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The rate of discount on public investments with future bias in an altruistic overlapping generations model☆

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ABSTRACT

A typical approach to evaluating public investment projects requires an appropriate rate for discounting future benefits and costs. In many developed countries, the practical discount rate for benefit–cost analysis has been calculated using the Ramsey formula in an optimal growth model, stretched back to the pioneering work presented by Frank Ramsey in 1928. Intergenerational altruism that is widely observed in our society generates future bias. Therefore, elected governments under democracies are future-biased. In such cases, the Ramsey formula is no longer valid. This paper derives a modified Ramsey formula for an appropriate social discount rate to be applied instead of the conventional Ramsey formula. We also examine properties of equilibrium policy. The degree of future bias depends on the political power of young people relative to elderly people. Demographic effects such as population aging negatively affect public investment through decreasing future bias effects. This paper contributes to the optimal theory of public investment for calculating an appropriate social discount rate, both theoretically and practically.

1. Introduction

A solid consensus prevails about the importance of public investment. For instance, it is broadly accepted in modern society that infrastructure investment is essential for supporting social life and for stimulating business activities. Naturally, public investment has long been examined closely in the literature on public economics and public finance. One of core issues is the analysis of optimal public investment. In principle, undertaking public investment optimally requires benefit–cost calculations for the investment project from the present day to some far future day. The key to such a calculation is a social discount rate used for evaluating the present values of the benefits and costs of public investment. Therefore, the issue of a social discount rate as an optimal discount rate has been debated in Economics since its early origins. The established formula of a social discount rate is well known as the *Ramsey formula*, first introduced long ago by Ramsey (1928). It is given as the rate of pure time preference plus the elasticity of marginal utility of consumption times

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the growth rate of consumption per capita.[1]

Intergenerational equity and social discounting are highlighted recently as new aspects of this literature on very long-term projects related to environmental change (e.g., Stern, 2006; Heal, 2007). The non-exponential discounting causes dynamically inconsistent preferences, leading to biases of some kinds (e.g., Strotz, 1956; Phelps and Pollak, 1968; Laibson, 1997).[2] Under a democracy, the governments elected by biased people inherit the same bias. Hence, democratic governments face self-control problems when conducting public policy (Laibson, 1997; Krusell et al., 2002). In particular, several studies show evidence that people exhibit *future bias* (Sayman and Önculer, 2009; Takeuchi 2011, 2012). Future bias is associated with increasing patience. Future-biased people tend to undervalue current benefits and to postpone taking rewards until future dates. Therefore, depending on the range and timing of benefits from public investment, future bias affects democratic governments' investment projects through social preference and discount.

This paper addresses an issue related to how the social discount rate should be set facing future bias, along with the literature on optimal public investment. Specifically, the purposes of this paper are to derive an appropriate social discount rate for optimal public investment under future bias, and to characterize the relationship between equilibrium policy, the social discount rate, and future bias. The existing literature on public investment is classifiable into two mutually related main categories. The first group presents analyses of the optimal policy of public investment theoretically, including its financing methods (e.g., Arrow, 1966; Sandmo and Dreze, 1971; Pestieau, 1974), whereas the second group describes theoretical and empirical analyses of the economic effects of public investment that are linked to costs and benefits of public investment using specific models to cope with their respective research issues (e.g., Aschauer, 1989; Barro, 1990; Futagami et al., 1993).[3]

Arrow (1966), as a pioneer, formulates the optimal problem of public investment (i.e., choice of social discount rate) based on optimal growth theory. In a book published a few years after that outstanding study, Arrow and Kurz (1970) provide general frameworks to analyze optimal investment policy including extensive issues of tax-financing and debt-financing. Existing studies of optimal public investment are based on these prominent works. Without any distortion including biases, the Ramsey formula is available for calculating the social discount rate (Arrow and Kurz, 1970; Turnovsky, 1997; Greiner, 1998; Tamai, 2008). Numerous factors that generate inconsistencies exist, such as risks and capital market imperfection (e.g., Arrow and Lind, 1970; Ogura and Yohe, 1977; Arrow, 1982), although these factors can be internalized by considering a premium. When incorporating future bias, the Ramsey formula should be modified for evaluation of future benefits and costs of public investment.

Empirical studies on the determinants of public investment also cast new light on the issue of intergenerational conflict and public investment through democratic decision processes of public policy. For instance, using data of 67 presidential and parliamentary democracies from 1975 to 2012, Gupta et al. (2016) found that elections affect public investment. Based on experimental evidence that senior citizens tend to discount future payoffs more heavily than working-age individuals do (Harrison et al., 2002; Read and Read, 2004), Jäger and Schmidt (2016) investigate the hypothesis that demographic change has led to budget cuts in public investment in economically advanced countries over the last four decades. They find that population aging decreases public investment through a rise in the voting power of senior people. The established empirical fact of the nexus between public investment and population aging is visible in the panel data of 35 countries and regions as advanced economies for 1960–2019.[4]

Focusing on the advanced economies, one can find a negative relationship between the government investment ratio and the age-dependency ratio in Fig. 1, which presents a scatter plot of the logarithms of the two ratios. The opposing trends in the government investment ratio and age-dependency ratio are observed similarly to Jäger and Schmidt (2016).[5] Naturally, population aging also affects private and public investment expenditure composition. Fig. 2 illustrates a scatter plot of the first-order differences in the logarithms of the public to private capital and age-dependency ratios. A change in public to private capital ratio corresponds to an expenditure composition change of private and public investment. Therefore, the positive relationship between the public to private capital and age-dependency ratios shows that a marginal decrease in public investment by population aging is smaller than that of private investment.[6] This evidence suggests an additional question of why population aging more severely influences private investment in an aged democratic society.

[1] Including the utility for capital stock, the Ramsey formula should be modified to add the product term that the elasticity of the marginal utility of consumption with respect to the capital stock is the multiplier. The growth rate of capital stock per capita is the multiplicand. If the cross derivative of utility function with respect to consumption and capital stock is zero, then the Ramsey formula becomes itself as appearing in the main text. Heal (2007) provides mathematical details considered here.

[2] Strotz (1956) presents the pioneering study of commitment mechanism and discounting. Phelps and Pollak (1968) formulate quasi-hyperbolic preferences ahead of Laibson (1997) and others.

[3] Pereira and Andraz (2013) and Välilä (2020) present surveys of the literature belonging to the second category. More recently, Papagni et al. (2021) supported the productivity effects of public investment, although changes in the quality of institutions lead to different effects of public investment on growth over time.

[4] The 35 countries and regions are Australia, Austria, Belgium, Canada, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Malta, Netherland, New Zealand, Norway, Portugal, Singapore, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Macao, Puerto Rico, San Marino, and Taiwan are excluded because several key variables are not available.

[5] The fitted line is based on the random effect model. Our panel regression results reported in Tables A1 show a statistically significant negative effect on the government investment ratio (Appendix A).

[6] The positive effect of population aging on a relative increase in public capital to private capital is statistically significant. Appendix A presents the relevant regression results and technical notes.

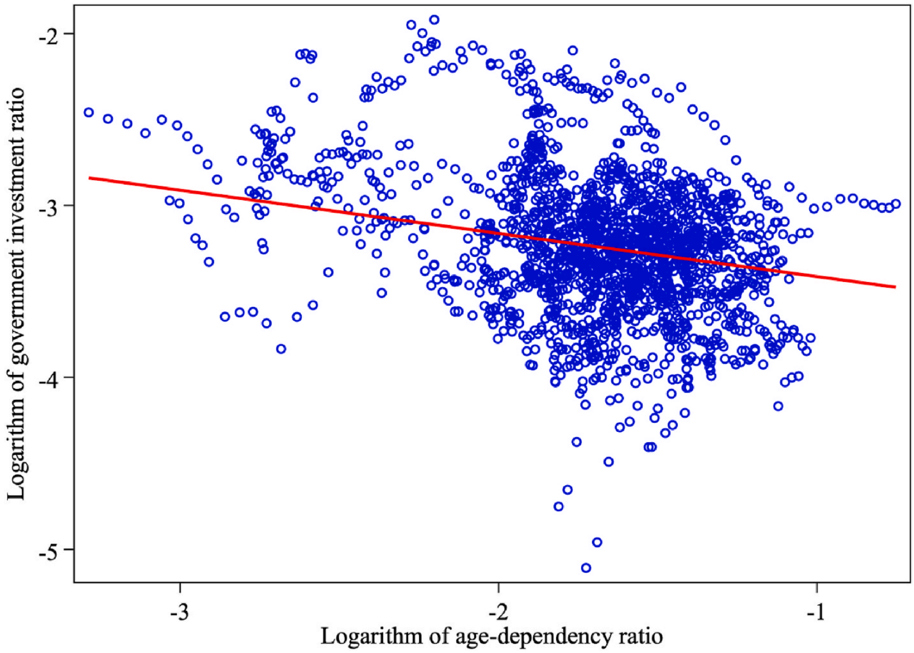

Fig. 1. General government investment and age-dependency ratio (log), 1960–2019
Data: General government investment (Investment and Capital Stock Dataset, IMF); Age-dependency ratio (World Development Indicators, World Bank).

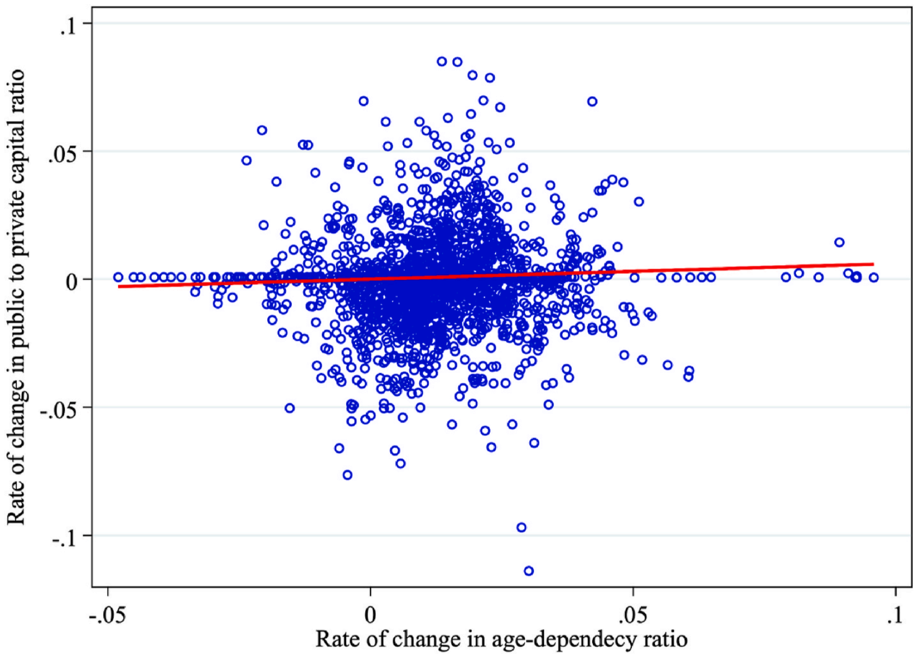

Fig. 2. Changes in the public to private capital ratio and in age-dependency ratio (1960–2019)
Data: General government capital stock (Investment and Capital Stock Dataset, IMF); Age-dependency ratio (World Development Indicators, World Bank).

To clarify the background mechanisms of these empirical facts, demographic changes and future bias are mutually essential for a determination of public investment under democracies. The dynamic framework must be modelled by incorporating future-biased people within realistic preference. Regarding such preference structures, Gonzalez et al. (2018) show that two-sided altruistic preferences derive future bias because young people discount the utility of their ancestors and do not discount the utility of themselves while they discount the youth utility relative to that of the coexisting old people in the future. In reality, filial and parental altruism are more important to consider the effects of public policy by inter vivos transfers including transfers to elderly people from their children (vice versa) as population aging becomes more pronounced worldwide. Two-sided altruism can account for observed upstream or downstream transfers in the family (Sloan et al., 2002; Kohli and Kunemund, 2003). This paper therefore develops an overlapping generations (OLG) model with intergenerationally altruistic preferences and public capital for business and consumption activities.

Democratic determination of public policy in our model is modelled as an analogue of a probabilistic voting model (Lindbeck and Weibull, 1987; Grossman and Helpman, 1998), which is sometimes used for the politico-economic analyses of fiscal policy in OLG models (e.g., Hassler et al., 2005; Gonzalez-Eiras and Niepelt, 2008; Song et al., 2012). To apply this framework, following Gonzalez et al. (2018) and Tamai (2022), we assume that governments are democratically elected and that they have a one-period administration era. Such one-period lived governments are obliged to provide public capital for consumers and business to maximize their democratic objectives. These democratic governments are delegates of young people who have future bias and old people who have no future bias. Therefore, the governments are future-biased. Furthermore, the aging population weakens future bias effects by decreasing the political power of young people relative to that of elderly people through demographic changes.

The nature of public investment policy under democracies with future bias is characterized as a result of self-control by governments. The working generation values future rewards from public investment more than the retired generation does. For that reason, the current government has an incentive to compensate the future resource misallocation from the present viewpoint by greater public investment. Future governments face the same problem as that of the current government. Therefore, they will make the same decisions about public investment as former governments did. However, this sequence can be true if and only if public investment generates direct benefits for future generations: public capital in the utility function. Without direct benefits of public capital, the governments have no incentive to manipulate the future resource allocation because public investment cannot affect the future resource misallocation only through their indirect benefits: productivity effects of public capital. In such case, there is no difference between private and public investment for the governments.

Demographic change can influence public investment through two channels: voting power and future bias. Smaller degrees of future bias yield less public investment, whereas larger degrees of future bias engender more public investment. Because young people have future bias, but senior citizens do not have it, population aging leads to a decline of the political power of future-biased people through democracies. Therefore, a society under population aging democratically chooses a cutting-back in public investment. On the other hand, the ratio of public investment to GDP can increase or decrease because population aging decreases both levels of public investment and GDP through deterioration of private and public capital accumulation. In a highly aged economy, the negative effect of population aging on public investment dominates the negative effect of population aging on GDP; population aging decreases public investment ratio. These nexuses are consistent with empirical evidence found by Jäger and Schmidt (2016) and the observations one can make from Fig. 1. Furthermore, it elucidates the relationship between declining public investment and an economic growth slowdown that occurs under population aging.

Finally, this paper derives the *modified* Ramsey formula for calculating the social discount rate, instead of the *conventional* Ramsey formula. The social discount rate under democracies with future bias must be below the social discount rate calculated from the conventional Ramsey formula. The social discount rate derived from the conventional Ramsey formula exceeds the optimal level. Therefore, government investment projects might be undervalued at the practical rate. Indeed, our numerical analysis based on the modified Ramsey formula suggests appropriate social discount rates at the rates below those derived using conventional Ramsey formula and authorities of Benefit–Cost Analysis in Japan, the United Kingdom, and the United States. Empirical studies have described the shortage of public capital relative to private capital in some countries (Ligthart and Suárez, 2011; Bom and Ligthart, 2014; Gupta et al., 2014). Our findings suggest that shortages of public capital for the countries are more severe than is usually pointed out. However, several countries that are regarded as possessing excess public capital stock might have a democratically optimal level of public capital under future bias.

The remainder of this paper is organized as follows. The next section elucidates the related literature of the present study. Section 3 describes our analytical framework of mathematical model and assumptions. Section 4 characterizes the nature of equilibrium policies and the effects of future bias on equilibrium outcomes. Section 5 derives the social discount rate in democracies under future bias and provides quantitative analyses based on recent data. Finally, Section 6 concludes this paper by explaining several policy implications and directions of future research.

2. Related literature

This section provides a general review of the literature related to our study. This paper follows at least three lines of the related literature on public investment: optimal theory of public investment, pure theory of social discounting, and benefit–cost analysis.[7] These three issues have been developed interactively. Naturally, this paper belongs to the first of those currents of research. The second of them is a basis for considering optimal investment policy through evaluation of future benefits and costs. The third one represents one practical aspect of the first and second lines.

Regarding the literature on optimal public investment, one of the most supplicated analytical frameworks was established by Arrow (1966) and by Arrow and Kurz (1970), along with optimal growth theory.[8] Under perfectly competitive markets, the Ramsey formula

[7] Zhuang et al. (2007) and Campos et al. (2015) present surveys of the literature on social discount rate and benefit–cost analysis.

[8] Their models are based on the optimal growth models developed by Ramsey (1928), Cass (1965), and Koopmans (1965).

holds with the equality of net marginal products of private and public capital as optimal conditions for public investment. This result provides the theoretical basis for the coincidence between social rate of time preference (SRTP) and social opportunity cost of capital (SOCC) in benefit–cost analysis. However, numerous factors such as risks and corporate income taxation affect such calculations in theoretical and practical ways.

Arrow and Lind (1970) considered effects of uncertainty on the evaluation of public investment, assuming that private and public capital are independent in the production function. Considering the complementarity or substitutability between private and public capital, Ogura and Yohe (1977) examined optimal public investment under capital market distortions including risks and corporate income taxation. Okuno and Yakita (1981) incorporated income distribution and Rawlsian welfare criterion into their analyses. Arrow (1982) considered capital market imperfection and optimal public investment as second-best policies. The extended studies derive modified formulas for optimal public investment with the disparity of the rates of return on private and public capital.

Another dynamic framework is also employed for analysis of (second-best) optimal public investment by many studies. These studies using an OLG model developed by Diamond (1965) are complements for the optimal growth approach rather than substitutes for them. In particular, using a simplified two-period OLG model and assuming that public investment does not affect the private sector activity, Sandmo and Dreze (1971) showed that the social discount rate should be a weighted average of the market interest rate and the marginal productivity of private capital under corporate income taxation. Pestieau (1974) derived the social discount rate when public investment yields production benefits. Yoshida (1986) extended Sandmo and Dreze (1971) into an OLG model with infinite horizons. Burgess (1988) incorporated the technological relationship between private and public capital in production, which is examined by Ogura and Yohe (1977). Yakita (1994) adopted the approach used by Arrow (1982) to derive the social discount rate. This group demonstrates that the second-best discount rate is the weighted average of the market interest rate and others including marginal productivities of private and public capital.

This paper is parallel to the flow of existing studies examining optimal public investment in the sense of deriving optimal formula for public investment. However, our framework differs from those of earlier related studies in two respects: the sustainable growth path with an endogenously determined positive growth rate and future bias are considered. These two features based on empirical and experimental evidence are theoretically and practically important for deriving a social discount rate. Regarding the first point, the sustainable growth path, it is necessary to part from neoclassical assumptions about the production function. Futagami et al. (1993) develop an endogenous growth model with public capital. Based on their model, several studies have examined optimal policies of public investment.

Turnovsky (1997) and Greiner (1998) show fundamentally the same formula of optimal public investment shown by Arrow and Kurz (1970) with an endogenously determined positive growth rate. Gómez (2004) examines irreversibility of public investment and demonstrates an optimal growth path with no public investment when public capital is over-accumulated. Tamai (2008) examines optimal public investment financed by combinations of distortionary taxes such as consumption, capital income, and labor income taxes. Furthermore, Tamai (2016) incorporates investment risks and derives a formula of optimal public investment resembling that presented by Ogura and Yohe (1977).[9] These studies using endogenous growth models are natural extensions of the analyses developed by Arrow and Kurz (1970) and others including their followers. However, future bias was beyond the respective scopes of their studies. One can envision the importance of considering future bias not only with experimentally obtained evidence but also with a pure theory of discounting.

The theory of discounting has a long history in the study of Economics. Modern mathematical approaches of a social discount rate stem from work presented by Ramsey (1928), but such approaches have since been developed in the literature on optimal economic growth.[10] Much controversy surrounds intergenerational equity for discounting future generations. In particular, Dasgupta and Heal (1979) point out this issue in relation to exhaustible resources. Furthermore, more recently, it has been heatedly argued since Stern (2006) on the economics of climate changes (e.g., Heal, 2007; Nordhaus, 2007). This difficulty stems fundamentally from how the far distant future should be discounted. A decreasing discount rate is one answer to the question (e.g., Weitzman 1998, 2001; Gollier 2002a, 2002b, 2008).[11] Indeed, several advanced countries have adopted a decreasing discount rate for evaluating government projects depending on the project horizon (Arrow et al., 2014).[12]

The decreasing discount rate tends to take the form of hyperbolic discounting. Non-constant discount rates result in a time-inconsistency problem (Strotz, 1956; Laibson, 1997). Consequently, governments might face time-inconsistency using hyperbolic discounting to evaluate their projects.[13] Considering intergenerational matters, our approach also faces the same difficulty because intergenerational altruism generates future bias as a result of non-constant discount rates for different generations. This bias might cause time-inconsistency of government policy. As a result of the self-enforcement mechanisms, our study derives a dynamically consistent social discount rate, but the discount rates facing governments in different time periods are not constant.

[9] The complementarity and substitutability between private and public capital is important for considering the effects of public investment on economic growth (e.g., An et al., 2019; Berg et al., 2019; Lowe et al., 2019).

[10] Frederick et al. (2002) and Heal (2007) present general reviews of time discounting and time preference.

[11] Zuber and Asheim (2012) consider social discount, suggesting alternatives to utilitarianism. Fleurbaey and Zuber (2015) show how different public investments should be discounted depending on various risks and inequality. They also discuss the computation of a discount rate.

[12] In the UK, the standard SRTP of 3.5% shown in Table 1 should be decreased over the long-term because of uncertainty about future values of its components (HM Treasury, 2020).

[13] Gollier et al. (2008) show that the government policy is time-consistent when the government maximizes the time-separable expected utility discounted at a constant exponential rate.

In relation to our study, Gonzalez et al. (2018) and Tamai (2022) also show that governments follow past governments' decisions as a result of the self-enforcement mechanisms. Gonzalez et al. (2018) demonstrate that two-sided altruistic preferences exhibit future bias. Future-biased people are willing to receive transfer benefits in the future rather than in the present. Furthermore, democratically elected governments inherit future bias from future-biased citizens. Hence, democratic governments have incentives to legislate and sustain a pay-as-you-go pension system, although it might involve an inefficiency of intertemporal resource allocation. Our study is based on the existence of future-biased preference of overlapping generations shown by Gonzalez et al. (2018). However, our perspective is to examine derivation of a dynamically consistent social discount rate and optimal investment policy specifically under future bias rather than legitimating the existing institutions or public interventions.

Tamai (2022) aims at elucidating why democratic governments continue to choose inefficient levels of public goods supply in reality in an endogenous growth model with two-sided altruistic preferences defined over private and public goods consumption and a linear production function with respect to private capital. In the model, the governments are future biased and have public goods expenditure policies (including welfare programs as social security) as an instrument of redistributing the intertemporal resource instead of public pensions. Tamai (2022) demonstrates that the level of public goods supply under future bias might have a positive growth effect, even if it has no productive effect, because manipulating resource allocation between private and public goods generates sources of additional investment. The previous model has a similar structure to our present model in a sense that both models incorporate direct benefits from pure public goods. By contrast, without the presence of public capital, one cannot consider the social discount rate and optimal public investment.

Furthermore, for the present study, we consider that the redistributive and productivity effects of public investment are indivisible. This indivisibility postpones the opportunity of intergenerational redistribution through public investment to a future date because the current government determines only the future redistribution by determining the size of public investment today. By contrast, in Tamai (2022), the current government chooses the current size of redistribution by adjusting welfare expenditures to control the public goods provision today, not at some future time. Hence, another different feature between the earlier and present models is the timing of redistribution. If the government were able to control public capital allocation today, then our model approximates the ealier model. Because public capital such as infrastructure is not divisible, such reallocation of public capital today is impossible in reality. Therefore, our study has different perspectives from those examined by Tamai (2022), although both the earlier and present studies shed light on different aspects with respect to different types of government expenditure policy.

In the practice of benefit–cost analysis, various methodological approaches have been adopted for calculating social discount rate and for evaluating government projects. Major methods are classified as the following three: SRTP, SOCC, and their weighted average. Controversy persists about the appropriate method of calculating social discount rate in the literature on benefit–cost analysis. Burgess and Zerbe (2011, 2013) advocate the use of SOCC, whereas Moore et al. (2013a, b) recommend the use of SRTP. Moore et al. (2013b) state that "The SRTP approach contends that governments set targets for deficits and public debt, so that a marginal government project will be tax-financed, largely crowding out current consumption. The SOCC belief is that governments set revenue targets, so that any government project will be deficit-financed on the margin, which will largely crowd out private investment." Therefore, they conclude that the SRTP is appropriate for benefit–cost analysis of self-financing government projects. Our study specifically examines such a case for deriving optimal social discount rate under future bias.

International differences in methodology are presented in Table 1. If the long-term growth rate of consumption per capita is zero and if there is no distortion in capital markets, then the interest rate coincides with the rate of pure time preference. Therefore, no discrepancy exists between SRTP and SOCC. The SRTP and SOCC approaches respectively represent calculation methods from the viewpoint of consumers and of investors. The SOCC includes the possible existence of market distortions. In fact, SRTP has been adopted widely as the calculation method. Therefore, considering the modified Ramsey formula instead of conventional Ramsey formula is important for evaluating benefits and costs in practical benefit–cost analyses.

This paper addresses issues related to optimal public investment along the research on optimal theory of public investment by incorporating recent findings of pure theory of social discounting. It practically contributes to the benefit–cost analysis for providing policy implications. In particular, our study makes two contributions to the existing literature: deriving the modified Ramsey formula for optimal social discount rate and clarifying the nature of equilibrium policy including the relationship between public investment and future bias under democracies. Therefore, our analyses cover new topics to provide complements to theoretical, empirical, and experimental findings obtained from the earlier studies.

3. The model

This section establishes the analytical framework used for this study. Consider a closed economy with altruistic overlapping generations and a homogeneous good that is producible using private and public capital. Time is indexed by subscript t; it is discretely counted as periods. In this economy, public capital is treated as joint products (Arrow and Kurz, 1970, Ch.4). For example, a highway is useful by firms and households for business and consumption activities. Therefore, it will appear in production and utility functions. Household behavior is based on overlapping generations models with two-sided altruism toward ancestors and descendants (Kimball, 1987; Hori and Kanaya, 1989), which causes future bias (Gonzalez et al., 2018).

Table 1
International differences in the calculation methods of social discount rate.

Country	Method	Discount rate
Canada	SOCC: Weighted average	8%
France	SRTP: Ramsey formula	4.5%
Germany	SRTP: Long-term interest rate of national debt	3%
Japan	SOCC: Long-term interest rate of national debt	4%
European Union	SRTP: Ramsey formula	3–5%
United Kingdom	SRTP: Ramsey formula	3.5%
United States	SOCC: Rate of return on private capital	7%
	SRTP: Long-term interest rate of national debt	4%

Sources: Canadian Cost-Benefit Analysis Guide Regulatory Proposals, 2007 (Treasury Board of Canada Secretariat) for Canada; Discount Rate in Project Analysis (2017) (France Strategie) for France; The 2030 Federal Transport Infrastructure Plan, 2018 (Federal Ministry of Transport and Digital Infrastructure) for Germany; Technical Guidelines for Cost-Benefit Analysis of Public Project Evaluation (2009) (Ministry of Land, Infrastructure, Transport and Tourism) for Japan; Guide to Cost-Benefit Analysis of Investment Projects—Economic appraisal tool for Cohesion Policy 2014–2020– (European Commission), 2014 for EU; The Green Book—Central Government Guidance on Appraisal and Evaluation, 2020 (HM Treasury) for UK; Circular A-94 Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs (1993) (Office of Management and Budget, OMB) and Circular A-4 Regulatory Analysis, 2003 (OMB) for US.

Production. Following Futagami et al. (1993), the production function is formulated as[14]

$$y_t = F(k_t, g_t), \tag{1}$$

where y_t represents the output of a homogeneous good, k_t stands for the private capital, and g_t denotes public capital. Production function F is assumed to be a concave function, homogeneous of degree one, and twice differentiable. Let x_t be the ratio of public to private capital (i.e., $x_t \equiv g_t/k_t$). Using this notation, Eq. (1) can be rewritten as

$$y_t = F\left(1, \frac{g_t}{k_t}\right) \equiv f(x_t),$$

where f is a positive-real (or non-negative-real) function with the domain of $x_t \in \mathbb{R}_0^+$.

The following assumption about the properties of $f(x_t)$ is retained throughout this study:

Assumption 1. *$f'(x_t) > 0$, $f''(x_t) < 0$, and $f'(\infty) \geq 0$.*

Using data of 151 countries for 1960–2014, An et al. (2019) find substitutability between private and public capital in production, which exhibits violation of the Inada condition partly in the constant elasticity of substitution (CES) functional forms.[15] Therefore, it requires that the Inada condition not be imposed. Excluding that, Assumption 1 is commonly used in the literature on economic growth and public investment (e.g., Arrow and Kurz, 1970; Futagami et al., 1993).

The partial derivatives of Eq. (1) with respect to k_t and g_t yield

$$F_k(k_t, g_t) \equiv \frac{\partial F(k_t, g_t)}{\partial k_t} = f(x_t) - x_t f'(x_t),$$

$$F_g(k_t, g_t) \equiv \frac{\partial F(k_t, g_t)}{\partial g_t} = f'(x_t).$$

The marginal product of private capital F_k increases with an increase in the ratio of public to private capital x_t, whereas the marginal product of private capital F_g decreases with an increase in x_t. One can verify that

$$\frac{\partial F_k}{\partial x} = -x f''(x) > 0,$$

$$\frac{\partial F_g}{\partial x} = f''(x) < 0.$$

Marginal products F_k and F_g respectively correspond to the rates of return on private and public investment.

[14] Barro (1990) considers the same production function by assuming a productive government expenditure as a factor of production instead of public capital stock. The background of equation (1) is the productivity effects of public investment supported by Aschauer (1989) and succeeding empirical studies.

[15] Berg et al. (2019) examine macroeconomic effects of public investment efficiency using the CES production function. Their study demonstrates that when the elasticity of substitution is not equal to unity, the observations over different income countries are consistent with their theoretical results. Therefore, we should not exclude cases in which the Inada condition is violated.

Households. Each generation lives during two periods. The first and second period respectively correspond to the young and the old phases of life. The generation born in period-t benefits from private consumption when young c_t^y, that when old c_{t+1}^o, with public capital at period-t and the next period, and altruistic preferences toward parents and children. A certain reason exists why we should consider that the period-t generation preference exhibits two-sided altruism. Two-sided altruism is a key factor to explain the observed downstream and upstream intergenerational transfers (Sloan et al., 2002; Kohli and Kunemund, 2003).

Assuming additive separability of the instantaneous utility at each period, the lifetime utility function is formulated as

$$U_t = u\left(c_t^y, g_t\right) + \beta u\left(c_{t+1}^o, g_{t+1}\right) + \mu U_{t-1} + \lambda U_{t+1},$$

where U_t stands for the period-t generation's utility level, u denotes the instantaneous utility function, β expresses the discount factor of the second period's utility, μ represents the degree of (filial) altruism toward their parents ($\mu > 0$), and λ denotes the degree of (parental) altruism toward their children ($\lambda > 0$). If $\mu = 0$ is allowed, then preference U_t exhibits only parental altruism, which takes the similar form of the Barro (1974) altruistic preference. However, $\mu = 0$ should be excluded because Koda and Uruyos (2018) found some evidence that individuals have altruism to both parents and children.

Following Arrow and Kurz (1970, Ch.4), the instantaneous utility function is concave, homogeneous of degree $(1-\sigma)$, and twice differentiable. More specifically, for $i = y, o$, we use

$$u\left(c_t^i, g_t\right) = \begin{cases} \dfrac{\left(c_t^i\right)^{1-\sigma}}{1-\sigma} + \chi \dfrac{g_t^{1-\sigma}}{1-\sigma} & \text{for } \sigma \neq 1 \text{ and } \sigma > 0. \\ \log c_t^i + \chi \log g_t & \text{for } \sigma = 1. \end{cases}$$

In the utility function, χ denotes the taste of public capital ($\chi \geq 0$). For the utility function, the following assumption will be made for this study:

Assumption 2. $\sigma \geq 1$.

Parameter σ represents an inverse of the elasticity of intertemporal substitution (EIS). It can be interpreted as a degree of inequality aversion in consumption between the young and the old in the sense that it measures the effect of a 1% increase in the consumption dispersion on a σ% decrease in the ratio of marginal utilities (i.e., the elasticity of marginal utility or the welfare cost of the inequality).[16] Although numerous estimations with different values of σ have been reported, we specifically examine the case of Assumption 2, based on studies by Barro (2009), Ai (2010), and Colacito and Croce (2011).[17] Furthermore, considering the discussion presented above, Assumption 2 implies that the society is unwilling (or neutral) to allow consumption inequality because of the large welfare cost of inequality $\sigma > 1$ ($\sigma = 1$).

As shown by Kimball (1987) and Hori and Kanaya (1989), the lifetime utility function U_t can be rewritten as

$$U_t = \sum_{s=1}^{\infty} \theta^s \left[u\left(c_{t-s}^y, g_{t-s}\right) + \beta u\left(c_{t-s+1}^o, g_{t-s+1}\right)\right] + u\left(c_t^y, g_t\right) + \beta u\left(c_{t+1}^o, g_{t+1}\right) + \sum_{s=1}^{\infty} \delta^s \left[u\left(c_{t+s}^y, g_{t+s}\right) + \beta u\left(c_{t+s+1}^o, g_{t+s+1}\right)\right],$$

where

$$\theta \equiv \frac{1-\sqrt{1-4\mu\lambda}}{2\lambda} \in (0,1), \delta \equiv \frac{1-\sqrt{1-4\mu\lambda}}{2\mu} \in (0,1), \mu + \lambda \in (0,1).$$

At period-t, there are people born at period-t (young generation) and their parental generation (old generation). Disregarding deceased ancestors, young and old generations respectively have

$$U_t \simeq -\left(\delta^{-1} - \theta\right)\beta u\left(c_t^o, g_t\right) + \sum_{s=0}^{\infty} \delta^s \left[u\left(c_{t+s}^y, g_{t+s}\right) + \delta^{-1}\beta u\left(c_{t+s}^o, g_{t+s}\right)\right], \tag{2a}$$

$$U_{t-1} \simeq \delta \sum_{s=0}^{\infty} \delta^s \left[u\left(c_{t+s}^y, g_{t+s}\right) + \delta^{-1}\beta u\left(c_{t+s}^o, g_{t+s}\right)\right]. \tag{2b}$$

Equation (2a) shows that the utility of elderly people $u(c_{t+s}^o, g_{t+s})$ is discounted by the discount factor subject to the sequence $\{\theta, 1, \delta, \delta^2, \cdots\}$ ($s = 0, 1, 2, \cdots$), which differs from the sequence $\{1, \delta, \delta^2, \cdots\}$ for that of young people $u(c_{t+s}^y, g_{t+s})$. For instance, $u(c_t^o, g_t)$ is discounted at the rate of θ for $s = 0$, $u(c_{t+1}^o, g_{t+1})$ is not discounted for $s = 1$, and $u(c_{t+s}^o, g_{t+s})$ is discounted at the rate of δ^{s-1} for $s \geq 2$. In contrast, Equation (2b) shows that the sequence of the utilities of young people and elderly people are, respectively, discounted by the same discount factor that follows the sequence $\{1, \delta, \delta^2, \cdots\}$. The period-$t$ generation discounts the utility of its ancestor and does not discount the utility of themselves, whereas the period-t young discounts the utility of young people relative to that of the coexisting

[16] Myles (1995, Ch.3) presents a general view of this concept. Tamai (2022) also presents specific discussion about inequality aversion related to our study.

[17] The reported values of EIS have varied among published studies. According to Havranek et al. (2015), the mean of reported values of EIS is 0.5 (in 169 published studies).

elderly people in the future. Incorporating $\theta \in (0,1)$ and $\delta \in (0,1)$ implies that the young people are unwilling to transfer resources from themselves to the living elderly people than from young people to elderly people in the future. Therefore, as demonstrated by Gonzalez et al. (2018), future bias arises.

Government. Following Gonzalez et al. (2018), we consider a sequence of a one-period government. The government provides public capital services, which are accumulated by public investment. Moreover, the government finances investment expenditure in non-distortional ways. The period-t government's objective function is formulated as

$$W_t = U_{t-1} + \eta U_t,$$

where $\eta > 0$. This objective function can be interpreted as a probabilistic voting model (Lindbeck and Weibull, 1987; Grossman and Helpman, 1998).[18] The setting described here is commonly used in the OLG framework for application of a probabilistic voting model (e.g., Hassler et al., 2005; Gonzalez-Eiras and Niepelt, 2008; Song et al., 2012). Inserting (2a) and (2b) into W_t leads to

$$W_t \simeq (\delta + \eta)\left\{u\left(c_t^y, g_t\right) + \psi u\left(c_t^o, g_t\right) + \sum_{s=1}^{\infty} \delta^s\left[u\left(c_{t+s}^y, g_{t+s}\right) + \varphi u\left(c_{t+s}^o, g_{t+s}\right)\right]\right\}, \tag{3}$$

where

$$\psi \equiv \frac{(1+\eta\theta)\beta}{\delta+\eta} \text{ and } \varphi \equiv \frac{\beta}{\delta}.$$

Comparison between ψ and φ yields $\psi < \varphi$. This inequality indicates that the weight of the old to the young in the current period is less than that in the future; the period-t government is not willing to transfer resources from the young to the elderly at present than from the young to the elderly in the future. Therefore, social preference (3) has future bias because the period-t generation preference exhibits future bias. Since ψ depends on η, demographic effects such as population aging or a baby boom affect the degree of future bias $(\varphi - \psi)$. Indeed, the partial derivative gives

$$\frac{\partial(\varphi - \psi)}{\partial \eta} = \frac{(1-\delta\theta)\beta}{(\delta+\eta)^2} > 0.$$

The degree of future bias $(\varphi - \psi)$ is positively associated with η. Hence, the equation described above also implies that population aging (i.e., a decrease in η) weakens the future bias effects, whereas rejuvenating the population (i.e., an increase in η) strengthens them.

Capital accumulation and resource constraint. Let i_t^k and i_t^g respectively represent investment in private and capital at period-t. Private and public capital are accumulated as

$$k_{t+1} = i_t^k = (1 - s_t)i_t,$$

$$g_{t+1} = i_t^g = s_t i_t,$$

where total investment denoted by $i_t \equiv i_t^k + i_t^g$ and s_t represents the share of public investment to total investment. By the definition of x_{t+1}, it follows that

$$x_{t+1} = \frac{g_{t+1}}{k_{t+1}} = \frac{i_t^g}{i_t^k} = \frac{s_t}{1-s_t}.$$

Because output at period-t is allocated between consumption and investment, the resource constraint is given as

$$y_t = c_t^y + c_t^o + i_t.$$

Using (1), the resource constraint becomes

$$F(k_t, g_t) = c_t + k_{t+1} + g_{t+1}, \tag{4}$$

where

$$c_t \equiv c_t^y + c_t^o.$$

Hereinafter, subscript t will be omitted throughout this study. Variables with a single prime are used as the next period's variables (i.e., x and x' respectively denote x_t and $x_{t+1} = x'$). However, it is noteworthy that a single (double) prime for a function is used as a conventional mathematical notation of first-order (second-order) derivative.

[18] It is the same as the populational welfare function (Hori, 1997; Aoki and Nishimura, 2017).

4. Equilibrium policy

This section characterizes a relationship between equilibrium policy and future bias. To address these issues, we presume that governments directly choose an allocation of aggregate resources between consumption and investment and choose an allocation of the aggregate consumption between young people and elderly people during period t. In other words, the period-t government can achieve the desired period-t allocation, although the government might not directly choose the future allocation. First, we characterize the planned economy with democratically elected governments. Subsequently, we consider a planned economy with non-biased planner and provide comparative analysis between two economies to clarify basic characteristics of the democratically planned economy.

4.1. Democratically planned economy

Consider that democratically elected short-lived governments plan intertemporal resource allocation to maximize their objective functions. Hereinafter, the economy is designated as a *democratically planned economy*. First, let us examine the consumption allocation between the young and the old at each period. Such decision-making corresponds to a choice of $\pi \equiv c^y/c$ and $1-\pi \equiv c^o/c$. Following Hori (1997), Aoki and Nishimura (2017), and Gonzalez et al. (2018), the period-t government's static problem is choosing the consumption allocation in period t for a given weight of the old to the young ($z = \psi, \varphi$) as

$$\max_{\pi}[u(\pi c, g) + z \cdot u((1-\pi)c, g)]. \tag{SP}$$

Solving the static optimization problem (SP) yields

$$\pi = \frac{1}{1+z^{\frac{1}{\sigma}}}. \tag{5}$$

Equation (5) implies that the consumption share of young people to total consumption is decreasing in the weight of elderly people.

The government at each period is obliged to maximize objective function (3) subject to (4) and (5). The governments have duration of one period. Therefore, they can choose their amount of total investment (i.e., the sum of private and public investment) in their own administration. Let $w \equiv k + g$. Then, total capital w should be treated as one state variable, described by Arrow and Kurz (1970, Ch.4). As shown in (3), the government perceives that the welfare weights of the current benefits of private consumption and public investment differ from those in the next government's administration.

Considering these matters, the period-t government's dynamic optimization problem is formulated as

$$V_0(w) = \max_{s,w'}\{v(c, g, \pi, \psi) + \delta V(w')\} \tag{DP1}$$

with (4), (5), the anticipated values of future young's consumption share $\widehat{\pi}$,

$$V(w) = v(c, g, \widehat{\pi}, \varphi) + \delta V(w'),$$

$$v(c, g, \pi, z) = q(\pi, z)\frac{c^{1-\sigma}}{1-\sigma} + X(z)\frac{g^{1-\sigma}}{1-\sigma},$$

$$q(\pi, z) \equiv \pi^{1-\sigma} + (1-\pi)^{1-\sigma} z,$$

$$X(z) \equiv (1+z)\chi.$$

Also, $v(c, g, \pi, z)$ is the instantaneous welfare function with the weight for aggregate private consumption $q(\pi, z)$ and that for direct benefits of public capital $X(z)$. The weight $q(\pi, z)$ is increasing in the weight for elderly people z for given π, and

$$\frac{\partial q(\pi, z)}{\partial \pi} = (\sigma-1)(1-\pi)^{-\sigma}\left[\left(\frac{1-\pi}{\pi}\right)^{\sigma} - z\right] \gtreqless 0 \Leftrightarrow \pi \lesseqgtr \frac{1}{1+z^{\frac{1}{\sigma}}}.$$

Because the society is inequality-averse, that is $\sigma > 1$, a greater increase in π increases (decreases) the weight for aggregate consumption if inequality exists with the poor young (old) relative to the rich old (young).

For different values of π and z, the instantaneous welfare functions become

$$v(c, g, \pi, \psi) = q(\pi, \psi)\frac{c^{1-\sigma}}{1-\sigma} + X(\psi)\frac{g^{1-\sigma}}{1-\sigma},$$

$$v(c, g, \widehat{\pi}, \varphi) = q(\widehat{\pi}, \varphi)\frac{c^{1-\sigma}}{1-\sigma} + X(\varphi)\frac{g^{1-\sigma}}{1-\sigma}.$$

Furthermore, the first-order partial derivatives of $v(c, g, \pi, z)$ are denoted as

$$v_c(c,g,\pi,z) \equiv \frac{\partial v(c,g,\pi,z)}{\partial c} = q(\pi,z)c^{-\sigma},$$

$$v_g(c,g,\pi,z) \equiv \frac{\partial v(c,g,\pi,z)}{\partial g} = X(z)g^{-\sigma}.$$

The first-order conditions of the period-t government optimization problem (DP1) for private consumption and aggregate investment choice are

$$v_c(c,g,\pi,\psi) = \delta\frac{\partial V(w')}{\partial w'}, \tag{6a}$$

$$F_g + \frac{v_g(c',g',\widehat{\pi},\varphi)}{v_c(c',g',\widehat{\pi},\varphi)} = F_k. \tag{6b}$$

Equation (6a) requires that the marginal value of total investment equals the marginal utility of the weighted sum of private consumption. Equation (6b) makes a similar statement to that of (6a) related to public capital investment. The results imply that the marginal benefits of public capital investment are equal to its marginal costs (i.e., the marginal benefit of private capital investment) at all periods. An alternative interpretation is possible using the converted form of (6b) such that $v_g/v_c = F_k - F_g$. In other words, equation (6b) requires that the marginal rate of substitution (MRS) of public capital for private consumption in the utility equals the private investment return net of public investment return.

The partial derivatives of the value function $V(w)$ and the policy function $c(w)$ must satisfy

$$\frac{\partial V(w)}{\partial w} = v_c(c,g,\widehat{\pi},\varphi)\frac{\partial c}{\partial w} + v_g(c,g,\widehat{\pi},\varphi)\frac{\partial g}{\partial w} + \delta\left[\frac{\partial V(w')}{\partial w'}\,\frac{\partial w'}{\partial w}\right], \tag{7a}$$

$$\frac{\partial c}{\partial w} = F_k + \left(F_g - F_k\right)\frac{\partial g}{\partial w} - \frac{\partial w'}{\partial w}. \tag{7b}$$

Equations (6a), (6b), (7a), and (7b) yield an Euler equation. The definition of w and (4) constitute the transition equation of total capital.

If government policy is determined, then the dynamic system consists of the Euler and transition equations, giving the dynamics of key economic variables k, g, and c. To study such an equilibrium government policy, the following definition is made for discussion in this section.

Definition 1. *A Markov strategy for the period-t government is a triplet of* $\{c_t(k_t,g_t), i_t(k_t,g_t), s_t(k_t,g_t)\}$. *A Markov perfect equilibrium is a set of sequences* $\{c_t(k_t,g_t), i_t(k_t,g_t), s_t(k_t,g_t)\}_{t=0}^{\infty}$, *satisfying* Equations (4)–(6b) *and* $\{c_t(k_t,g_t), i_t(k_t,g_t), s_t(k_t,g_t)\} = \{c(k_t,g_t), i(k_t,g_t), s(k_t,g_t)\}$ *for all* $t \geq 0$.

To derive the equilibrium policy, the period-t government requires information about the future government's investment policy. Therefore, we infer that total investment function takes the form of

$$k' + g' = \begin{cases} (1+\gamma)(k+g) \text{ for period } t. \\ (1+\widehat{\gamma})(k+g) \text{ for the periods after } t. \end{cases}$$

In this equation, coefficient $1+\gamma$ is the ratio of total investment to total capital and therefore, the growth factor of total capital. In addition, γ is the growth rate of total capital; $\widehat{\gamma}$ is the anticipated value for the periods after t. By the definition of w, the investment function can be rewritten as

$$w' = \begin{cases} (1+\gamma)w \text{ for period } t. \\ (1+\widehat{\gamma})w \text{ for the periods after } t. \end{cases} \tag{8}$$

Because governments will sequentially face (SP) and (DP1), the current government must respond to future governments' (anticipated) actions satisfying (4), (6a), (6b), (7a), (7b), and (8). By the definition of s, s is a function of x'. Therefore, equation (6b) gives the best response of a government's choice of s or x' with respect to $\widehat{\gamma}$: $x' = H(\widehat{\pi},\widehat{\gamma})$. Incorporating $x' = H(\widehat{\pi},\widehat{\gamma})$, equations (4) and (6a)–(8) will provide the best response of a government's choice of γ with respect to $\widehat{\gamma}$. Equations (4) and (6a)–(8) imply best-response mapping $\gamma = B(\pi,\widehat{\pi},\widehat{\gamma})$, which represents the best response of a government that allocates a share π of aggregate consumption c to the current young c^y and anticipates that future governments will respectively set $\widehat{\pi}$ and $\widehat{\gamma}$ as their consumption allocation and investment policies (see Appendix B).

The government's optimization problem for the current consumption allocation is (SP) with $z = \psi$ and identical not only for the current government but also for all future governments as

$$\max_{\pi}[u(\pi c, g) + \psi \cdot u((1-\pi)c, g)].$$

Consequently, from (5), the consumption allocation policy for all periods is given as

$$\pi^* = \frac{1}{1+\psi^{\frac{1}{\sigma}}},$$

so that the governments set $\pi = \widehat{\pi} = \pi^*$. Therefore, the best response mapping becomes $\gamma = B(\pi^*, \pi^*, \widehat{\gamma})$. Because a Markov perfect equilibrium condition requires $\gamma = \widehat{\gamma}$ and $x = x' = \widehat{x}$, the equilibrium outcome of a game is derived from $\gamma = B(\pi^*, \pi^*, \widehat{\gamma})$ and $x' = H(\pi^*, \widehat{\gamma})$.

Using (4) and (6a)–(8) with $\pi = \widehat{\pi} = \pi^*$, $x = x' = \widehat{x}$, and $\gamma = \widehat{\gamma}$, we formally obtain the following proposition (Appendix B presents the proof of Proposition 1).

Proposition 1. *For a democratically planned economy, if $\delta[f'(\overline{x})]^{1-\sigma} < 1$ with $\overline{x}$ such that $f'(\overline{x}) = f(\overline{x}) - \overline{x}f'(\overline{x})$, then there exists a unique Markov perfect equilibrium in linear strategies with*

$$\pi^* = \frac{1}{1+\psi^{\frac{1}{\sigma}}}, \tag{9a}$$

$$x^* = x^*(\psi, \varphi), \tag{9b}$$

$$c^*(w) = \left[\frac{f(x^*)}{1+x^*} - (1+\gamma^*)\right]w, \tag{9c}$$

$$i^*(w) = (1+\gamma^*)w, \tag{9d}$$

satisfying

$$F_g^* + \frac{X(\varphi)}{q(\pi^*, \varphi)}\left[\frac{f(x^*) - (1+x^*)(1+\gamma^*)}{x^*}\right]^{\sigma} = F_k^*, \tag{10a}$$

$$1+\gamma^* = \left\{\frac{q(\pi^*, \varphi)}{q(\pi^*, \psi)}F_k^* + \left[1 - \frac{q(\pi^*, \varphi)}{q(\pi^*, \psi)}\right](1+\gamma^*)\right\}^{\frac{1}{\sigma}}\delta^{\frac{1}{\sigma}}, \tag{10b}$$

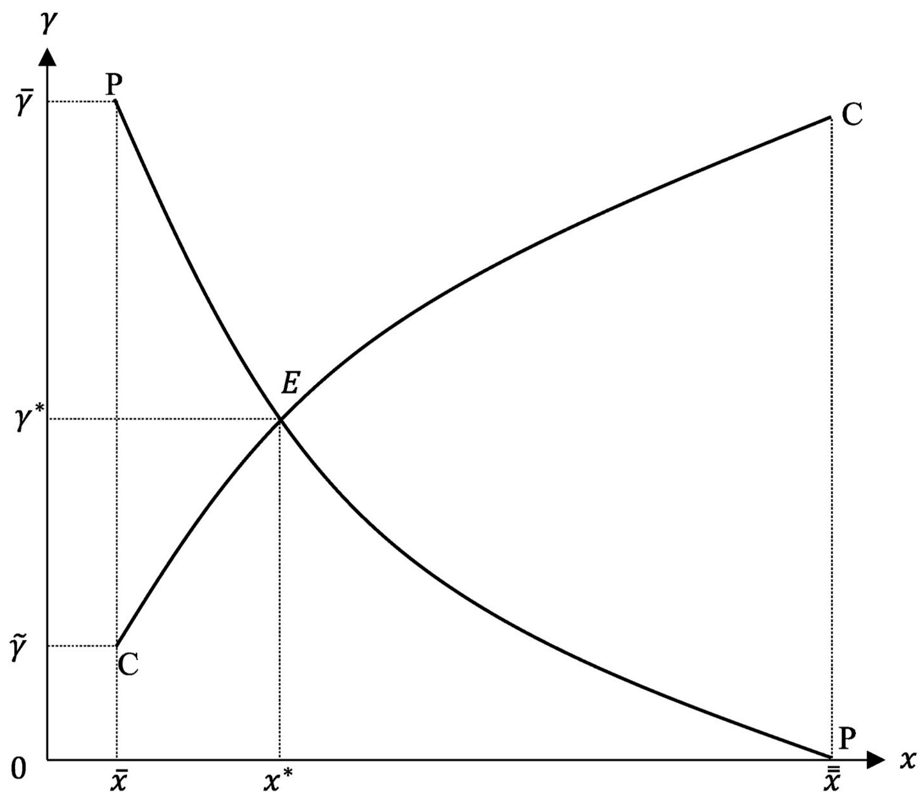

Fig. 3. Determination of the equilibrium values of x and γ

$F_k^* \equiv f(x^*) - x^* f'(x^*)$, *and* $F_g^* \equiv f'(x^*)$.

Equation (9a) gives the equilibrium share of youth consumption to aggregate private consumption. The young generation's share of private consumption decreases as the relative utility weight of the old period, ψ, increases. Equations (9b), (9c) and (9d) respectively give the equilibrium ratio of public capital to private capital, aggregate private consumption, and investment functions, satisfying (10a) and (10b). Equation (10a) represents the equalization of the marginal benefits and costs of public investment as the equilibrium condition of private and public investment for the period-t government. Finally, equation (10b) corresponds to the Euler equation in equilibrium, which is related to the Ramsey formula. It will meet a standard Euler equation if $\varphi = \psi$ would hold. With future bias and noting $q(\pi^*, \varphi) > q(\pi^*, \psi)$, the return on investment is more highly weighted to compensate future misallocation of consumption resources (i.e., the coefficient of F_k^*). However, transferring the current resources to the future will affect future misallocation. Therefore, aggregate investment mitigates the incentive for more investment in the future itself (i.e., the coefficient of $(1+\gamma^*)$ on the right-hand side).

The values of x^* and γ^* are derived from (10a) and (10b). Determination of these two values is presented in Fig. 3. The downward-sloping curve P–P represents the graph of (10a), which shows equilibrium conditions for private and public investment. The private investment return net of the public investment return increases as x increases. Therefore, keeping the equality of (10a), the MRS of public capital for private consumption must increase if x increases. Because MRS is decreasing in each of x and γ, γ must decrease to keep the equality of the MRS and the private investment return net of the public investment return. The upward-sloping curve C–C stands for the graph of (10b), which represents the consumption growth equation or equilibrium condition for private consumption and aggregate investment. Because the return on private investment increases as x increases, an increase in x stimulates aggregate investment. The P–P curve and C–C curve respectively have a negative and a positive gradient and because each curve is defined over a positive orthant, the P–P and C–C curves intersect at a single point with positive values of x and γ.

4.2. Effects of future bias

A benchmark case is necessary to verify characteristics of the dynamic equilibrium with future bias shown in Proposition 1. Naturally, one can consider a planning economy by which the period-t government can commit to all allocations from period $t+1$ onward (Gonzalez et al., 2018; Tamai, 2022). For instance, the government can do so if no difference exists between the parameters of welfare weighted for current and future old people. Because elderly people can constitute such a government, this benchmark economy is designated as an *elderly planned economy*.

A standard dynamic optimization is applicable to the equilibrium in the planned economy with a nonbiased planner (e.g., elderly people). The equilibrium concept explained in Definition 1 is also applicable to this case. Replacing φ with all the ψ s in equations of Proposition 1 establishes the following proposition.

Proposition 2. *Under the first-best scenario by which the period-t government can commit to all allocations from period* $t+1$ *onward, there exists a unique Markov perfect equilibrium in linear strategies with*

$$\pi^\dagger = \frac{1}{1+\varphi^{\frac{1}{\sigma}}}, \tag{11a}$$

$$x^\dagger = x^\dagger(\varphi), \tag{11b}$$

$$c^\dagger(w) = \left[\frac{f(x^\dagger)}{1+x^\dagger} - (1+\gamma^\dagger)\right] w, \tag{11c}$$

$$i^\dagger(w) = (1+\gamma^\dagger) w, \tag{11d}$$

satisfying

$$F_g^\dagger + \frac{X(\varphi)}{q(\pi^\dagger, \varphi)} \left[\frac{f(x^\dagger) - (1+x^\dagger)(1+\gamma^\dagger)}{x^\dagger}\right]^\sigma = F_k^\dagger, \tag{12a}$$

$$1+\gamma^\dagger = \left(\delta F_k^\dagger\right)^{\frac{1}{\sigma}}, \tag{12b}$$

$F_k^\dagger \equiv f(x^\dagger) - x^\dagger f'(x^\dagger)$, *and* $F_g^\dagger \equiv f'(x^\dagger)$.

Equations (11a)–(12b) respectively correspond to (9a)–(10b).[19] Comparison between key equations in Proposition 1 and 2

[19] These equilibrium conditions are fundamentally the same as the optimal conditions derived by Arrow and Kurz (1970, Ch.4) with linear homogeneity of private and public capital in production. If there is no direct benefit of public capital ($\chi = 0$), then it follows that $F_g^\dagger = F_k^\dagger$ is one of the first-best conditions derived by Turnovsky (1997) and Greiner (1998). Gómez (2004) and Tamai (2016) respectively present optimality conditions under irreversible investment and uncertainty.

indicates some important properties of equilibrium policies. The difference between a democratically planned economy and the benchmark lies in the difference between the welfare weight for the current old generation and the welfare weight for the future old generation. First, all governments in a democratically planned economy facing the identical optimization problem (SP) with $z = \psi$ choose the same consumption allocation during their administration. However, such allocation is not optimum for subsequent governments. Because $\varphi > \psi$ holds, $\pi^* > \pi^\dagger$ is obtained from (9a) and (11a). The consumption misallocation remains throughout all periods.

Second, consumption misallocation affects the investment allocation between private and public capital if there are direct benefits of public capital ($\chi > 0$). The present investment in public capital is reflected in the future stock of public capital. With $\chi > 0$, benefits from the redistributive and productivity effects of public investment arise at a future date. The current government determines only the redistribution at the future date by controlling the size of public investment today. The future government will do so. Therefore, the current and future governments in the democratically planned economy have the same need to maintain (10a). However, the welfare weight for the future old is miscalculated under the misallocated allocation of consumption resources. That miscalculation implies that investment resources of private and public capital are also misallocated. The key to this misallocation between private and public capital is the timing of intergenerational redistribution effect of public policy. If $\chi > 0$, then the redistributive effect through public investment is postponed to a future date because current public investment is effective at the future date. However, if $\chi = 0$, there is no redistribution effect. Therefore, no timing problem of redistribution occurs.

Finally, a choice of aggregate investment in the democratically planned economy deviates from that under the first-best scenario. This intertemporal misallocation of resources results from the consumption misallocation and the misallocation of investment resources between private and public capital. To compensate the consumption misallocation, the current government strategically uses aggregate investment and investment allocation between private and public capital. However, transferring the resource to the future date causes misallocation of consumption and investment resources in the future through the effects described above. Because the opposite effects are mixed, the effects of future bias on aggregate investment are ambiguous without further analysis.

To characterize the misallocations of consumption and investment, we now consider the effects of a change in ψ on x^* and γ^*. The loci of P–P and C–C curves in Fig. 3 depend on the value of ψ; each of x^* and γ^* is the function of ψ and φ (i.e., (9c)). Indeed, a rise in ψ moves the P–P curve upward and the C–C curve downward; These changes in loci of the two curves indicate that x^* is increasing in ψ and that γ^* might be increasing or decreasing in ψ. How a deviation of ψ from φ affects the equilibrium values is fundamentally important to interpretation of the effects of the presence of future bias. Specifically examining the equilibrium ratio of public capital to private capital and the equilibrium growth factor, the comparative statics of the system consisting of (10a) and (10b) yields the following result (Appendix C presents the proof of Proposition 3).

Proposition 3. *(i) Effects of a change in ψ on x^* and γ^* are given as*

$$\frac{\partial x^*}{\partial \psi} > 0 \text{ and } \frac{\partial \gamma^*}{\partial \psi} < 0.$$

(ii) *Furthermore, the effects of a rise in ψ on investment ratios are*

$$\frac{\partial}{\partial \psi}\left(\frac{i^k}{y}\right) < 0 \text{ and } \frac{\partial}{\partial \psi}\left(\frac{i^g}{y}\right)\bigg|_{\psi=\varphi} < 0 \text{ if } \delta(1+\gamma^\dagger)^{1-\sigma} \ll 1.$$

In those expressions, ψ is *inversely* related to a degree of future bias. Because a decrease in η caused by population aging generates an increase in ψ, the result (i) of Proposition 3 reveals that the equilibrium ratio of public capital to private capital is negatively associated with the presence of future bias, whereas the equilibrium growth rate is positively related to the future bias. This result is consistent with the empirical fact that population aging has a statistically significant positive effect on the ratio of public to private capital shown in Fig. 2 (see Table A2 in Appendix A). Furthermore, the result (ii) of Proposition 3 is derived from the result (i). In terms of the investment ratio, population aging decreases private and public investment ratio if the society is highly aged. This result is consistent with the empirical evidence established in Fig. 1 and findings from earlier studies.

The background mechanism for the result of Proposition 3 can be inferred using the best response mapping and the anticipated relationship between the current and future governments' policies. One can infer that the deviation of ψ from φ elucidates effects of future bias. For x_0, the initial equilibrium point of γ is given at point E_0 in Fig. 4. The B–B curve in the first orthant represents the best response mapping of γ, $B(\pi^*, \pi^*, \widehat{\gamma})$. The P–P curves in the second and fourth orthants denote the MRS of public capital for private consumption. The 45-degree lines, $\gamma = \widehat{\gamma}$ and $x = x' = \widehat{x}$, in the first and third orthants correspond to the equilibrium conditions.

Now presuming that the people are more future biased, ψ is more distant from φ. By a decrease in ψ, the curve of $B(\pi^*, \pi^*, \widehat{\gamma})$, the B–B curve in Fig. 4, moves to the B′–B′ curve. This movement implies that the period-t government can compensate for the loss from future consumption misallocation ($\pi^* > \pi^\dagger$) by strategically increasing aggregate investment to increase future consumption. The expected future growth rate is updated by anticipation with $\gamma = \widehat{\gamma}$. The initial equilibrium point E_0 moves to new point E^* along with 45-degree line in the first orthant in Fig. 4; The anticipated growth rate $\widehat{\gamma}$ is changed from γ_0 to γ^*. Therefore, the future growth rate will be increased; the equilibrium growth rate will also be increased.

At the same time, a decrease in ψ affects the MRS curve, which is the best response curve of x'; the marginal benefit of public capital measured by private consumption decreases because $q(\pi^*, \varphi)$ increases as ψ decreases. This change implies that increasing aggregate investment reduces the return on private investment to maintain the equality of the net return on private investment and the MRS of

Table 2
Parameters.

	Japan	United Kingdom	United States
β	0.567	0.213	0.285
η	0.300	0.500	0.700
θ	0.493	0.493	0.495
μ	0.332	0.332	0.332
σ	1.100	1.300	1.350
φ	0.575	0.216	0.288
ψ	0.506	0.179	0.227
γ	0.012; 0.015; 0.018	0.012; 0.017; 0.022	0.016; 0.019; 0.022

Note: For the value of γ, low growth, baseline, and high growth scenarios are shown in the order: left to right.

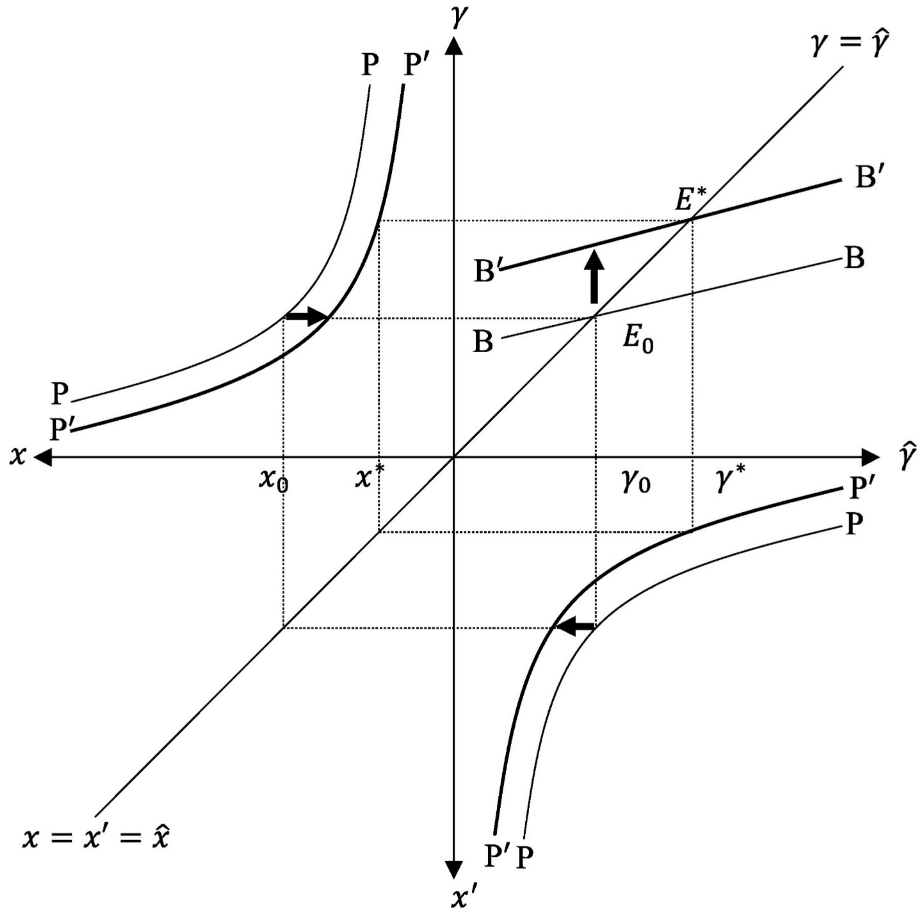

Fig. 4. Effects on x^* and γ^* of a decrease in ψ

public capital for private consumption. The MRS curve, the P–P curve, moves to the left (right) in the fourth (second) orthant in Fig. 4 because the government's anticipation is that this movement will be true subsequently. The new loci of the curves are updated to the P′–P′ curves. Then, transferring resources to the future has a lower return by future consumption misallocation if the ratio of public to private capital is stick to x_0. However, the equilibrium growth rate at present and future dates is pulled up to γ^*. For the new level of the growth rate, the new equilibrium ratio of public capital to private capital x^* is determined along with the P′–P′ curve.

The result (ii) of Proposition 3 is explained as presented below. Along with population aging, a rise in ψ decreases γ^*, leading to slowdowns of private and public capital accumulation. By the presence of direct benefits, the governments decrease private investment relative to public investment in terms of growth rates because a decrease in public investment harms future generation's welfare more. Private capital accumulation is more negatively affected than public capital accumulation; x^* increases. Since the capital growth rates of private and public capital are decreased by a rise in ψ, whether private and public investment ratios are increased or not depends on changes in output–capital ratios (i.e., y/k or y/g) through x^*. The ratio of output to private capital is increasing in ψ, whereas the ratio of output to public capital is decreasing in ψ. Therefore, an increase in ψ decreases the private investment ratio and has an ambiguous effect on the public investment ratio. Within plausible parameters to ensure low capital share and growth rate, if ψ is sufficiently large, then the effect of a rise in ψ on public capital growth rate dominates the effect on the ratio of output to public capital. Then, a rise in ψ reduces the public investment ratio.

5. Rate of return and social discount rate

This section analyzes the relationship between the social discount rate, the rates of return on private and public investment, and future bias. First, the analysis characterizes the equilibrium rates of return on private and public investment in the democratically planned economy. Second, we develop comparative analyses of the democratically planned economy and the benchmark economy to elucidate the relationship between the social discount rate and future bias.

5.1. Rates of return on private and public investment

Propositions 1 and 2 include formulae for the equilibrium rates of return on private and public investment so that the social discount rate can be calculated. Indeed, equations (10a) and (12a) show the relations between the rates of return on private and public investment. Equations (10a) and (12a) can be written as presented below:

$$F_k^* - F_g^* = \frac{X(\varphi)}{q(\pi^*, \varphi)}\left[\frac{f(x^*) - (1+x^*)(1+\gamma^*)}{x^*}\right]^{\sigma} > 0, \tag{13a}$$

$$F_k^\dagger - F_g^\dagger = \frac{X(\varphi)}{q(\pi^\dagger, \varphi)}\left[\frac{f(x^\dagger) - (1+x^\dagger)(1+\gamma^\dagger)}{x^\dagger}\right]^{\sigma} > 0. \tag{13b}$$

Equations (13a) and (13b) imply that the disparity in the two rates of return depends on the marginal direct benefit of public investment measured by the marginal benefit of private consumption, or equivalently, the MRS of public capital for private consumption in the two economies. In other words, the rate of return on public investment equals the rate of return on private investment minus the marginal direct benefit of public investment measured by the marginal benefit of private consumption. The divergence of (13a) from (13b) is identifiable as differences in the choice of π and γ.

Incorporating these facts and the results of Propositions 1 and 2 yields the following proposition (Appendix D presents the proof of Proposition 4):

Proposition 4. *With future bias and direct benefits of public investment, the following relation holds:*

(i) $F_g^* < F_k^*$.

(ii) $\frac{\partial(F_g^* - F_k^*)}{\partial\psi} = (1+x^*)f''(x^*)\frac{\partial x^*}{\partial\psi} < 0$, *or equivalently,*

$$\frac{\partial}{\partial\psi}\left\{\frac{X(\varphi)}{q(\pi^*, \varphi)}\left[\frac{f(x^*) - (1+x^*)(1+\gamma^*)}{x^*}\right]^{\sigma}\right\} > 0.$$

The result (i) of Proposition 4 is similar to that derived by Arrow and Kurz (1970) in the sense that the inequalities have the same sign. However, the degree of disparity is affected by the presence of future bias. Indeed, the result (ii) of Proposition 4 indicates that a disparity between the rates of return on two investments increases as the degree of future bias increases through a decrease in the direct benefit of public investment.

The presence of direct benefits of public investment pushes up the rate of return on private investment relative to that on public investment (i.e., $F_k^* > F_g^*$ and $F_k^\dagger > F_g^\dagger$). Without direct benefits of public investment (i.e., $\chi = 0 \Leftrightarrow X(\varphi) = 0$), no deviation exists between two rates of return such that $F_g^* = F_k^*$ holds even if future bias exists. If $\chi = 0$, then the future bias brings about no distortion about choices of private and public investment because the timings of private and public investment are simultaneous. Indeed, $x^\dagger = x^* = \overline{x}$ if and only if $F_g^* = F_k^*$. However, the direct benefits of public investment are not independent of the future bias. Future bias affects the direct benefit of public investment through the value of π^* (or equivalently ψ). Therefore, it also influences the disparity in the two rates of return. Because $\pi^* > \pi^\dagger$ so that $q(\pi^*, \varphi) < q(\pi^\dagger, \varphi)$, the direct benefits of public capital would be raised by future bias if γ were fixed. However, future bias stimulates aggregate investment and an investment increase reduces the marginal utility of public capital as measured by the marginal utility of private consumption.

Empirical studies showed evidence of $F_g > F_k$ (e.g., Ligthart and Suárez, 2011; Bom and Ligthart, 2014; Gupta et al., 2014).[20] Based on a conventional theory of public investment, results of those studies suggested underinvestment in public capital relative to private capital. This finding is qualitatively true even if one considers direct benefits of public capital and future bias. However, following our

[20] Gupta et al. (2014) reported that ignoring public investment inefficiencies brings about underestimation of the marginal productivities of both private and public capital.

theoretical findings, the direct benefits of public capital and future bias quantitatively affect the level of public capital stock relative to the private capital stock. Consequently, the result implies that countries with $F_g > F_k$ are more adversely affected by underinvestment in public capital.

More recently, Tamai (2016) found that $F_k > F_g$ holds in France and Japan whereas $F_k < F_g$ is true for other G5 countries.[21] Furthermore, it was shown that $F_k \approx F_g$ in the United Kingdom. As a result, the optimality of public investment in the UK was attained based on the theoretical prediction that the risk-adjusted returns on two investments were equalized. Lowe et al. (2019) estimated that Japan and Singapore have $F_k \approx F_g$, and that $F_k > F_g$ holds for Jordan, Malaysia, and Uruguay. They reported that these countries almost efficiently invest or slightly overinvest in public capital. However, our theoretical result implies that $F_k > F_g$ is derived from sub-optimal decisions made by democratic governments. Even if the choice under future bias deviates from the first-best scenario, one can infer that countries for which $F_k > F_g$ show more care about current and future generations through their choices of policies.

5.2. Social discount rate for public investment

To evaluate the optimality of public investment, one must investigate the relation between sub-optimal criteria and those under the first-best scenario. As shown in earlier studies of the literature, with no biases, the social discount rate is the utility discount rate plus a term that measures the rate of change of marginal utility. This rate coincides with the rate of return in equilibrium.[22] Therefore, we should first consider that these key variables are affected by future bias. Characteristics of the relationship between the sub-optimal and first-best equilibrium should be examined. The difference between equations appearing in Propositions 1 and 2 originates in the difference between the values of welfare weight for old people: ψ and φ. Propositions from 1 through 4 formally engender the following result.

Proposition 5. *In a democratically planned economy, (i) the equilibrium growth rate exceeds its first-best level ($\gamma^* > \gamma^\dagger$). (ii) The rate of return on public investment is greater than the first-best rate of return on public investment, whereas the rate of return on private investment is less than the first-best rate of return on private investment ($F_g^* > F_g^\dagger$ and $F_k^* < F_k^\dagger$). Therefore, the direct marginal benefit of public capital measured by marginal benefit of private consumption is below its first best level. (iii) The equilibrium ratio of public to private capital is less than its first-best level ($x^* < x^\dagger$) if there are direct benefits of public capital ($\chi > 0$). However, if there are no direct benefits of public capital ($\chi = 0$), then the ratio of public to private capital is equal to its first-best level ($x^* = x^\dagger = \bar{x}$).*

The intuition underpinning Proposition 5 can be explained as follows. Result (i) is obtained from Proposition 3. Future bias causes a misallocation of resources in the current and future periods (i.e., $\pi^* > \pi^\dagger$). The current government strategically chooses more aggregate investment to compensate the welfare loss of resource misallocation. Therefore, the equilibrium growth rate is higher than its optimal level. Result (ii) is derived from Propositions 3 and 4. The current government, which anticipates the future misallocation of resources, enables manipulation of the investment allocation between private and public investment for treating future resource misallocation. Because future public capital is overprovided by the presence of consumption misallocation, the current government allocates more resources to private investment relative to public investment. Therefore, the rate of return on private investment falls to a level below its rate under the first-best scenario. Finally, result (iii) naturally follows results (i) and (ii).

From the result (ii) of Proposition 5, the politically determined social discount rate is separated from the social discount rate under the first-best scenario by the presence of future bias. To elucidate this point, one can begin deriving a social discount rate that is chosen by a non-biased planner. Approximately, equation (12b) can be the following equation (Appendix D presents its derivation):

$$r^\dagger \approx \rho + \sigma\gamma^\dagger, \tag{14}$$

where $\rho \equiv (1-\delta)\delta^{-1}$. In Equation (14), the first and second term on the right-hand side respectively denote *time preference* and *wealth effect*. Also, ρ is the rate at which consumption (direct benefit of public capital) is discounted over time, assuming no change in per-capita consumption. γ reflects the expected growth in per-capita consumption (public capital) over time, where future consumption (direct benefit of public capital) will be higher relative to current consumption. It is expected to have a lower utility. Equation (14) gives a well-known formula of the social discount rate (i.e., conventional Ramsey formula). Under the first-best scenario, the period-t government can commit to all allocations from period $t+1$ onward. It reveals the equality between the consumption discount rate (consumption rate of interest) and the return on private capital.

However, particularly addressing a practical aspect of social discount, the social discount rate is chosen through political determination. As described in this paper, from (10b), the politically determined social discount rate satisfies the following formula (Appendix D presents its derivation) as

$$r^* \approx \rho + \sigma\gamma^* - \Delta, \tag{15}$$

where

[21] Tamai (2016) examines the effects of investment uncertainty. Therefore, the marginal productivities are risk-adjusted.

[22] Arrow and Kurz (1970) and Heal (2007) present relevant details.

$$\Delta \equiv \log \frac{v_c(c', g', \pi^*, \varphi)}{v_c(c', g', \pi^*, \psi)} - \left[\frac{v_c(c', g', \pi^*, \varphi)}{v_c(c', g', \pi^*, \psi)} - 1 \right] \delta(1+\gamma^*)^{1-\sigma} = (\varphi - \psi)\psi^{\frac{1-\sigma}{\sigma}} \left[1 - \delta(1+\gamma^*)^{1-\sigma}\pi^*\right] > 0.$$

Equation (15) denotes the modified Ramsey formula. The third term, Δ, on the right-hand side of (15) represents future bias effects. Because $\gamma^* > \gamma^\dagger$ (by result (i) of Proposition 5) and $q(\pi^*, \varphi) > q(\pi^*, \psi)$, it is apparent that $r_k^* < r_k^\dagger$ holds (Proposition 5, (ii)). Therefore, our final theoretical finding can be stated as presented below.

Proposition 6. *The social discount rate in a democratically planned economy should be set to* $r^* = \rho + \sigma\gamma^* - \Delta$, *whereas that under the first-best scenario should be* $r^\dagger = \rho + \sigma\gamma^\dagger$. *The social discount rate in a democratically planned economy is set to the level below the social discount rate under the first-best scenario.*

Proposition 6 presents an important policy implication for benefit–cost analysis. As described in Section 2, SRTP has been used widely for benefit–cost analysis in developed countries. The main approach of SRTP adopts the conventional Ramsey formula (14) for calculating the social discount rate (e.g., France, European Union, United Kingdom). However, with future bias, this conventional form of Ramsey formula yields a biased discount rate based on the biased equilibrium growth rate and consumption distribution. Therefore, an appropriate social discount rate should be the social discount rate based on the modified Ramsey formula (15).

5.3. Numerical examples

To complete the analyses developed in this section, we provide numerical examples by calculating the appropriate social discount rate under future bias for several countries based on reasonable parameters that are consistent with the empirical findings of earlier studies. We select Japan, the United Kingdom, and the United States because Japan has an extremely aged population, the UK has been a pioneer in adopting SRTP based on the Ramsey formula, and the United States adopts different discounting methods for different investment projects. Our model is constructed from a normative view of optimal public investment. It therefore has the difficulty of the direct estimation of the key parameters. The procedure of determining the values of the parameters is explained as presented below.

First, we should find appropriate parameters related to the degree of altruism toward ancestors and descendants. In a usual sense, parental altruism is extended toward all of their children. For that reason, λ is greater than at least number of children times μ. Because the fertility rate (births per woman) is 1.6 for OECD members and 2.4 for World, this implies $1.6\mu \le \lambda \le 2.4\mu$.[23] Assuming that $\lambda = 2\mu$ holds, one can obtain

$$\delta = \frac{1 - \sqrt{1 - 8\mu^2}}{2\mu} = 2\theta.$$

Once the value of δ is fixed, then these equations give values of μ and θ simultenously. For our purposes, the discount factor δ can be set appropriately from the time preference rate found from earlier studies or used for government reports of benefit–cost analysis because of $\delta = (1+\rho)^{-1}$.

Next, we require equilibrium values of π^* and γ^* and parameter values of σ, ψ, and φ for the modified Ramsey formula. Fortunately, π^* and γ^* can be inferred from the data. One can find the value of σ from the earlier literature or the government's benefit–cost analysis report. If the values of π^* and σ are fixed, then one obtains the value of ψ. With fixed values of δ and θ, the definition of ψ engenders the value of β if and only if one assigns the value of η. Even though it is difficult to obtain the value of η directly, we have a turnout rate as one proxy of η. Therefore, it is possible to ascertain the value of φ (i.e., $\varphi = \delta^{-1}\beta$) similarly to the value of β. To compare the social discount rates in our model and those used for benefit–cost analysis, all values of key parameters and equilibrium variables are annualized.

Japan. The time preference rate is set to the 1.5% used by Zhuang et al. (2007) for calculating the social discount rate for Japan. The average value of γ for the period 1961–2019 is calculated as 3.0% using the data of World Bank Indicator. Since it becomes 1.4% for the more recent period of 2009–2019, we use 1.5% for the baseline of γ^*. Furthermore, we set 1.2% and 1.8% respectively as values of γ^* for low and high growth scenarios. According to Havranek et al. (2015), the value of EIS in Japan is about 0.9 derived from 109 estimation samples. Based on this value, we set $\sigma = 1.1$. To determine the value of η, the turnout rate is helpful. For almost all countries, the ratio of the voter turnout rate for ages 16–35 to age over 55 is available from Society at a Glance 2016 by OECD. However, the OECD report does not include the data of Japan. Instead of referring to Society at a Glance, one can refer to a survey report of the 2021 House of Representatives general election in Japan, which has indicated that the ratio of ages 18–35 to age over 55 in Japan is 0.35. Therefore, we set $\eta = 0.3$. Finally, $\pi^* = 0.65$ is obtained using household expenditure by age of household reference person in the 2019 Family Income and Expenditure Survey.[24] Then, the values of β, θ, μ, ψ, and φ are given respectively as 0.567, 0.493, 0.332, 0.506, and 0.575. All parameters used for Japan are presented in Table 2.

The calculated social discount rates for Japan are shown in Table 3. The social discount rates based on (15) are varied in the range 0.2–0.8%, whereas the social discount rates based on (14) are 2.8–3.5%. Differences in two rates are in the range of 2.6–2.7 percentage

[23] The values for Japan, the United Kingdom, and the United States are respectively 1.3, 1.6, and 1.6. The data is from World Bank Indicators (World Bank).

[24] The ratio of total consumption expenditure of under age 65 to that of over age 65 is calculated as 0.648. The same method is applied to the UK and the US.

points. For the baseline case, the social discount rate based on the modified Ramsey formula and that based on the conventional Ramsey formula are 0.5% and 3.2%, respectively. The gap separating two rates for the baseline case is 2.7 percentage point. The social discount rate based on (15) takes a quite small value relative to the social discount rate based on (14) because the young people's consumption share is small relative to the old people's consumption share.

A highly aged economy such as that of Japan (i.e., small η) tends to have small bias as for weak political power for the young generation, which has future bias. However, at the same time, consumption distribution is deviated to the old generation in the aged society, leading to a large future bias effect with small σ. In our numerical analyses, the effect of consumption distribution on future bias dominates the political effect of aging on future bias. Since one generation has about 30 years, the one percentage point difference in the social discount rate strongly affects the benefit–cost analysis results. In Japan, using the SOCC method engenders the practical social discount rate at 4.0%. In addition, SOCC yields a social discount rate higher than SRTP. Therefore, our result suggests that a social discount rate of less than 1.3% should be used for benefit–cost analysis, even if a social discount rate of 4% is calculated appropriately using SOCC.

United Kingdom. HM Treasury (2020) reports that the time preference rate is 1.5% and that the growth rate is 2.0%, based on the 2003 Green Book. The average value of γ for the period 1949–2016 is 2.2%; it is lower at 1.7%, particularly addressing the more recent period of 1996–2016 (2020 Green Book). Hence, 1.7% is set as the baseline. Furthermore, 2.2% is used for γ^* in a high growth scenario, whereas 1.2% is used for γ^* in a low growth scenario. In the 2020 Green Book, the values of σ are 1, 1.3, or 1.5 (HM Treasury, 2003; Layard et al., 2008; Groom and Maddison, 2019).[25] We adopt the intermediate case for which $\sigma = 1.3$. The ratio of the voter turnout rate for ages 16–35 to age over 55 is 0.54 (OECD, 2016). Based on this value, we use $\eta = 0.5$. Finally, π^* is set to the value calculated using household expenditures by the age of the household reference person (Family Spending in the UK: April 2019 to March 2020, Office for National Statistics): $\pi^* = 0.79$. These determine the values of β, θ, μ, ψ, and φ, respectively, as 0.213, 0.493, 0.332, 0.179 and 0.216. Table 2 presents the values of parameters for the UK.

The social discount rates calculated using Equations (14) and (15) for the UK are reported in Table 3. Depending on the growth scenario, the modified Ramsey formula (15) yields the social discount rates of 1.8–3.1%, whereas the conventional Ramsey formula (14) gives social discount rates of 3.1–4.4%. For the baseline case, the (15)-based social discount rate and (14)-based social discount rate are 2.5% and 3.7%, respectively. In each case, the difference between two social discount rates is about 1.2 percentage points, which is smaller than that of Japan, but which cannot negligible. Since η is larger than that of Japan and π^* is smaller than that of Japan, these work to generate a small future bias effect relative to Japan. In benefit–cost analysis for the UK, the social discount rate is set as 3.5%. Therefore, incorporating the future bias, the practical social discount rate is expected to be less than the level below 2.5%.

United States. Several studies have considered the social discount rate based on SRTP. In particular, Moore et al. (2013a) set the values of parameters to $\rho = 1\%$, $\gamma^* = 1.9\%$, and $\sigma = 1.35$ for their baseline.[26] They use $\gamma^* = 1.6\%$ and $\gamma^* = 2.2\%$, respectively, to represent low-growth and high-growth scenarios. We also adopt these parameters for deriving the social discount rates. We now consider the other parameters. Since the ratio of the voter turnout rate for ages 16–35 to age over 55 in the US is 0.72 (Society at a Glance, 2016), we set $\eta = 0.7$. Calculations based on total average annual expenditures by Age in the Consumer Expenditure Survey give ratios of consumption under age 65 to that over age 65 in 2018, 2019, and 2020, respectively, as 0.75, 0.76, and 0.77. Therefore, we use $\pi^* = 0.75$ for US. Using all these values, β, θ, μ, ψ, and φ are determined respectively as 0.285, 0.495, 0.332, 0.227, and 0.288. These parameter values are presented in Table 2.

Table 3 shows the social discount rates for the US using Equations (14) and (15). Results of our analysis suggest a social discount rate of the range 0.8–1.6% based on the modified Ramsey formula (15), whereas the social discount rate based on the conventional Ramsey formula (14) is in 3.2–4.0%. The disparity between the two rates is around 2.4%, which is nearly equal to that of Japan because the young generation in US has great political power and consumption share relative to Japan. In benefit–cost analysis, the calculations provided by the Office of Management and Budget give the social discount rate at 7% by SOCC and at 4% by SRTP. Burgess and Zerbe (2011) suggest a 6–8% SOCC-social discount rate, whereas Moore et al. (2013a) suggest a 3.5% SRTP-social discount rate. Even though some controversy related to the calculation methodology persists, our results imply that the social discount rate currently used for benefit–cost analysis should be reduced to 1.2% (social discount rate based on (15) for the baseline).

6. Conclusion

This paper examined an optimal public investment policy and an appropriate discount rate for public investment under future bias using an OLG model with two-sided altruism. Future bias affects optimal public investment under democracies through the political power balance between the young and the elderly. Since the young generation has future bias, they are willing to undertake more public investment for obtaining future benefits from public investment including its redistribution effects. By contrast, the old generation has no future bias and therefore has no intention to require more public investment. The declining political power of young people that occurs along with population aging weakens the degree of future bias in the society. As a result, public investment decreases with population aging, leading to an economic growth slowdown. In per GDP terms, the public investment ratio decreases with aging population within realistic economic situation of a highly aged society. This result is consistent with empirical evidence.

[25] Havranek et al. (2015) found 0.5 as the value of EIS for UK using 251 estimation samples. The EIS of 0.5 corresponds to $\sigma = 2$.

[26] Based on Havranek et al. (2015), the EIS of US is 0.6 from 1429 estimation samples. This EIS yields $\sigma = 1.67$. Moore et al. (2013a) use $\sigma = 1.35$ as the simple average of estimated values including $\sigma = 1.65$ near the value of Havranek et al. (2015).

Table 3
Calculation results of social discount rates.

Scenario	Ramsey formula	Japan	United Kingdom	United States
Low growth	Conventional form	2.8%	3.1%	3.2%
	Modified form	0.2%	1.8%	0.8%
Baseline	Conventional form	3.2%	3.7%	3.6%
	Modified form	0.5%	2.5%	1.2%
High growth	Conventional form	3.5%	4.4%	4.0%
	Modified form	0.8%	3.1%	1.6%

Note: Conventional and modified forms of Ramsey formula respectively correspond to (14) and (15).

Furthermore, future bias influences the rates of return on private and public investment and the ratio of public to private capital. This result suggests a more severe shortage of public investment than the facts revealed by earlier studies.

Results of our study also indicate that the optimal social discount rate under future bias is calculable from the modified Ramsey formula, which is the conventional Ramsey formula with the additional term of the future bias effect. The modified Ramsey formula implies that the conventional Ramsey formula has upward bias by the presence of future bias. In recent years, many developed countries have adopted SRTP methods, especially the conventional Ramsey formula, for calculating social discount rates. Using data of household consumption surveys and parameter values used for benefit–cost analysis by the authorities, the social discount rate based on conventional Ramsey formula has upward bias of 2.6–2.7 percentage points for Japan, 1.2–1.3 percentage points for UK, and around 2.4 percentage points for US. Their differences cannot be ignored to apply it to benefit–cost analysis for medium-term and long-term public projects. In practice, the social discount rates based on the conventional Ramsey formula are nearly equal to the social discount rate for benefit–cost analysis, except for US using social discount rate by SOCC in some cases. Considering this fact, the present social discount rate used for benefit–cost analysis is set to a higher level than its appropriate rate.

The present paper contributes to the literature on optimal public investment and its policy implications by incorporating future bias observed in empirical and experimentation-based studies into the analyses. However, some room remains for extending our study by incorporating realistic settings such as the probability of death as demographic characteristics, the financial sources of public investment, and government expenditure composition. First, issues of public investment financing such as distortionary taxation and public debt must be considered. Our study, which specifically examined public investment financed by non-distortionary tax, leads an appropriate discount rate under future bias. As discussed in the literature on public investment theory and benefit–cost analysis, financial sources of public investment affect the choice of calculation methodology to obtain the appropriate discount rate. Therefore, examining the financing method of public investment is worthwhile. Furthermore, under such a framework, comprehensive numerical analysis based on parameter estimations is necessary to contribute practically to benefit–cost analysis.

Second, population aging and government expenditure composition should be examined. Felice (2016) examines growth effects of public expenditure size and composition in an endogenous growth model with modern and traditional sectors, where the government provides public capital stock for the modern sector and a flow of productive goods and services for the traditional sector. In the two-sector framework, the characteristics of growth-maximizing expenditure size and composition differ from those of the one-sector framework; the growth-maximizing government size and share of infrastructure investment to government revenue are larger than those in a one-sector growth model. Based on results obtained from our analysis, population aging reduces private and public investment because population aging weakens future bias effects and therefore decreases the needs for redistribution caused by future bias. If sectoral distribution of public service is incorporated, then the share of public investment for amenity to government revenue will be increased relative to the share of public investment for production.

Effects of population aging on structural change such as sectoral composition in matured economies have also been reported by Cravino et al. (2022). They show that population aging is a causal factor for the observed increase in the service share in consumption using the household-level data from the United States. Furthermore, they demonstrate that the increase in the service share caused by population aging contributes to the same size of increase in real income growth. These findings imply that population aging changes the sectoral shares of different goods and services. Therefore, the optimal investment policy in a two-sector framework differs from that in a one-sector framework through at least two channels such as the effects of population aging on future bias and sectoral composition change. Based on these facts, we infer that population aging affects the optimal investment expenditure and composition in a two-sector framework through mutual effects of future bias and sectoral composition change on the needs of redistribution. This paper provides an analytical basis for these avenues of future studies.

Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix

A. Data and regression results

Nexus between public investment and population aging

The scatter plot is illustrated based on the following data: Government investment ratio is the government investment ratio (general government gross fixed capital formation, % of GDP) from Investment and Capital Stock Dataset (ICSD). Furthermore, age-dependency ratio (old-age population, % of working-age population) comes from World Development Indicators. For deriving the fitted line in Fig. 1, we use the following econometric model to examine the relationship between public investment and population aging:

$$y_{it} = \alpha + \beta x_{it} + v_i + \varepsilon_{it}, \tag{A1}$$

where i and t respectively denote the country and time dimension. Also, y_{it} is the logarithm of the government investment ratio as the dependent variable, x_{it} is the logarithm of the age-dependency ratio as the main explanatory variable, and $v_i + \varepsilon_{it}$ is the error term (v_i is the unit-specific error term). It is noteworthy that α, β, y, and x are not directly relevant to those appearing in the main text.

We use panel data of 35 countries and regions for the period between 1960 and 2019. The data include over 2000 observations for the government investment ratio and age-dependency ratio. The data for several countries include missing values for several years. The regression results are presented in Table A1. Statistical tests suggest that the random effect models are statistically supported for all the models (1)–(4). The estimated results by the random effect model are shown in Table A1 (the estimated results by the fixed effect model are also reported as a reference). Shown in Table A1, the effect of age-dependency on government investment ratio is statistically significant and consistent with usual hypothesis or the existing studies. Therefore, we conclude that the government investment ratio is negatively associated with aging population. The random effect model provides the fitted line in Fig. 1.

Marginal effect of population aging on investment composition

Investment composition is approximately equal to the ratio of public to private capital. If the investments follow the forms of (A1), we can estimate the marginal effect of population aging on investment composition by estimation of the first-order difference in (A1):

$$\widetilde{y}_{it} = \gamma \widetilde{x}_{it} + \widetilde{\varepsilon}_{it}. \tag{A2}$$

Therein, $\widetilde{y}_{it}$ is the first-order difference in the logarithm of the ratio of general government capital stock to private capital stock as the dependent variable, whereas the control variable is that of age-dependency ratio. General government capital stock and private capital stock are from ICSD. Because the marginal effect of population aging on private capital differs from that on public capital (i.e., β in (A1)), γ captures the marginal effect of population aging on investment composition.

Using the first-order lags in the logarithms of the ratio of public to private capital and age-dependency ratio, the scatter plot is illustrated in Fig. 2. Taking the first-order difference, the unit-specific error terms are removed. All the variables (except for age-dependency) have no unit-root supported by the panel unit-root tests (IPS and Fish er types). However, serial correlation is suspected because the explanatory variables for private investment might be dropped here. Therefore, the fitted line is illustrated by using the estimated results based on Prais–Winsten regression because we assume that the panel-specific autocorrelations exist the disturbances exhibit panel-level heteroskedastic. The fitted line appearing in Fig. 2 corresponds to the model based on Prais–Winsten regression, where $\widetilde{y}_{it} = 0.061\widetilde{x}_{it} + \widetilde{\varepsilon}_{it}$. Table A2 reports the estimation result including that estimated by the OLS.

B. Proof of Proposition 1 and derivation of $B(\pi, \widehat{\pi}, \widehat{\gamma})$

Proof of Proposition 1: In equilibrium, the following condition holds:

$$\frac{k'}{k} = \frac{g'}{g} = \frac{c'}{c}. \tag{B1}$$

Using (B1) and $x \equiv g/k$, we obtain

$$x' = x. \tag{B2}$$

Equation (8), (B1), and (B2) yield

$$\frac{c'}{g'} = \frac{\frac{g'}{g}c}{g'} = \frac{c}{g} = \frac{F(k,g) - k' - g'}{g} = \frac{f(x) - (1+x)\gamma}{x}. \tag{B3}$$

Using equations (6a)–(8) with $\pi = \widehat{\pi} = \pi^*$ and $\gamma = \widehat{\gamma}$, we have

$$\frac{q(\pi,\psi)c^{-\sigma}}{\delta} = \frac{\partial V(w')}{\partial w'} = q(\pi,\varphi)(c')^{-\sigma}\frac{\partial c'}{\partial w'} + X(\varphi)(g')^{-\sigma}\frac{\partial g'}{\partial w'} + q(\pi,\psi)(c')^{-\sigma}(1+\gamma)$$
$$= q(\pi^*,\varphi)(c')^{-\sigma}[F_k - (1+\gamma)] + q(\pi^*,\psi)(c')^{-\sigma}(1+\gamma).$$

By (8), (B1), and (B2), this equation can be transformed to

$$\left(\frac{c'}{c}\right)^{\sigma} = (1+\gamma)^{\sigma} = \left\{\frac{q(\pi^*,\varphi)}{q(\pi^*,\psi)}[f(x) - xf'(x)] + \left[1 - \frac{q(\pi^*,\varphi)}{q(\pi^*,\psi)}\right](1+\gamma)\right\}\delta. \tag{B4}$$

Using (B1), (B2), (B3), and (B4), Eq. (6b) becomes

$$f'(x) + \frac{X(\varphi)}{q(\pi^*,\varphi)}\left[\frac{f(x) - (1+x)(1+\gamma)}{x}\right]^{\sigma} = f(x) - xf'(x). \tag{B5}$$

Equations (B4) and (B5) provide the following nonlinear simultaneous equations system of equations with respect to x and γ as

$$0 = \left\{\frac{q(\pi^*,\varphi)}{q(\pi^*,\psi)}[f(x) - xf'(x)] + \left[1 - \frac{q(\pi^*,\varphi)}{q(\pi^*,\psi)}\right](1+\gamma) - \frac{(1+\gamma)^{\sigma}}{\delta}\right\}q(\pi^*,\psi) \equiv P_1(x,\gamma;\psi,\varphi), \tag{B6}$$

$$0 = X(\varphi)[f(x) - (1+x)(1+\gamma)]^{\sigma} - [f(x) - (1+x)f'(x)]q(\pi^*,\varphi)x^{\sigma} \equiv P_2(x,\gamma;\psi,\varphi). \tag{B7}$$

Consider the properties of (B6) and (B7). First, total differentiation of P_1 gives

$$\frac{d(1+\gamma)}{dx} = -\frac{\frac{\partial P_1}{\partial x}}{\frac{\partial P_1}{\partial(1+\gamma)}} > 0,$$

where

$$\frac{\partial P_1}{\partial x} = -xf''(x)q(\pi^*,\varphi) > 0,$$

$$\frac{\partial P_1}{\partial(1+\gamma)} = -[q(\pi^*,\varphi) - q(\pi^*,\psi)] - \sigma q(\pi^*,\psi)\frac{(1+\gamma)^{\sigma-1}}{\delta} < 0.$$

These properties of P_1 show that P_1 is an upward-sloping curve in the x–γ plane (C–C curve in Fig. 3). Next, we characterize P_2. Its total differentiation yields

$$\frac{d(1+\gamma)}{dx} = -\frac{\frac{\partial P_2}{\partial x}}{\frac{\partial P_2}{\partial(1+\gamma)}} < 0,$$

where

$$\frac{\partial P_2}{\partial x} = \sigma X(\varphi)[F_g - (1+\gamma)][f(x) - (1+x)(1+\gamma)]^{\sigma-1} + [(1+x)xf''(x) - (F_k - F_g)\sigma]q(\pi^*,\varphi)x^{\sigma-1}$$
$$= \left\{(1+x)f''(x) - \frac{[F_k - (1+\gamma)](F_k - F_g)\sigma}{[f(x) - (1+x)(1+\gamma)]x}\right\}q(\pi^*,\varphi)x^{\sigma} < 0,$$

$$\frac{\partial P_2}{\partial(1+\gamma)} = -\sigma X(\varphi)[f(x) - (1+x)(1+\gamma)]^{\sigma-1}(1+x)$$
$$= -\sigma\left[\frac{(F_k - F_g)(1+x)}{f(x) - (1+x)(1+\gamma)}\right]q(\pi^*,\varphi)x^{\sigma} < 0.$$

From (B8) with Assumption 1, there exists a lower limit of x such that $f(\overline{x}) - (1+\overline{x})f'(\overline{x}) = 0$, which is the equilibrium value of x when $\chi = 0$ (i.e., $X = 0$). Similarly, an upper limit of x exists such that $\gamma = 0$ or equivalently $P_2(x,0;\psi,\varphi) = 0$ because γ is monotonically decreasing in x on the P_2 curve. This relation implies that $x \in [\overline{x}, x]$ for $\gamma \geq 0$. Therefore, we obtain $P_2(\overline{x},\overline{\gamma};\psi,\varphi) = 0 \Leftrightarrow 1+\overline{\gamma} = f'(\overline{x})$ and $P_2(x,0;\psi,\varphi) = 0 \Leftrightarrow \gamma = 0$. Furthermore, $\delta[f'(\overline{x})]^{1-\sigma} < 1$ must be satisfied for the bounded value function. These characteristics of P_2 show that P_2 is a downward-sloping curve in the x–γ plane (P–P curve in Fig. 3).

Because the domain of P_1 is $x \in \mathbb{R}^+$, we must check the value of γ on P_1 at $x = \overline{x}$. Inserting $x = \overline{x}$ into (B4), it requires that

$$1+\gamma = \Gamma(\gamma;\overline{x},\varphi,\psi), \tag{B8}$$

where

$$\Gamma(\gamma;\bar{x},\varphi,\psi)=\left\{\frac{q(\pi^*,\varphi)}{q(\pi^*,\psi)}f'(\bar{x})+\left[1-\frac{q(\pi^*,\varphi)}{q(\pi^*,\psi)}\right](1+\gamma)\right\}\delta(1+\gamma)^{1-\sigma}.$$

If $1+\gamma=1+\bar{\gamma}=f'(\bar{x})$, then it becomes $1=\delta[f'(\bar{x})]^{1-\sigma}$, leading to a contradiction. Because the right hand-side of (B8), Γ, is decreasing in γ, it follows that

$$f'(\bar{x})>\Gamma(f'(\bar{x});\bar{x},\varphi,\psi)>1+\tilde{\gamma},$$

where $\tilde{\gamma}$ is the fixed point of (B8) such that $\tilde{\gamma}=\Gamma(\tilde{\gamma};\bar{x},\varphi,\psi)-1$. Therefore, P_1 (C–C curve) is located below P_2 (P–P curve) in the x–γ plane at $x=\bar{x}$. Because P_1 and P_2 curves respectively have a positive and a negative gradient P_1 and P_2 curves have a single crossing point. Therefore, the system of P_1 and P_2 has a unique solution satisfying (B6) and (B7).

Derivation of $B(\pi,\widehat{\pi},\widehat{\gamma})$: Equation (B5) gives $x'=H(\widehat{\pi},\widehat{\gamma})$. Using the partial derivatives of P_2, one can verify that $\partial H(\widehat{\pi},\widehat{\gamma})/\partial(1+\gamma)<0$. Solving the recursion in the value function $V(w)$, it follows that

$$V(w')=\frac{v(c',g',\widehat{\pi},\varphi)}{1-\delta(1+\widehat{\gamma})^{1-\sigma}}. \tag{B9}$$

Differentiation of (B9) with respect to w' and inserting (6b) and (7b) (with one period delay) into it leads to

$$\frac{\partial Vw'}{\partial w'}=\frac{q(\widehat{\pi},\varphi)(c')^{-\sigma}\frac{\partial c'}{\partial w'}+X(\varphi)(g')^{-\sigma}\frac{\partial g'}{\partial w'}}{1-\delta(1+\widehat{\gamma})^{1-\sigma}}=\frac{q(\widehat{\pi},\varphi)(c')^{-\sigma}[f(x')-f'(x')x'-(1+\widehat{\gamma})]}{1-\delta(1+\widehat{\gamma})^{1-\sigma}}. \tag{B10}$$

Using (6a) and (B10), we obtain

$$\left[\frac{A(x')-(1+\widehat{\gamma})}{A(x)-(1+\gamma)}\right]^{\sigma}(1+\gamma)^{\sigma}=\delta\frac{q(\widehat{\pi},\varphi)}{q(\pi,\psi)}\frac{[f(x')-f'(x')x'-(1+\widehat{\gamma})]}{1-\delta(1+\widehat{\gamma})^{1-\sigma}}, \tag{B11}$$

where $A(x)=f(x)/(1+x)$. Because $x'=H(\widehat{\pi},\widehat{\gamma})$, equation (B11) gives $\gamma=B(\pi,\widehat{\pi},\widehat{\gamma})$.

C. Proof of Proposition 3

Inserting π^* into $q(\pi,\psi)$ and $q(\pi,\varphi)$ provides

$$q(\pi^*,\psi)=\left(\frac{1}{1+\psi^{\frac{1}{\sigma}}}\right)^{1-\sigma}+\psi\left(\frac{\psi^{\frac{1}{\sigma}}}{1+\psi^{\frac{1}{\sigma}}}\right)^{1-\sigma}=\left(1+\psi^{\frac{1}{\sigma}}\right)^{\sigma},$$

$$q(\pi^*,\varphi)=\left(\frac{1}{1+\psi^{\frac{1}{\sigma}}}\right)^{1-\sigma}+\varphi\left(\frac{\psi^{\frac{1}{\sigma}}}{1+\psi^{\frac{1}{\sigma}}}\right)^{1-\sigma}=\frac{1+\varphi\psi^{\frac{1-\sigma}{\sigma}}}{\left(1+\psi^{\frac{1}{\sigma}}\right)^{1-\sigma}}.$$

Using these two equations and Assumption 2, we have

$$\frac{\psi}{d(\pi^*,\psi)}\frac{dq(\pi^*,\psi)}{\partial\psi}=\frac{\psi^{\frac{1}{\sigma}}}{1+\psi^{\frac{1}{\sigma}}}=1-\pi^*>0,$$

$$\frac{\psi}{q(\pi^*,\varphi)}\frac{dq(\pi^*,\varphi)}{d\psi}=-\left(\frac{\sigma-1}{\sigma}\right)\left[\frac{\varphi\psi^{\frac{1-\sigma}{\sigma}}}{1+\varphi\psi^{\frac{1-\sigma}{\sigma}}}-\frac{\psi^{\frac{1}{\sigma}}}{1+\psi^{\frac{1}{\sigma}}}\right]<0,$$

$$\frac{d\log\left[\frac{q(\pi^*,\varphi)}{q(\pi^*,\psi)}\right]}{d\log\psi}=-\left(\frac{\sigma-1}{\sigma}\right)\frac{\varphi\psi^{\frac{1-\sigma}{\sigma}}}{1+\varphi\psi^{\frac{1-\sigma}{\sigma}}}-\sigma^{-1}\frac{\psi^{\frac{1}{\sigma}}}{1+\psi^{\frac{1}{\sigma}}}<0.$$

Total differentiation of the system of P_1 and P_2 gives

$$\begin{pmatrix}\frac{\partial P_1}{\partial x} & \frac{\partial P_1}{\partial(1+\gamma)}\\ \frac{\partial P_2}{\partial x} & \frac{\partial P_2}{\partial(1+\gamma)}\end{pmatrix}\begin{pmatrix}dx^*\\ d(1+\gamma^*)\end{pmatrix}=-\begin{pmatrix}\frac{\partial P_1}{\partial\psi}\\ \frac{\partial P_2}{\partial\psi}\end{pmatrix}d\psi, \tag{C1}$$

where

$$\frac{\partial P_1}{\partial\psi}=[f(x)-xf'(x)-(1+\gamma)]q(\pi^*,\psi)\frac{d}{d\psi}\left[\frac{q(\pi^*,\varphi)}{q(\pi^*,\psi)}\right]<0,$$

$$\frac{\partial P_2}{\partial \psi} = -[f(x) - (1+x)f'(x)]x^{\sigma}\frac{dq(\pi^*, \varphi)}{d\psi} > 0,$$

$$|P| = \overbrace{\frac{\partial P_1}{\partial x}}^{(+)}\overbrace{\frac{\partial P_2}{\partial(1+\gamma)}}^{(-)} - \overbrace{\frac{\partial P_2}{\partial x}}^{(-)}\overbrace{\frac{\partial P_1}{\partial(1+\gamma)}}^{(-)} < 0.$$

Applying Cramer's rule to (C1) leads to

$$\frac{\partial(1+\gamma^*)}{\partial \psi} = -\frac{\begin{vmatrix} \frac{\partial P_1}{\partial x} & \frac{\partial P_1}{\partial \psi} \\ \frac{\partial P_2}{\partial x} & \frac{\partial P_2}{\partial \psi} \end{vmatrix}}{|P|} = -\frac{\overbrace{\frac{\partial P_1}{\partial x}}^{(+)}\overbrace{\frac{\partial P_2}{\partial \psi}}^{(-)}\overbrace{\frac{\partial P_2}{\partial x}}^{(-)}\overbrace{\frac{\partial P_1}{\partial \psi}}^{(-)}}{|P|}, \tag{c2}$$

$$\frac{\partial x^*}{\partial \psi} = -\frac{\begin{vmatrix} \frac{\partial P_1}{\partial \psi} & \frac{\partial P_1}{\partial(1+\gamma)} \\ \frac{\partial P_2}{\partial \psi} & \frac{\partial P_2}{\partial(1+\gamma)} \end{vmatrix}}{|P|} = -\frac{\overbrace{\frac{\partial P_1}{\partial \psi}}^{(-)}\overbrace{\frac{\partial P_2}{\partial(1+\gamma)}}^{(-)} - \overbrace{\frac{\partial P_2}{\partial \psi}}^{(+)}\overbrace{\frac{\partial P_1}{\partial(1+\gamma)}}^{(-)}}{|P|} > 0. \tag{c3}$$

The sign of (C2) is ambiguous. To verify this ambiguity, one must calculate the numerator of the rightmost part of (C2) as

$$\frac{\partial P_1}{\partial x}\frac{\partial P_2}{\partial \psi} - \frac{\partial P_2}{\partial x}\frac{\partial P_1}{\partial \psi} = \frac{q(\pi^*, \psi)[F_k - (1+\gamma)]^2 (F_k - F_g)\sigma}{[f(x) - (1+x)(1+\gamma)]x}\frac{\partial}{\partial \psi}\left[\frac{q(\pi^*, \varphi)}{q(\pi^*, \psi)}\right]$$
$$+ \left\{[f(x) - (1+x)(1+\gamma)]\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\varphi\psi^{\frac{1-\sigma}{\sigma}}}{1+\varphi\psi^{\frac{1-\sigma}{\sigma}}} - \frac{\psi^{\frac{1}{\sigma}}}{1+\psi^{\frac{1}{\sigma}}}\right) + \frac{(1+x)[F_k - (1+\gamma)]\psi^{\frac{1}{\sigma}}}{1+\psi^{\frac{1}{\sigma}}}\right\} \times \frac{q(\pi^*, \varphi)f''(x)x^{\sigma}}{\psi} < 0.$$

Therefore, the sign of (C2) is negative:

$$\mathrm{sgn}\left(\frac{\partial(1+\gamma^*)}{\partial \psi}\right) = \mathrm{sgn}\left[\frac{\partial P_1}{\partial x}\frac{\partial P_2}{\partial \psi} - \frac{\partial P_2}{\partial x}\frac{\partial P_1}{\partial \psi}\right] \Rightarrow \frac{\partial \gamma^*}{\partial \psi} < 0.$$

Using Equations (C2) and (C3), we obtain

$$\frac{\partial}{\partial \psi}\left(\frac{i^k}{y}\right) = \frac{\partial}{\partial \psi}\left(\frac{\gamma^*}{f(x^*)}\right) = \frac{1}{[f(x^*)]^2}\left[f(x^*)\frac{\partial \gamma^*}{\partial \psi} - \gamma^* f'(x^*)\frac{\partial x^*}{\partial \psi}\right] < 0.$$

$$\frac{\partial}{\partial \psi}\left(\frac{i^g}{y}\right) = \frac{\partial}{\partial \psi}\left(\frac{\gamma^* x^*}{f(x^*)}\right) = \frac{1}{[f(x^*)]^2}\left[f(x^*)\left(\frac{\partial \gamma^*}{\partial \psi}x^* + \gamma^*\frac{\partial x^*}{\partial \psi}\right) - \gamma^* x^* f'(x^*)\frac{\partial x^*}{\partial \psi}\right]. \tag{C4}$$

The sign of Equation (A15) is given as

$$\mathrm{sgn}\frac{\partial}{\partial \psi}\left(\frac{i^g}{y}\right) = \mathrm{sgn}\left\{\frac{\psi}{\gamma^*}\frac{\partial \gamma^*}{\partial \psi} + \left[1 - \frac{x^* f'(x^*)}{f(x^*)}\right]\frac{\psi}{x^*}\frac{\partial x^*}{\partial \psi}\right\}.$$

The calculation shows that

$$\frac{\psi}{\gamma^*}\frac{\partial \gamma^*}{\partial \psi}\bigg|_{\psi=\varphi} + \varepsilon_k \frac{\psi}{x^*}\frac{\partial x^*}{\partial \psi}\bigg|_{\psi=\varphi} = \underbrace{\frac{\varphi\frac{\partial P_1}{\partial \psi}\big|_{\psi=\varphi}}{|P|}}_{(+)}\left\{\frac{1}{\gamma^*}\frac{\partial P_2}{\partial x}\bigg|_{\psi=\varphi} - \varepsilon_k \frac{1}{x^*}\frac{\partial P_2}{\partial(1+\gamma)}\bigg|_{\psi=\varphi}\right\},$$

where

$$\varepsilon_k \equiv \frac{F_k k}{F} = \frac{xf'(x)}{f(x)} \in (0,1).$$

Regarding the term in curly brackets on the RHS of the equation above, we have

$$\mathrm{sgn}\left\{\frac{1}{\gamma^*}\frac{\partial P_2}{\partial x}\bigg|_{\psi=\varphi} - \left[1 - \frac{x^* f'(x^*)}{f(x^*)}\right]\frac{1}{x^*}\frac{\partial P_2}{\partial(1+\gamma)}\bigg|_{\psi=\varphi}\right\}$$

$$= \mathrm{sgn}\left\{\frac{(1+x)f''}{\sigma\gamma} + \left[\left(1-\frac{xf'}{f}\right)\left(\frac{1+x}{x}\right) - \frac{F_k-(1+\gamma)}{\gamma x}\right]\frac{F_k-F_g}{f-(1+x)(1+\gamma)}\right\}$$

$$= \mathrm{sgn}\left\{\frac{(1+x)f''}{\sigma\gamma} + \frac{1}{(1+\gamma)\gamma x}\left[1+\varepsilon_k\left(\frac{1+x}{1+\gamma}\right)\gamma - \frac{1}{\delta(1+\gamma)^{1-\sigma}}\right]\frac{F_k-F_g}{f(x)-(1+x)(1+\gamma)}\right\}.$$

Hence, if $\delta(1+\gamma^{\dagger})^{1-\sigma} \ll 1$, then Equation (C4) becomes

$$\frac{\partial}{\partial\psi}\left(\frac{i^g}{y}\right) < 0.$$

D. Derivation of (14) and (15)

Equation (12b) can be written as $\log F_k^{\dagger} = \sigma\log(1+\gamma^{\dagger}) - \log\delta$ by taking the logarithm of both sides. Approximately, it follows that

$$r_k^{\dagger} \approx \sigma\gamma^{\dagger} + \rho.$$

Using (10b), we have

$$\delta^{-1}v_c(c,g,\pi^*,\psi) = v_c(c',g',\pi^*,\varphi)\left[F_k^* - (1+\gamma^*)\right] + (1+\gamma^*)v_c(c',g',\pi^*,\psi).$$

Rearranging this equation, it follows that

$$\delta^{-1}\frac{v_c(c,g,\pi^*,\psi)}{v_c(c',g',\pi^*,\psi)} = \frac{v_c(c',g',\pi^*,\varphi)}{v_c(c',g',\pi^*,\psi)}\left[F_k^* - (1+\gamma^*)\right] + 1 + \gamma^* \Leftrightarrow \delta^{-1}\gamma^{\sigma}$$
$$+\left[\frac{v_c(c',g',\pi^*,\varphi)}{v_c(c',g',\pi^*,\psi)} - 1\right](1+\gamma^*) = \frac{v_c(c',g',\pi^*,\varphi)}{v_c(c',g',\pi^*,\psi)}F_k^* \Leftrightarrow \left\{1+\left[\frac{v_c(c',g',\pi^*,\varphi)}{v_c(c',g',\pi^*,\psi)} - 1\right]\delta(1+\gamma^*)^{1-\sigma}\right\}\delta^{-1}$$
$$(1+\gamma^*)^{\sigma} = \frac{v_c(c',g',\pi^*,\varphi)}{v_c(c',g',\pi^*,\psi)}F_k^*.$$

Taking the logarithm of both sides yields the following:

$$\sigma\log(1+\gamma^*) - \log\delta + \left[\frac{v_c(c',g',\pi^*,\varphi)}{v_c(c',g',\pi^*,\psi)} - 1\right]\delta(1+\gamma^*)^{1-\sigma} - \log\frac{v_c(c',g',\pi^*,\varphi)}{v_c(c',g',\pi^*,\psi)} = \log F_k^*.$$

Approximately, this becomes

$$r_k^* \approx \sigma\gamma^* + \rho - \Delta.$$

Table A1
Estimation results of (A1)

	Random effect model	Fixed effect model
Age-dependency ratio	−0.251** (−2.12)	−0.245* (−2.00)
Constant	−3.665*** (−20.02)	−3.645*** (−17.73)
Observations	1916	1916

Note: The dependent variable is the logarithm of the government investment ratio. Values in parenthesis are *z*-statistics and *t*-statistics, respectively, for the random and fixed effect models. The random effect model is supported as results of the Breusch and Pagan Lagrangian multiplier and Hausman tests. ***, **, and * respectively denote significance at the 1%, 5%, and 10% levels.

Table A2
Estimation results of (A2).

	Prais–Winsten regression	OLS
Age-dependency ratio	0.061** (1.98)	0.003 (0.13)
R-squared	0.0022	0.0000
Observations	2063	2063

Note: The dependent variable is the first-order lag of logarithm of the ratio of public to private capital. The explanatory variable is the first-order lag of age-dependency ratio. Values in parentheses respectively represent z-statistics and t-statistics for Prais–Winsten regression and OLS. ** denotes significance at the 5% level.

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