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A Model of Endogenous Targeting in Duopoly

Christian Siemering

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Highlights

- The model takes into account that firms can influence the precision of targeting by investing in the technologies used.
- Ex ante symmetric duopolists always use different levels of precision in each equilibrium of the model proposed.
- In contrast to the high-technology firm, the low-technology firm may increase the industry profit by using a less precise technology.

A Model of Endogenous Targeting in Duopoly*

Christian Siemering‡

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Abstract

The paper investigates welfare effects of targeted advertising in a duopoly. To this end, a game-theoretical model is proposed in which firms can make costly investments in their targeting technology. It can be shown that ex ante identical firms use different technologies in every pure-strategy equilibrium of the technology game. If firms target the same group of consumers, the low-technology firm could increase overall welfare by using a better technology. However, this leads to lower industry profits due to tougher competition among firms.

JEL classification: D43, M37, L13.

Keywords: targeted advertising, technology investments, excessive targeting, welfare.

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†Leibniz University Hannover, School of Economics and Management, Institute of Microeconomics, Königsworther Platz 1, 30167 Hannover. E-mail: siemering@mik.uni-hannover.de.

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1 Introduction

Targeted advertising technologies and big data allow companies to send advertising messages to groups of consumers with common characteristics that are correlated with their preferences for goods and services. For example, targeted advertising based on browser histories, location and time data collected by consumers' smartphones can display advertisement messages to smartphone owners only if they are interested in the advertised product or service and only if they are in proximity to a location where it is offered. This paper defines better targeting technology as the ability to direct informative advertising messages to potential customers with greater precision. The advantage of more precise targeting technology is that costly advertisements can be avoided for individuals who are unlikely to purchase the product or service being offered. This is one reason why investments in targeting technologies, efforts to collect more data, and spending money to acquire additional data have become important decisions for firms that use targeted advertising while competing against each other for the same group of customers.

So far, the welfare effects of targeting are not fully understood. This paper investigates the impact of endogenous targeting under competition on social welfare. Specifically, I analyze a game in which two firms make simultaneous investments in targeting technology, which determines the likelihood that advertising messages will reach their intended consumers. After these investments have been made, the firms choose their advertising strategies and then set prices.

Advertising in the proposed model is assumed to provide not only information about the advertised product's characteristics and its price but also about existence of the advertised product, which is in the tradition of the seminal Butters (1977) model. An originally uninformed consumer who receives the advertising message can decide whether to buy or not. This role of advertising is consistent with advertisement campaigns for new products that should create awareness among consumers but is not limited to this case. The advertisement strategy in this paper is simply the choice of whether to target the market segment a or b. The market segments are characterized by two representative consumers that differ in their taste and willingness to pay. If a consumer receives one or two advertisement messages which are tailored to him or her, he or she will purchase the cheapest product as long as the price is not higher than his or her willingness to pay. I assume that a consumer who receives an advertisement message from a firm targeting a different consumer segment will not buy the advertised product.[1] This assumption can be interpreted in various ways. One possibility is that the targeting decision is made in conjunction with a decision on product differentiation. Another possibility is that the same product is marketed with very different advertising messages, each of which only appeals to one of the two consumer groups to buy.

An interesting feature of my model is that it explains firms technology dispersion as an equilibrium phenomenon. In fact, it turns out that my model has no symmetric equilibrium in which firms use the same pure strategy in the technology game - the first stage of the model. Thus, firms that are ex-ante symmetric may choose different levels of the targeting technology. This is surprising and has important implications for the effect that targeting has on the industry profit and the consumers' surplus.

The paper contributes to a growing literature on the economics of targeting. Several studies, beginning with Butters (1977), have analyzed the effect of advertising on product information and pricing. Grossman and Shapiro (1984) extended this model to markets with horizontal differentiation and examined the impact of informative advertising on competition and product variety. Iyer et al. (2005) also investigated targeting and pricing in a duopoly with horizontal product differentiation. They showed that firms target consumer groups that have a higher preference for their respective product and, therefore, targeted advertisement can be interpreted as a tool to create more horizontal product differentiation. By contrast, in my paper, firms are ex-ante identical and produce homogeneous goods when they target the same consumer group.

Iyer et al. (2005) also argued that targeted advertising can make all competing firms better off. By contrast, Brahim et al. (2011) showed that targeting may also lead to lower profits since it results in fiercer price competition among firms. The analysis of the present paper shows that industry profits may react differently depending on whether a high-technology firm uses a better targeting technology or a low-technology firm.

Similar to my model, Roy (2000) and Galeotti and Moraga-González (2008) study

[1] This assumption also implies that targeting in the present paper cannot be interpreted as a price discrimination device, with is in contrast to the models in, e.g., Esteves and Resende (2016, 2019).

targeting and pricing in a duopoly where firms are ex ante symmetric. One of the main differences to my model is that in these papers, the advertisement technology is given exogenously and perfect, as all targeted consumers receive the ad message at a fixed cost. By contrast, I study endogenous investments into an imperfect targeting technology. The closest related paper is from Karle and Reisinger (2019). They investigate whether exogenous improvements of a common targeting technology may increase or decrease the industry profit in a duopoly. In contrast to their work, the present paper considers investments into a targeting technology which allows that the probability of reaching a targeted consumer to vary across two firms. Other important contributions in this literature are, e.g., Chandra (2009), Esteban et al. (2001), Johnson (2013), Bergemann and Bonatti (2011). The paper is also related to the literature that studies asymmetric price competition and advertising in oligopoly. Closest to my work in this literature is the paper by Anderson et al. (2015), which investigates personalized price competition with costly advertising. In contrast to the present paper, the advertisement cost is fixed, firms can have different cost of quality per assumption, and the offered goods are heterogeneous.

The remainder of this paper is organized as follows. In the next section, I present a three-stage model, in which two firms sequentially choose a technology, decide which consumer they target, and make a pricing decision. Section 3 derives all subgame-perfect equilibria of this game and section 4 provides a welfare analysis. In section 5, I briefly discuss variations and extensions of the basic model. Section 6 concludes. Some of the proofs can be found in the appendix.

2 Model Setup

There are two consumers a and b which are characterized by their willingness to pay for a specific good which can be produced by firms 1 and 2. Let consumer a's willingness to pay A and consumer b's willingness to pay B.[2] Suppose $\{A, B\} \in \mathbb{R}^2$, $B > A$. The firms 1 and 2 are symmetric and compete in a three-stage model with perfect information. The

[2]The parameters used in the paper for different willingness to pay in the different segments can be more generally interpreted measures for the profitability of each segment. An increase in A can be interpreted as an increase in willingness to pay, an increase in demand, or a decrease in production costs for product features that are unique for segment a.

order of play and possible actions are as follows:

Stage 1: firms simultaneously choose technologies $\alpha_1, \alpha_2 \in [0, 1]$,

Stage 2: firms simultaneously choose targets $t_1, t_2 \in \{a, b\}$,

Stage 3: firms simultaneously set prices $p_1, p_2 \geq 0$.

The endogenous technology choice at stage 1 may be interpreted as that firms use big data and targeting technologies themselves or alternatively that they may use different platforms to advertise their products. At stage 2 of the game (the targeting decision) firms design their products and advertising messages so that they suit to a specific consumer. Accordingly, there are three cases: firms both target consumer a, firms both target consumer b or they target different consumers (market segmentation). I assume that that each firm cannot reach both consumers at the same time.[3] The firms' payoff depend on whether the final good market is segmented or not. Under market segmentation, i.e. $t_1 \neq t_2$, firm i's payoffs is given by

$$\pi_i = \begin{cases} \alpha_i p_i - c(\alpha_i), & p_i \leq T_i \\ -c(\alpha_i), & p_i > T_i, \end{cases}$$

where $T_i \in \{A, B\}$ denotes the willingness to pay of firm i's targeted consumer, $i \in \{1, 2\}$. In contrast to the market-segmentation case, let $t_1 = t_2$ and suppose that $\max\{p_1, p_2\}$ is at most the willingness to pay of the targeted consumer. Then we have

$$\pi_i = \begin{cases} \alpha_i p_i - c(\alpha_i), & p_i < p_{-i} \\ [\frac{\alpha_i \alpha_{-i}}{2} + \alpha_i[1 - \alpha_{-i}]]p_i - c(\alpha_i), & p_i = p_{-i} \\ \alpha_i[1 - \alpha_{-i}]p_i - c(\alpha_i), & p_i > p_{-i}. \end{cases}$$

The cost function $c(\alpha_i)$ represents both investments in the targeting technology (e.g. time spend for the development of new classification algorithms) and the cost of data collection. Alternatively, the cost function represents prices to pay for vertically differentiated platforms. To simplify the analysis, let the cost function satisfy the following standard assumptions: $c > 0$, $c' > 0$ for $\alpha_i > 0$ and $c = 0$, $c' = 0$ for $\alpha_i = 0$,

[3]This assumption is relaxed in Section 5.1.

$\lim_{\alpha_i \to 1} c'(\alpha_i) = \infty$, $c'' > 0$. For ease of exposition, I will hereafter refer to the firm with the respectively higher targeting technology as firm 1 and the lower-technology firm as firm 2.

For the case of a symmetric, exogenously given targeting technology, the stages 2 and 3 of the present model are analyzed in Karle and Reisinger (2019). I extend the model of Karle and Reisinger with the costly technology choice in stage 1 and allow for asymmetric competition at stages 2 and 3. As will become clear in the next section, this extension leads to significant changes of the equilibrium analysis and the welfare effects targeting technologies have.

3 Analysis

A pure strategy of firm i in the context of the model is the complete contingent plan $\{\alpha_i, t_i(\alpha_1, \alpha_2), p_i(\alpha_1, \alpha_2, t_1, t_2)\}$. That is, it specifies the technology $\alpha_i \in [0, 1]$ to be chosen at the first stage of the game, then for each chosen pair of technologies $(\alpha_1, \alpha_2) \in [0, 1]^2$ it prescribes whether firm i targets either consumer a or b at stage 2, and finally, for each reachable information set $h \in [0, 1]^2 \times \{a, b\}^2$ at the beginning of stage 3, it predetermines the price $p_i(\alpha_1, \alpha_2, t_1, t_2)$ for firm i to be chosen.

The solution concept is subgame-perfect Nash equilibrium (SPE). That is, each profile of strategies that induces a Nash equilibrium in each of the three subgames is considered to be an equilibrium of the three-stage game. The game is solved with a generalized backward induction procedure.[4]

3.1 Stage 3: Pricing Game

The analysis starts with the simultaneous pricing decision, which is the unique final subgame beginning with the information set where both firms know the targeting and technology choices from previous stages, i.e. $(\alpha_1, \alpha_2, t_1, t_2)$ can be observed by both firms.

First, suppose firm 1 targets consumer b and firm 2 targets consumer a. Then, charging the respective monopoly price $p_1^* = B$ and $p_2^* = A$ is the unique optimal action, leading

[4] As described in Mas-Colell et al. (1995), for instance.

to the payoffs $\pi_1^* = \alpha_1 B - c(\alpha_1)$ and $\pi_2^* = \alpha_2 A - c(\alpha_2)$.

Second, suppose that both firms target consumer b. By charging the price B, firm i earns at least the profit $\alpha_i[1-\alpha_{-i}]B$. Accordingly, undercutting firm $-i$'s price remains profitable as long as $p_{-i} \geq [1-\alpha_{-i}]B$. It has been shown by Karle and Reisinger (2019) that the pricing game has a symmetric mixed-strategy equilibrium for the case $\alpha_1 = \alpha_2 = \alpha$. I will briefly provide the analysis. Suppose firm 2 plays a mixed strategy according to the cumulative distribution $G(p)$ with the range $[[1-\alpha]B, B]$. Then $\{G(p), G(p)\}$ constitutes a Nash equilibrium of the pricing game if

$$\alpha[[1-G(p)] + [1-\alpha]G(p)]p = \alpha[1-\alpha]B. \tag{1}$$

Solving for $G(p)$ gives $\frac{p-[1-\alpha]B}{\alpha p}$.

Now let $\alpha_1 > \alpha_2$. It is easy to show that there is no pure-strategy equilibrium in this case:

(1) If $[1-\alpha_2]B < \min\{p_1, p_2\} \leq B$, undercutting the respectively lowest price is profitable for the firm charging the highest price.

(2) If $[1-\alpha_2]B = p_1 = p_2$, undercutting is profitable for firm 2.

(3) If $p_2 < [1-\alpha_2]B \leq p_1$, it is profitable for firm 1 to set B. Charging higher prices is profitable for firm 2 (while undercutting p_1).

In the following, I identify a mixed strategy Nash equilibrium. First, the analysis can be simplified by iterative elimination of dominated strategies. For firm 1, all prices below $[1-\alpha_2]B$ are dominated by $p_1 = B$. Accordingly, $p_1 \in [[1-\alpha_2]B, B]$ is the rationalizable set of firm 1's prices. By common knowledge, prices that are never a best reply to one of firm 1's remaining prices can be eliminated from firm 2's action space. The remaining set of firm 2's prices is then also $[[1-\alpha_2]B, B]$. No further prices can be eliminated.

Notice that $\{G(p), G(p)\}$ is not a Nash equilibrium in this asymmetric case. The reason is that firm 2 has a profitable deviation by charging $p_2 = [1-\alpha_2]B$, which leads to the payoff $\alpha_2[1-\alpha_2]B > \alpha_2[1-\alpha_1]B$. Firm 1's best response to $p_2 = [1-\alpha_2]B$ is then $p_1 = B$. However, as (1) shows, firm 1 is indifferent between all remaining prices if firm 2 draws its price from $G(p)$. Firm 2 is also indifferent if

$$\alpha_2[[1 - F(p_2)] + [1 - \alpha_1]F(p_2)]p_2 = \alpha_2[1 - \alpha_2]B \tag{2}$$

Solving for $F(p)$ gives

$$F(p) = \begin{cases} \frac{p-[1-\alpha_2]B}{\alpha_1 p}, & \text{if } [1-\alpha_2] \leq p < B \\ 1, & \text{if } p = B. \end{cases}$$

Note that $p_2 = B$ is not a best reply to $F(p)$ although this price is in the range of $G(p)$. However, this technical problem can be ignored because $p_2 = B$ is a zero-probability event if p_2 is drawn from the continuous distribution $G(p)$. Accordingly, $\{F(p), G(p)\}$ constitutes a mixed Nash equilibrium for the asymmetric case in which $\alpha_1 > \alpha_2$, leading to the firm 1's revenue $\alpha_1[1 - \alpha_2]B$ and the revenue $\alpha_2[1 - \alpha_2]B$ for firm 2. Notice that the firm with the better targeting technology earns the higher profit net of targeting technology cost.

The case where both firms target consumer a can be analyzed by the same steps. The analysis of the pricing game is summarized in the following proposition.

Proposition 1. *Suppose the targeting technologies are given by $\alpha_1 \geq \alpha_2$. Then the price setting is as follows.*

- *If firms segment the market, the firm targeting consumer b charges the price B while the firm targeting consumer a charges the price A. The payoffs net of targeting technology costs are $\alpha_i B$ and α_{-i} A, respectively.*

- *If both firms target the same consumer, they play the mixed strategy pricing equilibrium with cumulative distribution functions*

$$F(p) = \begin{cases} \frac{p-[1-\alpha_2]T}{\alpha_1 p}, & \textit{if } [1-\alpha_2] \leq p < T \\ 1, & \textit{if } p = T. \end{cases} \quad \textit{and} \quad G(p) = \frac{p - [1-\alpha_2]T}{\alpha_2 p},$$

where $T \in \{A, B\}$ denotes the willingness to pay of the targeted consumer. The payoffs net of targeting technology costs are $\alpha_1(1-\alpha_2)T$ for firm 1 and $\alpha_2(1-\alpha_2)$ T for firm 2.

3.2 Stage 2: Targeting Game

The targeting subgame is the subset of the game that starts with the information set (α_1, α_2) for both firms, contains the simultaneous targeting decision $t_1, t_2 \in \{a, b\}$, and all subsequent pricing subgames for given (α_1, α_2) and arbitrary (t_1, t_2). This extensive form game can be reduced by replacing all pricing subgames with the respective Nash equilibrium payoffs specified in Proposition 1. The result is a normal form game which can be described by the following payoff matrix[5]:

		Firm 2: a	Firm 2: b
Firm 1	a	$\alpha_1[1-\alpha_2]A,\ \alpha_2[1-\alpha_2]A$	$\alpha_1 A, \alpha_2 B$
	b	$\alpha_1 B, \alpha_2 A$	$\alpha_1[1-\alpha_2]B, \alpha_2[1-\alpha_2]B$

Table 1: Payoff Matrix of the Targeting Game

Suppose $A \leq [1-\alpha_2]B$. Then $\{b, b\}$ is an equilibrium in dominant strategies for the targeting subgame - unique if the inequality is strictly fulfilled. In this case both firms compete in the more profitable market segment, which is the reason why I will refer to this equilibrium as *competition-for-cherries* equilibrium. If $A \geq [1-\alpha_2]B$, the game becomes a coordination game with three Nash equilibria: the two *market-segmentation* equilibria $\{a, b\}$, $\{b, a\}$, and a *mixed-targeting* equilibrium in which both firms choose b with probability $\frac{B-[1-\alpha_2]A}{\alpha_2[A+B]}$, respectively.[6] The expected payoffs of firm 1 and 2 in the mixed Nash equilibrium are $\frac{\alpha_1[2-\alpha_2]AB}{A+B}$ and $\frac{\alpha_2[2-\alpha_2]AB}{A+B}$, respectively.

3.3 Stage 1: Technology Game

At the first stage of the overall game, firms simultaneously choose technologies $\alpha_1, \alpha_2 \in [0, 1]$. As the choice is made simultaneously, the unique proper subgame of this stage is the overall three-stage game itself. We can continue to apply the generalized backwards

[5]Note that we can ignore the technology cost $c(\alpha_1)$ for firm 1 and $c(\alpha_2)$ for firm 2 at this stage. The reason for this is that the costs of the targeting technology are sunk at the time of the targeting decision.

[6]Note that in Karle and Reisinger (2019), the authors identify "competition for cherries" and "market segmentation" targeting regimes. The naming used in this paper is adopted from these authors.

induction procedure by reducing the extensive form by replacing the targeting-subgames with their respective Nash equilibrium outcomes. As we have potentially multiple equilibria at the targeting stage, we have to do this for each possible equilibrium regime, that is, competition-for-cherries, market-segmentation, and mixed-targeting. This comes along with the implicit assumption that firms have consistent expectations about the equilibrium to be played at the next stage of the game, so that there are no coordination failures. Of course, this may be an assumption that is not always fulfilled in reality. However, this is a general and well-known criticism of the concept of subgame perfection equilibrium and also of the Nash equilibrium concept.

At the first stage, firms simultaneously choose technologies $\alpha_1, \alpha_2 \in [0, 1]$. The equilibrium analysis depends on the firms' belief about the equilibrium in the targeting subgame, i.e., whether they expect a market-segmentation, competition-for-cherries or mixed-targeting equilibrium to be played at the second-stage subgame.

In what follows, each of these three cases is analyzed.

Market segmentation

If firms expect a $\{b, a\}$-equilibrium to be played at stage 2, they solve

$$\max_{\alpha_1} \Pi_1 = \alpha_1 B - c(\alpha_1)$$

$$\max_{\alpha_2} \Pi_2 = \alpha_2 A - c(\alpha_2)$$

Let the inverse function of the marginal cost curve denoted by $\gamma(\cdot) := [c']^{-1}(\cdot)$. Notice that by $c'' > 0$, $\gamma(\cdot)$ is well defined. The Nash equilibrium candidate of the technology game under market segmentation is then $\{\alpha_1, \alpha_2\} = \{\gamma(B), \gamma(A)\}$.

Competition for Cherries

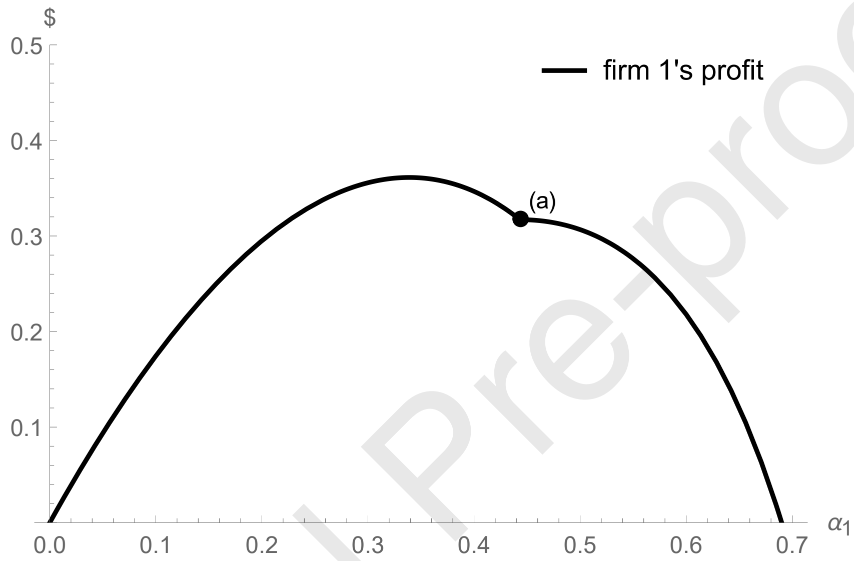
Suppose firms expect the competition-for-cherries equilibrium to be played at stage 2. Firm 1's profit maximization problem is then given by

$$\max_{\alpha_1} \Pi_1 = \begin{cases} \alpha_1[1-\alpha_2]B - c(\alpha_1) & \alpha_1 \geq \alpha_2 \\ \alpha_1[1-\alpha_1]B - c(\alpha_1) & \alpha_1 < \alpha_2. \end{cases}$$

First, it can be shown that there is no symmetric equilibrium.

Lemma 1. *There exists no SPE involving competition-for-cherries in which firms use the same targeting technology.*

The intuition for this result is that changes in the lower technology used by either of the two firms lead to a negative price effect, because the less precise technology determines the lower bound of rationalizable prices (see the analysis in section 3.1). When a firm deviates unilaterally by choosing a less precise technology starting from a symmetric equilibrium, this price effect must be taken into account. This always leads to a profitable deviation, starting from the symmetric equilibrium candidate. Figure 1 illustrates a numerical example.



The figure shows firm 1's profit in a market characterized by competition-for-cherries provided that firm 2 chooses the technology $\alpha_2 = \alpha^*$, where a^* is the technology of a symmetric equilibrium candidate. Point (a) indicates the location where firm 1's objective function is kinked, i.e., $(\alpha^*, \Pi_1(\alpha^*, \alpha^*))$. Clearly, firm 1 has a profitable (unilateral) deviation by choosing a less precise technology. Cost function and parameter values in this example are given by $A = 1,\ B = 2,\ c(\alpha) = 0.5\frac{\alpha^2}{1-\alpha}$.

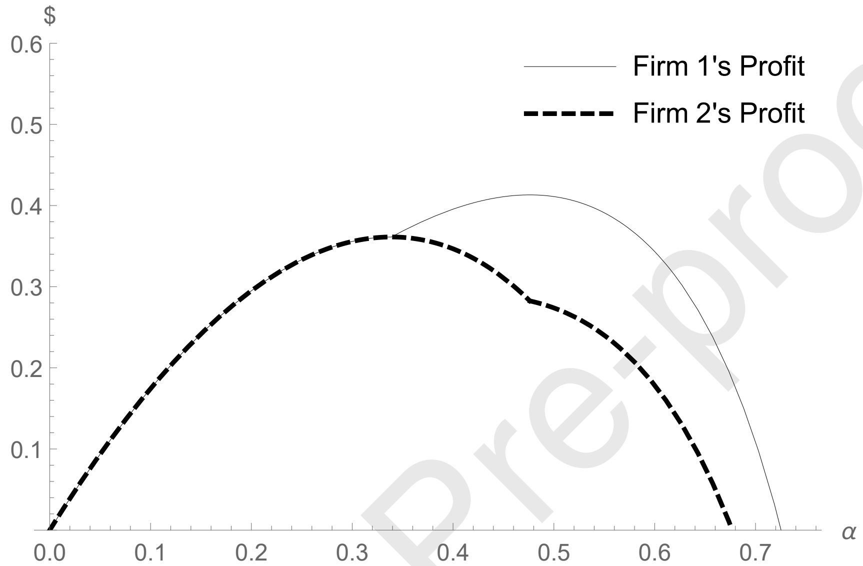
Figure 1: Profitable Deviation under competition-for-cherries

Suppose there exists a SPE with $\alpha_1^* > \alpha_2^*$. Then

$$[1 - \alpha_2^*]B = c'(\alpha_1^*), \tag{3}$$

$$[1 - 2\alpha_2^*]B = c'(\alpha_2^*) \tag{4}$$

are the necessary conditions for technologies α_1^* and α_2^* to be mutual best responses. The intuition is as follows. Increasing the preciseness of the high technology firm only has a quantity effect on the high-technology firm's revenue, which amounts to $[1-\alpha_2^*]B$. As we know from the analysis of the pricing game, the lowest technology used by the firms determines the lower bound of prices that can be rationalized. Accordingly, improving the lower technology α_2 has the same quantity-effect and additionally a negative price effect of $-\alpha_2^* B$ on both firms, since the price competition is intensified. Figure 2 illustrates a numerical example for competition-for-cherries equilibrium.



The figure shows firm 1's (2's) profit in a market characterized by competition-for-cherries provided that firm 2 (1) chooses the low (high) equilibrium technology. Cost function and parameter values in this example are given by $A = 1,\ B = 2,\ c(\alpha) = 0.5\frac{\alpha^2}{1-\alpha}$.

Figure 2: Equilibrium under competition-for-cherries

Mixed Targeting

Suppose firms expect the mixed-targeting equilibrium to be played at stage 2. Then firm 1's objective function is given by

$$\max_{\alpha_1} \Pi_1 = \begin{cases} \frac{\alpha_1[2-\alpha_2]AB}{A+B} - c(\alpha_1) & \alpha_1 \geq \alpha_2 \\ \frac{\alpha_1[2-\alpha_1]AB}{A+B} - c(\alpha_1) & \alpha_1 < \alpha_2. \end{cases} \tag{5}$$

Again, it can be shown that there is no symmetric equilibrium.

Lemma 2. *There exists no SPE involving mixed targeting in which firms use the same targeting technology.*

Next, suppose there exists a SPE in which $(\alpha_1, \alpha_2) = (\alpha_1^*, \alpha_2^*)$, $\alpha_1^* > \alpha_2^*$. Then

$$\frac{[2-\alpha_2^*]AB}{A+B} = c'(\alpha_1^*), \tag{6}$$

$$\frac{2[1-\alpha_2^*]AB}{A+B} = c'(\alpha_2^*) \tag{7}$$

for technologies α_1^* and α_2^* to be mutual best responses.

3.4 Equilibrium

Define $\bar{\alpha}$, $\tilde{\alpha}$ and $\hat{\alpha}$, such that

$$\bar{\alpha} = \gamma(A), \tag{8}$$

$$\tilde{\alpha} = \gamma\left(\frac{[2-2\tilde{\alpha}]AB}{A+B}\right), \text{ and} \tag{9}$$

$$\hat{\alpha} = \gamma\left([1-2\hat{\alpha}]B\right). \tag{10}$$

Recall that $\gamma(\cdot)$ is the well-defined inverse of the marginal cost curve, which ensures that $\bar{\alpha}$, $\tilde{\alpha}$ and $\hat{\alpha}$ are unique. Summarizing the results of the last subsections, conditions for the existence of all types of subgame-perfect equilibria can be identified. They are summarized in the following proposition.

Proposition 2. *Suppose firms first choose their targeting technology, second announce their target (sending one message each) and then set prices. For each B, there exist values $\underline{A}, \bar{A}$ so that:*

1. ***Market Segmentation:*** *If $A \geq \underline{A}$, there exist SPE in which the targeting technologies are given by $\{\alpha_1, \alpha_2\} = \{\gamma(B), \gamma(A)\}$, firm 1 targets consumer b and firm 2 targets consumer a, the prices are $p_1 = B$ and $p_2 = A$. The sum of expected profits is $\gamma(A)A + \gamma(B)B - c(\gamma(A)) - c(\gamma(B))$.*

2. ***Competition for Cherries:*** *If $A < \bar{A}$, there exists a unique SPE in which the targeting technologies are given by $\{\alpha_1, \alpha_2\} = \{\gamma([1-\hat{\alpha}]B), \hat{\alpha}\}$. Both firms target consumer b and charge prices according to Proposition 1. The sum of expected profits is*

$$[\alpha_1 - \alpha_1\alpha_2 + \alpha_2 - \alpha_2^2]B - c(\alpha_1) - c(\alpha_2).$$

3. ***Mixed Targeting:*** *If $A \geq \underline{A}$, there exist SPE in which the targeting technologies are given by $\{\alpha_1, \alpha_2\} = \left\{\gamma\left(\frac{[2-\tilde{\alpha}]AB}{A+B}\right), \tilde{\alpha}\right\}$ and firms play the mixed strategy equilibrium in the targeting game. Prices are charged according to Proposition 1, depending on the targeting game outcome. The sum of expected profits is*

$$[2\alpha_1 - \alpha_1\alpha_2 + 2\alpha_2 - \alpha_2^2]\frac{AB}{A+B} - c(\alpha_1) - c(\alpha_2).$$

The proposition states that if the two market segments are similarly profitable, there is an SPE that involves market-segmentation. In this case, each firm becomes a monopolist in its chosen market segment. The firm in market segment a has no incentive to switch to segment b, as intense price competition would lead to a greater loss of profits than the increase in profits associated with a higher willingness to pay by consumers in segment b.[7] If the markets have a similar profitability, there is also a SPE that involves a targeting equilibrium in mixed strategies.

By contrast, if market segment b is much more profitable compared to segment a, there will be a unique SPE with competition-for-cherries targeting equilibrium.[8] In this case, one firm chooses a more precise targeting-technology at a higher cost, which is compensated by the ability to charge higher prices in expected terms.

The analysis from above also leads to this surprising result:

Proposition 3. *Suppose symmetric firms choose technologies $\alpha_1, \alpha_2 \in [0, 1]$ simultaneously at the first stage. Then there exist no SPE in which firms choose the same targeting technology.*

[7] The value $\underline{A}$ describes the critical lower limit for the profitability of segment a given the profitability of segment b, so that if $A \geq \underline{A}$, there is no incentive to switch from a to b. $\underline{A}$ depends on B and can generally only be implicitly defined.

[8] $\bar{A}$ is the critical upper limit for the profitability of segment a, so that if $A < \bar{A}$, no firm has a profitable deviation by switching to segment a. $\bar{A}$ depends on B and can generally only be implicitly defined.

The finding of Proposition 3 has important implications for the welfare analysis. Under asymmetric competition, technology improvements of the same magnitude may or may not be desirable from a welfare perspective depending on which firm adopts the new technology.

4 Welfare

4.1 Industry Profit and Excessive Targeting

Can the targeting technology be excessive, where excessive means that the industry profit can be increased by using a less precise targeting technology? In what follows I answer this question while using two different measures for the industry profit: the sum of second-stage continuation profits (net of the first-stage targeting technology costs) and the sum of overall profits (considering the first-stage costs). I refer to these measures as ex post and ex ante industry profits, respectively. It seems most natural to look at the ex ante industry profit as this measure considers all the costs that are present in my model. The reason for considering also the ex post industry profits is that this measure is more appropriate for comparing my results with welfare analyses of related papers that assume an exogenous targeting technology.

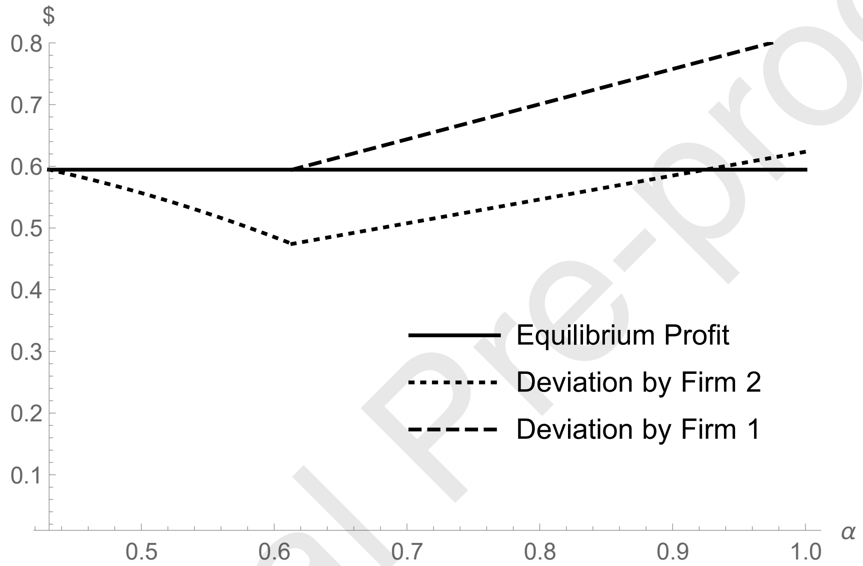
Ex post excessive targeting

Let the industry profit be measured by the sum of stage-two continuation profits of both firms. It is straightforward that there is no excessive targeting under market-segmentation since there are no adverse effects a technology improvement could impose on a competing firm.

Let's turn to the competition-for-cherries equilibrium in which the industry profit is given by $[\alpha_1 - \alpha_1\alpha_2 + \alpha_2 - \alpha_2^2]B$. Partial differentiation of the industry profit respect to α_1 and under the condition that $\alpha_2 = \hat{\alpha}$ leads to $[1-\hat{\alpha}]B > 0$. Hence, firm 1's technology is not excessive because the industry profit increases when firm 1 deviates unilaterally by using a better targeting technology than the equilibrium level. By contrast, partial differentiation with respect to α_2 conditioned on $\alpha_1 = \gamma([1-\hat{\alpha}]B)$ gives

$$[1-\gamma([1-\hat{\alpha}]B) - 2\hat{\alpha}]B \gtreqless 0. \tag{11}$$

In general it is ambiguous whether firm 2's technology is excessive or not. The shape of the marginal cost curve $c'(\cdot)$ is the decisive factor. If the marginal cost curve is flat enough, both equilibrium technologies are rather high so that (11) is less than zero. In this case, firm 2's targeting technology is excessive. The opposite is true if the marginal cost curve is sufficiently steep. Figure 3 illustrates how the ex post industry profit changes if firm 1 or firm 2 deviates unilaterally from the equilibrium by using a more precise targeting technology.



The figure shows the industry's continuation profit in equilibrium (solid line), for unilateral increasements of the targeting technology of firm 1 (dashed line), and firm 2 (dotted line). Cost function and parameter values in this example are given by $A = 0.3,\ B = 1.5,\ c(\alpha) = 0.1\frac{\alpha^2}{1-\alpha}$.

Figure 3: Ex-Post Excessive Targeting in the competition-for-cherries Equilibrium

Consider now the mixed-targeting equilibrium. The ex post industry profit is given by $[2\alpha_1 - \alpha_1\alpha_2 + 2\alpha_2 - \alpha_2^2]\frac{AB}{A+B}$. Partial differentiation of the industry profit respect to α_1 and α_2, given that the respective competitor plays the equilibrium strategy, gives $[2-\tilde{\alpha}]\frac{AB}{A+B} > 0$ for firm 1's targeting technology and $\left[2 - 2\tilde{\alpha} - \gamma\left(\frac{[2-\tilde{\alpha}]AB}{A+B}\right)\right]\frac{AB}{A+B} \gtreqless 0$ for firm 2's targeting technology. Again, firm 1's technology is not excessive. Firm 2's technology is excessive if both technologies are rather high as it is if the marginal cost curve is flat enough, and not excessive otherwise.

Ex ante excessive targeting

Clearly, targeting cannot be excessive in any equilibrium that involves market-segmentation since both firms choose their technology so that their respective monopoly profit is maximized while there is no external effect on the competitor's profit.

Next, we turn to the competition-for-cherries equilibrium. Obviously, if $\alpha_1 \geq \alpha_2$ in the equilibrium of the subgame, the technology of firm 1 cannot be excessive. The reason is that by (3)-(4), for α_2 given, α_1 maximizes π_1 while π_2 is independent from α_1. We have

$$\frac{\partial \Pi_{1+2}}{\partial \alpha_1} = \frac{\partial \pi_1}{\partial \alpha_1} + \frac{\partial \pi_2}{\partial \alpha_1} = 0$$

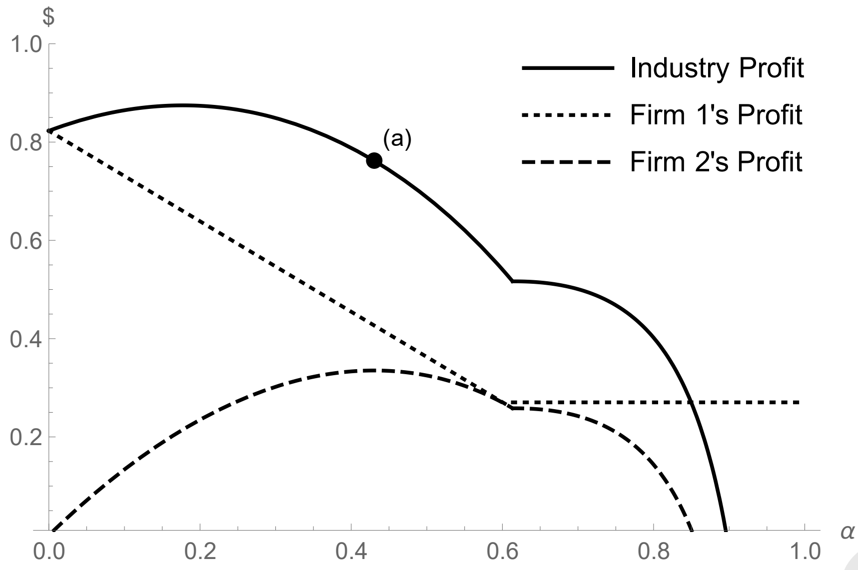
By contrast, α_2 is always excessive in this equilibrium. The reason is that firm 2 does not consider the negative external effect an increase in α_2 imposes on π_1. We have

$$\frac{\partial \Pi_{1+2}}{\partial \alpha_2} = \frac{\partial \pi_1}{\partial \alpha_2} + \frac{\partial \pi_2}{\partial \alpha_2} = \frac{\partial \pi_1}{\partial \alpha_2} = -\gamma\left([1-\hat{\alpha}]\right) B < 0.$$

The analysis for the mixed-targeting equilibrium can be done by the same steps. We then have $\frac{\partial \Pi_{1+2}}{\partial \alpha_1} = 0$ and $\frac{\partial \Pi_{1+2}}{\partial \alpha_2} = -\gamma\left(\frac{[2-\tilde{\alpha}]AB}{A+B}\right)\frac{AB}{A+B} < 0$.

Proposition 4. *There is no SPE which involves excessive targeting by the high-technology firm. In competition-for-cherries equilibria and in the mixed-targeting equilibria, the technology of the low-technology firm can be excessive if the marginal cost curve is sufficiently flat. There is no excessive targeting by any firm when the market is segmented.*

This result is different from Karle and Reisinger (2019), who found excessive targeting by both firms in a competition-for-cherries equilibrium. This equilibrium type arises in their model when the exogenously given targeting technology is sufficiently imprecise. However, excessive targeting by both firms in a competition-for-cherries equilibrium can only occur if the firms are symmetric, i.e., using the same technology. As the present paper has shown, that such equilibria do not exist in my model of endogenous technology choice since the first stage leads to an ex-post asymmetry. Similar as in Karle and Reisinger



The figure shows the industry profit, firm 1's profit, and firm 2's profit depending on firm 2's technology α_2 provided firm 1 plays the equilibrium technology $\gamma([1-\hat{\alpha}]B)$. Point (a) indicates the industry profit in an equilibrium, i.e., the sum of firm 1's and firm 2's profits when $\alpha_2 = \hat{\alpha}$. Clearly, point (a) lies in a region with negative slope, which shows that there is excessive targeting by firm 2. Cost function and parameter values in this example are given by $A = 0.3,\ B = 1.5,\ c(\alpha) = 0.1\frac{\alpha^2}{1-\alpha}$.

Figure 4: Ex-Ante Excessive Targeting in the competition-for-cherries Equilibrium

(2019), my model illustrates that changes in the technology can have a non-monotone effect on the industry profit.

4.2 Social Welfare

Analogue to the industry profit, Social welfare can be defined either by the sum of the expected consumers' surplus and the firms' second stage continuation payoffs, or by the sum of the expected consumers' surplus and firms' overall profits. The first definition does not consider the technology costs and can be used to compare my results to other models where the targeting technology is exogenously given. I will refer to the first definition as ex post social welfare. By contrast, I call the second welfare measure, which considers the cost of the targeting technology, ex ante social welfare. In what follows, I analyze for all equilibrium classes – market-segmentation, competition-for-cherries, and mixed-targeting – whether social welfare can be increased if firms deviate from their equilibrium technology. With a slight abuse of notation, I will always refer to the respective social-welfare measure as SW.

Consumers' Surplus

The consumers' surplus is zero in the case of market-segmentation because each firm is able to extract the whole consumer surplus by charging a price equal to the consumer's willingness to pay.

The consumers' surplus in the competition-for-cherries equilibrium is given by $\alpha_2 B[2-\alpha_2]$.[9] Differentiation with respect to α_2 gives $2B(1-\alpha_2) > 0$. Hence, consumers benefit from better targeting technologies used by firm 2, because a smaller technological advantage of firm 1 leads to tougher competition among firms which in turn results in lower prices. By contrast, the rents of firm 1's technology are completely skimmed. The expected consumers' surplus in the mixed-targeting equilibrium is given by

$$CS = \left[\frac{B-[1-\tilde{\alpha}]A}{\tilde{\alpha}[A+B]}\right]^2 B\tilde{\alpha}[2-\tilde{\alpha}] + \left[\frac{A-[1-\tilde{\alpha}]B}{\tilde{\alpha}[A+B]}\right]^2 A\tilde{\alpha}[2-\tilde{\alpha}].$$

It is easy to show that the derivative of this expression with respect to $\tilde{\alpha}$ is strictly positive. Hence, not surprisingly, consumers are also better off in the mixed-targeting equilibrium if the lower targeting technology becomes more precise.

Market Segmentation

Under market segmentation, it is clear that ex-post social welfare is maximized if both firms would use a perfect targeting technology, i.e. $\alpha_1 = \alpha_2 = 1$. Since the equilibrium technologies fulfil $\{\alpha_1, \alpha_2\} = \{\gamma(B), \gamma(A)\}$, $\gamma(A) < \gamma(B) < 1$, both firms underinvest in the targeting technology from the perspective of ex post social welfare. By contrast, ex ante social welfare is always maximized in the equilibrium described by Proposition 2. The reason is that each firm chooses the technology that maximizes its respective monopoly profit while the consumers' surplus is always zero as firms set prices equal to the consumers willingness to pay.

[9] The derivation of this expression can be found in the appendix.

Competition for Cherries

Under the competition-for-cherries equilibrium, we have

$$\frac{\partial SW}{\partial \alpha_1} = B[1-\alpha_2] \tag{12}$$
$$\frac{\partial SW}{\partial \alpha_2} = B[3-\alpha_1-4\alpha_2]. \tag{13}$$

Equations (12) and (13) imply that welfare is maximized at $\{\alpha_1, \alpha_2\} = \{1, 1/2\}$.[10] Notice that (10) implies that $\hat{\alpha}$ is in the range $(0, 1/2)$. Accordingly, both technologies in the competition-for-cherries equilibrium are too low from the perspective of ex post social welfare.

When the technology costs are taken into account, we have

$$\frac{\partial SW}{\partial \alpha_1} = B[1-\alpha_2] - c'(\alpha_1) \tag{14}$$
$$\frac{\partial SW}{\partial \alpha_2} = B[3-\alpha_1-4\alpha_2] - c'(\alpha_2). \tag{15}$$

The effect of improving firm 2's technology starting from the competition-for-cherries equilibrium value on ex ante social welfare, therefore, is:

$$\begin{aligned} \left.\frac{\partial SW}{\partial \alpha_2}\right|_{\alpha_2=\hat{\alpha}} &= B[3-\alpha_1-4\hat{\alpha}] - c'(\hat{\alpha}) \\ &= B[2-2\hat{\alpha}-\alpha_1] > 0. \end{aligned}$$

The last inequality stems from the fact that $\hat{\alpha} < \frac{1}{2}$. Hence, firm 2's technology in the competition-for-cherries equilibrium is too low from the perspective of ex ante social welfare. Next we consider firm 1's technology. Setting expression (14) equal to zero leads to equation (3), which is the first order condition in firm 1's profit maximization problem. Hence, under the condition that $\alpha_2 = \hat{\alpha}$, firm 1's chooses a welfare maximizing technology. However, as $\alpha_2 = \hat{\alpha}$ is too low from the perspective of ex ante social welfare, (14) implies that firm 1's technology is too high.

The welfare analysis of the mixed targeting case is very similar and can be found in the proof of the following proposition listed in the appendix.

[10] The expression in (12) is strictly positive for all $\alpha_2 < 1$, which implies that welfare is strictly increasing in α_1 and maximized by $\alpha_1 = 1$, which is the highest feasible value. Substituting $\alpha_1 = 1$ into equation (13) and setting it to zero yields the interior solution $\alpha_2 = 1/2$.

Proposition 5. *Consider a SPE in which $\alpha_1 > \alpha_2$. Then:*

1. *From the perspective of ex post social welfare, the investments of firm 1 and 2 are too low, with the exception of firm 2's investment in a mixed-targeting equilibrium, which can also be too high.*

2. *From the perspective of ex ante social welfare, firm 1's (2's) investment in the targeting technology is optimal under market-segmentation, and too high (too low) under competition-for-cherries. In a mixed-targeting equilibrium, it is ambiguous whether the firms' technologies are too low, optimal or too high.*

5 Variations and Extensions

5.1 Price discrimination

The assumption from the basic model that potential customers from one segment will not buy the advertised product if they receive the wrong advertising message is associated with a limitation of the model used. If one considers a market with homogeneous goods and price differentiation between the two segments, it is plausible that firms would want to penetrate both segments simultaneously and simply use the targeting technology to send different advertising messages to customers from different segments.

For the following analysis, I make the simplifying assumption that the pricing decision in one customer segment can be separated from the pricing decision in the other customer segment. The targeting subgame that starts with the information set (α_1, α_2) for both firms can then be described by the following payoff matrix.

		firm 2		
		a	b	ab
firm 1	a	$\alpha_1[1-\alpha_2]A,\ \alpha_2[1-\alpha_2]A$	$\alpha_1 A, \alpha_2 B$	$\alpha_1[1-\alpha_2]A,\ \alpha_2[1-\alpha_2]A + \alpha_2 B$
	b	$\alpha_1 B, \alpha_2 A$	$\alpha_1[1-\alpha_2]B, \alpha_2[1-\alpha_2]B$	$\alpha_1[1-\alpha_2]B, \alpha_2 A + \alpha_2[1-\alpha_2]B$
	ab	$\alpha_1[1-\alpha_2]A + \alpha_1 B, \alpha_2[1-\alpha_2]A$	$\alpha_1 A + \alpha_1[1-\alpha_2]B, \alpha_2[1-\alpha_2]B$	$\alpha_1[1-\alpha_2][A+B], \alpha_2 A + \alpha_2[1-\alpha_2][A+B]$

Table 2: Payoff Matrix of the extended Targeting Game

It is easy to see that the above game has the unique equilibrium in dominant strategies $\{ab, ab\}$, so that both firms advertise and target both consumer segments. In fact, the payoffs in this equilibrium are identical to the payoffs in the competition-for-cherries case of the basic model if B is replaced by $A + B$. Let $\hat{\alpha}_{ab}$ be defined implicitly by

$$\hat{\alpha}_{ab} = \gamma\left([1 - 2\hat{\alpha}_{ab}][A + B]\right)$$

Then, Proposition 2.2 can easily be modified to characterize the optimal technology choice in the extended game.

Proposition 6. *If firms can target both consumer segments without additional costs, there is a unique SPE in with both firms target both market segments a and b. The targeting technologies are given by* $\{\alpha_1, \alpha_2\} = \{\gamma\left([1 - \hat{\alpha}_{ab}][A + B]\right), \hat{\alpha}_{ab}\}$.

The main message of this section is that some key results of the basic model are robust when firms can target both segments. In particular, firms continue to choose different targeting technologies. Furthermore, all results regarding a competition-for-cherries equilibrium can be directly transferred. One may arrive at different results if sending different advertising messages were associated with significant additional costs. When these additional costs are very high, the basic model is applicable. For very low costs, the above analysis is valid and leads to the unique equilibrium described in Proposition 6. For intermediate values, more complex equilibria are conceivable, in which, for example, firm 1 targets both segments and firm 2 only segment a. In an extension of their basic model, Karle and Reisinger (2019) analyze these cases for an exogenously given targeting technology.

5.2 Targeting before technology choice

In the basic model firms first invest in the targeting technology and then decide which market segment to target. Of course, this is not the only way. Alternatively, it is conceivable that firms first select their targets and subsequently choose the precision of the targeting technology. Which of the two alternatives is more realistic in reality depends on many factors. For example, if firms develop specific classification algorithms with internal experts and at the same time offer products with a relatively short life cycle, it is more realistic to assume that the technology choice is more long-term than the targeting decision and therefore will be made first. By contrast, if the collection of very specific data about consumers of the targeted segment is of much greater importance than the algorithms used, it certainly makes more sense to first determine the target and then decide how high the precision or the amount of collected data should be. Fortunately, the order of the first two stages within the context of the present paper does not matter. The reason for this is that I use SPE as an equilibrium concept and provide conditions so that firms have no incentive at the first stage to enforce a different targeting regime than the expected one at the second stage (see Proposition 2 and the associated proof in the appendix). I would like to illustrate this briefly using the example of a SPE that involves market segmentation $\{b, a\}$ and the technologies $\{\alpha_1^*, \alpha_2^*\}$ for firm 1 and 2, respectively. At the second stage of the basic model, market segmentation is an equilibrium of the targeting subgame if $\alpha_2^* \leq 1 - A/B$. As it is argued in the proof of Proposition 2, there is no profitable deviation at the first stage of the basic model if there is no α_2' such that $\alpha_2'[1 - \alpha_2']B - c(\alpha_2') > \alpha_2^* A - c(\alpha_2^*)$. If the alternative ordering is considered, the second stage subgame starts with the information set $\{T_1, T_2\}$ and firms simultaneously choose their respective technology precision. The analysis of these subgames is identical to the analysis conducted in section 3.3. Thus, in the $\{b, a\}$ subgame, the unique subgame-perfect technologies are again α_1^*, α_2^*. Applying backwards induction, firms anticipate the technology choice at the first stage, when choosing their targets. Given that firm 2 chooses a, firm 1 has no profitable deviation since it earns the monopoly profit on the most profitable market by choosing b. Firm 2 has no first-stage profitable deviation if there is no α_2' such that $\alpha_2'[1 - \alpha_2']B - c(\alpha_2') > \alpha_2^* A - c(\alpha_2^*)$. This is exactly the same condition as with the alternative ordering.[11]

6 Conclusions

The paper presents a three-stage model that allows to analyze welfare effects an endogenous choice of a costly targeting technology have on a duopoly market with heterogeneous groups of consumers.

[11]Theoretically, there it is also possible that after firm 2 deviates, it also chooses the more precise technology at the second stage of the alternative stage game. However, this would merely be a switch from one SPE to another, where both SPE also exist in the original game.

I identify conditions under which the targeting equilibria on this market are characterized by market-segmentation, competition-for-cherries, or mixed-targeting. Although firms are assumed to be ex ante symmetric, there exists no symmetric equilibrium in my model in which firms choose a targeting technology of the same precision.

I also show that the technology of the firm with the more precise technology is never excessive. In other words: starting from any equilibrium of the game, lower investments in the targeting technology by the high-technology firm cannot increase total industries profits. By contrast, investments by the low-technology firm can have non-monotone effects on industry profits. If the market is not segmented, consumers benefit from further investments of the low-technology firm, because they lead to tougher price competition. However, industry profits may decrease if the marginal cost of production is sufficiently low around equilibrium.

From the perspective of ex post social welfare, which does not consider the cost of the targeting technology, the equilibrium targeting technologies are too low in most of the cases. When the technology cost are considered, ex ante welfare is maximized under market segmentation while the consumers' surplus is zero. By contrast, in a market characterized by competition-for-cherries, the high-technology firm's technology is too high and the low-technology firm's technology is too low from the welfare perspective. In the mixed-targeting equilibrium, the welfare effects depend on the slope of the technology cost curve and are, therefore, ambiguous in general.

One limitation of my model is clearly that the technology choice is a one-time fixed investment. The results should be transferred with caution on industries in which technologies can be changed easily, for example, if targeted advertisement is conducted by a platform and not by firms themselves. Future research will investigate the robustness of my results in settings with more dynamic investment choices.

A Proofs

Proof of Lemma 1 Suppose a symmetric equilibrium exists such that $\alpha_1 = \alpha_2 = \alpha^*$. Then α^* solves

$$[1-\alpha^*]B - c'(\alpha^*) = 0. \tag{16}$$

Conditioned on firm 2 playing α^*, firm 1's objective function becomes

$$\Pi_1|_{\alpha_2=\alpha^*} = \begin{cases} \alpha_1[1-\alpha^*]B - c(\alpha_1) & \alpha_1 \geq \alpha^* \\ \alpha_1[1-\alpha_1]B - c(\alpha_1) & \alpha_1 < \alpha^*. \end{cases} \tag{17}$$

Clearly, $\Pi_1|_{\alpha_2=\alpha^*}$ is kinked at $\alpha_1 = \alpha^*$. The derivative from the right side is zero, i.e.,

$$\lim_{\substack{\alpha_1\to\alpha^*,\\ \alpha_1>\alpha^*}} \left.\frac{d\Pi_1}{d\alpha_1}\right|_{\alpha_2=\alpha^*} = 0$$

Differentiation from the left gives

$$\begin{aligned}\lim_{\substack{\alpha_1\to\alpha^*,\\ \alpha_1<\alpha^*}} \left.\frac{d\Pi_1}{d\alpha_1}\right|_{\alpha_2=\alpha^*} =& [1-2\alpha^*]B - c'(\alpha^*) \\ =& -\alpha^*B + [1-\alpha^*]B - c'(\alpha^*).\end{aligned}$$

By (16), we have

$$\lim_{\substack{\alpha_1\to\alpha^*,\\ \alpha_1<\alpha^*}} \left.\frac{d\Pi_1}{d\alpha_1}\right|_{\alpha_2=\alpha^*} = -\alpha^*B < 0.$$

This means that firm 1's profit function is strictly decreasing in the left neighborhood of $\alpha_1 = \alpha^*$. Accordingly, firm 1 has a profitable deviation by decreasing α_1. Since $(\alpha_1, \alpha_2) = (\alpha^*, \alpha^*)$ is the unique symmetric equilibrium candidate, this contradicts the existence of a symmetric equilibrium.[12]

[12]In their two-stage game, where firms first invest in advertising and then compete in prices and inventories, Montez and Schutz (2021) also show that there are no SPEs that are both pure and symmetric (see section VII.5 of their online appendix), which means that there will always be asymmetric ex-post investments in equilibrium.

Proof of Lemma 2 If there is a symmetric equilibrium such that $\alpha_1 = \alpha_2 = \alpha^*$, then α^* solves

$$\frac{[2-\alpha^*]AB}{A+B} - c'(\alpha^*) = 0. \tag{18}$$

Conditioned on firm 2 playing α^*, firm 1's objective function becomes

$$\Pi_1|_{\alpha_2=\alpha^*} = \begin{cases} \frac{\alpha_1[2-\alpha^*]AB}{A+B} - c(\alpha_1) & \alpha_1 \geq \alpha^* \\ \frac{\alpha_1[2-\alpha_1]AB}{A+B} - c(\alpha_1) & \alpha_1 < \alpha^*. \end{cases} \tag{19}$$

Differentiation gives

$$\begin{aligned} \lim_{\substack{\alpha_1 \to \alpha^*, \\ \alpha_1 < \alpha^*}} \frac{d\Pi_1}{d\alpha_1}|_{\alpha_2=\alpha^*} =& \frac{[2-2\alpha^*]AB}{A+B} - c'(\alpha^*) \\ =& \frac{[2-\alpha^*]AB}{A+B} - c'(\alpha^*) - \frac{\alpha^* AB}{A+B}. \end{aligned}$$

By (18), we have

$$\lim_{\substack{\alpha_1 \to \alpha^*, \\ \alpha_1 < \alpha^*}} \frac{d\Pi_1}{d\alpha_1}|_{\alpha_2=\alpha^*} = -\frac{\alpha^* AB}{A+B} < 0.$$

By the continuity of the objective function, firm 1 then has a profitable deviation by decreasing α_1. This contradicts the existence of the symmetric equilibrium.

Proof of Proposition 2 We start with the competition-for-cherries case. By the characteristics of $c'(\cdot)$, there is a unique α_2^* that solves (4). Then there is also a unique α_1^* that solves (3). By $[1-\alpha_2^*]B > [1-2\alpha_2^*]B$ and $c''(\cdot) > 0$, $\alpha_1^* > \alpha_2^*$ is fulfilled. Sufficient conditions for α_1^*, α_2^* to be part of a SPE involving competition-for-cherries are then:

$$\alpha_1^*[1-\alpha_2^*]B - c(\alpha_1^*) \geq \alpha_2^*[1-\alpha_2^*]B - c(\alpha_2^*) \quad \text{and} \tag{20}$$

$$\alpha_2^*[1-\alpha_2^*]B - c(\alpha_2^*) \geq \alpha_1^*[1-\alpha_1^*]B - c(\alpha_1^*), \tag{21}$$

which can be rearranged to

$$[\alpha_1^* - \alpha_2^*][1-\alpha_2^*]B - \int_{\alpha_2^*}^{\alpha_1^*} c'(\alpha) d\alpha \geq 0 \quad \text{and} \tag{22}$$

$$\int_{\alpha_2^*}^{\alpha_1^*} c'(\alpha) - [1-2\alpha]B d\alpha \geq 0. \tag{23}$$

The first order conditions (3)-(4) imply that both conditions are fulfilled. Accordingly, neither firm has a profitable deviation.

We continue the analysis with the mixed-targeting case. By the characteristics of $c'(\cdot)$, there is a unique α_2^* that solves (7). Notice that α_2^* is independent of α_1^* and, therefore, can be treated as parameter in (6). Then there is also a unique α_1^* that solves (6). By $\frac{[2-\alpha_2^*]AB}{A+B} > \frac{2[1-\alpha_2^*]AB}{A+B}$, we have $\alpha_1^* > \alpha_2^*$. Accordingly, there exist $\{\alpha_1^*, \alpha_2^*\}$ so that $\alpha_1^* > \alpha_2^*$ and $\Pi_i(\alpha_1^*)$, $\Pi_2(\alpha_2^*)$ are local maxima. Sufficient conditions for α_1^*, α_2^* to be part of a SPE under mixed targeting are then:

1. firm 1 does not deviate profitably by using the technology $\alpha_1 = \alpha_1' < \alpha_2^*$.

2. firm 2 does not deviate profitably by using the technology $\alpha_2 = \alpha_2' \geq \alpha_1^*$.

The first condition is fulfilled if

$$\begin{aligned}
\frac{\alpha_1^*[2-\alpha_2^*]AB}{A+B} - c(\alpha_1^*) \geq& \frac{\alpha_2^*[2-\alpha_2^*]AB}{A+B} - c(\alpha_2^*) \\
\Leftrightarrow [\alpha_1^* - \alpha_2^*]\frac{[2-\alpha_2^*]AB}{A+B} \geq& c(\alpha_1^*) - c(\alpha_2^*) \\
\Leftrightarrow [\alpha_1^* - \alpha_2^*]\frac{[2-\alpha_2^*]AB}{A+B} - \int_{\alpha_2^*}^{\alpha_1^*} c'(\alpha)d\alpha \geq& 0.
\end{aligned}$$

Let the LHS of this condition be Δ_1. Then conditions (6) and (7) imply that $\Delta_1 > 0$, which is illustrated by Figure 5. Hence, firm 1 has no profitable deviation.

firm 2 has no profitable deviation if

$$\begin{aligned}
\frac{\alpha_2^*[2-\alpha_2^*]AB}{A+B} - c(\alpha_2^*) \geq& \frac{\alpha_1^*[2-\alpha_1^*]AB}{A+B} - c(\alpha_1^*) \\
\Leftrightarrow [c(\alpha_1^*) - c(\alpha_2^*)] - \frac{AB}{A+B}[\alpha_1^*[2-\alpha_1] - \alpha_2^*[2-\alpha_2])] \geq& 0 \\
\Leftrightarrow \int_{\alpha_2^*}^{\alpha_1^*} c'(\alpha) - \frac{AB}{A+B}[2[1-\alpha]]d\alpha \geq& 0.
\end{aligned}$$

Let the LHS of this condition be Δ_2. Then conditions (6) and (7) imply that also $\Delta_2 > 0$, which is also illustrated in Figure 25.

It remains to be checked whether firms have unilateral profitable deviations by switching from one targeting regime to another. First suppose the firms expect a market segmentation equilibrium to be played at the second stage. The consistent technology choice

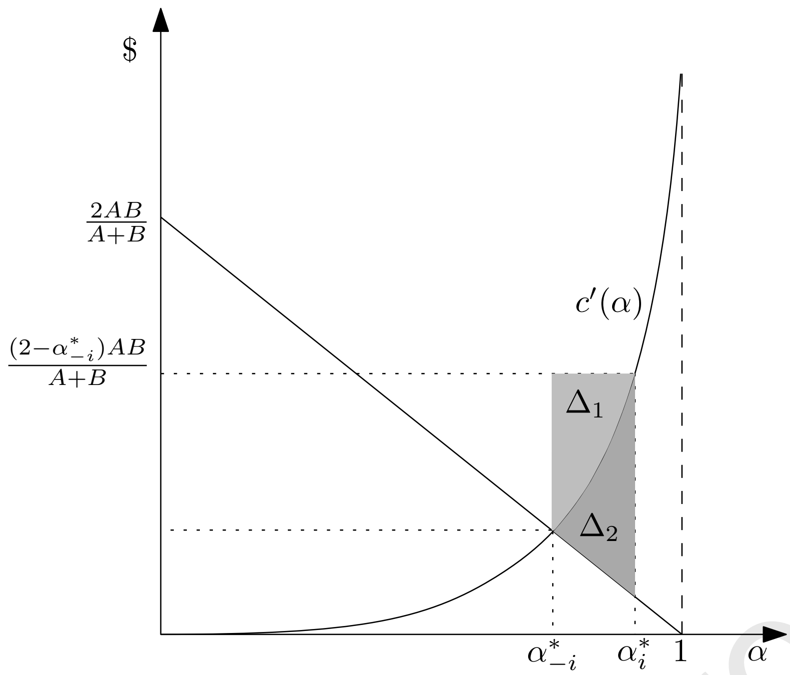


Figure 5: No profitable Deviations.

is then $\{\alpha_1, \alpha_2\} = (\gamma(B), \gamma(A))$, where the condition

$$1 - A/B \leq \gamma(A) < \gamma(B) \tag{24}$$

is fulfilled. Given that firm 2 targets consumer a, firm 1 is the monopolist on the most profitable market and can make the biggest profit possible in the overall game. Therefore, firm 1 has no profitable deviation. Firm 2 can enforce a competition-for-cherries equilibrium by choosing the technology $\alpha'_2 \leq 1 - A/B$. The deviation is profitable if

$$\alpha'_2[1 - \alpha'_2]B - c(\alpha'_2) > \gamma(A)A - c(\gamma(A)). \tag{25}$$

If A is close to B, there exist no $\alpha'_2 \in [0, 1]$ such that the inequality (25) is fulfilled. By contrast, if A is close to zero, there always exist α'_2 so that there is a profitable deviation. Then, by the intermediate value theorem, there exist $\underline{A} \in [0, B)$ so that only if $A \geq \underline{A}$, there is no strictly profitable deviation and there exist a SPE with market segmentation. Second, suppose the firms expect a competition-for-cherries equilibrium to be played. Since the firm with lower technology has the lower profit in this equilibrium and the profit from targeting consumer a would be the same for both firms, it is sufficient to find conditions that firm 2 has no profitable unilateral deviation. If firm 1's technology is sufficiently imprecise, i.e., $\gamma([1-\hat{\alpha}]B) < 1 - A/B$, there does not exist a profitable deviation since a market segmentation targeting equilibrium cannot by enforced by a stage 1 unilateral deviation. Otherwise, there is a profitable deviation if there exist α_2' so that

$$\alpha_2' \geq 1 - A/B \quad \text{and} \tag{26}$$

$$\alpha_2' A - c(\alpha_2') > \hat{\alpha}(1-\hat{\alpha})B - c(\hat{\alpha}), \tag{27}$$

where the first inequality ensures the existence of a market segmentation equilibrium and the second inequality the profitability of firm 2's deviation. If A is close to 0, there exist no $\alpha_2' \in [0,1]$ such that the conditions are fulfilled. By contrast, if A is close to B, there obviously exist α_2' so that there is a profitable deviation. In particular, deviating to $\alpha_2 = \hat{\alpha}$ would be strictly profitable. By the intermediate value theorem, there exist $\bar{A} \in [0, B)$ so that only if $A \leq \bar{A}$, there exist a SPE with a competition-for-cherries targeting regime. Also, by applying the intermediate value theorem, it is straightforward to prove the existence of a mixed targeting equilibrium for sufficiently high values of A.

Proof of Proposition 5 To prove the proposition, the mixed-targeting case has to be analyzed. Under the mixed-targeting equilibrium, we have

$$\frac{\partial SW}{\partial \alpha_1} = \frac{AB[2-\alpha_2]}{A+B} \tag{28}$$

$$\frac{\partial SW}{\partial \alpha_2} = \frac{AB[4+\alpha_2^2[4-\alpha_1-4\alpha_2]] - 2A^2 - 2B^2}{[A+B]\alpha_2^2}. \tag{29}$$

Improvements of the α_1-technology clearly increase welfare in this case while the α_2-welfare effect is ambiguous.

Considering the technology cost, we get

$$\frac{\partial SW}{\partial \alpha_1} = \frac{AB[2-\alpha_2]}{A+B} - c'(\alpha_1) \tag{30}$$

$$\frac{\partial SW}{\partial \alpha_2} = \frac{AB[4+\alpha_2^2[4-\alpha_1-4\alpha_2]] - 2A^2 - 2B^2}{[A+B]\alpha_2^2} - c'(\alpha_2). \tag{31}$$

If firm 1 chooses a best response to a given firm 2's technology, (7) implies that (30) equals zero. The effect of improving firm 2's technology starting from the mixed-targeting equilibrium value on ex ante social welfare is then:

$$
\begin{aligned}
\frac{\partial SW}{\partial \alpha_2}\Big|_{\alpha_2=\tilde{\alpha}} =& \frac{AB[4+\tilde{\alpha}^2[4-\alpha_1-4\tilde{\alpha}]]-2A^2-2B^2}{[A+B]\tilde{\alpha}^2} - c'(\tilde{\alpha}) \\
=& \frac{AB[4+\tilde{\alpha}^2[2-\alpha_1-2\tilde{\alpha}]]-2A^2-2B^2}{[A+B]\tilde{\alpha}^2} + \frac{2[1-\tilde{\alpha}]AB}{A+B} - c'(\tilde{\alpha}) \\
=& \frac{AB[4+\tilde{\alpha}^2[2-\alpha_1-2\tilde{\alpha}]]-2A^2-2B^2}{[A+B]\tilde{\alpha}^2}.
\end{aligned}
$$

It is ambiguous whether this expression is positive or negative.

Consumers' surplus under competition-for-cherries The consumers' surplus in a SPE involving competition-for-cherries is given by:

$$
\begin{aligned}
CS =& \alpha_1[1-\alpha_2]\,[B-\mathbb{E}_F(p_1)] \\
&+ \alpha_2[1-\alpha_1]\,[B-\mathbb{E}_G(p_2)] \\
&+ \alpha_1\alpha_2\,[B-\mathbb{E}(\min\{p_1,p_2\})] \\
=& \alpha_1[1-\alpha_2]\left[B - B\frac{1-\alpha_2}{\alpha_1}\ln\left(\frac{1}{[1-\alpha_2]}\right) - \frac{\alpha_1-\alpha_2}{\alpha_1}B\right] \\
&+ \alpha_2[1-\alpha_1]\left[B - B\frac{1-\alpha_2}{\alpha_2}\ln\left(\frac{1}{[1-\alpha_2]}\right)\right] \\
&+ \alpha_1\alpha_2\left[B - \frac{[1-\alpha_2][2-\alpha_1-\alpha_2]}{\alpha_1\alpha_2}\ln\left(\frac{1}{[1-\alpha_2]}\right)\right] \\
=& \alpha_2 B[2-\alpha_2].
\end{aligned}
$$

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