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Unconventional monetary policy and the bond market in Japan: A new Keynesian perspective

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ABSTRACT

Using the lens of a medium scale DSGE model, we analyze macroeconomic effects of Japan's unconventional monetary policy which is known as Qualitative and Quantitative Easing (QQE). Our focus is on the bond market. The model features: (i) commercial bank's demand for excess reserve in response to liquidity risk and (ii) linkage among central bank, commercial banks and the government via government bonds and bank reserve. We simulate the policy effects of a quantitative easing (QE) shock and a negative shock to the interest rate on excess reserve (IOER). The QE multiplier for real GDP is 1.94 and it has substantial effect on lowering the bond yield in line with the policy target of QQE. On the other hand, an IOER cut has qualitatively similar effects on the real and financial sectors but quantitatively its effect is of second order importance. In light of these policy simulations, we evaluate Japan's recent yield curve control policy.

1. Introduction

During the last two decades, the Japanese economy experienced several episodes of monetary policy changes. Starting from an era of near zero interest rate, Bank of Japan (BoJ) officially implemented the Quantitative Easing (QE) policy to inject liquidity into the banking system from the beginning of the millennium. Following this, after seven years of experiment with a conventional monetary policy of interest rate targeting, BoJ adopted a Qualitative and Quantitative Easing (QQE). This policy features an explicit inflation target, GDP, short rate and long term bond yield targets which are the cornerstones of the unconventional monetary policy (UMP).

The aim of this research is to analyze the effects of this unconventional monetary policy on the real economy and the Japanese bond market using the lens of a dynamic stochastic general equilibrium (DSGE) model. Understanding the effect of monetary policy on bond market is important in the context of Japan. The amount of outstanding Japanese government bonds (JGB) is much bigger than that of equities. It is 982.6 trillion yen while the market capitalization of equities is 619.7 trillion yen as of September 2019. Also the debt/GDP ratio in Japan is 238% which is by far the highest among advanced economies. As the majority of Japanese debt is financed by JGB, it has important implications for both monetary and fiscal policies. The changes in prices and yields of JGBs affect the balance sheet of both the government as a debtor and BoJ (47% of JGB is held by BoJ) and commercial banks (15% of JGB is held by commercial banks) as creditors. Thus the banks as a bond holder play an important role in the real world while the literature has mainly focused on the role of household as the holder of bonds.[2]

Our model is a stylized medium scale new Keynesian model similar to the extant models such as Smets and Wouters (2007), Gerali et al. (2010), Banerjee et al. (2019) and others. The advantage of using a DSGE model is that it enables us to see the linkage between the real and financial sectors of the economy when a policy shock hits the economy. The model has standard frictions such as aggregate habit persistence, investment adjustment cost, portfolio adjustment cost, monopolistic price formation, and nominal stickiness.

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2 See the appendix for the source of the data.

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There are two nonstandard features of our DSGE model. First, we model banking friction resulting from a liquidity risk similar to Chang et al. (2014) and Banerjee et al. (2019). A liquidity risk in the form of an anticipated negative cash flow shock disciplines commercial banks to hold precautionary excess reserve and not to push loans recklessly. This precautionary demand for bank reserve depends on two key policy rates, namely a penalty rate set by the Central Bank and the interest rate on reserve (IOER). We model the commercial bank's demand for reserve with a motivation to study the effect of a negative IOER which is a characteristic feature of QQE.

Second, we explicitly study the dynamic link among the government, central bank (CB hereafter) and commercial banks via the long term government bonds and bank reserves by formulating the CB's resource constraint in line with the recent work of Hall and Reis (2015). This helps us study the general equilibrium consequences of QQE policy for the bond market when the CB creates reserve. The government in our model plays a passive role in spending an exogenous stream of final goods and finances it from lump sum tax as well as consumption tax on households.[3]

In the related literature, the real effects of QE arise from the limit to arbitrage. This limit to arbitrage can arise from lender's moral hazard as in Gertler and Karadi (2013) or some market segmentation due to preferred habitat (Vayanos and Vila, 2020) or transaction cost as in Chen et al. (2012). In our model, instead of moral hazard and preferred habitat, portfolio adjustment costs drive wedges between bond and loan rates in the short run. Households in our model do not trade in long term government bonds and hold short term bank deposits which are perfect substitutes for short term government bonds. Bank deposits provide direct convenience utility to households as in Hansen and Imrohoroglu (2016) in addition to transaction utility of money. This assumption gives rise to a natural steady state borrowing-lending spread in our model. On the other hand, commercial banks specialize in dealing with long term government bonds and loans. The assumption of bond market segmentation in our model is motivated by the Japanese financial structure where commercial banks and CB are major holders of JGBs.

We evaluate the relative effectiveness of two distinct monetary interventions to stimulate the economy. These two policies are namely, (i) a pure quantitative easing (QE) which lasted for about five years since 2001 and (ii) a cut in the IOER which was experimented during the most recent QQE phase. Given that the interest rate was near zero during the QE period, we set the call rate at zero. Within such a zero interest environment, QE is modeled as a stochastic open market operation with a steady state positive inflation target. Such a policy entails a positive shock to monetary base with an accompanying increase in the share of CB holding of long term JGB. Higher inflationary expectation triggered by a positive monetary base shock gives rise to an inflation tax on bank reserve. QE also opens up the credit channel of monetary transmission. This together with the new Keynesian price-marginal cost channel fueled by inflationary expectations has the potential to stimulate real growth in the economy.

On the bond market front, QE purchase of bonds lowers commercial bank's bond holding. While depleting the bond holding, commercial banks experience higher portfolio adjustment cost which lowers its cash flow. Banks smooth this loss of cash flow by subsequent bond purchases. This shows up as a decline in yield to maturity which accords well with the low yield target of the Bank of Japan. The overall effect of QE is thus positive on the economy. QE alone can achieve all three targets of stimulating growth, inflation and lowering the bond yield, at least in the short run.

[3] As for the fiscal year 2019, the consumption tax is around 33.4% of tax revenue for the government. Thus it constitutes an important part of tax income.

Since the IOER changes happened as a discrete policy intervention after 2016, we model this as a deterministic policy change. We trace out the time path of the economy following such a discrete policy intervention while holding other forcing variables at their steady state levels. Since IOER shock is a tax on bank's holding of excess reserve, such a rate cut opens the credit channel and stimulates growth. It also lowers the nominal bond yield. Our simulations predict that an IOER cut could have nontrivial real and financial effects on the economy although the effects are quantitatively smaller than a QE operation.

There are several policy lessons from our DSGE model. First, our policy simulation shows that the real effect of QE is substantial and the impact QE multiplier is close to 2.0 as far as real GDP is concerned. The stimulative effect of monetary policy operates primarily via the inflationary expectations. The credit channel of monetary policy is not as substantial as the inflation channel but it is stronger for QE than for IOER changes. This finding is consistent with the data as discussed in Section 5. Second, QE experiment successfully held the long term bond yield at a near zero level with a slight dip. This accords well with the yield curve policy experiment of BoJ during the QQE era. Third, our model points out that zero bond yield is not a sustainable monetary policy target, because it is inconsistent with a long run positive growth and inflation targets. Such a contradiction arises from the immutable violation of the Fisher's effect.

Although our paper focuses on the BoJ policy experiment, it has relevance for the present post pandemic environment. In the past, ECB had implemented IOER cut as documented in the next section. The Bank of England also contemplated in the past to cut IOER (Financial Times, May 20, 2020). In addition, recently Fed is debating on the yield curve control experiment.[4] The policy lessons that we get from our DSGE model may thus provide some useful insights about the post Covid recovery in general.

The paper is organized as follows. In the next section, we briefly review the extant literature on the modeling of the Japanese economy. In Section 3, we give an overview of the monetary policy history of Japan. In Section 4, our basic DSGE model is laid out. Section 5 is devoted to present quantitative analysis of the model. Section 6 concludes.

2. Connections to literature

There is a growing literature on DSGE modeling of the Japanese economy. Sugo and Ueda (2008) is one of the first articles that estimate a DSGE model of the Japanese economy. Although they model monetary policy rule and use call rate as a proxy for the short term nominal interest rate, they do not explicitly model the role of CB and abstract from any analysis of monetary or fiscal policy effects on bond market except that there is an interest rate shock through a discount bond. Iwata (2009) focuses on the fiscal policy under DSGE setting. Hirose (2020) estimates a DSGE model with a deflationary steady state for Japan and considers whether several shocks to the economy have an inflationary effect. McNelis and Yoshino (2016) compare the performance of three policy rules on reducing the government debt using a DSGE model. However, they do not explicitly model the role of CB and there is no government bond in the model. Fueki et al. (2016) set up a DSGE model to analyze potential output and output gap for the Japanese economy.

There is a growing volume of empirical literature on QE effects. Adjemian and Juillard (2010) estimate a DSGE model with a zero lower bound for nominal interest rate. Michaelis and Watzka (2017) consider

[4] It is important to note that Japan is one of the first countries that implemented yield curve control. While BoJ targeted the 10 year bond yield, Fed is contemplating targeting three year bond yield. See https://www.reuters.com/article/us-usa-fed-yieldcurve-analysis/yield-control-bets-increase-as-investors-wait-for-fed-idUSKBN23H1HU

the change in the effectiveness of quantitative easing policy at the zero lower bound. Although there are liquidity shocks in their model, they do not have a DSGE model. Instead they estimate time varying parameter using VAR analysis and do not study the effect of monetary policy on bonds. Hayashi and Koeda (2019) consider the effect of QE on macroeconomy under the framework of regime-switching structural VAR. They show that a higher reserve raises inflation and output when the nominal policy rate is close to the effective lower bound of interest rate on excess reserve. Nagao et al. (2021) measures the magnitude of both conventional and unconventional monetary policy shocks from the term structure of interest rate and show that the magnitude of monetary policy shocks on the macroeconomic variables are modest in a VAR setting when both the short and long term interest rates are close to the lower bound of zero. Koeda (2019) estimates a 5 variable structural VAR with an effective zero lower bound and showed that QQE increased output.

There is a third strand of literature which focuses on portfolio behavior of Japanese banks involving loan and reserve. Ogura (2020) models the static profit maximizing behavior of regional banks and regional loan demand function by individuals. He shows empirically that increase in liquidity ratio of banks caused by quantitative easing led to more competition among banks. Shioji (2019) investigates the effect of QE on bank loan using panel data of bank balance sheets. He finds that banks did extend loans but the estimated effect of on loan/asset ratio is very close to zero. Shioji (2020) also estimates the effect of QE on bank based on the panel data of regional banks and finds that the result depends on the sample period. However even in the sample period where banks extended loan due to QE, the effect of QE on bank loan was not large.

In the backdrop of these extant studies, our study has two novel features. First, we explicitly model the role of CB and the nexus between the government budget constraint, the CB budget constraint and the commercial bank's budget constraint in a DSGE framework. We focus on the transmission channels of BoJ's QQE policy to the Japanese bond market by exploiting the dynamic linkage among CB, commercial banks and the government through bond holdings and the bank reserve as in Hall and Reis (2015). Second, we analyze the effect of negative IOER on the aggregate economy.[5] Such an exercise is also internationally relevant because before BoJ introduced IOER in 2016, ECB had already introduced on June 5, 2014 a negative rate on excess bank reserve of the Eurosystem.[6]

While a plethora of literature exists on various applications of DSGE models, what is less understood is its bond yield implications. Rudebusch and Swanson (2012) show some innovative applications of a DSGE model to understand bond pricing behavior. However, they do not focus on the monetary policy effects on the bond market behavior, which is the scope of our study. Chen et al. (2012) is one of the few studies that uses DSGE modeling to assess the effects of UMP on long term bond yields in the US who find that the QE in the US has rather insignificant effects on long term bond yield. They, however, do not formulate the CB balance sheet and commercial banks' asset portfolio which we do. Moreover, their focus is on the QE operation in the US, while we focus on QQE in Japan which involves additional monetary policy instruments including IOER.

Our paper is closest to Sudo and Tanaka (2021) who investigate the effect of QE on long term and short term yields in a DSGE model with segmented bond market. However, they do not explicitly model dynamic portfolio behavior of banks involving the choice of precautionary excess reserve, loans and government bonds with portfolio adjustment cost. Our monetary policy transmission channel, therefore, works via the change in the asset mix of banks and its interaction with the portfolio adjustment cost. In Sudo and Tanaka (2021), the QE effects work via the differential responses of households in a segmented bond market where a group of households do not hold any short term government bonds. We model the bond market segmentation by assuming that only banks hold long term government bonds and deal with BoJ when QE operation takes place. This feature is also consistent with the data (see Fig. 2 later). The monetary transmission channel in our setting is thus fundamentally different from Sudo and Tanaka (2021).

[5] Koeda (2019) investigates the effect of exit from IOER intervention in a structural VAR setting while we explore the effect of IOER drop on the aggregate economy within DSGE setting.

[6] https://www.ecb.europa.eu/press/pr/date/2014/html/pr140605_3.en.html

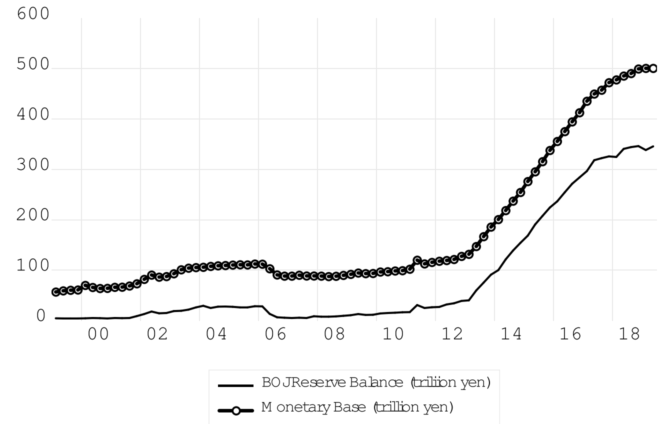

Fig. 1. BoJ reserve balance and monetary base.

3. An overview of BoJ monetary policy

To gain perspective, in this section we give an overview of the BoJ Monetary Policy during the last 20 years. During these years, BoJ has implemented four different monetary policy regimes: (i) zero interest rate policy, (ii) quantitative easing monetary policy (QQE), (iii) call rate policy and (iv) qualitative and quantitative monetary easing policy (QQE). During the fourth regime, there were three sub-regimes, namely a pure QQE, regime, QQE with a negative interest rate on excess reserve and QQE with yield curve control.

BoJ implemented zero interest rate policy on February 12, 1999 and the call rate dropped from around 0.3% to around 0.05% after the announcement.[7] On March 19, 2001, BoJ adopted quantitative easing monetary policy. It changed the operation target from the current uncollateralized overnight call rate to the outstanding balance of the current accounts[8]and set the inflation target of zero percent. There was also a change in current-account balance at the Bank of Japan and the outright purchase of long-term government bonds. We interpret this as an era of pure quantitative easing (QE). On March 9, 2006, BoJ changed the operating target from the reserve back to uncollateralized overnight call rate.[9] After the announcement, there was a gradual increase in the level of the overnight call rate. The reserve balance also declined initially reaching 7.6 trillion yen in May 2008 but it has increased afterwards reaching 58.1 trillion yen in March 2013. Fig. 1 plots the time series of BoJ reserve balance.

[7] See the press release on the same date "Announcement of the Monetary Policy Meeting Decisions" and "New Procedures for Money Market Operations and Monetary Easing".

[8] We use the phrase current account and reserve synonymously here. In practice there is a difference of the order of 2%.

[9] See the press release on the same date "Change in the Guideline for Money Market Operations".

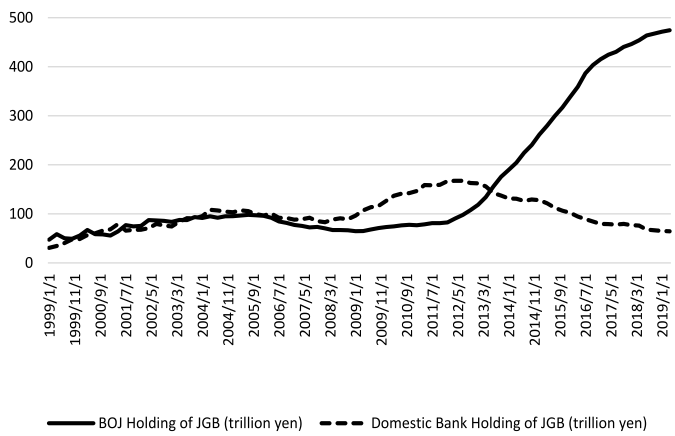

Fig. 2. JGB holding by BoJ and commercial banks.

On April 4, 2013, BoJ introduced Qualitative and Quantitative Monetary Easing Policy (QQE) in order to achieve an annual CPI inflation target of 2 percent. It implemented the monetary base control with an announcement to double the monetary base and the outstanding amounts of JGB in two years, and more than double the average remaining maturity of JGB purchases.[10] The monetary base nearly doubled between April 2013 and April 2015 (Fig. 1). Similar trend was reflected in the reserve balances held at the BoJ by banks as seen in Fig. 2. From 2009 onward, long term bonds held by BoJ exponentially increased while the holding of the same by the banks decreased. This reflects a large scale purchase of long term government bonds by the BoJ which is the very essence of QQE.

On January 29, 2016, BoJ implemented QQE with a negative interest rate (minus 10 bp) on excess reserve in order to achieve the inflation target of two percent at the earliest possible time. They adopted a three-tier system in which the outstanding balance of each bank's current account at BoJ would be divided into three tiers offering positive, zero and negative interest rates respectively.[11]

In recent years, two major policy shifts are worth mentioning. First is yield curve control in September 2016 in which BoJ would control short-term and long-term interest rates. The second is an inflation-overshooting commitment in which BoJ would commit itself to expanding the monetary base until the year-to-year CPI inflation rate exceeds the inflation target of two percent and stays above the target in a stable manner.[12] Therefore, yet another target was added and this time it was the long term yield target in addition to the short term interest rate target introduced in January of the same year. The target maturity of the JGB was 10 year and target rate of JGB was zero percent. Such a target yield is achieved by BoJ's open market operations of JGB with approximately 10 year maturity. Table 1 summarizes these various policy changes.

In a nutshell, since 1999 BoJ kept switching between two operating targets, namely call rate and the monetary base. During the second phase of QQE starting 2016, BoJ added new legs to QE. First, it introduced negative interest rate on excess reserve and then a zero yield on the 10 year JGB. In light of these different policy experiments, in the following DSGE model our main focus is on QE. We also compare QE with an IOER cut in a zero call rate environment to see which would have fared better in achieving the BoJ goal. Finally we evaluate whether a long run zero yield target is compatible with positive growth and inflation targets.

4. Model

The building blocks of the model are similar to Banerjee et al. (2019). We have seven players in the economy: the representative household, three types of firms, commercial banks, CB and the government. Household owns all productive units and save in the form of short term bank deposits (which are perfect substitutes for short-term government bonds). They supply labour to wholesale goods firms. Their income consists of labour, interest income from deposit and cash flows generated from the ownership of firms and the commercial banks.

There are three types of firms: wholesale, capital goods and retail firms. Competitive risk neutral one period lived wholesale firms finance their capital spending from banks. Competitive capital goods firms buy used capital from wholesalers and refurbish it to new capital using investment goods bought from retail firms. Retail firms costlessly convert wholesale goods to differentiated final goods and have some monopoly power of price fixing. Final goods can be used for household consumption, capital goods producers' investment and government use.

Retails banks collect household deposit to intermediate this to wholesale firms and also hold long term government bonds and interest earning excess reserve since they anticipate an aggregate liquidity risk in the form of negative cash flow shock. If the size of this liquidity shock exceeds the current bank reserve, banks borrow from the lender of the last resort, CB at a penalty rate.

The government consumes some final goods financed by lump sum taxes and consumption taxes on households and borrowing from the commercial banks and the CB via issuing long term government bonds. The CB finances its government bond holding by reserve creation, seigniorage and the revenue earned from banks resulting from penalty loans.

4.1. Households

The representative household solves the following maximization problem:

$$\max_{\{c_{t+j},D_{t+j},M^{TD}_{t+j},H_{t+j}\}} E_t\sum_{j=0}^{\infty}\beta^j[U(c_{t+j}-\gamma_c C_{t+j-1})+V(D_{t+j}/P_{t+j}) + W(M^{TD}_{t+j}/P_{t+j})-\Phi(H_{t+j})]$$

subject to the flow budget constraint:

$$P_t c_t(1+\tau_c)+D_t+M^{TD}_t\le W_t H_t+(1+i^D_t)D_{t-1}+M^{TD}_{t-1}+TR_t \tag{1}$$

where c_t is the representative agent's consumption basket after adjusting for the previous period's aggregate consumption C_{t-1} up to a fraction γ_c which means a habit persistence relative to aggregate consumption,[13] D_t is one period nominal deposit which are perfect substitutes for short term government bonds (as in (Gertler and Karadi, 2013)), P_t is aggregate price index, M^{TD}_t is nominal transaction demand for cash, H_t is labour hours, W_t is nominal wage, i^D_t is the risk-free nominal interest rate on deposits and TR_t is the nominal lump sum transfer to the household which includes cash flows from capital goods firms, retail goods firms, commercial banks as well as transfer from the government. The household pays a consumption tax at a flat rate τ_c. We assume that household receives direct utility from

[10] See the press release on the same date "Introduction of the Quantitative and Qualitative Monetary Easing".

[11] See the press release on the same date "Introduction of Quantitative and Qualitative Monetary Easing with a Negative Interest Rate".

[12] See the press release on the same date "New Framework for Strengthening Monetary Easing: Quantitative and Qualitative Monetary Easing with Yield Curve Control".

[13] As in Gali (2015), we assume that the household receives utility from a CES consumption aggregator of continuum of differentiated goods over a unit interval with elasticity of substitution equal to ε^Y which characterizes the price elasticity of demand of the ith differentiated good.

Table 1
Four monetary policy regimes of BoJ, 1999–2019.

Regime	Date	Event
1	1999/02/12	Zero Interest Rate Policy
2	2001/03/19	Quantitative Easing Monetary Policy (QE)
3	2006/03/09	Call Rate Target Policy
4–1	2013/04/04	Qualitative and Quantitative Monetary Easing Policy (QQE)
4–2	2016/01/29	QQE with a Negative Interest Rate on Excess Reserve
4–3	2016/09/21	QQE with Yield Curve Control

bank deposits and cash holding.[14] $U(.), V(.), W(.)$ are instantaneous continuous, strictly concave utility functions from consumption, real deposit and real money balance with the usual regularity conditions and $\Phi(H_t)$ is the continuous disutility function from work.

The first order conditions are:

$$D_t : \; U_{ct} = V'(d_t) + \beta E_t U_{ct+1}(1 + i^D_{t+1})/(1 + \pi_{t+1}) \tag{2}$$

$$M^{TD}_t : \; U_{ct} = W'(m^{TD}_t) + \beta E_t U_{ct+1}/(1 + \pi_{t+1}) \tag{3}$$

$$H_t : \Phi'(H_t) = (W_t/P_t)\, U_{ct} \tag{4}$$

where U_{ct} is the derivative of $U(c_t - \gamma_c C_{t-1})$ with respect to c_t, $d_t = D_t/P_t$ is the real deposit, $\pi_{t+1} = (P_{t+1}/P_t - 1)$ is the net inflation rate and $m^{TD}_t = M^{TD}_t/P_t$ is the real transaction demand for cash. Eq. (2) shows that marginal utility cost of holding a dollar of deposit balances the temporal marginal utility of liquidity service from deposits and the discounted utility benefits of the interest on deposit adjusted for inflation tax. Likewise Eq. (3) shows the marginal equivalence condition of cost and benefit of holding a dollar money balance. Eq. (4) is the standard static efficiency condition for labour supply.

4.2. Production sector

4.2.1. Capital goods producing firms

Capital goods producers buy last period's used capital $\left\{(1-\delta_k) K_{t-1}\right\}$ from the wholesale firms/entrepreneurs at a real price Q_t. They produce new capital stock K_t by investing I_t of final goods using a linear investment technology:

$$K_t = (1-\delta_k)K_{t-1} + Z_{xt} I_t \tag{5}$$

where δ_k is the physical rate of depreciation of capital and Z_{xt} is an investment specific technology shock which evolves as follows:

$$Z_{xt} = \overline{Z_x}^{1-\rho_z} Z^{\rho_z}_{xt-1}\; exp(\xi^z_t)$$

where $\overline{Z_x}$ is its the steady state level, ρ_z is the serial correlation coefficient and ξ^z_t is a stationary noise to be specified later. After investment this new capital is sold to the wholesalers at a real price Q_t. For one unit investment, the capital goods producers purchase $[1 + \Xi\left(\frac{I_t}{I_{t-1}}\right)]$ of final goods where $\Xi(.)$ is a convex flow investment adjustment cost function with $\Xi(1) = \Xi'(1) = 0$ and $\Xi''(1) = \kappa$.[15] The capital goods producer then solves

$$\max_{\{I_{t+j}\}} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} CF^k_{t+j}$$

where $\Omega_{t,t+j}$ is the inflation adjusted stochastic discount factor[16] between t and $t+j$ which is equal to $\frac{\beta^j U_{ct+j}}{U_{ct}} \cdot \frac{1}{1+\pi_{t+j}}$. CF^k_t is the cash flow of the capital goods producer given by:

$$CF^k_t = P_t\left[Q_t I_t - \left\{1 + \Xi\left(\frac{I_t}{I_{t-1}}\right)\right\} I_t\right]$$

The first order condition gives the following Euler equation similar to Gertler and Karadi (2013):

$$Q_t = 1 + \Xi\left(\frac{I_t}{I_{t-1}}\right) + \Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} - E_t \frac{\beta U_{ct+1}}{U_{ct}}\left[\Xi'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{6}$$

4.2.2. Wholesale goods producing firms

There are continuum of risk neutral wholesale firms over the unit interval. The ith wholesale firm produces intermediate goods $(Y^W_t(i))$ for the ith final goods producing retailer. For doing so, it hires labour from the households and purchases new capital from the capital good producing firms. This firm borrows $L_t(i)$ from the bank in order to cover the cost of new capital, $Q_t K_t(i)$. We assume that all capital spending is debt financed. Used capital at date t is sold at the resale market at the price Q_t.

Balance sheet condition of the typical wholesale firm is:

$$Q_t K_t(i) = \frac{L_t(i)}{P_t} \tag{7}$$

The wholesale goods production function is specified as follows:

$$Y^W_t(i) = A_t K^{\alpha}_{t-1}(i)(\Theta_t H_t(i))^{1-\alpha} \tag{8}$$

where A_t is the TFP shock, $0 < \alpha < 1$, and Θ_t is a labour augmenting technical progress component. The TFP shock evolves as follows:

$$A_t = \overline{A}^{1-\rho_A} A^{\rho_A}_{t-1}\; exp(\xi^A_t)$$

where $\overline{A}$ is its steady state level, ρ_A is a serial correlation coefficient and ξ^A_t is a stationary noise to be specified later. We assume that Θ_t grows at a deterministic gross rate Λ which is the balanced growth rate of the economy. The equilibrium real wage is $W_t/P_t = (1-\alpha)\frac{(P^W_t/P_t)Y^W_t(i)}{H_t(i)}$, where P^W_t is the nominal price of the wholesale good.

The gross rate of return from capital is given by,

$$\begin{aligned} 1 + r^k_{t+1}(i) &= \frac{(P^W_{t+1}/P_{t+1})Y^W_{t+1}(i) - (W_{t+1}/P_{t+1})H_{t+1}(i) + \left(1-\delta_k\right)Q_{t+1}K_t(i)}{Q_t K_t(i)} \\ &= \frac{(P^W_{t+1}/P_{t+1})\left(\frac{Y^W_{t+1}(i)}{K_t(i)}\right) - (1-\alpha)\frac{(P^W_{t+1}/P_{t+1})Y^W_{t+1}(i)}{H_{t+1}(i)}\left(\frac{H_{t+1}(i)}{K_t(i)}\right) + \left(1-\delta_k\right)Q_{t+1}}{Q_t} \\ &= \frac{(P^W_{t+1}/P_{t+1})MPK_{t+1}(i) + (1-\delta_k)Q_{t+1}}{Q_t} \end{aligned}$$

[14] We put both real cash balance and real deposits in the utility function motivated by the fact that both money and short term bank deposits provide different kinds of transaction convenience to the household. Putting real cash balance in the utility function has a long tradition following Sidrauski (1967). The idea of real deposits in the utility function is borrowed from Hansen and Imrohoroglu (2016) who put short term government bonds in the utility function. Since households value the liquidity service of short term bank deposits, they are willing to accept a lower rate on bank deposits than the loan rate the banks charge to the wholesale goods firms which are also owned by households. A natural borrowing-lending spread or limits to arbitrage thus arises in our model (see footnote 24).

[15] Note that this investment adjustment cost is incurred before investment is undertaken to install new capital K_t. That is why it does not appear in the linear investment technology (5).

[16] Since the household owns all firms and banks, these firms and banks also share the same stochastic discount factor.

where $MPK_{t+1}(i)$ denotes the ith firm's marginal product of capital at date $t+1$. Defining i_t^L as the net nominal interest rate on loans, the optimality condition for firms' demand for capital (or the no arbitrage condition) can be written as

$$1+r_{t+1}^k(i) = \left(1+i_{t+1}^L\right)/(1+\pi_{t+1})$$

which yields,

$$1+i_{t+1}^L = \frac{P_{t+1}^W MPK_{t+1}(i) + \left(1-\delta_k\right)P_{t+1}Q_{t+1}}{P_t Q_t}$$

In other words,

$$1+i_{t+1}^L = \left[\left(\frac{P_{t+1}^W}{P_{t+1}}\right)\frac{MPK_{t+1}(i)}{Q_{t+1}} + 1 - \delta_k\right]\left[\frac{(1+\pi_{t+1})Q_{t+1}}{Q_t}\right] \quad (9)$$

Since all firms face the same loan rate, i_t^L and capital price, Q_t, they all produce the same output in equilibrium.

4.2.3. Retail firms

Similar to Bernanke et al. (1999), there are continuum of retail firms over a unit interval. The ith retailer buys intermediate goods at price P_t^W and packages them into final goods and operates in a monopolistically competitive environment. The ith retailer converts the ith variety of the intermediate goods, $Y_t^W(i)$, one-to-one into differentiated final good, $Y_t(i)$ at zero cost. Each retailer sells his unique variety of final product after applying a markup over the wholesale price, and factoring in the market demand condition which is characterized by price elasticities $\left(\varepsilon^Y\right)$.[17] Retailer's prices are sticky and indexed to past and steady state inflation as in Gerali et al. (2010) and Banerjee et al. (2019) based on the indexation parameter $\theta_p \in (0,1)$. Retailers bear a quadratic adjustment cost given by ϕ_p if they want to change their price over and above what indexation allows.[18]

The first order condition after imposing a symmetric equilibrium is standard:

$$1-\varepsilon^Y + \varepsilon^Y(P_t/P_t^W)^{-1} - \phi_p\left\{1+\pi_t - (1+\pi_{t-1})^{\theta_p}(1+\overline{\pi})^{1-\theta_p}\right\}$$

$$+E_t\Omega_{t,t+1}\phi_p\left\{1+\pi_{t+1} - (1+\pi_t)^{\theta_p}(1+\overline{\pi})^{1-\theta_p}\right\}(1+\pi_{t+1})^2\frac{Y_{t+1}}{Y_t} = 0 \quad (10)$$

where $Y_t = \int_0^1 Y_t(i)di$.

In the steady state, when $\pi_{t+1} = \pi_t = \pi$, the above price equation reduces to a simple static markup equation:

$$\frac{P_t}{P_t^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1}. \quad (11)$$

4.3. Banks

As in Banerjee et al. (2019) the banking problem is nonstandard in our setting. Commercial banks solve a dynamic portfolio choice problem involving three assets, namely reserve holding, government bonds (JGB) and loans. Denote outstanding nominal loans issued at date $t-1$ as L_{t-1} and the corresponding outstanding nominal value of government bonds held by the commercial banks as B_{t-1}^P. Likewise, let M_{t-1}^{RD} be commercial banks' outstanding reserve holding at date $t-1$. Banks are subject to a statutory reserve requirement as follows:

$$M_t^{RD} \geq \alpha_r D_t \text{ for all } t \quad (12)$$

where α_r is the legal reserve ratio. As in Banerjee et al. (2019) and Chang et al. (2014), banks plan to hold excess reserve because they face a liquidity risk of a negative cash flow shock ζ_t. The size of this cash flow shock is bounded by deposit at date t which means $\zeta_t \in [0, D_t]$. Such a liquidity risk necessitates a demand for excess reserve by the banks.

Bank's cash flow at date t can be rewritten as:

$$\begin{aligned} CF_t^b =& (1+i_t^L)L_{t-1} + (1+i_t^R)(M_{t-1}^{RD} - \alpha_r D_{t-1}) + (1+nhpr_t)B_{t-1}^P \\ &- (1+i_t^D)D_{t-1} \\ &- (1+i_t^p)\chi_t(\zeta_t - M_{t-1}^{RD}) - (1-\chi_t)\zeta_t - B_t^P \\ &- \frac{\bar{\iota}\ (1+\bar{\lambda})P_t}{2}\left[\frac{L_t}{L_{t-1}}.\frac{B_{t-1}^p}{B_t^p} - 1\right]^2 \\ &- L_t - M_t^{RD} + D_t \end{aligned} \quad (13)$$

Few clarifications about bank's cash flow are in order. First, banks earn interest only on its excess reserve $(M_{t-1}^{RD} - \alpha_r D_{t-1})$ at a deterministic rate i_t^R. At date t, banks first make decisions about loans (L_t), bond holding (B_t^P) and reserve holding (M_t^{RD}) after observing the deposit (D_t) banks. Banks expect that a liquidity shock (ζ_t) may hit the banking sector in the form of a negative cash flow. If this shock exceeds banks' existing reserve M_{t-1}^{RD}, banks have to approach the lender of last resort, CB for an emergency loan at a punitive rate i_t^p known in advance. Bank pays back the emergency loan and the penalty $(1+i_t^p)(\zeta_t - M_{t-1}^R)$ at the end of period t.[19] Let χ_t, be an indicator function that takes the value unity if $\zeta_t - M_{t-1}^{RD} > 0$ and zero otherwise.[20] At date t, banks make decision about the end of period reserve holding (M_t^{RD}) based on their prediction of the liquidity shock (ζ_t).

Second, as in Harrison (2017) banks incur a quadratic flow portfolio adjustment cost (parameterized by ι) for changing bond to loan ratio from its steady state target $\bar{\lambda}$. Such an adjustment cost prevents fire sale of loan or long term bonds in response to a sudden cash flow shock. It also allows the bank to smooth a shock to cash flow by adjusting loan and bond holdings around the target proportion and prevents abrupt movements in bond yields.

Third, S_t is the date t price of a nominal default free long term bond which gives a geometrically decaying coupon sequence $\{1, \nu, \nu^2, \ldots\}$ at the end of each period as in Woodford (2001) and Lansing (2015).[21] Then the nominal holding period return denoted as $nhpr_t$ is $(1 + \nu S_t)/S_{t-1} - 1$ as in Lansing (2015).

Given the assets at date t, and deposit sequence $\{D_t\}$ determined by the household's problem, banks choose M_t^{RD}, B_t^P, L_t which solve the following dynamic optimization:

$$\max_{\{M_{t+j}^{RD}, B_{t+j}^P, L_{t+j}\}} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} CF_{t+j}^b$$

[17] As in Rotemberg (1982), each retail firm continuously adjusts its price subject to a quadratic price adjustment cost and maximizes the present value of cash flows subject to differentiated final demand function. We omit the details of the decision problem of the retail firms, which are quite standard. See Basu and Sarkar (2016) for details of the retailer's problem.

[18] As in any standard new Keynesian model, the nominal rigidity is quite crucial for generating real effects of a monetary policy.

[19] Here is an example of bank's overrunning reserve and incurring contingent penalty. Suppose ζ_t = \$30 dollars and $M_{t-1}^{RD} = 10$ dollars. The bank falls short of reserve by 20 dollars and thus takes an emergency loan from the CB. The bank incurs 10 percent penalty (i_t^p) on its loan. Thus at the end of the day bank's payment to the CB with penalty is 22 dollars which includes the principal and interest. On the other hand, if the bank reserve is 40 dollars instead of 10 dollars, the bank does not need to approach the CB for an emergency loan but the bank's cash flow still falls by 30 dollars. Taking this into consideration, bank chooses the reserve holding optimally at the start of date t.

[20] Notice that this liquidity shock has no effect on household's first order conditions because if it materializes, banks suffer a negative shock to their cash flows which are translated into a negative shock to the lump sum transfer to the households.

[21] Rudebusch and Swanson (2008) employ similar bonds with geometrically decaying coupon payments but the coupon is paid at the start of the period. Our quantitative results do not change much if we assume a geometrically decaying coupon as in Rudebusch and Swanson (2008).

s.t. the statutory reserve requirement (12).

As in Banerjee et al. (2019), the Euler equation for bank reserve is given by:

$$M_t^{RD}: \ 1 = E_t\Omega_{t,t+1}\left[(1+i_{t+1}^R) + (1+i_{t+1}^p)Prob(\zeta_t/D_t \geq M_t^{RD}/D_t)\right] + \varkappa_t \tag{14}$$

The first term in the square bracket in (14) is the bank's interest income from reserve and the second term is the expected saving of penalty because of holding more reserve. $Prob(\zeta_t/D_t \geq M_t^{RD}/D_t)$ is the probability of the liquidity shock exceeding reserve. $\varkappa_t$ is the Lagrange multiplier associated with the reserve constraint (12).[22]

The Kuhn–Tucker condition states that

$$\frac{M_t^{RD}}{D_t} = \alpha_r \text{ if } \varkappa_t > 0 \tag{15}$$

Otherwise

$$E_t\Omega_{t,t+1}\left[(1+i_{t+1}^R) + (1+i_{t+1}^p)\text{Prob}(\zeta_t/D_t \geq M_t^{RD}/D_t)\right] = 1 \tag{16}$$

Assuming a rectangular distribution for ζ_t, (16) reduces to[23]:

$$M_t^{RD}: 1 = E_t\Omega_{t,t+1}\left[(1+i_{t+1}^R) + (1+i_{t+1}^p)(1 - \frac{M_t^{RD}}{D_t})\right] \tag{17}$$

Solve $\frac{M_t^{RD}}{D_t}$ as follows:

$$\frac{M_t^{RD}}{D_t} = 1 - \frac{1-(1+i_{t+1}^R)E_t\Omega_{t,t+1}}{(1+i_{t+1}^p)E_t\Omega_{t,t+1}} \tag{18}$$

Since $(1+i_{t+1}^R)E_t\Omega_{t,t+1} < 1$, given the stochastic discount factor, $\Omega_{t,t+1}$, a higher i_{t+1}^R or i_{t+1}^p means a higher proportion of deposits held as reserve (M_t^{RD}/D_t) by the banks.

Next the bank solves a recursive problem of choosing B_t^P and L_t given B_{t-1}^P and L_{t-1} which were chosen in the previous period. This is a dynamic allocation problem. The Euler equations for bonds (b_t^p) and loans (l_t) in real terms (denoted as lower cases) are written as follows:

$$b_t^P: 1 = E_t\Omega_{t,t+1}\left[(1+nhpr_{t+1}) - adj_{t,t+1}^b\right] + adj_{t-1,t}^b \tag{19}$$

$$l_t: \ 1 = E_t\Omega_{t,t+1}[(1+i_{t+1}^L) + adj_{t,t+1}^l] - adj_{t-1,t}^l \tag{20}$$

and various incremental portfolio adjustment costs are given by:

$$adj_{t,t+1}^b = \iota \bar{l}(1+\bar{\lambda})\left(\frac{l_{t+1}}{l_t}\frac{b_t^p}{b_{t+1}^p} - 1\right)\left(\frac{l_{t+1}}{b_{t+1}^p}\right)\left(\frac{1}{l_t}\right)(1+\pi_{t+1}) \tag{21}$$

$$adj_{t-1,t}^b = \iota \bar{l}(1+\bar{\lambda})\left(\frac{l_t}{l_{t-1}}\frac{b_{t-1}^p}{b_t^p} - 1\right)\left(\frac{l_t}{b_t^p}\right)\left(\frac{b_{t-1}^p}{l_{t-1}}\right)b_t^{p-1} \tag{22}$$

$$adj_{t,t+1}^l = \iota \bar{l}(1+\bar{\lambda})\left(\frac{l_{t+1}}{l_t}\frac{b_t^p}{b_{t+1}^p} - 1\right)\left(\frac{l_{t+1}}{b_{t+1}^p}\right)\left(\frac{b_t^p}{l_t^2}\right)(1+\pi_{t+1}) \tag{23}$$

$$adj_{t-1,t}^l = \iota \bar{l}(1+\bar{\lambda})\left(\frac{l_t}{l_{t-1}}\frac{b_{t-1}^p}{b_t^p} - 1\right)\left(\frac{b_{t-1}^p}{l_{t-1}}\right)b_t^{p-1} \tag{24}$$

Note that $adj_{t,1+1}^b$, $adj_{t-1,t}^b$ are the flow bond portfolio because they both pertain to flow adjustment costs. Likewise $adj_{t,1+1}^l$, $adj_{t-1,t}^b$ are flow loan adjustment costs. All these adjustment costs are zero in the steady state which means the long run returns on bond and loans are equal. In the short run, they may differ due to changes in adjustment costs thus creating dynamic wedges between loan and bond rates.[24]

[22] We assume that the cash flow shock is bounded by deposits to rule out the possibility of a sudden stop of the banking system when depositors lose all their deposits.

[23] See Banerjee et al. (2019) technical appendix for derivation.

[24] Note that although in the long run bond and loan rates are equal, there is a borrowing-lending spread because deposit appears in the utility function and provides transaction convenience to the household. To see it combine (2) and (20) to get the following steady state borrowing-lending spread:

$$i^L - i^D = \frac{(1+\pi)}{\beta}\frac{V'(d)}{U_c(c)} > 0$$

4.4. Central bank and government

QE involves open market purchase of JGB by the BoJ. Due to such operation commercial banks undergo transformation of its assets by having more reserves and less JGB as seen in Fig. 2. To show it clearly, we now specify the BoJ's budget constraint. Define the supply of bank reserve as M_t^R and supply of currency as M_t^T. BoJ must create enough reserve to pay for the interest and principal on existing commercial bank reserves and also to cover the purchase of government bonds (B_t^{CB}) net of bond income held from the previous period (which is $(1+nhpr_t)B_{t-1}^{CB}$). The BoJ pays the rest to the government as dividend (Div_t) after netting out the seigniorage revenue from printing cash ($M_t^T - M_{t-1}^T$). In other words, BoJ's nominal budget constraint is given by:

$$M_t^R = (1+i_t^R)M_{t-1}^R + B_t^{CB} - (M_t^T - M_{t-1}^T) - (1+nhpr_t)B_{t-1}^{CB} + Div_t$$

The idea of dividend payment by the BoJ to the government is borrowed from Hall and Reis (2015). Literally, the BoJ does not pay such dividend but it should generate sufficient revenue to cover the deficits of the government. Thus the dividend is the link between BoJ and the government.

The real dividend to the government can be written as:

$$div_t = m_t^R - (1+i_t^R)\frac{m_{t-1}^R}{(1+\pi_t)} + m_t^T - \frac{m_{t-1}^T}{(1+\pi_t)} - b_t^{CB} + (1+nhpr_t)\frac{b_{t-1}^{CB}}{(1+\pi_t)} + xr_t \tag{25}$$

where $div_t = Div_t/P_t$, $m_t^R = M_t^R/P_t$, $m_t^T = M_t^T/P_t$, $b_t^{CB} = B_t^{CB}/P_t$ and xr_t is the real penalty revenue that the BoJ receives from the commercial banks due to emergency loans.

We next specify the government budget constraint. The government spends exogenous stream (G_t) of final goods. This spending is financed by lump sum taxes on households (T_t) and the dividends (Div_t) received from the BoJ. All government borrowing is in the form of long term government bonds with a nominal value B_t^G. The government budget constraint in nominal form is given by:

$$P_tG_t + (1+nhpr_t)B_{t-1}^G = P_tT_t + \tau_cP_tc_t + B_t^G + Div_t \tag{26}$$

The real government budget constant is thus:

$$G_t + (1+nhpr_t)\frac{b_{t-1}^G}{1+\pi_t} = T_t + \tau_c c_t + b_t^G + div_t \tag{27}$$

where $b_t^G = B_t^G/P_t$.

The real government spending (G_t) has the following exogenous law of motion:

$$G_t = \Lambda^t \, \widetilde{G}_t$$

where Λ is the balanced growth rate as specified in (8) and the stationarized government spending shock ($\widetilde{G}_t$) follows the process[25]:

$$\widetilde{G}_t = \overline{G}^{1-\rho_G}\widetilde{G}_{t-1}^{\rho_G}\exp(\xi_t^G) \tag{28}$$

with $\overline{G}$ as its steady state level, ρ_G is the serial correlation coefficient and ξ_t^G is a stationary noise to the government spending which will be specified later.

[25] The stationarized level variables are written with Tilda. See the appendix for the stationarized equation system.

4.5. Integrated government budget constraint

Plugging (25) into (27) and using the bond market equilibrium condition (39), we get the integrated government budget constraint as follows:

$$G_t-\overline{T_t}=b_t^p-(1+nhpr_t)\frac{b_{t-1}^p}{(1+\pi_t)}+m_t^R-(1+i_t^R)\frac{m_{t-1}^R}{(1+\pi_t)}+m_t^T-\frac{m_{t-1}^T}{(1+\pi_t)} \quad (29)$$

where $\overline{T}_t=T_t+xr_t+\tau_c c_t$. The integrated government budget constraint basically states that the government finances the fiscal deficit $(G_t-\overline{T_t})$ and outstanding interest payment on its debt by issuing new bonds to the commercial banks and the seigniorage revenues from bank reserve (m^R) and the transaction money (m^T).

4.6. QE operation

QE in our model means an open market purchase of JGB by the BoJ from the commercial banks. While doing this open market operation, BoJ keeps an inflation target in mind. Keeping these features, we formulate the QE operation as a stochastic shock to monetary reserve around a long run target inflation $\overline{\pi}$ of BoJ as follows:

$$\frac{1+\mu_t}{1+\overline{\pi}}=\left(\frac{1+\mu_{t-1}}{1+\overline{\pi}}\right)^{\rho_\mu}\exp(\xi_t^\mu) \quad (30)$$

where $1+\mu_t=M_t^R/M_{t-1}^R$, $\rho_\mu\in(0,1)$ and ξ_t^μ is a QE shock with a forcing process to be specified later. Such a money supply process imposes restriction on the short run growth rate of real reserve and inflation as follows:

$$\frac{(1+\pi_t)(m_t^R/m_{t-1}^R)}{1+\overline{\pi}}=\left(\frac{(1+\pi_{t-1})(m_{t-1}^R/m_{t-2}^R)}{1+\overline{\pi}}\right)^{\rho_\mu}\exp(\xi_t^\mu) \quad (31)$$

Since real reserve is connected to deposit as shown in the bank's reserve demand function (18), it also imposes restriction on the dynamics of deposits, interest rate on loans and consumption.

What is the implication of such a QE shock for the balance sheets of BoJ and commercial banks? When BoJ buys JGBs from the commercial banks, its asset increases by the purchase of JGBs. Simultaneously BoJ creates more liability by increasing monetary base. Thus the BoJ's balance sheet grows. On the other hand, commercial bank's assets just undergo a maturity transformation from long term JGB to short term bank reserves which means that commercial banks reduce the JGB holding and boosts bank reserve. This basically means

$$(\mu_t-\bar{\pi})M_{t-1}^R=-(\varsigma_t-\bar{\pi})B_{t-1}^p \quad (32)$$

where ς_t is the rate of decrease of nominal bonds of the commercial banks starting from the steady state path.[26] Due to QE, ς_t is endogenously determined by the law of motion of money supply. In other words,[27]

$$\varsigma_t=\bar{\pi}-\frac{(\mu_t-\bar{\pi})M_{t-1}^R}{B_{t-1}^p} \quad (33)$$

4.7. Forcing processes

We assume the following specifications for the TFP shock ξ_t^A, IST shock ξ_t^z, government spending shock ξ_t^G, QE shock ξ_t^μ:

$$\xi_t^j=\theta_j\xi_{t-1}^j+\epsilon_t^j,\ \text{for } j=A,z,\widetilde{G},\mu \quad (34)$$

where $\{\epsilon_t^j\}$ is an $i.i.d.$ process for all $j=A,z,G,\mu$.

4.8. Yield to maturity

As far as the bond market is concerned, our central interest in this paper is to understand the behavior nominal yield to maturity in response to alternative monetary policy shocks. For the bond with a geometrically decaying coupon that is paid at the end of the period, the nominal yield to maturity denoted as $nytm_t$ is defined as follows:

$$S_t=\sum_{j=1}^{\infty}\frac{\nu^{j-1}}{(1+nytm_t)^j} \quad (35)$$

which can be rewritten as:

$$nytm_t=\frac{1+\nu S_t}{S_t}-1 \quad (36)$$

4.9. Market clearing conditions

In equilibrium, the following market clearing conditions hold:

1. Goods market clears which means that GDP equals the sum of consumption, private investment including adjustment costs, government spending, and price adjustment costs.

$$C_t+\left\{1+\Xi\left(\frac{I_t}{I_{t-1}}\right)\right\}I_t+G_t+\frac{\phi_p}{2}\left[\left\{(1+\pi_t)-(1+\pi_{t-1})^{\theta_p}(1+\overline{\pi})^{1-\theta_p}\right\}^2Y_t\right]=Y_t \quad (37)$$

2. The loan market clears in the sense that the balance sheet constraint (7) of the wholesaler binds:

$$L_t/P_t=Q_tK_t \quad (38)$$

3. Given that all public debt is domestically held, the bond market equilibrium requires that JGB held by banks and the CB sum to the government issued bonds

$$b_t^P+b_t^{CB}=b_t^G \quad (39)$$

4. Money market clears which means that the demand for bank reserve (M_t^{RD}) equals the supply of bank reserve (M_t^R) and the transaction demand for money (M_t^{TD}) equals the corresponding supply (M_t^T):

$$m_t^{RD}=m_t^R \quad (40)$$

$$m_t^{TD}=m_t^T \quad (41)$$

5. Estimation and simulation

5.1. Baseline calibration

Our sample period is from January 1999 to April 2019. The observation frequency is quarterly so that all the parameter values are set at the quarterly values and all the rates of return are calculated at the quarterly rates. The details of the data are explained in the appendix. The model parameters are classified in three groups. The first group includes calibrated parameters whose values are set at conventional levels. The second category includes parameters which are backed out using relevant steady state (SS hereafter) macroeconomic ratios. The third group of parameters for which no conventional values are available are estimated using Bayesian procedure.

Our target key SS macroeconomic ratios are as follows: consumption/GDP ratio, investment/GDP ratio, government spending/GDP ratio, the ratio of JGBs held by domestic banks to GDP ratio, consumption/deposit ratio and consumption/cash ratio. Since we have a model

[26] To see this clearly, note that in our closed economy model, a positive shock to M_t^R means a positive shock to B_t^{CB} in BoJ's balance sheet due to its open market purchase of government securities. Since in equilibrium, $B_t^G=B_t^{CB}+B_t^p$, given B_t^G, a positive shock to B_t^{CB} means an offsetting negative shock to B_t^p. In other words, $\Delta B_t^{CB}+\Delta B_t^p=0$. A QE operation means $\Delta M_t^R=\Delta B_t^{CB}=-\Delta B_t^p$ as in (32).

[27] Note that in the steady state, $\mu_t=\varsigma_t=\bar{\pi}$.

of a closed economy, we abstract from international trade and define GDP as $Y = C + I + G$.[28]

The SS deposit rate $\overline{i^d}$ is fixed at the average quarterly short term deposit rate during the sample period which is converted from the rate quoted at an annual frequency. The SS interest rate on excess reserve is set at 10 bp.[29] The consumption tax rate is fixed at 8% which is the rate at the end of the sample period. The habit persistence parameter γ_c was calibrated to match our target SS ratios. The balanced growth rate Λ was fixed at the quarterly per capita real growth rate of closed economy GDP of 13 basis points.

We specialize our simulation to a utility function: $\ln(c_t - \gamma_c C_{t-1}) + \eta_1 \ln d_t + \eta_2 \ln m_t^T - H_t$ and quadratic investment adjustment cost function: $\Xi(I_t/I_{t-1}) = 0.5\kappa(I_t/I_{t-1} - \Lambda)^2$. The preference parameters η_1 and η_2 are backed out using C/d and C/m^{TD} ratios. Doing so, we exactly match these two targets.[30] The investment adjustment cost parameter, κ is fixed at 12 following Sugo and Ueda (2008).

The steady state IST level $\overline{Z}_x$ is normalized at unity. The SS TFP, $\overline{A}$ and government expenditure levels $\overline{G}$ are calibrated to match the target SS government spending ratio. The target inflation $\overline{\pi}$ is set equal to the quarterly average of CPI (all items without fresh food) change during the sample period. The quarterly time preference β is set at the conventional level of 0.995 which is the same value as in Sugo and Ueda (2008). The quarterly physical rate of depreciation δ_k is fixed at 0.009 with a view to target the SS I/Y ratio which is close to the conventional 1.5 percent estimate (Sugo and Ueda, 2008). The geometrically decaying coupon rate ν is set at 0.9799 so that the duration of bond is 40 quarters.[31] Since we employ the yield of zero coupon JGB in the estimation, we model the behavior of zero coupon bond with the maturity of 10 years. The capital share in the production function α is set at 0.314 which is close to the conventional level in Sugo and Ueda (2008). The price markup ratio ϵ^Y is set at 6 to match the near zero BoJ target of the nominal yield to maturity. The long run bond to loan ratio $(\overline{\lambda})$ in portfolio adjustment costs in bank's cash flow Eq. (13) is calibrated at 0.05 in order to match the SS value of b^P/Y ratio of 0.85 which was calculated by the sample average of the ratio of quarterly JGB holding of commercial banks divided by quarterly closed economy GDP. Finally the quadratic price adjustment cost ϕ_p is fixed at 178.76 from Basu and Sarkar (2016).[32] Table 2 presents the baseline calibrated parameter values and Table 3 reports both the observed and calibrated values for SS ratios.[33]

We undertake an estimation to compute the remaining second moment parameters as well as investment adjustment cost, κ, inflation indexation, θ_p and the bank portfolio adjustment cost parameter, ι for which we have no readily available estimate. We pick three major macroeconomic variables as observable for Bayesian estimation, namely (i) consumption/GDP ratio, (ii) the investment/GDP ratio and (iii) inflation. Since we have matched the SS value of the consumption, investment and government spending ratios, use of these observable for estimation means that these three major macroeconomic ratios and the inflation rate are perfectly matched by our model. Details of the data sources are given in the appendix.

Our selection of the probability density functions for the priors are based on educated guesses and available estimates from extant studies. For prior, the Beta distribution is used for the fractions while the Inverse Gamma distribution is specified for the parameters with non-negativity constraints in line with Smets and Wouters (2007).

The joint posterior distribution of the estimated parameters is obtained by standard procedure. First, the model equation system is log-linearized around the balanced growth path of the economy and written in a linear rational expectation recursive form.[34] Second, the system of equations is written in a Kalman filter observation equation form. Third, using this observation equation, the log-likelihood function of the relevant parameter vector is constructed. Fourth, the log posterior kernel is expressed using the prior density of the parameter. Fifth, the mode of this posterior kernel is computed using standard numerical optimization routines. All the computations are done by using Dynare 5.2 version.[35] For the baseline model where there is only QE shock, call rate is set at the sample average of 0.000126 while interest rate on excess reserve is set at 0 in line with the no arbitrage condition for banks $i_p > i_R$.

Table 4 reports the baseline second moment parameter estimates from the Bayesian estimation routine. Most of the posterior estimates are apart from the chosen prior means which suggest that the model is identified. Small changes in prior means do not significantly change the results.

5.2. Variance decomposition

Table 5 shows the variance decompositions of fundamental shocks. Several points are in order. First, government spending shock has nearly insignificant contribution to any real and financial variables except GDP. Second, TFP shock picks up the lion's share of output variations in line with Sugo and Ueda (2008). Not surprisingly, the IST shock accounts for major fluctuations of investment while QE shock explains little of investment fluctuation which is in line with Shioji (2020). Nontrivial components of the variances of output and inflation are explained by QE shock making it a potent monetary policy instrument. Similar relative importance of shocks is observed for aggregate consumption, although QE shock accounts more and IST shock explains less for the consumption variance than GDP variance. Third, on the bank asset allocation and financial market fronts, IST shock plays a significant role. The stock price (Q) and nominal yield to maturity ($nytm$) are primarily influenced by IST shock, although QE shock also explains about 29% of variation in the nominal yield of bonds.

5.3. Effect of a QE shock

We formulate a positive QE shock as a positive innovation ξ_t^μ to the monetary base Eq. (30). We use the historical average inflation

[28] As of March 2019, only 12.7% of the long term JGB is held by foreign entities. Also net trade is only −0.24% of GDP as of the 1st quarter of 2019 and about -1% on average during the sample period. Moreover, given that interest rate in Japan is close to zero, and yen exchange rate is volatile, there is an exchange rate puzzle about which our model has no implications. Besides, the monetary transmission of QQE is really a domestic issue. Given these considerations, we make a closed economy assumption for our analysis.

[29] The interest rate on excess reserve was zero until 2008 and then it was increased discretely by 10 basis points. With the advent of QQE in 2016, it was cut to negative 10 basis points. Taking this history into account, we set the steady state IOER at zero basis point.

[30] It is straightforward to verify that $\eta_1 = (1 - \frac{\beta}{\Lambda}\frac{1+i^D_{t+1}}{1+\pi_t})\frac{\Lambda}{\Lambda-\gamma_c}(\frac{c_t}{d_t})^{-1}$ and $\eta_2 = (1 - \frac{\beta}{\Lambda}\frac{1}{1+\pi_t})\frac{\Lambda}{\Lambda-\gamma_c}(\frac{c_t}{m_t})^{-1}$.

[31] The formula for pricing kernel based duration is given by $(1 - \beta\nu)^{-1}$in Woodford (2001). Lansing (2015) derives a similar formula for duration based on a different functional form of the utility function.

[32] The impulse responses and variance decompositions for the baseline model are reasonably robust to small changes in the values of the structural parameters which we report later.

[33] For these baseline values, we find that the marginal benefit of holding an extra excess reserve exceeds the cost. Thus it is reasonable to assume that all banks start off with an excess reserve meaning that the Lagrange multiplier $\varkappa_t = 0$ in the steady state.

[34] All nonstationary macroeconomic variables are deflated by the balanced growth rate Λ^t to make the model stationary. The stationarized equations and recursive steady states are presented in the appendix.

[35] The details of this optimization routine is explained in https://git.dynare.org/Dynare/dynare/-/wikis/mode-compute-6.

Please note that estimates reported in Table 4 are based on the posterior mode only. We do not do any further Metropolis Hastings draws for estimation. Results are very similar if we do MH for 100,000 replications.

Table 2
Baseline parameter values.

Parameter	Description	Value	Source
$\overline{i^D}$	quarterly deposit rate	0.0000947	data
$\overline{i^p}$	quarterly call rate	0.000126	data
$\overline{i^R}$	quarterly reserve interest rate	0.00	data
τ_c	consumption tax rate	0.08	data
Λ	quarterly GDP growth rate	1.0013	data
γ_c	habit persistence	0.67	calibration
$\overline{G}$	SS government consumption	1.85	calibration
$\overline{\pi}$	quarterly steady sate inflation	0.000126	data
$\overline{A}$	SS TFP	1.01	calibration
$\overline{Z_x}$	SS investment specific shock	1	normalization
β	quarterly time preference	0.995	Sugo and Ueda (2008)
δ_k	quarterly capital depreciation	0.009	Sugo and Ueda (2008)
ν	quarterly decaying coupon rate	0.9799	duration of 10 year bond
α	capital share	0.314	Sugo and Ueda (2008)
ϵ^Y	price markup ratio	6	calibration
$\overline{\lambda}$	long run bond/loan ratio	0.05	calibration
η_1	bank deposit preference	0.1229	data
η_2	cash preference	0.0286	data
ϕ_p	quadratic price adjustment cost	178.76	Basu and Sarkar (2016)

Table 3
Target SS ratios; observed values vs calibrated values.

	Observed values	Calibrated values
consumption/GDP	0.624	0.629
investment/GDP	0.167	0.176
government consumption/GDP	0.208	0.195
domestic bank bond holding/GDP	0.850	0.853
Nominal yield to maturity	0.0026	0.0065
consumption/deposit	0.155	exact match
consumption/cash	0.678	exact match

Table 4
Prior densities and posterior estimates for baseline model.

Parameters	Prior mean	Posterior mode	Posterior std.	Distribution	Prior std.
θ_p	0.200	0.0925	0.0263	beta	0.05
κ	12.00	12.5207	1.2812	normal	1.5
ρ_A	0.9	0.8170	0.0690	beta	0.05
ρ_{zx}	0.9	0.9434	0.0408	beta	0.05
ρ_G	0.9	0.9007	0.0646	beta	0.05
ρ_μ	0.9	0.7935	0.0492	beta	0.05
θ_A	0.5	0.4671	0.0503	beta	0.05
θ_{zx}	0.5	0.5126	0.0537	beta	0.05
θ_G	0.5	0.4468	0.0463	beta	0.05
θ_μ	0.5	0.4730	0.0485	beta	0.05
ι	0.3	0.2911	0.0425	beta	0.05
Shocks					
$std(\epsilon^A)$	0.01	0.0492	0.0100	invgamma	0.2
$std(\epsilon^{zx})$	0.01	0.2457	0.1701	invgamma	0.2
$std(\epsilon^G)$	0.01	0.0113	0.0012	invgamma	0.2
$std(\epsilon^\mu)$	0.01	0.0027	0.0006	invgamma	0.2

Table 5
Variance decomposition for baseline model.

	ϵ^A	ϵ^G	ϵ^{zx}	ϵ^μ
GDP	54.25	2.80	21.69	21.25
Consumption	48.95	0.02	14.56	36.47
Investment	22.46	0.00	74.64	2.89
Inflation	64.84	0.02	11.61	23.53
Reserve/Asset	16.45	0.03	66.73	16.79
Loan/Asset	11.06	0.03	69.97	18.95
Bond/Asset	34.49	0.01	57.69	7.81
Loan Rate	8.21	0.03	54.15	37.61
Tobins'Q	26.89	0.01	61.97	11.14
Nominal Yield	12.63	0.02	58.51	28.83

rate of 1.26 basis points as the SS (see Table 2) because Japan has never achieved 2% inflation target by BoJ during the sample period. If inflation rises substantially from SS level towards the 2% target following a QE shock, we view it as a successful policy for achieving the inflation target. Fig. 3 summarizes the impulse responses of the QE shock.

Due to open market purchase of JGB by the BoJ, a positive one standard deviation (27 bp) shock to the monetary base growth (ξ_t^μ) means about a 138 bp increase in JGB holding of BoJ. This is shown in the second figure in the panel. The increase in monetary base immediately translates into a positive inflation shock (40 bp) via the money supply rule (30). Higher inflation raises the real marginal cost via the staggered price adjustment cost Eq. (10) as in any standard new Keynesian model which means P_t^w/P_t rises. Higher real marginal cost makes the value of the marginal product of capital and labour shift out, which means wholesale firms buy more capital and hire more labour. This translates into a sharply higher real price of capital Q_t. Retail output supply also rises along the standard new Keynesian channel as real marginal cost rises. A wealth effect due to higher output and resulting higher wages promotes consumption. Overall, QE has a significant positive real effect on the economy. As far as the impact effect is concerned, the QE multiplier for GDP is 1.94 and these for consumption and investment are 2.46 and 2.26 respectively.

On the banking front, as a result of this QE operation banks undergo a major portfolio shift. Even though BoJ injects reserves into the banking system via bond purchase, banks do not hold these reserves because of a higher anticipated inflation tax resulting from the QE operation. The ratio of reserve to bank's total asset (comprising loans, reserve and JGB) thus falls. On the other hand, when BoJ purchases bonds, commercial bank's holding of JGB declines by $(\mu_t - \overline{\pi})M_{t-1}^R/B_{t-1}^p$ as shown in (33). This raises the bond price and the nominal yield to maturity. sharply falls by 26 bp. Due to the portfolio adjustment cost, commercial banks experience a reduction in cash flow at date t as seen in Eq. (13) following the QE bond purchase. In order to smooth the cash flow, banks slowly buy back bonds in the following periods. This explains why the ratio of bonds to total asset first drops and then

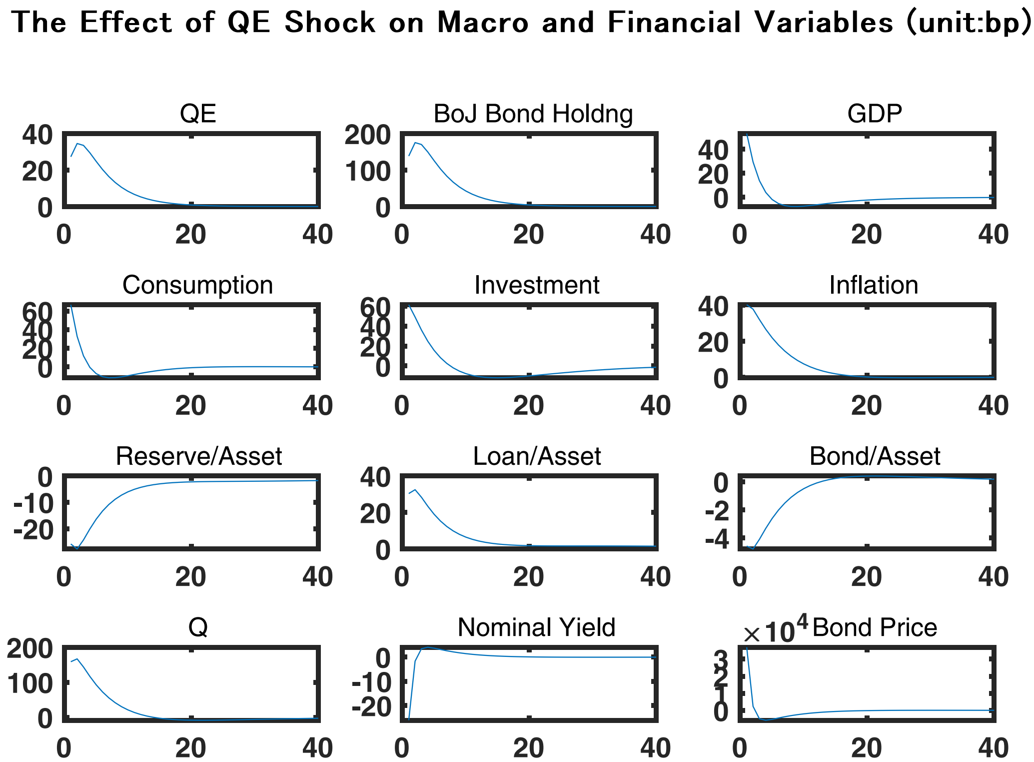

Fig. 3. IRFs with a positive one standard deviation of QE shock.

steadily rises. On the credit front, there is a surge in loans by the commercial banks (30 bp) following the QE but it steadily declines due to increased bond holding by the commercial banks. The immediate response of loan/asset ratio is about 24% smaller than that of inflation following a QE shock. Sudo and Tanaka (2021) report the effect of long-term bond purchase on the term premium. The largest decline of 19 bp happens at the impact period. In our paper, we have a fixed short term rate so that the decline in the nominal yield of long term bond leads to the decline in the term premium. The 26 bp decline in nominal yield in our paper is in line with the 19 bp decline in Sudo and Tanaka (2021). Chen et al. (2012) reports the estimated total impact effect of LASAPs on the same 10 year Treasury yield in the literature and the these values range from −13 bp to −107 bp.

The credit channel does not work stronger than inflation channel for the QE operation due to the countervailing buying back of bonds by the commercial banks. The portfolio adjustment cost is quite crucial to weaken the credit channel of QE transmission.[36] Overall, QE has a stimulative impact effect on the economy from two channels, namely the new Keynesian price-marginal cost channel and the credit channel.

5.4. Robustness check

How do the results of IRF and VD respond to small changes in key structural parameters? Table 6 reports the sensitivity of both the impact effects (IE) and VD of two major macroeconomic variables of interest, namely output growth and inflation and two relevant financial variables, namely loan/asset ratio and the nominal yield to maturity. Recall that we have three classes of parameters, namely calibrated, composite and estimated. All five calibrated parameters (τ_c, $\overline{\pi}$, ϕ_p, $\overline{i^R}$, $\overline{i^P}$) are perturbed and the model is re-estimated. For estimated parameters such as ρ^{μ} and ι, the prior mean is perturbed and the model is re-estimated. The composite parameter η_1 is decreased by 10% and the model is also re-estimated. Raising the smoothing parameter ρ^{μ} by 10% affects both the IE and VD of all the relevant variables. Changes in other parameters have negligible effects.

5.4.1. Identifying the key assumption for QE having effects on output and the bond yield

Our principal finding in this paper is that QE has a nontrivial effect on output and has a significant effect on the bond yield. Since our model has several structural parameters, a natural question arises as to which of these parameters are responsible for these key results?[37] To this end, based on theoretical considerations we narrow down our search and identify three key structural parameters which are namely the nominal rigidity parameter ϕ_p, the deposit preference parameter η_1 which proxies the preferred habitat in the model and the portfolio adjustment cost parameter ι which reflects bond market friction. Muting ϕ_p makes the model a flexible price model and the real effect of monetary policy is wiped out. QE becomes superneutral in the sense that its effects on output, consumption and investment are zero.[38]

Second, setting the deposit preference parameter η_1 close to zero (0.0001) has no effect on the real sector. The VD of y, c, i,remain unaffected and so is the bond yield.($nytm$). The only minor change that

[36] Between April 2001 (the starting quarter of QE) and June 2001, there was a 295 bp increase in loan/asset ratio and between April 2013 (the starting quarter of the 1st phase of QQE) and June 2013, there was an increase of 247 bp in loan/asset ratio. Thus the qualitative effect of QE on loan/asset ratio in our model is consistent with the observed pattern.

[37] We thank a referee for raising this question.

[38] Our baseline ϕ_p is fixed at 178.76 as in Basu and Sarkar (2016) which is higher than the level as in Iwasaki et al. (2021). It is not surprising that the price adjustment cost coefficient (ϕ_p) is high in our model because we do not have any nominal wage adjustment cost as in Iwasaki et al. (2021). The brunt of nominal rigidity is thus borne by prices. For our chosen values of ϕ_p and other relevant parameters, the implied Calvo nominal price rigidity parameter is in line with Sugo and Ueda (2008). Moreover, fixing ϕ_p at 30.8 as in Iwasaki et al. (2021), lowers the variance of output due to TFP shock to an implausibly low level.

Table 6
The effect of parameter changes on IE (basis point) and VD (percent) for baseline model.

	Change	Y (IE)	Y (VD)	π (IE)	π (VD)	$nytm$ (IE)	$nytm$ (VD)	$\frac{loan}{asset}$ (IE)	$\frac{loan}{asset}$ (VD)
	Baseline	53	21.25	40	23.53	−26	28.83	30	18.95
ρ^{μ}	10%	41	13.73	31	14.86	−20	19.57	21	7.71
τ_c	−10%	53	21.10	40	23.50	−26	28.84	30	18.81
$\overline{i^R}$	−10 bp	53	21.25	40	23.53	−26	28.83	30	18.95
$\overline{i^p}$	−10 bp	53	21.25	40	23.53	−26	28.83	30	18.95
$\overline{\pi}$	−10%	53	21.24	40	23.52	−26	28.82	30	18.94
ι	−10%	53	21.29	40	23.54	−24	29.06	31	18.99
η_1	10%	53	21.25	40	23.53	−26	28.83	29	19.05
ϕ_p	−10%	52	20.85	42	24.64	−26	28.46	30	18.55

happens is in the portfolio composition of the banks because deposit demand of household is nearly muted.

Third, setting the portfolio adjustment cost (ι) to near zero value (0.0001) has some mild real effects on y and c. The noteworthy effect is on the IRF of nominal bond yield with respect to QE shock. The shape of the IRF of nominal yield is flipped. There is an immediate negative effect of QE on nominal yield in the baseline model (see Fig. 3) but there is a positive effect of QE on nominal yield when we mute the effect of ι. Thus portfolio adjustment cost plays an important role on the effect of QE on nominal yield.[39]The intuition for this result is explained in the preceding section.

Based on this sensitivity analysis, we identify nominal rigidity as fundamentally responsible for QE having a real effect on the aggregate economy. Portfolio adjustment cost is responsible mainly for QE having a negative effect on the bond yield.

5.5. Effects of IOER changes

We finally do a policy simulation of a negative deterministic shock to IOER only. All other forcing variables are held at their respective steady states. The deterministic modeling of IOER shock stems from the fact that IOER is discretely lowered to negative 10 bp in 2016 from the steady state 0 bp. We treat the change in IOER as an once-for-all discrete intervention in a deterministic policy setting. The structural parameters are estimated from a model where IOER is the only policy instrument.[40] The time paths of the relevant macro and financial variables following this negative IOER shock are then traced out in Fig. 4.

A cut in IOER has the same directional effects on bank reserve and bank's bond holding as in QE. Unlike QE shock, a cut in IOER directly impacts the relative returns on bank's assets. It penalizes the banks to hold excess reserve which show up as an immediate reduction in bank reserves to asset ratio. What do the banks do with the released reserve? They loan out more to business which also happens in line with BoJ objective. In other words, an IOER cut also opens up the credit channel. The effect on commercial bank's bond holding is, however, nonlinear because of countervailing effects on the cash flow. What happens to bond holding crucially depends on the nominal holding period return on bond (*nhpr*). Because IOER sets the lower bound on asset returns, a sharp drop of 10 bp IOER lowers the holding period return by 1.8 bp. Banks, therefore, reduce bond holding. The decline in nhpr translates into a decline in yield to maturity and a higher bond price. On the real front, GDP, consumption, inflation also show a positive responses as in QE.

[39] We have also done further sensitivity analysis by setting ι at a much larger value 0.6 in which case the nominal bond yield drops by 43 bp which is almost double the baseline reported in Fig. 3. Thus portfolio adjustment cost is important in our model in causing the drop in bond yield in response to a QE shock.

[40] The parameter values are similar to Table 2 as in the QE model which are not reported for brevity. Details are available from the authors upon request.

Overall, the quantitative effects of an IOER cut on the macroeconomy are of second order importance because of miniscule impact effects. It is important to note that the estimated QE shock is larger than the observed IOER shock. For five quarters, the cumulative QE (149 bp) change is about fifteen times larger than the one time 10 bp IOER shocks. Moreover, QE is allowed to last longer than the IOER shock in our policy simulation. Thus it is not surprising that IOER shock has a smaller macroeconomic effect. However, even after dividing the cumulative changes in all the endogenous variables by the cumulative change in each policy shock (149 bp vs 10 bp), we find that QE has substantially larger effect on the macro and financial variables than IOER shocks.

5.5.1. Consistency of QE and yield curve control:

How does BoJ's QE policy fare with its yield curve control? Are these two policies mutually consistent? The primary goal of QE is to inject liquidity in the banking system through open market purchase of JGBs from the commercial banks. Such an operation will drive the bond price up and lower its yield below the SS yield. The yield curve control, on the other hand, sets a zero yield target. Thus as long as the SS yield is above this zero yield target, BoJ should keep using QE as a policy tool to drive the bond yield below the SS to reach the target yield of zero percent. In our estimated DSGE model, this is indeed the case. The nominal SS bond yield is 65 bp. Given that the BoJ target bond yield is zero percent, our impulse response chart in Fig. 3 shows that the impact effect of BoJ's QE was a 26 bp drop in nominal yield from SS level. Viewed from this perspective, QE is a successful experiment to lower the bond yield towards the zero target.

However, it is important to note that such a yield curve control by continuous QE operation is not sustainable in the long run because it conflicts with positive inflation and growth targets. To see this use the yield to maturity Eq. (36) and check that in the SS it reduces to[41]:

$$1 + nytm = (1 + \pi)\Lambda\beta^{-1}$$

For zero yield, it is necessary that $\Lambda = \beta/(1 + \pi)$ which means that BoJ has to either sacrifice a positive growth or positive inflation target. This inconsistency of goals arises due to the violation of Fisher's relationship between nominal yield and inflation.

6. Conclusion

Hardly any country has ever experienced so many monetary policy rules and regime switches within a short period of time as Japan. On the other hand, fiscal policy has been relatively stable. In this paper, we set up a monetary business cycle model of the Japanese economy with a particular focus on the bond market. Using this model, we assess the macro financial effects of QE vis-a-vis other monetary policy experiments. Quantitative easing is modeled as a positive shock to monetary base with simultaneous purchase of long term government bonds by BoJ which causes maturity transformation of commercial

[41] It is straightforward to verify from (19) that in the steady state $S = \Omega/(1 - \nu\Omega)$.

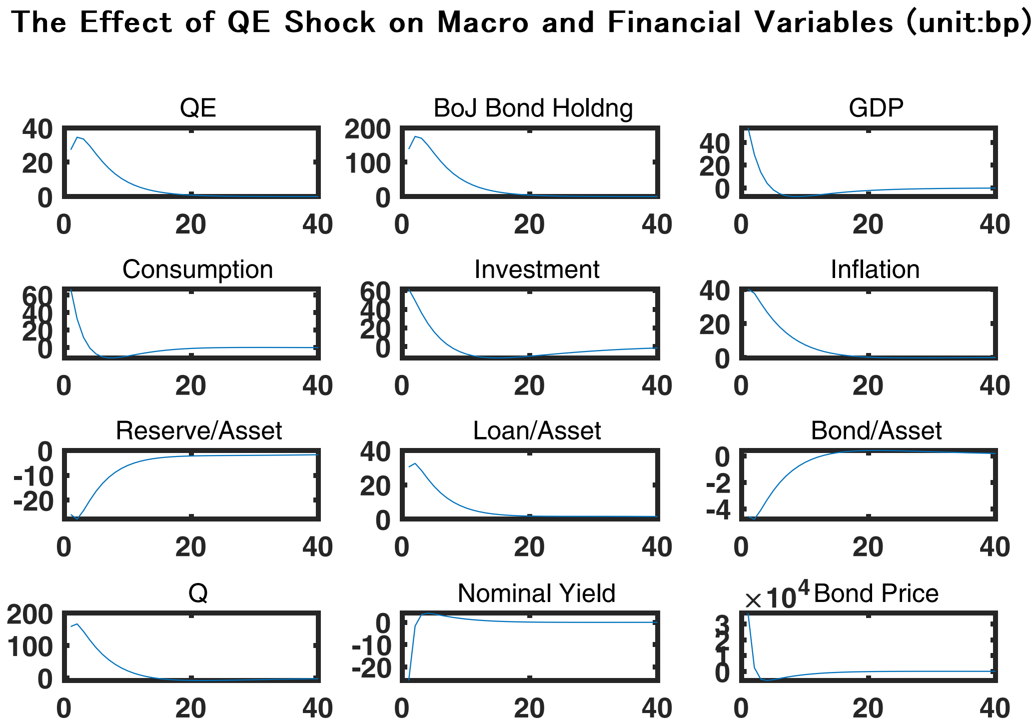

Fig. 4. IRFs with a 10 bp negative IOER shock.

bank assets. Our study spans the period 1999:Q1 to 2019:Q1 over which the Japanese monetary policy underwent several transitions which include switches between interest rate control and monetary base control.

Our study has several lessons for the efficacy of monetary policy in Japan as well as the rest of the world. Japan experimented with alternative instruments of monetary control which include conventional call rate, QE and IOER. Among all these three policy instruments, we focus primarily on QE and IOER. We find that the QE is a potent policy tool because it alters the monetary base of the economy, fuels inflationary expectations, and opens the credit channel of monetary transmission. Our calibrated DSGE model predicts that QE is an adequate policy tool for attaining the BoJ target of (i) boosting the economy, (ii) inflation targeting, and (iii) lowering the yield of long term JGB. In contrast, the macroeconomic effects of negative IOER are of second order importance. About the bond market targets of BoJ, our DSGE model predicts that although QE is an effective instrument for lowering the yield to maturity of JGB in the short run, a zero percent yield target for 10 year JGB is not sustainable because it conflicts with a positive long run growth and inflation targets. Our study has relevance for the post pandemic recovery of the world economy where leading Central Banks often resort to QE operation and negative IOER to rejuvenate their economies.

Data availability

Data will be made available on request.

Appendix. Short run equation system

All level variables are stationarized by the growth rate Λ^t. The short run equation system is given by:

$$\Omega_{t,t+1} = \frac{\beta(\widetilde{C}_t - \Lambda^{-1}\gamma_c\widetilde{C}_{t-1})}{(\widetilde{C}_{t+1} - \Lambda^{-1}\gamma_c\widetilde{C}_t)}(1+\pi_{t+1})^{-1} \tag{A.1}$$

$$D_t \;:\; (\widetilde{C}_t - \Lambda^{-1}\gamma_c\widetilde{C}_{t-1})^{-1} = \eta_1\widetilde{d}_t^{-1} + \beta(\widetilde{C}_{t+1} - \Lambda^{-1}\gamma_c\widetilde{C}_t)^{-1}(1+i^D_{t+1})(1+\pi_{t+1})^{-1} \tag{A.2}$$

$$M^T_t \;:\; (\widetilde{C}_t - \Lambda^{-1}\gamma_c\widetilde{C}_{t-1})^{-1} = \eta_2\widetilde{m}^T_t + \beta(\widetilde{C}_{t+1} - \Lambda^{-1}\gamma_c\widetilde{C}_t)^{-1}(1+\pi_{t+1})^{-1} \tag{A.3}$$

$$H_t \;:\; 1 = (1-\tau^h_t)(W_t/P_t)(\widetilde{C}_t - \Lambda^{-1}\gamma_c\widetilde{C}_{t-1})^{-1} \tag{A.4}$$

$$\widetilde{K}_t = \Lambda^{-1}(1-\delta_k)\widetilde{K}_{t-1} + \widetilde{I}_t \tag{A.5}$$

$$Q_t = 1 + \Xi\left(\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}}\right) + \Xi'\left(\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}}\right)\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}} - E_t\Omega_{t,t+1}(1+\pi_{t+1})\left[\Xi'\left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_t}\right)\left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_t}\right)^2\right] \tag{A.6}$$

$$b^P_t \;:\; 1 = E_t\Omega_{t,t+1}\left[(1+nhpr_{t+1}) - adj^b_{t,t+1}\right] + adj^b_{t-1,t} \tag{A.7}$$

$$l_t \;:\; 1 = E_t\Omega_{t,t+1}[(1+i^L_{t+1}) + adj^l_{t,t+1}] - adj^l_{t-1,t} \tag{A.8}$$

$$\frac{\widetilde{m}^R_t}{\widetilde{d}_t} = 1 - \frac{1-(1+i^R_t)E_t\Omega_{t,t+1}}{(1+i^P_t)E_t\Omega_{t,t+1}} \tag{A.9}$$

$$1+i^L_t = \left[\left(\frac{P^w_t}{P_t}\right)(MPK_t/Q_t) + (1-\delta_k)\right]\left[\frac{(1+\pi_t)Q_t}{Q_{t-1}}\right] \tag{A.10}$$

$$\frac{W_t}{P_t} = (1-\alpha)\frac{\widetilde{Y}_t}{H_t}\frac{P^w_t}{P_t} \tag{A.11}$$

$$\frac{P_t}{P^W_t} = \left(\frac{\varepsilon^Y}{\varepsilon^Y-1}\right).$$

$$\left[1 + \frac{\phi_p}{\varepsilon^Y-1}\left(\frac{1+\pi_t}{1+\overline{\pi}}\right)\left\{\frac{(1+\pi_t)}{(1+\overline{\pi})} - 1\right\} - E_t\Omega_{t,t+1}\frac{\phi_p}{\varepsilon^Y-1}\left\{\frac{\widetilde{Y}_{t+1}}{\widetilde{Y}_t}\frac{(1+\pi_{t+1})^2}{(1+\overline{\pi})}\left[\frac{(1+\pi_{t+1})}{(1+\overline{\pi})} - 1\right]\right\}\right]^{-1} \tag{A.12}$$

$$\widetilde{C}_t+\widetilde{I}_t+\Xi\left(\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}}\right)\widetilde{I}_t+\phi_p\left[\left\{\frac{1+\pi_t}{(1+\overline{\pi})}-1\right\}^2\widetilde{Y}_t\right]+\widetilde{G}_t=A_t\Lambda^{-\alpha}\widetilde{K}^{\alpha}_{t-1}H_t^{1-\alpha} \tag{A.13}$$

$$\widetilde{G}_t+(1+\nu S_t)\frac{\widetilde{b}^p_{t-1}}{\Lambda(1+\pi_t)}=\widetilde{\widetilde{T}}_t+S_t\widetilde{b}^p_t+\widetilde{m}^R_t-(1+i^R_t)\frac{\widetilde{m}^R_{t-1}}{\Lambda(1+\pi_t)}+\widetilde{m}^T_t-\frac{\widetilde{m}^T_{t-1}}{\Lambda(1+\pi_t)} \tag{A.14}$$

$$i^D_t=\overline{i^D}\ Exogenous \tag{A.15}$$

$$\widetilde{b}^p_t=\overline{\lambda}\widetilde{K}_t \tag{A.16}$$

$\widetilde{G}_t$ given by (28) (A.17)

$$\frac{1+\mu_t}{1+\overline{\pi}}=\left(\frac{1+\mu_{t-1}}{1+\overline{\pi}}\right)^{\rho_\mu}\exp(\xi^\mu_t) \tag{A.18}$$

There are 18 short run equations with 18 endogenous variables as follows.

$i^{L,}_t\ \widetilde{K}_t, H_t, \widetilde{C}_t, \widetilde{I}_t, \widetilde{d}_t, \widetilde{m}^R_t, \widetilde{m}^T_t, S_t, \widetilde{b}^p_t, i^D_t, \pi_t, Q_t, W_t/P_t, P_t/P^w_t, \widetilde{G}_t, \mu_t, \widetilde{\widetilde{T}}_t$

A.1. SS block written recursively

SS equations can be solved recursively.

$$\pi=\overline{\pi} \tag{A.19}$$

$$\Omega=\frac{\beta}{(1+\pi)\Lambda} \tag{A.20}$$

$$1+i^L=\Omega^{-1} \tag{A.21}$$

$$\frac{P}{P^W}=\frac{\varepsilon_Y}{\varepsilon_Y-1} \tag{A.22}$$

$$1+i^L=\left[\left(\frac{P^w}{P}\right)\frac{\alpha\widetilde{Y}}{\widetilde{K}}+\left(1-\delta_k\right)\right](1+\pi)\text{ solves }\widetilde{k}/H \tag{A.23}$$

$$W/P=(1-\alpha)\,\Lambda^{-\alpha}\left(\frac{\varepsilon^Y-1}{\varepsilon^Y}\right)\left(\frac{\widetilde{K}}{H}\right)^{\alpha} \tag{A.24}$$

$$\widetilde{C}=W/P \tag{A.25}$$

$$\widetilde{d}=\frac{\eta_1(1+\pi)\widetilde{C}}{1+\pi-\beta(1+i^D)} \tag{A.26}$$

$$\widetilde{m}^T=\frac{\eta_2(1+\pi)\widetilde{C}}{1+\pi-\beta} \tag{A.27}$$

$$\widetilde{I}=\delta_k\widetilde{K} \tag{A.28}$$

$$\widetilde{G}=\overline{\widetilde{G}} \tag{A.29}$$

$$\widetilde{C}+\widetilde{I}+\widetilde{G}=\overline{A}\Lambda^{-\alpha}\widetilde{K}^{\alpha}H^{1-\alpha}\text{ solves }\widetilde{K}\text{ using (A.23)} \tag{A.30}$$

$$S=\Omega/(1-\nu\Omega) \tag{A.31}$$

$$\widetilde{m}^R=1-\frac{1-(1+i^R_t)\Omega}{(1+i^p)\Omega} \tag{A.32}$$

$$Q=1 \tag{A.33}$$

$\widetilde{\widetilde{T}}$ determined by (A.14) (A.34)

$$i^D=\overline{i^D} \tag{A.35}$$

$$\widetilde{b}^p=\overline{\lambda}\widetilde{K} \tag{A.36}$$

A.2. Data sources

The market capitalization of equities is from Japan Exchange Group. The debt/GDP ratio in Japan is from IMF World Economic Outlook. The CPI (all items less fresh food) series is from Statistics Bureau, Ministry of Internal Affairs and Communications (a proxy for price level). The following series are from the Japanese Ministry of Finance: (i) the amount outstanding of JGB, (ii) the percentage of JGB held by both BoJ and commercial Banks and (iii) the nominal estimated yield to maturity of zero coupon bonds with ten year maturity (a proxy for the nominal ten year yield for JGB). We used the BoJ sources to get the series for (i) the reserve balance, (ii) seasonally adjusted monetary base (average amounts outstanding), (iii) the holding of JGB (central government securities) by BoJ and domestically licensed banks, (iv) uncollateralized overnight call rates, (v) the deposit amount at domestically licensed banks and Shinkin Banks, (vi) bank notes plus coins (a proxy for cash) and (vii) loan/asset ratio of domestically licensed banks (domestic branch). The Economic and Social Research Institute, Cabinet Office is used as a source for: (i) nominal seasonally adjusted private consumption (a proxy for the closed economy consumption), (ii) nominal seasonally adjusted private non-residential investment (a proxy for the closed economy investment), and (iii) nominal seasonally adjusted government consumption (a proxy for the closed economy government expenditure).

References

Adjemian, S., Juillard, M., 2010. Dealing with ZLB in DSGE Models an Application To the Japanese Economy. ESRI Discussion Paper Series, No. 258.

Banerjee, S., Basu, P., Ghate, C., 2019. A monetary business cycle model for India. Econ. Inq. 56 (3), 1362–1386.

Basu, P., Sarkar, A., 2016. Partial inflation indexation and long-run inflation targeting in a growing economy: A comparison of calvo and rotemberg pricing models. J. Macroeconomics 50, 293–306.

Bernanke, B., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In: Handbook of Macroeconomics, Vol 1, Part C. pp. 1341–1393.

Chang, S., Contessi, S., Francis, J., 2014. Understanding the accumulation of bank and thrift reserves during the U.S. financial crisis. J. Econom. Dynam. Control 43, 78–106.

Chen, H., Curdia, V., Ferrero, A., 2012. The macroeconomic effects of large-scale asset purchase programs. Econ. J. 122 (November), F289–F315.

Fueki, T., Fukunaga, I., Ichiue, H., Shirota, T., 2016. Measuring potential growth with an estimated DSGE model of Japan's economy. Int. J. Central Bank. 12 (1), 1–32.

Gali, J., 2015. Monetary Policy, Inflation, and the Business Cycle, second ed. Princeton University Press.

Gerali, A., Neri, S., Sessa, L., Signoretti, F., 2010. Credit and banking in a DSGE model of the euro area. J. Money Credit Bank. 42 (6), 107–141.

Gertler, M., Karadi, P., 2013. QE 1 vs. 2 vs.3: A framework for analyzing large-scale asset purchases as a monetary policy tool. Int. J. Central Bank. 9 (S1), 5–53.

Hall, R., Reis, R., 2015. Maintaining Central-Bank Financial Stability under New-Style Central Banking. National Bureau of Economic Research Working Paper, No. 21173.

Hansen, G., Imrohoroglu, S., 2016. Fiscal reform and government debt in Japan: A neoclassical perspective. Rev. Econ. Dyn. 21, 201–224.

Harrison, R., 2017. Optimal Quantitative Easing. Bank of England working paper 678.

Hayashi, F., Koeda, J., 2019. Exiting from quantitative easing. Quant. Econ. 10, 1069–1107.

Hirose, Y., 2020. An estimated DSGE model with a deflation steady state. Macroecon. Dyn. 24, 1151–1185.

Iwasaki, Y., Muto, I., Shintani, M., 2021. Missing wage inflation? Estimating the natural rate of unemployment in a nonlinear model. Eur. Econ. Rev. 132, 103626.

Iwata, Y., 2009. Fiscal Policy in an Estimated DSGE Model of the Japanese Economy: Do Non-Richardian Households Explain All?. ESRI Working Paper Series, No. 216.

Koeda, J., 2019. Macroeconomic effects of quantitative and qualitative monetary easing measures. J. Japan. Int. Econ. 52, 121–141.

Lansing, K., 2015. Asset pricing with concentrated ownership of capital and distribution shocks. Am. Econ. J.: Macroecon. 7 (4), 67–103.

McNelis, P., Yoshino, N., 2016. Finding stability in a time of prolonged crisis: Unconventional policy rules for Japan. J. Financial Stab. 27, 122–136.

Michaelis, H., Watzka, S., 2017. Are there differences in the effectiveness of quantitative easing at the zero-lower bound in Japan over time? J. Int. Money Finance 70, 204–233.

Nagao, R., Kondo, Y., Nakazono, Y., 2021. The macroeconomic effects of monetary policy: Evidence from Japan. J. Japan. Int. Econ. 61, 101149.

Ogura, Y., 2020. Intensified lending competition and search-for-yield under prolonged monetary easing. J. Japan. Int. Econ. 56, 101076.

Rotemberg, J.J., 1982. Monopolistic price adjustment cost and aggregate output. Rev. Econom. Stud. 57, 7–531.

Rudebusch, G., Swanson, E., 2008. Examining the bond premium puzzle with a DSGE model. J. Monetary Econ. 55 (S), S111–S126.

Rudebusch, G., Swanson, E., 2012. The bond premium in a DSGE model with long-run real and nominal risks. Am. Econ. J.: Macroecon. 4 (1), 105–143.

Shioji, E., 2019. Quantitative 'flooding' and bank lending: Evidence from 18 years of near-zero interest rate. J. Japan. Int. Econ. 52, 107–120.

Shioji, E., 2020. Response of bank loans to the bank of Japan's quantitative and qualitative easing policy: A panel data analysis. Seoul J. Econ. 33 (3), 355–394.

Sidrauski, M., 1967. Rational choice and patterns of growth in a monetary economy. Amer. Econ. Rev. 57 (2), 534–544.

Smets, F., Wouters, R., 2007. Shocks and frictions in US business cycles: A Bayesian DSGE approach. Amer. Econ. Rev. 97 (3), 586–606.

Sudo, N., Tanaka, M., 2021. Quantifying stock and flow effects of QE. J. Money Credit Bank. 53 (7).

Sugo, T., Ueda, K., 2008. Estimating a DSGE model for Japan: Evaluating and modifying a CEE/SW/LOWW model. J. Japan. Int. Econ. 22 (4), 476–502.

Vayanos, D., Vila, J., 2020. A Preferred-Habitat Model of the Term Structure of Interest Rates. Working Paper, London School of Economics.

Woodford, M., 2001. Fiscal Requirements for Price Stability. National Bureau of Economic Research Working Paper, No. 8072.