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Insuring longevity risk and long-term care: Bequest, housing and liquidity

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ABSTRACT

We study the impact of housing wealth and individual preferences on demand for annuities and long-term care insurance (LTCI). We build a multi-state lifecycle model that includes longevity risk and health shocks. The preference is represented by a recursive utility function that separates risk aversion and elasticity of intertemporal substitution (EIS). When health shocks are considered, a higher level of risk aversion lowers the annuity demand, while a lower level of the EIS has the opposite effect. The impact diminishes with a weaker bequest motive, more liquid wealth, or access to LTCI, all of which increase the demand for annuities. The presence of home equity can enhance annuity demand, but the enhancement is marginal when LTCI is available. The presence of home equity has a crowding-out effect on LTCI demand, and the effect is strengthened by a lack of bequest motives or a lower degree of risk aversion. The cash poor but asset rich may demand more LTCI coverage than their renter counterparts to preserve bequests. When both life annuities and LTCI are available, we find that the product demand is robust to changes in risk aversion and the EIS, providing insights into product designs that bundle annuities and LTCI.

1. Introduction

The occupation pension funds worldwide have undergone massive transitions from defined benefit (DB) to defined contribution (DC) schemes. DB schemes provide lifetime income streams by design, whereas DC schemes often pay out retirement benefits in capital form. Whilst pension funds can theoretically offer to transform capital into a lifetime payment stream to protect individuals against longevity risk, the payout phase remains largely under-developed with limited product offerings (Rocha et al., 2010). The under-development of the payout phase is in contrast to the accumulation phase for which fund managers have implemented multiple default investment strategies that achieve good overall performance (Duque et al., 2021). It is more challenging to design the payout phase due to, among other difficulties, a high level of heterogeneity that makes it hard to tailor products to individual needs. Retirees vary in wealth levels, homeownership status, risk tolerance, bequest motives, all of which can affect their retirement planning. Investment strategies in the accumulation phase, by contrast, can be based solely on age, account balances, and stock market participation with minimal welfare loss (Dahlquist et al., 2018).

The paper helps address the challenge pertinent to life annuities and long-term care insurance (LTCI). We focus on these two products due to their importance in tackling some of the most common retirement risks. Life annuities insure against longevity risk (i.e., the risk of outliving one's financial resources), which is the key consideration in designing the payout phase (Rocha et al., 2011). LTCI protects against unexpected healthcare costs, which can be the single most severe spending shock for retirees (Brown and Finkelstein, 2011). When retirees have access to life annuities and LTCI at the same time, they can create their own life care annuities, which are more flexible than the life care annuities that

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prescribe a top-up (i.e., long-term care) amount (see e.g., Wu et al., 2022).

We study the impact of housing wealth and individual preferences (including bequest motives and risk aversion) on demand for life annuities and LTCI. We consider longevity risk together with health shocks in a multi-state lifecycle model that starts at retirement. The preference is represented by an Epstein-Zin-Weil-type utility (Epstein and Zin, 1989, 1991; Weil, 1989) that separately identifies risk aversion and elasticity of intertemporal substitution (EIS), more flexible than the commonly-used power utility. We use alternative parameter values for risk aversion, the EIS, and bequest motives to investigate their impact on product demand. We assume different wealth levels and proportions of net worth in home equity to capture heterogeneous financial profiles. Although we do not intend to explain the low voluntary annuitization rate or the small private LTCI market, our results will help DC pension funds design personalized retirement products and advance the development of the payout phase. Our research also provides insights, from the supply-side perspective, into designing products that combine long-term care insurance and longevity insurance (see e.g., Chen et al., 2022; Hieber and Lucas, 2022). Ameriks et al. (2020) also study the importance of individual preference for insuring longevity risk and long-term care. They use state-dependent utility, whereas we focus on recursive utility.

We do not allow borrowing against home equity through equity release products (e.g., reverse mortgages) since our focus is on home equity as a means of funding long-term care. Separate studies that consider the impact of housing on reverse mortgages and LTCI show that reverse mortgages bring forward consumption and that individuals rely on LTCI to fund healthcare costs (see e.g., Davidoff, 2010; Shao et al., 2019). Abstracting from reverse mortgages is likely to reduce consumption in earlier years of retirement. These studies also show that home equity substitutes LTCI, which will be reflected in our results.

The presence of home equity complicates the decision on life annuities and LTCI due to its size and illiquidity. This issue is relevant to the majority of older Americans who have high homeownership rates, hovering around 80% over the last few decades (U.S. Census Bureau, 2022). The elderly homeowners have a large fraction of household portfolios held in home equity. The median ratio of home equity to all assets is close to 60% in the U.S. (Davidoff, 2009; Flavin and Yamashita, 2002). The presence of home equity can pose a liquidity constraint that limits one's capacity to pay for the insurance premiums or to support general consumption. In countries where the homeownership rates are high, their retirees are often asset rich and relatively cash poor (see e.g., Bradbury, 2010; McCarthy et al., 2002).

The impact of illiquid housing wealth on annuity demand is investigated in Pashchenko (2013) who finds that illiquid housing wealth decreases the annuity market participation rates because it reduces the amount of disposal wealth. If housing wealth provides a source of liquidity, Peijnenburg et al. (2017) find a slight increase in the optimal annuitization rate. Both papers do not explicitly consider the interaction between housing wealth and health shocks. In practice, housing wealth is rarely drawn upon to finance non-durable consumption, and selling the house is often associated with losing spouses or moving into a nursing home (Walker, 2004; Venti and Wise, 2004). This means housing wealth can be a significant source of funding for costly long-term care, thus reducing the need to keep a liquid wealth buffer. Precautionary savings for health shocks are known to affect annuity decisions (see e.g., Davidoff et al., 2005; Pang and Warshawsky, 2010; Peijnenburg et al., 2017; Turra and Mitchell, 2008), but how the liquidity released from home equity in the event of health shocks can affect annuity demand remains largely unexplored. The present study fills this gap.

Home equity can substantially reduce demand for LTCI, provided that home equity is not liquidated unless the homeowner moves to a long-term care facility. This result is proved by Davidoff (2010) in a one-period model and later confirmed by Shao et al. (2019) in a multi-period setting. Housing wealth can also reverse the complementarity between life annuities and LTCI (Davidoff, 2009). A common feature of these studies is to use a power utility function that imposes an inverse relationship between risk aversion and the EIS. Since empirical experiments find no such correlation (Barsky et al., 1997) and that individuals have relative risk aversion greater than the reciprocal of the EIS (Brown and Kim, 2013), the power utility function is unlikely to capture heterogeneous preferences of retirees.

We investigate the impact of housing wealth as well as risk aversion and EIS on demand for life annuities and LTCI by analyzing the optimal annuity and LTCI decisions for retirees of different characteristics. The choice variables include purchasing life annuities and LTCI at the point of retirement (i.e., a one-off decision) and consumption for each period while alive. We assume both products have actuarially fair prices and abstract from product loading that is often used to explain the thin empirical demand (see e.g., Brown and Finkelstein, 2011; Mitchell et al., 1999), which is beyond the scope of this paper. The house value is the source of investment risk in the model, and we abstract from equity risk due to limited investment in stocks among the elderly.[1] We use a Markov process to model health state transitions and fit the model to the data collected by the U.S. Health and Retirement Study. We explicitly consider the link between home equity liquidation and health shocks by assuming that entering into the state of requiring long-term care automatically triggers house selling. This assumption is based on the empirical evidence that home equity is rarely spent before death except for moving into a nursing home, also in line with the assumption made in Davidoff (2009).

We show that housing wealth significantly enhances annuity demand when LTCI is not available. The presence of home equity can increase the optimal annuitization rate as a fraction of total wealth even though its presence reduces the proportion of total wealth that can be annuitized. The result is stronger than that in Peijnenburg et al. (2017) and contrasts with Pashchenko (2013) due to our assumption of housing wealth liquidation in the event of illness. We confirm the intuitive explanation in Peijnenburg et al. (2017) that home equity lowers liquid wealth buffer for health shocks, thereby increasing the amount of wealth available for annuitization. Moreover, we find that the enhancement effect of home equity is marginal when retirees can access LTCI. As a result, the optimal annuitization rate as a proportion of total wealth decreases with housing wealth in this case.

We reveal that housing wealth interacts with preferences to affect LTCI demand, extending the literature that considers the impact of single factors, such as housing wealth (Davidoff, 2010; Shao et al., 2019) or bequest motives (Lockwood, 2018; Pauly, 1990). We find that the crowding-out effect of home equity on LTCI demand is stronger with a weaker bequest motive or a lower degree of risk aversion. The presence of bequest motives may also reverse the crowing-out effect of home equity. We find that LTCI helps homeowners in the low wealth groups to preserve their bequests, thereby improving the demand for LTCI. Furthermore, we find a minimal impact of the EIS on LTCI demand, robust to changes in homeownership status and amount of housing wealth.

Using the recursive utility allows us to capture a wider degree of heterogeneity in preferences and pinpoint the impact of risk

[1] The median wealth invested in stocks is $7,000 in 2006 dollars among single households, and less than a quarter of the single households own any stocks (Love and Smith, 2010).

aversion and the EIS on product demand. When LTCI is not available, we find a higher degree of risk aversion and a lower degree of the EIS drives the optimal annuitization rate in the opposite direction. This suggests the power utility can confound the impact of risk aversion on annuity demand. When both products are offered in the market, we find the demand for annuities as well as LTCI is relatively robust to changes in risk aversion and the EIS if we control for the total wealth level and the ratio of home equity to net worth. This result provides a new incentive to bundle life annuities and LTCI for it can greatly simplify the choice menus of retirement products.

The rest of the paper is organized as follows. Section 2 presents the lifecycle model and the model input in detail. Section 3 discusses how the demand for annuities and LTCI varies with wealth, homeownership, and individual preferences. Section 5 concludes.

2. Lifecycle model in retirement

We set up a discrete-time lifecycle model starting at retirement. The model consists of a series of one-year period that is indexed by $t \in \{1, 2, \ldots, T, T+1\}$.[2] The individual retires at $t = 1$ aged 65, and her maximum attainable age is 100, so $T = 36$. The maximum attainable age is determined by the data that is used to estimate the health dynamics. We discuss this more in Section 2.7. All variables are defined in real terms.

2.1. Health dynamics and costs

In each period, the retiree can either be healthy, sick, or dead. We follow Ai et al. (2017), Ameriks et al. (2011) and Shao et al. (2017) to consider two sick states, one requiring long-term care and the other not. The two states vary significantly in healthcare costs, and LTCI only pays benefits when one requires long-term care. We refer to the sick state that does not require long-term care as mildly disabled, and the one requiring long-term care as severely disabled. The categorization of the alive states is based on the number of difficulties in independently performing activities of daily living (ADLs). There are usually a total of six ADLs: dressing, walking, bathing, eating, transferring, and toileting. Mildly disabled state is defined as having difficulties in 1 – 2 ADLs, and severely disabled state is defined as having difficulties in 3 – 6 ADLs. The health state at period t is denoted as s_t.

The health state transitions are modeled using a Markov process. Fong et al. (2015) show a significant proportion of the elderly can recover from the disabled state to the healthy state. On the other hand, severe disability is usually chronic in nature that substantially reduces the possibility of recovery (Ferri and Olivieri, 2000; Olivieri and Pitacco, 2001). We, therefore, allow for transition from the mildly disabled state to the healthy state and do not allow for recoveries from the severely disabled state. Fig. 1 depicts the health state transitions, where '1' means healthy, '2' mildly disabled, '3' severely disabled, and '4' dead. The notation $\sigma_t(j,k)$ $(j \in \{1,2,3\}, k \in \{1,2,3,4\})$ denotes the transition intensity from state j to state k at time t. A more comprehensive approach is to model both care state and health state and allow for all possible transitions (see e.g., Friedberg et al., 2015), where care state is defined by the type of care one receives (e.g., home care, assisted living, and nursing home) and health state is defined by an individual's health status (e.g., number of difficulties in ADLs). We abstract from additional features, and in particular, the recovery from the state of requiring long-term care. In practice, the recovery rate from the long-term care state is very low, and insurers usually do not consider recovery when pricing LTCI, which is a prudent approach.

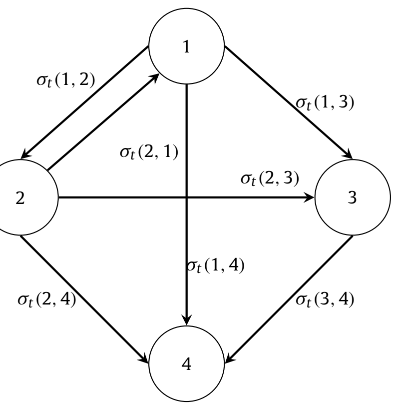

Fig. 1. Four-state Markov process that models health state transitions. 1, 2, 3, and 4 refer to the health states of healthy, mildly disabled, severely disabled, and dead, respectively. $\sigma_t(j,k)$ $(j \in \{1,2,3\}, k \in \{1,2,3,4\})$ denotes the transition intensity from state j to state k at time t.

Given the transition intensities, $\sigma_t(j,k)$, the single-period transition probabilities, $\pi_t(j,k) \equiv \Pr(s_{t+1} = k | s_t = j)$, can be solved through Kolmogorov equations. In particular, we assume the transition intensities are constant within an integer age. Then the annual transition probabilities for each period are given by

$$\begin{pmatrix} \pi_t(1,1) & \pi_t(1,2) & \pi_t(1,3) & \pi_t(1,4) \\ \pi_t(2,1) & \pi_t(2,2) & \pi_t(2,3) & \pi_t(2,4) \\ \pi_t(3,1) & \pi_t(3,2) & \pi_t(3,3) & \pi_t(3,4) \\ \pi_t(4,1) & \pi_t(4,2) & \pi_t(4,3) & \pi_t(4,4) \end{pmatrix} = \exp\left[\begin{pmatrix} \sigma_t(1,1) & \sigma_t(1,2) & \sigma_t(1,3) & \sigma_t(1,4) \\ \sigma_t(2,1) & \sigma_t(2,2) & \sigma_t(2,3) & \sigma_t(2,4) \\ 0 & 0 & -\sigma_t(3,4) & \sigma_t(3,4) \\ 0 & 0 & 0 & 0 \end{pmatrix}\right],$$

where

$$\sigma_t(1,1) = -\big(\sigma_t(1,2) + \sigma_t(1,3) + \sigma_t(1,4)\big),$$
$$\sigma_t(2,2) = -\big(\sigma_t(2,1) + \sigma_t(2,3) + \sigma_t(2,4)\big),$$
$$\exp(X) = \sum_{k=0}^{\infty} \frac{1}{k!} X^k, \; X^0 \text{ is the identity matrix with the same dimensions as } X.$$

Given the single-period transition probabilities, the n-period transition probability, $\pi_t^n(j,k) \equiv \Pr(s_{t+n} = k | s_t = j)$, can be obtained through the Chapman-Kolmogorov equations. When $n = 1$, it reduces to the single-period transition probability, i.e., $\pi_t^1(j,k) = \pi_t(j,k)$.

We follow Ameriks et al. (2011) to model the out-of-pocket health expenditure as a deterministic process given the health state, s_t. The deterministic process is preferred over a stochastic model (see e.g., De Nardi et al., 2010) for its simplicity and its ability to capture the characteristics of empirical medical expense risk. Since the healthcare inflation usually exceeds that of the consumer price index (CPI), it is assumed that the relative price of healthcare increases at a rate of q per annum.

2.2. Housing and financial assets

Given that a large majority of retired homeowners have paid off their mortgages, the model assumes the individual lives in a mortgage-free home at retirement. In addition, empirical data

[2] Note that the latest possible consumption occurs at $t = T$. The last time index $T + 1$ is for the purpose of bequest only.

shows that housing assets are rarely drawn upon unless the retiree moves to a long-term care facility (see e.g., Venti and Wise, 2004). It is assumed that the retiree will liquidate the house when she becomes severely disabled and subsequently moves to a nursing home. The house has a gross rate of return $R_{H,t+1}$ from time t to time $t+1$, where $\ln(R_{H,t+1})$ follows a normal distribution with mean μ_H and variance σ_H^2. The liquid assets earn a constant risk-free return of R_f. We abstract from the equity market.

2.3. Retirement products

At retirement, the individual has access to two types of retirement products, immediate life annuities and reimbursement LTCI, both of which are offered by private companies and have actuarially fair premiums. The retiree decides the proportion (α) of liquid assets to annuitize and the percentage coverage (λ) of LTCI to purchase, which means a λ proportion of healthcare cost in the severely disabled state will be reimbursed by LTCI. The decisions are made at retirement only. The public offering of similar products is not explicitly considered in the model. Nevertheless, the individual's endowment at retirement can be perceived as including the expected present value of public pension paid during retirement, and the out-of-pocket health expenditure can be seen as net of any publicly funded schemes.

We do not consider deferred annuities in our analysis. If the individual delays annuitization, there will not be a large difference from our analysis since mortality credits are low at these ages and there is only a risk-free asset to invest in and drawdown from. Delaying annuitization when allowing for risk is dominated in utility terms by purchasing annuities from the start because an actuarially fair annuity provides a higher risk-adjusted return due to mortality credits (Yaari, 1965).

The life annuity provides annual level payment for the remaining lifetime of the annuitant. The payment starts at the beginning of the first period. Given an α proportion of liquid assets annuitized at retirement, the annual income from annuities is given by

$$\mathsf{Y} = \frac{\alpha \mathsf{B}}{1+\sum_{n=2}^{T} R_f^{-(n-1)} \pi_1^{n-1}(s_1, s_n \neq 4)}, \tag{1}$$

where B denotes the initial endowment of liquid assets, $\pi_1^{n-1}(s_1, s_n \neq 4)$ denotes the probability that a 65-year-old individual with health state s_1 will survive for the next $(n-1)$ years.

LTCI covers healthcare costs when the policyholder is severely disabled (i.e., health state 3). We assume a lump-sum premium and exclude any loading on the product. The actuarially fair price (P) for a full coverage LTCI policy is given by

$$\mathsf{P} = \sum_{n=2}^{T} R_f^{-(n-1)} \pi_1^{n-1}(s_1, s_n = 3) h(s_n = 3, n), \tag{2}$$

where $h(s_n, n)$ represents the out-of-pocket health expenditure at time n in health state s_n.

2.4. Budget constraints and wealth dynamics

In the first period, the retiree is endowed with liquid wealth of B and housing wealth of $\mathsf{W^H}$, and the retiree is in the healthy state (i.e., health state 1). She then decides the proportion of liquid assets to annuitize and LTCI coverage to purchase. After that, she receives income from annuities (if any), incurs the healthcare costs, and decides how much to consume. Let B_1 denote the amount of liquid wealth available after purchasing the retirement products. It is given by

$$B_1 = (1-\alpha)\mathsf{B} - \lambda \mathsf{P}, \; B_1 \geq 0, \tag{3}$$

where α and λ are chosen such that $B_1 \geq 0$. We do not allow retirees to take out a loan to purchase LTCI or annuities. Since the use of equity release is very low in practice, this assumption has a limited impact on our results.

Starting from the second period, the retiree enters the period t with health state s_t and wealth W_t, which consists of housing wealth W_t^H and liquid wealth B_t. Note that W_t, W_t^H, and B_t denote the amount available at the beginning of the period t (i.e., before any action is taken) except for B_1, which is specified in Equation (3). The timing of events is as follows.

1. If $s_t = 4$, the individual is deceased, so the wealth W_t is bequeathed.
2. If $s_t < 4$, one of the following events will occur.
   (a) If $s_t = 3$ and $s_{t-1} \in \{1, 2\}$, the individual will liquidate the home equity and move into a residential care facility.
   (b) If $s_t = 3$ and $s_{t-1} = 3$, the individual will remain staying at the residential care.
   (c) If $s_t < 3$, the individual will remain living at home.
3. If $s_t < 4$, the health costs $h(s_t, t)$ are incurred and a consumption decision (C_t) is made. The remaining liquid assets earn a risk-free return R_f.

The chosen consumption level must not fall below the consumption floor C^f to ensure a minimum standard of living. If the individual's budget cannot support the minimum consumption level, we assume the government will provide subsidies to increase the consumption level to C^f. The liquid wealth in the next period is subsequently set to zero. The consumption floor plays the role of Medicaid in our model. It aims to provide a subsistence level, so the consumption floor is assumed independent of one's wealth at retirement. Furthermore, since the means-tested Medicaid typically excludes the applicant's home (U.S. Department of Health and Human Services, 2005), we assume that the government subsidizes the person even if she owns a house.

The budget constraint for liquid assets B is given by

$$\begin{aligned} &B_2 = \left(B_1 + \mathsf{Y} - h(s_1, 1) - C_1\right)^+ R_f; \\ &\text{for } t \in \{2, 3, \ldots, T\}, \\ &B_{t+1} = \begin{cases} \left(B_t + \mathsf{Y} - h(s_t, t) - C_t\right)^+ R_f \\ \quad \text{if } s_t \in \{1, 2\} \\ \left(B_t + \mathsf{Y} + W_t^H \mathbb{1}_{\{s_{t-1} \in \{1,2\}\}} - (1-\lambda) h(s_t, t) - C_t\right)^+ R_f \\ \quad \text{if } s_t = 3, \end{cases} \end{aligned} \tag{4}$$

where $(\cdot)^+$ is defined as $\max(\cdot, 0)$. The non-negativity constraint is binding when the liquid wealth available for consumption is below the consumption floor, and the government subsidizes the retiree to cover the consumption shortfall.

The budget constraint for total wealth W is given by

$$\begin{aligned} &W_2 = B_2 + W_1^H R_{H,2}, \;\text{ where } W_1^H = \mathsf{W^H}; \\ &\text{for } t \in \{2, 3, \ldots, T\}, \\ &W_{t+1} = \begin{cases} B_{t+1} + W_{t+1}^H & \text{if } s_t \in \{1, 2\} \\ B_{t+1} & \text{if } s_t = 3 \end{cases} \text{ and } W_{t+1}^H = W_t^H R_{H,t+1}. \end{aligned} \tag{5}$$

2.5. Preferences

Individuals in the model are assumed to have Epstein-Zin-Weil-type preferences (Epstein and Zin, 1989, 1991; Weil, 1989) over non-housing consumption and a bequest. Although the housing service consumption is not directly included in the utility function, the housing wealth contributes to the utility through bequests or home equity liquidation that alleviates the budget constraint caused by excessive medical care costs.

The Epstein-Zin model generalizes the power utility model in that it can separately identify the risk aversion and the EIS. The two elements are intrinsically different. Risk aversion describes an individual's willingness to substitute consumption across different states of the world, whereas the EIS describes an individual's willingness to substitute consumption over time. When the individual's EIS is the reciprocal of the coefficient of relative risk aversion, the Epstein-Zin model reduces to the power utility model.

The preferences are specified by

$$V_t \equiv V(B_t, W_t^H, s_t, t) = \max_{O_t}\Bigg\{(1-\beta)C_t^{1-\rho} + \beta\Bigg[\mathbb{E}_t\Bigg[\sum_{k\neq 4}\pi_t(s_t, s_{t+1}=k)V(B_{t+1}, W_{t+1}^H, s_{t+1}=k, t+1)^{1-\gamma} + \pi_t(s_t, s_{t+1}=4)b^{\gamma}W_{t+1}^{1-\gamma}\Bigg]\Bigg]^{\frac{1}{\theta}}\Bigg\}^{\frac{1}{1-\rho}}, \quad \theta = \frac{1-\gamma}{1-\rho};$$

$$O_t = \begin{cases}\{\lambda, \alpha, C_t\}, & \text{for } t = 1;\\ \{C_t\}, & \text{for } t = 2, \ldots, T.\end{cases} \tag{6}$$

The notation V_t is the indirect utility value at time t, $\beta \in (0, 1)$ the subjective discount factor, $\rho > 0$ the inverse of the EIS (i.e., $\rho = 1/\psi$), $\mathbb{E}$ the expectation operator, $\gamma > 1$ the coefficient of relative risk aversion, $b \geq 0$ the strength of bequest motive. The subjective discount factor (β) measures an individual's impatience to defer consumption, with a lower value representing less willingness to postpone the consumption. The strength of bequest motives increases with the value of b.

2.6. Optimization problem and solution method

Individuals optimize over consumption, annuitization rate, and LTCI coverage to maximize the expected lifetime utility in (6), subject to conditions (1) to (5). The consumption can come from annuity income, liquid wealth, and liquidated housing wealth. Therefore, by choosing their optimal consumption, retirees also optimize their withdrawal strategy. We set up grid points on liquid wealth, housing wealth, and current health state to solve the optimization problem. The method of endogenous grid points (Carroll, 2006) is used to set up the grid points for the liquid assets. The grid points on housing wealth are given exogenously. The log-normal distribution of house price growth is discretized by the Gauss-Hermite quadrature. The first-order condition for consumption can be solved analytically to speed up the solution process. The analytical form is derived in Appendix A. The optimization problem is solved backward, starting from the last period. For the points not lying on the grid, a hybrid interpolation method introduced in Ludwig and Schön (2018) is used to find the optimal consumption and the indirect utility value.

The optimal annuitization rate and LTCI coverage are solved in the first period using the following steps. First, we set up the grid points on annuitization rate and LTCI coverage. On each grid point, we solve the optimal consumption and indirect utility levels backward from the last period to the first period. Given the initial liquid wealth and housing wealth, the indirect utility value in the first period for a healthy individual can be found through the hybrid interpolation method. The optimal annuitization rate and LTCI coverage are found by searching for the grid point that gives the highest value of indirect utility.

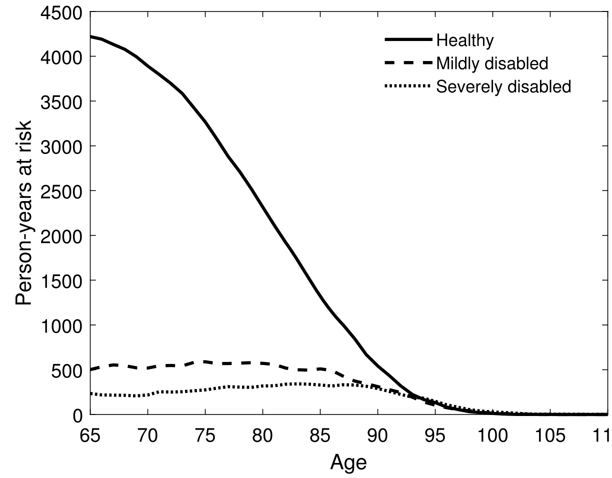

Fig. 2. Person-years at risk in healthy, mildly disabled, and severely disabled states. We use the female experience between 1998 and 2018.

After solving the optimal decision rules defined on the state space, the time-series profiles of a retiree's optimal consumption can be obtained through simulation. Specifically, we first simulate house price growths and health states, and then use the optimal policy rules to calculate the optimal consumption. The corresponding liquid and total wealth levels can also be obtained. The simulation is run 10,000 times.

2.7. Model parameterization

We set the liquid wealth endowment at between \$50K and \$1 million, with an increment of \$50K. When retirees are endowed with home equity, we consider home equity comprises a quarter, a third, or a half of total wealth. The housing wealth proportions are lower than those reported in Davidoff (2009) and Flavin and Yamashita (2002) because the pre-annuitized wealth is implicitly included in the total wealth. The varieties of liquid wealth levels and home equity proportions allow us to investigate the impact of housing wealth and liquidity on demand for life annuities and LTCI.

We proceed to discuss the remaining inputs to the lifecycle model: health state transitions in Section 2.7.1, and preference parameters in Section 2.7.2.

2.7.1. Health state transitions

The health state transition is estimated using the data from the U.S. Health Retirement Study that surveys a nationally representative sample of Americans over age 50 every two years, starting from 1992. The data before 1998 is removed due to inconsistent question structure. We use the data between 1998 and 2018, the latest data available. We focus on the female experiences since they have longer life expectancy than males, and tend to spend more years in the disabled state (Fong et al., 2015). Fig. 2 shows the exposed-to-risk (measured in person-years) of the selected female sample. The exposed-to-risk is close to zero beyond age 100, so we are unable to obtain reliable estimates of health transition probabilities beyond age 100. We, therefore, set the maximum attainable age of our lifecycle model at 100.

We follow the method in Fong et al. (2015) to estimate the health state transitions using a generalized linear model (GLM) with the log link function. The number of transitions at age x is assumed to follow a Poisson distribution with mean (m_x) defined as a polynomial function of age. The mean is given by

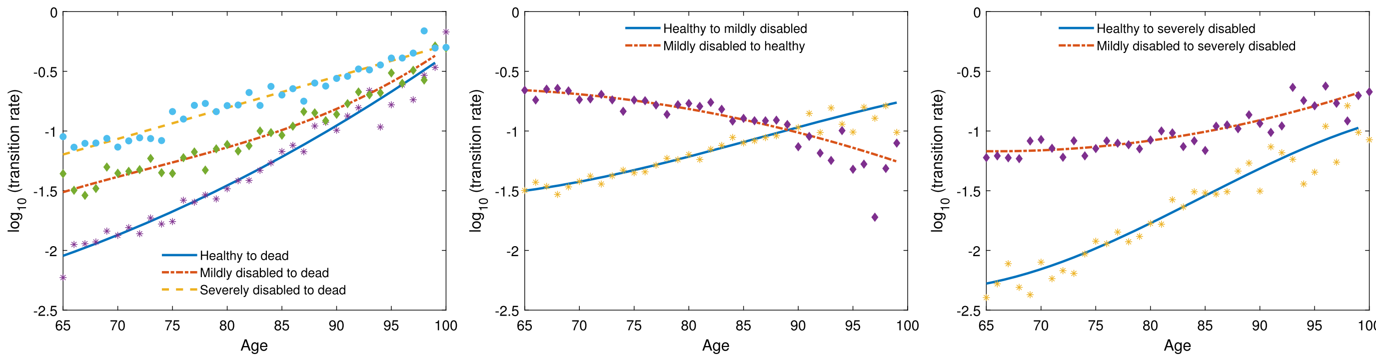

Fig. 3. Crude and estimated health transition rates. The scattered points are the crude rates and the curves show the estimated rates.

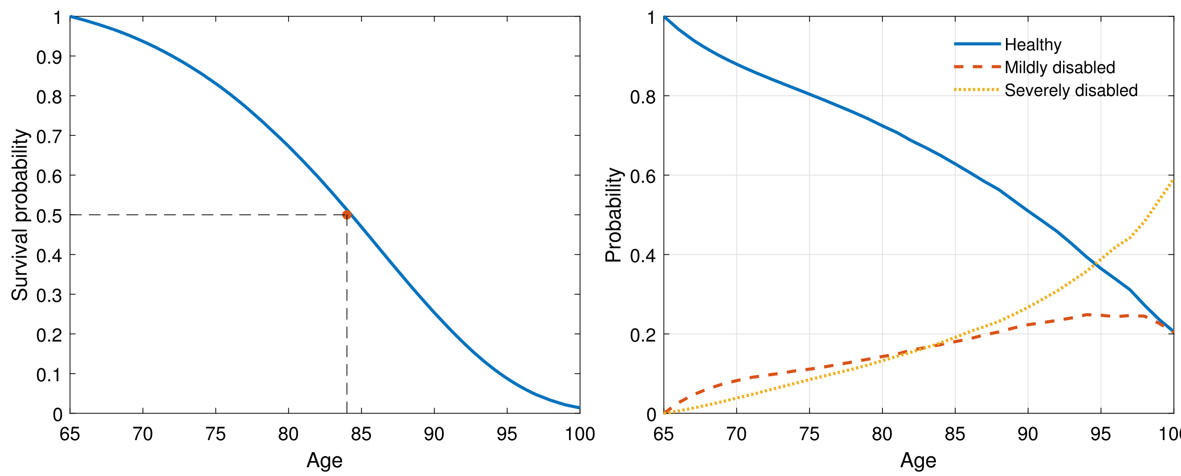

Fig. 4. (Left panel) Survival curve and (right panel) probability of being in each health state conditional on being alive for a 65-year-old healthy female.

$$m_x = e_x \sum_{k=0}^{K} \eta_k x^k, \tag{7}$$

where e_x is the central exposure to risk for x-year-old individuals, K the degree of the polynomial, η_k the coefficients of the polynomial. We use the Akaike information criterion corrected for sample size (AICc), Bayesian information criterion (BIC), and the likelihood ratio test to select the degree of polynomials. The detailed results are presented in Appendix B. Fig. 3 compares fitted transition rates with the crude ones, and shows that the estimation achieves a good fit.

We calculate the survival probability and the probability of being in each health state based on the estimated transition rates. Fig. 4 shows that a 65-year-old healthy female has a more than 50% chance of living to the mid-80s, and that the probability of being severely disabled increases substantially after age 85, so the overall risk of requiring long-term care is high. We follow Yogo (2016) to set the risk free rate at 2.5%. As a result, the actuarially fair price of life annuities for a healthy 65-year-old individual is \$14.98 per \$1 of annual income, and that of LTCI is \$89,524.28 for the full coverage.

2.7.2. Preference parameters

The preference parameters used in the numerical simulation take the commonly used values in the literature. Their baseline values are displayed in Table 1 along with other parameter values. The sources of the parameters, unless otherwise specified, are listed in the brackets. To study the impact of bequest motives, we consider two cases: no bequest motives ($b = 0$) and a certain bequest motive ($b = 2$). We will separately change the value of γ and ψ to examine the impact of risk aversion and the EIS. Kaplow (2005) estimates that the coefficient of relative risk aversion is at least 2. Hence, we choose the low value of γ to be 2. We set the high value of γ at 8 since the probability that the relative risk aversion exceeds 8 is very low (Halek and Eisenhauer, 2001). The empirical estimates of ψ can be as low as between 0.1 and 0.2 and as high as close to 1 (Guvenen, 2006). The meta-analyses performed by Havranek et al. (2015) show the mean estimate of ψ in the U.S. is 0.594 with a standard error of 0.036. We, therefore, set alternative values of ψ at 0.2 (close to the low end of the empirical estimates) and 0.7 (approximately the mean plus three standard deviations based on the meta-analysis results).

3. Variations in demand for life annuity and LTCI

This section presents the optimal decisions on immediate life annuities and reimbursement LTCI based on the lifecycle model described in the previous section. Due to the possible interaction between life annuities and LTCI (see e.g., Ameriks et al., 2011; Koijen et al., 2016), we consider the following three scenarios: *1)* annuities alone are offered (Section 3.1), *2)* LTCI alone is offered (Section 3.2), and *3)* both annuities and LTCI are offered in the market (Section 3.3). We begin each subsection by verifying prior results in the literature before discussing the impact of housing wealth and preferences on product demand.

3.1. Annuities

Our model verifies some well-established results in the literature of optimal annuitization that abstracts from home equity. First and foremost, it is long recognized in the literature that full annuitization is optimal for those who have no bequest motives and

Table 1
The parameter values used for the base case.

Parameter	Explanation	Value
Preference (Pang and Warshawsky, 2010)		
b	Strength of bequest motive	0 and 2
β	Subjective discount factor	0.96
γ	Coefficient of relative risk aversion	5
ψ	Elasticity of intertemporal substitution (EIS)	0.5
Asset returns (Yogo, 2016)		
R_f	Risk free rate	1.025
μ_H	Parameters of the lognormal distribution	0.34%
σ_H^2	of house price growth	3.5%
Consumption floor (Ameriks et al., 2011)		
c^f	Floor for healthy and mildly disabled states	$4,630
	Floor for severely disabled states	$5,640
Health expenditure (Ameriks et al., 2011)		
$h(s_1, 1)$	Initial cost for healthy state	$1,000
$h(s_2, 1)$	Initial cost for mildly disabled state	$10,000
$h(s_3, 1)$	Initial cost for severely disabled state	$50,000
$q^{\dagger}$	Health expenditure inflation in excess of CPI inflation	1.90%

† *Source*: Yogo (2016).

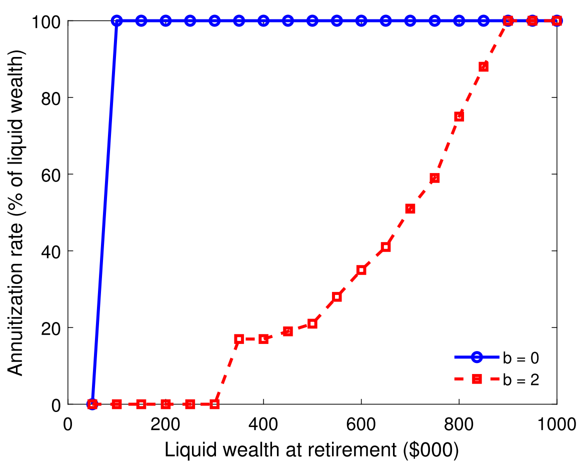

Fig. 5. Optimal annuitization rates for retirees endowed with liquid wealth and no housing wealth at retirement. The legend represents the strength of bequest motives. The other preference parameters are $\gamma = 5, \psi = 0.5$. We assume no LTCI is offered in the market.

face no uncertainty other than their future lifetime (Yaari, 1965; Davidoff et al., 2005). Full annuitization remains optimal in the presence of uncertain healthcare expenditures, provided that they occur later in life (Davidoff et al., 2005; Peijnenburg et al., 2016), while the presence of bequest motives reduces the annuity demand (Lockwood, 2012). Fig. 5 shows that our model reproduces the same set of results. The only exception is for those in the lowest wealth band who purchase no life annuities since they rely heavily on government transfers. In addition, Fig. 5 shows that higher wealth can increase the optimal annuitization rate, a result also found in Ai et al. (2017).

That individuals save from annuity income explains the optimality of full annuitization in the presence of uncertain healthcare costs (Peijnenburg et al., 2016). We verify this result by simulating the optimal consumption of a fully annuitized retiree endowed with $600K liquid wealth and no housing wealth.[3] Fig. 6 shows some summary statistics of the simulated consumption paths. The mean and almost all of the quantiles are consistently below the annuity income until late in life, indicating that the annuitants save from annuity income to build up precautionary savings.

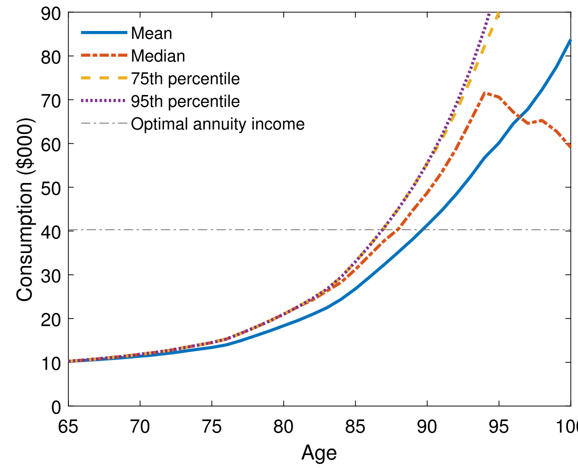

Fig. 6. Simulated optimal consumption for retirees endowed $600K liquid wealth and no housing wealth. The preference parameters are $b = 0, \gamma = 5, \psi = 0.5$. We assume no LTCI is offered in the market, and the resulting optimal annuitization rate is 100%.

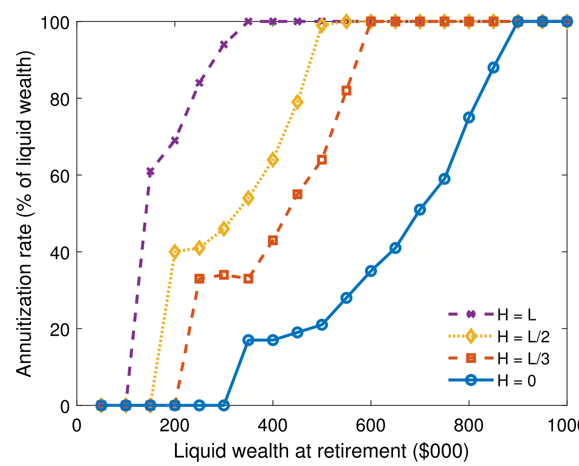

Fig. 7. Optimal annuitization rates for retirees endowed with liquid wealth and housing wealth. The legend represents the ratio between housing wealth (H) and liquid wealth (L) at retirement. The preference parameters are $b = 2, \gamma = 5, \psi = 0.5$. We assume no LTCI is offered in the market.

3.1.1. *Housing wealth enhances annuity demand*

Having replicated the well-known results in the literature, we extend our model to include housing wealth endowment. Without bequest motives, the optimal annuitization rates are again 100% except for the very poor, so we henceforth focus on the case with bequest motives (i.e., $b = 2$). Due to the illiquidity of housing wealth, retirees can only annuitize their liquid assets if they are unable to access equity release products (e.g., reverse mortgage). To assess the impact of housing wealth on annuity demand, we investigate how the amount of annuitized wealth as a proportion of liquid wealth varies with housing wealth at retirement. Fig. 7 shows that as the ratio between housing and liquid wealth grows, partial annuitization starts at a lower wealth level, and the minimum wealth required for full annuitization is also reduced. Therefore, the presence of housing wealth enhances the annuity demand.

We find that the presence of housing wealth lowers the precautionary savings from liquid wealth, thereby allowing retirees to annuitize a greater proportion of their liquid wealth. To examine the interaction between precautionary savings and housing

[3] The amount of $600K is chosen for illustrative purposes, and the same result can be found using larger or smaller amount so long as the full annuitization is optimal. We use the total wealth endowment of $600K in later numerical illustrations as well. The results can be extended to other wealth levels.

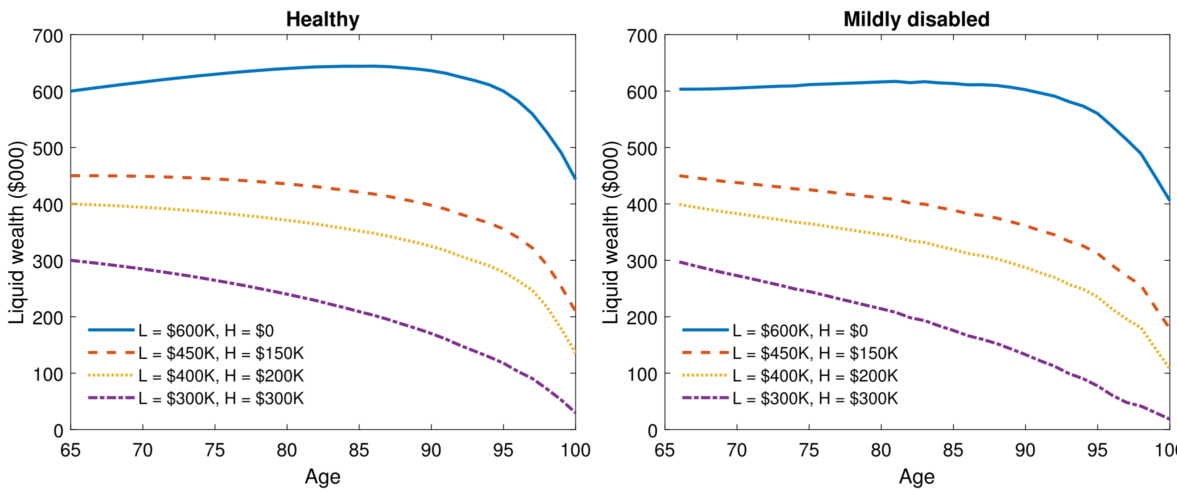

Fig. 8. Simulated average optimal liquid wealth paths in (left panel) healthy and (right panel) mildly disabled states. The legend represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are $b = 2$, $\gamma = 5$, $\psi = 0.5$. We assume retirees do not purchase life annuities or LTCI.

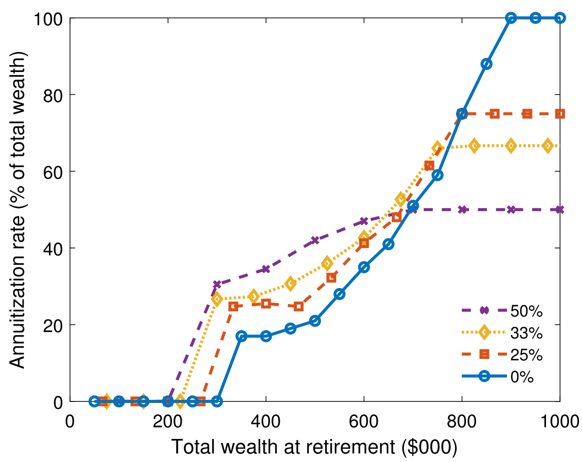

Fig. 9. Optimal annuitization rates (as a percentage of total wealth) for retirees with different levels of total wealth. The legend represents the proportion of total wealth in home equity at retirement. The preference parameters are $b = 2$, $\gamma = 5$, $\psi = 0.5$. We assume no LTCI is offered in the market.

wealth, we simulate the optimal liquid wealth paths assuming one does not purchase life annuities or LTCI. Fig. 8 plots the average paths in the healthy and mildly disabled states, in which retirees hold precautionary savings. When the retirement endowment has no housing component, the average liquid wealth increases slightly before declining. With a higher proportion of net worth in housing wealth, the curve first flattens and then becomes steeper. This suggests that as housing wealth increases, retirees draw down their liquid wealth at a faster pace and employ less liquid wealth relative to total wealth as precautionary savings.

While housing wealth can increase the annuity demand, its presence imposes a liquidity constraint that reduces the proportion of total wealth that can be annuitized. To investigate the net effect, we plot the optimal annuitization rate as a percentage of total wealth in Fig. 9. We see that the annuitization rates are capped at the proportion of liquid wealth. Before such constraint becomes binding, the enhancement effect outweighs liquidity constraint and housing wealth increases annuity demand that is measured by the fraction of total wealth.

3.1.2. Risk aversion and EIS both affect annuity demand

In addition to housing wealth, we find that both risk aversion and the EIS affect the annuity demand. Fig. 10 shows that a higher degree of risk aversion generally reduces the optimal annuitization rate. Individuals with stronger risk aversion are more averse to substituting consumption across different health states, so they set aside more liquid wealth to smooth health shocks. This in turn reduces the optimal annuitization level. Fig. 10 also shows that the differences shrink with a higher level of liquid wealth or housing wealth as both factors enhance the annuity demand.

Our finding is in contrast to those in Inkmann et al. (2010) and Pashchenko (2013). Both find that more risk-averse retirees should purchase more annuities. Inkmann et al. (2010) consider a different setting where one can invest in the stock market and has no healthcare costs. More risk-averse individuals invest less in equities and subsequently purchase more annuities. In fact, after removing the component of healthcare costs, we also find that the demand for annuities increases with risk aversion (left panel of Fig. 11). Pashchenko (2013) employs a power utility function where a higher degree of risk aversion is tied to a lower degree of the EIS. Our finding does not contradict hers to the extent that we find the demand for annuity generally increases with a smaller EIS as we will discuss next.

Individuals with a higher level of the EIS are known to have higher current consumption and lower savings if the time-preference-adjusted return on savings is negative (Campbell and Viceira, 1999). We replicate this result using a simplified version of our model that assumes a certain finite lifespan and no healthcare costs. Furthermore, we find that a higher level of the EIS is associated with a larger amount of bequests based on the same set of assumptions. The detailed results are presented in Appendix C.

After incorporating mortality risk back to the model while still abstracting from the uncertain healthcare expenditure, we find that the optimal annuitization rates are similar among retirees with different levels of the EIS (right panel of Fig. 11). However, we find noticeable differences in the optimal consumption paths. The left panel of Fig. 12 shows that individuals with a higher degree of the EIS tend to have less current consumption and a flatter consumption path. Consequently, they tend to leave larger bequests (right panel of Fig. 12), consistent with our prior finding in the case of no mortality risk or healthcare costs.

When facing health shocks, retirees will normally choose to hold precautionary savings if LTCI is not offered in the market. They can either annuitize less to set aside more liquid wealth upfront, or save from annuity income to build up the buffer. Since our health transition model predicts that the risk of requiring long-term care increases significantly after age 85 (Fig. 4), retirees have time to accumulate liquid wealth by spending less than the annuity income during early retirement. For someone without bequest

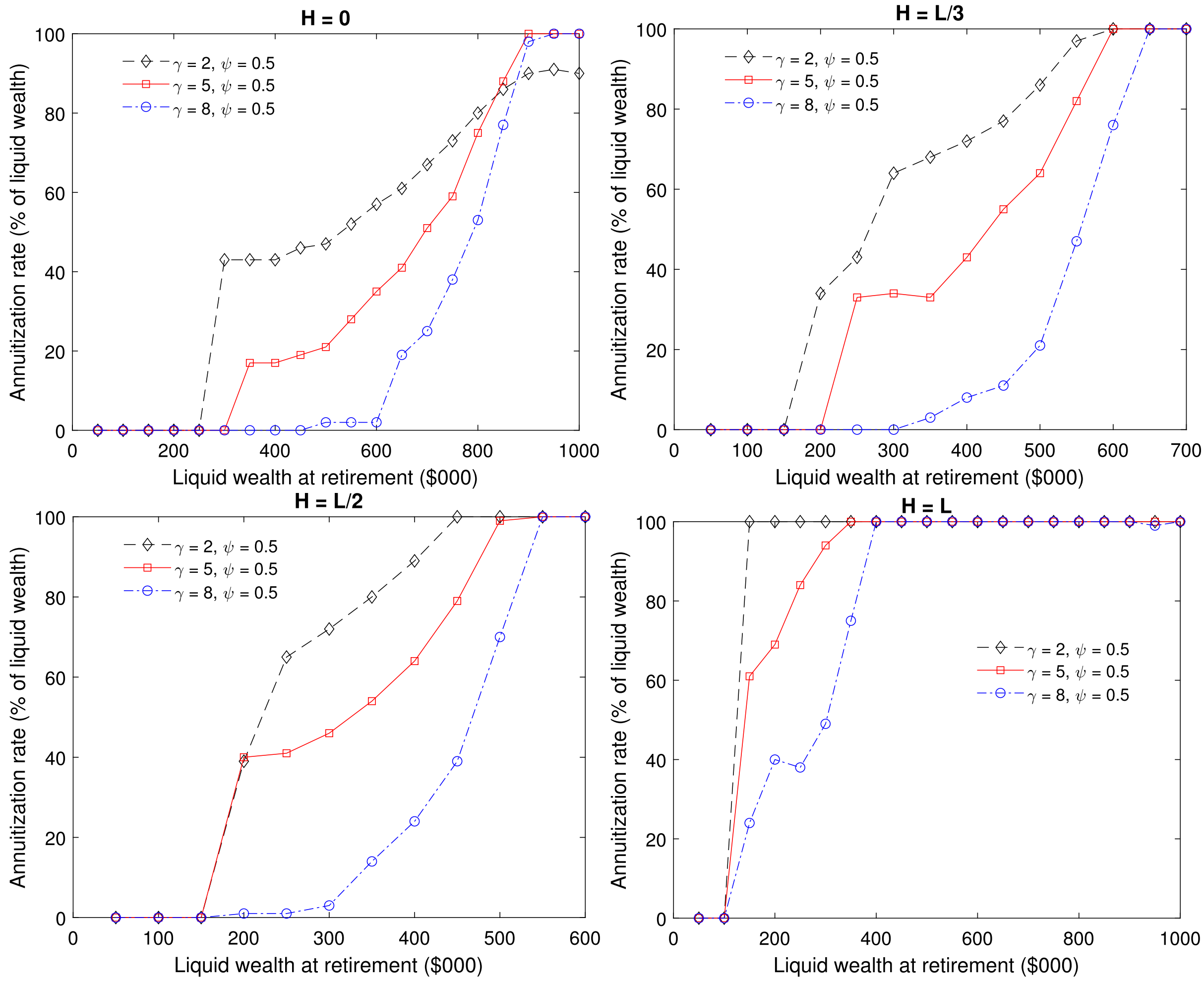

Fig. 10. Optimal annuitization rates for retirees with different levels of risk aversion and a certain bequest motive ($b = 2$). The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. We assume no LTCI is offered in the market.

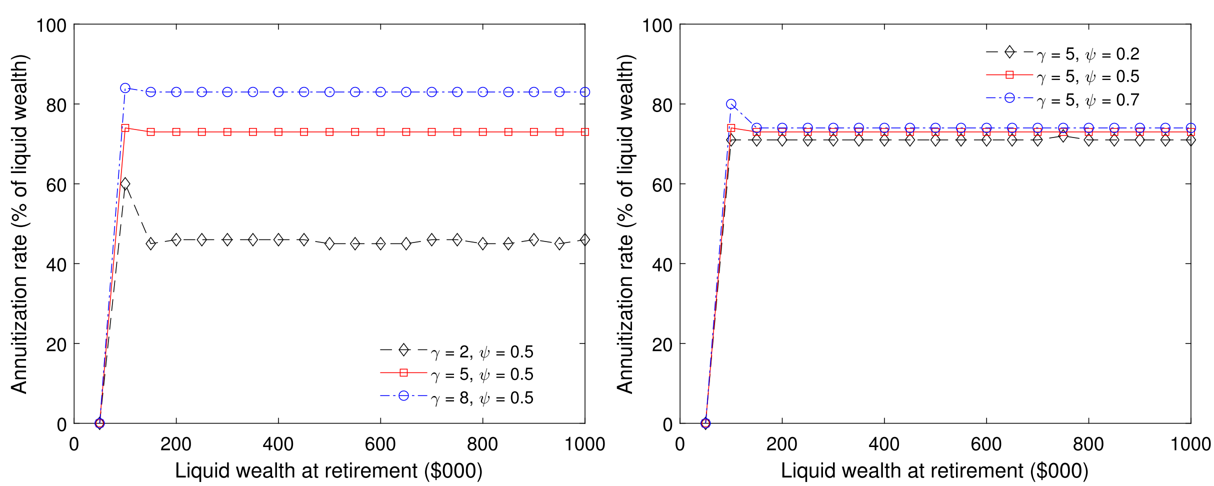

Fig. 11. Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS in the absence of uncertain healthcare costs. The strength of bequest motives is given by $b = 2$. Retirees are endowed with liquid wealth and no housing wealth.

motives, this is a more efficient strategy since wealth, if left unconsumed, generates no utility. For those with bequest motives, using a mixture of upfront savings and annuity income to build a buffer becomes optimal. Their desire to leave bequests lowers the opportunity cost of using liquid wealth as precautionary savings (Lockwood, 2018). We have shown that retirees with a higher level of the EIS are likely to leave a larger amount of bequests, which implies a lower opportunity cost of holding liquid wealth. As a result, Fig. 13 shows that retirees with a higher degree of the EIS tend to annuitize less of their wealth. Similar to the case in Fig. 10, the variations in the optimal annuitization rate diminish with more liquid or housing wealth.

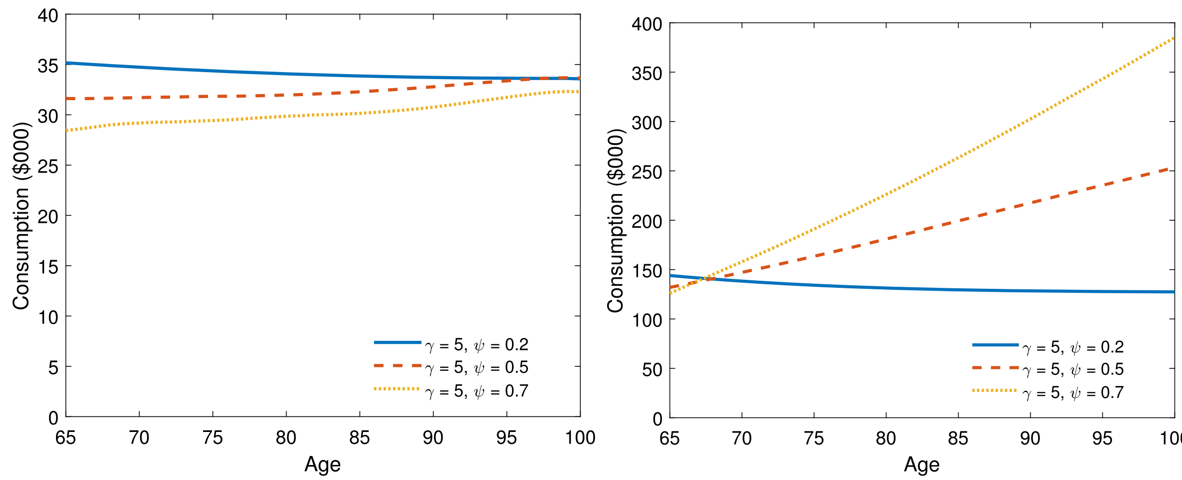

Fig. 12. Simulated average (left panel) optimal consumption paths and (right panel) optimal liquid wealth paths for retirees with different levels of the EIS and a certain bequest motive ($b = 2$). Retirees are endowed with $600K liquid wealth and no housing wealth at retirement. They purchase the optimal amount of annuities at retirement and have no access to LTCI.

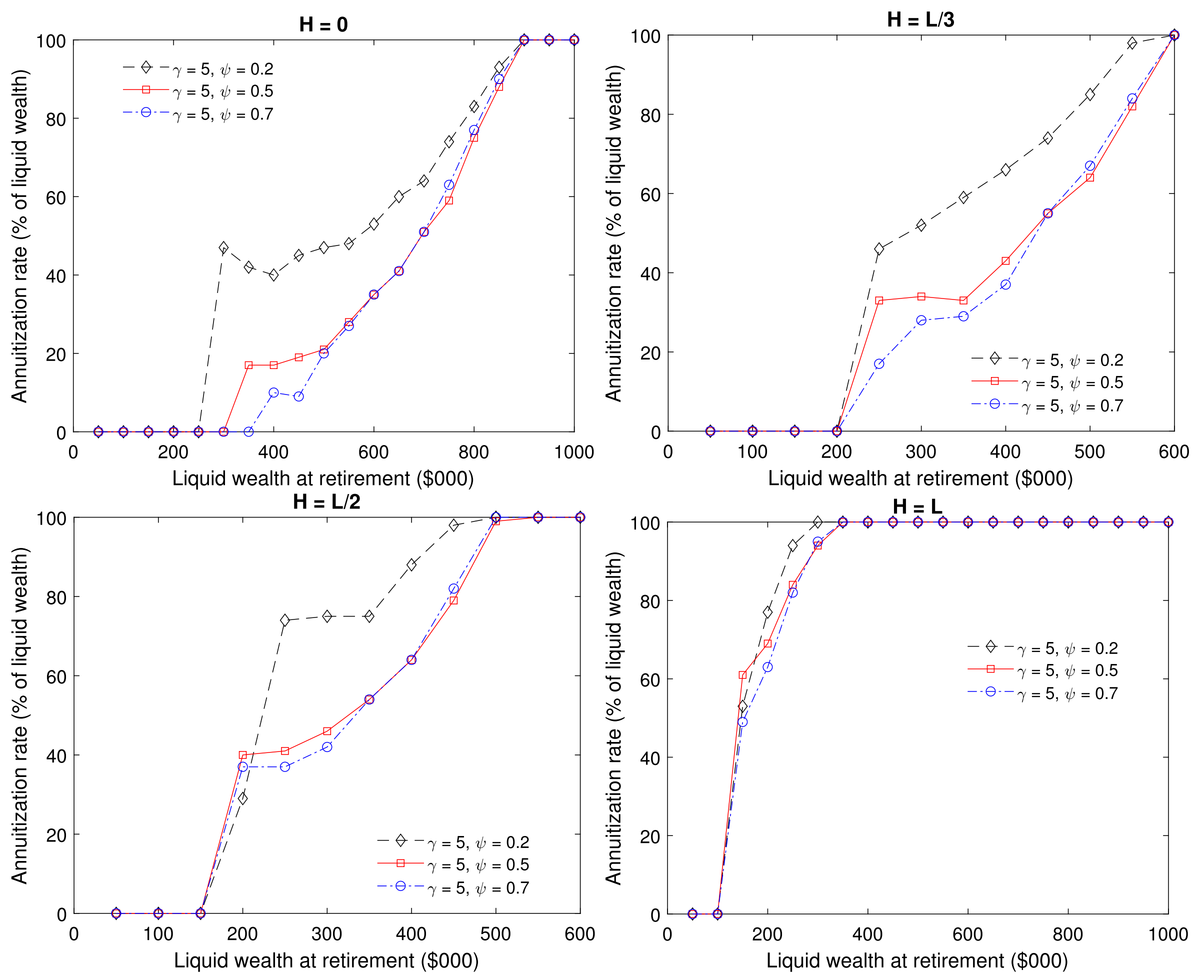

Fig. 13. Optimal annuitization rates for retirees with different levels of the EIS and a certain bequest motive ($b = 2$). The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. We assume no LTCI is offered in the market.

3.2. LTCI

LTCI is an effective instrument in managing the sizable health-care costs. Fig. 14 shows retirees endowed with liquid wealth and no housing wealth demand nearly full LTCI coverage once their wealth levels exceed a certain threshold. Those who optimally choose to purchase no LTCI coverage rely on government transfers that provide some form of LTCI through the minimum consumption guarantee. The impact of bequest motives is marginal, which is not surprising given the two offsetting effects of bequest motives. On the one hand, the desire to leave bequests can increase the demand for LTCI since the insurance coverage will add to the

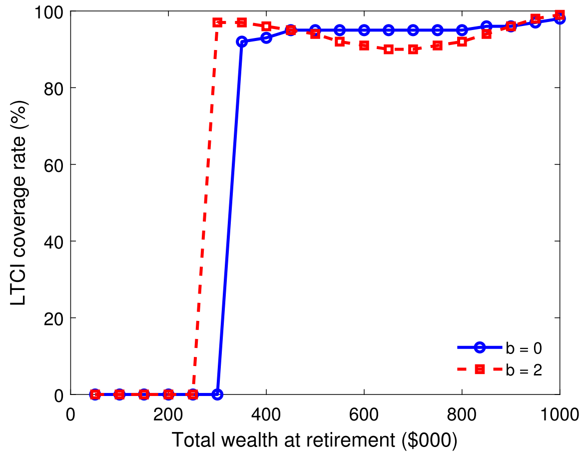

Fig. 14. Optimal LTCI coverage rates for retirees endowed with liquid wealth and no housing wealth at retirement. The legend represents the strength of bequest motives. The other preference parameters are $\gamma = 5, \psi = 0.5$. We assume no life annuity is offered in the market.

bequests left by those who died after becoming severely disabled (Pauly, 1990). On the other hand, bequest motives can lower the opportunity cost of precautionary savings, thereby reducing the demand for LTCI (Lockwood, 2018).

The jump in the optimal LTCI coverage rate shown in Fig. 14 is not unusual. Shao et al. (2019) also find strong demand for LTCI at different wealth levels, although they set the lowest wealth level at $240K, below which retirees are likely to have a minimal demand due to government transfers. The jump can be explained by the non-linear effect of LTCI on consumption. Fig. 15 shows the average optimal consumption paths in the severely disabled state under different LTCI coverage rates. Regardless of bequest motives, the increment in the optimal consumption grows considerably when LTCI coverage increases in equal steps from zero to 100%. By contrast, the average consumption in the healthy and mildly disabled states changes more or less evenly with LTCI coverage rate. The figures are displayed in Appendix D.1. The non-linear effect on consumption implies that, if one does not completely rely on government transfers and purchases some LTCI coverage, the marginal benefit of an extra coverage rate can easily exceed its marginal cost when the coverage is not high. Therefore, the optimal coverage rate for non-homeowners is either at the high end or the low end.

3.2.1. Housing wealth interacts with bequest motives

Fig. 16 shows that more housing wealth in proportion to total wealth generally lowers the optimal LTCI coverage rate regardless of the desire to leave bequests. This is due to the substitution effect that comes from the overlap between LTCI payment and housing wealth liquidation. Similar result is also found in Davidoff (2010) and Shao et al. (2019). The comparison between the two panels in Fig. 16 shows that the gaps between the curves in the left panel are larger than those in the right panel, suggesting that bequest motives lessen the impact of housing wealth on the optimal LTCI coverage rate. This implies that, between the two offsetting effects of bequest motives, the enhancement effect dominates in the presence of home equity.

The right panel of Fig. 16 shows homeowners endowed with less than $300K total wealth demand far more LTCI coverage than non-homeowners endowed with the same amount of total wealth. This is because purchasing LTCI helps preserve bequests for homeowners more than non-homeowners in the low wealth bands, while the reduction in consumption due to LTCI purchase is limited due to the minimum consumption guarantee. Fig. 17 compares the average amount of bequests under no LTCI coverage and full coverage.[4] The left panel shows that the two curves are almost parallel before the solid line falls to zero, suggesting that LTCI has a limited effect in slowing the wealth drawdown for non-homeowners. By contrast, the middle and right panels of Fig. 17 show that increasing LTCI coverage flattens the curve for homeowners. The average amount of bequests under the full coverage almost levels off after age 85. Fig. 18 shows the extent of consumption reduction caused by purchasing LTCI. The difference in annual consumption between no LTCI coverage and full coverage is, on average, around $1,000 for the first 20 years into retirement. Afterward, the gap closes due to the increased risk of requiring long-term care that incurs substantial costs and triggers LTCI payment. For homeowners, the right two panels of Fig. 18 show that the average consumption with the full LTCI coverage eventually overtakes that of no LTCI coverage.

3.2.2. Risk aversion more important than the EIS in affecting LTCI demand

We find that the EIS has a minimal impact on the optimal LTCI coverage rate for retirees with and without bequest motives alike (Fig. 19). The same result holds for homeowners with different levels of housing wealth (see Appendix D.1 for more details). That the EIS has little effect on the demand for LTCI is intuitive. Unlike life annuities which provide a constant stream of income throughout one's lifetime, LTCI provides income only when one is severely disabled, limiting its ability to smooth consumption over time. We previously argued that a higher level of the EIS strengthens the role of bequest motives in lowering the opportunity cost of liquid wealth buffers, which can reduce the demand for LTCI. The effect is offset by the enhancement made to the bequests by a higher LTCI coverage rate.

Fig. 20 shows how the demand for LTCI varies with risk aversion in the absence of housing wealth. Although it appears that a higher risk aversion leads to a lower optimal LTCI coverage rate, it is not necessarily the case for homeowners, which will be discussed later. In addition, the optimal coverage rates (conditional on purchasing LTCI) in Fig. 20 are all close to 100%. Fig. 21 shows that the relative difference between the optimal and the full coverage rate, in terms of the objective function, is well below 5%, suggesting that the utility lost from purchasing the full LTCI coverage is minimal.

To further explain the result in Fig. 20 that a higher risk aversion drives down LTCI demand, we plot the simulated average optimal consumption paths in each health state along with the overall average (Fig. 22). Deviating away from the optimal LTCI coverage to purchase the full amount widens the gap in consumption between the severely disabled state and other health states. Since more risk-averse individuals prefer a smoother consumption between different health states, retirees with a relatively high level of risk aversion optimally choose to avoid the full LTCI coverage.

Furthermore, we find that risk aversion interacts with housing wealth in affecting LTCI demand. Fig. 23 and Fig. 24 compare the impact of risk aversion on the optimal LTCI coverage among retirees endowed with various levels of housing wealth. As housing wealth grows, the lower the level of risk aversion, the greater the reduction in the optimal LTCI coverage rate. In one case where retirees are endowed with an equal amount of liquid and housing wealth and have no bequest motives (right panel of Fig. 23), the optimal LTCI coverage rate increases with risk aversion, reversing the order in Fig. 20.

[4] We select the total wealth endowment amount of $200K for illustrative purposes. The results can be extended to other wealth levels below $300K.

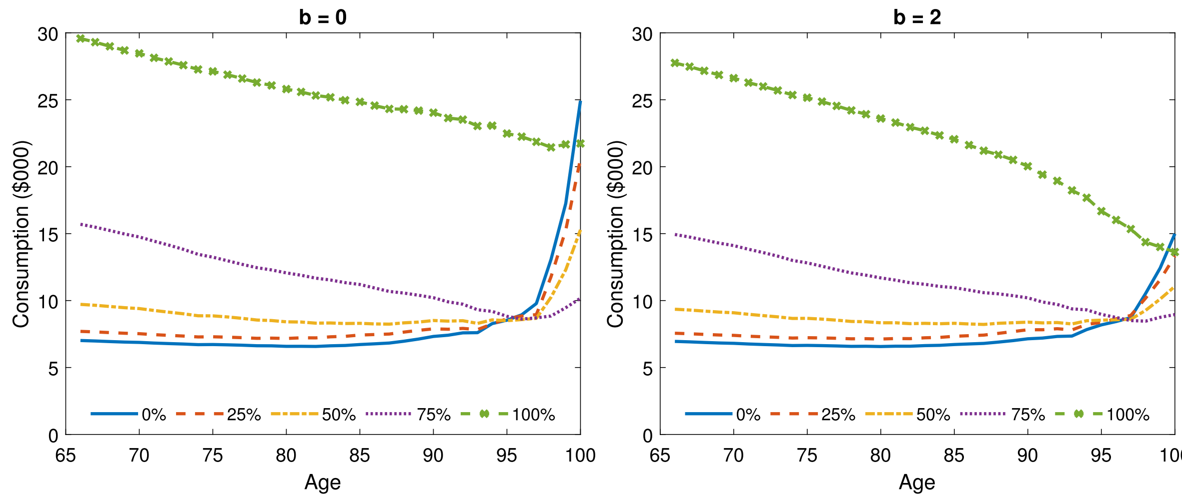

Fig. 15. Simulated average optimal consumption paths in the severely disabled state. The legend represents different LTCI coverage rates. Retirees are endowed with $600K liquid wealth and no housing wealth at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$. We assume no life annuity is offered in the market.

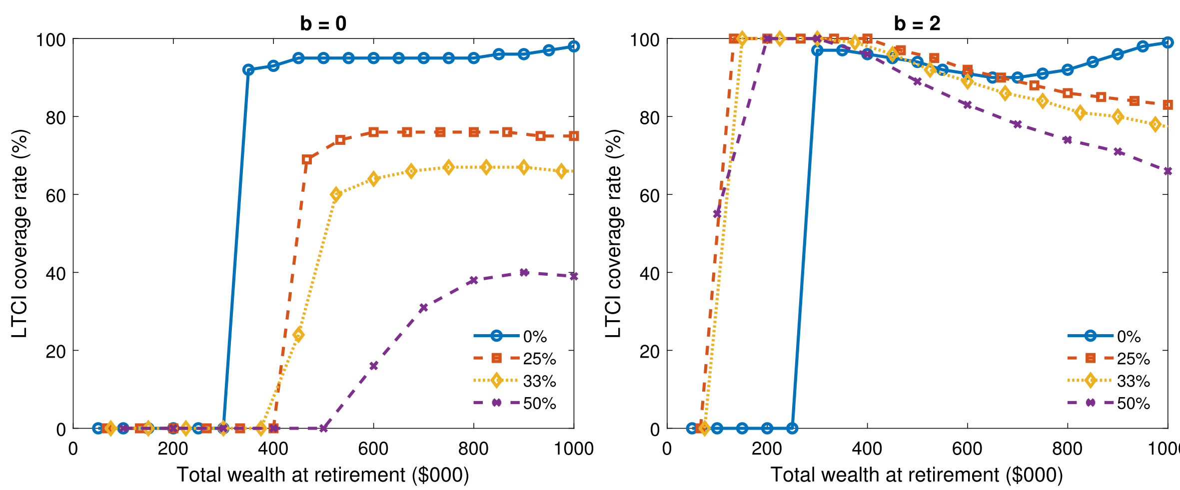

Fig. 16. Optimal LTCI coverage rates for retirees endowed with liquid wealth and housing wealth. The legend represents the proportion of total wealth in home equity at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$. We assume no life annuity is offered in the market.

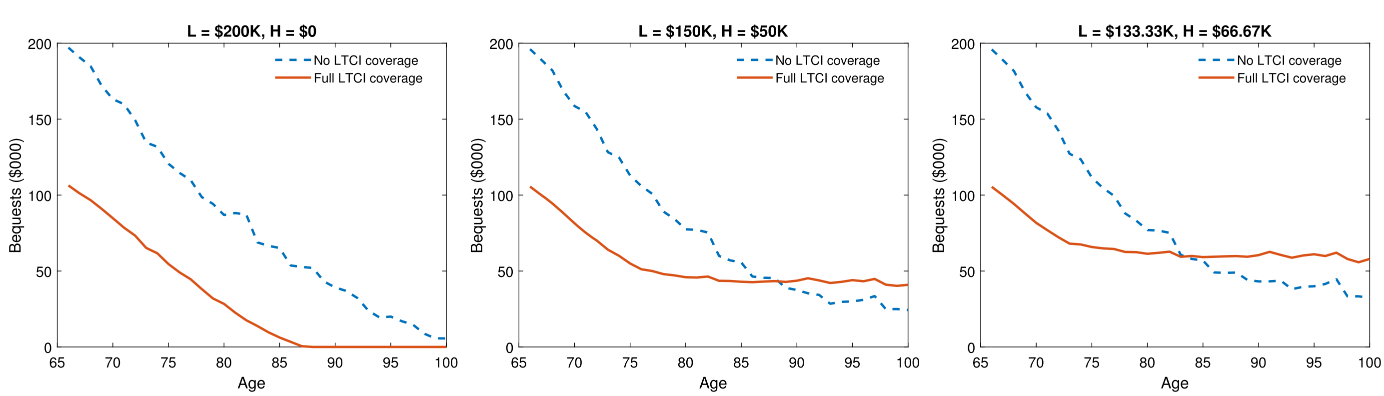

Fig. 17. Simulated average bequests for retirees who purchase no LTCI coverage or full LTCI coverage. The title of each panel represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are $b = 2$, $\gamma = 5$, $\psi = 0.5$. We assume no life annuity is offered in the market.

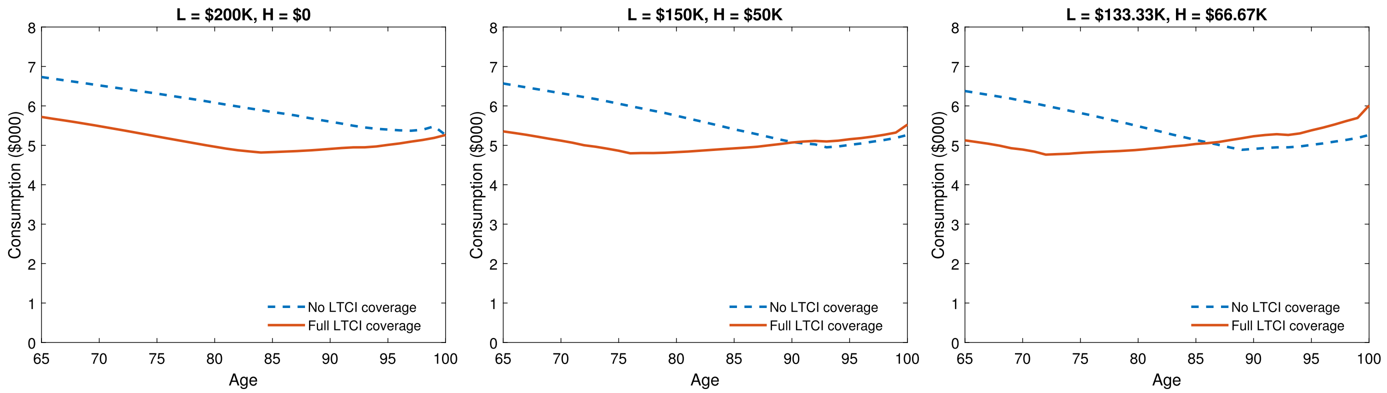

Fig. 18. Simulated average optimal consumption paths for retirees who purchase no LTCI coverage or full LTCI coverage. The title of each panel represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are $b = 2$, $\gamma = 5$, $\psi = 0.5$. We assume no life annuity is offered in the market.

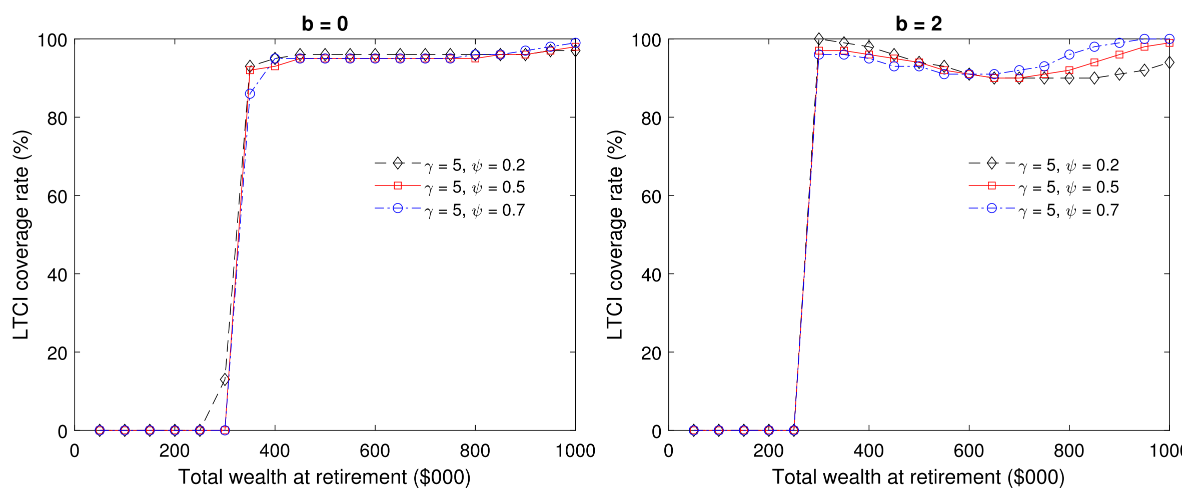

Fig. 19. Optimal LTCI coverage rates for retirees with different levels of the EIS. Retirees have no housing wealth. We assume no life annuity is offered in the market.

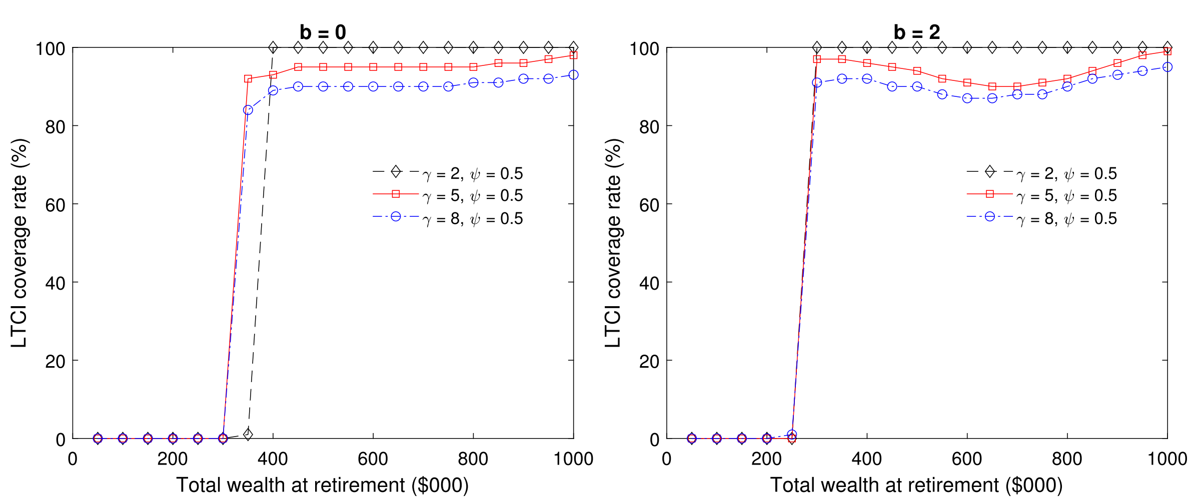

Fig. 20. Optimal LTCI coverage rates for retirees with different levels of risk aversion. Retirees have no housing wealth. We assume no life annuity is offered in the market.

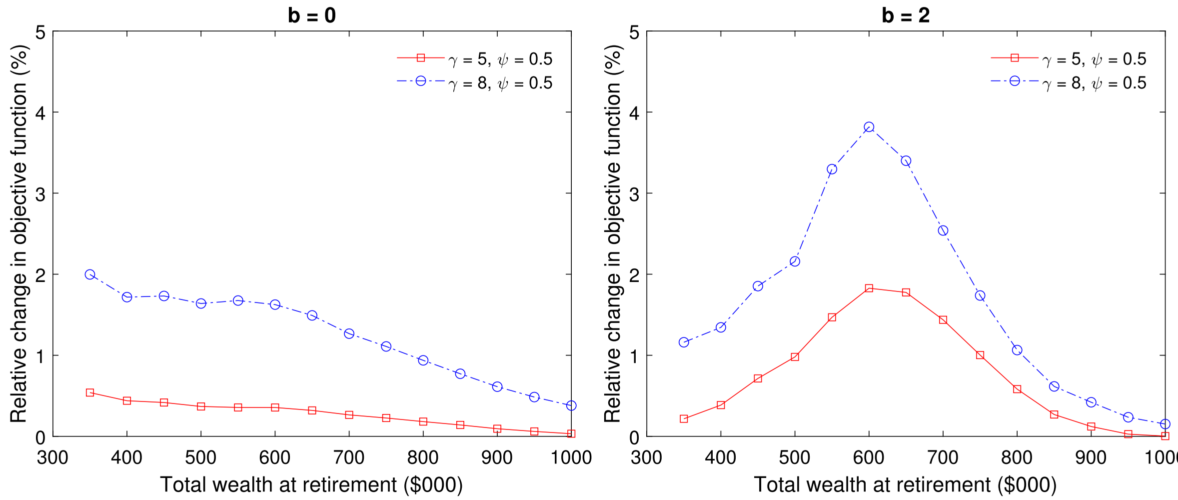

Fig. 21. Relative difference in the value of objective function between the full LTCI coverage and the optimal LTCI coverage. Retirees have no housing wealth. We assume no life annuity is offered in the market.

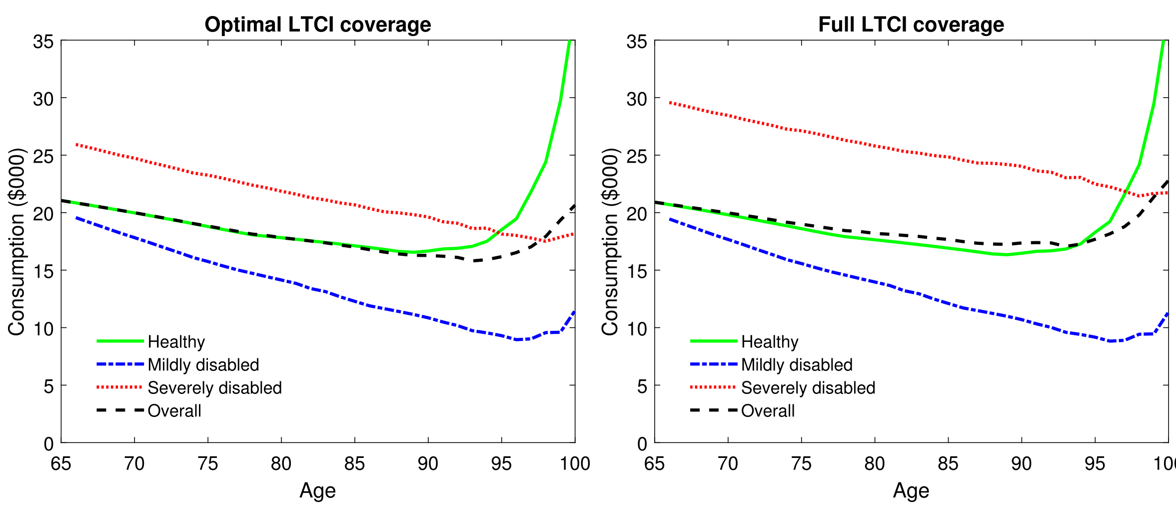

Fig. 22. Simulated average optimal consumption paths in each health state and the overall average. Retirees are endowed with \$600K liquid wealth and no housing wealth at retirement. The preference parameters are $b = 0$, $\gamma = 5$, $\psi = 0.5$. We assume no life annuity is offered in the market.

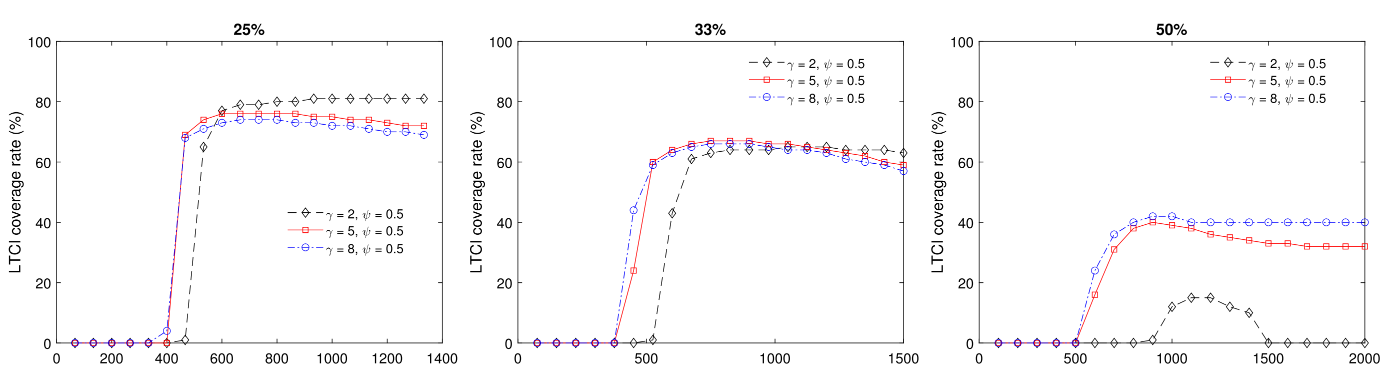

Fig. 23. Optimal LTCI coverage rates for retirees with different levels of risk aversion and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. We assume no life annuity is offered in the market.

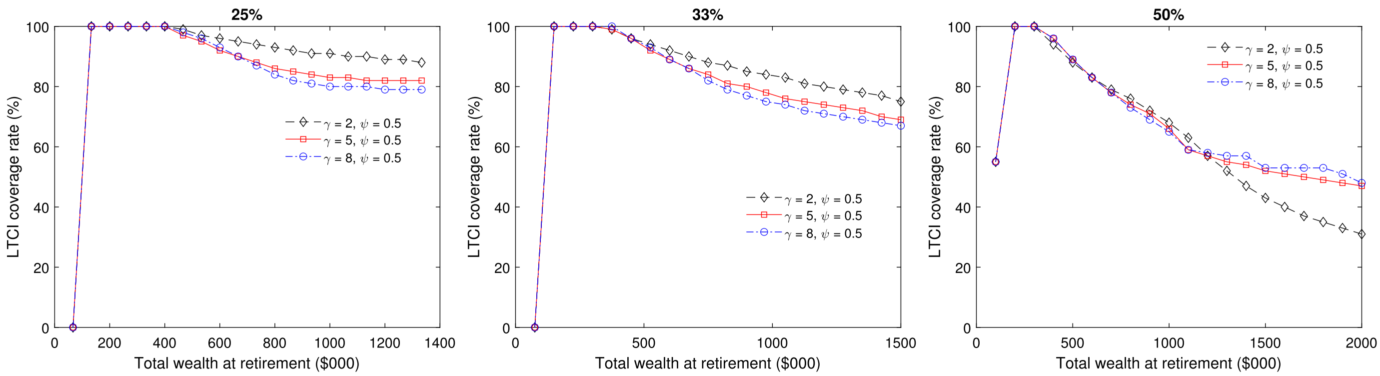

Fig. 24. Optimal LTCI coverage rates for retirees with different levels of risk aversion and a certain bequest motive ($b = 2$). The title above each panel denotes the proportion of total wealth in home equity at retirement. We assume no life annuity is offered in the market.

3.3. Both annuities and LTCI

LTCI is known to enhance the demand for annuities in the absence of housing wealth (see e.g., Ameriks et al., 2008; Wu et al., 2022), and the complementarity between life annuities and LTCI can be reversed by illiquid housing wealth (Davidoff, 2009). We replicate this pair of results and present the details in Appendix D.2.

When the life annuity or LTCI alone is offered in the market, we have shown that the product demand is affected by both housing wealth and preferences. When both products are offered, we find that one's wealth level and homeownership status are more important than her risk aversion or the EIS. In addition, the bequest motives remain an important factor in determining the product demand.

3.3.1. Housing wealth and liquidity

We have shown that housing wealth increases annuity demand for retirees with bequest motives when the life annuity alone is offered in the market. For retirees without bequest motives, the improvement is disguised by the optimality of full annuitization in the absence of housing wealth. When LTCI becomes accessible, retirees without bequest motives do not always find full annuitization optimal. Among those who partially annuitize their wealth, the optimal annuitization rates show slight improvement with housing wealth (left panel of Fig. 25). For retirees with bequest motives, homeownership remains an important factor in affecting the annuity demand. The right panel of Fig. 25 shows that homeowners tend to have higher optimal annuitization rates than non-homeowners. There is, however, little variation among homeowners endowed with the same level of liquid wealth.

Fig. 26 displays the optimal annuitization rate as a proportion of total wealth. The housing wealth almost always reduces the annuity demand when retirees can access LTCI, in contrast to the case of no LTCI access (Fig. 9). That housing wealth enhances annuity demand is unable to offset the liquidity constraint introduced by its presence. The reasons are twofold. Firstly, the presence of LTCI reduces precautionary savings, thereby narrowing the gap between homeowners and non-homeowners in terms of their optimal annuitization rates as a proportion of liquid wealth (Fig. 25). Secondly, that some liquid wealth is allocated to purchase LTCI further lowers the amount of wealth that can be annuitized.

Fig. 27 compares the optimal LTCI coverage rate between homeowners and non-homeowners, and among homeowners with different levels of housing wealth. There are noticeable declines in the optimal LTCI coverage rates with higher housing wealth proportions, so the result that housing wealth typically weakens LTCI demand remains the same regardless of the access to life annuities. The right panel of Fig. 27 shows that housing wealth increases LTCI demand for retirees in the low wealth bands, similar to the case of no access to life annuities.

Compared to Fig. 16 where life annuities are not offered in the market, the curves in Fig. 27 move more abruptly with total wealth. This is due to the result that retirees generally use all the liquid wealth to purchase the two products, which we will discuss later. Since a one percentage point increase in LTCI coverage rate generally requires less liquid wealth than the same percentage point increase in annuitization rate, the capacity to purchase LTCI is more sensitive to changes in liquid wealth. As the optimal annuitization rate increases steadily with liquid wealth, the optimal LTCI coverage rate might land in a higher or lower position compared to that of the closest wealth band depending on the budget constraint.

Fig. 28 and Fig. 29 show the allocation of liquid wealth endowment in the absence and presence of bequest motives, respectively. For retirees without bequest motives, both homeowners and non-homeowners usually spend all liquid wealth on life annuities and LTCI. The proportion allocated to LTCI decreases as housing wealth grows, reflecting a weakening LTCI demand. For retirees with bequest motives, the homeownership status significantly affects the liquid wealth allocation. Homeowners generally exhaust their liquid wealth on purchasing the two products and leave little cash on hand at the point of retirement. In contrast, the top left panel of Fig. 29 shows that non-homeowners usually have some cash on hand after the product purchases. For homeowners with bequest motives, the allocation to LTCI shows an exponential decay with liquid wealth. This is mainly driven by the high LTCI coverage in the low wealth levels among homeowners (right panel of Fig. 27).

3.3.2. Preference

We find that risk aversion and the EIS play a far less important role in determining product demand compared to housing wealth. Fig. 30 compares the optimal annuitization rate among different levels of risk aversion and the EIS in the case of no bequest motives. Fig. 31 performs the same comparison for retirees with bequest motives. In both figures, the curves almost overlap with each other. The only exception is for retirees who have a relatively low risk aversion and some bequest motives (left panel of Fig. 31). They show significantly less annuity demand. The comparison of the optimal LTCI coverage rate shows a similar result. In Fig. 32 and Fig. 33, the optimal levels vary little with the risk aversion or the EIS. We find similar results for homeowners and that the results are robust to different levels of housing wealth. The figures are displayed in Appendix D.2.

In the single product case, we have shown the strong impact of bequest motives on annuity demand (Fig. 5) and on LTCI demand for homeowners (Fig. 16). When both products are available, bequest motives continue to play an important role in product

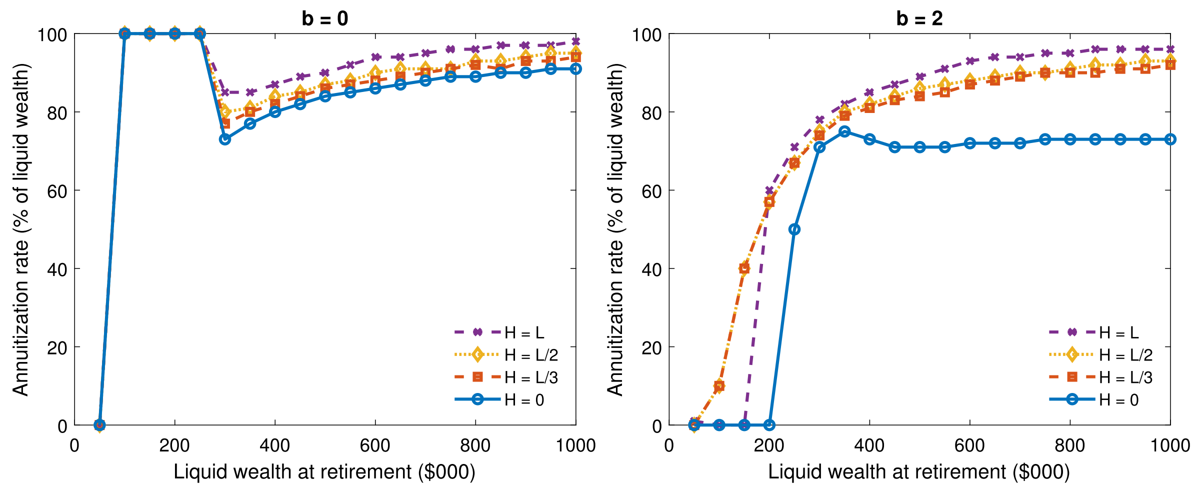

Fig. 25. Optimal annuitization rates (as a percentage of liquid wealth) for retirees who have access to LTCI. The legend represents the ratio between housing wealth (H) and liquid wealth (L) endowment at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$.

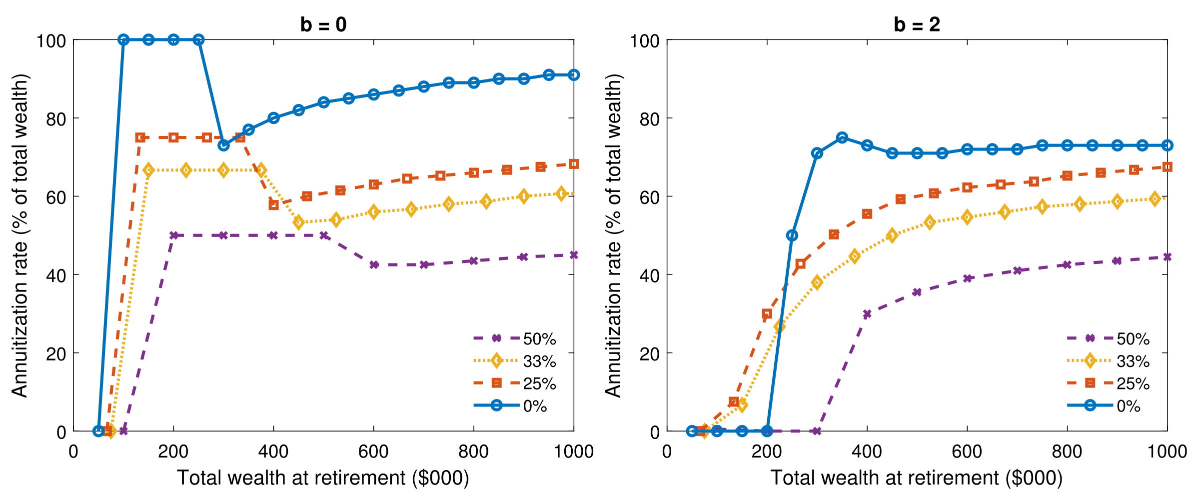

Fig. 26. Optimal annuitization rates (as a percentage of total wealth) for retirees who have access to LTCI. The legend represents the proportion of total wealth in housing at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$.

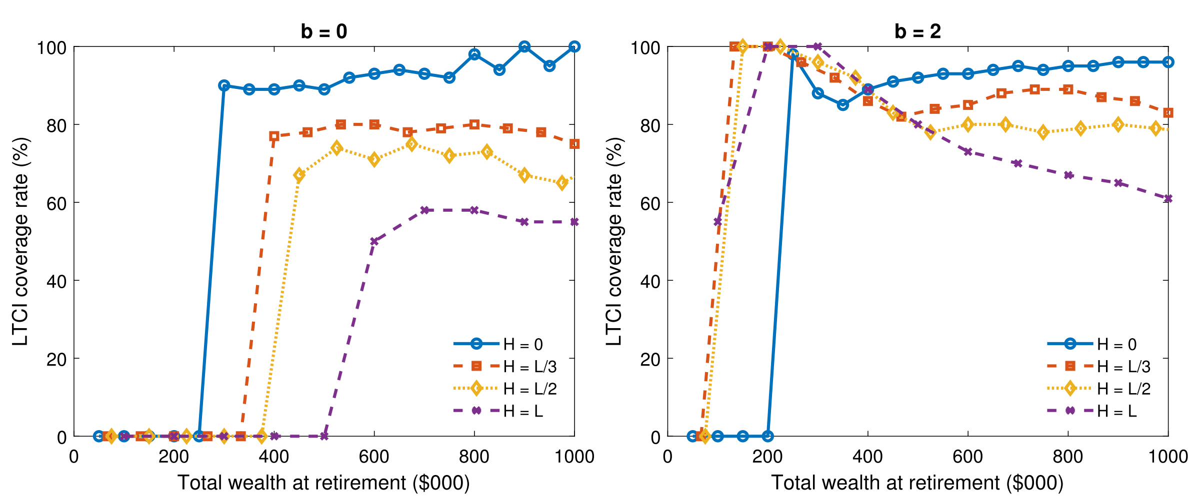

Fig. 27. Optimal LTCI coverage rates for retirees who have access to life annuities. The legend represents the ratio between housing wealth (H) and liquid wealth (L) endowment at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$.

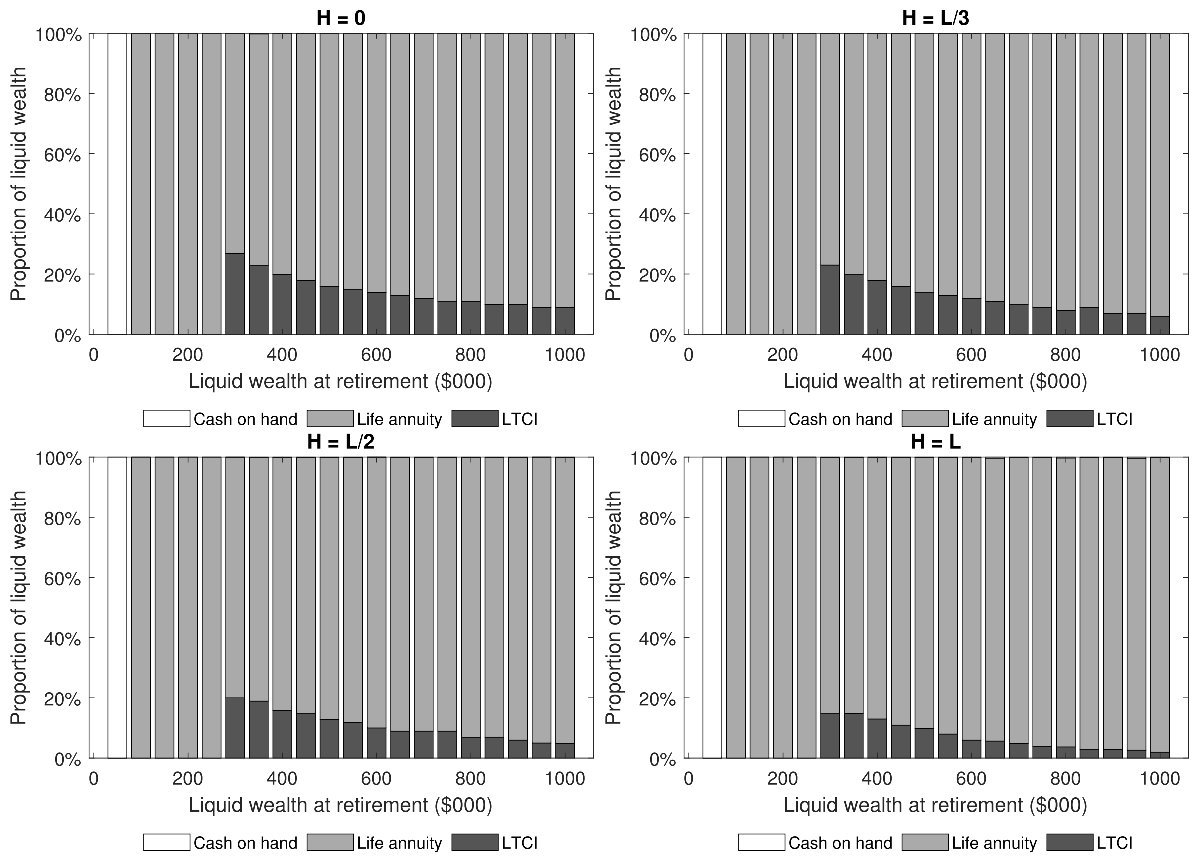

Fig. 28. The optimal allocation of liquid wealth endowment in the absence of bequest motives. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. The preference parameters are $b = 0$, $\gamma = 5$, $\psi = 0.5$.

decisions. Fig. 25 shows that bequest motives discourage annuity purchase, especially for those in the low wealth bands or do not have housing wealth. Fig. 27 shows a similar result to Fig. 16 that bequest motives improve the optimal LTCI coverage rate. Moreover, Figs. 28 and 29 show that bequest motives affect the liquid wealth spending, especially for non-homeowners. While retirees with no bequest motives tend to spend up their liquid wealth on the two products, non-homeowners with bequest motives leave some cash on hand.

4. Discussion

The real interest is an important assumption that affects the product prices and liquid asset returns. We perform sensitivity analysis on the real interest rate by assuming a lower (1.5%) and a higher (3.5%) rate compared to the baseline (2.5%). While a lower real interest rate is more relevant based on recent experiences (e.g., an era of low interest rates followed by recent high inflation), a higher interest rate might reoccur in the longer term. We find a lower real interest rate generally makes LTCI less attractive due to a higher price. In addition, a lower interest rate slows down the pace at which the optimal annuitization rate grows with liquid wealth. In terms of the impact of housing wealth and individual preference on product demand, we find our results robust to varying levels of real interest rate. The detailed results are available upon request.

We abstract from equity risk, which is a background risk in our setting. Peijnenburg et al. (2016) show that background risk does not significantly reduce the optimal annuitization levels. With equity risk, individuals annuitize (almost) fully and use excess annuity income over consumption to invest in equity according to their risk aversion. Therefore, including equity risk only changes the investment allocation, not the optimal annuity or LTCI decisions.

We abstract from borrowing against housing wealth through equity release products such as reverse mortgages. Having access to reverse mortgages will increase consumption in early retirement and enhance LTCI demand (Shao et al., 2019). For wealthy homeowners, their annuitized wealth is constrained by the amount of liquid wealth (Fig. 9). Offering reverse mortgages to them will alleviate the liquidity constraint and increase the optimal annuitization rate as a proportion of total wealth. Future research can solve a more comprehensive portfolio decision problem that includes life annuities, LTCI, and reverse mortgages.

We have made a few simplified assumptions in our model that can be modified in future research. We assume that home equity liquidation is automatically triggered by entering into the state of requiring long-term care. While such an assumption is backed by empirical findings and helps keep our model more tractable, future research could relax this assumption and make whether to sell the property as a choice variable. We also assume actuarially fair pricing of life annuities and LTCI, which is the standard approach in the literature on optimal portfolio choices for retirement. Adding loads makes life annuities and LTCI more expensive, reducing the optimal annuitization rate and optimal LTCI coverage. Since we are interested in how demand changes with housing wealth and individual preferences rather than the absolute value of demand, we expect no material changes to our results if we consider product loads. Nevertheless, product loads will have an impact if we explicitly consider life care annuities, which will be more affordable than purchasing life annuities and LTCI separately due to lower expense and risk loading. Future research can investigate the impact of product loads while considering a more comprehensive product menu.

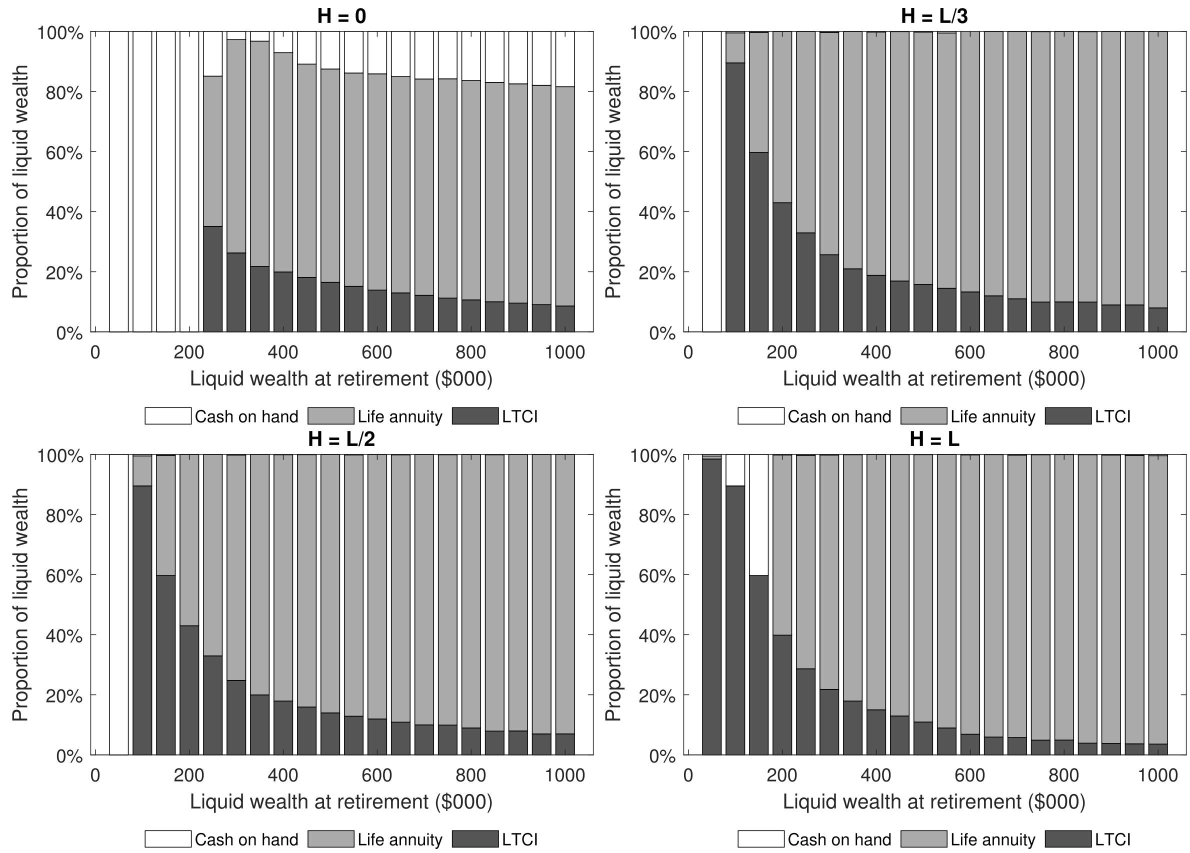

Fig. 29. The optimal allocation of liquid wealth endowment in the presence of bequest motives. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. The preference parameters are $b = 2$, $\gamma = 5$, $\psi = 0.5$.

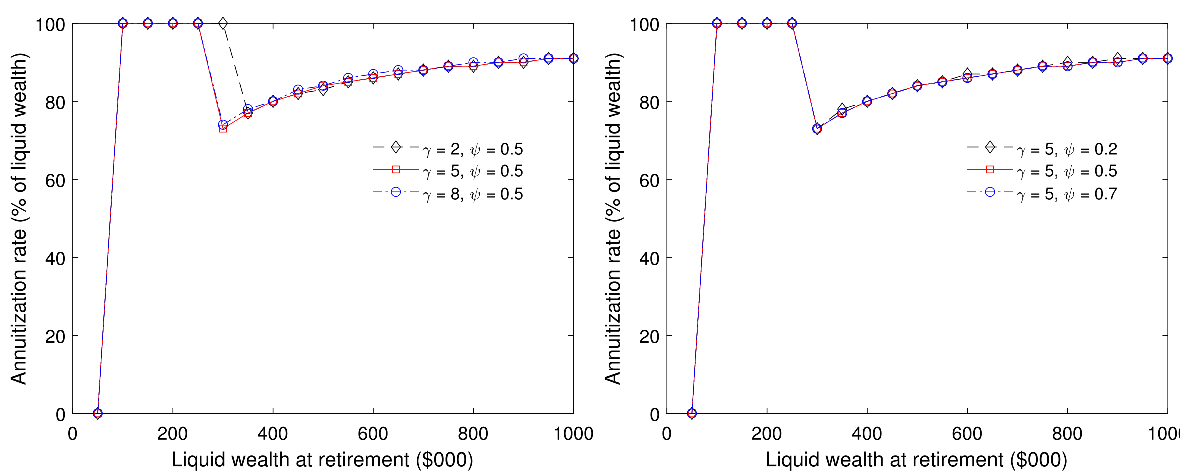

Fig. 30. Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when LTCI is offered in the market. Retirees have no housing wealth and no bequest motives.

5. Conclusions

The DC pension funds worldwide are reaching maturity as a growing number of members approach retirement. They need to convert a lump sum into income streams to support their retirement. However, the payout phase remains less developed than the accumulation phase, exposing retirees to longevity risk and health shocks, among other risks during retirement. A major difficulty in developing the payout phase is to design personalized retirement products that meet individual needs and circumstances. Our research offers new insights to help address the challenge.

We study the impact of housing wealth and individual preferences on demand for the products that insure against longevity risk and health shocks, i.e., life annuities and LTCI. Taking into account housing wealth makes the results relevant to homeowners, who make up the majority of retirees in the U.S. We use Epstein-Zin-Weil-type utility that separates risk aversion from the EIS to capture the preferences of more heterogeneous retirees compared to the commonly-used power utility function.

We find a higher level of risk aversion and a lower level of the EIS has opposite effects on annuity demand, highlighting the need to break their inverse relation imposed by the power utility

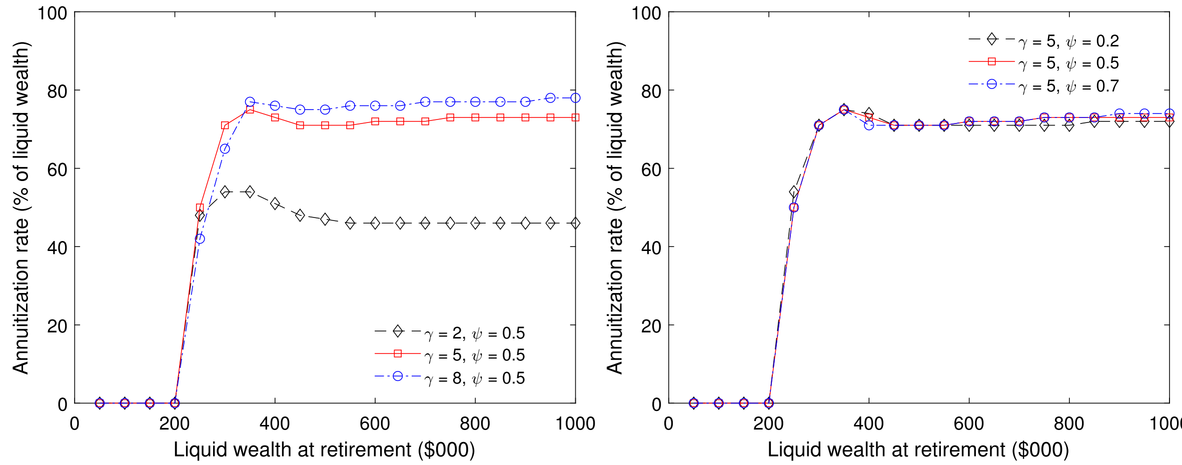

Fig. 31. Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when LTCI is offered in the market. The strength of bequest motives is given by $b = 2$. Retirees have no housing wealth.

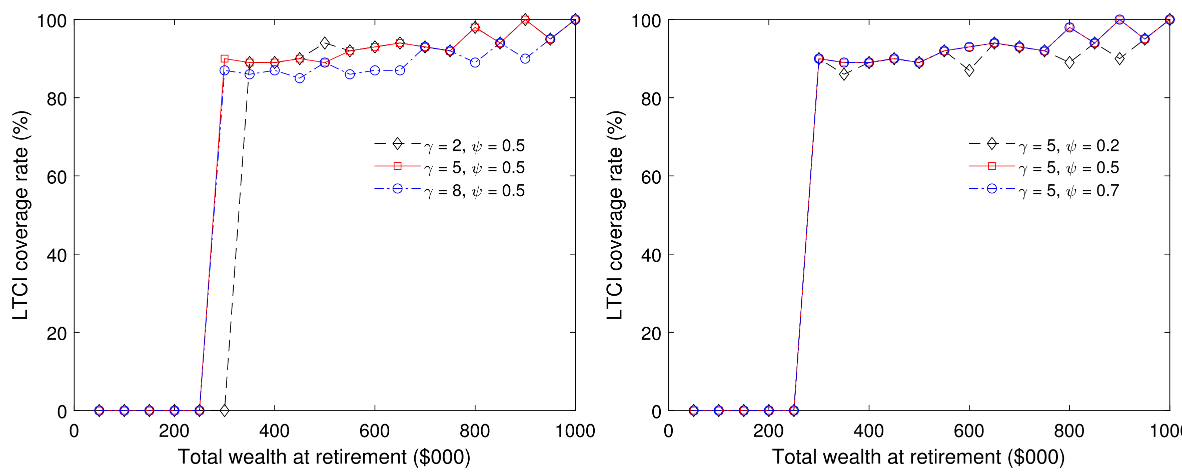

Fig. 32. Optimal LTCI coverage rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when life annuities are offered in the market. Retirees have no housing wealth and no bequest motives.

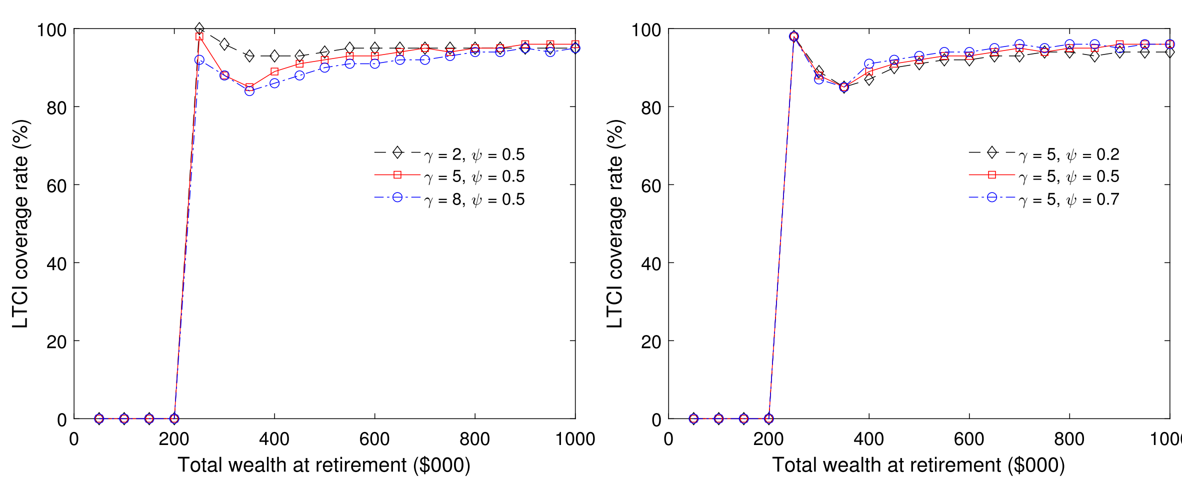

Fig. 33. Optimal LTCI coverage rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when life annuities are offered in the market. The strength of bequest motives is given by $b = 2$. Retirees have no housing wealth.

function. When health shocks are considered, a higher level of risk aversion or a higher level of the EIS decreases annuity demand. The impact diminishes with weaker bequest motives, a higher level of liquid wealth, or access to LTCI, all of which enhance annuity demand. The presence of home equity enhances annuity demand, albeit to a less extent when retirees can access LTCI.

Risk aversion and bequest motives interact with housing wealth to affect LTCI demand, while the impact of the EIS is limited. A lower degree of risk aversion strengthens the crowding-out effect of housing wealth on LTCI demand. In contrast, the crowding-out effect of housing wealth can be reduced or even reversed by bequest motives. Homeowners with limited wealth may demand higher LTCI coverage than renters endowed with the same amount of total wealth since LTCI can help preserve the bequests.

We find the demand for life annuities and LTCI is relatively robust to changes in risk aversion and the EIS when both products are offered simultaneously. Bequest motives, wealth levels, and homeownership status remain important factors in affecting product demand. Since the information about wealth and homeownership is far easier to obtain than risk aversion or the EIS, the finding implies that bundling life annuities with LTCI can substantially lower the cost of designing personalized retirement products.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.insmatheco.2023.03.004.

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