing the service on a yet exclusive basis (see the work of Nash, 2010). In addition, the state—procurer of public transport services—may impose a public service obligation (according to Regulation (EC) No 1370/2007), but in this case the transport company's burdens resulting from this service has to be balanced (Mándoki & Lakatos, 2018). In other words, it is a practical solution for a state to sustain public transport services through subsidisation, but this method of operation also fixes the dependence of the service providers on state resources.

Among the European Union member states, Hungary has one of the highest coverage rates of public bus transport (double of the EU-28 average, see the Statistical Office of the European Communities, 2019), a kind of service that is available at nearly all settlements. (Compare the Hungarian case study with the results and findings of Sakai and Shoji (2010).) This level of coverage is probably caused by the fact, that the motorisation level is well below the European average (around 2/3 of the EU-28, see the Statistical Office of the European Communities, 2019), moreover, the Hungarian State provides travel discounts to several social groups. For example, citizens under the age of 6 and over 65 can use almost all kind of public transport for free, and people with a student card also receive a 50% discount on their travel (Oszter, 2017).

Although this paper primarily deals with the development of methodology of calculating optimal public transport subsidies, the Hungarian adaptation of the modified calculation method was also an important element of the research. The application of the methodology for Hungary has been carried out on a national level, primarily in order to ensure that the local characteristics of the large-scale transport system do not significantly influence the results of the calculations.

Although Parry and Small applied their model to urban areas, London, Los Angeles and Washington, DC are three cities with different geographic and socio-economic characteristics (as the authors admit on page 702), and therefore we believe the model was designed to be robust enough under a variety of circumstances, such as, in our case, the Hungarian interurban public transport market.

2. Methodology

One of the most significant contributions of public transport economics is Parry and Small (2009) on optimal pricing and subsidisation in urban transport. The paper reflects on fierce debates in the United States on whether the subsidy rate of public transport companies needs to be reduced (or even abolished), or increased to ensure self-financing as in case of many other sectors of the transport industry; aviation and road and rail freight, for example. The authors stated that despite the differences between the subsidy rates of transport companies, there is no generally accepted, practicable calculation method that can be used to determine the ideal financial strategy for a given transport operator, i.e. the rate of subsidy.

The aim of Parry and Small was to create an analytical model that is simple enough to use without highly specific local data sources. Their model is general enough to rely only on data that are typically available at all transport providers, statistical offices, etc., and at the same time the model can manage the most important parameters describing the transport system. Their objectives also included that the calculation method should be applicable to transport systems of significantly different size, structure, technological features, and vehicle occupancy, as well as to cities and countries with different levels of development.

The relative simplicity of the model is also its major drawback. The aggregate model and data used for describing public transport networks cannot handle the local variation in spatial demand and supply characteristics (e.g. service frequency, wage rate, socio-demographic characteristics). Since the model does not describe and include information about the structure and capacity of the transportation system it cannot take bottlenecks or capacity constraints in account. Another issue often highlighted nowadays is the marginal cost of public funds (MCF, described in more depth by Kleven & Kreiner, 2006), which is also not included in the model—just mentioned as an issue among Parry and Small's conclusions. Due to the inefficiency of raising taxes elsewhere in the economy, our updated results may still overestimate the optimal subsidy; see a related general equilibrium model in Hörcher, De Borger, Seifu, and Graham (2020).

2.1. Model structure

The model includes two time periods (peak and off-peak) and three modes of transport: private automobile (H), rail (R), and bus (B). One of the model's unconventional methodological features is that many parameters and endogenous variables are normalised by the vehicle-miles supplied and travelled along a representative origin–destination pair.

Parry and Small define household behaviour by assuming that the representative agent maximises a utility function U of general consumption X, subutility M from passenger miles travelled, the generalised cost of travel Γ, and the magnitude of externalities (Z) which is exogenous in the consumer's decision. Further specifications include that M depends on the vector of peak and off-peak passenger miles by the three modes, and Γ has four arguments: in-vehicle travel time T, wait time W, access time A, and crowding disutility C. The cost of waiting is inversely proportional to service frequency f^{ij}, the access cost decreases in route density D^{ij}, and crowding increases in the load factor l^{ij} of services, where i and j and time period and mode indices, respectively. Naturally, W, A, and C are not applicable for the private car mode.

On the supply side, travel time on the road is endogenous in the model, congestion affects car as well as bus users, while rail is assumed uncongestible in the headway interval considered. In addition, bus and rail dwell times (time of boarding and alighting) are dependent on the vehicle occupancy of these modes, which will also be a source of a user externality in subsequent derivations. The average boarding and alighting time per passenger is θ^j in mode j. The operating cost of public transport services is governed by the available capacity (frequency and vehicle capacity). To (partly) cover this expense, the transport agency has two sources of revenue: fuel tax revenues paid after each vehicle mile travelled by car, and the fare payments of public transport users. The transport system is allowed to produce a deficit. To cover the deficit, households' budget constraint includes a lump-sum payment to the government responsible for transport provision.

2.2. The welfare effect of marginal price adjustments

The household's consumption problem is solved through the usual steps of utility maximisation subject to the budget constraint, and the solution leads us to demand functions $\{M^{ij}\}$, X, and indirect utility $\tilde{U}$. As the agency budget constraint feeds into consumer utility through the lump-sum tax above, the representative household's indirect utility can be used as a measure of aggregate social welfare, and the welfare effect of a marginal fare adjustment will be expressed through the total differentiation of $\tilde{U}$ with respect to one of the fare variables p^{ij}. Note that this modelling approach is based on a restrictive assumption: lump-sum taxes are rare in practice and it is more likely that transport subsidies are funded through income or consumption related taxes. These tax instruments cause distortions in the related markets by discouraging labour supply and demand for goods and services. The distortions caused by taxation can be captured by the *marginal cost of public funds* (MCPF); see Hörcher and Tirachini (2021) for a review of the relevant transport literature. The MCPF creates an incentive for the welfare maximising transport operator to raise fares and reduce the share of subsidies in the funding mix. This aspect could also be integrated into the Parry–Small model as part of future research efforts.

The differentiation of indirect utility governs the core pricing analysis in the model of Parry and Small (2009); as opposed to the vast majority of the public transport literature, they do not derive an analytical formula for the welfare maximising set of public transport fares. They derive the marginal welfare effect of fare adjustments instead, which is equally suitable to (i) compute the welfare maximising policy in a numerical implementation, and (ii) provide transparent interpretation of the efficiency implications of the pricing intervention.

To demonstrate the second benefit of this approach, they derive and interpret the welfare effect of a marginal reduction in the peak rail ($i = P$ and $j = R$) fare. In the analytical expression below, the change in indirect utility is normalised by the marginal utility of money (u_X) to express welfare in money terms.

$$MW^{PR} = -\frac{1}{u_X}\frac{d\tilde{U}}{dp^{PR}} = \\ = \left(MC^{PR}_{supply} - p^{PR}\right) M^{PR}_{PR} + \left(MC^{PR}_{occ} - MB^{PR}_{scale}\right) M^{PR}_{PR} + \\ + \sum_{ij=PR,iH} \left(MC^{ij}_{ext} M^{ij}_{PR}\right) + \\ + \sum_{ij=OR,PB,OB} \left(MC^{ij}_{supply} + MC^{ij}_{ext} + MC^{ij}_{occ} - MB^{ij}_{scale} - p^{ij}\right) M^{ij}_{PR} \quad (1)$$

All four additive elements in this marginal welfare effects emerge as a result of the impact of the peak rail fare (p^{PR}) on demand for transport services:

$$M^{ij}_{PR} = \frac{dM^{ij}}{dp^{PR}} \quad (2)$$

denotes the quantity of demand in mode j at period i induced or discouraged by the marginal peak rail fare reduction. Intuitively, $M^{PR}_{PR} < 0$ while $M^{ij}_{PR} \geqslant 0$ for other modes and time periods.

The first element in (1) captures the direct financial impact on the operator: it is the gap between the marginal operating cost of serving another user and the fare revenue from the same traveller at the actual fare level. The second element is the net impact on user cost: this is the difference between the marginal cost of higher vehicle occupancy (including crowding inconvenience and vehicle size dependent operating costs) and the marginal benefit perceived by users due to lower wait time and reduced access cost (Mohring effect). Third, the peak fare reduction increases welfare by reducing the externalities of car use in peak ($ij = PH$) and off-peak ($ij = OH$) periods, as long as substitution is possible through $M^{PH}_{PR} > 0$ and $M^{OH}_{PR} > 0$. This effect is partly offset by the marginal external cost of peak rail use itself, MC^{PR}_{ext}. Finally, the fourth additive term in (1) quantifies the welfare effect of substitution with other public transport markets in the model. This substitution effect becomes relevant when the prevailing fare in the related market deviates from the net marginal social cost of travel, i.e. $MC^{ij}_{supply}+MC^{ij}_{ext}+MC^{ij}_{occ}+MB^{ij}_{scale}$. Parry and Small provide detailed analytical formulae for all marginal cost and benefit terms above.

Eq. (1) is then used to derive the social welfare maximising peak rail fare iteratively; in a numerical implementation, welfare is maximised when $MW^{PR} = 0$.

2.3. Model extension: The disutility of crowding

In recent years, the user cost of crowding has become a focal point of the public transport literature (Hörcher & Tirachini, 2021). The seminal study of Parry and Small (2009) builds crowding into their model in a rather simplistic way, which contradicts recent evidence on the behavioural mechanisms behind crowding disutility and the magnitude of discomfort relative to other user cost elements in public transport. From a behavioural point of view, our criticism is that crowding enters their model as a unit cost on *passenger miles* travelled, so that it remains independent from the speed of buses and trains. This contradicts the intuitively plausible evidence that journey time (i.e. the time spent inside the crowded vehicle) matters more for passengers than the actual distance travelled (Wardman & Whelan, 2011).

In the Parry–Small model, travel time t on the representative passenger mile is independent from crowding cost c on the same mile. This unconventional modelling approach causes difficulties in finding empirical evidence for model calibration as well. The authors admit 'there was only limited empirical basis for quantifying access and crowding costs' at the time of their research project, and they proposed they would 'eliminate the need to do so' (Parry & Small, 2009, p. 708) by developing a detailed model of the public transport operator's optimal supply decisions. In practice, the need for a reliable crowding cost function could not be eliminated. Their alternative approach relies on a crucial parameter, the elasticity of crowding cost per mile with respect to the average load factor:

$$\eta_c = \frac{\mathrm{d}c(l)}{\mathrm{d}l}\frac{l}{c}. \tag{3}$$

This elasticity is defined for both time periods and public transport services. The authors correctly ascertain that an empirical estimate would be needed to calibrate the model, but 'there is little empirical basis for gauging η_c' (page 15 of their online Appendix B). They set $\eta_c = 1.5$ in the baseline for both modes and time periods arbitrarily. They claim that their results are not sensitive to different assumptions, adding that this might be because crowding costs are relatively small. This claim is somewhat surprising because recent findings suggest that crowding cost can be very substantial in dense public transport systems (Prud'homme, Koning, Lenormand, & Fehr, 2012) and supply-side models are often very sensitive to crowding cost parameters (see a Monte Carlo sensitivity analysis by Hörcher & Graham, 2018). Studies over the past 10 years have shown that perceived travel costs can rise by around 50%, or even more according to some measures at a crowding level of 3 passengers per square metre (Björklund & Swärdh, 2015; Hörcher, Graham, & Anderson, 2017; Kroes, Kouwenhoven, Debrincat, & Pauget, 2014; Tirachini, Sun, Erath, & Chakirov, 2016; Whelan & Crockett, 2009).

The key methodological aim of this paper is to redesign the Parry–Small model in line with recent trends in the public transport literature. Following the previous empirical evidence, we express the disutility of crowding as a load factor-dependent multiplier of the *perceived* travel time (Björklund & Swärdh, 2015; Hörcher et al., 2017; Kroes et al., 2014; Tirachini et al., 2016; Whelan & Crockett, 2009). That is, we integrate $c^{ij} = t^{ij} m\left(l^{ij}\right)$ into the model. Since the empirical studies above confirmed that the relationship between occupancy rate and crowding disutility is linear, we specify the multiplier function as $m(l) = \alpha_c \cdot l$.

The user cost of crowding becomes relevant in the Parry–Small model in two ways. First, crowding is a determinant of the public transport operator's capacity adjustment policy in response to any change in demand, which is governed by a trade-off between vehicle capacity and the average load factor of vehicles. Second, crowding affects the marginal welfare effect of pricing interventions, as summarised above in Eq. (1), through operating costs MC^{ij}_{supply} and the user cost of vehicle occupancy MC^{ij}_{occ}. In the rest of the Methodology section, we present the related methodological improvements before we turn to the numerical implementation of the model.

2.4. Optimal vehicle size

Parry and Small define the capacity policy of public transport provision through the following assumptions.

1. In response to any change in demand M^{ij}, they assume that a constant fraction ε_V of the new trips is accommodated through increased vehicle miles V^{ij}. The remaining $1 - \varepsilon_V$ fraction is served by an incremental increase in vehicle capacity (vehicle size) n and an increase in the average load factor of vehicles, l^{ij}.
2. The increase in vehicle miles is realised through the optimal combination of adjustments in route density and service frequency.
3. The trade-off between vehicle size adjustment and in-vehicle crowding is also optimised to maximise social welfare in the model.

Assumptions 2 and 3 lead to the derivation of first-order conditions of optimal route density and vehicle size, respectively. The specification of crowding costs affects the latter decision variable only. To reproduce the model with the proposed crowding multiplier approach, we derive new first-order conditions for vehicle size optimisation.

The public transport operator maximises the sum of household utility and environmental externalities subject to a resource constraint which captures that the total operating cost OC cannot exceed the monetary savings of households. Thus, its objective is to solve for

$$\tilde{U} = \max_{X,\{M^{ij}\},\lambda} u\left(X, \{M^{ij}\}, \Gamma(\cdot)\right) - Z + \lambda\left(I - X - OC(\cdot)\right). \tag{4}$$

Lagrange multiplier λ represents the social value of money, that is, the welfare gain of an incremental relaxation of the resource constraint. We incorporate crowding disutility in the generalised travel cost function as a multiplier of in-vehicle travel time.

$$\Gamma(T, W, A) = \Gamma\left(\sum_{ij} t^{ij}\left(1 + m(l^{ij})\right) M^{ij}, \sum_{ij\neq iH} w^{ij} M^{ij}, \sum_{ij\neq iH} a^{ij} M^{ij}\right) \tag{5}$$

The welfare-maximising vehicle size satisfies the following first-order condition. We derive this expression for one particular pair of i and j but we suppress the superscripts for the sake of transparency.

$$\frac{\partial\tilde{U}}{\partial n} = \frac{\partial u}{\partial\Gamma}\frac{\partial\Gamma}{\partial T} M t \frac{\partial m}{\partial l}\frac{\partial l}{\partial n} - \lambda\frac{\partial OC}{\partial n} = 0 \tag{6}$$

To simplify notation, we introduce ϱ^T as the monetary valuation of in-vehicle time

$$\varrho^T = -\frac{u_\Gamma \Gamma_T}{u_X} \tag{7}$$

where subscripts denote partial derivatives. In addition, assume that K_n denotes the vehicle size-dependent operating cost of a unit of vehicle-hour supplied. Thus, the operating cost related element in (6) becomes

$$\frac{\partial OC}{\partial n} = V t K_n. \tag{8}$$

Using (7) and (8), together with $l = o/n$ and $\partial l/\partial n = -l/n$, the optimality condition can be rewritten as

$$\frac{\partial \tilde{U}}{\partial n} = \varrho^T tm\frac{\eta_m}{n}M - VtK_n = 0 \tag{9}$$

in which the multiplier's elasticity is defined as $\eta_m = \frac{\partial m}{\partial l}\frac{l}{m}$. Note that with a linear multiplier function $m(l) = \alpha_c \cdot l$, this elasticity boils down to $\eta_m = 1$. Eq. (9) enables us to reach an analytical expression of the welfare maximising vehicle size:

$$n = \frac{\varrho^T m(l)M}{K_n V} = \frac{\varrho^T m(l)o}{K_n}. \tag{10}$$

That is, the optimal vehicle size linearly increases in the absolute occupancy of vehicles and passengers' crowding sensitivity measured by the value of in-vehicle travel time and the crowding multiplier; operating cost parameter K_n works in the opposite direction. Note, however, that (10) is an implicit function of vehicle size because load factor l on the right hand side also depends on the n set by the operator.

Further calculations require that we express the generalised price of travel with our crowding multiplier approach. With a pre-specified multiplier function, this is

$$q = p + \varrho^T t(1 + m(l)) + \varrho^W w + \varrho^A a \tag{11}$$

in which a and w are waiting and access costs per passenger mile and ϱ^W and ϱ^A transform them into monetary units, in line with Eq. (7). Note that the user cost of crowding, $\varrho^T tm(l)$, can also be expressed by rearranging the first-order condition in (9). Thus, generalised travel price becomes

$$q = p + \varrho^T t + \frac{tK_n}{\eta_m l} + \varrho^W w + \varrho^A a \tag{12}$$

with $\eta_m = 1$ in the linear crowding multiplier specification. Note that it would be possible to transform waiting and access costs as well into the equivalent in-vehicle time cost by introducing additional multiplier. However, in the numerical application in Section 3 we stick to the original specification of Parry and Small as we believe waiting and access costs were supported by sufficient empirical evidence.

2.5. Optimal public transport fares

Crowding plays a significant role is the derivation of optimal fares and subsidy rates as well, through the marginal welfare formula presented in Eq. (1). Recall that pricing policy p^{ij} will be optimised by solving $MW^{ij} = 0$ through iterative search.

Parry and Small show that crowding disutility appears in the marginal welfare equation in (1) via the terms MC^{ij}_{occ}; they provide the detailed derivation on page 6 of their online appendix. In our alternative specification of user costs introduced in Eq. (5), these terms can be rewritten as

$$MC_{occ} = \sum_{ij} MC^{ij}_{occ} MC^{ij}_{PR} = \sum_{ij}\left[-\frac{u_\Gamma \Gamma_T}{u_X}\frac{\mathrm{d}\left(m(l^{ij})t^{ij}\right)}{\mathrm{d}p^{PR}}M^{ij} + \frac{\lambda}{u_X}\frac{\mathrm{d}OC^{ij}}{\mathrm{d}p^{PR}}\right]. \tag{13}$$

Using the definitions in (7) and (8), this expression simplifies to

$$MC_{occ} = \sum_{ij}\left[\varrho^T\frac{\mathrm{d}\left(m(l^{ij})t^{ij}\right)}{\mathrm{d}p^{PR}}M^{ij} + t^{ij}V^{ij}K^{ij}_n\frac{\mathrm{d}n^{ij}}{\mathrm{d}p^{PR}}\right], \tag{14}$$

and the product rule and $M^{ij}_{PR} = \partial M^{ij}/\partial p_{PR}$ imply further transformation into

$$MC_{occ} = \sum_{ij}\left[\left(\varrho^T t^{ij}M^{ij}\frac{\partial m(l^{ij})}{\partial M^{ij}} + \varrho^T\frac{\partial t^{ij}}{\partial M^{ij}}M^{ij}m(l^{ij}) + t^{ij}V^{ij}K^{ij}_n\frac{\partial n^{ij}}{\partial M^{ij}}\right)M^{ij}_{PR}\right]. \tag{15}$$

That is, we account for the fact that the pricing intervention modifies (i) the crowding multiplier of each passenger hour travelled, (ii) the duration of time spent in crowded conditions through a potential increase in road congestion and dwell times, and (iii) operating costs to the extent that the operator adjusts vehicle size in response to changing demand. Item (ii) is a new element in the model because the original paper did not consider that travel delay implies an increase in crowding inconvenience as well.

Parry and Small introduce additional elasticities to simplify the results above.

$$\varepsilon_V = \frac{\partial V}{\partial M}\frac{M}{V} = \frac{\partial V}{\partial M}o; \qquad (1-\varepsilon_V) = \frac{\partial o}{\partial M}\frac{M}{o} = \frac{\partial o}{\partial M}V \tag{16}$$

$$\varepsilon_n = \frac{\partial n}{\partial o}\frac{o}{n} = \frac{\partial n}{\partial o}l; \qquad (1-\varepsilon_n) = \frac{\partial l}{\partial o}\frac{o}{l} = \frac{\partial l}{\partial o}n \tag{17}$$

With this notation, the first element of (15) simplifies to

$$\varrho^T t^{ij}M^{ij}\frac{\partial m(l^{ij})}{\partial M^{ij}} = \varrho^T t^{ij}m^{ij}(1-\varepsilon_V)(1-\varepsilon_n), \tag{18}$$

which, under the first-order condition of optimal vehicle size in (9), becomes

$$\varrho^T t^{ij}M^{ij}\frac{\partial m(l^{ij})}{\partial M^{ij}} = (1-\varepsilon_V)(1-\varepsilon_n)\frac{t^{ij}K_n}{l^{ij}}. \tag{19}$$

We derive a similar expression to replace the third element in (15):

$$t^{ij}V^{ij}K^{ij}_n\frac{\partial n^{ij}}{\partial M^{ij}} = (1-\varepsilon_V)\varepsilon_n\frac{t^{ij}K_n}{l^{ij}}. \tag{20}$$

Finally, the derivation of congestion and dwell time delays, i.e. $\partial t^{ij}/\partial M^{ij}$ in Eq. (15), follows the same steps as Parry and Small present in their online appendix.

In summary, our crowding multiplier approach implies that the crowding-related marginal welfare effects of a pricing intervention add up to

$$MC_{occ} = \sum_{ij}(1-\varepsilon_V)\frac{t^{ij}K_n}{l^{ij}}M^{ij}_{PR} + \sum_{ij\neq iR}\varepsilon_V V^{iB}\varrho^T m(l^{iB})\frac{\partial t^{iB}}{\partial V^{ij}}M^{ij}_{PR} + \sum_{ij\neq iH}(1-\varepsilon_V)\theta^j\varrho^T m(l^{ij})o^{ij}M^{ij}_{PR}. \tag{21}$$

The first item in this sum is the combined social cost of the crowding externality and vehicle size adjustment, the second item is the crowding disutility of bus users getting delayed due to road congestion, while the third item quantifies the excess crowding cost of the marginally increased dwell time on both rail and bus services.

In the final step of the modelling exercise it is necessary to provide a proper definition of load factor l. Since the crowding level is often expressed by the ratio of the number of passengers on board and the vehicle's interior floor area, vehicle size n will have to be quantified in this dimension.

Eq. (21) is suitable for the numerical implementation of the updated version of Parry and Small's analysis. Before we turn to the corresponding numerical results in Section 4, let us introduce a new set of parameters for the Hungarian application of the model.

3. Data: parameters of the Hungarian application

By the time of our research, the operating and budget data of the national service providers were available for the years 2016/17, so we were able to perform domestic calculations with quite recent data. To describe the main operating conditions of the Hungarian transport system we used the local parameter values shown in Table 1. For the Hungarian adaptation we relied on the following data sources. High-level socioeconomic data such as the median wage rate and the aggregate number of unlinked trips are provided by the Central Statistical Office (KSH). Transport-specific values including demand-side statistics and the estimates of operating expenditure were provided by the Ministry of Innovation and Technology (ITM), the ministry responsible for transport policy. As an additional information, the average speed of public transport came from MÁV-Start, the passenger operator of the Hungarian State Railways concern. Further data items

Table 1
Parameter values used by the calculations for the Hungarian interurban transport system.

Parameters	Hungary		Unit	Source of data
	Rail	Bus		
Median wage rate	1946	1946	HUF/hour	KSH, 2017
Number of unlinked trips	146.9	490.6	million/year	KSH, 2017
Annual passenger kilometres	4577	4596	million km	ITM, 2017
Annual rail car/bus kilometres	51	228	million km	ITM, 2017
Fleet size	1677	5239	–	ITM, 2017
Public transport speed	51.9	35.5	km/h	MÁV-Start, 2016; KTI
Purchase price of rail car and bus	350	62	million HUF	purchase price of previous transactions
Total operating cost	243	161	billion HUF	ITM, 2017
Total fare revenues	39	57	billion HUF	ITM, 2017
Parameters	Road transport		Unit	Source of data
	Peak	Off-peak		
Annual vehicle miles	18411		million/year	Magyar Közút, 2018
Average trip length	41.1		km	KTI, 2016
Fuel tax	228.0	228.0	HUF/litre	legally prescribed
Occupancy	1.32		pass/vehicle	KTI, 2016
Car average speed	50	60	km/h	own estimate
Proportion of vehicle kilometres	51	49	%	own estimate
Fuel efficiency	6	6	litres/100 km	own estimate
Constants	Peak	Off-peak	Unit	Source of data
Access elasticity	0.5	0.5	–	based on Parry and Small (2009)
External cost of accidents	0.77	0.77	HUF/vehicle-km	European Commission (2019)

on private road use were made available by Magyar Közút, the public roads operator, and the Institute for Transport Sciences (KTI).

When interpreting the forthcoming results, it should also be noted that the data used for the calculations mainly describe the characteristics of the interurban transport system. Note, however, that this market is dominated by suburban commuting services around Budapest and a couple of major cities where most of the demand for such services arises. Unfortunately, our data collection efforts were not successful in case of the peak and off-peak average speed of private cars and their fuel efficiency. These values are based on the authors' own judgement. Also, we relied on the original (Parry & Small, 2009) work by keeping the elasticity of demand with respect to access cost equal to 0.5.

Finally, the model's solution algorithm is based on a matrix of 'modal diversion ratios' defined as $m^{ij}_{PR} = -M^{ij}_{PR}/M^{PR}_{PR}$. This set of parameters describe the share of prior users of mode j in period i among new peak ($i = P$) rail ($j = R$) travellers when the peak rail fare is adjusted on the margin. That is, these parameters describe the substitution pattern between modes and time periods, also allowing for entirely new demand to be generated when the fare decreases, keeping aggregate demand elastic. The same ratios can be defined for off-peak and bus services identically. Our sensitivity analyses presented later confirmed that the matrix of modal diversion ratios has a significant impact on the numerical results, while their precise estimation poses significant empirical challenges.

Thus, we decided to deliver three sets of results that only differ in the assumed substitution pattern according to three scenarios: Scenarios 1 and 2 assume that substitution is equivalent to what Parry and Small used in case of Los Angeles (similar to Hungarian towns in urban density) and London (similar in public transport coverage), respectively. In Scenario 3 we assume that the substitution between private cars and public transport is lower due to historically lower car ownership rates in Central Europe, and we set a somewhat higher aggregate demand elasticity to reflect that public transport users in this region are more price sensitive than their peers in the US and the UK. Our parameters in Scenario 3 implies that induced peak public transport demand has the following breakdown: 30% car users, 10% from the same mode off-peak, 30% from the other public transport mode and same time period, 5% from the other mode and other period, while 25% are newly generate trips. Table 2 provides the remaining values for both time periods and all three scenarios; just like in Parry and Small (2009), bus and rail services are assumed to feature the same modal diversion characteristics.

4. Results

Parry and Small examined the transport systems of three cities having drastically different transport systems (London, Washington, DC, and Los Angeles). The paper's results show that in most cases significant rates of subsidies are recommended for transport agencies. In 10 out of 12 cases, the optimal value of subsidy is more than two-thirds of the operating cost, and in half of the cases it reaches 90%. The results of the model also show that the need for high subsidies primarily comes from scale economies and the externalities of underpriced road use. The model predicts a major increase in passenger miles, more than 50% increase in off-peak periods. This result seems plausible, as it facilitates the best use of capital (i.e. vehicles, stations and infrastructure) in less frequented periods. With significantly lower off-peak period public transport fares and due to greater spatial and temporal coverage, on account of the Mohring effect, more passengers will choose public transport.

4.1. Adaptation of crowding-related extensions

After the methodological improvements derived in Section 2, the cost of crowding can be included as a multiplier of in-vehicle travel time valuation, rather than a standalone user cost. Our new numerical results with the modified cost and crowding formulas are summarised in Table 3. For the sake of comparison, Tables 5, 6, and 7 in the Appendix show the changes in each variable for Washington, DC, Los Angeles and London, respectively.

The following tables are structured as the ones in Parry and Small (2009), again, for the sake of easy comparison. The tables include (i) 'current' subsidies; (ii) marginal welfare effects—welfare gains from a reduction in the passenger fare—broken down to five factors (price gap of marginal and average travel cost, net scale economy and crowding effects, externalities, other transport modes); (iii) optimal subsidy—broken down to the same factors again—and (iv) the modelled change in passenger miles.

Compared to the results of Parry and Small (2009), it can be seen that the discomfort caused by crowding already accounts for a significant proportion of travel costs even at moderate congestion levels.

Table 2
Modal diversion ratios used in our sensitivity analysis.

Modal diversion ratios [%]	Scenario 1		Scenario 2		Scenario 3	
	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
auto–same period	85	75	50	40	30	20
auto–other period	0	0	0	0	0	0
same transit mode–other time	10	10	10	10	10	10
other transit mode–same time	5	5	30	30	30	30
other transit mode–other time	0	0	0	0	5	5
fewer motorised trips	0	10	10	20	25	35

Table 3
Model re-evaluation with modified cost and congestion relationships.

	Washington, DC				Los Angeles				London			
	Rail		Bus		Rail		Bus		Rail		Bus	
	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Current subsidy, percent of operating costs	47	55	80	76	82	83	79	69	67	72	59	40
Marginal welfare effects												
MW/W at current subsidy	0.21	0.24	−0.51	−0.50	0.34	−0.10	−0.18	4.21	0.51	0.01	0.09	1.38
Marginal cost/price gap	−0.03	−0.16	−0.94	−1.42	−0.57	−1.22	−0.87	−2.36	−0.20	−0.55	−0.25	−0.09
Net scale economy	0.09	0.41	0.51	1.99	0.18	0.86	0.45	5.90	0.04	0.28	0.30	1.74
Crowding	−0.08	−0.06	−0.20	−0.14	−0.17	−0.14	−0.22	−0.55	−0.07	−0.06	−0.21	−0.17
Externalities	0.20	0.07	0.13	−0.05	0.79	0.32	0.46	0.44	0.57	0.35	0.14	−0.52
Other transport modes	0.04	−0.02	−0.02	0.13	0.10	0.07	−0.01	0.77	0.18	−0.01	0.12	0.41
MW/W at 50% subsidy	−0.21	−0.26	−0.05	−0.56	0.37	0.18	0.09	3.25	0.49	0.17	0.14	1.45
Optimum subsidy, percent of operating costs	>90	80	42	>90	>90	78	64	>90	>90	73	71	>90
Proportion of subsidy due to												
Average-marginal cost gap	0.45	0.57	0.40	0.40	0.38	0.48	0.42	0.49	0.31	0.55	0.50	0.39
Net scale economy	0.18	0.44	0.77	0.61	0.12	0.41	0.40	0.45	0.04	0.22	0.42	0.74
Crowding	−0.16	−0.06	−0.29	−0.04	−0.11	−0.07	−0.19	−0.04	−0.07	−0.04	−0.29	−0.07
Externalities	0.45	0.08	0.15	−0.02	0.54	0.15	0.38	0.04	0.55	0.28	0.20	−0.30
Other transport modes	0.08	−0.03	−0.02	0.04	0.07	0.03	0.00	0.07	0.17	−0.01	0.17	0.24
Percent change in pass-miles	23.9	34.1	−48.3	28.6	11.6	−12.8	−23.6	51.0	20.3	1.7	12.6	142.5

Table 4
Hungarian adaptation: Sensitivity analysis w.r.t. three sets of modal diversion ratios.

	Scenario 1				Scenario 2				Scenario 3			
	Rail		Bus		Rail		Bus		Rail		Bus	
	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Optimum subsidy [%]	>90	77	>90	81	>90	87	83	88	76	80	60	78
Change in pass-miles [%]	5.2	−18.5	21.8	14.4	5.2	0.0	10.0	21.4	−15.8	−13.6	−18.1	11.6

On-board crowding decreases the optimal subsidy rate by 4–29%. It can also be observed that a significant increase in crowding-related marginal cost is to be expected, regardless of city, transport mode and travel period. The growing number of trips in the off-peak period may be more likely to be caused by crowding in the peak, which was nearly neglected beforehand due to previous model assumptions. The results show that the optimal subsidies and the expected passenger numbers are both reduced in most of the studied cases. The direction (and magnitude) of the subsidy change is in line with prior expectations, as the transport system is able to optimally operate with fewer passengers due to the increase of travel costs, where the demand losses are mostly caused by crowding disutility.

The exception is the off-peak bus service of Los Angeles where the model predicts significant subsidy and passenger growth compared to previous results. This may be caused by the fact that decreasing peak-time subsidies and significantly increasing off-peak period subsidies may lead to a higher proportion of trips taking place in the latter time period. We also observe a minimal increase in London's peak rail marginal welfare gains.

Comparing the results with the original model, it is clearly seen that even in cases of moderate congestion and crowding level, user costs significantly increased. As a result, in most cases, a substantial part of the marginal benefit arising from scale economies is neutralised by the marginal external crowding cost. In all the modelled cities, the marginal crowding cost approaches the level of peak rail transport's marginal benefit coming from scale economies, and in the case of London it even outweighs it. We conclude the following: crowding is incorporated in the model according to the state-of-the-art empirical evidence, it is indeed not a negligible element of the marginal social cost of public transport trips, and its appropriate representation in the Parry–Small model has significant impact on numerical outcomes.

4.2. Results of the model's application in Hungary

Optimisation results for the Hungarian adaptation of the model show similar patterns in terms of the breakdown of marginal welfare effects and the components of the optimal subsidy; our detailed results are provided in Table 8 of the appendix. Similarly to the three cities of the original analysis, the benefits of reducing externalities play a decisive role in the Hungarian transport system as well.

More importantly, our sensitivity analysis of this application reveals that the optimal subsidy rate and the corresponding change in demand are highly dependent on the assumed substitution pattern between modes and time periods, and the aggregate elasticity of travel demand. Table 4 shows that with a relatively high level of substitution with car use in Scenarios 1 (Los Angeles parameters) and 2 (London parameters), the optimal peak subsidy rates are mostly above 90%. This would imply an increase in public transport demand in both the peak bus and rail markets. However, in Scenario 3 where we reduced the degree of substitution with car use and increased the overall price sensitivity of demand (refer back to Table 2 for the exact modal diversion ratios), the optimal subsidy rates are somewhat lower. For rail, these are 76% in the peak and 80% off-peak, while for buses, we get 60% and 78% for the two periods, respectively. Should the substitution pattern of Scenario 3 be the closest to reality, the optimal self-financing ratio would imply a double-digit reduction in demand for peak and off-peak rail services as well as peak buses, with the exception of off-peak bus travel where the model suggests an 11.6% increase under the optimal policy.

This sensitivity test highlights the importance of the appropriate measurement of modal substitution patterns in the context of policy decisions on funding. Such empirical estimates, even at the granularity of this aggregate model, do not exist in most cities and countries in Europe and elsewhere, leaving an obvious gap for future empirical research efforts.

Parry and Small (2009) mentioned that ‘these substantial operating subsidies for transit systems are warranted on efficiency grounds. The main caveat is that some of the subsidy may be lost to inefficiency or captured by increased wages and other costs following transit subsidies. Thus the analysis is most applicable to a transit agency with strong incentives to minimize costs.’ In case of Hungary, the transport market is limited to a few publicly owned service providers, effectively functioning as public monopolies. In this market structure, regulatory oversight, the public opinion, and potential political consequences provide the only incentives to ensure cost efficiency. Thus, the words of caution expressed by Parry and Small are applicable in this context as well. Market opening and competition may be an effective way to ensure cost minimisation and innovation in the public transport industry.

The results of the modelling with Hungarian data show many similarities with the characteristics of foreign cities, although the model aims to examine the transport system of the urban and suburban environment. The similarity between the results of the model variants (see a detailed comparison of the numerical values in the tables of the Appendix) shows that the model can be adapted and used in a geographical, social, economic, technical environment that is significantly different from the original cities. Naturally, the accuracy of the results could be improved by narrowing down the analysis to smaller geographical areas with granular data, or by a detailed review of the factors adjusted to the features of localities.

5. Conclusions

This paper revisits one of the foundational contributions of public transport economics by Parry and Small (2009). We identify one critical shortcoming in their modelling approach which substitutes a reliable estimate of the user cost of crowding with ad-hoc elasticities. The core aim of this paper is to fix this shortcoming with an extended methodology. The second contribution of the present work is that we apply numerical implementation of the model in a new case study area, the Hungarian interurban public transport system.

By solving the numerical optimisation of fares and subsidies for peak and off-peak bus and rail services, we show that (i) crowding is indeed a non-negligible element of the user cost of bus and rail use, and (ii) with appropriately calibrated crowding-dependent value of time, the model produces lower optimal subsidy rates in peak periods. This is caused by an increase in fares, thus resulting in fewer passengers and less crowding on board public transport vehicles. At the same time, higher off-peak subsidy levels and cheaper fares would allow some travels to be shifted to this period, which leads to a more even distribution of capacity utilisation between time periods.

The model adaptation for Hungary shows that although the model was originally designed for urban and suburban traffic, it can be applied to model interurban public transport services as well. Our analysis is based on data from state-owned railway and bus transport operators with a full national coverage. Intuition suggests that an important requirement for any implementation is that the data should come from a geographical area with more or less homogeneous socio-economic conditions, to avoid potential bias from averaging. We believe this condition is satisfied as Hungary is a relatively small country in both European and global standards.

The practical implementation of our model recommendations faces two obstacles. First, these provisions would require time-differentiated pricing which is currently unavailable in Hungary. It is also important to note that in Hungary this theory may be hampered by the fact that those social groups whose trips could be relatively easily rescheduled, such as people over the age of 65, currently do not have to pay fares, and hence the changes of pricing would have no effect on their travel habits.

The results for the Hungarian transport system are characterised by high subsidy rates in optimum. However, we have identified that the Parry–Small model is quite sensitive to the assumed substitution pattern with other transport modes (especially with private cars) which affects the optimal self-financing ratio as well. With the substitution patterns borrowed from Los Angeles and London (the original Parry–Small paper), less than 10% of the operating costs should be covered by fare revenues. By contrast, after reducing the assumed substitution with cars and increase the overall price sensitivity of travel demand, the optimal fares cover only 60 to 80 percent of the operating expenditure depending on the mode and time period. This insight highlights the importance of reliable empirical inputs in the optimisation of pricing policies.

Let us conclude the paper with a set of qualitative policy recommendations. We suggest that the state should also promote the more even use of capacities in other ways to prevent harmful levels of crowding. Staggered work hours could be introduced by state-owned institutions and service providers, including schools. Real-time information could be presented to employers on the usual occupancy rate of vehicles, on the basis of ticket sales data and vehicle sensors (Csiszár & Földes, 2015). Additionally, the Covid-19 pandemic might lead to new employment and commuting habits with beneficial consequences: accelerating digitalisation and the possibility of working from home provide an opportunity to a major review of transport strategies.

As one of our anonymous reviewers pointed out correctly, ‘these findings are very far from EU transport policy’. We agree with this assessment. Marginal cost pricing has been a flagship initiative of EU transport policy in the 1990s and early 2000s under the ‘user pays principle’, a more widely accessible terminology which avoids economic jargon. This principle has nearly disappeared from policy documents in the past decade, especially in the context of public transport. This article intends to showcase the power of economic modelling in informed policy making at a time when resource efficiency is particularly important due to geopolitical tensions and energy shortages, in all modes of transport.

By further improving the model, the impact of strategic pricing decisions could be predicted even more robustly at the level of cities and wider regions. Currently the model cannot really account for the capacity constraints of the transport system. Thus, for example with increasing peak frequencies the user cannot be certain that the system can manage the increasing demand without costly infrastructure (or traffic organisational and controlling) investments. Take the rail network of the capital city Budapest as an example, which is currently suffering from the lack of capacity, so the model would also require a parallel examination of road and rail capacities. Although the model can control the occupancy of a particular mode and in a given period by optimising frequencies and crowding, there may be situations where this method is no longer sufficient to control for infrastructural constraints.

We believe there is room for further improvements and extensions of the Parry–Small model as part of future research. One possible extension with high potential is already mentioned in our paper: the current

Table 5
Comparison of the original and modified models' results for Washington, DC.

Washington, DC	adjusted results, Parry and Small (2009)				with improved crowding multipliers			
	Rail		Bus		Rail		Bus	
	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Current subsidy, percent of operating costs	47	55	80	76	u.v.	u.v.	u.v.	u.v.
Marginal welfare effects								
MW/W at current subsidy	0.24	0.34	−0.45	0.64	0.21	0.24	−0.51	−0.50
Marginal cost/price gap	−0.04	−0.12	−0.94	−1.42	−0.03	−0.16	−0.94	−1.42
Net scale economy	0.09	0.41	0.51	1.99	0.09	0.41	0.51	1.99
Crowding	−0.04	0.00	−0.17	0.00	−0.08	−0.06	−0.20	−0.14
Externalities	0.20	0.07	0.17	−0.04	0.20	0.07	0.13	−0.05
Other transport modes	0.03	−0.02	−0.02	0.11	0.04	−0.02	−0.02	0.13
MW/W at 50% subsidy	0.24	0.35	−0.03	0.63	−0.21	−0.26	−0.05	−0.56
Optimum subsidy, percent of operating costs	>90	88	46	>90	>90	80	42	>90
Proportion of subsidy due to								
Average-marginal cost gap	0.43	0.54	0.37	0.39	0.45	0.57	0.40	0.40
Net scale economy	0.17	0.41	0.70	0.59	0.18	0.44	0.77	0.61
Crowding	−0.07	0.00	−0.23	0.00	−0.16	−0.06	−0.29	−0.04
Externalities	0.41	0.08	0.19	−0.01	0.45	0.08	0.15	−0.02
Other transport modes	0.07	−0.03	−0.02	0.03	0.08	−0.03	−0.02	0.04
Percent change in passenger miles	24.9	51.0	−48.1	28.7	23.9	34.1	−48.3	28.6

u.v.: unmodified values

Table 6
Comparison of the original and modified models' results for Los Angeles.

Los Angeles	adjusted results, Parry and Small (2009)				with improved crowding multipliers			
	Rail		Bus		Rail		Bus	
	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Current subsidy, percent of operating costs	82	83	79	69	u.v.	u.v.	u.v.	u.v.
Marginal welfare effects								
MW/W at current subsidy	0.36	0.14	−0.06	1.09	0.34	−0.10	−0.18	4.21
Marginal cost/price gap	−0.61	−1.12	−0.87	−0.93	−0.57	−1.22	−0.87	−2.36
Net scale economy	0.18	0.87	0.45	1.72	0.18	0.86	0.45	5.90
Crowding	−0.10	0.00	−0.15	0.00	−0.17	−0.14	−0.22	−0.55
Externalities	0.79	0.32	0.52	0.12	0.79	0.32	0.46	0.44
Other transport modes	0.10	0.08	0.00	0.17	0.10	0.07	−0.01	0.77
MW/W at 50% subsidy	0.35	0.28	0.14	1.01	0.37	0.18	0.09	3.25
Optimum subsidy, percent of operating costs	>90	89	74	>90	>90	78	64	>90
Proportion of subsidy due to								
Average-marginal cost gap	0.38	0.49	0.42	0.35	0.38	0.48	0.42	0.49
Net scale economy	0.11	0.34	0.33	0.55	0.12	0.41	0.40	0.45
Crowding	−0.06	0.00	−0.11	0.00	−0.11	−0.07	−0.19	−0.04
Externalities	0.51	0.13	0.37	0.04	0.54	0.15	0.38	0.04
Other transport modes	0.06	0.03	0.00	0.06	0.07	0.03	0.00	0.07
Percent change in passenger miles	11.5	21.2	−9.8	33.2	11.6	−12.8	−23.6	51.0

u.v.: unmodified values

Table 7
Comparison of the original and modified models' results for London.

London	adjusted results, Parry and Small (2009)				with improved crowding multipliers			
	Rail		Bus		Rail		Bus	
	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Current subsidy, percent of operating costs	67	72	59	40	u.v.	u.v.	u.v.	u.v.
Marginal welfare effects								
MW/W at current subsidy	0.49	0.02	0.52	1.76	0.51	0.01	0.09	1.38
Marginal cost/price gap	−0.20	−0.55	−0.25	−0.09	−0.20	−0.55	−0.25	−0.09
Net scale economy	0.04	0.28	0.30	1.74	0.04	0.28	0.30	1.74
Crowding	−0.05	0.00	−0.11	0.00	−0.07	−0.06	−0.21	−0.17
Externalities	0.57	0.35	0.48	−0.23	0.57	0.35	0.14	−0.52
Other transport modes	0.13	−0.06	0.10	0.34	0.18	−0.01	0.12	0.41
MW/W at 50% subsidy	0.46	0.17	0.51	1.93	0.49	0.17	0.14	1.45
Optimum subsidy, percent of operating costs	>90	73	>90	>90	>90	73	71	>90
Proportion of subsidy due to								
Average-marginal cost gap	0.31	0.55	0.35	0.33	0.31	0.55	0.50	0.39
Net scale economy	0.04	0.22	0.23	0.61	0.04	0.22	0.42	0.74
Crowding	−0.04	0.00	−0.08	0.00	−0.07	−0.04	−0.29	−0.07
Externalities	0.56	0.28	0.42	−0.11	0.55	0.28	0.20	−0.30
Other transport modes	0.13	−0.04	0.09	0.17	0.17	−0.01	0.17	0.24
Percent change in passenger miles	21.6	1.7	42.2	152.9	20.3	1.7	12.6	142.5

u.v.: unmodified values

Table 8
Numerical results: Public transport in Hungary, Scenario 1.

	Rail		Bus	
	Peak	Off-peak	Peak	Off-peak
Current subsidy, percent of operating costs	87	87	76	64
Marginal welfare effects				
MW/W at current subsidy	0.35	−0.20	0.35	0.29
Marginal cost/price gap	−0.85	−1.73	−0.71	−0.69
Net scale economy	0.13	0.26	0.38	0.75
Crowding	−0.11	−0.09	−0.15	−0.12
Externalities	1.06	0.52	0.76	0.08
Other transport modes	0.12	0.84	0.08	0.28
Optimum subsidy, percent of operating costs	>90	77	>90	81
Proportion of subsidy due to				
Average-marginal cost gap	0.38	0.46	0.37	0.44
Net scale economy	0.07	0.09	0.22	0.42
Crowding	−0.06	−0.03	−0.09	−0.07
Externalities	0.55	0.18	0.45	0.04
Other transport modes	0.06	0.30	0.05	0.16
Percent change in passenger miles	5.2	−18.5	21.8	14.4

model does not take the marginal cost of public funds into account, which might be another reason for a downward adjustment of optimal subsidies. Other possible extensions include a wider economic approach in which positive externalities such as agglomeration economies could be integrated into the model. Finally, the present paper as well as Parry and Small's original work is based on a representative household approach with no user heterogeneity. By introducing multiple income groups, the analysis could be extended to the distributional consequences of public transport subsidies.

CRediT authorship contribution statement

Tamás Strommer: Conceptualization, Methodology, Formal analysis, Resources, Writing – original draft, Visualization. **Daniel Hörcher:** Conceptualization, Methodology, Formal analysis, Writing – review & editing, Supervision. **András Munkácsy:** Investigation, Resources, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Table 5 compares the new modelling results for Washington, DC, with the original model. Several major changes can be identified among the results and attributed to the novel way we model in-vehicle crowding. In case of every transport mode and time period, the optimal subsidy decreased, giving an approximately 10% lower level for these values compared to the original paper.

Among the three cities, Washington, DC is the only where in one scenario, the model suggests that the 2009 level of subsidy should be substantially reduced (halved). Applying the modified crowding relationships to the model, the optimal level of subsidy decreases, along with the predicted change in passenger miles. In case of Los Angeles (Table 6) and London (Table 7), the new representation of crowding has a negative impact on optimal subsidies, once again.

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