A career like no one else can offer: On the conditions for two-party dominance

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ABSTRACT

The determinants of the number of parties competing in any given first-past-the-post election have been widely studied. Much less clear are the conditions required for two parties to dominate all elections across separate districts and at different levels of government. In this paper, I propose a novel model of party formation and show that two parties can only dominate all elections if they provide sufficient opportunities for members while limiting the success of defectors. More specifically, I establish three conditions for two-parties dominance: (*i*) parties must be divided into a left-wing and a right-wing camp in any two-party equilibrium, (*ii*) voters at the national level cannot be too concentrated in the centre relative to the most radical districts, and (*iii*) politicians need to be sufficiently motivated by the desire to win elections at higher levels of government. Furthermore, I establish the existence of a specific two-party equilibrium featuring a centre-left and a centre-right party. I use this equilibrium to illustrate that primaries can reduce the likelihood of entry of third parties. An extension that introduces regionalism shows that high salience of this second dimension of policy is by itself not enough to rule out two-party equilibria.

1. Introduction

Duverger's famous law states that first-past-the-post (FPTP) elections in combination with single member districts introduce a tendency towards competition among only two parties. The logic underlying this claim is that parties with no realistic chance of winning are either abandoned by voters or decide to drop out of the race until only two parties remain. However, this line of reasoning applies to a single election, but not to elections held across separate districts or at multiple levels of government (see Cox, 1994). Applying Duverger's law to the US, for example, we should expect to see two parties competing for the governorship of California and two parties competing for the presidency, but there is no reason why the same two parties should be competing in both of these elections. Similarly, two parties should be contesting each individual seat in Congress, but more than two parties may be represented in Congress overall. In fact, one does not have to look far to find political systems relying on FPTP elections where more than two parties attract significant vote shares: The Houses of Commons of the UK and Canada each currently feature four parties represented by more than ten MPs. National politics in India, on the other hand, are dominated by two parties, but many

more parties enjoy success at the regional level. What are the conditions required to enable two parties to dominate all elections across a country? When do more than two parties emerge?

In this paper, I propose a novel model of party formation and characterise the conditions under which equilibria with two parties exist. The model features elections at different levels of government and imposes no ad-hoc restrictions on the number of parties that compete across these elections. Politicians standing at the beginning of their career join parties in order to signal their policy preferences to voters. Joining a party enables politicians to do so since membership of a party is limited to specific types of politicians. Parties thus serve as "informative labels" (Snyder and Ting, 2002) that provide information about their members to voters. A politician who has won a regional election then has a chance to become their party's candidate at the national level. In addition, politicians also have the option of contesting elections as independents at any point in their career. Voters are allowed to vote strategically, which creates the possibility of coordination failure when more than two parties contest an election. The model is framed in the context of a federal system, with politicians initially running in state elections and later on possibly moving up to the federal level. However, the model can be applied more generally to political systems where elected offices exist at the sub-national level that serve as stepping stones for success on the national stage.

I start by considering a one-dimensional policy space. A key assumption of the model is that there is a minimum amount of heterogeneity in voter preferences across states. This heterogeneity implies that two parties can only form an equilibrium if they have a sufficiently broad ideological profile. Otherwise there are politicians who are well placed to win the election in their state but who are unable to join one of the existing parties. These politicians then have an incentive to form a third party. If parties allow a wide range of politicians to join, however, this creates internal competition for party nominations in the run-up to state elections. As a consequence, members potentially have an incentive to join smaller parties with a narrower ideological profile. In fact, successful entry of smaller parties is always possible in a two-party equilibrium. To prevent politicians from taking advantage of this possibility, joining one of the equilibrium parties needs to provide sufficient opportunities for members at the federal level while ensuring that the success of defectors does not extend beyond the state level. In a two-party equilibrium, parties thus implicitly take on additional roles beyond signalling the ideology of their members: parties need to enable and channel the careers of politicians, for example by coordinating voting behaviour or ensuring that all of their members have a shot at winning nominations. Paradoxically, the need to assume a broad ideological profile implies that parties fulfil their original purpose of revealing information about their members less effectively than they could.

The main results of the paper all follow from the logic laid out in the preceding paragraph. The first result that I present is that parties are divided into a (centre-)left and a (centre-)right camp in any two-party equilibrium. If, for example, both parties allowed left-wing politicians to join, this would create the possibility that both parties nominate such a candidate for the federal election. But since both candidates then fall on the same side of the federal median voter, this situation creates an opening for a moderate independent or third-party candidate. Based on this result, I derive necessary conditions for existence.

The first necessary condition for the existence of two-party equilibria follows from the requirement that even the most extreme party members have a shot at contesting the federal election. Such candidates can only win the federal election if facing a similarly extreme competitor. However, this situation raises the possibility of a centrist independent entering the race. Entry can only be prevented if voters continue to vote for the extremists even if entry occurs, perhaps because voting along party lines serves as a focal point. Parties thus implicitly take on the role of coordinating voting behaviour in equilibrium. The ability of parties to achieve coordination on their candidates has limits though. Moderate voters, in particular, have nothing to lose by voting for a moderate independent. The first necessary condition for the existence of two-party equilibria is therefore that voters in the federal election are not too concentrated in the centre of the policy space. A polarisation of the electorate, in contrast, is not a threat to a two-party equilibrium. While even radical voters may prefer victory of a moderate over a tie between extremists, voting for a moderate entrant instead of their own party's candidate creates the risk of an outright victory of the opposing camp. In the model, coordination on electing a third-party candidate does not occur under polarization.

The preceding results together imply that the equilibrium parties dominate the federal election even when a third candidate enters the race. These parties can thus compensate their members for the intense internal competition that they generate at the state level with the prospect of success on the federal stage. A necessary prerequisite for politicians to be willing to make this trade-off, however, is that they value opportunities at the federal level sufficiently strongly relative to winning at the state level. The second necessary condition for the existence of two-party equilibria is therefore that politicians have strong career concerns.

In a subsequent step, I establish the existence of an equilibrium with a centre-left and a centre-right party. This equilibrium exists if the previously established necessary conditions are satisfied, demonstrating that these conditions are also sufficient for the existence of a two-party equilibrium. I use this equilibrium to illustrate the role of candidate selection. The first result I establish is that parties maximise the number of elections won by nominating politicians located close to the federal median voter in *any* election. While it is clear that nominating candidates in this way makes sense in the federal election or in a state with a moderate median voter, the same applies to states where the electorate has a clear preference for one of the two parties. For example, in a state with a left-leaning median voter the centre-left party wins independently of the type of candidate it selects. Nominating a politician located close to the federal median voter therefore does not reduce the probability of winning the state election, but has the advantage that such politicians win with higher probability if subsequently nominated for the federal election. The drawback of always favouring moderates, however, is that this approach is likely to lead to defections by extremists. More specifically, extremists from states where the electorate favours such candidates can increase their chances of winning the election in their state by joining a party with a narrower ideological profile. In this context, the introduction of primaries can be thought of as a way in which parties can commit to nominating the politicians who are popular in their state, increasing the payoffs of the politicians who are most likely to have a profitable deviation. Consequently, the two-party equilibrium with a centre-left and a centre-right party exists

under weaker career concerns when parties use primaries to nominate candidates instead of simply maximising the number of elections won.

I then extend the model by introducing a second dimension of policy intended to capture a concern specific to a subset of regions, like the presence of an ethnic minority or an independence movement. Interestingly, regionalism in itself is not a threat to the existence of two-party equilibria, even if regionalism is the dominant issue for voters. This is the case because the presence of a regionalist party can stabilise a party representing all remaining voters, which would otherwise suffer from defections. However, the possibility of a regionalist candidate running for the federal election relaxes the conditions required for a moderate independent to win. In particular, two-party equilibria are unlikely to exist if regionalism is salient and there exists at least one state where a fairly homogeneous group of regionalist voters forms a majority while federally regionalist voters make up only a small share of the electorate. To understand this result, first note that salience of regionalism implies that regionalist candidates cannot win the federal election, at least in a two-party equilibrium, since a majority of voters would vote for any non-regionalist candidate. Career concerns therefore cannot keep regionalist politicians from joining parties with a narrow ideological profile since doing so maximises their chances of winning state elections. The existence of a regionalist state with a clear majority in favour of a particular type of policy then implies that a party catering specifically to these preferences must exist. The other party existing in a two-party equilibrium must consequently be a broad party representing all remaining voters. Under these conditions it must occur with positive probability that, for example, a regionalist candidate and a non-regionalist from the right of the ideological spectrum are competing in the federal election. Non-regionalist voters on the centre-left would then be willing to vote for a non-regionalist moderate. While these voters represent a majority of non-regionalist voters, they only form a majority overall if the share of regionalist voters in the electorate is sufficiently small.

The rest of the paper is organised as follows: In Section 2 I place the contributions of the paper in the context of the literature. Section 3 explains the details of the model, while Section 4 presents the theoretical results. Section 5 extends the model to allow for regionalism. Robustness of the results to relaxing some of the assumptions is discussed in Section 6. Section 7 concludes.

2. Related literature

A paper closely related to my work is (Aldrich and Lee, 2016), which also highlights the importance of political ambitions in explaining why only two parties exist in the US. To make this point, the authors specify a utility function for politicians and explain how the utility of joining a party that offers the highest probability of winning a state election can be lower than joining a national party as long as the national party offers a sufficiently high probability of winning elections at the federal level. My paper extends the analysis of Aldrich and Lee in several ways. In particular, I provide an explanation for why the chances of winning a state election should be lower as a member of the national party in the first place. In the model presented here, national parties are less attractive due to a higher level of internal competition for nominations, which arises endogenously. In addition, I present additional requirements for two-party dominance that are not discussed in Aldrich and Lee (2016).

This paper is also related to the literature on political competition with entry (Palfrey, 1984; Osborne, 1993, 2000; Callander, 2005), which analyses the effect that the threat of entry has on the equilibrium behaviour of two parties. Closest to the current paper is (Callander, 2005), who studies competition between two parties in multiple single-member districts with threat of entry of independent candidates at the district level. Parties, whose formation is not part of the model, are free to choose any platform. Callander (2005) finds that the threat of entry leads to the divergence of party platforms, similar to this paper. A key difference is that Callander establishes conditions that rule out entry at the district level, which would be equivalent to establishing conditions that rule out entry at the state level in the current paper. As it turns out though, entry at the state level is always possible in a two-party equilibrium in the current paper. The only reason that entry does not occur in equilibrium is that parties offer potential entrants a more attractive alternative. Eyster and Kittsteiner (2007) also present a model that features multiple districts, but take the number of parties as fixed. Neither of these papers mentions career concerns nor allows for regionalism.

Citizen candidate models as introduced by Osborne and Slivinski (1996) and Besley and Coate (1997) have previously been used to investigate the determinants of the number of parties competing in elections (See, for example, Dickson and Scheve, 2010). In these models, parties are identical to individual candidates. The current paper therefore requires a different approach, as parties have to be organisation that span multiple levels of government. Few papers have modelled parties as consisting of multiple politicians while endogenising the number of parties existing in equilibrium (Jackson and Moselle, 2002; Levy, 2004; Morelli, 2004; Osborne and Tourky, 2008; Eguia, 2011, 2012). To the best of my knowledge, I am the first to do so employing the concept of parties as informative labels.[2] Given the need to include multiple elections with separate electorates, affiliation choices of politicians, as well as assumptions about candidate selection, the model is necessarily relatively complex. Nevertheless, the model is tractable and naturally lends itself to the purpose of investigating other questions, such as the interplay between social diversity and electoral rules in determining the number of political parties (Dickson and Scheve, 2010; Milazzo et al., 2018). Indeed, my results suggest that a theoretical analysis of the number of parties competing in a particular district may be misleading if linkages across levels of government are not taken into account. In particular, career concerns can suppress the number of parties competing across states compared to a situation where politicians have no ambitions towards higher offices. The extension introducing regionalism indicates that the model can accommodate multi-dimensional policy spaces.

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[2] In contrast, Snyder and Ting (2002) as well as other contributions building on their approach (Ashworth and Bueno de Mesquita, 2008; Bernhardt et al., 2009) consider the behaviour of a given number of parties.

The insight that primaries can increase the stability of a two-party equilibrium is similar to the view of primaries as a unifying force proposed by Hortala-Vallve and Mueller (2015). The additional insight provided by my analysis is that primaries do not simply increase the probability that extremists within the party are more likely to be nominated, but provide a way to flexibly target candidates to the specific preferences of local electorates across various elections.

3. The model

A federal country consisting of $S \geq 3$ states selects federal and state governments through FPTP elections. Political parties nominate candidates for these elections, but politicians who are not nominated can decide to run as independents. Initially a large number of potential parties exists, but only those that attract members can compete in elections. The timing is as follows: In the beginning of the game, politicians decide which party to join. Once affiliation decisions have been made, parties nominate candidates in each state and state elections are held. Each winner of a state election then has a chance to become their party's candidate for the federal election. After the federal election the game ends. In any of these elections, any politician who has not been nominated by a party can run as an independent candidate. The following sections describe the elements of the model in detail.

3.1. Players

The strategic players of the game are politicians and voters.

3.1.1. Politicians

There are five politicians in each state. Every politician is endowed with a platform p and in each state there is one politician for each of the five possible platforms collected in the set $\mathcal{T} = \{-1, -.5, 0, .5, 1\}$. For brevity, $g_{s,p}$ refers to the politician with platform p from state s.

Whenever a politician wins an election, they are committed to implementing their platform. Politicians with platform -1 or 1 are referred to as extremists, while all remaining politicians are labelled as moderates. Politicians are office-motivated. The utility of a politician who does not win any elections is normalised to zero. In contrast, the winning candidate in an election at the state level receives a payoff of $y_s > 0$, while the utility of the winner of the federal election further increases by $y_f > 0$.

3.1.2. Voters

Each state s contains a set of voters that is large, finite, and odd. Let p_l denote the policy that is implemented in region $l \in \{1, \ldots, S, f\}$. Given beliefs over the platforms of candidates and the behaviour of other voters, the objective of a voter with ideal policy $i \in \mathbb{R}$ who participates in the election in region l is to maximise

$$\mathbb{E}[u(|p_l - i|)] ,$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly decreasing.[3] The set of voters in state s is described by a measure Λ_s that assigns to any subset of $\mathbb{R}$ the share of voters whose ideal policies lie in this subset. Let m_s denote the ideal policy of the median voter of state s.

I assume that there exists a minimal amount of heterogeneity in voter preferences across states: There exists at least one state such that a strict majority of voters in the state strictly prefers the platform -1 over any other platform, at least one state such that a strict majority of voters in the state strictly prefers the platform 0 over any other platform, as well as at least one state such that a strict majority of voters in the state strictly prefers the platform 1 over any other platform. As the labels of states are arbitrary, it is without loss of generality to refer to these states as states 1, 2, and 3, respectively.

All voters vote in the federal election and the federal electorate is described by a corresponding measure Λ_f with $m_f = 0$.

3.2. Political parties

A party consists of a "shape" that specifies which politicians are allowed to join the party and a set of functions determining how candidates are selected. The shape of a party P is a consecutive list of platforms denoted by I_P and only politicians with these platforms can join. A possible shape of a party is therefore $\{0, .5, 1\}$, while a party of shape $\{0, 1\}$ is ruled out for example.

Given a pool of potential nominees, the selection of candidates by parties occurs according to fixed probabilities. Denote by $\mathcal{M}_{P,l}$ the set of politicians who are eligible to be nominated for the election in region $l \in \{1, \ldots, S, f\}$ by party P. The probability that a politician with platform p who belongs to $\mathcal{M}_{P,l}$ is nominated for the election in region l is given by a function $\eta_{P,l}(p|\mathcal{M}_{P,l})$. The only restrictions placed on this function are that for any $\mathcal{M}_{P,l}$ the nomination probabilities across all members of $\mathcal{M}_{P,l}$ are strictly positive and sum to one.

The set of parties that exists in the beginning of the game is denoted by $\mathcal{P}$. Since any party can only compete in elections if joined by at least one politician, the set $\mathcal{P}$ is referred to as the set of potential parties. Any party that does attract members is referred to as an active party. The set of potential parties $\mathcal{P}$ is "large". In particular, for any possible shape I there exists at least one party $P \in \mathcal{P}$ such that $I_P = I$.

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[3] Elections at the state level determine who becomes a candidate at the federal level, but it is assumed that voters do not take this interdependence into account when voting in a state election. When there are more than just a handful of states, election outcomes in one state are unlikely to have a strong effect on events at the federal level, but are very difficult to keep track of when solving the model. Furthermore, it seems realistic to assume that voters do not take potential future events that are hard to predict into account when voting in a state election.

3.3. Timing

The game proceeds as follows:

(i) All politicians decide whether to join one of the parties in $\mathcal{P}$ or remain independent.
(ii) In every state, any party that has been joined by at least one politician in the state nominates a candidate. Only members who reside in the state are eligible to be nominated for the state election.
(iii) After observing the candidates put forward by parties, all politicians who have not been nominated decide simultaneously whether to run as independents for the election in their state.
(iv) Each voter casts a vote at the election in their state and the winner in each state is the candidate who receives the highest number of votes. Ties are broken randomly. Winners of state elections implement their platform as the state policy.
(v) Any party that has won at least one state election nominates one of their winning candidates as their candidate for the federal election.
(vi) After observing the candidates put forward by parties, all politicians who have not been nominated decide simultaneously whether to run as independents for the federal election.
(vii) All voters vote in the federal election. The winner is once more the candidate who receives the highest number of votes, who then implements their platform as the federal policy.[4]

3.4. Information

Voters have limited information about politicians. Specifically, it is assumed that the electorate cannot distinguish between different politicians and initially believes that the platform of a politician is equal to any of the five possible platforms with probability .2. Furthermore, voters can see which parties have nominated a candidate in their state, but not how many politicians have joined each party. Voters do know, however, how candidates are selected. This knowledge combined with a belief about which politicians have joined a particular party allows voters to update their beliefs about the platform of a party's candidate prior to casting their vote at the state-level election.

The winner of a state election implements their platform at the state level, thus revealing it to voters. Voters accordingly have full information about the platform of any candidate for the federal election who has previously won a state election.

Finally, all agents are fully informed about the distribution of voters in all states and at the federal level.

3.5. Equilibrium

The party-formation game described in the previous sections is a dynamic game of incomplete information and requires a corresponding equilibrium concept. I focus on the set of perfect Bayesian equilibria. Without further restrictions, this choice entails a huge number of equilibria due to the fact that voters are allowed to vote strategically instead of assumed to vote sincerely. For example, with three or more voters and two or more candidates it is always an equilibrium that all voters vote for any specific candidate, even if all voters strictly prefer a different candidate. This extreme form of coordination failure makes the existence of equilibria with two parties trivial, since it is always possible to find a voting equilibrium such that any third-party candidate loses. On the other hand, assuming perfect coordination among voters is not realistic either. In addition, one of the roles that parties implicitly take on in equilibrium is to coordinate voting behaviour. To strike the right balance, I impose the following restriction: if a candidate in some election is the unique most preferred option of a strict majority of voters based on their beliefs at the point when the election is held, then a voting equilibrium where this candidate wins the election is selected.[5] While such an equilibrium always exists under the stated conditions, there are typically additional equilibria where a different candidate wins. Nevertheless, it seems highly plausible that voters are able to coordinate on electing a candidate who is favoured by a strict majority.[6]

The following definition summarises the equilibrium concept:

Definition 1 (*Party-Formation Equilibrium*). A party-formation equilibrium is a perfect Bayesian equilibrium of the party-formation game that satisfies the following condition: If a candidate in some election is the unique most preferred option of a strict majority of voters based on their beliefs at the point when the election is held, then this candidate wins the election.

$\mathcal{P}^*$ will denote the set of parties that are active in an equilibrium.

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[4] Even in a presidential system, policymaking often requires passing legislation, which in turn requires a majority in parliament. With more than two parties competing the choice of policy may therefore require a process of coalition formation. I abstract from such issues here. At least the two-party equilibria presented below do not depend on what is assumed about the process of policy formation when no party achieves a majority. This is because voters are allowed to vote strategically, which implies that there always exists a voting equilibrium with one party winning with a strict majority, even off the equilibrium path when a third party has entered.

[5] When a strict majority of voters favours a particular candidate, the restriction imposed here yields the same winner as sincere voting. However, assuming sincere voting would require exact knowledge of the distribution of voters in order to determine the winner of an election when a strict majority in favour of one candidate does not exist. Under the assumptions made here, in contrast, any candidate can be the winner in such a situation, which actually makes the model more tractable.

[6] For example, Myatt (2007) models a three-candidate election as a global game. While coordination failure is generally a feature of equilibrium in this model, a candidate favoured by a majority of voters wins with certainty.

4. Results

The aim of this paper is to investigate conditions under which equilibria with two active parties exist. It is useful to introduce a formal definition of the type of equilibrium that is the main focus.

Definition 2 (*Two-Party Equilibrium*). A two-party equilibrium is a party-formation equilibrium such that the number of active parties in equilibrium is equal to two and no independent candidates run in any elections along the equilibrium path.

A two-party equilibrium thus requires not only that the number of active parties is equal to two, but also that these parties face no competition from independent candidates. Independent candidates are a common occurrence in most democratic countries, but independent candidates with a serious chance of winning are much rarer.

To begin, I will derive two basic properties that any two-party equilibrium must satisfy. Remember that a strict majority of voters in state 1 strictly prefers the platform -1 over any other platform. If the politician with platform -1 of the state joins a party of shape $\{-1\}$, voters know that a candidate nominated by this party must have the platform -1 since no other politicians are allowed to join. The party's candidate would therefore receive a strict majority of votes, no matter what other candidates may be running. Accordingly, the politician with platform -1 can always win the state election with certainty by joining a party of shape $\{-1\}$. The same logic applies to the politician with platform 0 in state 2 and the politician with platform 1 in state 3. A minimal requirement that any two-party equilibrium must fulfil is therefore that politicians with platforms -1, 0, and 1 are able to join one of the parties that are active in equilibrium. Otherwise at least one of the aforementioned politicians would win the election in their state with probability zero and therefore have an incentive to join a singleton party.

Lemma 1. *In any two-party equilibrium, politicians with platforms -1, 0 and 1 must be allowed to join at least one of the parties that are active along the equilibrium path.*

Proof. See Appendix. □

A second feature of any two-party equilibrium is that any politician who has a chance to join a party that can win the election in the politician's state must do so. A central ingredient of this result is the assumption that voters cannot observe the membership of a party at the state level. A deviation by a politician therefore does not affect voters' expectations regarding the platform of each party's candidate and voters continue to vote for the same parties. Even politicians who are not well aligned with the median voter of a state can thus join a party without jeopardising the electoral success of the party.

Lemma 2. *Consider any two-party equilibrium and suppose there is a politician in some state s who is eligible to join a party that wins the election in the state with positive probability along the equilibrium path. Then the equilibrium strategy of this politician must place zero probability on not joining a party or on joining a party that loses the state election with certainty.*

Proof. See Appendix. □

A consequence of Lemma 1 is that at least one party must allow multiple types of politicians to join. In combination with the fact that successful parties attract members, this observation implies that a necessary part of any two-party equilibrium is internal competition for nominations at the state level by different factions within a party. In isolation, this factor gives party members an incentive to defect to smaller parties with a narrower ideological profile in order to increase their chances of winning state elections. If there was no federal election, any equilibrium would feature at least three active parties with shapes $\{-1\}$, $\{0\}$, and $\{1\}$. Nevertheless, politicians can be persuaded to stay loyal to a broader party if this party provides a path to winning the federal election. Two parties can therefore only form an equilibrium if they offer their members sufficient success at the federal level while simultaneously limiting that of defectors. The main results presented in the next three sections all follow from this logic.

4.1. Left versus right

The first main result provides a partial characterisation of the parties that can be active in a two-party equilibrium. Specifically, it must be the case in any two-party equilibrium that there is one active party which does not admit members with positive platforms, while the other party does not admit members with negative platforms.

Proposition 1. *Consider any constellation of two parties A, $B \in \mathcal{P}$. Then a two-party equilibrium such that A and B are the active parties in equilibrium exists only if $I_A \subseteq \{-1, -.5, 0\}$ and $I_B \subseteq \{0, .5, 1\}$.*

Proof. See Appendix. □

Parties must thus be divided into clear ideological camps in any two-party equilibrium. Otherwise, there would be the possibility that both parties nominate candidates for the federal election whose platforms lie on the same side relative to the median voter, which would enable an independent with platform 0 to enter and win. While this logic seems straightforward, there are two complications. First of all, an independent candidate can only run successfully in the federal election if voters know the platform of this candidate, which requires that this politician has previously won a state election. By Lemma 1, politicians with platform 0 must

be allowed to join at least one of the equilibrium parties. These parties therefore themselves generate a pool of potential challengers. The second complication is the possibility that parties, for example, both allow politicians with negative platforms to join, but the sorting behaviour of politicians nevertheless prevents that both parties simultaneously nominate such a candidate for the federal election. The proof of Proposition 1 builds on the logic that such sorting behaviour is difficult to maintain in equilibrium. If one party has no members with positive platforms while the other party has no members with negative platforms, then the former party must win the election in state 1 while the latter party must win the election in state 3. But if a party wins a state election with certainty, then all eligible politicians of the state must join the party by Lemma 2. Accordingly, both parties must have members with, for example, negative platforms if both parties allow such politicians to join.

4.2. No outlier states

The second necessary condition for the existence of two-party equilibria states that voters in the federal election cannot be too concentrated in the centre of the policy space.

Proposition 2. *Consider any constellation of two parties $A, B \in \mathcal{P}$. Then a two-party equilibrium such that A and B are the active parties in equilibrium does not exist if there are more than half of all voters in the federal election located in at least two of the intervals $(-.5, .5)$, $(-.75, .25)$, and $(-.25, .75)$.*

Proof. See Appendix. □

As was already established by Lemma 1, the active parties in a two-party equilibrium have to allow extremists to join. As a consequence, the situation may arise that both parties nominate extremists for the federal election. According to Proposition 1, these candidates will necessarily be drawn from opposite ends of the political spectrum. Voters in the centre of the policy space would then prefer a moderate candidate. For example, a politician with known platform .5 is strictly preferred over any extremist by all voters in the interval $(-.25, .75)$. Under the condition set out by Proposition 2, at least some moderate winners of state elections, whose platforms are thus known to voters, have an incentive to run as an independent in the federal election when both parties nominate extremists.

Proposition 1 highlights the role that parties need to play in coordinating voter behaviour in any two-party equilibrium. In a situation where one party has nominated a candidate with platform -1 for the federal election while the other party has nominated a candidate with platform 1 and an independent with platform 0 joins the race, one may argue that even radical voters have an incentive to vote for the moderate to prevent the opposite camp from winning. Collectively this is certainly true. If voting along party lines is a focal point, however, partisan voters will likely prefer not to opt for the moderate candidate for fear of splitting the vote. Voters in the centre who dislike both types of extremists more or less equally, on the other hand, have little to lose by voting for the independent candidate. In the model, coordinating on electing the moderate candidate is only successful when the number of centrist voters is sufficiently high as specified by Proposition 2.

Importantly, the condition of Proposition 2 is only necessary for the existence of two-party equilibria due to the presence of the relatively extreme voters of states 1 and 3. Without such states, there would be two-party equilibria where the active parties do not allow extremists to join and Proposition 2 loses its bite. The assumption that a strict majority of voters in state 1 strictly prefers the platform -1 over any other platform implies that $m_1 < -.75$. Equivalently, it must be the case that $m_3 > .75$. Another way of expressing the condition in Proposition 2 is therefore that there cannot be states whose median voters are "outliers" in the distribution of voters overall in the sense that their position is more radical than the ideal policies of more than half of all voters.

Proposition 2 is interesting due to the situations that it does not rule out. In particular, the existence of two-party equilibria requires no restrictions on the shares of voters located below -.75 or above .75. A polarisation of the electorate therefore poses no threat to a two-party equilibrium. Even if a shift in voter preferences should push parties towards nominating more extreme candidates, such a situation does not create an opening for a moderate entrant as long as the number of remaining moderate voters is sufficiently small. This logic may help explain why the two-party system is alive and well in the US despite increasing polarization both of political elites and of the electorate (Iyengar et al., 2019).

4.3. Career concerns

As was already pointed out above, there are at least some politicians in any two-party equilibrium who could increase their chances of winning a state election by joining a third party. The results introduced in the preceding two sections follow from the necessity of ensuring that the success of defectors does not extend beyond the state level. If those conditions are satisfied, the relatively broad parties that are necessarily part of any two-party equilibrium can compensate their members for the internal competition that they generate with opportunities at the federal level. Politicians need to value these opportunities sufficiently strongly, otherwise defections cannot be prevented. For example, the politician with platform -1 in state 1 can always win the state election with certainty by joining a party of shape -1. If this politician instead joins a party that also allows other politicians to join, they win the state election with less than certainty. Joining the broader party may nevertheless provide a higher payoff if it is associated with chance of winning the federal election and the payoff y_f is sufficiently large relative to the payoff y_s.

Proposition 3. *For any constellation of two parties $A, B \in \mathcal{P}$, there exists a constant $\bar{y} > 0$ such that a two-party equilibrium in which A and B are the active parties in equilibrium exists only if $y_f / y_s \geq \bar{y}$.*

Proof. See Appendix. □

In words, Proposition 3 requires that the ratio between the payoff from winning the federal election and the payoff of winning a state election must exceed a certain threshold for a two-party equilibrium to exist. The level of this threshold depends on the specific equilibrium under consideration, but is generally driven by how likely different politicians are to win elections in equilibrium. These probabilities in turn depend on the candidates nominated by the opposing party and the intensity of internal competition. At the federal level, internal competition increases with the number of state elections won by a party: the greater the electoral success of a party at the sub-national level, the greater the number of party members with a public profile who would make viable candidates for the federal election. At the state level, internal competition increases with number of politicians allowed to join a party. Furthermore, the chances of any given politician of securing nominations depend on how their party selects candidates. A party that gives priority to moderates due to their electability is likely to suffer defections by extremists. If the party heavily favours the latter type of candidate, in contrast, moderates may be reconsidering their options. I explore the role of candidate selection more formally in Section 4.5.

While the payoffs y_s and y_f depend on the intrinsic motivations of politicians, other factors such as financial rewards, public visibility, or the competencies and powers associated with an office equally play a role. The strength of career concerns accordingly depends on the general setup of the political system.

4.4. An example of a two-party equilibrium

The following proposition shows that the necessary conditions introduced in the previous sections are also sufficient for the existence of a two-party equilibrium.

Proposition 4. *Consider a constellation of two parties, L and R, with $I_L = \{-1, -.5, 0\}$ and $I_R = \{0, .5, 1\}$ and suppose that parties use some combination of nomination technologies satisfying the assumptions made in Section 3.2. Then an equilibrium such that $\mathcal{P}^* = \{L, R\}$ exists if*

(i) *there are no more than half of all voters in the federal election located in each of the intervals $(-.75, .25)$, $(-.5, .5)$, and $(-.25, .75)$, and*

(ii) *$\frac{y_f}{y_s}$ is larger or equal to some threshold $\bar{y} > 0$.*

Proof. See Appendix. □

The type of equilibrium presented in Proposition 4 features a centre-left and a centre-right party that overlap in the centre of the policy space. This constellation is essentially the one that has been dominating US politics ever since the end of the civil war and also captures the party landscape in Britain in the decades after World War II well. I will refer to an equilibrium of this kind as an $L - R$ equilibrium.[7] Naturally, these equilibria feature one party that does not allow politicians with positive platforms to join and one party that does not allow politicians with negative platforms to join as required by Proposition 1. Proposition 2 reappears as Part (*i*) of Proposition 4. Since all types of moderate politicians are able to join the parties L and R, it must be true that neither of these politicians is preferred by strict majority of voters in the federal election over any extremist. Finally, Proposition 3, which requires sufficiently strong career concerns, is reflected in Part (*ii*) of Proposition 4.

While $L - R$ equilibria are appealing due to their similarity to two-party systems observed in reality, any other two-party equilibrium must similarly feature a (centre-)left party and a (centre-)right party. Lemma 1 and Proposition 1 in combination imply that the possible two-party equilibria feature a party of shape $\{-1, -.5, 0\}$ competing against a party of shape $\{0, .5, 1\}$, $\{.5, 1\}$, or $\{1\}$, or a mirror image of any of these constellations. Other types of two-party equilibria can be shown to exist under similar conditions to those of Proposition 4. An exception are constellations of parties where neither party allows politicians with platform $-.5$ or $.5$ to join, which entail the additional requirement that there is no state where a majority of voters strictly prefer the platform of the excluded type of politician over any other platform.

4.5. The role of candidate selection

The class of equilibria of Proposition 4 can be used to illustrate the role of candidate selection. The proposition below provides a benchmark by establishing which method of candidate selection maximises the expected number of elections won by a party. Calculating this number is not straightforward, since multiple equilibria consistent with Proposition 4 may exist under any given method of candidate selection. I thus compare the expected number of elections won by a party across different methods of candidate selection holding the behaviour of politicians fixed. To avoid some technicalities, I assume throughout the remainder of this section that indifferent voters in the federal election vote with equal probability for either of their most preferred candidates.

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[7] While very similar, strictly speaking more than one $L - R$ equilibrium exists. Most importantly, these equilibria can differ with respect to the behaviour of politicians with platform 0 in states with a moderate median voter. For example, the politician $g_{2,0}$ may join party L or party R or mix over joining either of these parties in equilibrium.

Proposition 5. *Fix a strategy profile for politicians such that only parties L and R attract members and no politicians run as independent candidates. Assume indifferent voters in the federal election vote for either of their most preferred candidates with equal probability.*

Then parties L and R respectively maximise the expected number of elections won along the equilibrium path by always nominating the candidate located closest to zero in any election.

It is clear that nominating the candidate closest to zero maximises a party's chances of winning the federal election in a two-party equilibrium, given that the federal median voter is located at zero. Conditional on politician behaviour, the same is true in a state election: If the median voter is located sufficiently far to the left of zero, for example, party L wins independently of how candidates are selected. If the median voter is located close to zero, on the other hand, nominating the politician closest to zero does not reduce the chances of winning. In addition, parties benefit from increasing the number of politicians with platform 0 in their federal pool of candidates.

While always nominating the candidate closest to zero is good for winning elections conditional on politician behaviour, it is straightforward to show that no two-party equilibrium exists if both parties choose candidates in this fashion. Either the politician with platform -1 in state 1 or the politician with platform 1 in state 3 would have no chance of ever winning an election and would therefore defect and join a third party. However, two-party equilibria may exist if parties aim to nominate the candidate closest to zero, but sometimes make mistakes. More specifically, assume that in a given election a party chooses the candidate closest to zero with probability $1 - \varepsilon$,[8] while with probability $0 < \varepsilon < 1$ the candidate is drawn uniformly at random from the set of eligible politicians. ε thus represents the probability of a mistake occurring. Refer to this method of candidate selection as "no commitment", since strategic parties that aim to win elections would select candidates in this fashion if they are unable to make a prior commitment to any other distribution of nomination probabilities.

The existence of two-party equilibria under no commitment requires strong career concerns, particularly if ε is small and extremists are unlikely to win nominations. Simply nominating extremists more frequently, on the other hand, may lead to defections by moderates. A mechanism for candidate selection that makes the existence of an $L - R$ equilibrium particularly likely is one that favours politicians located close to the median voter when choosing the nominee for a state election. After all, these are the politicians who can do well at the state level after defecting and are thus the ones that pose a threat to the two-party system. A credible commitment to such a nomination strategy may be achieved through a decentralisation of the nomination process, for example by introducing primaries or caucuses. To formally demonstrate the trade-off between winning elections and avoiding entry, let "primaries" refer to a mechanism for candidate selection that in any election nominates a random candidate with probability ε, but with probability $1 - \varepsilon$ nominates the politician located closest to the median voter of the election in question. Let $\bar{y}_{PR}$ be the threshold such that an $L - R$ equilibrium exists under primaries if and only if $y_f/y_s \geq \bar{y}_{PR}$, conditional on Part (*i*) of Proposition 4 being satisfied. Define $\bar{y}_{NC}$ as the analogous threshold under no commitment.

Proposition 6. *Assume that there are three states and that indifferent voters in the federal election vote for either of their most preferred candidates with equal probability. Then the threshold $\bar{y}_{PR}$ required for the existence of $L - R$ equilibria under primaries is strictly smaller than the equivalent threshold $\bar{y}_{NC}$ under no commitment.*

When there are three states, the politicians who can win a state election after deviating to joining a third party in an $L - R$ equilibrium are the politician with platform -1 in state 1, the politician with platform 0 in state 2, and the politician with platform 1 in state 3. Relative to no commitment, primaries increase the payoffs of all of these politicians. The extremists of states 1 and 3 benefit from a higher chance of being nominated for the election in their state, while the nomination probabilities in state 2 and at the federal level actually remain unchanged, at least conditionally on a given candidate pool. However, the politician with platform 0 in state 2 benefits from the fact that any potential competitor for the federal nomination, internally or externally, is more likely to be an extremist under primaries. Since all potential defectors are better off under primaries, nominating candidates in this fashion relaxes the restrictions on career concerns required for an $L - R$ equilibrium to exist. If primaries instead lead to a reduction of the likelihood that a politician with platform zero is nominated in state 2 or at the federal election, as might be the case under closed primaries, the proof of Proposition 6 would be less straightforward. However, the result would likely continue to hold since extremists generally win the federal election with lower probability than moderates and the payoffs of extremists are therefore typically the binding constraint on the existence of an $L - R$ equilibrium. Either way, the flexibility provided by primaries is likely to be a major factor in the stability of the two-party system of the US.

5. Regionalism

In this section, I introduce a second dimension of policy and derive an additional necessary condition for the existence of two-party equilibria in this extended version of the model. The second dimension of the policy space represents an issue or characteristic specific to some states, such as an independence movement or the presence of an ethnic minority that is concentrated in a subset of states. In line with these examples, the secondary issue is modelled as binary. Accordingly, the policy space is now given by $\mathbb{R} \times \{0, r\}$ with $r > 0$ and the set of possible platforms is $\{-1, -.5, 0, .5, 1\} \times \{0, r\}$. Voters and politicians located at r along the second

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[8] If there is more than one politician located closest to zero, as may happen in the federal election, assume that all of these politicians are chosen with equal probability.

dimension of policy will be referred to as regionalists and all others as non-regionalists. The terms extremist and moderate continue to describe the position of a politician along the first, ideological dimension of the policy space.

In line with the basic version of the model, I assume that the median voter along the ideological dimension in the federal election is located at zero:

$$\min\{\Lambda_f((-\infty, 0] \times \{0, r\}), \Lambda_f([0, \infty) \times \{0, r\})\} > .5 \ .$$

In addition, $\Lambda_f([-1, 1] \times \{0\}) > .5$ such that regionalism is a minority issue. In most states, there are only five politicians, one for each of the platforms $\{-1, -.5, 0, .5, 1\} \times \{0\}$, and there are no regionalist voters, that is, $\Lambda_s(\mathbb{R} \times \{0\}) = 1$. In a non-empty subset S^r of states, however, there are 10 politicians, one for each possible platform in $\{-1, -.5, 0, .5, 1\} \times \{0, r\}$, and the distribution of voters in the state is not restricted to $\mathbb{R} \times \{0\}$. To ensure that the results from the previous sections also apply to the extended version of the model, I assume that there are no regionalist voters in states 1, 2, and 3, that is, $\{1, 2, 3\} \cap S^r = \emptyset$.[9] In addition, the assumptions about the distributions of voters in these states are maintained. While regionalism is a minority issue in the federal election, there exists at least one state $s \in S^r$ such that $\Lambda_s([-1, 1] \times \{r\}) > .5$ and regionalist voters form the majority.

Given that the shapes of parties were restricted to contain consecutive platforms in the basic model, a natural generalisation is to require party shapes to be "convex" in the sense that if politicians with two distinct platforms can join a party then the same must be true for any politician located on a straight line between those two platforms.

The aim of a voter with ideal policy (i, j) in an election in some region $l \in \{1, \dots, S, f\}$ is now to maximise

$$\mathbb{E}[u(|p_{l,1} - i|) + u(|p_{l,2} - j|)] \ ,$$

where $(p_{l,1}, p_{l,2})$ is the policy implemented in region l.

The necessary conditions for the existence of two-party equilibria of Propositions 2 and 3 carry over to the extended model. Intuitively, one might expect that the extended model yields the additional requirement that the salience of regionalism is low enough to prevent entry of a regionalist party. Salience of the regionalist issue is determined by the parameter r. If r is small, the position of a politician along the regionalist dimension has a negligible impact on voters' utility relative to the ideological position. In this case there may even be equilibria in which no active party allows any regionalists to join.[10] If r is large, on the other hand, regionalism becomes the decisive issue for regionalist and non-regionalist voters alike. While the latter case does indeed imply that any equilibrium must feature a regionalist party, the following example illustrates that two-party equilibria can exist no matter how strongly voters care about regionalism.

Proposition 7. *Suppose there are four states with equal-sized populations, $S^r = \{4\}$, and*

$$\Lambda_1([-1, -.75] \times \{0\})$$
$$= \Lambda_2((-.25, .25) \times \{0\})$$
$$= \Lambda_3([.75, 1] \times \{0\})$$
$$= 1$$

while

$$\Lambda_4([-1, -.75] \times \{r\}) \ = \ \Lambda_4([.75, 1] \times \{r\}) \ = \ .5 \ .$$

Then there exists a two-party equilibrium such that $\mathcal{P}^* = \{N, R\}$ with $I_N = \{-1, -.5, 0, .5, 1\} \times \{0\}$ and $I_R = \{(-1, r)\}$ if $r > 2$ and y_f/y_s exceeds some threshold $\bar{y} > 0$.

Proof. See Appendix. □

A specific case satisfying the assumptions made in Proposition 7 is illustrated in Fig. 1, where grey discs indicate that voters are located at the centre of the disc. The size of each disc is proportional to the share of voters in the federal election in the specified location and grey numbers indicate which state the voters belong to. Party N is a non-regionalist party with a broad ideological profile, which wins the elections in states 1, 2, and 3. Party R, in contrast, is a regionalist party with a narrow profile, which wins the election in the sole regionalist state and never wins the federal election. The equilibrium requires that the salience of the regionalist issue is sufficiently high to ensure that regionalist voters vote en bloc for the regionalist candidate at the federal election. For lower values of r, a coalition of regionalist and non-regionalist voters would enable entry of a moderate independent in the federal election when both parties nominate candidates located at -1 along the ideological dimension.

A grey disc indicates that voters are located at the centre of the disc. The size of each disc is proportional to the share of voters in the federal election at the indicated location. Grey numbers give the state that the voters in question belong to. Rounded rectangles indicate the shapes of the parties that are active in equilibrium, that is, which of the politicians located at black dots are allowed to join each party.

Note that the voters of states 2 and 3 in the example of Proposition 7 would be willing to vote in favour of an independent candidate with platform $(-.5, 0)$ in the federal election if party N nominates a candidate with platform $(-1, 0)$. Similarly, the voters

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[9] There are therefore at least four states.

[10] Since there are states in which no regionalist politicians are present, the reverse case is not possible.
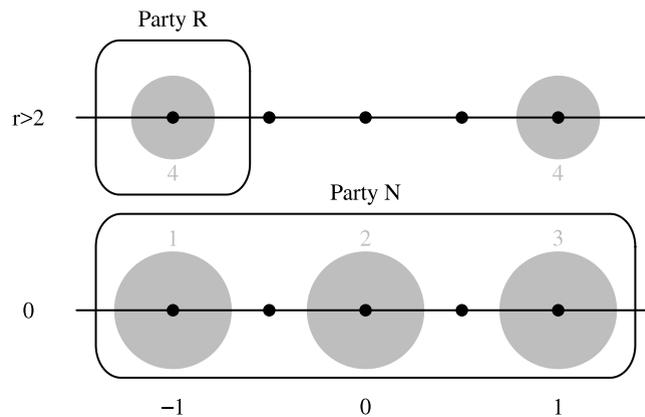
Fig. 1. Example of a two-party equilibrium that exists for arbitrarily large values of *r*.

of states 1 and 2 would prefer an independent with platform $(.5, 0)$ over the party candidates if party N nominates a politician with platform $(1, 0)$. If the coalitions of either the voters in states 1 and 2 or of the voters in states 2 and 3 made up a slightly larger share of the federal electorate, one of the aforementioned independents could win the federal election as they would be strictly preferred by a strict majority of voters. This observation forms the basis of the following result.

Proposition 8. *No two-party equilibrium exists if all of the following conditions hold:*

(i) $r > 2$,
(ii) *there exists at least one state where a strict majority of voters strictly prefers a platform $p \in \{-1, -.5, 0, .5, 1\} \times \{r\}$ over any other platform, and*
(iii) *the distribution of voters at the federal level satisfies either*

$$\Lambda_f((-\infty, .75) \times \{0\}) > .5$$

or

$$\Lambda_f((-.75, \infty) \times \{0\}) > .5 .$$

Proof. See Appendix. □

While part (*i*) of the preceding Proposition simply states that regionalism is highly salient, part (*ii*) requires that there is a state where a strict majority of voters is not only regionalist but also fairly homogeneous in terms of their preferences along the ideological dimension. In combination, these conditions ensures that there must be a party that allows only regionalist politicians to join in any two-party equilibrium: Due to the salience of regionalism, regionalist candidates cannot win the federal election and are therefore willing to join a narrow party targeted at the preferences of voters in a regionalist state. In a two-party equilibrium, the competing party must then be a broad party that allows at least all non-regionalists to join, which creates the chance that an extremist and a regionalist compete in the federal election. If part (*iii*) of Proposition 8 is satisfied, the preceding constellation of candidates creates an opportunity for entry of an independent candidate.

The homogeneous preferences in at least one regionalist district required by part (*ii*) of Proposition 8 are most likely to be found in political systems where offices that serve as starting points for a political career are controlled by relatively small electorates. The conditions of part (*iii*) of the Proposition, on the other hand, are particularly likely to be satisfied if regionalist voters make up a small share of the federal electorate due to the assumption that the median voter along the ideological dimension of policy is located at 0. The example of Proposition 7 illustrates this logic, where slightly decreasing the number of voters in state 4 would enable entry of independent candidates in the federal election. Intuitively speaking, a larger number of regionalist voters increases the risk that a split in the non-regionalist vote will produce a regionalist winner, thus complicating coordination on a non-regionalist independent.

Note that the conditions that enable entry of a moderate independent in the basic version of the model, as given by Proposition 2, imply that Part (*iii*) of Proposition 8 is satisfied, while the reverse is not true. Salience of the regionalist dimension of policy thus relaxes the conditions required to enable successful entry of independent candidates in the federal election.

6. Discussion and robustness

The basic model of party formation presented above requires a number of simplifying assumptions for tractability, while other assumptions are made merely for ease of exposition. I discuss some of these assumptions in more detail in this section.

An assumption that is certainly not standard is that politicians are committed to implementing a specific platform. I show that this assumption can be relaxed in a separate subsection at the end of this section.

Allowing for five different platforms is the smallest number that enables me to show all of the results. Additional platforms can easily be accommodated.

The assumption that the preferences of politicians are homogeneous can also be relaxed. The strength of career concerns required to keep a politician from joining a third party differs across politicians in any case and depends on how likely a politician is to win elections in equilibrium and how well they could do after joining a third party. Since some politicians are simply unable to win elections as member of a new party, they would not have an incentive to deviate even if they did not care about winning the federal election at all. However, there are always some politicians who can win a state election after joining a third party, such as the politician with platform -1 in state 1 for example, and a particular two-party equilibrium only exists if all members of this set of politicians are sufficiently motivated by the prospect of winning the federal election.

In principle, it would also be possible to relax the definition of a two-party equilibrium to include the possibility that independent candidates can win elections as long as the probability of this happening does not exceed a specific threshold. Proposition 1 would *not* continue to hold under this broader definition. For example, there may then exist two-party equilibria where both parties allow politicians with the platform $-.5$ to join. While the proof of Proposition 1 implies that an independent candidate wins the federal election with positive probability under these conditions, this event may nevertheless be highly unlikely. However, I believe that there are good reasons why parties may want to ensure that no independent can ever win the federal election. A key factor that stabilises any two-party equilibrium is the limited ability of voters to coordinate (see Section 4.2). An independent winner of the federal election may be able to use their public visibility to achieve coordination on a different voting equilibrium in subsequent periods, permanently upsetting the original two-party equilibrium. The victory of Emmanuel Macron in the French presidential election of 2017 and the success of his newly-formed party La République En Marche! in the subsequent legislative elections is a case in point. While such an extension is beyond the scope of this paper, it is imaginable to create a repeated version of the party-formation game with overlapping generations of politicians, where the winner of the federal election of the previous period can affect voter coordination. Stability of a two-party equilibrium over time would then likely require that no independent can ever win the federal election.

An assumption that may appear strong is that there can be parties that perfectly reveal the platforms of their members. However, the essential feature of the model is not that there can be such "singleton parties", but that there can be parties that reveal different amounts of information about their members. The Tea Party movement in the US (Arceneaux and Nicholson, 2012), while not a party of its own, illustrates that it is possible to send a more fine-grained signal to voters than, for instance, the Republican party label does.

Finally, it would also be possible to introduce an electoral college at the federal level to fit the model more closely to the US. In this case the results go through unchanged if the median voter of the state with the median electoral vote is assumed to be located at zero.[11]

6.1. Policy choices

The assumption that politicians are committed to implementing their platform is not satisfying. While a number of empirical studies indicates that preferences over policies are the main driver of the choices that politicians make in office (Levitt, 1996; Chattopadhyay and Duflo, 2004; Lee et al., 2004; Bhalotra and Clots-Figueras, 2014), it would be more appealing to see this behaviour emerge as part of an equilibrium rather than imposing it from the outset. Endowing politicians with policy preferences alongside their office motivations introduces two additional difficulties: First, politicians may want to choose a policy equal to or at least closer to zero than their actual ideal policy in an attempt to fool voters. If they succeed, this would increase their chances of winning the federal election. Second, politicians take into account how their choices affect the policies chosen by other politicians. Particularly the latter issue creates difficulties, since it is hard to track how the decision of a politician to join or not to join a party affects events in the federal election. However, it is possible to incorporate the role of state policies as signals of policy preferences without having to deal with the second type of complication. To do so, I follow (Snyder and Ting, 2002) and Ashworth and Bueno de Mesquita (2008) and assume that politicians only care about policy once elected. The utility of a politician with ideal policy $p \in \{-1, -.5, 0, .5, 1\}$ is now given by

$$\pi_s[y_s + \alpha \, v(|p_s - p|)] \; + \; \pi_f[y_f + \alpha \, v(|p_f - p|)] \,,$$

where α measures the relative weight that politicians attach to policy, v is a decreasing function with $v(0) = 0$, and the notation otherwise follows Section 3. Parties then allow only politicians with certain ideal policies to join. In addition, politicians can freely choose the policy they implement at any stage from the set $\{-1, -.5, 0, .5, 1\}$. All other elements of the game remain unchanged.

The following result shows that politicians are always willing to forgo a higher chance of winning the federal election in favour of implementing their own ideal policy at the state level if α is sufficiently large.

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[11] The median electoral vote can be calculated as follows: Create a distribution of electoral votes by assigning the electoral college votes of the state to the ideal policy of the median voter of the state. Then find the median of this distribution. When there are two parties competing at the federal election, the party closest to the median voter of the state with the median electoral vote wins a majority of electoral votes.

Proposition 9. *Suppose $\alpha > -y_f/v(.5)$. Then any politician must implement their own ideal policy at any point of the game in any equilibrium.*

Proof. See Appendix. ☐

When all politicians always select their ideal point when choosing policy, the utility function considered here simplifies to the one assumed in the benchmark model and all results go through unchanged.

7. Conclusion

Why are the same two parties competing in elections in the US across all levels of government, while more than two parties attract significant vote shares in other countries relying on FPTP such as the UK or Canada? The model of party formation presented in this paper provides a number of potential explanations. Specifically, any two-party equilibrium of the model requires that a number of conditions are satisfied: First, voters in the federal election cannot be too concentrated in the centre of the policy space, as otherwise a centrist independent candidate could successfully enter. Second, politicians need to be sufficiently motivated by career concerns to prevent them from joining parties more targeted at the preferences of voters in specific regions. Finally, an issue that splits regions into two camps, such as ethnic cleavages or an independence movement, must either be less salient than the classical left–right divide or, alternatively, it must be the case that regionalist voters either make up a sufficiently large share of the electorate or are divided regarding policies other than regionalism. If any of these conditions fail, only equilibria with three or more parties exist. In this sense, the necessary conditions for the existence of two-party equilibria suggest explanations for the emergence of new parties in different settings. An interesting prediction of the model is that only centrist parties should be expected to be formed top-down by politicians in advanced stages of their career,[12] while parties located in the political wings emerge as grassroots movements at the regional or local level.

In the absence of a regionalist movement and subject to the conditions given above, the model has an equilibrium featuring a centre-left and a centre-right party very much in line with the party system of the US. Even though some politicians have an opportunity to win a state election with higher probability by joining a third party in this equilibrium, they choose not to do so. The reason is that the equilibrium parties offer a more attractive career, since only their members can win the federal election. Parties thus use their dominance of the federal election to also dominate state elections.

An issue that warrants further investigation is the role of candidate selection. Here, I only provide some relatively basic results indicating that the introduction of primaries can increase the stability of a two-party equilibrium by increasing the payoffs of those politicians who are most likely to defect, namely the politicians who are particularly popular among the voters of their state. In doing so, I assume that primaries lead to the nomination of candidates close to the median voter of the election in question, but it is also conceivable that primaries generally lead to the nomination of more extreme candidates. In the latter case, parties might introduce primaries selectively in some states but not in others and the predictions of the model in this regard are likely to be empirically testable.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix. Proofs

Lemma 3. *The following politicians must win the election in their state with positive probability in any party-formation equilibrium: the politician with platform -1 in state 1, the politician with platform 0 in state 2, and the politician with platform 1 in state 3.*

Proof. Suppose that the politician $g_{1,-1}$ joins a party D of shape $I_D = \{-1\}$. Since no other politicians in the state are able to join this party, voters believe that the candidate of party D has platform -1 with certainty, in or out of equilibrium. Since a strict majority of voters in the state strictly prefers the platform -1 over any other platform, the politician with platform -1 thus wins the state election with certainty, receiving a payoff of at least y_s. If this politician was in a situation where they win the election in their state with zero probability, they would receive a payoff of zero and therefore prefer to deviate and join a party of shape $\{-1\}$. An analogous argument applies to the politician $g_{2,0}$ and the politician $g_{3,1}$. ☐

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[12] Strictly speaking, the model does not allow for the formation of parties by politicians who have already moved beyond the state level. However, the model does allow the possibility of independent candidates in the national election. In parliamentary systems, such as the UK, independent candidates would essentially be forced to form a new party since becoming prime minister requires support of a majority in Parliament.

Proof of Lemma 1. Consider a two-party equilibrium such that $\mathcal{P}^* = \{A, B\}$ and suppose there is a platform $p \in \{-1, 0, 1\}$ such that $p \notin I_A \cup I_B$. This implies that either the politician $g_{1,-1}$, the politician $g_{2,0}$, or the politician $g_{3,1}$ neither joins a party in equilibrium nor runs as an independent and thus wins the state elections with probability zero. This situation cannot be part of two-party equilibrium by Lemma 3. $\qquad\square$

Proof of Lemma 2. Recall that voters observe whether a party nominates a candidate for a state election, but not how many politicians have joined a party. Voting behaviour can therefore only be conditional on which parties have nominated candidates. Let party A be a party that wins the election in some state s with positive probability. If party A wins with positive probability due to the other party not nominating a candidate, then this event does not become any less likely due to additional politicians joining party A. Conditional on facing a competitor, the probability that the candidate of party A wins cannot decrease either due to additional members joining. Either voters cannot detect that a deviation has occurred or the second party has no members left. The probability that party A wins the state election therefore cannot decrease if any politician joins the party with higher probability. Joining party A in state s therefore yields a positive payoff and any politician who has this option can therefore never chose the strategies of remaining passive or joining a party that loses the state election with certainty, given that the latter two strategies lead to a payoff of zero. $\qquad\square$

Proof of Proposition 1. Constellations of two parties that are not consistent with the conditions set out in the statement of the proposition can be divided into two categories: constellations such that either both parties allow politicians with positive platforms to join or both allow politicians with negative platforms to join, and constellations such that one party has the shape $\{0\}$ while the other party admits members with positive and negative platforms. In the latter case it follows from Lemma 1 that the second party must have shape $\{-1, -.5, 0, .5, 1\}$. The two cases will be considered separately.

One party of shape $\{0\}$ and one party of shape $\{-1, -.5, 0, .5, 1\}$

Refer to the party with shape $\{-1, -.5, 0, .5, 1\}$ as party A and refer to the other party as party B. If party B wins any state elections with certainty, then this party also nominates a candidate with platform 0 for the federal election with certainty. As a consequence, any politicians with a platform other than 0 cannot win the federal election. The politician $g_{1,-1}$ would then prefer to join a singleton party to maximise their chance of winning the state election. If, on the other hand, party B does not win any state election with certainty, then there is the possibility that party A wins all state elections. Since a strict majority of voters in state 2 prefers the platform 0 over any other platform, party A can only win the state election if the politician with platform 0 of the state does not join party B. By Lemma 2, said politician must then join party A instead and wins the state election with positive probability. The politician $g_{1,-1}$, on the other hand, must be able to win the election in their state with positive probability as a member of party A by Lemma 3. It thus occurs with positive probability that party A wins all state elections and nominates a politician with platform -1 for the federal election while a politician with platform 0 has won a state election. The latter politician would then have an incentive to run as an independent in the federal election, contradicting equilibrium.

Both parties admit negative platforms or both parties admit positive platforms

Without loss of generality, the proof will focus on the case such that both parties allow at least one politician with a platform smaller than zero to join. Denote these parties as A and B. In equilibrium, the politician $g_{1,-1}$ must win the election in their state with positive probability as a member of either party A or party B by Lemma 3. Without loss of generality, assume that the politician can win the state election as a member of party A.

If party A and party B both nominate politicians with negative platforms for the federal election, a strict majority of voters would strictly prefer a candidate with platform 0, since the federal median voter is located at 0. The same is true if both parties nominate a politician with a positive platform, or if only one party nominates a candidate and this candidate has a platform other than 0. In these situations, a politician with platform 0 who has won a state election could therefore successfully run as an independent candidate in the federal election and would accordingly have an incentive to do so (recall that voters know the platform of any winner of a state election). That both parties allow politicians with negative platforms to join is therefore only possible in a two-party equilibrium if one of the party does not actually attract any members with negative platforms, or at least not in the states where the party can win the state election. However, the proof below shows that any attempt to construct the equilibrium such that only party A has members with negative platforms necessarily results in a situation where both parties may nominate politicians with positive platforms for the federal election, contradicting that no independent candidate can run in equilibrium. The proof is divided into multiple steps.

Moderate politicians must win the elections in states 2 and 3 with positive probability: That the politician $g_{2,0}$ must win the election of the state with positive probability follows from Lemma 3. Consider the politician with platform 0 of state 3. By Lemma 3, the politician with platform 1 in the same state can win the state election with positive probability. One of the equilibrium parties must therefore allow this politician to join while also admitting at least one negative platform. Since parties consist of consecutive platforms, politicians with platform 0 must accordingly also be able to join the party. Lemma 2 therefore implies that the latter politician wins the state election with positive probability.

Party B wins no state election with certainty: Since party A may nominate the politician $g_{1,-1}$ for the federal election, it follows that party B cannot have any members with a negative platform in any state where this party can win the state election

other than state 1. Otherwise the party would nominate this politician for the federal election with positive probability, giving the politician with platform 0 from state 2 or state 3 a chance to run as an independent when party A simultaneously nominates the politician $g_{1,-1}$. It further follows that party B does not win any state elections with certainty as it was assumed that party A can win the election in state 1 and party B would be joined by at least one politician with negative platform in any other state where it wins with certainty by Lemma 2.

A politician with positive platform wins the election in state 2 as a member of Party B with positive probability: Since party B does not win any state elections with certainty, it occurs along the equilibrium path that party A wins all state elections and nominates the politician $g_{1,-1}$ as the sole candidate for the federal election. If any moderate has simultaneously won a state election, this situation would enable the latter politician to run as an independent. It must therefore be the case that no moderates join party A in any state other than state 1. The politician with platform 0 of state 2 can win the state election with positive probability in equilibrium and must therefore do so as a member of party B. Suppose the politician with platform 0 was the only member of party B in state 2. Since a strict majority of voters in state 2 strictly prefers the platform 0 over any other platform, Party B would then win the state election with certainty, but this possibility was already ruled out above. Party B must therefore be joined by a politician with positive platform in the state.

A politician with positive platform wins the election in state 3 as a member of Party A with positive probability: Suppose party A is not joined by any politicians with positive platform in state 3. Since party B does not have any members with negative platform and the median voter of the state prefers positive platforms over negative platforms, this would imply that party B wins the state election with certainty unless it has no members in the state in equilibrium. However, equilibrium requires that the politician with platform 1 wins the state election with positive probability by Lemma 3 and must therefore do so as a member of party B. Furthermore, the politician must join party B with certainty by Lemma 2. However, it was already argued that party B does not win any state elections with certainty.

A profitable deviation exists: It may thus happen that party A nominates a politician with positive platform from state 3 for the federal election while party B nominates a politician with positive platform from state 2. The politician $g_{1,0}$ therefore cannot win the state election with positive probability, as they would otherwise have an incentive to run as an independent in the federal election. However, party A allows politicians with positive and with negative platforms to join and therefore also politicians with platform 0. Furthermore, party A wins the election in state 1 with positive probability. It therefore must occur that the politician with platform 0 of the state wins the state election by Lemma 2 and subsequently has an incentive to run as an independent candidate in the federal election. □

Proof of Proposition 2. Suppose three candidates with known platforms are running for the federal election, one with platform -1, one with platform 1, as well as a moderate with platform $p \in \{-.5, 0, .5\}$. Since voters prefer candidates with platforms close to their ideal points, a voter with ideal point i strictly prefers the moderate candidate over any other candidate if and only if $i \in (\frac{-1+p}{2}, \frac{1+p}{2})$. If $\Lambda_f(\frac{-1+p}{2}, \frac{1+p}{2}) > .5$, the moderate candidate must then win the federal election by the definition of party-formation equilibrium.

Now consider the following situation: in a two-party equilibrium, one party has nominated a candidate with platform -1 for the federal election while the other has nominated a candidate with platform 1. At the same time, there is a moderate politician with platform p who has won a state election. Suppose this moderate deviates and runs for the federal election as an independent candidate. Then the independent candidate wins if $\Lambda_f(\frac{-1+p}{2}, \frac{1+p}{2}) > .5$ by the previous paragraph, given that the state election has resolved any uncertainty about the identity of the politician in question. The deviation is therefore profitable.

To complete the proof, it will be shown that the situation described in the previous paragraph must arise with positive probability in any two-party equilibrium. Proposition 1 and Lemma 1 in combination imply that in any two-party equilibrium one party must allow politicians with platform -1 to join but not politicians with platform greater than zero, while the other party allows politicians with platform 1 to join but not politicians with platform smaller than zero. Refer to these parties as A and B, respectively. Party A (party B) must then win the election in state 1 (state 3) with positive probability with the politician with platform -1 (platform 1) as a member. Otherwise one of these politicians would have a profitable deviation by Lemma 3. As any winner of a state election is nominated by their party with positive probability for the federal election, it thus happens with probability greater than zero that party A nominates a politician with platform -1 while party B nominates a politician with platform 1. At the same time, the politician $g_{2,0}$ must win the state election with positive probability by Lemma 3. Since party shapes consist of consecutive platforms, the same must be true of at least one of the politicians with platforms $-.5$ and $.5$ in the same state. As the identities of state winners are independent of each other conditional on the set of existing parties, it thus occurs with positive probability that a candidate with platform -1 and a candidate with platform 1 are nominated for the federal election while a moderate has won the election in state 2.

A moderate politician therefore has a profitable deviation in any potential two-party equilibrium if there is a strict majority of voters in the federal election located in at least two of the intervals $(-.75, .25)$, $(-.5, .5)$, and $(-.25, .75)$. □

Proof of Proposition 3. Consider a two-party equilibrium such that $\mathcal{P}^* = \{A, B\}$. Any politician can only win a state election with certainty in a two-party equilibrium if they are member of a singleton party with certainty. Otherwise at least one of the parties that the politician joins with positive probability would attract other members with positive probability by Lemma 2. As a consequence, there would be a non-zero chance that the politician misses out on the nomination for the state election. Given that any politician can only win a state election with certainty by joining singleton parties, it is impossible that the politician $g_{1,-1}$, the politician $g_{2,0}$ and the politician $g_{3,1}$ all win the election in their state with certainty, since this would require that three singleton parties are

active. Given that no independent candidates run in a two-party equilibrium, at least one of these politicians therefore achieves a payoff in equilibrium of at most

$$\pi(y_s + y_f)\,,$$

where $0 < \pi < 1$ denotes the probability that the politician wins the state election.

The same politician can achieve a payoff of at least y_s by deviating and joining a singleton party since a strict majority of voters in their state is strictly in favour of their platform. This deviation is profitable if $y_s > \pi(y_s + y_f)$, which can be rewritten as $y_f/y_s < (1-\pi)/\pi$ where $(1-\pi)/\pi > 0$. A necessary condition for the existence of the equilibrium is therefore $y_f/y_s \geq (1-\pi)/\pi > 0$. □

Proof of Proposition 4. The aim of the proof is to demonstrate that an equilibrium consistent with the statement of the proposition exists under the specified conditions. The proof first provides a fuller description of such an equilibrium before systematically ruling out different types of deviations.

Description of the equilibrium: Suppose $\mathcal{P}^* = \{L, R\}$, every politician joins one of the two active parties, and no politician ever runs as an independent. In any state s such that $m_s \leq -.25$ ($m_s \geq .25$) the politician with platform 0 joins party L (party R) and the party joined by the politician wins the state election. In any other state the politician with platform zero mixes over joining either of the two parties such that the median voter of the state is indifferent, who then mixes over electing either party. Party L thus wins the election in state 1 and party R the election in state 3, so both parties compete in the federal election and one of the candidates closest to the federal median voter located at zero wins. Assume indifferent voters mix such that either candidate wins the federal election with equal probability when two candidates are equidistant from zero.

No profitable deviations by voters along the equilibrium path: In the federal election the candidates closest to the median voter wins. Given that voters are fully informed about the platforms of party candidates in the federal election, this behaviour is consistent with equilibrium. In any state s such that $m_s \leq -.25$ voters believe that the politician with platform 0 of the state has joined party L. The median voter therefore at least weakly prefers any potential candidate of party L over any potential candidate of party R and party L winning the state election is thus consistent with equilibrium. An analogous argument can be applied to any state s such that $m_s \geq .25$. It will be argued below that in any other state voter behaviour that is consistent with equilibrium exists.

No politician has an incentive to run as an independent: Part (*i*) of the statement of the proposition is constructed to ensure that no independent candidate in the federal election can be preferred by a strict majority of voters over the candidates of both parties. Accordingly, the equilibrium can be constructed such that independent candidates in the federal election do no win. Next consider a politician who decides to run as an independent in a state election. After this out-of-equilibrium event, it is possible to assign voters the belief that the politician running as an independent has a platform furthest from the location of the state median voter. This is possible since voters observe only which parties nominate candidates, but not the affiliation choices of individual politicians, thus putting no restrictions on voter beliefs. The equilibrium can thus be constructed such that any independent candidate in a state election does not win.

No politician has an incentive to increase their probability of joining party L or party R: Only politicians with platform 0 can switch between equilibrium parties. Holding all other behaviour constant and assigning continuation payoffs accordingly, the choice between party L and party R of the politician with platform 0 in a specific state can be seen as a finite game of incomplete information between this politician and the median voter of the state. An equilibrium of this game is guaranteed to exist.[13] In any such equilibrium the politician with platform 0 must win the state election with positive probability by Lemma 2. In any state s such that $m_s \leq -.25$ ($m_s \geq .25$) the politician with platform 0 would not benefit from joining party R (party L), given that this party loses the state election.

No politician can benefit from remaining passive: Not joining a party is never a profitable deviation since politicians' payoffs cannot be negative and remaining passive gives a payoff of zero.

No politician has an incentive to join a previously passive party: It is sufficient to show that no politician wants to join a singleton party. This is true as it is always possible to assign voters the same belief after a politician has deviated and joined a party that allows more than one type to join as in the case of a deviation to a singleton party. Suppose thus that a politician deviates and joins a singleton party instead of party $P \in \{L, R\}$.

Politicians who cannot win a state election in equilibrium: Consider a politician with platform -1 or $-.5$ in a state s where party R wins, which implies $m_s > -.25$. After the deviation, it is possible to assign voters the belief that the candidate of party R has the platform 0 while the candidate of party L has the platform -1 or $-.5$ (depending on the identity of the deviating politician) since the information set reached is off the equilibrium path. The median voter and any voter with a larger ideal policy thus prefer the candidate of party R and the equilibrium can be constructed such that the deviating politician loses the state election and the deviation is not profitable. An analogous argument applies to politicians with platform $.5$ or 1 who do not win a state election, while politicians with platform 0 always win the state election with positive probability.

Politicians who win a state election in equilibrium with positive probability: As a first step, it will be shown that no such politician can achieve a payoff greater than y_s by joining a singleton party. If the politician wins the state election after deviating and faces two competitors for the federal election, Part (*i*) of the statement of the proposition again ensures that the deviating politician can be made to lose the federal election. A special case is that a deviation has the effect that either party A or party B does not nominate a

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[13] It may be necessary that the politician with platform 0 plays a mixed strategy since it is possible that the act of joining a party affects nomination probabilities in such a way that the median voter always prefers the party *not* joined by the politician with platform 0.

candidate for the federal election, as may happen when state 1 is the unique state won by party L or state 3 is the unique state won by party R. Without loss of generality, focus on the latter case. If any politician of state 3 other than the politician with platform 1 joins a singleton party, voters may believe that the candidate of party R has the platform 1 and the entrant party loses the election. The politician with platform 1, on the other hand, cannot be strictly preferred by a strict majority of voters in the federal election even when competing against only one other candidate and may thus at most win the state election by joining a singleton party. Any politician who can win a state election in equilibrium can therefore achieve a payoff no higher than y_s by deviating. In equilibrium, on the other hand, such a politician wins the federal election with positive probability. This is true by the assumption that each member of the candidate pool for each election is nominated with non-zero probability and because each candidate wins the federal election with equal probability when two extremists compete. For a particular politician, who is nominated for and wins their state elections with probability $0 < \pi_s < 1$ and, conditional on doing so, is nominated for and wins the federal election with probability $0 < \pi_f < 1$, deviating to joining a singleton party is therefore not profitable if

$$\pi_s(y_s \ + \ \pi_f \ y_f) \geq y_s \Leftrightarrow \frac{y_f}{y_s} \geq \frac{1 - \pi_s}{\pi_s \pi_f} > 0 \ .$$

Given that the number of politicians is finite, there thus exists a threshold $\bar{y} > 0$ such that no politician has a profitable deviation if $y_f/y_s \geq \bar{y}$.   □

Proof of Proposition 5. Given the assumption that indifferent voters in the federal election vote for each of their most preferred candidates with equal probability along the equilibrium path, voting behaviour in the federal election is uniquely determined in equilibrium. In this case, always nominating the politician located closest to zero clearly maximises the probability of winning since the federal median voter is located at zero. Next, consider the election in some state s. Conditional on winning the state election, nominating the member of the party closest to zero maximises the probability of winning the federal election since this politician may later be nominated for the federal election and no other politician can win with higher probability. Furthermore, nominating the politician closest to zero also maximises the probability of winning the state election. Suppose, for example, that party L wins the election in state s with less than certainty despite nominating the politician closest to zero. This implies that the median voter of the state has an ideal policy greater than the platform of the candidate of party L: given that the expected platform of the candidate of party R must be greater than that of party L, the median voter would otherwise strictly prefer the candidate of party L over the candidate of party R. But if the median voter is located further to the right than the candidate of party L, the party cannot increase it's probability of winning by nominating a candidate further to the left.   □

Proof of Proposition 6. In general, a large number of different $L - R$ equilibria exists, but many of these differ only in off-path behaviour or in the behaviour of some voters while leaving the outcome of a particular election unchanged and generate the same payoffs. A profile of payoff-relevant behaviour is defined as strategy profile for politicians and a set of election outcomes along the equilibrium path, where an outcome of an election specifies which candidate wins or which candidates tie. The proof starts by showing that at most three profiles of payoff-relevant behaviour can be supported in any $L - R$ equilibrium and that these profiles are identical under no commitment and under primaries. Subsequently, the proof establishes that only the politicians $g_{1,-1}$, $g_{2,0}$ and $g_{3,1}$ may have profitable deviations in an $L - R$ equilibrium. In a third step, it is established that these politicians are more likely to win elections under primaries than under no commitment. Based on these preliminaries, the proof then concludes by showing that $\bar{y}_{PR} < \bar{y}_{NC}$.

A finite number of profiles of payoff-relevant behaviour can be supported across the set of $L - R$ equilibria and the set of such profiles is identical under no commitment and primaries: Given the assumption that indifferent voters in the federal election vote for each of their most preferred candidates with equal probability along the equilibrium path, the outcome of the federal election is uniquely determined in equilibrium, with all candidates located closest to zero winning with equal probability and all other candidates losing with certainty. In state 1, only party L winning the state election is consistent with equilibrium since the median voter of the state prefers any potential candidate of party L over any potential candidate of party R. All eligible politicians must therefore join party L with certainty in this state by Lemma 2, while the behaviour of all remaining politicians is irrelevant. The situation in state 3 is symmetric. In state 2, the politician with platform 0 must win the state election with positive probability in equilibrium by Lemma 3. Any party that this politician joins with positive probability in equilibrium must therefore win the state election with positive probability by Lemma 2 and any such party must accordingly attract all of the remaining eligible politicians by the same lemma. Given these preliminary results, up to three equilibria may exist in state 2: a pure strategy equilibrium where $g_{2,0}$ joins party L and party L wins, a pure strategy equilibrium where $g_{2,0}$ joins party R and party R wins, and a unique mixed strategy equilibrium where $g_{2,0}$ joins either party L or party R with positive probability and both parties win with equal probability. To see that there is at most one equilibrium in mixed strategies, first note that parties L and R are symmetric from the perspective of the politician with platform 0 if they use the same mechanism for candidate selection. Indifference of this politician therefore requires that each party is equally likely to win the state election, which requires mixing by the median voter. Given that the median voter prefers the platform 0 over any other platform, the utility of the median voter from electing party L is strictly larger if politician $g_{2,0}$ joins than if not. The expected utility of the median voter if party L wins is therefore a linear function that is increasing in the probability that $g_{2,0}$ joins the party. Conversely, the utility of the median voter if party R wins is a decreasing linear function of the probability that $g_{2,0}$ joins party L. There may therefore exist at most one mixed strategy of $g_{2,0}$ such that the median voter is indifferent. It has thus been established that at most three profiles of payoff-relevant behaviour can be supported across the set of $L - R$ equilibria, which only differ with respect to the behaviour of voters in state 2 and the choice

of party affiliation of the politician with platform 0 of that state. Importantly, the set of such profiles is identical if both parties us no commitment or both parties use primaries since the nomination probabilities in state 2 are the same in both cases.

Only the politicians $g_{1,-1}$, $g_{2,0}$ and $g_{3,1}$ may have profitable deviations in an $L - R$ equilibrium and the lowest deviation payoff of they achieve in any equilibrium is equal y_s: If Part (*i*) of Proposition 4 is satisfied, no independent or third-party candidates can be preferred by a strict majority of voters in the federal election over both candidates of parties L and R. Accordingly, the equilibrium can be constructed such that independent candidates in the federal election do no win. Furthermore, an $L - R$ equilibrium can be constructed such that independent candidates do not win state elections, since it is always possible to assign the belief to voters that the platform of the independent candidate is the one furthest from the ideal policy of the state median voter. The only profitable deviation a politician may therefore have is to join a previously passive party. If a politician of state 1 other than $g_{1,-1}$ joins a third party, it is possible to assign the belief to voters that only the politician with platform -1 has joined party L. Since a strict majority of voters strictly prefers party L in this case, the party must win the state election and the deviation is not profitable. A similar argument applies to any politician in state 2 other than $g_{2,0}$ and to any politician in state 3 other than $g_{3,1}$. The politicians $g_{1,-1}$, $g_{2,0}$ and $g_{3,1}$, on the other hand, can always win the election in their state with certainty by Lemma 3. Since the equilibrium can be constructed such that these politicians subsequently do not win the federal election as was argued above, they therefore achieve a payoff of y_s by joining a singleton party.

Conditional on a specific profile of payoff-relevant behaviour, the politicians $g_{1,-1}$, $g_{2,0}$ and $g_{3,1}$ are weakly more likely to win the election in their state and strictly more likely to win the federal election conditional on winning a state election: Fix one of the profiles of payoff-relevant behaviour established in the previous step. Start by considering the politician $g_{2,0}$. Since both parties are symmetric from this politician's perspective, consider the case that the politician joins party L. The probability of winning the state election is the same under primaries and no commitment. Conditional on the identity of the winner of the election in state 1, $g_{2,0}$ is equally likely to be nominated for the federal election under both mechanisms. However, primaries reduce the probability that $g_{1,0}$ wins the state election, which increases the probability that $g_{2,0}$ is nominated for the federal election. Primaries also reduce the probability that $g_{3,0}$ wins the election in state 3, making it less likely that party R nominates a politician with platform 0 for the federal election, which increases the probability that $g_{2,0}$ wins.

Next, consider the politician $g_{3,1}$. Primaries increase the probability that this politician is nominated for and wins the state election. Conditional on winning the state election, the probability that the politician is nominated for the federal election does not depend on the mechanism used for candidate selection since the politician with platform 0 and the one closest to the median voter coincide in state 2 and at the federal level. It thus remains to be shown that the politician $g_{3,1}$ wins the federal election with higher probability under primaries conditional on being nominated. Winning the federal election is only possible for this politician if party L either nominates an extremist by accident or if all potential candidates of party L are extremists. Both of these events become more likely under primaries.

Finally, the case of the politician $g_{1,-1}$ is symmetric to the case of the politician $g_{3,1}$.

The thresholds $\bar{y}_{NC}$ and $\bar{y}_{PR}$ exist and $\bar{y}_{PR} < \bar{y}_{NC}$: Denote by σ^* a strategy profile that forms an $L - R$ equilibrium. Let $\pi_{s,p}^s(\sigma^*)$ denote the probability that $g_{s,p}$ is nominated for and wins the election in state s under σ^*. Similarly, $\pi_{s,p}^f(\sigma^*)$ denotes the probability that $g_{s,p}$ is nominated for and wins the federal election under σ^* conditional on having won the election in state s. Since it was shown above that only the politicians $g_{1,-1}$, $g_{2,0}$ and $g_{3,1}$ may have profitable deviations, σ^* forms a two-party equilibrium if Part (*i*) of Proposition 4 is satisfied and

$$\pi_{s,p}^s(\sigma^*)[y_s + \pi_{s,p}^f(\sigma^*)y_f] \geq y_s$$
$$\Leftrightarrow \qquad \frac{y_f}{y_s} \geq \frac{1 - \pi_{s,p}^s(\sigma^*)}{\pi_{s,p}^s(\sigma^*)\pi_{s,p}^f(\sigma^*)} \tag{1}$$

holds for the politicians $g_{1,-1}$, $g_{2,0}$ and $g_{3,1}$. The $L - R$ equilibrium σ^* that exists most widely in the parameter space is the one that minimises the maximum of the right-hand side of the preceding inequality across these three politicians. Since it was shown above that any strategy profile σ^* that forms an $L - R$ equilibrium corresponds to one of at most three profiles of payoff-relevant behaviour, the preceding minimisation problem has a solution and this solution provides the threshold $\bar{y}_{NC}$ or $\bar{y}_{PR}$, depending on which mechanism for candidate nomination parties are using. Furthermore, it was argued above that the probability $\pi_{s,p}^s(\sigma^*)$ is no smaller under primaries than under no commitment for any of the politicians $g_{1,-1}$, $g_{2,0}$ and $g_{3,1}$ and any possible strategy profile σ^*, while the probability $\pi_{s,p}^f(\sigma^*)$ is strictly larger. Since the right-hand side of Inequality (1) is decreasing in $\pi_{s,p}^s(\sigma^*)$ and $\pi_{s,p}^f(\sigma^*)$, this shows that $\bar{y}_{PR} < \bar{y}_{NC}$. $\square$

Lemma 4. *If $r > 2$, any voter with ideal point (i, j) with $i \in [-1, 1]$ and $j \in \{0, r\}$ strictly prefers any candidate with platform (p, j) over any candidate with platform $(p', r - j)$.*

Proof. The statement of the lemma is true since

$$u(|p - i|) + u(0) \geq u(2) + u(0) > u(r) + u(0) \geq u(|p' - i|) + u(r) . \quad \square$$

Proof of Proposition 7. To complete the description of the equilibrium, assume that all eligible politicians join party N or party R, while all remaining politicians remain passive. Since all voters are located in the interval $[-1, 1]$ along the ideological dimension and $r > 2$, any non-regionalist voter prefers any non-regionalist candidate with platform $(p, 0)$ over any regionalist candidate with

platform (p', r) and vice versa by Lemma 4. Accordingly, party N wins the elections in states 1, 2, and 3, party R wins the election in state 4, and any candidate of party N wins the federal election.

It can now be verified that no politician has a profitable deviation. Given the above, the politician with platform $(-1, r)$ in state 4 achieves a payoff of y_s and cannot improve on this through any deviation as long as party N nominates a candidate for the federal election. Consider the possibility of any independent or third-party candidates other than the politician with platform $(-1, r)$ from state 4 running in the federal election. If a non-regionalist enters the race, a fourth of all voters at least weakly prefers the candidate of party R over the entrant while another fourth of voters at least weakly prefers the candidate of party N. If a regionalist enters, all voters in states 1, 2, and 3 strictly prefer the candidate of party N. Therefore, no strict majority strictly prefers any third candidate and a voting equilibrium can be constructed such that these candidates do not win the federal election. In state elections, on the other hand, beliefs over the platform of any independent candidates can be assigned such that these candidates do not win.

It remains to be verified that no politicians can gain from changing their party affiliation. Deviating to not joining a party is never profitable. In state 4, at least half of all voters strictly prefer the candidate of party R over any other candidate and no additional party can enter successfully. Suppose a politician in any other state could win the state election by joining a third party. This deviation achieves a payoff of at most y_s, since third party candidates cannot win the federal election by the argument above. In equilibrium, in contrast, each of the politicians in states 1, 2, and 3 wins the federal election with positive probability, receiving a payoff of $\pi_s(y_s + \pi_f y_f)$ where π_s and π_f are positive probabilities. The deviation is therefore not profitable if $y_s \leq \pi_s(y_s + \pi_f y_f)$, which can be rewritten as

$$\frac{y_f}{y_s} \geq \frac{1 - \pi_s}{\pi_s \pi_f} \ . \tag{2}$$

The right-hand side of the preceding inequality is strictly greater than zero since none of the politicians in states 1, 2, or 3 are nominated for the state election with certainty. □

Proof of Proposition 8. The proof proceeds in three steps: First, it is shown that under the conditions of the statement of the proposition, there must be one party that allows only one particular type of regionalist politician to join in any two-party equilibrium. Second, the proof demonstrates that the other party must then allow all non-regionalist politicians to join. The final step is to show that this constellation must create an opening for an independent candidate.

Assume $r > 2$ and let state s be a state such that a strict majority of voters strictly prefers a policy belonging to the set $\{-1, -.5, 0, .5, 1\} \times \{r\}$ over any other policy. The politician of the state with the corresponding platform is unable to win the federal election since a strict majority of voters is located in $[-1, 1] \times \{0\}$ and all of these voters strictly prefer any non-regionalist over any regionalist by Lemma 4. Therefore, a regionalist always loses against a non-regionalist and even if all parties nominate a regionalist candidate, an independent from a non-regionalist state (who is certain to be located at 0 along the regionalist dimension) could successfully enter the race. The politician from state s who is favoured by a strict majority of voters there must therefore choose a party affiliation that maximises their chances of winning the state election, which is achieved by joining a party that allows no other types of politicians to join. Denote this party as party A.

Let B be the party competing with party A in a two-party equilibrium. By Lemma 3, the politician with platform $(-1, 0)$ of state 1 and the politician with platform $(1, 0)$ must win the elections in their respective states with positive probability as members of party B in equilibrium. By convexity of party shapes, party B must allow all types on non-regionalist politicians to join.

Since party A cannot have any members in state 2, party B must win the state election there with certainty, which implies that all politicians of the state join the party by Lemma 2. Given that all eligible politicians are nominated with positive probability, it therefore occurs along the equilibrium path that party B nominates the politician with platform $(-1, 0)$ from state 1 for the federal election while party A nominates a regionalist and the politician with platform $(-.5, 0)$ from state 2 has won the state election there. If $\Lambda_f((-.75, \infty)) > .5$, the latter politician would be strictly preferred by a strict majority of voters in the federal election over the remaining candidates by Lemma 4 since $r > 2$. The winner of the election in state 2 could therefore successfully run as an independent. Similarly, it occurs with positive probability along the equilibrium path that party B nominates the politician with platform $(1, 0)$ from state 3 for the federal election while party A nominates a regionalist and the politician with platform $(.5, 0)$ from state 2 has won the state election there, giving the latter politician the opportunity to run as an independent if $\Lambda_f((-\infty, .75)) > .5$. □

Proof of Proposition 9. Consider a politician who has won the election in their state. If they implement their own ideal policy, this could imply at worst that they win the federal election with probability 0. The corresponding continuation payoff would be 0. In contrast, choosing any other policy at best yields a payoff of $\alpha v(.5) + y_f$, which would be the case if the politician subsequently wins the federal election with certainty and implements their own ideal policy federally. Choosing the own ideal policy at the state level leads to a strictly greater payoff if $\alpha > -y_f / v(.5)$, taking into account that $v(.5) < v(0) = 0$. □

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