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Asymmetric technologies in contests

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ABSTRACT

I consider a contest between two risk-neutral players over a common-value prize, in which one player has a linear cost-of-effort function and the other a strictly convex cost-of-effort function *f*. I show that if the value of the prize is above (below) a certain threshold level, then the equilibrium aggregate effort in this contest is larger (smaller) than in a contest in which both players are characterized by the strictly convex cost-of-effort function *f*, and smaller (larger) than in a contest in which both players are characterized by a linear cost-of-effort function. Therefore, in contrast to the general result in the literature, asymmetry in contests can increase competition.

1. Introduction

Contest models are widely used to analyze sports tournaments, political competition, R&D races, labor promotion tournaments, conflicts, and rent seeking (see Congleton et al., 2008; Konrad, 2009; Congleton and Hillman, 2015; Hillman and Long, 2019; Fu and Wu, 2019).

In general, models in the contest literature support the conventional wisdom that a symmetric match between two players is more competitive than an asymmetric one, implying that a contest organizer who wishes to maximize effort should level the playing field (see, for example, Franke, 2012; Corchón and Serena, 2018).[1] Furthermore, this implies that asymmetry is beneficial in rent-seeking contests in which effort is wasteful. Nevertheless, there are exceptions. In a quite general setting, Drugov and Ryvkin (2017) derive conditions under which introducing bias into a contest between two symmetric players increases aggregate effort and also achieves other objectives, such as maximizing the effort exerted by a single player.[2] Fu and Wu (2020) show that in the case of more than two players the aforementioned result often does not hold.

Loosely speaking, the asymmetry between players in these models is generated either by adding a constant to the effort of some of the players or by multiplying their effort by a constant. Other models have employed contests between players with different prize valuations or different linear cost functions (see Hillman and Riley, 1989; Nti, 1999; Stein, 2002; Ryvkin, 2007). In contrast, I consider a Tullock contest (Tullock, 1980) with a common-value prize between two risk-neutral players, where one of them faces a linear cost-of-effort function and the other a strictly convex cost-of-effort function (herein: a "linear-convex contest"). Such asymmetry is suggested by various real-world situations.

For instance, suppose that effort is in the form of money invested in the campaign of a political candidate. For a rich candidate, the cost of effort can be proxied by the rate of interest or the rate of return on some alternative economic activity, in which case the marginal cost of effort is constant. On the other hand, a poor candidate may need to inject her own funds at the expense of her consumption, which implies that her marginal cost of effort is increasing. Alternatively, the poor candidate may choose to invest time rather than money, where the

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[1] See Mealem and Nitzan (2016) for a comprehensive review of discrimination in contests.

[2] See Serena (2017) on these other objectives.

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alternative cost of time is usually considered to be increasing (for a detailed discussion, see Esteban and Ray, 2001).3 A similar situation would arise in the case of a costly legal battle over property rights, where one side is rich and the other poor.

A further example is conflict of interest between the public and an energy company that is trying to obtain a tax exemption on profits from natural gas drilling, or an exemption from Pigouvian taxes that it must pay for the pollution it produces. For this company, effort is usually in the form of the cost of lobbying and therefore the marginal cost of effort is constant, as already noted. By contrast, the public might spend time protesting against the company's activity and therefore for them the marginal cost of effort is increasing.[4]

Another example is a head of government who treats the government budget as his own personal wealth and uses it to maintain his power or to fund his own private investments. The marginal cost-of-effort to maintain power is constant, while the public's effort to depose him is in the form of time-consuming protest activity.

I show that if the value of the prize divided by the cost coefficient is above (below) a certain threshold level, then the equilibrium aggregate effort in a linear-convex contest is larger (smaller) than that in a contest in which both players are characterized by the same strictly convex cost-of-effort function as that of the "convex player" in the linear-convex contest (herein: the "convex-convex contest"), and smaller (larger) than that in a contest in which both players are characterized by a linear cost-of-effort function (herein: the "linear-linear contest). I therefore contribute to the contest literature by demonstrating a new result in which asymmetry can increase effort.[5]

The paper proceeds as follows. Section 2 sets up the model. Section 3 presents the results. Section 4 concludes.

2. The model

There are two risk-neutral players: player 1 and player 2, who compete for a prize commonly valued at $v>0$, which is allocated to the winner. Each player $i=1,2$ simultaneously exerts effort $x_i \geq 0$ at a cost of $cx_i^{a_i}$, where $a_i \geq 1$ and $c>0$. The probability that player $i \neq j$ wins the contest is:

$$p_i = \begin{Bmatrix} \frac{x_i}{x_i + x_j}, x_i + x_j > 0 \\ \frac{1}{2}, otherwise \end{Bmatrix} \quad (1)$$

Note that p_i is usually referred to as the Tullock "lottery" Contest Success Function (CSF), since a player's probability of winning is equal to her share of the aggregate effort. The Tullock functional form is one of the most commonly used classes of CSFs in contest theory and has been axiomatized by Skaperdas (1996). Below I will show that the model is similar to a different form of the Tullock contest.

Therefore, given x_j, each player $i=1,2$ $i \neq j$ chooses x_i to solve:

$$\max_{x_i} E\pi_i \quad (2)$$

where

$$E\pi_i = \frac{x_i}{x_i + x_j} v - cx_i^{a_i}$$

In what follows, let $X=x_1+x_2$ and $v/c=v'$.

3. Results

In this section, I first rewrite the (well-known) generic closed-form solution for the class of symmetric contests in which $a_2=a_1=a\geq 1$, and then prove that in the linear-convex contest in which $a_2=\hat{a}>a_1=1$, there exists a unique Nash equilibrium. I then show that if v' is above a certain threshold level (which depends on $\hat{a}$), the equilibrium aggregate effort in the linear-linear contest, in which $a_2=a_1=1$, is greater than the corresponding effort in the linear-convex contest, in which $a_2=\hat{a}>a_1=1$, which in turn is greater than the corresponding effort in the convex-convex contest in which $a=\hat{a}$, and vice versa for v' below that threshold level.

3.1. Symmetric contests

The following lemma presents the solution for the class of symmetric contests in which $a_2=a_1\geq 1$.

[3] It could be that the rate of interest or the rate of return on some alternative economic activity is not constant. Then the marginal cost of effort of the rich candidate can be also increasing.

[4] The public is expected to confront free-rider issues. On inhibitions to contributions to effort in groups when rents are public-goods benefit, see Ursprung (1990).

[5] In an important study, Yildirim (2005) examined a contest between two players with different effectiveness of effort. He analyzes a dynamic two-stage contest in which players can add effort, and, among other results, shows that, when players are asymmetric, there are multiple subgame perfect equilibria including the equilibrium effort of the one-shot contest. I compare that effort to the equilibrium effort in the one-shot symmetric contest (although I consider a slightly different CSF and cost function).

Lemma 1. *The individual and aggregate equilibrium efforts are:* $\left(\frac{v'}{4a}\right)^{\frac{1}{a}}$ *and* $2\left(\frac{v'}{4a}\right)^{\frac{1}{a}}$ *respectively, when* $a_2=a_1=a\geq 1$ *(Proposition 3, Pérez-Castrillo and Verdier, 1992).*[6]

3.2. The linear-convex contest

In the linear-convex contest:

Lemma 2. There exists a unique Nash equilibrium in pure strategies when $a_2=\widehat{a}>a_1=1$.

All proofs appear in appendix. I can now compare the equilibrium aggregate effort in the linear-convex contest, in which $a_2=\widehat{a}>a_1=1$, and the equilibrium aggregate effort in the two corresponding symmetric contests, namely the contest in which $a_2=a_1=\widehat{a}$ and the contest in which $a_2=a_1=1$. Recall that $2\left(\frac{v'}{4a}\right)^{\frac{1}{a}}$ is the equilibrium aggregate effort level in the contest in which $a_2=a_1=a$, and note that at $a=1$, $2\left(\frac{v'}{4a}\right)^{\frac{1}{a}}=\frac{v'}{2}$. Proposition 1 presents my main result:

Proposition 1. Let X* be the aggregate equilibrium effort when $a_2=\widehat{a}>a_1=1$. Then:

a. $\frac{v'}{2}>X^*>2\left(\frac{v'}{4\widehat{a}}\right)^{\frac{1}{\widehat{a}}}$ when $v'>\frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}$.

b. $2\left(\frac{v'}{4\widehat{a}}\right)^{\frac{1}{\widehat{a}}}>X^*>\frac{v'}{2}$ when $v'<\frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}$.

c. $2\left(\frac{v'}{4\widehat{a}}\right)^{\frac{1}{\widehat{a}}}=X^*=\frac{v'}{2}$ when $v'=\frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}$.

Note that the condition in Proposition 1 can be rewritten as $\widehat{a}\left(\frac{v'}{4}\right)^{\widehat{a}-1}\gtrless 1$. Recall that $v'/4$ is the player's equilibrium effort in a contest in which $a_2=a_1=1$. Proposition 1 therefore implies that if at $v'/4$ the marginal cost of effort of the "strictly convex player" is smaller (larger) than that of her "linear" rival, then the equilibrium aggregate effort will be smaller than it would be in the contest, in which both players are "strictly convex" ("linear").

3.3. The model's relationship to other forms of the Tullock contest

Note that, since $y_i(x_i)=x_i^{a_i}$ is strictly increasing in x_i, (2) can be written as: $E\pi_i=\frac{y_i^{1/a_i}}{y_j^{1/a_j}+y_i^{1/a_i}}v-cy_i$, where player i's choice variable is y_i instead of x_i.[7] This is a Tullock contest in which players have the same linear cost-of-effort function but may have different "effectiveness of effort", which in the context of the Tullock contest is usually referred to as different "r"s (specifically, $r_i=1/a_i$). Therefore, given Lemma 1, in a Tullock contest in which $r_2=r_1=r$, the effectiveness of the equilibrium aggregate effort is $y_j^{r_j}+y_i^{r_i}=2\left(\frac{rv'}{4}\right)^r$. Proposition 1 therefore implies the following regarding the effectiveness of the equilibrium aggregate effort:

Corollary 1. Consider the Tullock contest, in which $E\pi_i=\frac{y_i^{r_i}}{y_j^{r_j}+y_i^{r_i}}v-cy_i$, where $r_i\in\{\widehat{r},1\},\widehat{r}<1$ for i = 1,2 i≠j. In the unique Nash equilibrium:

a. $\frac{v'}{2}>y_j^{\widehat{r}}+y_i>2\left(\frac{\widehat{r}v'}{4}\right)^{\widehat{r}}$ when $v'>\frac{4}{\widehat{r}^{\left(\frac{\widehat{r}}{\widehat{r}-1}\right)}}$.

b. $2\left(\frac{\widehat{r}v'}{4}\right)^{\widehat{r}}>y_j^{\widehat{r}}+y_i>\frac{v'}{2}$ when $v'<\frac{4}{\widehat{r}^{\left(\frac{\widehat{r}}{\widehat{r}-1}\right)}}$.

c. $2\left(\frac{\widehat{r}v'}{4}\right)^{\widehat{r}}=y_j^{\widehat{r}}+y_i=\frac{v'}{2}$ when $v'=\frac{4}{\widehat{r}^{\left(\frac{\widehat{r}}{\widehat{r}-1}\right)}}$.

Corollary 1 therefore implies that in a Tullock contest, in which both players have a linear cost-of-effort function and a weakly concave "effectiveness-of-effort" function (i.e., $y_i^{r_i}$, $r_i\leq 1$), asymmetry in the r's can increase the effectiveness of equilibrium aggregate effort.[8] Hence, a contest designer who wishes to maximize this parameter may prefer an asymmetric contest.

3.4. Comparison

My results compare with Dari-Mattiacci et al. (2015), who consider a two-player Tullock contest in which players have the same cost-of-effort function, different effectiveness-of-effort and different prize (or rent) evaluation, as well as being allowed to trade rents.

[6] This can also be obtained by solving (4) (which is the F.O.C of player i in the appendix).

[7] This has been pointed out by Cornes and Hartley (2005), among many others.

[8] Note that when the "effectiveness-of-effort" function is sufficiently convex a Nash equilibrium in pure strategies may not exist (see Baye et al., 1994; Ewerhart, 2015).

They discuss some interesting properties of the equilibria and, more specifically, they derive a condition under which aggregate effort in equilibrium is equal to the highest valuation of the prize, which by contrast to my model requires that $r > 1$ for both players. The results in Corollary 1 above regarding the ranking of the effectiveness of the equilibrium aggregate effort in contests with different r's do not appear in Dari-Mattiacci et al. (2015) nor do the results in Proposition 1 regarding the ranking of the equilibrium aggregate effort in contests with different cost-of-effort functions. The model presented here can therefore be viewed as complementing Dari-Mattiacci et al. (2015) since it explicitly shows how the effectiveness of the equilibrium aggregate effort depends on the values of r, and how equilibrium aggregate effort depends on the cost-of-effort functions.

3.5. Understanding aggregate effort

Understanding aggregate effort in various circumstances as in the examples presented is particularly important since it is the measure of social loss in rent-seeking contests. Tullock (1989) asked why more rent-seeking effort is not observed. One possibility he suggests is that rents and the social costs of rent-seeking are hidden.[9] His argument implies that collecting data on rent-seeking effort in order to conduct empirical studies is difficult, and therefore, theoretical models such as the one presented here are particularly useful.

3.6. Implications of the results to different types of contests

In rent-seeking contests, effort is wasteful. Specifically, in these contests, rent dissipation, which is the ratio between the resources expended in contesting the rent and the value of the prize sought, is the measure of social loss. Therefore, my results suggest that "the right type of symmetric technologies" (i.e., strictly convex cost-of-effort function when the prize is large and linear cost-of-effort function when the prize is small) is beneficial in rent-seeking contests, since it reduces the extent of rent dissipation. In armed conflict, effort at harming others is reduced. In other types of contests, such as sports tournaments, the organizer wants to maximize productive effort, and therefore my results suggest in this context that leveling the playing field may not be beneficial for the objective sought.

4. Conclusions

In general, in the theory of contests, asymmetries among contenders reduce aggregate effort. The intuition is that contenders disadvantaged by asymmetries have reduced incentives to expend effort, which reduces the need for contenders who asymmetries advantage to expend effort. I have shown that technological asymmetries through different cost-of-effort functions can as in the usual asymmetric cases decrease aggregate effort but can also increase aggregate effort.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix

Proof of Lemma 2: The first-order conditions for an interior solution (F.O.Cs) for players 1 and 2 respectively are:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{x_2}{X^2} v' - 1 = 0 \tag{3}$$

and

$$\frac{\partial \pi_2}{\partial x_2} = \frac{x_1}{X^2} v' - \widehat{a} x_2^{\widehat{a}-1} = 0 \tag{4}$$

By (3):

[9] For a thorough discussion of this issue, including other possible answers suggested by Tullock such as political accountability and free-riding incentives in interest groups, see Hillman and Ursprung (2016).

$$x_2 = \frac{X^2}{v'} \tag{5}$$

Substituting (5) into (4) yields:

$$x_1 = \widehat{a}\frac{X^{2\widehat{a}}}{v'^{\widehat{a}}} \tag{6}$$

Summing up (6) and (5) and rearranging terms gives:

$$X^{2\widehat{a}-1} + \frac{v'^{\widehat{a}-1}}{\widehat{a}}X - \frac{v'^{\widehat{a}}}{\widehat{a}} = \tag{7}$$

Note that the LHS of (7) is negative at $X = 0$, positive at $X = v$', and increasing in X over the interval $X \in [0, v']$. Therefore, (7) has a unique interior solution denoted by X^* over the interval $X \in [0, v']$. Substituting X^* into (6) and (5) gives the unique solution to (3) and (4) such that $X \in [0, v']$. Note that $X \in (0, v')$ in an interior solution. This is because $E\pi_1 \leq 0$ iff $X \geq v'$. And if $X = 0$, then each of the players can capture the rent by exerting a small amount of effort. Therefore, since $E\pi_i$ is strictly concave given $x_j \in (0, v')$ for $i=1,2$ $i \neq j$,[10] X^* is the unique equilibrium aggregate effort. QED

Proof of Proposition 1: Substituting a=1 into $2(\frac{v'}{4a})^{\frac{1}{a}}$ results in $v'/2$. Substituting $X=v'/2$ into the LHS of (7) yields:

$$\frac{v'^{2\widehat{a}-1}}{2^{2\widehat{a}-1}} + \frac{v'^{\widehat{a}}}{2\widehat{a}} - \frac{v'^{\widehat{a}}}{\widehat{a}}, \tag{8}$$

where

$$\frac{v'^{2\widehat{a}-1}}{2^{2\widehat{a}-1}} + \frac{v'^{\widehat{a}}}{2\widehat{a}} - \frac{v'^{\widehat{a}}}{\widehat{a}} > 0$$

$$\leftrightarrow$$

$$v' > \frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}$$

As noted above, for any given v', the LHS of (7) is negative at $X = 0$, positive at $X = v$', and increasing in X over the interval $X \in [0, v']$. Therefore (8) implies that $v' > \frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}} \leftrightarrow X^* < \frac{v'}{2}$.

Substituting $X = 2(\frac{v'}{4\widehat{a}})^{\frac{1}{\widehat{a}}}$ into (7) and rearranging terms results in:

$$v'^{\widehat{a}}\left[(2^{2\widehat{a}-1})\left(\left(\frac{1}{4\widehat{a}}\right)^{\left(2-\frac{1}{\widehat{a}}\right)}\right)\left(v'^{-\frac{(\widehat{a}-1)^2}{\widehat{a}}}\right) + 2\frac{1}{\widehat{a}}\left(\frac{1}{4\widehat{a}}\right)^{\frac{1}{\widehat{a}}}\left(v'^{\left(\frac{1}{\widehat{a}}-1\right)}\right) - \frac{1}{\widehat{a}}\right] = 0 \tag{9}$$

Recall that $\widehat{a} > 1$. Note that the term in the square brackets in the LHS of (9) is strictly decreasing in v' and goes to infinity when $v' \to 0$ and to $-\frac{1}{\widehat{a}}$ when $v' \to \infty$. Therefore, for a given $\widehat{a} > 1$, (9) has exactly two solutions, one of which is v'=0. Note that substituting $v' = \frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}$ into the LHS of (9) also solves the equality.[11] Therefore, the two solutions of (9) are: v'=0 and $v' = \frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}$. Given that the derivative of the LHS of (9) with respect to v' is positive when $v' \to 0$, the LHS of (7) is positive only over the interval $v' \in \left(0, \frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}\right)$ when $X = 2(\frac{v'}{4\widehat{a}})^{\frac{1}{\widehat{a}}}$.[12]

Recall that for any given v', the LHS of (7) is negative at $X = 0$, positive at $X = v$', and increasing in X over the interval $X \in [0, v']$. Therefore, $v' < \frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}} \leftrightarrow X^* < 2(\frac{v'}{4\widehat{a}})^{\frac{1}{\widehat{a}}}$. QED

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[10] Note that this also implies that each player has a unique best response to any effort choice of her rival, which implies that an equilibrium in mixed strategies does not exist.

[11] Substituting $v' = \frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}$ into $2(\frac{v'}{4\widehat{a}})^{\frac{1}{\widehat{a}}}$ results in $2\widehat{a}^{\left(\frac{1}{1-\widehat{a}}\right)}$. Substituting $v' = \frac{4}{\widehat{a}^{\left(\frac{1}{\widehat{a}-1}\right)}}$ and $X = 2\widehat{a}^{\left(\frac{1}{1-\widehat{a}}\right)}$ into the LHS of (7) yields: $2^{(2\widehat{a}-1)}\widehat{a}^{\left(\frac{2\widehat{a}-1}{1-\widehat{a}}\right)} + 2\left(\frac{4^{\widehat{a}-1}}{\widehat{a}^2}\right)\widehat{a}^{\left(\frac{1}{1-\widehat{a}}\right)} - \frac{4^{\widehat{a}}}{\widehat{a}\,\widehat{a}^{\left(\frac{2\widehat{a}-1}{\widehat{a}-1}\right)}} = 2^{(2\widehat{a}-1)}\widehat{a}^{\left(\frac{2\widehat{a}-1}{1-\widehat{a}}\right)} + 2^{(2\widehat{a}-1)}\widehat{a}^{\left(\frac{2\widehat{a}-1}{1-\widehat{a}}\right)} - 2^{2\widehat{a}}\widehat{a}^{\left(\frac{2\widehat{a}-1}{1-\widehat{a}}\right)} = 0$.

[12] Note that after rearranging terms, taking the derivative of the LHS of (9) with respect to v' yields: $2^{(2\widehat{a}-1)}\left(\frac{1}{4\widehat{a}}\right)^{\left(2-\frac{1}{\widehat{a}}\right)}\left(2-\frac{1}{\widehat{a}}\right)v'^{\left(1-\frac{1}{\widehat{a}}\right)} + 2\frac{1}{\widehat{a}}\left(\frac{1}{4\widehat{a}}\right)^{\frac{1}{\widehat{a}}}\frac{(\widehat{a}-1)^2}{\widehat{a}}v'^{\left(\widehat{a}-2+\frac{1}{\widehat{a}}\right)} - v'^{(\widehat{a}-1)} > 0 \leftrightarrow +2^{(2\widehat{a}-1)}\left(\frac{1}{4\widehat{a}}\right)^{\left(2-\frac{1}{\widehat{a}}\right)}\left(2-\frac{1}{\widehat{a}}\right)v'^{-\frac{(\widehat{a}-1)^2}{\widehat{a}}} + 2\frac{1}{\widehat{a}}\left(\frac{1}{4\widehat{a}}\right)^{\frac{1}{\widehat{a}}}\frac{(\widehat{a}-1)^2}{\widehat{a}}v'^{\left(\frac{1}{\widehat{a}}-1\right)} - 1 > 0$ when $v' \to 0$.

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