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Optimal insurance design under mean-variance preference with narrow framing

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ABSTRACT

In this paper, we study an optimal insurance design problem under mean-variance criterion by considering the local gain-loss utility of the net payoff of insurance, namely, narrow framing. We extend the existing results in the literature to the case where the decision maker has mean-variance preference with a constraint on the expected utility of the net payoff of insurance, where the premium is determined by the mean-variance premium principle. We first show the existence and uniqueness of the optimal solution to the main problem studied in the paper. We find that the optimal indemnity function involves a deductible provided that the safety loading imposed on the "mean part" of the premium principle is strictly positive. Our main result shows that narrow framing indeed reduces the demand for insurance. The explicit optimal indemnity functions are derived under two special local gain-loss utility functions – the quadratic utility function and the piecewise linear utility function. As a spin-off result, the Bowley solution is also derived for a Stackelberg game between the decision maker and the insurer under the quadratic local gain-loss utility function. Several numerical examples are presented to further analyze the effects of narrow framing on the optimal indemnity function as well as the interests of both parties.

1. Introduction

The design of optimal (re)insurance has been at the forefront of actuarial and insurance research for decades. The main question arises from the trade-off between the *ex post* indemnity and the *ex ante* premium, which always move along the same direction. Usually, the premium is determined based on some principle or a function of the indemnity. In that case, the main problem boils down to finding the optimal indemnity function. The commonly used indemnity functions include, for example, quota-share and stop-loss functions. The usage of these two kinds of functions gets justified by Borch (1960) and Arrow (1974) in a theoretical way. These two seminal works ignite intensive interest from generations of scholars in such or related problems. We refer interested readers to Albrecher et al. (2017), who comprehensively review the history and some recent developments about optimal insurance problem.

To derive the optimal insurance policy, an optimization criterion is always needed. Based on the applied criteria, the majority of works could be categorized into two main streams – one maximizes the expected utility (EU) and the other one minimizes the risk. There are flourishing results in both streams. Since this paper is built on the EU maximization foundation (in particular the mean-variance criterion), we here first review some recent literature in the first stream, where great efforts are devoted to extending Arrow's classical result. To name a few, Chi and Wei (2018) revisit Arrow's problem by incorporating a background risk of the insurer and higher-order risk attitudes of the decision maker (DM) and establish the optimality of the stop-loss function under some specific dependence structures between the insurable risk and background risk. Chi (2019) extends Arrow's theorem of the deductible to the case of belief heterogeneity, which allows the DM and insurer to hold different subjective beliefs regarding the loss distribution. Jiang et al. (2019) extend Arrow's unilateral problem to the bilateral setting, which considers the interests of both the DM and insurer. They show that the optimal indemnity function

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is of some coinsurance form. For the study of risk-minimizing optimal insurance, interested readers may refer to Cai and Chi (2020) for a comprehensive review of the historical and recent developments in this stream.

Although the EU framework enjoys much popularity in the past quite a few decades, it bears some criticisms due to its failure in explaining human's behavior in some experiments, e.g. the famous Allais paradox and Ellsberg paradox, where DM's decision seems to be "irrational" as per the classical EU theory. Some alternatives or extensions to the EU framework have been proposed and applied, such as dual theory of choice (Yaari, 1987), rank dependent expected utility theory (Quiggin, 1982) and cumulative prospect theory (CPT) (Tversky and Kahneman, 1992). All the above-mentioned three alternative theories have been broadly applied in optimal insurance problems. We refer the interested readers to, for example, Doherty and Eeckhoudt (1995), Sung et al. (2011), Cheung et al. (2015a), Ghossoub (2019), Liang et al. (2022) and the references therein.

Besides the above well known theories, Bell (1982) and Loomes and Sugden (1982) initiate another way to look into those "irrational" behavior, which is called "regret theory". Essentially, regret theory could be understood as a modified EU theory where the DM would experience disutility when comparing the *ex post* optimal decision with the current decision. Afterwards, a similar theory called "disappointment theory" is developed by the same authors (Bell, 1985; Loomes and Sugden, 1986) where the DM experiences disappointment if the realized outcome is worse than the expected level. Both theories get applied in studying optimal insurance problems in some recent literature. See, for instance, Cheung et al. (2015b) and Chi and Zhuang (2022).

In disappointment theory, an important notion called "isolation effect", which is a special case of the more general phenomenon of "framing effect", is re-visited by Loomes and Sugden (1986). The effect happens when the decision problem is presented to the DM with different logical structures. Such notion closely relates to the notion of "narrow framing" in behavioral analysis. Generally speaking, narrow framing refers to the situation where the DM assesses a given risky position in an isolation way rather than mixing it with other risky positions. Such effect has been evidenced by many empirical studies (Guiso, 2015) and popularized among the economists who focus on behavioral studies.

In an insurance market, insurance buyers usually view the insurance itself as a gamble with the insurance company, which exactly falls within the framework of narrow framing meaning that the insured cares about the realized value of an insurance contract and views it in isolation when the real goal is to maximize the EU of her terminal wealth. See, for example, Barberis and Huang (2001) and Barberis and Huang (2009). The concept of narrow framing has already been widely used in insurance and related fields. Particularly, Zheng (2020) considers such gambling motive departure from the standard EU framework. He shows that narrow framing reduces insurance demand due to aversion to risk on the net insurance payoff, i.e. the difference between the insurance indemnity and premium. It is further shown that the optimal insurance contract involves a deductible and coinsurance above the deductible when the safety loading factor is strictly positive in the expected value premium principle. Later on, Chi et al. (2022) assume that the insured adopts an S-shaped local utility function when evaluating the gamble. Such S-shaped utility function stems from the CPT theory. They show that the policyholder under S-shaped narrow framing is more likely to underinsure more negatively skewed risks but to overinsure less negatively skewed risks when only coinsurance is offered.

In this paper, we re-visit the design of optimal insurance under narrow framing when the DM aims to optimize a mean-variance-based objective. The mean-variance model is different from the standard EU model as it cares about both the expected value and volatility of the terminal wealth. Moreover, we generalize the commonly adopted expected-value premium principle to the mean-variance premium principle. These extensions would bring non-trivial technical challenges to the study of optimal insurance problem. The main contributions of this paper are summarized in the following:

- We embed the notion of "narrow framing" into the traditional mean-variance-based profit maximization model, which complements the expected utility model studied in Zheng (2020). We assume that the insurer adopts the mean-variance premium principle, which generalizes the expected-value premium principle considered in Zheng (2020) and Chi et al. (2022).
- It finds that the "variance part" in the premium introduces a proportional coefficient to the stop-loss treaty, which differs in nature with the findings obtained by Zheng (2020). We also show that the existence of deductible can be characterized by the existence of safety loading imposed on the "mean part" in the premium principle.
- We prove the existence and uniqueness of the non-trivial optimal ceded loss function in a rigorous mathematical way, which makes the story of "narrow framing embedded in mean-variance preference" more intuitive and interesting.
- Under the expected-value premium principle and quadratic local gain-loss utility function, we derive the explicit Bowley solution for a one-period Stackelberg game between the DM and insurer. Our work differs from Li and Young (2021) in two aspects: (a) the DM picks the optimal insurance policy under narrow farming, and (b) the insurer also aims to optimize a mean-variance objective.

The rest of the paper is outlined as follows: Section 2 formulates the optimization problem of the DM under the mean-variance model and narrow framing when the insurer employs the mean-variance premium principle. Section 3 studies the DM's demand for insurance when the indemnity function is restricted to the coinsurance type. Section 4 solves the main problem formulated in Section 2. Besides, explicit forms of the ceded loss function are provided when the utility function of the narrow framing is quadratic or linear piece-wise. Section 5 investigates the Bowley solution between the DM and the insurer under the quadratic local gain-loss utility and the expected-value premium principle, serving as a comparison with the corresponding findings in Li and Young (2021) without narrow framing. Section 6 presents some numerical examples to show the implications of our main findings. Section 7 concludes the paper. The proofs of all the main results are delegated to Appendix B.

2. Problem formulation

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, and we assume that all the randomness (losses/risks) considered in this paper are defined on this space. Suppose that an individual (called decision maker, or DM) with constant initial wealth W_0 faces a random loss X, which is a non-negative random variable with support $[0, M]$, where $M < \infty$ represents the essential supremum of X. We assume that $S_X(x) > 0$ for all $x \in \mathbb{R}^+ := [0, \infty)$, where $S_X(x)$ denotes the survival function of X and $F_X(x) = 1 - S_X(x)$, for $x \in \mathbb{R}^+$.

To prevent the potential *ex post* moral hazard, where the DM might be incentivized to manipulate the losses, we follow the literature, such as Huberman et al. (1983) and Carlier and Dana (2003), and impose the incentive compatibility condition on the indemnity functions. That is, we only consider the indemnity functions from the class below:

$$\mathcal{I}=\left\{I:[0,M]\mapsto[0,M]\,\middle|\,I(0)=0,\ 0\le I(x)-I(y)\le x-y \text{ for all } 0\le y\le x\le M\right\}. \tag{2.1}$$

It is easy to see that all the functions in $\mathcal{I}$ are 1-Lipschitz continuous.

Suppose that the insurer charges premium according to some rule $\pi(I(X))$. In this paper, we assume that $\pi(I(X))$ admits the mean-variance premium principle given by

$$\pi(I(X))=(1+\theta)\mathbb{E}(I(X))+\frac{\eta}{2}\mathrm{Var}(I(X)), \tag{2.2}$$

in which $\theta\ge 0$ and $\eta\ge 0$ are safety loading parameters. Note that, the insurance premium in (2.2) reduces to the standard variance premium principle if $\theta=0$ and to the expected-value premium principle if $\eta=0$.

In this paper, we incorporate "*narrow framing*" into the optimal insurance design. Specifically, the DM views the insurance contract not only as a hedging instrument for reducing her risk but also as a gamble with the insurer. The net amount of money received by the DM after buying an insurance contract is $I(X)-\pi(I(X))$. Thus, the DM earns money from the insurer when $I(X)-\pi(I(X))$ is positive, while loses money when it is negative. More precisely, narrow framing means that, when the DM is deciding whether to accept a gamble, she uses another utility function to evaluate the outcome of the net payoff of insurance in isolation to the utility of her end-of-period wealth with insurance. Zheng (2020) studied an optimal insurance design problem under narrow framing with increasing and concave utility functions. Chi et al. (2022) extended the study by employing S-shaped local gain-loss narrow framing functions.

In this paper, we aim to maximize a mean-variance functional of the DM's terminal wealth under narrow framing. Specifically, we want to solve the following problem:

Problem 1 *(Main problem).*

$$\max_{I\in\mathcal{I}} V(I):=\left\{\mathbb{E}(W)-\frac{\gamma}{2}\mathrm{Var}(W)+k\mathbb{E}[g(I(X)-\pi(I(X)))]\right\},$$

where $W=W_0-X+I(X)-\pi(I(X))$ is the end-of-period wealth of the DM. Here $\gamma>0$ describes the DM's aversion to the volatility of her terminal wealth, and the function $g(\cdot)$ is assumed to be a continuous and twice differentiable utility function with $g(0)=0$, $g'(\cdot)>0$ and $g''(\cdot)<0$.

The formulation of above problem is in line with that of the main problem in Zheng (2020). The added term $k\mathbb{E}[g(I(X)-\pi(I(X)))]$ could be understood as the penalty term resulting from the narrow framing. Here, the parameter $k\ge 0$ represents the degree of penalty. A larger k means that the DM has a higher degree of narrow framing. When $k=0$, the objective degenerates to the traditional mean-variance criterion. We remark that for the piece-wise linear utility function employed later in (4.11), which is not twice differentiable everywhere, we shall re-prove our result via a slight modification of our methodology.

Problem 1 is closely related to the following constrained problem

$$\begin{cases}\max\limits_{I\in\mathcal{I}}\mathbb{E}(W)-\dfrac{\gamma}{2}\mathrm{Var}(W)\\ \ \text{s.t. } \mathbb{E}[g(I(X)-\pi(I(X)))]\ge G,\end{cases} \tag{2.3}$$

where the constraint describes the DM's minimum acceptable expected utility of the insurance net payoff, which arises from the narrow framing. The details about the connection between Problems 1 and (2.3) are put in Appendix A. Furthermore, as justified in Appendix A, if k is too large, we will end up with trivial solution – zero insurance for Problem 1. As our main interest is in the non-trivial solution for Problem 1, we adopt the following assumption throughout the rest of paper.

Assumption 2.1. The DM has a reasonable degree of narrow framing such that zero insurance is not the solution for Problem 1.

The range of k that leads to non-zero insurance for Problem 1 is also discussed in detail in Appendix A.

We conclude this section by presenting the following theorem, which shows the existence and uniqueness of the solution to Problem 1.

Theorem 2.1. *There exists a solution to Problem 1. Furthermore, the solution to Problem 1 is unique in the sense that* $\mathbb{P}(I_1(X)=I_2(X))=1$ *if both* I_1 *and* I_2 *solve Problem 1.*

Remark 2.1. Note that the objective of Problem 1 can be written as

$$\begin{aligned}&\mathbb{E}(W)-\frac{\gamma}{2}\mathrm{Var}(W)+k\mathbb{E}[g(I(X)-\pi(I(X)))]\\ &=\mathbb{E}[W_0-X]+\mathbb{E}[I(X)-\pi(I(X))]-\frac{\gamma}{2}\mathrm{Var}(W)+k\mathbb{E}[g(I(X)-\pi(I(X)))]\\ &=\mathbb{E}[W_0-X]+\mathbb{E}[u(I(X)-\pi(I(X)))]-\frac{\gamma}{2}\mathrm{Var}[X-I(X)],\end{aligned}$$

where $u(x)=x+kg(x)$ which still satisfies $u(0)=0$, $u'(\cdot)>0$ and $u''(\cdot)<0$. Thus, Problem 1 can be written as

$$\max_{I\in\mathcal{I}}\ \mathbb{E}[u(I(X)-\pi(I(X)))]-\frac{\gamma}{2}\mathrm{Var}[X-I(X)]. \tag{2.4}$$

Therefore, our Problem 1 can be read as a pure narrow-framing-based EU maximization problem subject to the DM's aversion to the volatility of its retained risk (i.e., $X - I(X)$). Apparently, if the DM is not concerned of the variance, i.e. $\gamma = 0$, then no insurance will be purchased (as per Jensen's inequality). Thus, for our Problem 1, it is the aversion to the volatility of the terminal wealth that pushes the DM to purchase insurance.

We also note that Problem (2.4) relates to the variance-constrained problem of Chi et al. (2020), where they aim to maximize the utility of the end-of-period wealth of the DM with variance constraint. Specifically, the optimization problem they considered is as follows:

$$\begin{cases} \max\limits_{I\in\mathcal{I}} \mathbb{E}[u(W)] \\ \quad \text{s.t. } \mathrm{Var}[I(X)] \le v, \end{cases} \tag{2.5}$$

where $v > 0$ is a prescribed constant. Note that the objective function of the above problem is concave w.r.t. I, and $I = 0$ is strictly feasible. By Slater's condition (cf. Boyd and Vandenberghe, 2004), solving Problem (2.5) is equivalent to solving the problem

$$\max_{I\in\mathcal{I}} \mathbb{E}[u(W)] - \beta \mathrm{Var}[I(X)] \tag{2.6}$$

for some $\beta \ge 0$. A comparison between (2.6) and (2.4) tells that: the variance constraint of Chi et al. (2020) is set by the insurer while our constraint is set by the DM reflecting the degree of aversion to the volatility of its retained risk.

3. Optimal insurance under coinsurance

In this section, we characterize the optimal decision of the DM under the mean-variance criterion with narrow framing when the insurance treaty is of the proportional form. The main task of this section is to provide some elementary characterizations and insights on the optimal ceded loss function investigated in the next section. Some findings would be very helpful for the later analysis as the coinsurance-type indemnity function is a special case in the set $\mathcal{I}$. Specifically, the DM can transfer αX to the insurer and only retains $(1-\alpha)X$, where $\alpha \in [0, 1]$. The insurance premium in this case becomes

$$\pi(I(X)) = (1+\theta)\alpha\mu + \frac{\eta}{2}\alpha^2\sigma^2, \tag{3.1}$$

in which $\mu = \mathbb{E}(X)$ and $\sigma^2 = \mathrm{Var}(X)$. Then, Problem 1 is reduced to the following one-variable optimization problem[1]

$$\max_{\alpha\in[0,1]} V(\alpha) = \max_{\alpha\in[0,1]} \left\{ -\theta\alpha\mu - \frac{\gamma}{2}(1-\alpha)^2\sigma^2 - \frac{\eta}{2}\alpha^2\sigma^2 + k\mathbb{E}g(h(\alpha, X)) \right\}, \tag{3.2}$$

where $h(\alpha, x) = \alpha x - (1+\theta)\alpha\mu - \frac{\eta}{2}\alpha^2\sigma^2$. Note that the second derivative of $V(\alpha)$ is

$$\begin{aligned} V''(\alpha) &= -(\gamma+\eta)\sigma^2 + k\mathbb{E}\left(g''(h(\alpha, X))\left[\frac{\partial h}{\partial \alpha}(\alpha, X)\right]^2 + g'(h(\alpha, X))\frac{\partial^2 h}{\partial \alpha^2}(\alpha, X) \right) \\ &= -(\gamma+\eta)\sigma^2 + k\mathbb{E}\left(g''(h(\alpha, X))\left[\frac{\partial h}{\partial \alpha}(\alpha, X)\right]^2 + g'(h(\alpha, X))(-\eta\sigma^2) \right) < 0. \end{aligned}$$

Thus, $V(\alpha)$ is strictly concave. By the first-order condition, we get that the optimal proportion α^* is given by $\alpha^* = \max\{0, \min\{1, \tilde{\alpha}\}\}$ where $\tilde{\alpha}$ satisfies

$$-\theta\mu + \gamma(1-\tilde{\alpha})\sigma^2 - \tilde{\alpha}\eta\sigma^2 + k\mathbb{E}\left(g'(h(\tilde{\alpha}, X))\frac{\partial h}{\partial \alpha}(\tilde{\alpha}, X) \right) = 0. \tag{3.3}$$

The strict concavity of $V(\alpha)$ guarantees the existence and uniqueness of α^*.

For the special case of $k = 0$ corresponding to the optimal coinsurance design without narrow framing, we denote by $\tilde{\alpha}_0$ the solution of Eq. (3.3). Obviously, one has $\tilde{\alpha}_0 = \frac{\gamma\sigma^2 - \theta\mu}{\sigma^2(\gamma+\eta)} < 1$, which yields the optimal quota-share coefficient $\alpha_0^* = \max\{0, \min\{1, \tilde{\alpha}_0\}\} = \max\{0, \tilde{\alpha}_0\}$. It is interesting to investigate how narrow framing affects the coinsurance policy. The following proposition concludes the main result of this section.

Proposition 3.1. *The optimal proportion of insurance coverage α^* is decreasing in the degree of narrow framing ($i.e.$ k), which further implies that the DM with a positive degree of narrow framing ($i.e.$ $k > 0$) strictly prefers partial insurance even when the insurance premium is actuarially fair ($i.e.$ $\theta = 0, \eta = 0$). As a direct consequence, it holds that $0 \le \alpha^* \le \alpha_0^*$, that is, the optimal quota-share coefficient without narrow framing serves as an upper bound of the one derived with narrow framing.*

Note that the local utility function g is strictly increasing, which results in that a DM with a positive degree of narrow framing should purchase less coinsurance than a DM without narrow framing. This finding is exactly what we obtained in Proposition 3.1, which echoes the finding in Zheng (2020). Although Proposition 3.1 only applies to the proportional insurance, we will show later that in general situations the marginal coverage for the loss in excess of the deductible point will be reduced in the presence of narrow framing. Another important message delivered by Proposition 3.1 is that full insurance can never be the solution to Problem 1. This conclusion will be helpful in proving Theorem 4.1 in the next section.

[1] We slightly abuse the notation V, which represents the objective function.

4. Optimal insurance design with mean-variance preference under narrow framing

In this section, we fully characterize the solution to Problem 1 under Assumption 2.1. It is also proved that the optimal indemnity I^* also has a deductible under certain mild conditions and is incentive compatible.

Theorem 4.1. *Under Assumption 2.1, the optimal indemnity $I^* \in \mathcal{I}$ for Problem 1 solves the equation*

$$I^*(x) = \left(\pi(I^*(X)) + L^{-1}\left(\frac{\gamma}{2\lambda_2}x - \frac{\lambda_1}{2\lambda_2} - \pi(I^*(X))\right)\right)_+, \tag{4.1}$$

in which $L^{-1}(\cdot)$ is the inverse function of $L(\cdot)$, which is defined as

$$L(z) := z - \frac{k}{2\lambda_2}g'(z).$$

Moreover, if $I^*(x) > 0$, then the marginal coverage $I^{*\prime}(x)$ satisfies

$$I^{*\prime}(x) = \frac{\gamma}{2\lambda_2}\left(1 - \frac{k}{2\lambda_2}g''(I^*(x) - \pi(I^*(X)))\right)^{-1} \in [0,1], \tag{4.2}$$

where λ_1 and λ_2 are two KKT coefficients determined by Eqs. (B.10)∼(B.13). Further, the marginal coverage is increasing if $g'''(\cdot) \geq 0$, i.e., $(I^*)''(x) \geq 0$. Moreover, for the optimal indemnity I^*, there is a deductible point $D \geq 0$.[2] In particular, $D = 0$ if the safety loading factor $\theta = 0$, and $D > 0$ if the safety loading factor $\theta > 0$.[3]

Compared with Proposition 3 in Zheng (2020), our Theorem 4.1 presents a more clear structure for the optimal indemnity function. Some other findings are similar to those in Zheng (2020), such as that the slope of I^* is within [0, 1] (thus I^* is in $\mathcal{I}$), and the marginal coverage is increasing if $g'''(\cdot) \geq 0$. Another interesting finding is that, though we adopt the mean-variance premium principle, the deductible is also proven to be 0 if the safety loading imposed on the mean part is 0 regardless of the safety loading applied to the variance part. This partially generalizes the statement in Proposition 3(i) of Zheng (2020) to the situation that the deductible can still be 0 if one adopts the standard variance premium principle.

Remark 4.1. Now, we examine two extreme cases with the results of Theorem 4.1. First, when $k \to \infty$ (which will be greater than k_0 eventually), it is not hard to observe that $I^{*\prime}(x) \to 0$. Since $I^*(0) = 0$, we have $I^*(x) \equiv 0$, which explains that when the individual places an extremely high weight on the expected utility of the net payoff of insurance, zero insurance will be purchased.

Second, when $k = 0$, we have $L(z) = z$. Then, $L^{-1}(z) = z$, which results in

$$I^*(x) = \left(\frac{\gamma}{2\lambda_2}x - \frac{\lambda_1}{2\lambda_2}\right)_+ = \frac{\gamma}{2\lambda_2}\left(x - \frac{\lambda_1}{\gamma}\right)_+.$$

Based on Eqs. (B.12) and (B.13), it is easy to find that $2\lambda_2 = \gamma + \eta$ and $\lambda_1 = \theta + \gamma\mathbb{E}[X] - (\eta + \gamma)\mathbb{E}[I^*(X)]$. Then, the above optimal indemnity function could be written as

$$I^*(x) = \frac{\gamma}{\gamma + \eta}(x - d)_+, \tag{4.3}$$

where $d = \frac{\lambda_1}{\gamma}$ solves the equation $\theta = \gamma\int_0^d F_X(x)dx$, which agrees with the corresponding result of Corollary 3.1 in Li and Young (2021).

Remark 4.2. Although Theorem 4.1 characterizes the optimal indemnity function for Problem 1, the explicit structure might not be available if the utility function g is not specified. Nevertheless, we can still determine the upper and lower bounds for I^* in Theorem 4.1, which provide extra insights into the structure of I^*.

First, note that when $k > 0$, $L(z) = z - \frac{k}{2\lambda_2}g'(z) < z$. In other words, $L^{-1}(z) > z$, which results in

$$I^*(x) > \left(\frac{\gamma}{2\lambda_2}x - \frac{\lambda_1}{2\lambda_2}\right)_+ = \frac{\gamma}{2\lambda_2}\left(x - \frac{\lambda_1}{\gamma}\right)_+.$$

Thus, I^* is bounded from below by the proportional stop-loss function $I_L(x) := \frac{\gamma}{2\lambda_2}(x - \frac{\lambda_1}{\gamma})_+$.

Second, Theorem 4.1 tells that a deductible point D exists for I^* if $\theta > 0$. By Eq. (B.14), we can derive that $D = \frac{\lambda_1 - kg'(-\pi(I^*))}{\gamma}$. Moreover, when $k > 0$, Eq. (4.2) tells that $I^{*\prime}(x) < \frac{\gamma}{2\lambda_2}$. Hence, I^* is bounded from above by another proportional stop-loss function $I_U(x) := \frac{\gamma}{2\lambda_2}(x - \frac{\lambda_1 - kg'(-\pi(I^*))}{\gamma})_+$. See Fig. 1.

When $k > 0$, the proportion $\frac{\gamma}{2\lambda_2}$ is strictly smaller than $\frac{\gamma}{\gamma+\eta}$ according to Eq. (B.13). Thus, in the presence of narrow framing, the DM will under-insure the loss in excess of the deductible point (as compared with the coverage under (4.3) when narrow framing is absent). This agrees well with the result in Section 3. However, generally it is difficult to compare d in (4.3) with D in I^* of Theorem 4.1 due to the implicit forms of both deductible points. Such comparison could be done by using specific examples.

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[2] $I^*(x) = 0$ for $x \leq D$, and $I^*(x) > 0$ for $x > D$.

[3] Note that whether D is zero only depends on the expected-value safety loading θ, but independent on the variance safety loading η. Similar observations can be also found in Li and Young (2021).

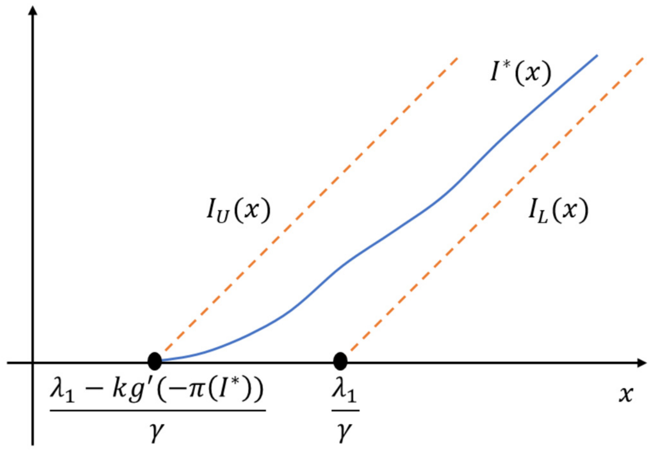

Fig. 1. The lower and upper bounds for I^*.

In the next two sub-sections, we present some closed or semi-closed solutions for Problem 1 when the local gain-loss function g is taking some specific forms. This allows a more straightforward comparison between our results and those in Zheng (2020) and Li and Young (2021).

4.1. g is a quadratic local gain-loss function

In this section we consider a special case when g is a quadratic local gain-loss function

$$g(w) = w - bw^2, \tag{4.4}$$

where $0 < b < \frac{1}{2M}$ is a constant.[4] In this case, g is continuous and twice differentiable everywhere. As we see in the following proposition, the optimal indemnity is in the form of proportional stop-loss. For simplicity, in the following we use $\mathbb{E}(I^*)$ and $\pi(I^*)$ to denote $\mathbb{E}(I^*(X))$ and $\pi(I^*(X))$ respectively.

Corollary 4.1. *Under the quadratic local gain-loss utility $g(\cdot)$ as given by (4.4), the optimal indemnity function for Problem 1 is given by*

$$I^*(x) = \frac{\gamma}{\lambda + 2bk}\left(x - \frac{d}{\gamma}\right)_+, \tag{4.5}$$

where λ, d are determined by the following two equations

$$\lambda = \eta + \gamma - 2kb\eta\mathbb{E}(I^*) + k\eta(1 + 2b\pi(I^*)),$$
$$d = \Big(-(\gamma + \eta) - \eta k(1 + 2b\pi(I^*)) + 2bk\eta\mathbb{E}(I^*) - 2bk(1+\theta)\Big)\mathbb{E}(I^*) + \theta + \gamma\mathbb{E}(X) + \theta k(1 + 2b\pi(I^*)).$$

If the expected-value premium principle is used, the above result can be further simplified.

Corollary 4.2. *Under the quadratic local gain-loss utility function $g(\cdot)$ as given by (4.4) and the expected-value premium principle $\pi(I(X)) = (1+\theta)\mathbb{E}[I(X)]$, the optimal indemnity function for Problem 1 is given by*

$$I^*(x) = \frac{\gamma}{\gamma + 2bk}\left(x - \frac{d_1}{\gamma}\right)_+, \tag{4.6}$$

where

$$d_1 = \inf\{d \in [0, \gamma M] : \kappa_1(d) \ge 0\}, \tag{4.7}$$

and[5]

$$\kappa_1(d) = d + \frac{\gamma + 2bk(1-\theta^2)}{\gamma + 2bk}\gamma\int_{\frac{d}{\gamma}}^{M} S_X(x)dx - (1+k)\theta - \gamma\mathbb{E}[X].$$

[4] The constraint on b is used to guarantee that the local gain-loss utility $g(\cdot)$ is increasing with the local payoff $I(x) - \pi(I(X))$, since M is the essential supremum of loss X.

[5] It is easy to verify that $\kappa_1'(d) \ge 0$.

Note that, after some rearrangements,

$$\kappa_1(d) = d + (1-\theta^2)\gamma \int_{\frac{d}{\gamma}}^{M} S_X(x)dx - (1+k)\theta - \gamma\mathbb{E}[X] + \frac{\gamma^2\theta^2}{\gamma+2bk}\mathbb{E}\left[\left(X-\frac{d}{\gamma}\right)_+\right],$$

which shows that $\kappa_1(d)$ is decreasing in k. This indicates that a larger k leads to a larger d_1, which shows that the deductible point increases w.r.t. the degree of narrow framing. This specific example shows that the DM will under-insure its risk under narrow framing.[6]

Comparing the optimal indemnity functions in Corollaries 4.1 and 4.2, it is not difficult to see that

$$\lambda = \gamma + \eta(1+k) + 2kb\eta(\pi(I^*) - \mathbb{E}[I^*]) > \gamma$$

if $\eta > 0$. Hence, the marginal indemnity function for the loss in excess of deductible is reduced if using the mean-variance premium principle. If $k = 0$ (in the absence of narrow framing), Li and Young (2021) has shown that the deductible point is not affected by the safety loading for the variance component in the mean-variance premium principle. This may no longer hold in the presence of narrow framing (i.e., $k > 0$).

The results in Corollaries 4.1 and 4.2 are close to the result in Corollary 1 of Zheng (2020), where they considered an example in which both the DM's terminal wealth and the net payoff of insurance are assessed by quadratic utility functions under the expected-value premium principle. However, they did not provide explicit optimal deducible point D. To further compare the optimal indemnity function under the mean-variance preference and the one under the quadratic utility, we consider the following problem

$$\max_{I\in\mathcal{I}}\left\{\mathbb{E}(W) - \frac{\hat{\gamma}}{2}\mathbb{E}(W^2) + k\,\mathbb{E}[g(I(X) - \pi(I(X)))]\right\}, \tag{4.8}$$

with $0 < \hat{\gamma} \le \frac{1}{W_0}$, and $g(\cdot)$ is a quadratic function given by (4.4). The following proposition gives the explicit solution to Problem (4.8).

Proposition 4.1. *Under the expected-value premium principle $\pi(I(X)) = (1+\theta)\mathbb{E}(I(X))$, the optimal indemnity function for Problem (4.8) is given by*

$$I^*(x) = \frac{\hat{\gamma}}{\hat{\gamma}+2bk}\left(x - \frac{d_2}{\hat{\gamma}}\right)_+, \tag{4.9}$$

where

$$d_2 = \inf\{d \in [0, \hat{\gamma}M] : \kappa_2(d) \ge 0\}, \tag{4.10}$$

and[7]

$$\kappa_2(d) = d + (1-\theta^2)\hat{\gamma}\int_{\frac{d}{\hat{\gamma}}}^{M} S_X(x)dx - (1+k)\theta + \hat{\gamma}W_0\theta - \hat{\gamma}(1+\theta)\mathbb{E}[X].$$

Generally, optimizing the mean-variance objective is different from optimizing the expected utility under the quadratic utility function as the former is linear in the mean while the latter is quadratic in the mean (Collins and Gbur, 1991). By comparing Corollary 4.2 and Proposition 4.1, we have the following findings.

- If $\hat{\gamma} = \gamma$, i.e. the DM adopts the quadratic utility function with the same averse coefficient as the mean-variance preference setting, the optimal indemnity functions (4.6) and (4.9) have the same slope but different deductible points. Notably, d_1 in (4.6) does not depend on the DM's initial wealth[8] while d_2 in (4.9) depends on the DM's initial wealth. To compare d_1 and d_2 when $\hat{\gamma} = \gamma$, we re-write $\kappa_1(d)$ and $\kappa_2(d)$ as

$$\kappa_1(d) = d + (1-\theta^2)\gamma\int_{\frac{d}{\gamma}}^{M} S_X(x)dx - (1+k)\theta - \gamma\mathbb{E}[X] + \gamma\theta\frac{\gamma\theta}{\gamma+2bk}\mathbb{E}\left[\left(X-\frac{d}{\gamma}\right)_+\right],$$

$$\kappa_2(d) = d + (1-\theta^2)\gamma\int_{\frac{d}{\gamma}}^{M} S_X(x)dx - (1+k)\theta - \gamma\mathbb{E}[X] + \gamma\theta\left(W_0 - \mathbb{E}[X]\right).$$

[6] In Remark 4.2, we only show that the marginal indemnity above the deductible will be reduced if the DM has narrow framing. In the case where g is a quadratic function, we further show that the deductible point becomes larger.

[7] It is easy to verify that $\kappa_2'(d) \ge 0$.

[8] This is well explained in Remark 2.1, where Problem 1 can be re-formulated as (2.4).

Thus, if $W_0 > \mathbb{E}[X] + \frac{\gamma\theta}{\gamma+2bk}\mathbb{E}[(X - \frac{d_1}{\gamma})_+]$ (the DM is wealthier), then $d_2 < d_1$, which means that the DM will purchase more coverage under the quadratic utility than under the mean-variance preference. On the contrary, if $W_0 < \mathbb{E}[X] + \frac{\gamma\theta}{\gamma+2bk}\mathbb{E}[(X - \frac{d_1}{\gamma})_+]$ (the DM is poorer), then $d_2 > d_1$, which means that the DM would like to purchase less coverage under the quadratic utility than under the mean-variance preference.

- Note that (4.9) can also be written as

$$I^*(x) = \frac{\hat{\gamma}}{\hat{\gamma} + 2bk}\left(x - \hat{d}_2\right)_+,$$

where $\hat{d}_2 = \inf\{d \in [0, M] : \hat{\kappa}_2(d) \geq 0\}$, where

$$\hat{\kappa}_2(d) = d + (1-\theta^2)\int_d^M S_X(x)dx - \frac{(1+k)\theta}{\hat{\gamma}} + W_0\theta - (1+\theta)\mathbb{E}[X].$$

Since $\hat{\kappa}_2(d)$ is increasing in $\hat{\gamma}$, a larger $\hat{\gamma}$ leads to a smaller $\hat{d}_2$. Note that $\hat{\gamma} \in (0, 1/W_0]$, under the quadratic utility function the DM will at most purchase

$$I^*(x) = \frac{1}{1 + 2bkW_0}(x - d_{min})_+,$$

where $d_{min} = \inf\{d \in [0, M] : \tilde{\kappa}_2(d) \geq 0\}$, where

$$\tilde{\kappa}_2(d) = d + (1-\theta^2)\int_d^M S_X(x)dx - k\theta W_0 - (1+\theta)\mathbb{E}[X].$$

In other words, if the DM is wealthy enough (e.g., $W_0 \to \infty$), she will not purchase any insurance.
The mean-variance preference is not dependent on the DM's initial wealth, and the coefficient γ has no upper bound. If $\gamma \to \infty$, then the variance component in (2.4) will play the dominating role in the objective function, which pushes the DM to purchase full insurance (i.e., $I^*(x) = x$). This phenomenon can also be seen more clearly in (4.6) under the expected-value premium principle.

The above comparisons show that the decision of the DM whose preference is captured by expected utility can heavily depend on her initial wealth while the decision of the DM who has mean-variance preference is mostly affected by her aversion to the volatility. For a wealthy DM who is very concerned of the volatility, applying expected utility and mean-variance may yield significantly different demands for insurance.

4.2. g is a piece-wise linear function

In this section we slightly weaken the assumption that the utility function $g(\cdot)$ is twice differentiable everywhere by taking into account possible loss aversion. Specifically, the local gain-loss utility function $g(\cdot)$ has the following piece-wise linear form:

$$g(I(x) - \pi(I(X))) = \begin{cases} I(x) - \pi(I(X)), & \text{if } I(x) \geq \pi(I(X)); \\ -\beta[\pi(I(X)) - I(x)], & \text{if } I(x) < \pi(I(X)), \end{cases} \tag{4.11}$$

where $\beta > 1$ measures the degree of loss aversion, which implies that $\lim_{x\to 0^-} g'(x) > \lim_{x\to 0^+} g'(x)$. Also, note that even though $g(\cdot)$ is not continuously differentiable at zero, $g(\cdot)$ is still globally concave. In the following proposition, we present the optimal indemnity under this type of utility function.

Proposition 4.2. *Under the local gain-loss utility $g(\cdot)$ as in (4.11), the optimal indemnity function for Problem 1 is given by*

$$I^*(x) = \begin{cases} \frac{\gamma}{2\lambda_2}(x - \overline{D}), & \text{if } x \geq \overline{D} + \frac{2\lambda_2}{\gamma}\pi(I^*(X)); \\ \pi(I^*(X)), & \text{if } \underline{D} + \frac{2\lambda_2}{\gamma}\pi(I^*(X)) \leq x < \overline{D} + \frac{2\lambda_2}{\gamma}\pi(I^*(X)); \\ \frac{\gamma}{2\lambda_2}(x - \underline{D}), & \text{if } \underline{D} \leq x < \underline{D} + \frac{2\lambda_2}{\gamma}\pi(I^*(X)); \\ 0, & \text{if } x < \underline{D}. \end{cases} \tag{4.12}$$

where $\underline{D} = \dfrac{\lambda_1 - k\beta}{\gamma}$ and $\overline{D} = \dfrac{\lambda_1 - k}{\gamma}$. When $\theta = 0$, the lower deductible barrier $\underline{D} = 0$. When $\theta > 0$, $\underline{D}$ is strictly positive.

Under the expected utility theory with the local gain-loss utility $g(\cdot)$ as in (4.11), it was shown in Proposition 4 of Zheng (2020) that the optimal indemnity function also has two layers. However, our result shows that the optimal ceded loss function is based on the mixture of proportional insurance and stop-loss insurance as displayed in Proposition 4.2. We attribute such difference to the mean-variance premium principle being used here.[9]

[9] Generally, $2\lambda_2 > \gamma$ if $\eta > 0$. See Eq. (B.13).

5. Bowley solution under the expected-value premium principle and narrow framing

Section 4 presents the main results of this paper by assuming that the safety loading factors θ and η are given. In a monopolistic market where the insurer has the absolute power to adjust these safety loading factors, a Stackelberg game between the DM and the insurer is interesting to investigate. That is, the DM first selects the optimal indemnity function in response to the given safety loading factors, and then the insurer adjusts the safety loading factors to optimize its objective. The solution to such a Stackelberg game is called the Bowley solution. For sake of comparison, in what follows we denote the DM's level of volatility aversion γ by γ_1. Since the general solution to Problem 1 is quite complex, to make the problem tractable, we focus on a simple setting in this section where the DM's local gain-loss utility function is of the quadratic form (4.4) and the variance safety loading factor $\eta = 0$.

The DM's problem:

As per Corollary 4.2, given the safety loading factor $\theta \geq 0$, the optimal indemnity I^* is given by

$$I^*(x) = \alpha(x-\xi)_+, \tag{5.1}$$

where $\alpha = \frac{\gamma_1}{\gamma_1+2bk}$ and

$$\xi = \inf\{d \in [0, M] : \kappa(\xi) \geq 0\} \tag{5.2}$$

where

$$\kappa(\xi) = \gamma_1\left(\xi - \int_0^\xi S_X(x)dx\right) - 2\alpha bk\theta^2 \int_\xi^M S_X(x)dx - (1+k)\theta,$$

and $\inf \emptyset = M$. Note that

$$\kappa'(\xi) = \gamma_1(1 - S_X(\xi)) + 2\alpha bk\theta^2 S_X(\xi) > 0 \text{ for } \xi \in (0, M)$$

and

$$\begin{aligned} \kappa(0) &= -2\alpha bk\theta^2\mathbb{E}[X] - (1+k)\theta \leq 0, \\ \kappa(M) &= \gamma_1(M - \mathbb{E}[X]) - (1+k)\theta. \end{aligned}$$

Here k is treated as a fixed parameter representing the DM's degree of narrow framing, then if $\kappa(M) = \gamma_1(M - \mathbb{E}[X]) - (1+k)\theta \leq 0$, we have $\xi = M$ and the DM purchases zero insurance. Thus, it suffices to discuss the case when $\kappa(M) = \gamma_1(M - \mathbb{E}[X]) - (1+k)\theta \geq 0$, which also includes zero insurance when the equality holds. In other words, the insurer only needs to focus on $\theta \in \left[0, \frac{\gamma_1(M-\mathbb{E}[X])}{1+k}\right]$, and then ξ uniquely satisfies

$$\gamma_1\left(\xi - \int_0^\xi S_X(x)dx\right) - 2\alpha bk\theta^2 \int_\xi^M S_X(x)dx = (1+k)\theta. \tag{5.3}$$

The insurer's problem:

Knowing the DM's optimal choice of the indemnity function I^* (as shown by (5.1)) for a given $\theta \geq 0$, we assume that the insurer would choose the optimal safety loading factor θ^* that maximizes a mean-variance functional of her terminal wealth W^i:

$$\theta^* = \arg\max_\theta \mathbb{E}(W^i) - \frac{\gamma_2}{2}\mathrm{Var}(W^i), \tag{5.4}$$

where $\gamma_2 > 0$ describes the insurer's volatility aversion level, and

$$W^i = W_0^i - I^*(X) + \pi(I^*(X)) \tag{5.5}$$

with W_0^i being the initial surplus of the insurer.

With $\pi(I^*(X)) = (1+\theta)\mathbb{E}(I^*(X))$, Problem (5.4) could be written as

$$\begin{cases} \max\limits_{(\theta,\xi)\in[0,\frac{\gamma_1(M-\mathbb{E}[X])}{1+k}]\times[0,M]} & \theta\mathbb{E}[(X-\xi)_+] - \alpha\frac{\gamma_2}{2}\mathrm{Var}[(X-\xi)_+] \\ & \text{s.t. } (\theta, \xi) \text{ satisfies Eq. (5.3).} \end{cases} \tag{5.6}$$

Note that Eq. (5.3) is a quadratic equation regarding θ. Since $\theta \geq 0$, for a selected deductible point $\xi \in [0, M]$, we can figure out the unique corresponding safety loading factor θ from Eq. (5.3):

$$\begin{aligned} \theta &= \frac{\sqrt{(1+k)^2 + 8b\alpha\gamma_1 k(\xi - \mathbb{E}[X\wedge\xi])\mathbb{E}[(X-\xi)_+]} - (1+k)}{4b\alpha k\mathbb{E}[(X-\xi)_+]} \\ &= \frac{2\gamma_1(\xi - \mathbb{E}[X\wedge\xi])}{\sqrt{(1+k)^2 + 8b\alpha\gamma_1 k(\xi - \mathbb{E}[X\wedge\xi])\mathbb{E}[(X-\xi)_+]} + (1+k)} \in \left[0, \frac{\gamma_1(M-\mathbb{E}[X])}{1+k}\right]. \end{aligned} \tag{5.7}$$

Thus, there is a one-to-one relationship between θ and ξ.

By inserting (5.7) into (5.6), we see that Problem (5.6) becomes

$$\max_{\xi\in[0,M]} f(\xi) := \frac{2\gamma_1(\xi - \mathbb{E}[X\wedge\xi])\mathbb{E}[(X-\xi)_+]}{\sqrt{(1+k)^2 + 8b\alpha\gamma_1 k(\xi - \mathbb{E}[X\wedge\xi])\mathbb{E}[(X-\xi)_+]} + (1+k)} - \alpha\frac{\gamma_2}{2}\mathrm{Var}[(X-\xi)_+]. \tag{5.8}$$

We conclude this section by presenting the following proposition, which shows the Bowley solution for this section in the presence of narrow framing. The proof is omitted since it involves only the use of first order condition.

Proposition 5.1. *The optimal deductible point ξ^* for Problem (5.8) satisfies the first order condition $f'(\xi^*) = 0$, where*

$$f'(\xi) = \gamma_1 \frac{(\mathbb{E}[X\wedge\xi] - \xi)S_X(\xi) + (1 - S_X(\xi))\mathbb{E}[(X-\xi)_+]}{\sqrt{(1+k)^2 + 8b\alpha\gamma_1 k(\xi - \mathbb{E}[X\wedge\xi])\mathbb{E}[(X-\xi)_+]}} + \alpha\gamma_2(1 - S_X(\xi))\mathbb{E}[(X-\xi)_+]. \tag{5.9}$$

Then, the pair (θ^*, ξ^*), where θ^* is derived by inserting ξ^* into Eq. (5.7), is the Bowley solution to the Stackelberg game.

Remark 5.1. Notably, in Eq. (5.7) the safety loading factor θ decreases in k. In other words, for a given deductible point ξ, the insurer needs to lower the safety loading factor to attract or keep a DM with higher degree of narrow framing.

Remark 5.2. In the work of Li and Young (2021), they explored the Bowley solution of a one-period mean-variance Stackelberg game in insurance. One can observe that if we let $k \to 0$, the optimal indemnity function in (5.1) is equal to the one in Corollary 3.1 in Li and Young (2021) with $\eta = 0$. Moreover, Eq. (5.9) implies that the optimal deductible ξ^* is independent of the parameter k when $\gamma_2 = 0$, and the derived optimal deductible ξ^* is as the same as the optimal one in Li and Young (2021) with $\eta = 0$.

6. Numerical examples

In this section, we present some numerical examples to further analyze the effect of narrow framing on the indemnity function, efficient frontier and the Bowley solution.

6.1. The effect of narrow framing on the indemnity function

In this subsection, we investigate the effect of narrow framing on the demand for insurance using the results in Section 4. Due to the simple form of the indemnity function when assuming a quadratic utility function g, we confine ourselves to this specific utility function. More specifically, we adopt the following setup for the numerical example:

- The ground-up loss X follows a truncated exponential distribution,[10] i.e.

$$f_X(x) = \frac{\frac{1}{100}e^{-\frac{x}{100}}}{1 - e^{-10}}, \quad x \in [0, 1000].$$

- $g(x) = x - \frac{x^2}{2000}$.
- $\theta = 0.2$, $\eta = 0.2$ and $\gamma = 0.1$.

Apparently, in the absence of narrow framing, or when $k = 0$, we recover the result of Li and Young (2021) (see Eq. (4.3)). When $k > 0$, we already know from Remark 4.2 that the marginal coverage for the loss in excess of the deductible will be reduced. However, it is not clear whether the deductible point will be larger or not. We thus change the value of k from 0 to 0.7 to see the effect of narrow framing on the overall coverage. Fig. 2 shows the optimal indemnity functions for $k = 0, 0.3$ and 0.7, and Fig. 3 shows the optimal slopes and deductible points for different values of k. These results indicate that a higher degree of narrow framing will result in a lower demand for insurance, which echoes the main result in Section 3.

6.2. The effect of narrow framing on the efficient frontier

Let $L_I = X - I(X) + \pi(I(X))$, which is the terminal loss faced by the DM. Then, the objective of Problem (2.3) could be written as

$$\min_{I\in\mathcal{I}} \mathbb{E}[L_I] + \frac{\gamma}{2}\mathrm{Var}[L_I]. \tag{6.1}$$

Note that

$$\begin{aligned}\mathbb{E}[L_I] =& \mathbb{E}[X - I(X) + \pi(I(X))] \\ =& \mathbb{E}[X] - \mathbb{E}[I(X)] + (1+\theta)\mathbb{E}[I(X)] + \frac{\eta}{2}\mathrm{Var}[I(X)] \\ =& \mathbb{E}[X] + \theta\mathbb{E}[I(X)] + \frac{\eta}{2}\mathrm{Var}[I(X)] \geq \mathbb{E}[X]\end{aligned}$$

[10] Note that here the truncation point is set to be 1000. It could be verified that $\mathbb{P}(X > 1000) \approx 0$ if the mean of the original exponential distribution is 100.

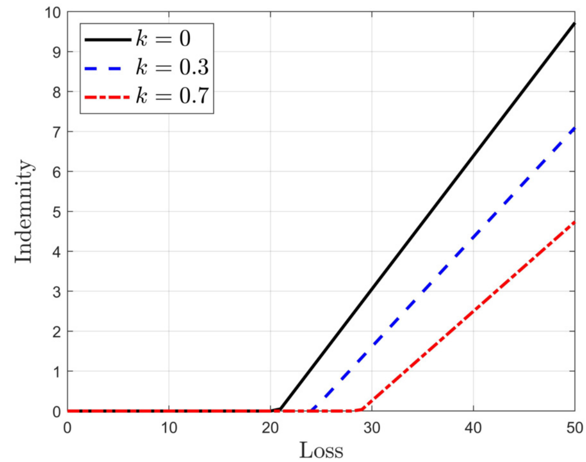

Fig. 2. The optimal indemnity functions for $k = 0, 0.3$ and 0.7.

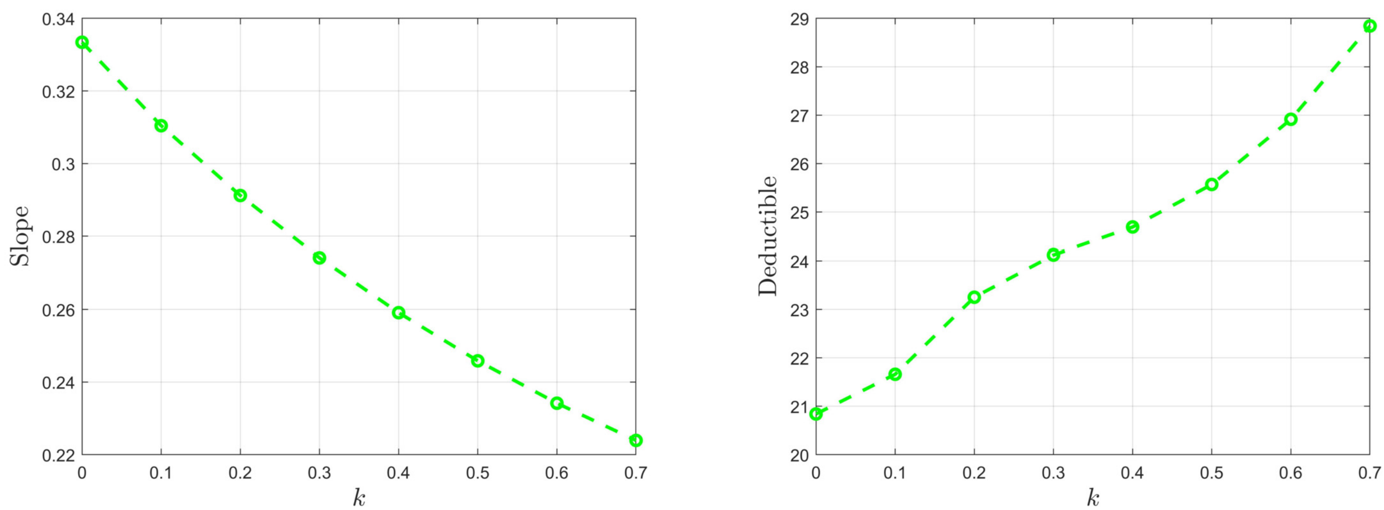

Fig. 3. The optimal slopes and deductible points for different degrees of narrow framing.

and

$$\begin{aligned}\text{Var}[L_I] =& \text{Var}[X - I(X) + \pi(I(X))]\\ =& \text{Var}[X - I(X)]\\ =& \text{Var}[X] - \text{Var}[I(X)] - 2\text{Cov}[X - I(X), I(X)] \le \text{Var}[X].\end{aligned}$$

Thus, the DM is sacrificing its expected terminal loss for the reduction of the variance of its terminal loss. Since the DM is a mean-variance user, for different volatility aversion levels (i.e. different values of γ), minimizing the weighted average of $\mathbb{E}[L_I]$ and $\text{Var}[L_I]$ will result in a Pareto-optimal indemnity function I^* such that no other indemnity function $\tilde{I} \in \mathcal{I}$ can further reduce both the expected value and variance of the terminal wealth, i.e.

$$\mathbb{E}[L_{\tilde{I}}] \le \mathbb{E}[L_{I^*}] \quad \text{and} \quad \text{Var}[L_{\tilde{I}}] \le \text{Var}[L_{I^*}]$$

with at least one inequality being strict (for details, see Miettinen (2012)). By varying the value of γ, we can obtain different Pareto-optimal indemnity functions, whose resulting $(\mathbb{E}[L_I], \text{Var}[L_I])$ constitutes the so-called efficient frontier. Under the setting of our paper, it is interesting to investigate the effect of narrow framing on such efficient frontier.

Using the general setup of Section 6.1, we change the value of γ under $k = 0$ (no narrow framing) and $k = 1$ (with narrow framing) and plot the efficient frontiers in Fig. 4, from which we can clearly see that the presence of narrow framing reduces the expected terminal loss but increases the volatility of the terminal loss. Therefore, with narrow framing the DM pursues more the reduction of its expected terminal loss and puts less weight on the volatility component of its mean-variance objective.

6.3. The effect of narrow framing on the Bowley solution

Under the same setting of Section 6.1 and by assuming that $\gamma_2 = 0.1$ (the insurer's volatility aversion level), we calculate the Bowley solution for the Stackelberg game as described in Section 5. Fig. 5 shows the plots of $f'(\xi)$ under different degrees of narrow framing.

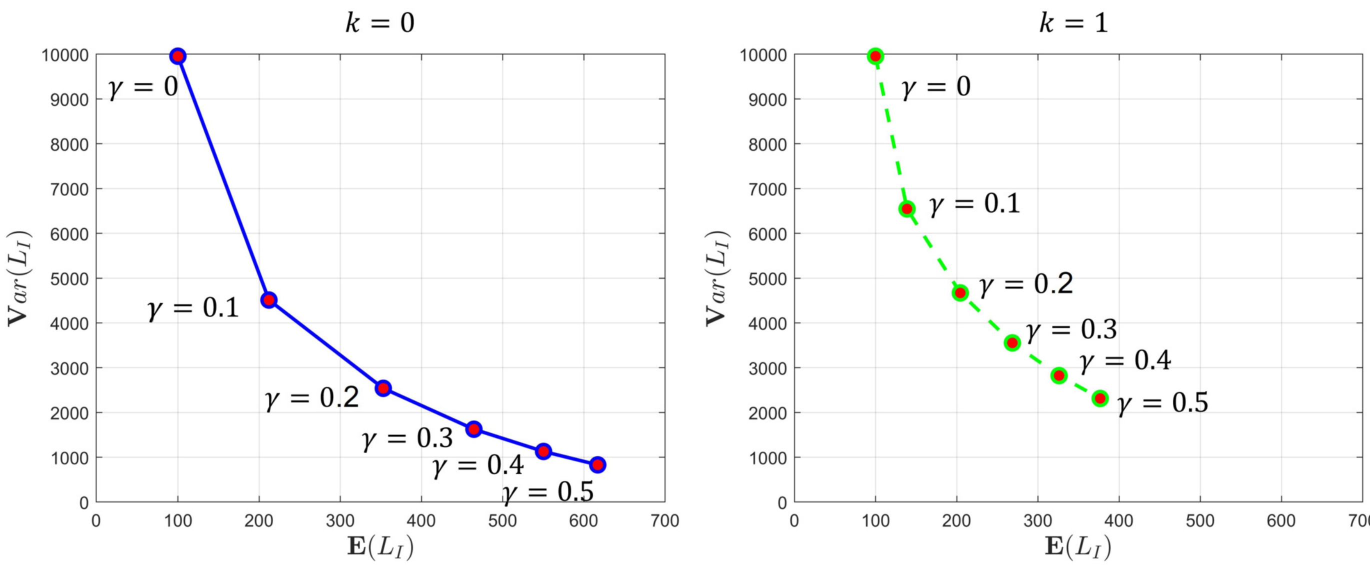

Fig. 4. The comparison between the efficient frontiers for $k = 0$ and $k = 1$.

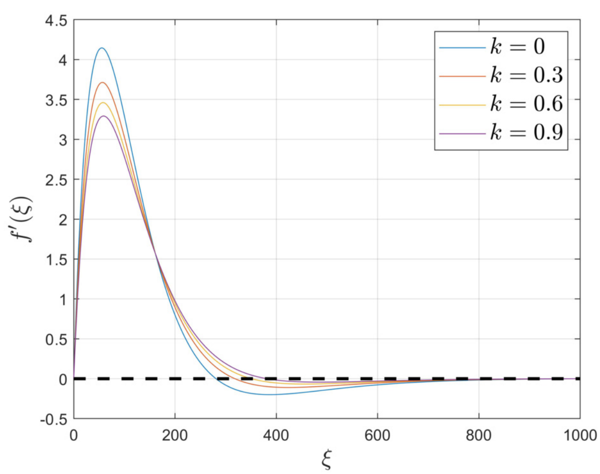

Fig. 5. The plots of $f'(\xi)$ for different values of k. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Under our setting, it is found that $f(\xi)$ is firstly increasing and then decreasing on the range $[0, 1000]$, and thus admits only one maximum point. This facilitates the calculation of ξ^* for Problem (5.8). Fig. 6 shows the Bowley solution (θ^*, ξ^*) for different degrees of narrow framing. Interestingly, under our setting the optimal deductible point ξ^* still increases w.r.t. k, which shows that the DM will still underinsure its risk even in a monopolistic market. At the meantime, θ^* decreases w.r.t. k, which implies that the insurer would reduce the safety loading factor to keep or attract the DM with higher degree of narrow framing.

In addition to analyzing the change of (θ^*, ξ^*) w.r.t. k, we are also interested in the change of the objectives of the DM and insurer w.r.t. k. Note that the DM's problem, similar to Section 6.2, could be written as

$$\min_{I \in \mathcal{I}} \text{DM's objective} := \mathbb{E}[X - I(X) + \pi(I(X))] + \frac{\gamma_1}{2}\text{Var}[X - I(X)],$$

while the insurer's problem could be written as

$$\min_{I \in \mathcal{I}} \text{Insurer's objective} := \mathbb{E}[I(X) - \pi(I(X))] + \frac{\gamma_2}{2}\text{Var}[I(X)].$$

This way, we get rid of the influence of their initial wealth on their objectives. Under the setting of this section, the Bowley solution in a monopolistic market depends on k. We plot the value of the DM's objective versus the value of the insurer's objective in Fig. 7. It shows that the insurer is always worse off when the DM's degree of narrow framing increases, while the DM is first better off and then worse off when its degree of narrow framing increases.

7. Conclusions

In this paper, we study the problem of optimal insurance design under mean-variance criterion by incorporating the narrow framing of the DM. With the intention to gamble with the insurer, the optimal indemnity function is characterized when the DM aims to maximize a mean-variance objective of her terminal wealth, which extends the work of Zheng (2020) under the expected utility model.

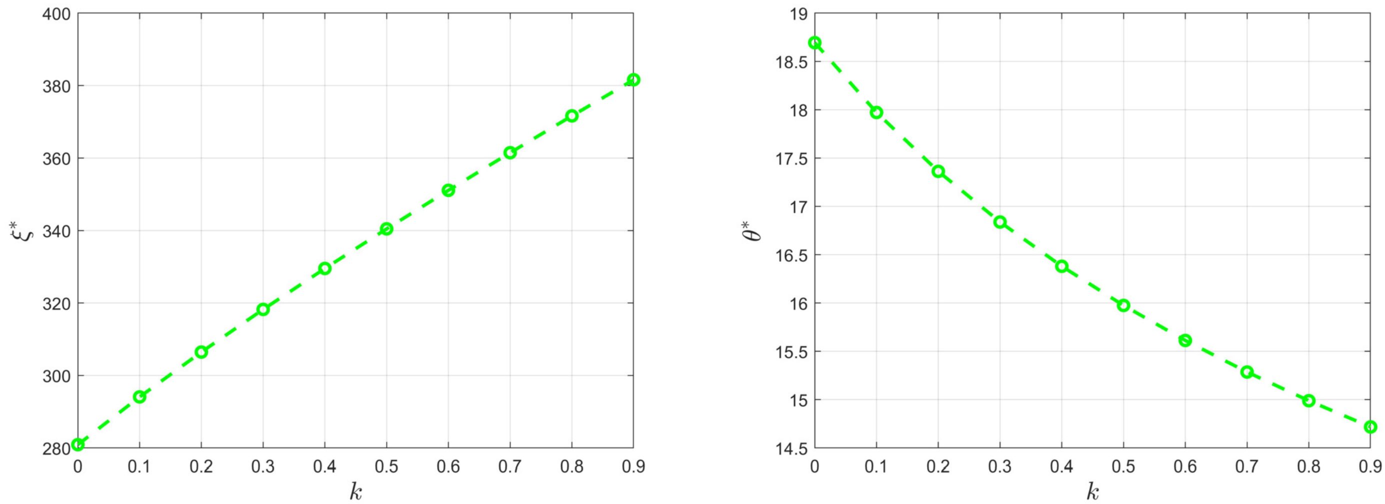

Fig. 6. The Bowley solutions (θ^*, ξ^*) for different degrees of narrow framing.

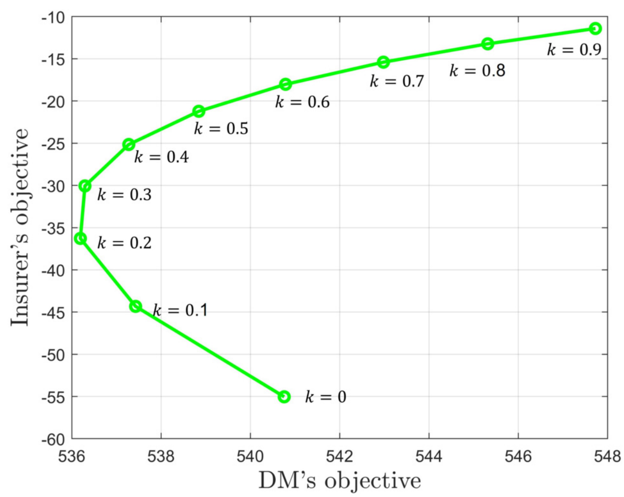

Fig. 7. The comparison between the objectives of the DM and insurer under different degrees of narrow framing.

Assuming that the insurer adopts the mean-variance premium principle, which generalizes the commonly adopted expected-value premium principle, we study analytically the DM's demand for insurance when the indemnity function is restricted to the coinsurance type. Our result tells that the DM's demand decreases with respect to her degree of narrow framing, which echoes the result in Zheng (2020). Without restricting the type of indemnity function, we show that the optimal indemnity function involves a deductible point. Our main results further confirm that when narrow framing is incorporated, the DM's insurance demand will be reduced and partial insurance will always be purchased. We also present explicit forms for the optimal indemnity functions when the local gain-loss utility function is quadratic or piecewise linear. As a spin-off result, the explicit Bowley solution is derived for a Stackelberg game between the DM and insurer under the quadratic local gain-loss utility and the expected-value premium principle, which extends the results of Li and Young (2021) without narrow framing. We finally provide several numerical examples to further analyze the effects of narrow framing on the optimal indemnity function, efficient frontier and the Bowley solution.

As a future research, motivated by the study of Chi et al. (2022), it would be very interesting to extend the current results to the case when the DM is a S-shaped utility user. Besides, how to build the notion of "narrow framing" upon the framework of modern risk management is also worth investigating.

Declaration of competing interest

No potential competing interests were reported by the authors.

Data availability

No data was used for the research described in the article.

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Appendix A. The connection between Problems 1 and (2.3)

To facilitate the subsequent discussions, we adopt the following assumptions for Problem (2.3).

Assumption A.1. $0 > G > \min_{I\in\mathcal{I}} \ \mathbb{E}[g(I(X)-\pi(I(X)))]$.

By using Jensen's inequality, we note that

$$G \le \mathbb{E}[g(I(X)-\pi(I(X)))] \le g\left(\mathbb{E}[I(X)]-\pi(I(X))\right) = g(-\theta\mathbb{E}[I(X)] - \frac{\eta}{2}\mathrm{Var}[I(X)]) \le g(0) = 0.$$

Thus, $G < 0$ implies that zero insurance is strictly feasible to Problem (2.3). If $G \ge 0$, Problem (2.3) has no solution or only the trivial solution – zero insurance. The condition $G > \min_{I\in\mathcal{I}} \ \mathbb{E}[g(I(X)-\pi(I(X)))]$ implies that the utility arising from the gambling against the insurer should not be too small, as otherwise the constraint of (2.3) becomes useless.

However, imposing only Assumption A.1 cannot guarantee that we will not end up with the trivial solution – zero insurance. The following assumption is therefore adopted in the rest of this paper.

Assumption A.2. $\theta + \gamma\mathbb{E}[X] < \gamma M$.

Assumption A.2 says that the safety loading for the premium should not be too large in order to get a non-trivial solution to Problem (2.3) without constraint. With Assumptions A.1 and A.2, we have the following lemma.

Lemma A.1. *Under Assumptions A.1 and A.2, zero insurance can never be optimal for Problem (2.3).*

Proof. Notice that Problem (2.3) without the constraint is exactly the buyer's problem studied by Li and Young (2021), for which the optimal indemnity function is given by

$$I^*(x) = x - \left(\frac{\eta x + \lambda}{\eta + \gamma} \wedge x\right)$$

where λ satisfies

$$\lambda = \theta + \gamma \int_0^{\frac{\lambda}{\gamma}} S_X(x)dx.$$

Under Assumption A.2, we have

$$\lambda \le \theta + \gamma \int_0^{\infty} S_X(x)dx = \theta + \gamma\mathbb{E}[X] < \gamma M.$$

Thus,

$$I^*(M) = M - \left(\frac{\eta M + \lambda}{\eta + \gamma} \wedge M\right) > 0.$$

Since I^* is continuous, we get that I^* is not zero insurance.

Now consider another indemnity function $I_\alpha = \alpha I^*$, where I_0 is zero insurance. Note that

$$\begin{aligned} V(I_\alpha) =& \mathbb{E}[W_0 - X + I_\alpha(X) - \pi(I_\alpha(X))] - \frac{\gamma}{2}\mathrm{Var}[W_0 - X + I_\alpha(X) - \pi(I_\alpha(X))] \\ =& W_0 - \mathbb{E}[X] - \frac{\gamma}{2}\mathrm{Var}[X] - \theta\mathbb{E}[I_\alpha(X)] - \frac{\eta+\gamma}{2}\mathrm{Var}[I_\alpha(X)] + \gamma\mathrm{Cov}[X, I_\alpha(X)] \\ =& V(I_0) - \theta\mathbb{E}[I_\alpha(X)] - \frac{\eta+\gamma}{2}\mathrm{Var}[I_\alpha(X)] + \gamma\mathrm{Cov}[X, I_\alpha(X)]. \end{aligned}$$

Now let

$$f(\alpha) := -\theta\mathbb{E}[I_\alpha(X)] - \frac{\eta+\gamma}{2}\text{Var}[I_\alpha(X)] + \gamma\text{Cov}[X, I_\alpha(X)]$$
$$= -\theta\alpha\mathbb{E}[I^*(X)] - \frac{\eta+\gamma}{2}\alpha^2\text{Var}[I^*(X)] + \gamma\alpha\text{Cov}[X, I^*(X)],$$

which is a quadratic function of α. Since I^* is strictly better than zero insurance when optimizing the mean-variance objective function, it must follow that

$$V(I^*) > V(I_0),$$

that is

$$V(I_0) - \theta\mathbb{E}[I^*(X)] - \frac{\eta+\gamma}{2}\text{Var}[I^*(X)] + \gamma\text{Cov}[X, I^*(X)] > V(I_0),$$

which implies that

$$f(1) = -\theta\mathbb{E}[I^*(X)] - \frac{\eta+\gamma}{2}\text{Var}[I^*(X)] + \gamma\text{Cov}[X, I^*(X)] > 0.$$

Hence

$$f(\alpha) = \alpha\left[f(1) + (1-\alpha)\frac{\eta+\gamma}{2}\text{Var}[I^*(X)]\right] > 0, \quad \text{for all } \alpha \in (0,1],$$

from which we immediately have $f'(0) > 0$.

Note that I_0, i.e. zero insurance, satisfies the constraint of (2.3) under Assumption A.1. Since the mapping

$$\alpha \mapsto \mathbb{E}[g(I_\alpha(X) - \pi(I_\alpha(X)))]$$

is continuous w.r.t. α, there exists an $\tilde{\alpha} > 0$ such that $\mathbb{E}[g(I_{\tilde{\alpha}}(X) - \pi(I_{\tilde{\alpha}}(X)))] > G$ and $f(\tilde{\alpha}) > 0$ (since $f'(0) > 0$), which results in

$$V(I_{\tilde{\alpha}}) = V(I_0) + f(\tilde{\alpha}) > V(I_0).$$

Therefore, zero insurance cannot be optimal to Problem (2.3). □

It is easy to check that the objective function of (2.3) is concave w.r.t. the indemnity function I. Under Assumption A.1, strong duality holds due to Slater's condition. This leads to the following lemma (see also Chapter 5 of Boyd and Vandenberghe (2004)).

Lemma A.2. *The indemnity function I^* solves Problem (2.3) if and only if there exists a $k^* \geq 0$ such that (k^*, I^*) solves the following problem*

$$\min_{k\in\mathbb{R}^+}\left\{\max_{I\in\mathcal{I}}\left\{\mathbb{E}(W) - \frac{\gamma}{2}\text{Var}(W) + k\left(\mathbb{E}[g(I(X) - \pi(I(X)))] - G\right)\right\}\right\}. \tag{A.1}$$

Now let

$$L(k) = \max_{I\in\mathcal{I}}\left\{\mathbb{E}(W) - \frac{\gamma}{2}\text{Var}(W) + k(\mathbb{E}[g(I(X) - \pi(I(X)))] - G)\right\}. \tag{A.2}$$

It is easy to verify that $L(k)$ is a convex function. Under Assumptions A.1 and A.2, the following lemma presents the range of k such that zero insurance cannot be the solution to the maximization problem of (A.2).

Lemma A.3. *Let Assumptions A.1 and A.2 hold and define*

$$k_0 := \inf\{k \in \mathbb{R}^+ : \mathbb{E}[W_0 - X] - \frac{\gamma}{2}\text{Var}[X] - kG \geq L(k)\}, \tag{A.3}$$

where $\inf\emptyset = \infty$. If $k \in [0, k_0)$, then zero insurance cannot be the solution to

$$\max_{I\in\mathcal{I}}\left\{\mathbb{E}(W) - \frac{\gamma}{2}\text{Var}(W) + k(\mathbb{E}[g(I(X) - \pi(I(X)))] - G)\right\}. \tag{A.4}$$

If $k \in [k_0, \infty)$, then zero insurance is the solution to Problem (A.4).

Proof. First, since $\mathbb{E}[g(I(X) - \pi(I(X)))] \leq 0$, we have

$$L(k) \leq \max_{I\in\mathcal{I}}\left\{\mathbb{E}[W] - \frac{\gamma}{2}\text{Var}[W] - kG\right\} = \max_{I\in\mathcal{I}}\left\{\mathbb{E}[W] - \frac{\gamma}{2}\text{Var}[W]\right\} - kG := l_1(k) \tag{A.5}$$

for $k \in \mathbb{R}^+$, where $l_1(k)$ is a linear function of k with $l_1'(k) = -G > 0$. Since $L(k)$ is convex, the inequality (A.5) implies that $L'(k) \leq l_1'(k) = -G$ at a differentiable point k.[11]

[11] A proper convex function is almost everywhere differentiable.

Let

$$l_2(k) = \mathbb{E}[W_0 - X] - \frac{\gamma}{2}\text{Var}[X] - kG.$$

Since $L(0) = \max_{I\in\mathcal{I}} \left\{\mathbb{E}[W] - \frac{\gamma}{2}\text{Var}[W]\right\} > \mathbb{E}[W_0 - X] - \frac{\gamma}{2}\text{Var}[X] = l_2(0)$ under Assumption A.2, we have $k_0 > 0$. When $k = k_0$, we have

$$\begin{aligned}
&L(k_0) = l_2(k_0)\\
\Longrightarrow &\max_{I\in\mathcal{I}}\left\{\mathbb{E}[W] - \frac{\gamma}{2}\text{Var}[W] + k_0(\mathbb{E}[g(I(X) - \pi(I(X))) - G)\right\} = \mathbb{E}[W_0 - X] - \frac{\gamma}{2}\text{Var}[X] - k_0 G\\
\Longrightarrow &I^*(x) = 0 \text{ on } [0, M],
\end{aligned}$$

where the uniqueness of I^* is due to Theorem 2.1. Now we claim that when $k > k_0$, we always have

$$\max_{I\in\mathcal{I}}\left\{\mathbb{E}[W] - \frac{\gamma}{2}\text{Var}[W] + k(\mathbb{E}[g(I(X) - \pi(I(X))) - G)\right\} = \mathbb{E}[W_0 - X] - \frac{\gamma}{2}\text{Var}[X] - kG.$$

Otherwise, $L(k) < l_2(k)$ for some $k > k_0$ due to $L'(k) \le -G$. However, from the definition of $L(k)$, we see $L(k) \ge l_2(k)$, for any k. Hence, we get the contradiction. Therefore, $I^*(x) = 0$ for $x \in [0, M]$ when $k \ge k_0$.

Since $L(k)$ is convex and $L(0) > l_2(0)$, we have $L(k) > l_2(k)$ when $k \in [0, k_0)$. Therefore $I^*(x)$ is not always equal to zero on $[0, M]$ when $k \in [0, k_0)$. This completes the proof. □

Appendix B. Proofs of the main results in the paper

Proof of Theorem 2.1

First, we define a metric for the set $\mathcal{I}$:

$$d(I_1, I_2) = \max_{x\in[0,M]} |I_1(x) - I_2(x)| \tag{B.1}$$

for any two indemnity functions $I_1, I_2 \in \mathcal{I}$. With this metric, it is easy to check that the objective function of Problem 1 is continuous w.r.t. I. Thus, the maximum is attainable if the admissible set for I, i.e. $\mathcal{I}$, is compact. Note the following facts about the functions in $\mathcal{I}$:

- They are all 1-Lipschitz continuous, which means they are equicontinuous.
- They are uniformly bounded (by M).
- The 1-Lipschitz continuity is preserved under the uniform convergence.

Thus, by applying Arzelà-Ascoli Theorem, we get that the set $\mathcal{I}$ is sequentially compact, or equivalently, $\mathcal{I}$ is compact. This ends the proof of existence.

To prove the uniqueness of the solution to Problem 1, we first recognize that Problem 1 could be written as

$$\min_{I\in\mathcal{I}} \mathbb{E}[L(I)] + \frac{\gamma}{2}\text{Var}[L(I)] - k\mathbb{E}[g(I(X) - \pi(I(X)))] \tag{B.2}$$

where $L(I) = X - I(X) + \pi(I(X))$. Note that the variance $\text{Var}[X]$ is convex in X. That is, for $\epsilon \in [0, 1]$ and X_1, X_2 whose second moments exist,

$$\begin{aligned}
\text{Var}[\epsilon X_1 + (1-\epsilon)X_2] &= \mathbb{E}[(\epsilon X_1 + (1-\epsilon)X_2 - \mathbb{E}[\epsilon X_1 + (1-\epsilon)X_2])^2]\\
&\le \mathbb{E}[\epsilon(X_1 - \mathbb{E}[X_1])^2 + (1-\epsilon)(X_2 - \mathbb{E}[X_2])^2]\\
&= \epsilon\text{Var}[X_1] + (1-\epsilon)\text{Var}[X_2].
\end{aligned}$$

Then for $I_1, I_2 \in \mathcal{I}$, we have for any $\epsilon \in [0, 1]$

$$\pi(\epsilon I_1 + (1-\epsilon)I_2) \le \epsilon\pi(I_1) + (1-\epsilon)\pi(I_2).$$

Now if $I_1, I_2 \in \mathcal{I}$ both solve Problem (B.2), then we have

$$\mathbb{E}[L(I_1)] + \frac{\gamma}{2}\text{Var}[L(I_1)] - k\mathbb{E}[g(I_1(X) - \pi(I_1(X)))] = \mathbb{E}[L(I_2)] + \frac{\gamma}{2}\text{Var}[L(I_2)] - k\mathbb{E}[g(I_2(X) - \pi(I_2(X)))].$$

We now claim that

$$\mathbb{E}[L(I_1)] = \mathbb{E}[L(I_2)],\ \text{Var}[L(I_1)] = \text{Var}[L(I_2)] \text{ and } \mathbb{E}[g(I_1(X) - \pi(I_1(X)))] = \mathbb{E}[g(I_2(X) - \pi(I_2(X)))]. \tag{B.3}$$

If not, then we construct another indemnity function $I_3 = \epsilon I_1 + (1-\epsilon)I_2$ for some $\epsilon \in (0, 1)$. Note that

$$\begin{aligned}
&\mathbb{E}[L(I_3)] \le\ \mathbb{E}[\epsilon L(I_1) + (1-\epsilon)L(I_2)] = \epsilon\mathbb{E}[L(I_1)] + (1-\epsilon)\mathbb{E}[L(I_2)],\\
&\text{Var}[L(I_3)] =\ \text{Var}[\epsilon L(I_1) + (1-\epsilon)L(I_2)] \le \epsilon\text{Var}[L(I_1)] + (1-\epsilon)\text{Var}[L(I_2)]
\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}[g(I_3(X)-\pi(I_3))] &\geq \mathbb{E}[g(\epsilon I_1+(1-\epsilon)I_2-\epsilon\pi(I_1)-(1-\epsilon)\pi(I_2))]\\ &= \mathbb{E}[g(\epsilon(I_1-\pi(I_1))+(1-\epsilon)(I_2-\pi(I_2)))]\\ &\geq \epsilon\mathbb{E}[g(I_1-\pi(I_1))]+(1-\epsilon)\mathbb{E}[g(I_2-\pi(I_2))].\end{aligned}$$

Therefore, we have

$$\begin{aligned}&\mathbb{E}[L(I_3)]+\frac{\gamma}{2}\mathrm{Var}[L(I_3)]-k\mathbb{E}[g(I_3(X)-\pi(I_3(X)))]\\ &\leq \epsilon\left(\mathbb{E}[L(I_1)]+\frac{\gamma}{2}\mathrm{Var}[L(I_1)]-k\mathbb{E}[g(I_1(X)-\pi(I_1(X)))]\right)\\ &\quad+(1-\epsilon)\left(\mathbb{E}[L(I_2)]+\frac{\gamma}{2}\mathrm{Var}[L(I_2)]-k\mathbb{E}[g(I_2(X)-\pi(I_2(X)))]\right)\\ &= \mathbb{E}[L(I_1)]+\frac{\gamma}{2}\mathrm{Var}[L(I_1)]-k\mathbb{E}[g(I_1(X)-\pi(I_1(X)))],\end{aligned}$$

where the inequality is strict if $I_1 \neq I_2$. This gives rise to the contradiction to the fact that I_1 solves the problem (B.2). Thus (B.3) holds, from which we get

$$\mathbb{E}[g(I_1(X)-\pi(I_1))]=\mathbb{E}[g(I_2(X)-\pi(I_2))].$$

Since 0 is in the support of X, by using the similar arguments in the proof of Lemma 2.1 of Chi and Zhuang (2020), we can reach the conclusion that $\mathbb{P}(I_1(X)=I_2(X))=1$. This ends the proof of uniqueness.

Proof of Proposition 3.1

Note that α^* is monotone in $\tilde{\alpha}$, so we only need to show that $\tilde{\alpha}$ is decreasing in k. Since $g''<0$, thus $V'(\alpha)$ is strictly decreasing with respect to α. If we can prove that $V'(\alpha)$ decreases with k, then one can immediately derive that $\tilde{\alpha}$ decreases with k, which implies that an individual with a higher degree of narrow framing will purchase less insurance. Thus, we only need to check

$$\mathbb{E}\left(g'(h(\alpha,X))\frac{\partial h}{\partial\alpha}(\alpha,X)\right)<0. \tag{B.4}$$

We first rewrite the left side of (B.4) as an integration form:

$$\begin{aligned}&\int_0^M g'\left(\alpha x-(1+\theta)\alpha\mu-\frac{\eta}{2}\alpha^2\sigma^2\right)[x-(1+\theta)\mu-\alpha\eta\sigma^2]\,dF_X(x)\\ &=\int_0^{[(1+\theta)\mu+\alpha\eta\sigma^2]^-} g'\left(\alpha x-(1+\theta)\alpha\mu-\frac{\eta}{2}\alpha^2\sigma^2\right)[x-(1+\theta)\mu-\alpha\eta\sigma^2]\,dF_X(x)\\ &\quad+g'\left(\frac{\eta}{2}\alpha^2\sigma^2\right)\times 0\times\mathbb{P}(X=(1+\theta)\mu+\alpha\eta\sigma^2)\\ &\quad+\int_{[(1+\theta)\mu+\alpha\eta\sigma^2]^+}^{M} g'\left(\alpha x-(1+\theta)\alpha\mu-\frac{\eta}{2}\alpha^2\sigma^2\right)[x-(1+\theta)\mu-\alpha\eta\sigma^2]\,dF_X(x).\end{aligned} \tag{B.5}$$

Because $g''(\cdot)<0$, we get

$$g'\left(\alpha x-(1+\theta)\alpha\mu-\frac{\eta}{2}\alpha^2\sigma^2\right)<g'\left(\frac{\eta}{2}\alpha^2\sigma^2\right),\qquad \forall x>(1+\theta)\mu+\alpha\eta\sigma^2. \tag{B.6}$$

By substituting (B.6) into (B.5), we obtain

$$\begin{aligned}&\int_0^M g'\left(\alpha x-(1+\theta)\alpha\mu-\frac{\eta}{2}\alpha^2\sigma^2\right)[x-(1+\theta)\mu-\alpha\eta\sigma^2]\,dF_X(x)\\ &<g'\left(\frac{\eta}{2}\alpha^2\sigma^2\right)\int_0^{[(1+\theta)\mu+\alpha\eta\sigma^2]^-}\frac{g'\left(\alpha x-(1+\theta)\alpha\mu-\frac{\eta}{2}\alpha^2\sigma^2\right)}{g'\left(\frac{\eta}{2}\alpha^2\sigma^2\right)}[x-(1+\theta)\mu-\alpha\eta\sigma^2]\,dF_X(x)\\ &\quad+g'\left(\frac{\eta}{2}\alpha^2\sigma^2\right)\int_{[(1+\theta)\mu+\alpha\eta\sigma^2]^+}^{M}[x-(1+\theta)\mu-\alpha\eta\sigma^2]\,dF_X(x).\end{aligned} \tag{B.7}$$

Again, because the strictly concavity of g, we have

$$\frac{g'\left(\alpha x-(1+\theta)\alpha\mu-\frac{\eta}{2}\alpha^2\sigma^2\right)}{g'\left(\frac{\eta}{2}\alpha^2\sigma^2\right)}>1, \qquad \forall\, 0<x<(1+\theta)\mu+\alpha\eta\sigma^2.$$

Thus, we can rewrite (B.7) as

$$\begin{aligned}&\int_0^M g'\left(\alpha x-(1+\theta)\alpha\mu-\frac{\eta}{2}\alpha^2\sigma^2\right)[x-(1+\theta)\mu-\alpha\eta\sigma^2]\,dF_X(x)\\&<g'\left(\frac{\eta}{2}\alpha^2\sigma^2\right)\int_0^M [x-(1+\theta)\mu-\alpha\eta\sigma^2]\,dF_X(x)\\&=-(\theta\mu+\alpha\eta\sigma^2)g'\left(\frac{\eta}{2}\alpha^2\sigma^2\right)\le 0.\end{aligned}$$

As we see, even $\theta=\eta=0$, inequality (B.4) still holds. Hence, the optimal proportion α^* is decreasing with k.

With Eq. (B.4), we can get that

$$V'(1)=-\theta\mu-\eta\sigma^2+k\mathbb{E}\left(g'(h(1,X))\frac{\partial h}{\partial\alpha}(1,X)\right)<0.$$

Thus, $\alpha^*<1$ since $V(\alpha)$ is concave. In other words, the DM strictly prefers partial insurance. This ends the proof.

Proof of Theorem 4.1

By expanding $\mathbb{E}(W)-\frac{\gamma}{2}\mathrm{Var}(W)+k\,\mathbb{E}[g(I(X)-\pi(I))]$ with $W=W_0-X+I(X)-\pi(I)$ and $\pi(I)$ given in (2.2), one can see that Problem 1 is equivalent to[12]

$$\begin{aligned}\max_{I\in\mathcal{I}}\ &-\theta\mathbb{E}(I)-\frac{\eta}{2}\mathrm{Var}(I)-\frac{\gamma}{2}\left\{\mathbb{E}(I^2)-2\mathbb{E}(XI)-(\mathbb{E}(I))^2+2\mathbb{E}(X)\mathbb{E}(I)\right\}\\&+k\mathbb{E}\left[g\left(I(X)-(1+\theta)\mathbb{E}(I)-\frac{\eta}{2}\mathrm{Var}(I)\right)\right].\end{aligned}$$

We adopt a step-by-step procedure to solve the above problem. In the first step, we fix $\mathbb{E}(I)=m\in[0,\mathbb{E}(X)]$, and $\mathbb{E}(I^2)=n\in[0,\mathbb{E}(X^2)]$, and solve the above problem subject to these two constraints. In the second step, we search for the optimal m and n. In summary, we want to solve

$$\max_{(m,n)\in[0,\mathbb{E}[X]]\times[0,\mathbb{E}[X^2]]}\left\{\min_{\lambda_1,\lambda_2}\max_{I\in\mathcal{I}} f(I,\lambda_1,\lambda_2,m,n)\right\} \tag{B.8}$$

where

$$\begin{aligned}f(I,\lambda_1,\lambda_2,m,n)=&-\theta m-\frac{\eta}{2}(n-m^2)-\frac{\gamma}{2}[n-2\mathbb{E}(XI)-m^2+2m\mathbb{E}(X)]\\&+k\mathbb{E}\left[g\left(I-(1+\theta)m-\frac{\eta}{2}(n-m^2)\right)\right]\\&-\lambda_1(\mathbb{E}(I)-m)-\lambda_2(\mathbb{E}(I^2)-n).\end{aligned} \tag{B.9}$$

Here, $\lambda_1,\lambda_2\in\mathbb{R}$ are two KKT multipliers. As proved by Theorem 2.1, there exists a solution to Problem (B.8). Based on Proposition 3.1 and Lemma A.1, we know that zero insurance and full insurance are not optimal to Problem 1. Thus, the optimal solution $(I^*,\lambda_1^*,\lambda_2^*,m^*,n^*)$ is an interior point of the set

$$\left\{(I,\lambda_1,\lambda_2,m,n)\,\middle|\,(I,\lambda_1,\lambda_2,m,n)\in\mathcal{I}\times(-\infty,\infty)\times(-\infty,\infty)\times[0,\mathbb{E}[X]]\times[0,\mathbb{E}[X^2]]\right\}.$$

Since $f(I,\lambda_1,\lambda_2,m,n)$ is continuously differentiable w.r.t. all the arguments, the following first-order conditions are necessary conditions for the optimality of $(I^*,\lambda_1^*,\lambda_2^*,m^*,n^*)$:

$$\frac{\partial f}{\partial\lambda_1}=0\iff\mathbb{E}[I^*]=m^*, \tag{B.10}$$

$$\frac{\partial f}{\partial\lambda_2}=0\iff\mathbb{E}[I^{*2}]=n^*, \tag{B.11}$$

$$\frac{\partial f}{\partial m}=0\iff-\theta+(\eta+\gamma)m^*-\gamma\mathbb{E}(X)+k\mathbb{E}[g'(I^*(X)-\pi(I^*))](\eta m^*-(1+\theta))+\lambda_1^*=0, \tag{B.12}$$

$$\frac{\partial f}{\partial n}=0\iff 2\lambda_2^*=\eta+\gamma+k\eta\mathbb{E}[g'(I^*(X)-\pi(I^*))]. \tag{B.13}$$

[12] For simplicity, in this appendix, we write I to mean $I(X)$, for example, we use $\mathbb{E}(I)$, $\mathbb{E}(I^2)$, $\mathbb{E}(XI)$, and $\pi(I)$ to mean $\mathbb{E}(I(X))$, $\mathbb{E}((I(X))^2)$, $\mathbb{E}(XI(X))$, and $\pi(I(X))$.

Note that

$$\begin{aligned}
& f(I,\lambda_1,\lambda_2,m,n) \\
=& -\theta m - \frac{\eta}{2}(n-m^2) - \frac{\gamma}{2}\left[n-m^2+2m\mathbb{E}[X]\right] + \lambda_1 m + \lambda_2 n \\
& + \gamma\mathbb{E}[XI] + k\mathbb{E}\left[g(I-(1+\theta)m - \frac{\eta}{2}(n-m^2)\right] - \lambda_1\mathbb{E}[I] - \lambda_2\mathbb{E}[I^2] \\
=& -\theta m - \frac{\eta}{2}(n-m^2) - \frac{\gamma}{2}\left[n-m^2+2m\mathbb{E}[X]\right] + \lambda_1 m + \lambda_2 n \\
& + \int_0^M \left\{\gamma x I(x) + kg(I(x)-(1+\theta)m - \frac{\eta}{2}(n-m^2)) - \lambda_1 I(x) - \lambda_2 I(x)^2\right\} dF_X(x).
\end{aligned}$$

Let $\tilde{f}(I,\lambda_1,\lambda_2,m,n) := \gamma x I(x) + kg(I(x)-(1+\theta)m - \frac{\eta}{2}(n-m^2)) - \lambda_1 I(x) - \lambda_2 I(x)^2$. It is easy to verify that, when the optimum is attained, i.e. $m=m^*$, $n=n^*$, $\lambda_1=\lambda_1^*$ and $\lambda_2=\lambda_2^*$, maximizing $f(I,\lambda_1^*,\lambda_2^*,m^*,n^*)$ is equivalent to maximizing $\tilde{f}(I,\lambda_1^*,\lambda_2^*,m^*,n^*)$ for each $x\in[0,M]$. Note that

$$\frac{\partial^2\tilde{f}}{\partial I^2} = kg''(I(x)-\pi(I^*)) - 2\lambda_2^* < 0,$$

thus $\tilde{f}(I,\lambda_1^*,\lambda_2^*,m^*,n^*)$ is strictly concave w.r.t. I. Due to the strict concavity of $\tilde{f}$ w.r.t. I, let $\tilde{I}$ (not necessarily in $\mathcal{I}$) be the function that satisfies the following first-order condition

$$\gamma x - \lambda_1^* - 2\lambda_2^*\tilde{I}(x) + kg'(\tilde{I}(x)-\pi(I^*)) = 0. \tag{B.14}$$

Taking derivative on both sides of (B.14) w.r.t. x, we get

$$\tilde{I}'(x) = \frac{\gamma}{2\lambda_2^*}\left(1-\frac{k}{2\lambda_2^*}g''(\tilde{I}(x)-\pi(I^*))\right)^{-1}. \tag{B.15}$$

Since $2\lambda_2^*\geq\gamma$ (from (B.13)) and $g''(\cdot)<0$, we obtain that $0\leq\tilde{I}'(x)\leq 1$. If $g'''(\cdot)\geq 0$, then taking derivative on both sides of (B.15) w.r.t. x again leads to

$$\tilde{I}''(x) = \frac{\gamma k}{4\lambda_2^{*2}}\frac{g'''(\tilde{I}(x)-\pi(I^*))}{\left(1-\frac{k}{2\lambda_2^*}g''(\tilde{I}(x)-\pi(I^*))\right)^2}\tilde{I}'(x)\geq 0.$$

Define a function $L(z) := z - \frac{k}{2\lambda_2^*}g'(z)$, then $L'(z) = 1-\frac{k}{2\lambda_2^*}g''(z) > 1$, since $g''(\cdot)<0$. Thus, $L(\cdot)$ is strictly increasing and $L^{-1}(\cdot)$ exists. Then, the function $\tilde{I}(x)$ is given by

$$\tilde{I}(x) = \pi(I^*) + L^{-1}\left(\frac{\gamma}{2\lambda_2^*}x - \frac{\lambda_1^*}{2\lambda_2^*} - \pi(I^*)\right). \tag{B.16}$$

Since $I^*\in\mathcal{I}$, which implies that $I^*(x)\in[0,x]$. It is easy to check that

$$I^*(x) = \min\{x,\max\{0,\tilde{I}(x)\}\} = \underset{I\in\mathcal{I}}{\arg\max}\ \tilde{f}(I,\lambda_1^*,\lambda_2^*,m^*,n^*), \tag{B.17}$$

since $I^*(0)=0$ and $I^{*\prime}(x)=1$ or 0 or $\tilde{I}'(x)$ $(\in[0,1])$, and I^* element-wisely maximizes the function $\tilde{f}(I,\lambda_1^*,\lambda_2^*,m^*,n^*)$.

We now proceed to show that there exists a deductible point $D\geq 0$ for the optimal indemnity function I^*. Based on Eq. (B.17), it is easy to see that the deductible exists if and only if $\tilde{I}(0)<0$. Suppose now we have $\tilde{I}(0)>0$, then since $\tilde{I}'(x)\in[0,1]$, we have $\tilde{I}(x)>0$ for each $x\in[0,M]$. Then, we get from (B.17) that $I^*(x)=\min\{x,\tilde{I}(x)\}\leq\tilde{I}(x)$. Now define

$$x_1 := \inf\{x\in[0,M] : I^*(x)\geq\tilde{I}(x)\},$$

then $I^*(x)<\tilde{I}(x)$ for $x\in[0,x_1)$ and $I^*(x)=\tilde{I}(x)$ for $x\in[x_1,M]$. Since $\tilde{f}(I,\lambda_1^*,\lambda_2^*,m^*,n^*)$ is strictly concave in I, the first order condition (B.14) implies that

$$\gamma x - \lambda_1^* - 2\lambda_2^* I^*(x) + kg'(I^*(x)-\pi(I^*)) \geq 0, \tag{B.18}$$

where the strict inequality holds for $x\in[0,x_1)$. With Eqs. (B.12) and (B.13), we can get that

$$\begin{aligned}
& \gamma x - \lambda_1^* - 2\lambda_2^* I^*(x) + kg'(I^*(x)-\pi(I^*)) \geq 0 \\
\Longrightarrow\ & \gamma x - 2\lambda_2^* I^*(x) + kg'(I^*(x)-\pi(I^*)) \geq \lambda_1^* \\
\Longrightarrow\ & \gamma x - (\eta+\gamma+k\eta\mathbb{E}[g'(I^*(X)-\pi(I^*))])I^*(x) + kg'(I^*(x)-\pi(I^*)) \geq \lambda_1^*
\end{aligned}$$

$$\Longrightarrow \int_0^M \left\{\gamma x-(\eta+\gamma+k\eta\mathbb{E}[g'(I^*(X)-\pi(I^*))])I^*(x)+kg'(I^*(x)-\pi(I^*))\right\}dF_X(x)>\lambda_1^*$$
$$\Longrightarrow \gamma\mathbb{E}[X]-(\eta+\gamma+k\eta\mathbb{E}[g'(I^*(X)-\pi(I^*))])\mathbb{E}[I^*(x)]+k\mathbb{E}[g'(I^*(X)-\pi(I^*))]>\lambda_1^*$$
$$\Longrightarrow \gamma\mathbb{E}[X]-(\eta+\gamma+k\eta\mathbb{E}[g'(I^*(X)-\pi(I^*))])m^*+k\mathbb{E}[g'(I^*(X)-\pi(I^*))]$$
$$>\theta-(\eta+\gamma)m^*+\gamma\mathbb{E}(X)-k\mathbb{E}[g'(I^*(X)-\pi(I^*))](\eta m^*-(1+\theta))$$
$$\Longrightarrow 0>\theta\left(1+k\mathbb{E}[g'(I^*(X)-\pi(I^*))]\right),$$

which gives rise to the contradiction. Thus, $\tilde{I}(0)\leq 0$, which indicates the existence of the deductible point.

Moreover, if the safety loading factor $\theta>0$, then we claim that the deductible point $D>0$. Otherwise, if $D=0$, or equivalently $\tilde{I}(0)=0$, we can derive that $I^*(x)=\tilde{I}(x)$. Following the same steps presented above, we can get that

$$0=\theta\left(1+k\mathbb{E}[g'(I^*(X)-\pi(I^*))]\right),$$

which contradicts with our assumption $\theta>0$. Hence, we obtain that the deductible point $D>0$.

If $\theta=0$, then we claim that the deductible point $D=0$. Otherwise, if $D>0$, or equivalently $\tilde{I}(0)<0$, we can derive that $I^*(x)=\max\{0,\tilde{I}(x)\}\geq\tilde{I}(x)$. Following the similar approach as above, we can get that

$$0<\theta\left(1+k\mathbb{E}[g'(I^*(X)-\pi(I^*))]\right),$$

which contradicts with our assumption $\theta=0$. Hence, we obtain that the deductible point $D=0$.

Proof of Proposition 4.2

Similar to the proof of Theorem 4.1, by substituting the gain-loss utility function $g(\cdot)$ in (4.11) into (B.14), we see that the candidate indemnity function $\tilde{I}(x)$ equals to

$$\tilde{I}(x)=\begin{cases}\frac{\gamma}{2\lambda_2}(x-\overline{D}), & \text{if } \tilde{I}(x)\geq\pi(I^*(X));\\ \pi(I^*(X)), & \text{if } \tilde{I}(x)=\pi(I^*(X));\\ \frac{\gamma}{2\lambda_2}(x-\underline{D}), & \text{if } \tilde{I}(x)<\pi(I^*(X)),\end{cases}$$

with λ_1 and λ_2 satisfy the following two equations:

$$\lambda_1=k\beta-(\eta(1+k\beta)+\gamma)\mathbb{E}(I^*)+\gamma\mathbb{E}(X)+k(\beta-1)(\eta\mathbb{E}(I^*)-1)\mathbb{P}(I^*(X)>\pi(I^*(X)))$$
$$+\theta+k\theta(\beta\mathbb{P}(I^*(X)<\pi(I^*(X)))+\mathbb{P}(I^*(X)\geq\pi(I^*(X)))); \tag{B.19}$$
$$2\lambda_2=\eta(1+k\beta)+\gamma-k\eta(\beta-1)\mathbb{P}(I^*(X)\geq\pi(I^*(X))). \tag{B.20}$$

Now, we prove the last assertion that $\underline{D}>0$ if $\theta>0$. Otherwise, $\underline{D}\leq 0$, or equivalently, $\tilde{I}(0)\geq 0$. Then from the proof of Theorem 4.1, we see there must exist a point $x_1\geq 0$, such that $I^*(x)\leq\tilde{I}(x)$ for $x\in[0,x_1)$ and $I^*(x)=\tilde{I}(x)$ for $x\in[x_1,M]$, where $I^*(x)=\min\{x,\tilde{I}(x)\}$. Since the objective function in this case is strictly concave in I, the inequality (B.18) still holds, that is,

$$\gamma x-\lambda_1-2\lambda_2 I^*(x)+k1_{\{I^*(x)\geq\pi(I^*)\}}+\beta k1_{\{I^*(x)<\pi(I^*)\}}\geq 0.$$

With Eqs. (B.12) and (B.13), we can get that

$$\gamma x-\lambda_1-2\lambda_2 I^*(x)+k1_{\{I^*(x)\geq\pi(I^*)\}}+\beta k1_{\{I^*(x)<\pi(I^*)\}}\geq 0$$
$$\Longrightarrow \gamma x-2\lambda_2 I^*(x)+k1_{\{I^*(x)\geq\pi(I^*)\}}+\beta k1_{\{I^*(x)<\pi(I^*)\}}\geq\lambda_1$$
$$\Longrightarrow \gamma x-\left[\eta(1+k\beta)+\gamma-k\eta(\beta-1)\mathbb{P}(I^*(X)\geq\pi(I^*(X)))\right]I^*(x)$$
$$+k1_{\{I^*(x)\geq\pi(I^*)\}}+\beta k1_{\{I^*(x)<\pi(I^*)\}}\geq\lambda_1$$
$$\Longrightarrow \gamma\mathbb{E}(X)-\left[\eta(1+k\beta)+\gamma-k\eta(\beta-1)\mathbb{P}(I^*(X)\geq\pi(I^*(X)))\right]\mathbb{E}(I^*(X))+k\mathbb{P}(I^*(x)\geq\pi(I^*))$$
$$+\beta k\mathbb{P}(I^*(x)<\pi(I^*))\geq\lambda_1$$
$$\Longrightarrow \gamma\mathbb{E}(X)-\left[\eta(1+k\beta)+\gamma-k\eta(\beta-1)\mathbb{P}(I^*(X)\geq\pi(I^*(X)))\right]\mathbb{E}(I^*(X))+k\mathbb{P}(I^*(x)\geq\pi(I^*))$$
$$+\beta k\mathbb{P}(I^*(x)<\pi(I^*))\geq k\beta-(\eta(1+k\beta)+\gamma)\mathbb{E}(I^*)$$
$$+\gamma\mathbb{E}(X)-k(1-\beta)(\eta\mathbb{E}(I^*)-1)\mathbb{P}(I^*(X)\geq\pi(I^*(X)))$$
$$+\theta+k\theta\big(\beta\mathbb{P}(I^*(X)<\pi(I^*(X)))+\mathbb{P}(I^*(X)\geq\pi(I^*(X)))\big)$$
$$\Longrightarrow 0\geq\theta+k\theta\big(\beta\mathbb{P}(I^*(X)<\pi(I^*(X)))+\mathbb{P}(I^*(X)\geq\pi(I^*(X)))\big),$$

which contradicts with the assumption $\theta>0$, thus, $\underline{D}>0$. If $\theta=0$, we claim that $\underline{D}=0$. Otherwise, if $\underline{D}<0$, then one can get the contradiction from the above discussion; if $\underline{D}>0$, or equivalently, $\tilde{I}(0)<0$, we see $I^*(x)=\max\{0,\tilde{I}(x)\}\geq\tilde{I}(x)$. Following the similar approach as above, one can derive the contradiction.

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