Japan & The World Economy 65 (2023) 101175

Contents lists available at ScienceDirect

Japan & The World Economy

journal homepage: www.elsevier.com/locate/jwe

Elderly long-term care policy and sandwich caregivers' time allocation between child-rearing and market labor

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ARTICLE INFO

JEL Classification:
H51
J13
J14
J22

Keywords:
Double carers
Family labor supply
Fertility
Long-term care policy
Sandwich caregivers

ABSTRACT

Using an overlapping generations model, we present analyses of public long-term care provision effects on fertility and time allocation decisions of sandwich caregivers, those caring for young children and old parents simultaneously. If the public long-term care level runs short of the necessary level, then working children must compensate for the difference by spending their time. Reportedly, about a third of university students' parents are sandwich caregivers in Japan, although Japan has a Long-Term Care Insurance system, which is a mandatory system with universal coverage. With a rapidly aging population, demand for long-term care is predicted to increase, thereby affecting family time allocation, e.g., fertility decisions, in Japan. Results show that if public long-term care production is costly relative to family care provision, then increases in public care provision lower the fertility rate. If labor productivity in the public long-term care sector improves, then it increases the fertility rate by freeing caregivers' time from family care provision. It will also increase social welfare. The effects on labor employment in the goods production sector are generally ambiguous because the increased public care provision requires more labor.

1. Introduction

In most economically developed countries, both life expectancy and the first-birth age have increased simultaneously. Therefore, the probability that working adults will be burdened with responsibilities to care for both young children and old parents has increased. Generations caring for both parents and children simultaneously are called "sandwich generations" by Miller (1981) and "double carers" by Soma and Yamashita (2017).[1] Based on a study conducted by Parker and Pattern (2013), Suh (2016) reports that about half of American people aged between 47 and 59 care for older parents aged 65 and older and simultaneously for children under age 18 or provide financial support to children older than 18 in 2012. Yamashita and Soma (2020) report, from a 2012–2018 sample survey of parents who have children of university student age and younger, that about 30 % of Japanese people have experienced caring for both parents and children.

This paper presents analyses, using an overlapping generations model, of time allocation of sandwich caregivers or double carers among childrearing, family long-term care provision, and market labor supply. Then we describe the dynamics of the model economy. Such theoretical analyses of sandwich or double caregiving have not been presented sufficiently in the literature of economic dynamics models. This is the first feature of this paper.[2]

Reports of the literature often describe that changes in family values, mobility of children, and increasing labor participation of women can weaken family solidarity, thereby inducing people to rely on public assistance schemes. The demands of childcare are predictable because parents can determine the number of their children to a considerable extent and because care demands decline as children age. By contrast, demands for elderly care are less predictable, although every individual has biological parents. The aging process affects individuals quite differently. Whether an individual becomes dependent, what degree of

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[1] Parker and Pattern (2013) define the sandwich generation as those adults with at least one living parent of age 65 or older and who is either raising a child younger than 18 or providing financial support to a grown child aged 18 or older.

[2] Using household health production functions by which one's old-age health status depends on one's own input and one's child's input, Tabata (2005) and Mizushima (2009) analyze how population aging affects economic growth. Hashimoto and Tabata (2010) consider fertility decisions in addition to health investment. However, they do not consider family care provision, as we do in the present study.

https://doi.org/10.1016/j.japwor.2023.101175
Received 25 August 2022; Received in revised form 16 November 2022; Accepted 12 January 2023
Available online 19 January 2023
0922-1425/

care becomes necessary when becoming dependent, and how long the care is necessary are all uncertain. Furthermore, demands for elderly care increase as elderly parents age (Suh, 2016).[3] Therefore, elderly long-term care provision is a more important issue than childcare provision for sandwich or double caregivers.[4]

We assume that a minimum level of long-term care is necessary for dependent elderly people to live. Children might not afford to provide more than the minimum level of elderly long-term care for parents, although that level depends on the degree of dependence. The number of workers who quit their jobs to care for family member(s) became twice the typical number doing so in Japan in the 2010s (Ishibashi, 2019).[5] These workers apparently have no choice other than providing a 'minimum level' of care by quitting their jobs. Barigozzi et al.(2020) show that informal care provided by daughters, the major caregivers, exerts a negative externality on daughters providing less informal care than the average level as a social norm. Public long-term care provision might only replace informal family care, leaving the total amount of elderly care almost unchanged. Tamiya et al. (2011) and Sugawara and Nakamura (2014) report that, after introduction of the Long-Term Care Insurance system, the average informal care time has decreased considerably in Japan. Therefore, a minimum level apparently exists. This is the second feature of this paper.

A vast literature on the Japanese Long-Term Care Insurance system, including Tamiya et al. (2011) and Sugawara and Nakamura (2014), has been presented. The Japanese system was introduced as a mandatory system with universal coverage in 2000. Kato (2018) uses a numerical dynamic general equilibrium model incorporating the Long-Term Care Insurance system to explore the impacts of rapid population aging in Japan. He obtains results indicating that the burdens on elderly persons aged 65 and over and on workers aged 40–64 become more than 1.7 times and more than 2.7 times, respectively, over the next 40 years. However, Kondo (2019) reports that changes in the Long-Term Care Insurance payments might not increase the number of employees of the system and their wages. Using Data for Japan, Niimi (2021) reports that caregiver leave prevents workers from leaving their jobs despite their caregiving responsibilities. However, these studies do not consider fertility decisions of families simultaneously. Particularly, Japan has experienced, and is still experiencing, both rapid population aging and severely declining fertility. Long-term care policies are expected to contribute to mitigate their negative effects caused by both phenomena. Aya (2014) and Tian and Wang (2019) estimate the labor productivity of workers in the Japanese Long-Term Care Insurance sector. Then they apparently infer that the productivity is lower than those in other manufacturing industries in Japan. Because long-term care provision is regarded as labor intensive, the efficiency might be explored from the supply side of long-term care provision. This is also an important concern in our study.

For the analyses conducted for this paper, we assume that government provides constant care per dependent elderly person, financing the cost through taxes on workers. Uncertain and weakened family solidarity imposes great pressure on governments to provide long-term care benefits, substituting for family long-term care (Cremer and Pestieau, 2014; Yakita, 2020).[6] Public long-term care provision involves management costs of care workers and facility costs in addition to labor costs, although labor efficiency in public care production is higher than that of family informal care. If the level of public long-term care runs short of the minimum level, then children must compensate for their mutual differences. They are certainly (forcedly) altruistic to their parents in the sense that they are ready to help their parents with assistance as soon as their parents lose their autonomy (Pestieau and Sato, 2008). Sandwich or double caregivers also rear children at home simultaneously. Both child and parent caregiving are labor intensive. The burden of caregiving to children and parents is measured in care time. For analytical purposes, we do not assume childrearing services outside of the family.[7]

The main results are the following. If the non-labor cost of public long-term care provision is sufficiently great, i.e., if public care is sufficiently unit-cost inefficient, then increased public long-term care services require more tax revenues to finance them, thereby lowering the fertility rate through negative income effects on young workers. Increases in costly public care provision might lower social welfare, which is defined as the average lifetime utility of individuals. Although the market labor supply of young workers increases, whether the labor employment per worker in the consumption goods production sector increases is ambiguous. If the labor productivity in the public long-term care sector improves, then time that is freed from family care provision raises the fertility rate and the market labor supply of young workers. Although labor employment in the public long-term care sector decreases, labor employment per worker in the goods production sector might not increase if the workers' preference for having children is sufficiently strong. However, the improved labor productivity in the public long-term care sector will increase social welfare.

The paper is structured as follows. The next section introduces an overlapping generations model in which individuals are prepared to give necessary support to their elderly parents when they become dependent. Section 3 presents an analysis of the steady-state equilibrium with a given amount of public long-term care. The steady-state effects of an increase in public long-term care and the improved labor productivity in the public long-term care sector on fertility and the dynamic paths are analyzed in Section 4. Section 5 provides a numerical example to present calibration results. Most parameters are set to reflect the situation in Japan. The last section presents conclusions of these analyses.

2. Model

A three-period overlapping generations model is assumed. Individuals certainly live for three periods: childhood, and young and old adulthood. Individuals are fed by their parents during childhood. They work, save for their old period, rear children, and provide family long-term care for their dependent parents during young adulthood. Later, they retire in their old age, receiving long-term care from their children's generation if they become dependent. We assume that elderly people cannot live without the minimum level of assistance if they become dependent. The minimum support consists of both instrumental activities of daily living IADL (e.g., shopping and handling phone calls) and activities of daily living ADL (e.g., eating, bathing, and changing clothes). This study includes no consideration of bargaining between

[3] Hammersmith and Lin (2019), using the American Time Use Survey during 2012 and 2013, demonstrate that caregivers of parents report lower well-being than caregivers of children.

[4] To emphasize analyses of the effects of elderly care provision on the time allocation of families, we do not consider child policy. Most reports of empirical studies show that child policy increases fertility rates (Luci-Greulich and Thévenon, 2013; Bauernschuster et al., 2016). The analyses are complicated by consideration of such family policies simultaneously. This study assesses the unavoidability of long-term care for parental generations.

[5] When market labor employment has indivisibility, the young worker is forced to quit compensated employment to care for the parents. Ishibashi (2019) reports that 1.3% of workers who quit a job are explained by family elderly care in Japan in 2017.

[6] Cremer and Roeder (2017) and Yakita (2020) regard family (social) norms as forced altruism. Yakita (2020) does not consider fertility choices of individuals. If individuals face risks of having children with a low degree of altruism, then public long-term provision might lead to inefficiency, e.g., crowding out of family care (Canta and Cremer, 2021). However, Yamashita and Soma (2020) report that 43% of "double carers" reveal that they wanted to care for their dependent parents in Japan in 2017.

[7] Most studies reported in the literature, such as those by Day (2016) and Yakita (2018) indicate that childcare outside the home raises the fertility rate.

children and their elderly parents. Instead, we assume that altruism of children toward their parents might be social norms or forced altruism in the sense that they are ready to care for their dependent parents. Children are responsible for long-term care of their parents. Whether an individual becomes dependent or not when old is uncertain. An individual who remains independent in old age consumes the fruits of lifetime savings without receiving long-term care. An individual who otherwise becomes dependent receives the necessary level of elderly long-term care in addition to third-period consumption.[8] Government provides each dependent person with a constant level of care that is less than the necessary level of long-term care by financing the cost through lump-sum taxes imposed on the working generation.[9] We do not consider other government expenditures. The aggregate production technology of consumption goods is represented as a Cobb–Douglas production function. Markets are all perfectly competitive.

2.1. Individuals

We assume that children recognize whether parents become dependent or not after they decide the number of children they have and make a consumption–savings choice. For this study, we regard a unitary couple as an individual. Most reports in the literature explain that women care for elderly family members in economically developed countries (e.g., Pestieau and Sato, 2008). Therefore, we consider the life stages of a woman.[10] For instance, in Japan, the average age of the first birth of a woman is 30.7, the second-birth age is 32.7, and the third birth age is 33.8 in 2019 (Ministry of Health, Labour and Welfare, 2021). The ratio of women who care for their parents is 25.9% for ages 40-49 and 46.6% for ages 50–59 in 2018 (National Institute of Population and Social Security Research, 2018). These generations might care for both their parents and children before their children graduate from university. Therefore, it is realistic to assume that young workers choose the number of children and a lifetime consumption plan for a given probability of their parents' health status at the beginning of the period before the parents' health status is revealed to them.[11] After the parents' health status is revealed, some workers must care for their parents. The others do not do so even if their plans are the same *ex ante*.

Letting the necessary minimum level of long-term care for dependent old individuals be $\bar{h}$ and letting public long-term care services provided by government be h^G, then children have a responsibility to provide family (informal) long-term care $\bar{h} - h^G \equiv h^L$ for parents if their parents become dependent ($\bar{h} \geq h^G$).[12] Letting the number of siblings of an individual be n_{t-1} and letting the time efficiency of family long-term care time l_t^L of the individual be θ, then we have family care production as

$$h^L = \theta l_t^L n_{t-1} \tag{1}$$

Labor productivity of family care θ is assumed to be constant over time.[13] The number of siblings was already determined at the beginning of period t. Children are equally altruistic toward parents. Therefore, they share long-term care time for parents equally among siblings. Each child is compelled, altruistically, to devote care time for their parents as $l_t^L = (h^L/\theta)/n_{t-1}$ to provide the minimum level of long-term care. If parents become dependent, then the young workers must spend their time on family care provision l^L for the dependent parents. Otherwise, if their parents are still independent, then the young workers spend their time on the market labor supply after rearing their children.

Omitting second-period consumption for expositional simplicity, the lifetime utility of an individual working in period t can be described as.[14]

$$u_t = \varepsilon \ln n_t + (1-\varepsilon) \ln c_{t+1}^2, \ (0 < \varepsilon < 1) \tag{2}$$

where n_t stands for the number of children, and c_{t+1}^2 represents third-period consumption. Parameter ε represents the relative utility weight of their children. The log-linear utility function is commonly postulated in the literature of economic growth to obtain explicit solutions. The utility of caring for parents is not expressed explicitly because of the minimum long-term care level provided.

The expected lifetime utility of a worker in period t can be given as

$$EU_t = (1-\pi)\left[\varepsilon \ln n_t^a + (1-\varepsilon)\ln c_{t+1}^{2a}\right] + \pi\left[\varepsilon \ln n_t^d + (1-\varepsilon)\ln c_{t+1}^{2d}\right], \tag{3}$$

where π represents the probability of parents to be independent when they are old ($0 < \pi < 1$). Similarly to many reports of the literature such as those of Cremer and Pestieau (2014) and Cremer et al. (2017), we assume that the probability that parents become dependent is given exogenously. Superscripts $a$and d respectively designate variables when parents are independent or autonomous and when they are dependent. The budget constraints for each parent's health status can be written as

$$\left[w_t(1 - z n_t^a) - T_t\right](1 + r_{t+1}) = c_{t+1}^{2a} \tag{4a}$$

and

$$\left[w_t(1 - z n_t^d - l_t^L) - T_t\right](1 + r_{t+1}) = c_{t+1}^{2d}. \tag{4b}$$

Herein, w_t and r_{t+1} respectively represent the wage rate in period t and the interest rate in period $t+1$. In addition, T_t stands for a lump-sum tax in period t. The tax is determined before the health status of parents is revealed. Parameter z stands for the per-child rearing time, which is assumed to be constant (de la Croix and Doepke, 2003).[15] The time endowment for the young period is assumed to be unity. If parents

[8] The utility of third-period consumption might depend on the revealed health status during the period. However, we do not infer that dependence because our emphasis is on examination of the effects of public long-term care policy on fertility and growth paths for an exogenously given constant probability of becoming dependent.

[9] The public long-term care might not be burdened equally by workers. For example, only workers aged 40–65 pay premiums for long-term care insurance in addition to taxes in Japan. Half of the Long-Term Care Insurance costs is covered by revenues of the national, prefectural, and municipal governments. To reflect this fact, we do not assume a wage tax for the finance of public long-term care. We also assume that government does not confiscate a dependent's assets or income, as it does for the public long-term care security system of Japan. Whether government confiscates or not depends on the long-term care insurance system of countries (Cremer, Gahvari, & Pestieau, 2017).

[10] The average age difference between partners in a couple has been about 1.7 years in Japan since 2000 (Annual Health, Labour and Welfare Report 2020, Japan).

[11] The ratio of old people aged 65–69 who are certified as needing long-term care was 2.9% in Japan in 2019 (Ministry of Health et al., 2020); that of people aged 80–84 is 27.2%.

[12] For instance, the Japanese Long-Term Care Insurance system provides formal services only, with no cash benefit. The care is provided by trained, qualified, licensed, and supervised care workers. Tamiya et al. (2011) report that Germany started a care management program in 2008 based partly on Japan's experience. We do not consider individuals' purchases of private long-term care from the market. The market for long-term care insurance is mostly negligible in economically developed countries (Cremer et al., 2012).

[13] Caring skills might advance as they accumulate in society, although externalities are not considered explicitly.

[14] Under a log-linear utility function, this assumption is innocuous. Second-period consumption is a fixed multiple of the disposable income $w_t(1 - zn_t - \pi l_t^L) - T_t$. Explicit consideration of second-period consumption does not alter the conclusions qualitatively.

[15] The child-rearing time includes various caring time before children start working in the labor market.

become dependent, then the young workers must spend their time on family care provision l^L for the dependent parents. Otherwise, if their parents are still independent, then the young workers spend their time on the market labor supply after rearing their children.

From the first-order conditions for expected lifetime utility maximization at the beginning of period t, we obtain the optimal plans as

$$n_t^a = \frac{\varepsilon}{z}\left(1 - \frac{T_t}{w_t}\right) \text{ and } c_{t+1}^{2a} = \left(1-\varepsilon\right)\left(w_t - T_t\right)\left(1 + r_{t+1}\right), \tag{5a}$$

$$n_t^d = \frac{\varepsilon}{z}\left(1 - l_t^L - \frac{T_t}{w_t}\right) \text{ and } c_{t+1}^{2d} = \left(1-\varepsilon\right)\left[w_t\left(1 - l_t^L\right) - T_t\right]\left(1 + r_{t+1}\right) \tag{5b}$$

The lifecycle savings during the young period are, respectively, $s_t^a = (1-\varepsilon)(w_t - T_t)$ and $s_t^d = (1-\varepsilon)[w_t(1-l_t^L) - T_t]$. A shorter (longer) altruistically forced elderly care time l_t^L increases (decreases) the number of children and third-period consumption through the income effect when the parents become dependent.

Denoting the number of young workers in period t by N_t, the total market labor supply is given as $N_t\pi(1 - zn_t^d - l_t^L) + N_t(1-\pi)(1 - zn_t^a) = N_t(1 - zn_t - \pi l_t^L)$, where $(1-\pi)n_t^a + \pi n_t^d \equiv n_t$ is the average fertility rate, and $1 - zn_t - \pi l^L$ is equal to the average market labor supply in per-worker terms. The average savings per worker is given as $(1-\pi)s_t^a + \pi s_t^d \equiv s_t = (1-\varepsilon)[w_t(1-\pi l_t^L) - T_t]$.[16] We assume here that workers pay taxes even when their parents are revealed to be dependent.

2.2. Public long-term care

Public long-term care services are assumed to be provided by government in accordance with the following production function $h^G(\pi N_{t-1}) = \mu L_t^G$, where N_{t-1} and L_t^G respectively represent the old age population and the government employment of young workers in period t, and where μ represents labor productivity in the public long-term care production. We assume that labor productivity in the public long-term care sector, μ, is higher than that in family production, i.e., $\mu > \theta$. In addition, labor demand in the public care sector is obtained from the production function in per-worker terms as

$$l_t^G = \frac{\pi}{\mu n_{t-1}} h^G, \tag{6}$$

where $l_t^G = L_t^G / N_t$ stands for labor input in per-worker terms and $N_t = n_{t-1}N_{t-1}$.

The production cost of public long-term care services can be expressed as $w_t L_t^G(1+M)$, where parameter $M(>0)$ is designated as a management cost factor, as explained hereinafter. The management cost includes training and re-training costs of care workers in response to increases in the number and maintenance of the quality of formal care workers, thereby increasing the management cost. It also includes the costs for nursing facilities. We designate the sum of various costs except the wage payments the management costs in this paper.[17] We measure the management cost in terms of labor costs of formal long-term care provision. Letting the per-worker burden be lump-sum tax T_t in period t, the budget constraint of the government can be written in per-worker terms as

$$T_t = w_t l_t^G \left(1 + M\right) \tag{7}$$

At this stage, we briefly discuss whether public long-term care provision is willingly acceptable to the economy on a cost-efficiency basis. The labor time necessary to produce a unit of family long-term care is $1/\theta$ from (1), whereas that of public long-term care is $1/\mu$. Therefore, the unit cost of family care is w_t/θ. That of public care is $w_t(1+M)/\mu$, reflecting the management cost. If $w_t/\theta = w_t(1+M)/\mu$ or $\mu = (1+M)\theta$, then the cost per long-term care is the same in both family and public care. If $\mu \geq \theta(1+M)$, then public care production is more unit-cost efficient, i.e., less costly than family care. If, alternatively, $\mu < \theta(1+M)$, then public care production is unit-cost inefficient, i.e., more costly. However, individuals might have no choice about the public care service burden. For instance, workers aged 40–65 must pay long-term insurance premium compulsorily in Japan. All workers are covered by the public insurance system, irrespective of cost efficiency.

2.3. Goods production

Letting the aggregate production function of consumption goods be $Y_t = AK_t^\alpha L_t^{1-\alpha}$ $(A > 0,\ 0 < \alpha < 1)$, we have $Y_t = AK_t^\alpha l_t^{1-\alpha} N_t^{1-\alpha}$, where $L_t = l_t N_t$ and where l_t is the number of young employees of the goods production sector in period t. In addition, K_t is the aggregate capital stock in period t. In per-worker terms, the production function can be rewritten as

$$y_t = A l_t^{1-\alpha} k_t^\alpha, \tag{8}$$

where $y_t = Y_t/N_t$ and $k_t = K_t/N_t$.

Assuming perfectly competitive factor and goods markets, the profit maximization conditions are

$$w_t = \left(1-\alpha\right) A l_t^{-\alpha} k_t^\alpha \tag{9}$$

and

$$1 + r_t = \alpha A l_t^{1-\alpha} k_t^{\alpha-1} \tag{10}$$

The factor price of each production factor equals the marginal product.

2.4. Temporary market equilibrium

First, we consider the labor market clearing condition. The labor supply of the working generation is $(1 - zn_t - \pi l_t^L)N_t$, whereas labor demand from the goods production and public long-term care production sectors are signified respectively by $l_t N_t$ and $l_t^G N_t$. Therefore, the labor market equilibrium condition in period t is in per-worker terms as

$$1 - zn_t - \pi l_t^L = l_t + l_t^G \tag{11}$$

Labor is assumed to be perfectly substitutable between goods production and public long-term care production sectors, as described on the right-hand side of (11). Therefore, the wage rate must be equal between goods production and public long-term care production. In other words, the wage rate in the public sector must be equal to the wage rate for goods production. If the wage rate in public care production is set as lower than that for goods production, then no worker would want to go

[16] We assume that *ex post* after-tax income of workers remains non-negative when they care for their parents in families.

[17] The numbers of nursing and caring staff members per care center generally increased in Japan during 2015–2016: from 7.4 to 7.3 for home-visiting care, from 5.5 to 7.4 for day-care, and from 19.8 to 20.0 for specified nursing home occupant care. The numbers of long-term care facilities also increased overall (Ministry of Health, Labour and Welfare of Japan, 2017). These increases imply that various costs increase to maintain the quality of long-term care provision (e.g., training) and to schedule and arrange their activities as the number of staff members increases.

to the public sector.[18] Government is assumed to minimize the production cost because the cost is financed through taxation.

Next, the equilibrium condition in the capital market is given as equality between savings of the working generation of the prior period and the capital demand of the goods production sector as $K_{t+1} = s_t N_{t-1}$. The condition can be rewritten in per-worker terms as

$$n_t k_{t+1} = (1-\varepsilon)\left[w_t\left(1-\pi l_t^L\right) - T_t\right] \tag{12}$$

3. Dynamic system

The dynamic system of this model economy is given as a set of simultaneous difference equations of (n_t, k_t). From (5), (6), (7), and $\overline{h} = h^G + h^L$, we can obtain

$$n_t = \frac{\varepsilon}{z}\left[1 - \frac{\pi}{n_{t-1}}\left(\frac{\overline{h}}{\theta} - \frac{[\mu - \theta(1+M)]h^G}{\mu\theta}\right)\right] \tag{13}$$

From (5), (10), and (12), we obtain

$$k_{t+1} = \frac{(1-\varepsilon)z(1-\alpha)A}{\varepsilon} k_t^{\alpha} l_t^{-\alpha}, \tag{14}$$

where condition (11) can be rewritten as the equation below.[19].

$$l_t = (1-\varepsilon)\left[1 - \frac{\pi}{n_{t-1}}\left(\frac{\overline{h}}{\theta} - \frac{[\mu - \theta(1+M)]h^G}{\mu\theta}\right)\right] + \frac{M\pi h^G}{\mu n_{t-1}} \equiv l(n_{t-1}) \tag{15}$$

These Eqs. (13)–(15) determine (n_t, k_{t+1}) for given (n_{t-1}, k_t).

The steady-state equilibrium of the economy is obtained as follows. As might be readily apparent, Eq. (13) is an independent difference equation of n_t. Therefore, we first analyze the fertility dynamics.[20] The steady-state fertility rate n satisfies $n = \frac{\varepsilon}{z}[1 - \frac{\pi}{n}(\frac{\overline{h}}{\theta} - \frac{[\mu-\theta(1+M)]h^G}{\mu\theta})] > 0$. This is a quadratic equation of n. One can show readily that two steady states exist if the discriminant condition of $1 - \frac{4z\pi}{\varepsilon}(\frac{\overline{h}}{\theta} - \frac{[\mu-\theta(1+M)]h^G}{\mu\theta}) > 0$ is satisfied. Assuming that this condition is satisfied, we are concerned with the stable steady state $n*$ in the following.

$$n* = \frac{\varepsilon}{z}\left[1 - \frac{\pi}{n*}\left(\frac{\overline{h}}{\theta} - \frac{[\mu - \theta(1+M)]h^G}{\mu\theta}\right)\right] \tag{16}$$

The stability condition is $\frac{dn_t}{dn_{t-1}} = \frac{\varepsilon}{z}\frac{\pi}{n^2}(\frac{\overline{h}}{\theta} - \frac{[\mu-\theta(1+M)]h^G}{\mu\theta}) < 1$ at $n*$. We assume that the stability condition is satisfied. Fig. 1 presents the two solutions, where $n*$ is stable and $\underline{n}$ is unstable. We assume that $n_{-1} > \underline{n}$, where n_{-1} is the initial value.

Next, Eq. (15) shows that the labor employment per worker in goods production l_t depends solely on the fertility rate. Therefore, if the fertility rate is in a steady state, then the labor per worker in the goods production sector is also in a steady state as

$$l* = (1-\varepsilon)\left[1 - \frac{\pi}{n*}\left(\frac{\overline{h}}{\theta} - \frac{[\mu - \theta(1+M)]h^G}{\mu\theta}\right)\right] + \frac{M\pi h^G}{\mu n*} \equiv l(n*), \tag{17}$$

which is positive when $n* > 0$.

Finally, the steady-state per-worker capital $k*$ is obtained from (14) as

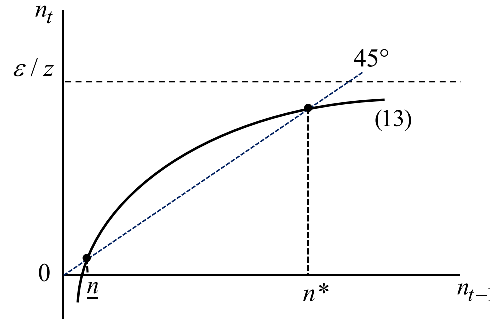

Fig. 1. Fertility dynamics.

$$k* = \left[\frac{(1-\varepsilon)z(1-\alpha)A}{\varepsilon}(l*)^{-\alpha}\right]^{1/(1-\alpha)} \tag{18}$$

The steady-state equilibrium of this economy is characterized by $(n*, k*)$ satisfying (16)–(18). Given the initial values (n_{-1}, k_0), the time paths converge to the stable steady state.

4. Changes in public long-term care provision

Assuming the existence of such a stable steady state satisfying conditions (16)–(18), in this section, we apply comparative statics to assess changes in the level of public long-term care in the first subsection. Ishibashi (2019) reports that only about 30% of dependent elderly people aged 65 and older are cared for at nursing facilities in Japan in 2016.[21] An aging population caused by longer life expectancies will demand more public long-term care services at nursing care facilities. The second subsection presents the effects of increases in labor productivity in public long-term care provision. Aya (2014) and Tiang and Wang (2019) present empirics indicating that the labor productivity of the long-term care sector is about 40–50% lower than the labor productivity of the manufacturing industry in Japan. The next section presents a numerical example to illustrate the results.

4.1. Increase in public long-term care provision

First, we present effects of increases in public long-term care. From (16), we obtain the result for the fertility rate as

$$\frac{dn}{dh^G} = \left[1 - \frac{4z\pi}{\varepsilon}\left(\frac{\overline{h}}{\theta} - \frac{[\mu - \theta(1+M)]h^G}{\mu\theta}\right)\right]^{-1/2} \frac{\pi[\mu - \theta(1+M)]}{\mu\theta}. \tag{19}$$

Whether the fertility rate increases with the level of public long-term care provision depends on the relative unit cost of family long-term care to public long-term care: $w/\theta - w(1+M)/\mu$. If public long-term care is more costly than family care, i.e., if $\mu - \theta(1+M) < 0$, then increases in public long-term care provision decrease the fertility rate. The tax burden for costly public long-term care depresses the (expected) disposable income of young workers, negatively affecting their fertility decisions. By contrast, if public long-term care is less costly relative to family care, i.e., if $\mu - \theta(1+M) > 0$, then the increased long-term care level increases the fertility rate by freeing time from family elderly care. Young workers will increase both the child-rearing time and the market working time. If the costs per unit of care are equal in family and public care provision, then the increased public long-term care does not affect

[18] We assume here that the wage rates are equalized between production sectors in this simple model. This assumption might not hold, apparently. Hanaoka (2009) demonstrates that the relative wage rates of care workers affect the turnover rates only in some employment patterns. However, Aya (2014) reports that the wage level of care workers is equal to or only slightly lower than those of other industries in Japan in 2012. Folbre and Nelson (2000) describe that wage rates in most care industries are typically lower than those of other industries in the US because the employees are disproportionately women and people of color.

[19] Appendix A1 provides a derivation of (15).

[20] Appendix A2 provides an explanation of fertility dynamics.

[21] The utilization rates of the three long-term insurance facilities of the long-term care insurance system of Japan (special elderly nursing homes, long-term care health facilities and designated medical long-term care sanatoriums) have accounted for nearly 90% or more of the capacity since the start of Long-Term Care Insurance System (Ministry of Health, Labour and Welfare, Survey of Long-Term Care Facilities and Care Centers).

the fertility rate. It is noteworthy that labor productivity in public long-term care provision higher than that for family care, i.e., condition $\mu > \theta$, *per se* does not warrant the result of the increased fertility rate.

Next, from (17), we obtain

$$\frac{dl}{dh^G} = \frac{\pi}{n*^2}\left\{\left(1-\varepsilon\right)\left(\frac{\bar{h}}{\theta} - \frac{[\mu - \theta(1+M)]h^G}{\mu\theta}\right) - \frac{Mh^G}{\mu}\right\}\frac{dn}{dh^G} + \frac{\pi}{n*}\left[\left(1-\varepsilon\right)\frac{\mu - \theta(1+M)}{\mu\theta} + \frac{M}{\mu}\right] \tag{20}$$

The second term on the right-hand side of (20), which reflects the tax hike effect through management cost increases, is positive when $\mu - \theta > 0$. The first term is the indirect effect of public care increases on market labor supply through fertility changes. Its sign is ambiguous even when $dn/dh^G > 0$. When ε is sufficiently small, the coefficient of dn/dh^G on the right-hand side of (20) is positive. In this case, we have $dn/dh^G > 0$ and $dl/dh^G > 0$ when public long-term care is less costly than family care, i.e., when $\mu > \theta(1 + M)$. Family care time that is freed up by the increased public long-term care provision increases the market labor supply and the time for childrearing. However, it is not the case in which public long-term care is more costly, i.e., when $\mu < \theta(1 + M)$. In this case, the inefficient public sector requires great tax revenues for the increased long-term provision. To compensate for the tax burden, they inevitably increase the market labor supply more than the increased public employment, thereby reducing child-rearing time.[22]

Finally, from (18), we obtain

$$\frac{dk}{dh^G} = \left(\frac{-\alpha}{1-\alpha}\frac{k*}{l*}\right)\frac{dl}{dh^G} \tag{21}$$

Therefore, it follows that $\text{sgn}(dk/dh^G) = \text{sgn}(-dl/dh^G)$. We might have $dk/dh^G < 0$ when the utility weight on having children is sufficiently small and the public care sector is unit-cost efficient.

The argument presented above can be summarized as the following proposition.

Proposition 1. (*a*)*When the public long-term care sector is more (less) unit-cost efficient, then increases in the long-term care level raise (lower, respectively) the steady-state fertility rate.* (*b*)*If the public long-term care sector is unit-cost inefficient and the individual's utility weight on having children is large, or if the public long-term care sector is unit-cost efficient and the individual's utility weight on having children is small, then the increased public long-term care increases the labor employment per worker in the goods production sector.* (*c*)*The effect on per-worker capital in the goods production sector has the opposite sign to that of the labor employment per worker in the goods production sector.*

An increase in public long-term care frees young workers' time from family care provision. If public long-term care is sufficiently unit-cost efficient, then the freed time increases the child-rearing time, thereby raising the fertility rate. It also increases the labor employed for goods production, thereby lowering the per-worker capital and hence the wage rate. By contrast, if public long-term care is unit-cost inefficient, then the increased public care decreases the fertility rate through a negative income effect caused by a heavier tax burden. It is noteworthy that, when the public long-term care sector is sufficiently unit-cost inefficient, the effect of the increased public care initially increases the market labor supply of young workers more than the increased public employment to compensate for the increased tax burden.

4.2. Improvement of labor productivity in long-term provision

This subsection presents consideration of increases in labor productivity in public long-term care provision, which is represented by parameter μ in (6). If the quality levels of care workers are improved through human resource development, then the care levels can be maintained with fewer care workers in public long-term care provision.

From (16) and using the stability condition, we obtain

$$\frac{dn}{d\mu} = \frac{\varepsilon}{z}\frac{\pi(1+M)h^G}{n*\mu^2}\bigg/\left\{1 - \frac{\varepsilon}{z}\frac{\pi}{n*^2}\left(\frac{\bar{h}}{\theta} - \frac{[\mu - \theta(1+M)]h^G}{\mu\theta}\right)\right\} > 0 \tag{22}$$

Increases in the labor productivity in public long-term care provision raise the fertility rate. The increased labor productivity reduces the number of workers employed in the public long-term care sector, thereby enabling the government to reduce taxes. Tax cuts increase the number of children through a positive income effect.

From (17) we have

$$\frac{dl}{d\mu} = \frac{\pi h^G}{n*\mu^2}\left[1 - \varepsilon\left(1+M\right)\right] + \frac{\pi}{n*^2}\left\{\left(1-\varepsilon\right)\left(\frac{\bar{h}}{\theta} - \frac{[\mu + \theta(1+M)]h^G}{\theta\mu}\right) - \frac{Mh^G}{\mu}\right\}\frac{dn}{d\mu} \tag{23}$$

Effects on labor employment per worker in the goods production sector is ambiguous. The coefficient of $dn/d\mu$ of the second term on the right-hand side of (23) is the same as the coefficient of dn/dh^G of the first term on the right-hand side of (20), which is likely to be positive when ε is small. The first term on the right-hand side of (23) is also ambiguous in sign. When both ε and M are sufficiently small, the first term is positive. Therefore, effects of the increased labor productivity in the long-term care sector on the goods production labor employment per worker are likely to be positive when the utility weight on having children is small. By contrast, when the caregivers' utility weight and the management cost of public long-term care are large, then the effect on labor employment per worker in the goods production sector is ambiguous.

Finally, from (18), we obtain

$$\frac{dk}{d\mu} = \left[\frac{-\alpha}{(1-\alpha)l*}\right]\left[\frac{(1-\varepsilon)z(1-\alpha)A}{\varepsilon}(l*)^{-\alpha}\right]^{1/(1-\alpha)}\frac{dl}{d\mu} \tag{24}$$

The effect of the improved labor productivity in the long-term care sector on the per-worker capital has the opposite sign to that of labor employment per worker in the goods production sector. When the improved labor productivity in the long-term care sector increases labor employment per worker in the goods production sector, it decreases the per-worker capital in the sector.

The arguments presented above can be summarized as shown below.

Proposition 2. (*a*)*Improvement in labor productivity in the public long-term care sector increases the steady-state fertility rate.* (*b*)*If the management cost factor in the public long-term care sector and the caregivers' utility weight of having children are sufficiently small, then the improved productivity in the public long-term care sector increases the labor employment per worker in the goods production sector, thereby oppositely affecting the per-worker capital.*

5. Numerical example

To demonstrate the results of the Propositions virtually, one can consider a numerical example. Although the population size aged 65 and older is 35.58 million, the number of persons who are assessed as being of care need levels 3–5 in the Long-Term Care Insurance system are 2.275 million in Japan in 2018 (Ministry of Health et al., 2020). Therefore, the probability of being dependent when old is assumed as $\pi = 0.065$ for this study.

The minimum level of long-term care for dependent elderly persons $\bar{h}$ and the ratio of public provision $h^G$are difficult to calculate. What is important are their relative sizes. The number of dependent persons who need care levels 3–5 assessed to the Long-Term Care Insurance system is 2.275 million, and the number of those admitted in care facilities is 1.33 million in Japan in 2018 (Ministry of Health, Labour and Welfare in Japan, 2020). However, some dependent persons are not in nursing

[22] Appendix A3 provides an explanation for the sign of (20).

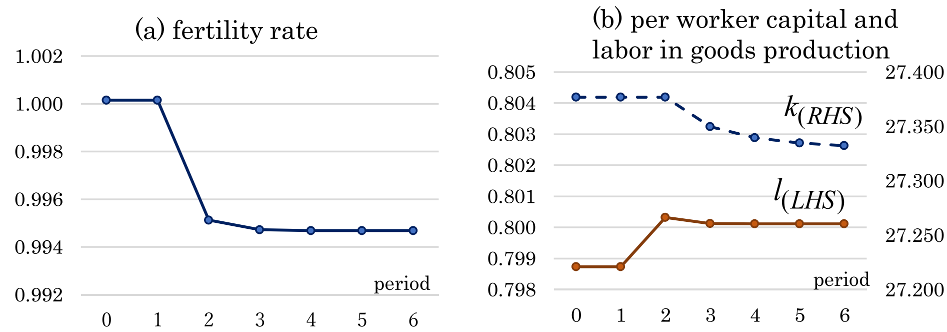

Fig. 2. Effects of an increase in public long-term care.

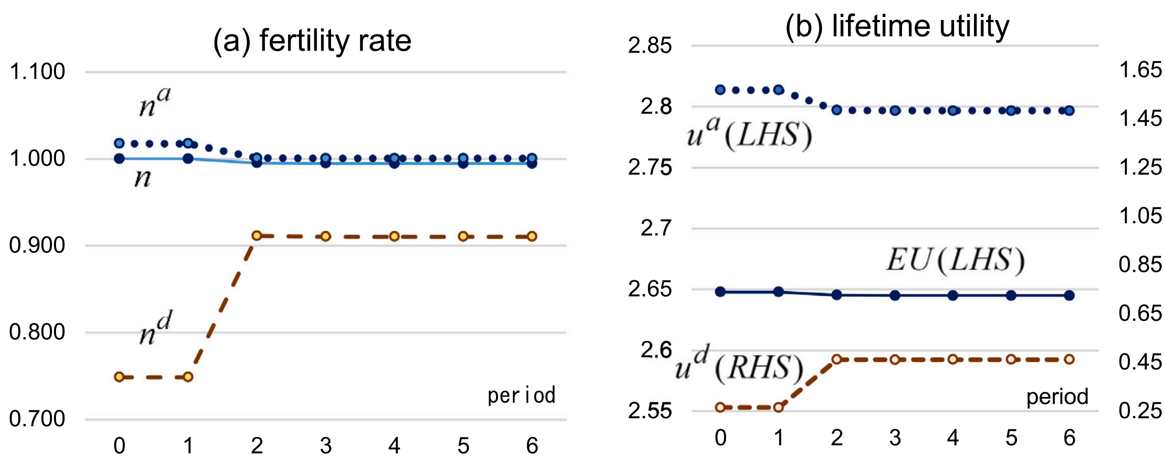

Fig. 3. Fertility rate and lifetime utility of workers with independent and dependent parents.

facilities, but they might use other public care services, e.g., in-home services. Therefore, we assume that $(\bar{h}, h^G) = (1, 0.7)$ for these analyses.

The total costs of three elderly care facilities in the Long-Term Care system in Japan, including long-term care welfare facilities, long-term care health facilities and long-term care medical facilities, were 3353.545 billion yen in 2018 (Ministry of Health, Labour and Welfare in Japan, 2020). In concurrence with values reported by Aya (2014), we assume the ratio of admitted persons care workers as 2.2 and assume the share of labor cost in the total cost as 65%. Based on these assumptions, we can infer the care worker's wage as 3.606 million yen per year. This wage level is approximately equal to the women's average wage of 3.609 million yen per year in 2018 (Ministry of Health, Labour and Welfare in Japan, 2022).[23] The management cost is calculable as $3353.545 \times (1 - 0.65)/[3.606 \times (1329.9/2.2)]$, where the number of the admitted dependent persons in the three elderly care facilities is 1.33 million. From these considerations, we infer that the management cost factor other than labor cost relative to the wage is $M = 0.55$. In this numerical example, we assume that the cost factor remains constant.

It is also difficult to determine labor productivity in long-term care. We assume that those are $(\theta, \mu) = (1.2, 1.3)$, respectively, for the family care time and for the public care time.

The rearing time per child z is set as 0.15, as assumed by de la Croix and Doepke (2003). The parameter reflecting altruism toward children, i.e., the workers' utility weight of having children, is set to keep the total population size constant following de la Croix and Doepke (2003). In our case, we set $\varepsilon = 0.1614$ to make the initial population size equal to one.

The capital share in the goods production is assumed as $\alpha = 0.5$, which is higher than that which is often assumed in the literature. However, to provide a positive interest rate, we assume it. The scale parameter in the goods production is assumed to be $A = 12$.

Given these assumptions, we have the initial steady-state equilibrium $(n, l, k) = (1.000, 0.799, 27.377)$. In the initial equilibrium, we obtain $(y, w, 1 + r, T, l^L) = (56.115, 35.127, 1.025, 1.905, 0.250)$ for given parameters $(\mu, h^G) = (1.3, 0.7)$ and realized income of workers who care for dependent parents is 20.497, whereas that of workers with independent parents is 27.860.[24]

5.1. Increases in public long-term care provision

First, we consider a change in the level of public long-term care from $h^G = 0.7$ in period 1 to $h^G = 0.9$ in period 2, assuming that the minimum long-term care level is constant. Initially the economy is in equilibrium at period 0. The increased long-term care provision lowers the fertility rate. In this case, the management cost factor is large relative to labor productivity in the public long-term care sector, i.e., $\mu - \theta(1 + M) = -0.56 < 0$. The increased public long-term care immediately and considerably decreases the family care provision. The decreased child-rearing time might increase the labor employment per worker in the goods production sector, thereby offsetting the increase in public employment. However, as periods proceed, the lower number of children increases the family care time per young worker. Therefore, the increased goods-production labor employment per worker becomes lower and consequently approaches a new steady-state equilibrium$l = 0.800$ (as shown in Fig. 2(b)), i.e., an overshoot in the goods-production labor. The per-worker capital in goods production consequently decreases to a new steady-state equilibrium $k = 27.330$. In this

[23] This is apparently consistent with a report by Aya's (2014) report (see note 17).

[24] We can demonstrate that the simulation results are not altered qualitatively when parameters θ, M, and π change slightly.

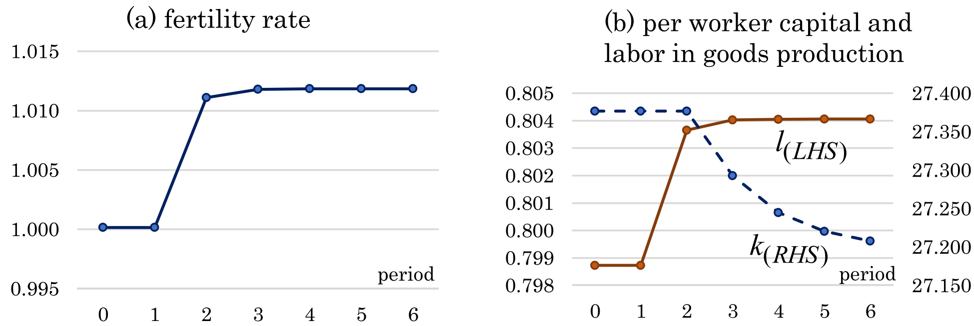

Fig. 4. Effects of an increase in the labor efficiency of the public long-term care.

case, the goods production labor employment per worker swings to the opposite direction before converging to the new long-term equilibrium. The paths are presented in Fig. 2. Assuming a period of 30 years in this paper, we do not present the new steady-state equilibrium in Figs. 2 through 5.[25]

At this stage, we present the lifetime utility values and the fertility rates of workers with independent and with dependent parents. When public long-term care services increase from $h^G = 0.7$ to $h^G = 0.9$, the fertility rate of workers with independent parents decreases from $n^a = 1.018$ to $n^a = 1.001$, whereas the fertility rate of worker with dependent parents increases from $n^d = 0.749$ to $n^d = 0.910$. The increased public long-term care provision narrows the difference of the fertility rates from $n^a - n^d = 0.269$ to $n^a - n^d = 0.090$. The increased tax burden reduces the fertility rate, whereas the increased public long-term care service provision increases the fertility rate, freeing time from family caregiving. Workers with dependent parents are favorably affected by the latter effect, although those with independent parents are negatively affected by the former effect. These are shown in Fig. 3(a), in which the average fertility rate is also written.

The lifetime utility changes are depicted in Fig. 3(b). The lifetime utility of workers with independent parents decreases with increased public long-term care provision, i.e., $u^a = 2.814$ to $u^a = 2.797$, although that of workers with independent parents increases, i.e., $u^d = 0.265$ to $u^d = 0.462$. The difference between the lifetime utility shrinks from $u^a - u^d = 2.548$ to $u^a - u^d = 2.335$: the real income distribution (i.e., the distribution in lifetime utility) becomes more equal. However, the average lifetime utility decreases slightly from $EU = 2.648$ to $EU = 2.645$, where $EU = (1 - \pi)u^a + \pi u^d$. The average lifetime utility can be regarded as utilitarian social welfare. Therefore, the increased long-term care provision lowers the social welfare level when the management cost factor is large relative to labor productivity in the public long-term care sector.

At this point in the discussion, we present a remark: if their parents become dependent without the public long-term care program, then double carers must care for their parents by spending $l^L\big|h^G = 0 = 0.983$, instead of $l^L\big|h^G = 0.7 = 0.250$ with the tax burden of $T = 1.905$ for public long-term care. The tax is smaller than the forgone wage income $wl^L\big|h^G = 0 - wl^L\big|h^G = 0.7 = 25.748$. In this sense, the public long-term care system provides insurance against the parental health risk.

[25] In this new steady-state equilibrium, we obtain $(y, w, 1+r, T, l^L) = (56.115, 35.067, 1.027, 2.459, 0.084)$ and caregiver's after-tax income becomes 24.881.

5.2. Improvement in labor productivity of public long-term care provision

Next, we consider an increase in labor productivity of the public long-term care sector from $\mu = 1.3$ to $\mu = 1.6$.[26] The time paths of the fertility rate and the per-worker capital and the labor employment per worker in the goods production sector are presented in Fig. 3. As Proposition 2 shows, improved labor productivity in the public long-term care sector raises the fertility rate: The new steady-state equilibrium is $n = 1.012$. With those assumed parameters, we have $1 - \varepsilon(1 + M) = 0.750 > 0$, although the management cost factor is not so small.[27] The steady-state labor employment per worker in the goods production sector increases to $l = 0.804$. The per-worker capital decreases to the new steady-state equilibrium value of $k = 27.196$.

The greater number of children reduces the family's per-child long-term care time. The improved labor productivity in the public long-term care sector enables the government to reduce public employment. Therefore, young workers can increase both the child-rearing time and the labor supply to the goods production sector, both in the short term and in the long term. The improvement of labor productivity in the public long-term care sector affects the economy similarly to a windfall. The time paths of variables approach the new steady-state equilibrium values monotonically. Those time paths are portrayed in Fig. 4.[28]

Next, we also show the changes in the lifetime utility and the fertility rate of workers with independent and with dependent parents when the labor productivity of the long-term care sector increases. When the labor productivity of public long-term care services increase from $\gamma = 1.3$ in period 1 to $\gamma = 1.6$ in period 2, the fertility rate of workers with independent parents increases from $n^a = 1.018$ to $n^a = 1.029$, whereas the fertility rate of workers with dependent parents increases from $n^d = 0.749$ to $n^d = 0.763$.[29] The increased labor productivity increases the fertility rate of all workers, thereby freeing time from family caregiving. The difference of the fertility rates changes from $n^a - n^d = 0.269$ to $n^a - n^d = 0.266$. The transition paths are depicted in Fig. 5(a).

The lifetime utility changes are depicted in Fig. 5(b). The lifetime utility of workers with independent parents increases concomitantly with increased labor productivity of the public long-term care sector from $u^a = 2.814$ to $u^a = 2.825$, whereas that of workers with independent parents also increases from $u^d = 0.265$ to $u^d = 0.289$. The improvement in labor productivity in public sector also equalizes the

[26] This change corresponds to an increase in the ratio of dependent persons to caregivers in the facilities from 2.20 to 2.57. Although the actual ratio of dependent persons per care worker is actually about 2 in Japan, the upper limit of the ratio is stipulated as 3 by laws qualifying acknowledged nursing-care institutions.

[27] After the change in labor productivity in the public long-term care sector, we still have $\mu - \theta(1 + M) = -0.26 < 0$.

[28] In the new steady-state equilibrium, we have $(y, w, 1 + r, T, l^L) = (56.115, 34.895, 1.032, 1.520, 0.247)$. The caregiver's after-tax income is 20.758.

[29] The difference in fertility rates between workers with independent parents and with dependent parents shrinks slightly.

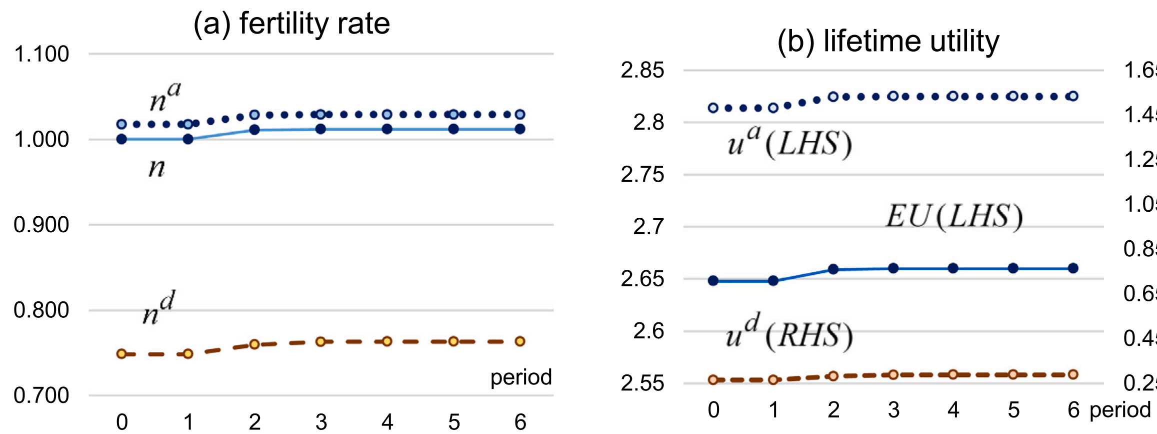

Fig. 5. Fertility rate and lifetime utility of workers with independent and dependent parents.

real income distribution from $u^a - u^d = 2.548$ to $u^a - u^d = 2.536$. With improved labor productivity in public long-term care provision, social welfare increases from $EU = 2.648$ to $EU = 2.660$. The labor-productivity improvement frees long-term care time, for the public and the family, to increase child-rearing time and the market labor supply of workers.

6. Concluding remarks

Analyses of long-term effects of public long-term care provision on fertility and economic growth paths through the time allocation of sandwich caregivers have been presented for an overlapping generations model. The analyses were based on the assumptions that necessary long-term care is provided by family and government when older parents become dependent.

Results demonstrate that increases in public long-term care provision lower the fertility rate if public long-term care production is inefficient relative to family long-term care provision on a unit-cost efficiency basis. When the unit-cost inefficiency of public care sector is severe and the young workers' utility weight on having children is large, then the increased public long-term care provision increases steady-state labor employment per worker in the goods production sector. In this case, the increased public care provision might immediately increase the goods-production labor employment per worker above the new steady-state equilibrium, i.e., overshooting the equilibrium. By contrast, when the public care sector is unit-cost efficient and the workers' utility weight on having children is small, the increased public long-term care provision raises the fertility rate. It increases the goods-production employment per worker. Results obtained from the numerical example indicate the possibility that differences of lifetime utility between young workers with autonomous and with dependent parents shrink, even with increasingly costly public long-term care, although such care lowers the average fertility rate and social welfare.

Improvements in labor productivity in the public long-term care sector raise the steady-state fertility rate. Young workers' time that is freed up from family care provision by the improved labor productivity in the public sector increases both the child-rearing time and the market labor supply. Nevertheless, the effects of improved long-term care labor productivity on the labor employment per worker in the goods production sector are ambiguous. When the workers' utility weight of having children and the management cost factor of the public care sector are small, then the improved labor productivity of public care provision increases the labor employment and decreases capital per worker in the goods production sector. The results obtained when using the numerical example indicate that labor productivity improvement of public long-term care provision can narrow differences in lifetime utility between workers with autonomous and with dependent parents, thereby improving social welfare.

The realistic relevance of the prediction of this paper is the following: The 2006 reform of the Japan's long-term care insurance program introduced a lower-cost preventive care system, which decreased spending for every person aged 75 years or older from 550,000 yen in 2004 to 500,000 yen in 2006; it then reached a plateau for a short while (Tamiya et al., 2011). The improved cost-efficiency might contribute to the total fertility rebound in Japan.[30] The total fertility rate rebound was brought about by increased fertility rates of women aged 30–49: potential sandwich caregivers.[31] This fact is apparently consistent with the results of our analyses.

The simple model presented herein has some deficiencies. First, we have not considered any tradeoff or interaction with childrearing services outside of the family. The availability of these outside-of-the-home childcare services likely increases the fertility rate by freeing parents' time from childrearing at home (e.g., Yakita, 2018). Second, pay-as-you-go pensions might impose interactive effects with long-term care policy (Yasuoka, 2018). Third, we have assumed that leisure is not an individual choice. If the time which is freed up by public long-term provision is consumed for leisure, then public long-term care might neither raise fertility nor increase market labor. Finally, and more importantly, we have not assumed exchanges between generations. Many reports have described the possibility of intergenerational exchange motives and family bargaining in addition to intergenerational altruism (e.g., Horioka et al., 2018; Barigozzi et al., 2020). Inclusion of these factors remains as a subject for future research.

Statements and declaration

The author declares that he has no competing financial interest or personal relationship that might influence the work reported in this paper.

Acknowledgments

The author thanks two anonymous referees and Shin-ichi Fukuda, the Editor, for their valuable comments and suggestions. An earlier version of this paper was read at the 2021 Spring Meeting of the Japanese Association for Applied Economics. The author also thanks Masaya Yasuoka and the meeting participants for their comments and suggestions. Financial support from the Japan Society for the Promotion of Science (KAKENHI Grant Nos. 19H01503 and 22K01544) is gratefully

[30] Although the dependent elderly population grew along with population aging, the reform apparently worked to prevent a further decline of fertility.

[31] The increase more than offset the declining fertility rates of women under age 29.

acknowledged.

Appendix A

A1. Derivation of (15)

From the labor market clearing condition, we have $l = 1 - zn - \pi l^L - l^G$. From (5) and (7) we produce $zn = \varepsilon[1 - \pi l^L - (1+M)l^G]$. Therefore, it follows that

$$l = 1 - zn - \pi l^L - l^G = (1-\varepsilon)\left[1 - \pi l^L - (1+M)l^G\right] + Ml^G. \tag{A1}$$

Using the formal and informal long-term care production functions and $\overline{h} = h^L + h^G$, we obtain

$$\pi l^L + \left(1+M\right)l^G = \frac{\pi}{n_{-1}}\left[\frac{\overline{h}}{\theta} - \frac{\mu - \theta(1+M)}{\mu\theta}h^G\right] \tag{A2}$$

Herein, n_{-1} denotes the fertility rate in the previous period. Therefore, by inserting (A2) into (A1), we obtain (15).

A2. Fertility dynamics

From (16) we have a quadratic equation of the fertility rate as

$$(z/\varepsilon)n^2 - n + \pi Q = 0, \tag{A3}$$

where $Q = \frac{\overline{h}}{\theta} - \frac{[\mu-\theta(1+M)]h^G}{\mu\theta} = \frac{\overline{h}-h^G}{\theta} + (1+M)\frac{h^G}{\mu} > 0$ because $\overline{h} \geq h^G$. From (A3), we obtain two steady-state fertility rates as

$$n = \frac{1 \pm [1 - 4\pi(z/\varepsilon)Q]^{1/2}}{2(z/\varepsilon)} \tag{A4}$$

Therefore, if the discriminant condition $1 - 4\pi(z/\varepsilon)Q > 0$ is satisfied, then the two roots are real: n_- and $n*$ $(0 < n_-, n*)$.

Regarding the stability of fertility rate $n*$, by inserting $n*$ from (A4) into the stability condition, we obtain

$$\begin{aligned} 1 - \frac{\varepsilon\pi Q}{zn^2} &= 1 - \frac{4\pi Q(z/\varepsilon)}{1 + 2[1 - 4\pi Q(z/\varepsilon)]^{1/2} + \left[1 - 4\pi Q\left(z/\varepsilon\right)\right]} \\ &= \frac{2\left\{[1 - 4\pi Q(z/\varepsilon)]^{1/2} + \left[1 - 4\pi Q\left(z/\varepsilon\right)\right]\right\}}{1 + 2[1 - 4\pi Q(z/\varepsilon)]^{1/2} + \left[1 - 4\pi Q\left(z/\varepsilon\right)\right]} > 0 \end{aligned} \tag{A5}$$

Therefore, the stability condition is satisfied at the stable long-term equilibrium $n*$. The stability and time path of fertility dynamics can be considered graphically as shown in Fig. 1.

A3. Sign of (20)

The coefficient of dn/dh^G can be rewritten as

$$\begin{aligned} &\left(1-\varepsilon\right)\left(\frac{\overline{h}}{\theta} - \frac{\left[\mu - \theta\left(1+M\right)\right]h^G}{\mu\theta}\right) - \frac{Mh^G}{\mu} \\ &= \frac{\overline{h} - h^G}{\theta} + \frac{h^G}{\mu} - \varepsilon\left(\frac{\overline{h}}{\theta} - \frac{\left[\mu - \theta\left(1+M\right)\right]h^G}{\mu\theta}\right) \end{aligned} \tag{A6}$$

If the motion of the fertility rate is given as presented in Fig. 1, then the coefficient of ε on the right-hand side is positive. Therefore, if ε is sufficiently small (large), then the coefficient is positive (negative).

The second term on the right-hand side of (20) is

$$\left(1-\varepsilon\right)\frac{\mu - \theta(1+M)}{\mu\theta} + \frac{M}{\mu} = \frac{\mu - \theta}{\mu\theta} - \varepsilon\frac{\mu - \theta(1+M)}{\mu\theta}, \tag{A7}$$

which is positive if $\mu - \theta > 0$, irrespective of the sign of $\mu - \theta(1+M)$.

Therefore, summing up the results, one obtains the following:

	when $\mu - \theta(1+M) \geq 0$	when $\mu - \theta(1+M) < 0$
for sufficiently small ε	$\frac{dn}{dh^G} \geq 0$ & $\frac{dl}{dh^G} > 0$	$\frac{dn}{dh^G} < 0$ & $\frac{dl}{dh^G} \gtreqless 0$
for sufficiently large ε	$\frac{dn}{dh^G} \geq 0$ & $\frac{dl}{dh^G} \gtreqless 0$	$\frac{dn}{dh^G} < 0$ & $\frac{dl}{dh^G} > 0$

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