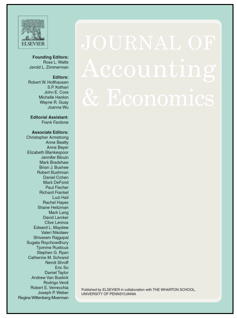
Journal Pre-proof



Strategic complexity in disclosure

Cyrus Aghamolla, Kevin Smith

PII: S0165-4101(23)00059-9

DOI: https://doi.org/10.1016/j.jacceco.2023.101635

Reference: JAE 101635

To appear in: *Journal of Accounting and Economics*

Received Date: 5 July 2022

Revised Date: 12 May 2023

Accepted Date: 2 August 2023

Please cite this article as: Aghamolla, C., Smith, K., Strategic complexity in disclosure, *Journal of Accounting and Economics*, https://doi.org/10.1016/j.jacceco.2023.101635.

Strategic complexity in disclosure*

Cyrus Aghamolla† Kevin Smith‡

May 12, 2023

Abstract

Extensive evidence suggests that managers strategically choose the complexity of their descriptive disclosures. However, their motives in doing so appear mixed, as complex disclosures are used to obfuscate in some cases and to provide information in others. Building on these observations, we first identify a novel stylized fact: disclosure complexity is *non-monotonic* in firm performance. We develop a model of disclosure complexity that incorporates the dual roles of complexity and can explain this stylized fact. In the model, a manager discloses to investors of heterogeneous sophistication and can adjust the complexity of the disclosure to either provide more precise information or to obfuscate. When the firm's investor base is largely unsophisticated, the manager issues a complex disclosure only upon observing negative news. In contrast, when the firm's investor base is more sophisticated, the manager issues a complex disclosure upon observing either highly positive or negative news. As a result, the market may react more positively to complex information releases than to simple releases, complex disclosures generate heightened return volatility, and firms with more inherently complex information are more likely to use their discretion to simplify their disclosures.

Keywords: Financial reporting complexity, disclosure, complexity, obfuscation, disclosure informativeness, sophisticated investors, MD&A.
JEL classification: C72, D82, D83, G14, M41.

*We thank Snehal Banerjee, Jeremy Bertomeu (discussant), Jungho Choi, John Core (the editor), Carlos Corona (discussant), Ed deHaan, Brandon Gipper, Ilan Guttman, John Kepler, Becky Lester, Iván Marinovic, Maureen McNichols, Beatrice Michaeli, Suzie Noh, Evgeny Petrov (discussant), Delphine Samuels, and Richard Thakor, as well as participants at the University of Zurich, 2021 Stanford Accounting Summer Camp, 2022 Journal of Accounting and Economics Conference, Hong Kong University of Science and Technology, and the Asian Bureau of Finance and Economic Research 10th Annual Conference for helpful comments and suggestions. We also thank Minjae Kim for exceptional research assistance.

†University of Minnesota. E-mail: caghamol@umn.edu.
‡Stanford GSB. E-mail: kevinsm@stanford.edu.

Strategic complexity in disclosure*

Cyrus Aghamolla† Kevin Smith‡

August 14, 2023

Abstract

Extensive evidence suggests that managers strategically choose the complexity of their descriptive disclosures. However, their motives in doing so appear mixed, as complex disclosures are used to obfuscate in some cases and to provide information in others. Building on these observations, we first identify a novel stylized fact: disclosure complexity is *non-monotonic* in firm performance. We develop a model of disclosure complexity that incorporates the dual roles of complexity and can explain this stylized fact. In the model, a manager discloses to investors of heterogeneous sophistication and can adjust the complexity of the disclosure to either provide more precise information or to obfuscate. When the firm's investor base is largely unsophisticated, the manager issues a complex disclosure only upon observing negative news. In contrast, when the firm's investor base is more sophisticated, the manager issues a complex disclosure upon observing either highly positive or negative news. As a result, the market may react more positively to complex information releases than to simple releases, complex disclosures generate heightened return volatility, and firms with more inherently complex information are more likely to use their discretion to simplify their disclosures.

Keywords: Financial reporting complexity, disclosure, complexity, obfuscation, disclosure informativeness, sophisticated investors, MD&A.
JEL classification: C72, D82, D83, G14, M41.

*We thank Snehal Banerjee, Jeremy Bertomeu (discussant), Jungho Choi, John Core (the editor), Carlos Corona (discussant), Ed deHaan, Brandon Gipper, Ilan Guttman, John Kepler, Becky Lester, Iván Marinovic, Maureen McNichols, Beatrice Michaeli, Suzie Noh, Evgeny Petrov (discussant), Delphine Samuels, and Richard Thakor, as well as participants at the University of Zurich, 2021 Stanford Accounting Summer Camp, 2022 Journal of Accounting and Economics Conference, Hong Kong University of Science and Technology, and the Asian Bureau of Finance and Economic Research 10th Annual Conference for helpful comments and suggestions. We also thank Minjae Kim for exceptional research assistance.
†University of Minnesota. E-mail: caghamol@umn.edu.
‡Stanford GSB. E-mail: kevinsm@stanford.edu.

"For more than forty years, I've studied the documents that public companies file. Too often, I've been unable to decipher just what is being said or, worse yet, had to conclude that nothing was being said. [...] Maybe we simply don't have the technical knowledge to grasp what the writer wishes to convey. Or perhaps the writer doesn't understand what he or she is talking about. In some cases, moreover, I suspect that a less-than scrupulous issuer doesn't want us to understand a subject it feels legally obligated to touch upon."

—Warren Buffett, 1998.[1]

1 Introduction

Corporate disclosures often include descriptive information regarding firm performance. For example, in the MD&A, press releases, and conference calls, firms may explain losses and discuss long-term strategy, products in development, or changes in contractual terms. Managers generally have considerable latitude in how they convey this information to the capital market, influencing not just its breadth and precision, but also its complexity, i.e., how accessible the information is to investors. Managerial discretion over the complexity of narrative disclosures—which accounts for roughly 80% of the content of annual reports—has recently been of significant interest in the empirical literature. In some cases, managers add complexity to disclosures in order to convey more detailed and precise information (e.g., Loughran and McDonald (2014), Lang and Stice-Lawrence (2015), Guay et al. (2016), Bushee et al. (2018), Chychyla et al. (2019), Cohen et al. (2020)), while in others, managers do so to reduce investors' ability to understand the content of disclosures (e.g., Li (2008), Ertugrul et al. (2017), Lo et al. (2017), Kim et al. (2018), deHaan et al. (2020)).[2]

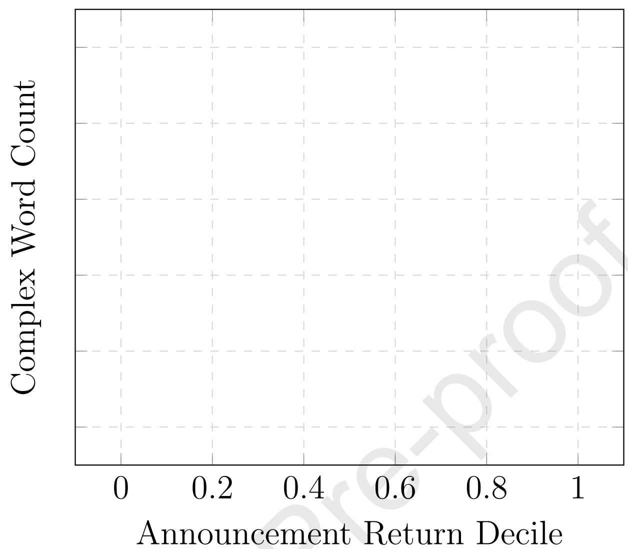
Managers' use of disclosure complexity in these distinct ways suggests that classical disclosure theory likely will fail to explain observed patterns in reporting complexity. This theory considers situations in which managers choose between two types of disclosure, one of which is always more informative to *all* investors than the other, and predicts that managers with better news will choose the more informative disclosure (e.g., Dye (1985)). Indeed, in Figure 1, we show that this intuitive prediction cannot readily explain managers' choices of

[1]The excerpt is from the preface of Securities and Exchange Commission (1998).

[2]Relatedly, in a large-scale survey of public company executives, Graham et al. (2005) document that "some CFOs admit that they do not mind 'fuzziness' in bad news disclosures" (p. 65). Likewise, Solomon (2012), Cohen et al. (2013), and Dzielinski et al. (2016) find that managers "spin" or obscure bad news releases.

Figure 1: Disclosure Complexity and Firm Performance.

We examine the three-day cumulative abnormal return around firms' 10-Q filing dates. We separate firm-quarters from 1994Q1 to 2019Q3 into deciles based on these returns, from low to high. Next, for each firm-quarter, we obtain the residual from regressing complex word count onto size, market-to-book, leverage, historical return volatility, and industry and year fixed effects. We then calculate the average residual for each bin using the respective measure. Appendix B provides further detail regarding the construction of this figure.



disclosure complexity. This figure identifies a new stylized fact, showing that managers who possess both highly positive and negative news, as proxied by contemporaneous returns, issue quarterly reports that are more complex than those of firms with intermediate news.[3] This pattern demonstrates the need for a novel theory to explain managers' choice of disclosure complexity.[4]

In this paper, we develop a model in which a manager chooses the complexity of a disclosure, where complexity can be used as a means either to convey more precise information or to reduce investor understanding. In addition to addressing why managers with extreme news may be more likely to issue complex disclosures, our model aims to shed light on the

[3]As sophisticated investors are generally able to process the manager's disclosure despite its complexity, contemporaneous abnormal returns around the 10-Q filing date should partially reflect the manager's information. This pattern holds if we instead use returns of the following quarter and also holds for other commonly used complexity measures, such as the fog index, average words per paragraph, the rix index, and the smog index.

[4]This demand has also been espoused by a recent review of the empirical complexity literature, Blankespoor et al. (2020). In particular, Blankespoor et al. (2020) note that "the literature has spent little time modeling the effects of disclosure processing costs on managers' disclosure decisions or other corporate actions. [...] More broadly, the effects of processing costs on disclosure decisions is an area where empirical research (reviewed below) has moved substantially beyond the existing analytical literature" (p. 11, 34), and that such models "can help researchers develop more well-grounded and complete empirical predictions, including predictions that incorporate non-monotonic relations and competing influences of multiple processing costs" (p. 10). Our study helps to fill this gap in the theoretical literature.

following questions: What are the expected market reactions to simple versus complex disclosure? How does discretionary disclosure complexity depend upon the sophistication of a firm's investor base and how does it vary across industries?

In our setting, a firm manager is obligated to disclose a piece of news to the market, but she can adjust both the informativeness of this disclosure and whether it is simple or complex. The manager seeks to maximize the short-term market price of the firm. The market consists of sophisticated and unsophisticated investors. Both classes of investors understand simple information, but only sophisticated investors understand complex information. The manager is subject to a natural constraint: in order to raise the disclosure's informativeness, she must increase its complexity. This captures the need to provide technical details or to use complex language in order to convey additional information. The manager can also raise the disclosure's complexity without increasing its informativeness. This reflects her ability to "obfuscate" the disclosure by adding irrelevant details or pseudo-signals, or by using unnecessarily technical language. In sum, the manager can choose among three types of disclosure: (*i*) simple disclosure, (*ii*) "complex informative" disclosure, which is complex and more informative than simple disclosure, and (*ii*) "complex uninformative disclosure" disclosure, which is complex and relatively uninformative.

Importantly, all investors in our model observe whether the firm's disclosure is simple or complex. This feature captures investors' ability to readily observe the length of and diction used in a disclosure, even if they do not fully understand its implications for firm value. Thus, in equilibrium, the manager's choice of disclosure complexity serves as a signal of the manager's information to investors. For instance, investors might infer that a manager using complex language possesses positive news and is seeking to communicate this news precisely. Alternatively, investors might infer that the manager possesses negative news and is seeking to obfuscate this news.

As is common in disclosure models, to prevent an unravelling equilibrium, we assume that with some probability the manager is constrained in her disclosure choice. This can reflect, for example, the presence of a board-level committee that determines the firm's disclosure policies or constraints embedded in the manager's contract. In a similar vein, the manager may be compelled to adhere to regulations that, in conjunction with the firm's (unobservable) transactions, require them to disclose in a particular way (e.g., Lang and Stice-Lawrence (2015), Guay et al. (2016), Aghamolla et al. (2021)). Conversely, discretion over the complexity choice can be attributed to a lack of board control over disclosure or the absence of disclosure constraints in the manager's contract.

As a concrete example of a scenario captured by these assumptions, consider a manager who has qualitative information regarding the potential success of a new technology in development and is compelled to discuss the technology due to market pressure or litigation concerns. The manager can attempt to convey her information to the market in a simple way by providing high-level projections without accompanying detail to support the assertions. By omitting potentially complicated information concerning the specifics of the technology, investors are left with a limited understanding of the project's impact on future performance. Alternatively, the manager can provide a more technical and complete description of the technology's promise. While this enables industry experts to fully assess the manager's news, it renders the disclosure uninterpretable to other investors. Finally, the manager can instead provide excessive technical detail that is largely irrelevant to the situation at hand. While more adept investors can parse through such detail and recognize its insignificance, unsophisticated investors would be unable, or find it too costly, to do so.

As illustrated by this example, complex informative disclosure has two offsetting effects relative to simple disclosure: sophisticated investors find such disclosure more informative, but unsophisticated investors find it entirely uninformative. We separate our analysis based upon whether complex informative or simple disclosure is more informative to the *average* investor. When the firm's investor base is relatively sophisticated or its information is complex in nature, complex informative disclosure is the most informative to the average investor. We begin by focusing on this case and show that any equilibrium takes the form of a *strategic complexity equilibrium,* whereby the manager chooses complex informative disclosure when she observes sufficiently positive news, simple disclosure when she observes intermediate news, and complex uninformative disclosure when she observes sufficiently negative news. That is, consistent with Figure 1, managers with more extreme information issue more complex disclosures.

The manager's disclosure choice in such an equilibrium reflects a trade-off between the informativeness of her disclosure to the average investor and investors' inferences from her disclosure choice. When the manager has extremely positive or negative news, her decision is transparent: in this case, she primarily aims to maximize or minimize the market's reaction to this news, respectively. Thus, the manager chooses the most and least informative disclosures, i.e., complex informative and complex uninformative disclosure, respectively. The reason that the manager chooses simple disclosure when she has intermediate news is more subtle. In fact, we show that there can exist two types of strategic complexity equilibria that are distinguished by the range of signals that prompt the manager to choose simple disclosure and her incentives for doing so.

In the first type of equilibrium, which always exists, the manager provides a simple disclosure when she has moderately *negative* news. Intuitively, upon observing such news, the manager aims to temper the reaction to her disclosure. This inclines her towards issuing a simple disclosure or obfuscating. While obfuscating would lead to a weaker response to the disclosure, in equilibrium, investors know that the manager obfuscates whenever she has very negative news. Moreover, sophisticated investors would recognize that the disclosure was uninformative and discount the firm. Therefore, the manager instead prefers to issue a simple disclosure. Surprisingly, this implies that the average price reaction to simple news is negative, while the average response to complex news is positive. Furthermore, in this equilibrium, complex disclosure is more likely to reflect information provision, as opposed to obfuscation. These findings are at odds with the conventional wisdom that bad news is more often presented in a complex manner.

A second equilibrium also exists when simple and complex informative disclosure provide a similar amount of information to the average investor. In this equilibrium, the manager provides a simple disclosure when she has, on average, moderately *positive* news. In this case, unsophisticated investors draw a negative inference upon observing a complex disclosure, causing the firm's price to decline on average following complex news. Because simple and complex informative disclosure provide roughly the same amount of information, when the manager observes intermediate news, she is primarily concerned with avoiding this negative inference. This leads the manager to select simple disclosure.

We next perform numerical analyses that reveal that both types of strategic complexity equilibria exhibit common features. First, when a firm's investor base is less sophisticated or the information loss from conveying information in a simple manner is small, the firm is *more* likely to issue complex disclosure. Intuitively, in any equilibrium, the manager is on the margin between simple and complex uninformative disclosure when she possesses moderately negative news. Thus, when simple information becomes relatively more informative, the manager is more inclined to obfuscate to reduce the reaction to her news.

Next, even when complex disclosures are typically uninformative, they generate more price volatility than simple disclosure. The reason is that, in equilibrium, the manager issues complex disclosure when she has either highly positive or negative news, which merits a large price reaction. Finally, we show that despite offering little information, complex uninformative disclosure can generate more disagreement among investors than complex informative disclosure. This results from the fact that only sophisticated investors recognize that such disclosure is likely to have been obfuscated by a manager with discretion, which offers them an additional information advantage over unsophisticated investors.

We conclude by considering the case in which simple disclosure is more informative to the average investor than complex informative disclosure. This occurs when either the firm's investor base is highly unsophisticated or when the firm's information is inherently simple, so that presenting the information in a simple manner entails little information loss. In this case, we find that an intuitive equilibrium largely consistent with Li et al. (2010)'s "obfuscation hypothesis" arises. Firms with positive information issue simple discloses, which maximizes the price reaction to their information by facilitating investor understanding. Firms with mildly negative news may issue a complex informative disclosure, while firms with very bad news obfuscate. As a result, the average price reaction to complex disclosure is negative.

1.1 Related Literature

Our study relates to the stream of literature that considers disclosure and complexity. Carlin (2009) examines strategic price complexity in a model where multiple firms independently choose the difficulty for consumers to understand their price of a homogeneous financial product. The composition of expert consumers (analogous to sophisticated investors in the current setting) is a decreasing function of the aggregate difficulty in understanding prices within the industry. Firms follow a mixed strategy in equilibrium over prices and difficulty, which generates price dispersion for the identical product. Among other differences, our study varies as we allow complexity to increase informativeness of the disclosure for sophisticated investors, and we allow the firm to have private information when making the complexity decision.

Similar to Carlin (2009), obfuscation in prices has also been investigated by Carlin and Manso (2011), Ellison and Wolitzky (2012), and Gu and Wenzel (2014).[5] These studies generally consider firm incentives to obfuscate prices within a consumer search framework. Our setting adds to this literature as we consider firm incentives when complexity may be information-increasing, in light of the potential for mimicry through obfuscation. A few papers consider the connection between the precision of information and disclosure choices. Langberg and Sivaramakrishnan (2008) consider a manager's voluntary disclosure decision in the presence of an analyst who can potentially learn and reveal the precision of the disclosed information. The equilibrium is one where the manager has a greater tolerance for imprecision when disclosing good news relative to bad news. Hughes and Pae (2004) and Lee (2019) investigate voluntary disclosure of the precision of a public signal when there is uncertainty as to the manager's endowment of such information, and Penno (1996) analyzes precision choice of subsequent mandatory disclosure following the release of a public signal. Titman and Trueman (1986) consider a model of IPOs where going-public firms can provide more precise information at a cost by using a high-quality auditor. Our model varies

[5] Carlin et al. (2013) present experimental evidence of the effect of complexity on asset trading behavior. They find that participants were significantly less likely to engage in trade in the complex treatment, suggesting that adverse selection concerns are amplified if the agent believes they face a more informed counterparty.

from these studies as we examine the trade-off between informativeness and accessibility of disclosure in a setting with heterogeneous investors.

In a Bayesian persuasion framework, Michaeli (2017) examines a setting where a manager with misaligned preferences can choose both the ex ante precision of disclosure and the fraction of investors who observe this signal. Michaeli (2017) finds that the manager makes an informative signal observable to only a subset of investors. In contrast, in our model we assume that the manager makes the disclosure decision when she has private information regarding the firm value and that, while all investors observe the disclosure, investors are heterogeneous in their ability to process the information.

Myatt and Wallace (2012), Chen et al. (2017), Avdis and Banerjee (2019), and Liang and Zhang (2019) consider models in which certain disclosures are exogenously "clearer" or "more objective" than others, in that agents' posterior means given such disclosures are more highly correlated. In these studies, the signals investors derive from the disclosure are of identical quality independent of the clarity of the disclosure. The notion of simplicity versus complexity we consider is related but distinct: simpler signals in our setting lead investors to receive signals of more homogeneous quality. This not only implies that their posterior means are more highly correlated, but also that their expected posterior variances are more similar.

As unsophisticated investors in our setting have uncertainty regarding the quality of complex disclosures, or model relates to studies that examine disclosure with uncertainty over precision, such as Subramanyam (1996), Kirschenheiter and Melumad (2002), and Beyer (2009). Our paper also relates to studies that entail signaling in disclosure, such as Teoh and Hwang (1991), Beyer and Dye (2012), and Aghamolla et al. (2021). In our setting, complexity is a decision by the manager after she has observed private information, and thus the choice of complexity itself conveys information.[6] Chen et al. (2020) examine the interaction of manipulation and disclosure accessibility; investors can exert costly effort to uncover manipulation if supplementary disclosure is made accessible. They show a separating equilibrium where only bad firms manipulate and make their disclosures inaccessible, but this equilibrium is sensitive to the degree of information asymmetry. Our model differs as we allow complexity to increase informativeness for one group of investors and we do not consider manipulation.

Our paper is also related to the literature that incorporates heterogeneous investors in disclosure. Dye (1998) extends the Dye (1985) framework to allow some investors to observe if the manager has received information. Another class of models examine disclosure incentives

[6]Relatedly, Bertomeu and Cheynel (2013) present a theory of standard setting and find that higher quality standards chosen under competition carry a positive signaling value.

when some investors may be better informed than others, such as Fishman and Hagerty (2003), Bertomeu et al. (2011), Kumar et al. (2016), Einhorn (2018), Petrov (2020), and Banerjee et al. (2022). The current setting incorporates a similar feature, as sophisticated investors are better able to interpret complex information, thus being more informed for certain disclosure choices. In contrast to these models, however, we allow discretion over the quality of disclosure, which permits the manager to affect the degree of heterogeneity among investors.

2 Model

We consider a firm whose manager receives a private signal regarding the firm's value that she must disclose to the market. We let $\tilde{y}$ denote the expected firm value given this signal, and, moving forward, refer to $\tilde{y}$ as the manager's private information. The manager faces a market composed of a continuum of investors. Investors are heterogeneous in the sense that a fraction $\chi \in [0,1]$ are sophisticated, while the remaining portion $1-\chi$ are unsophisticated. We denote the density function of $\tilde{y}$ as $f(\cdot)$, its distribution function as $F(\cdot)$, and its mean by $\mu \equiv \mathbb{E}(\tilde{y})$. We assume $\tilde{y}$ has support on $[y_L, y_H]$, where y_L and y_H can be arbitrary real numbers, or can be $-\infty$ or ∞, respectively.

We seek to capture the phenomenon that managers often have latitude to present their information in a complex or simple manner; however, to communicate information precisely, managers must increase its complexity. For example, technology firms may possess obscure details on their product development. Moreover, managers can observe and disclose performance metrics whose value is ambiguous to those not familiar with their industry. Managers may also discuss the legal details of their contracts with large customers or their derivative hedging practices. At the same time, we wish to capture the potential for managers to artificially add complexity to their disclosures without conveying additional details in order to obfuscate their information. To capture these possibilities parsimoniously and to allow for tractable analysis, we introduce the following three disclosure choices:

- *Simple disclosure.* The manager can choose to disclose the information through a simple or uncomplicated disclosure. In this case, sophisticated and unsophisticated investors both observe a signal Δ_S, which takes the following form. With probability $\rho_S \in (0,1)$, the signal Δ_S reveals the manager's private information y and otherwise provides no information. This feature captures the notion that, due to its simplicity, information is lost to the capital market. For example, a disclosure with insufficient details prevents investors from making an informative judgment on the future impact of the signal.[7] We assume that all investors are aware of the *type* of disclosure; that is, the fact that the disclosure is "simple" is common knowledge.[8]

- *Complex informative disclosure.* The manager can alternatively choose to provide sufficient detail such that the implications of the disclosed information can be adequately understood. However, the additional complexity in disclosure prevents unsophisticated investors from understanding the information. Formally, when the manager chooses complex informative disclosure, sophisticated investors observe a signal Δ_C that reveals the manager's private information with probability one. On the other hand, unsophisticated investors do not observe Δ_C. Hence, the information is fully revealed to a fraction χ of investors who are sufficiently sophisticated to parse the disclosure. In contrast to simple disclosure, all investors only observe that the information communicated is "complex" and are unable to distinguish it with the other kind of complex disclosure discussed next.

- *Complex uninformative disclosure.* The manager can instead choose to complexify the information release without making it more informative. When the manager strategically chooses this form of disclosure, we can think of it as being "obfuscated" with additional irrelevant details and explanations with the purpose of clouding the information. Sophisticated investors observe a signal Δ_U, which reveals the manager's private information with probability $\rho_U \in [0, 1)$, and otherwise provides no information. Unsophisticated investors are again unable to interpret the disclosure due to the complexity. Moreover, as above, all investors understand that the disclosure is "complex," but cannot disentangle between informative and uninformative complexity. As we see later, the lack of distinction between the two kinds of complex disclosure becomes salient only for unsophisticated investors.[9] A natural special case is $\rho_U = 0$, in which case complex uninformative disclosure is equivalent to uninformative "babbling."

This structure allows us to capture variation in the amount and quality of information communicated with complex versus simple disclosure in a parsimonious manner. The ad-

[7] We may expect a low ρ_S, for instance, in technical or high-growth industries where information is naturally complex, rendering simple disclosure to be ineffective for conveying the nuance and potential of the technology on the firm's future performance. In contrast, less technical and more stable industries can more easily convey information simply without significant information loss, implying a higher ρ_S.

[8] Support for the two types of investors has been documented by Kielty et al. (2022), who find that simplifying complex mortgage disclosures allowed unsophisticated (inexperienced) borrowers to better understand the disclosures. In contrast, sophisticated (experienced) borrowers were able to understand complex disclosures prior to their simplification.

[9] As sophisticated investors can always understand complex disclosure, they can tell whether it is informative.

vantage of this informational structure is that it avoids distributional features that make the analysis intractable, and allows us to cleanly demonstrate the economic insights that arise from the model. A simplification embedded in this setting is that information can be lost to the capital market (e.g., Dewatripont and Tirole (2005), Guttman and Marinovic (2018)).[10] While disclosures are generally not completely uninformative, we interpret this as a metaphor for noise or information loss in disclosure. For example, an unnecessarily complex and garbled disclosure can hinder the market's ability to fully understand the information conveyed.[11]

As complex uninformative disclosures are meant to be the least informative type of disclosure, we incorporate the following assumption throughout the analysis:

Assumption 1. *The parameters ρ_S, ρ_U, and χ satisfy:*

$$\rho_S > \chi \cdot \rho_U.$$

This condition states that simple disclosure is more informative than complex uninformative disclosure to the average investor. To see why, note that a fraction χ of investors can understand complex uninformative disclosure and this disclosure is informative with probability ρ_U. Thus, the likelihood that a randomly-selected investor observes y given a complex uninformative disclosure is $\chi \cdot \rho_U$. Likewise, since all investors can understand simple disclosure but it is only successfully communicated with probability ρ_S, the likelihood that a randomly-selected investor observes y given a simple disclosure is ρ_S. A natural, sufficient condition for this assumption to hold is that complex uninformative disclosure is no more informative than simple disclosure, i.e., $\rho_S \geq \rho_U$.

In our main analysis, we further impose the following parameter restriction.

Assumption 2. *The parameters ρ_S and χ satisfy:*

$$\chi > \rho_S.$$

This condition states that the manager is able to communicate more information to the average investor by choosing complex informative disclosure than by choosing simple disclosure. As mentioned above, the likelihood a randomly-selected investor observes y given simple disclosure is ρ_S. Since complex informative disclosure is always informative but understood by only a fraction χ of investors, the likelihood that a randomly-selected investor observes y

[10] This information structure also resembles models of probabilistic investor learning such as Goldstein et al. (2020) and Banerjee and Breon-Drish (2021).

[11] A similar notion is captured in the precision disclosure model of Hughes and Pae (2004), where the manager can disclose or withhold the precision level of an information release.

given complex informative disclosure is χ. We focus our analysis on this case given our interest in the potential for complexity to enable managers to communicate more information to the market. In Section 5.2, we return to consider the alternative case in which $\rho_S > \chi$, consistent with a setting in which a firm's investor base is unsophisticated.

To prevent an unravelling equilibrium, we assume that, with probability $\beta \in (0, 1)$, the manager does not have discretion over the disclosure choice. Conditional on the manager having no discretion, the probabilities that she is constrained to issuing a simple, complex uninformative, and complex informative disclosure are $\omega_S \in (0, 1)$, $\omega_U \in (0, 1)$, and $\omega_C = 1 - \omega_S - \omega_U \in [0, 1)$ respectively.[12] Moreover, investors cannot observe whether the manager has discretion.

Our results do not rely on the presence of non-discretionary managers that issue complex informative disclosures (i.e., $\omega_C > 0$), and we allow for this type only for completeness. However, if ω_S and ω_U were equal to zero, the equilibrium would "unravel" to one in which all managers issue complex informative disclosure. The reason is that, given Assumptions 1 and 2, this is the most informative disclosure type, and so managers with better news would be more inclined to issue such disclosure. Thus, the market would hold pessimistic beliefs given less informative disclosures, prompting managers with more mild news to also shift to complex informative disclosure.[13]

Following similar logic, the equilibrium would also unravel if the market could perfectly distinguish between managers with and without discretion. Uncertainty over managerial discretion allows managers with discretion to issue a relatively uninformative disclosure (simple or complex uninformative) when they have bad news, and "pool" with managers without discretion. Although we do not do so, we could allow the likelihood of each non-discretionary type, ω_C, ω_S, and ω_U, to depend on the realized signal y without qualitatively affecting the results.

The manager's potential lack of discretion can reflect, for example, constraints set on the manager through her contract with the firm or through the presence of a board-level

[12]The presence of no-discretion types is similar to that of Dye (1985) and Jung and Kwon (1988). While in the Dye (1985) framework, a manager without discretion is constrained to non-disclosure, the analogous assumption in our setting is that the manager does not have discretion over the complexity and informativeness of the disclosure. Furthermore, the disclosure models of Acharya et al. (2011) and Beyer and Dye (2012) similarly extend the Dye (1985) framework to multiple disclosure types, and Aghamolla et al. (2021) relatedly consider disclosure types without discretion. This assumption also implies that off-equilibrium-path beliefs do not arise in our setting, which allows for a cleaner characterization of the results.

[13]A proof of this result is available upon request. Note that only *partial* unravelling occurs if either ω_S or ω_U is equal to zero. For example, if $\omega_S = 0$ and $\omega_U > 0$, then unravelling occurs only with respect to simple disclosure—that is, no manager would choose simple disclosure in equilibrium, but managers with discretion would continue to choose complex uninformative disclosure (and vice versa if $\omega_S > 0$ and $\omega_U = 0$). Hence, partial unravelling occurs as long as only one of ω_S or ω_U is zero.

committee that determines the firm's disclosure policies.[14] Likewise, a manager's discretion over her firm's disclosure choice can be due to the lack of board control over disclosure or an absence of the firm's disclosure policies in the manager's contract.[15] More specifically, a manager who lacks discretion and issues a simple disclosure can capture one who aims (or is contractually obligated) to be forthcoming but lacks the additional, informative details necessary to present a complex informative disclosure. Alternatively, past literature has shown that shareholders' optimal level of disclosure informativeness trades off several forces including the cost of capital and risk-sharing efficiency (Diamond (1985), Verrecchia (2001)). Thus, some firms may impose that managers do not issue the most informative disclosure.

A manager who is constrained to issuing a complex uninformative report can possess complex information and aim to issue an informative report, but the manager and her reporting team may lack the ability to effectively communicate this information.[16] Consistent with this possibility, Chychyla et al. (2019) shows that financial experts on a firm's board of directors can be essential to effectively communicate complex transactions in the annual report, and that not all firms possess such experts.

Alternatively, a manager who is constrained to issue a complex uninformative report can capture one who includes distracting, boilerplate language due to regulatory or litigation concerns. Empirical evidence suggests that such concerns have contributed to the documented increase in the complexity of firms' reports over time (Bloomfield (2008), Lang and Stice-Lawrence (2015), Dyer et al. (2017)).[17] Some firms may be acutely sensitive to these concerns when preparing their reports, either because the transactions they engage in are

[14]For anecdotal evidence of a board of directors determining its firm's disclosure policies, see "The Best CFOs in America," *Institutional Investor*, February 12, 2004.

[15]As noted by Verrecchia (1990), if "a manager continues to exercise discretion in the disclosure of information ex post, there must be some unstated, unmodeled, and/or unresolved agency problem or efficiency consideration that lurks in the background" (p. 147).

[16]A manager who is an ineffective communicator resembles Buffett's reference in our leading quote to a manager who "doesn't understand what he or she is talking about." Anecdotal evidence also indicates that effectively communicating complex information is a valued skill among firm managers: "Financial transparency is not necessarily a matter of providing more information, however. Less can be more, suggests [Goldman Sachs Group CFO David] Viniar: 'If we can say in one page what someone else says in five pages, that's better because it's easier to read.' And for what it's worth, [former Enron CFO Andrew] Fastow and Enron produced tomes of thoroughly misleading disclosure" (Osterland (2004)).

[17]For example, following the implementation of the 2002 Sarbanes-Oxley Act, *Institutional Investor* notes: "Although the winning CFOs acknowledge the importance of restoring confidence in financial reporting, many question whether the extra cost and effort have resulted in better corporate disclosure. 'I don't believe there's a discernible difference in the information we disclose,' says Comcast Corp. co-CFO John Alchin, who notes that the new requirement for an audit of internal controls will cost his company more than $5 million this year. Adds his co-CFO and fellow honoree in Media, Lawrence Smith: 'You could argue that they've gone backwards, because investors are now confronted with more complex information and have to dig a lot deeper to make an informed decision.'" See "The Best CFOs in America," *Institutional Investor*, February 12, 2004.

subject to greater disclosure regulation, or because their reporting team views regulatory concerns as more important. As such, they may be willing to include boilerplate language even if it causes their report to resemble that of firm that is strategically obfuscating bad news.

The manager aims to maximize the firm's price P, which we assume reflects a weighted average of investors' beliefs regarding firm value:[18]

$$P \equiv \chi\mathbb{E}_I(\tilde{y}) + (1-\chi)\mathbb{E}_N(\tilde{y}), \tag{1}$$

where $\mathbb{E}_I(\tilde{y})$ and $\mathbb{E}_N(\tilde{y})$ denote the sophisticated and unsophisticated investors' conditional expectations given their information sets, respectively. As we will see, the key features of price for our main results are that: (i) price reflects a weighted average of sophisticated and unsophisticated investors' beliefs, and (ii) price reacts more strongly to complex informative than simple disclosure. We capture these features using the reduced-form representation of price above because it renders our results and analysis parsimonious. In the Appendix, we prove that our primary results continue to hold in a model in which price is determined by market clearing among risk-averse investors.[19]

Alternatively, we can interpret equation (1) as the manager's concern with the beliefs of both classes of investors, proportional to their mass of the shareholder base, in which case the manager aims to maximize aggregate, or average, shareholder beliefs.[20] One can also view the manager as maximizing the beliefs of a representative investor who is sophisticated with probability χ.

The sequence of the model is summarized as follows:

Stage 1: The manager privately observes y and whether or not she has discretion.

Stage 2: If the manager has discretion, she chooses the type of disclosure: simple, complex

[18]Note that, by the law of iterated expectations and the fact that $\tilde{y}$ is the expected value of the firm given the manager's signal, investors' expectations of firm value are equal to their expectations of $\tilde{y}$.

[19]Specifically, in the Appendix, we show that, when investors have mean-variance preferences and face residual uncertainty following disclosure, all equilibria in our model continue to be "strategic complexity equilibria" (defined below) and that there always exists an equilibrium in which simple disclosure leads to a negative price reaction.

[20]A microfoundation for this alternative interpretation is, for example, the manager's interest in raising capital from investors. More specifically, the payoff function (1) can represent the manager's interest in raising capital from the two types of investors. Under this specification, investors incur a private cost of investment and determine their investment level:

$$i = \arg\max_i \mathbb{E}(y|\Omega)i - \frac{i^2}{2} = \mathbb{E}(y|\Omega),$$

where Ω is the investors' information set depending on their sophistication. The capital raised from sophisticated investors is therefore $\chi\mathbb{E}_I(y)$ and the capital raised from unsophisticated investors is $(1-\chi)\mathbb{E}_N(y)$.

uninformative, or complex informative.

Stage 3: Investors observe whether disclosure is simple or complex. All investors observe the signal Δ_S under simple disclosure, while only sophisticated investors observe the signal Δ_C or Δ_U under complex informative or complex uninformative disclosure, respectively.

Stage 4: Investors form beliefs and the manager's payoff is realized.

We note that disclosure complexity choice is a topic that has received little attention in the literature, and there are potentially alternative ways to think about this choice. In Section 7, we discuss potential alternative modeling choices that may help to guide future work on this topic.

3 Equilibrium

In this section, we derive the model's equilibria. For ease of exposition, we introduce the notation $x \in \{S, C, U\}$ to denote the manager's choice of complexity and informativeness in disclosure, where S, C, and U represent simple, complex informative, and complex uninformative disclosure, respectively. Recall that the manager's goal is to maximize price, which reduces to maximizing the (weighted) average investor belief regarding firm value. Given the assumption that $\chi > \rho_S > \chi\rho_U$, the average investor reacts most strongly to complex informative and most weakly to complex uninformative disclosure. This suggests that the manager will choose $x = U$ upon observing sufficiently negative news and will choose $x = C$ upon observing sufficiently positive news. This motivates us to consider the following class of equilibria.

Definition 1. *Let a strategic complexity equilibrium refer to an equilibrium in which, for two thresholds T_L and T_H with $y_L < T_L < T_H < y_H$, a manager with discretion chooses complex uninformative disclosure, $x = U$, when she observes $\tilde{y} < T_L$, chooses simple disclosure, $x = S$, when she observes $\tilde{y} \in [T_L, T_H]$, and chooses complex informative disclosure, $x = C$, when she observes $\tilde{y} > T_H$.*

In a strategic complexity equilibrium, two thresholds determine the manager's disclosure choice. The manager chooses complex uninformative disclosure upon observing sufficiently bad news, complex informative disclosure upon observing sufficiently good news, and simple disclosure when observing moderate news. Our first key result is that any equilibrium *must* be a strategic complexity equilibrium.

Proposition 1. *Any equilibrium of the model is a strategic complexity equilibrium.*

Proposition 1 shows that any equilibrium in the model is consistent with both the empirically-documented patterns that firms with negative news obfuscate (e.g., Li (2008), Lo et al. (2017)), and that complex disclosure can be informative (Loughran and McDonald (2014), Lang and Stice-Lawrence (2015), Bushee et al. (2018)). It thus provides an equilibrium foundation for these two roles of complexity.

The intuition underlying this result is as follows. The manager's disclosure choice affects the expected price in two ways. First, it directly affects the informativeness of the disclosure to the average investor. Second, the manager's disclosure choice, in equilibrium, sends a signal to investors regarding the information she possesses. For instance, if the manager chooses more complex disclosure when her information is negative, investors will update their beliefs downward when observing such disclosure. When the manager's information is extreme (in either direction), the disclosure's informativeness has a very large impact on the price, so that the first effect dominates. Thus, the manager chooses the least informative disclosure, $x = U$, given highly negative news and the most informative disclosure, $x = C$, given highly positive news.

The reason that the manager chooses simple disclosure upon observing intermediate news is perhaps more surprising and subtle. In fact, as we will see in the next section, there can exist two equilibria that differ in the range of signals that lead the manager to issue a simple disclosure. The intuition for why this holds is as follows. Suppose by contradiction that there is an equilibrium in which the manager always chooses either $x = U$ or $x = C$. Then, the manager selects between two disclosure choices, one that leads to a stronger response to her news than the other. Therefore, the equilibrium resembles that from a classical Dye (1985) disclosure model.

Standard arguments thus imply that the equilibrium involves a unique threshold such that the manager chooses $x = U$ ($x = C$) when her signal falls below (above) the threshold. Moreover, this threshold lies below the firm's ex-ante expected cash flows, i.e., the manager provides a complex informative disclosure when observing moderately *negative* news (e.g., Jung and Kwon (1988)). Suppose now that the manager deviates to issue a simple disclosure when observing moderately negative news. By doing so, all investors assess the firm's expected value to be μ in the event that the disclosure is uninformative, as any simple disclosure is believed to result from a manager with no discretion. Moreover, because simple disclosure is less informative than complex informative disclosure, this reduces the average investor's reaction to the manager's negative news. Thus, the manager is strictly better off under this deviation, which implies that such an equilibrium cannot exist.[21]

[21]Formally, the payoff to the manager who observes $\tilde{y} = \tau$ from selecting $x = C$ is $\pi_C(\tau) \equiv \chi\tilde{y} + (1-\chi)\mu$, while the payoff from $x = S$ is $\pi_S(\tau) \equiv \rho_S\tilde{y} + (1-\rho_S)\mu$. Observe that $\pi_C(\tau) < \pi_S(\tau)$ since $\chi > \rho_S$ and

Figure 2: This figure depicts the firm's expected price as a function of the manager's private information $\tilde{y}$ in a strategic complexity equilibrium. The dashed lines represent the equilibrium thresholds T_L and T_H. In the figure, we let $\tilde{y}$ be uniformly distributed on $[0, 1]$. The parameters held constant in the plot are: $\beta = 0.5$; $\omega_S = 0.8$; $\omega_U = 0.1$; $\chi = 0.5$; $\rho_U = 0.3$; $\rho_S = 0.3$.

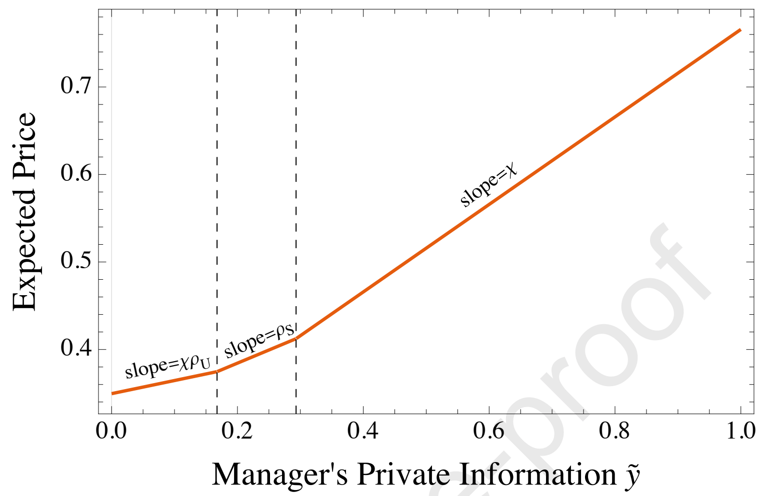


Figure 2 illustrates the firm's expected price in a strategic complexity equilibrium. It shows that, in such an equilibrium, the marginal reaction to the manager's information is increasing, i.e., price is a convex function of y. This arises because, as the manager's information grows more positive, her disclosure becomes more informative.

We next show that a strategic complexity equilibrium always exists, but need not be unique. To do so, we first derive the investors' beliefs and the manager's incentives in such an equilibrium.

Investors' Conditional Beliefs

We begin by characterizing investors' beliefs when the manager discloses simple information. In this case, both sophisticated and unsophisticated investors hold the same beliefs, which are determined by the disclosed signal $\tilde{\Delta}_S$:

$$\mathbb{E}_N\left(\tilde{y}|\tilde{x} = S\right) = \mathbb{E}_I\left(\tilde{y}|\tilde{x} = S\right) = \mathbb{E}\left(\tilde{y}|\tilde{\Delta}_S\right)$$

$$= \begin{cases} \tilde{y} & \text{if } \tilde{\Delta}_S = \tilde{y}, \\ \frac{\beta\omega_S\mu + (1-\beta)(F(T_H)-F(T_L))\mathbb{E}(\tilde{y}|\tilde{y}\in[T_L,T_H])}{\beta\omega_S + (1-\beta)(F(T_H)-F(T_L))} & \text{if } \tilde{\Delta}_S = \emptyset. \end{cases}$$

$\tau < \mu$.

We see above that investors make a rational inference given uninformative disclosure. Specifically, investors realize that such disclosure either arises from a manager without discretion, in which no inference can be made, or a manager with discretion, in which case it can be inferred that the manager's information belongs to the interval on which she chooses simple disclosure, i.e., $\tilde{y} \in [T_L, T_H]$.

Next, if the manager issues a complex uninformative disclosure, $x = U$, investors' beliefs depend upon whether they are sophisticated. Unsophisticated investors form their beliefs purely based upon the inference they make in equilibrium. As these investors cannot distinguish whether the disclosure is complex informative or uninformative, they can only infer that $\tilde{y} \notin [T_L, T_H]$, if the manager has discretion; this leads to:

$$\mathbb{E}_N\left(\tilde{y}|\tilde{x}=U\right) = \frac{\beta\left(1-\omega_S\right)\mu + \left(1-\beta\right)\left(1-F\left(T_H\right)+F\left(T_L\right)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\notin[T_L,T_H]\right)}{\beta\left(1-\omega_S\right)+\left(1-\beta\right)\left(1-F\left(T_H\right)+F\left(T_L\right)\right)}. \quad (2)$$

In contrast, sophisticated investors observe the disclosure signal $\tilde{\Delta}_U$. When this signal is informative, they learn $\tilde{y}$; otherwise, sophisticated investors are able to infer that, if the manager has discretion, $\tilde{y} < T_L$. Thus, we have:

$$\mathbb{E}_I\left(\tilde{y}|\tilde{x}=U\right) = \mathbb{E}\left(\tilde{y}|\tilde{\Delta}_U\right) = \begin{cases} \tilde{y} & \text{if } \tilde{\Delta}_U = \tilde{y}, \\ \frac{\beta\omega_U\mu+(1-\beta)F(T_L)\mathbb{E}(\tilde{y}|\tilde{y}<T_L)}{\beta\omega_U+(1-\beta)F(T_L)} & \text{if } \tilde{\Delta}_U = \emptyset. \end{cases}$$

Finally, if the manager issues a complex informative disclosure, unsophisticated investors again believe that $\mathbb{E}_N\left(\tilde{y}|\tilde{x}=C\right) = \mathbb{E}_N\left(\tilde{y}|\tilde{x}=U\right)$, as given in expression (2). In contrast, sophisticated investors always learn $\tilde{y}$:

$$\mathbb{E}_I\left(\tilde{y}|\tilde{x}=C\right) = \mathbb{E}\left(\tilde{y}|\tilde{\Delta}_C\right) = \tilde{y}.$$

With these results at hand, we move to deriving the manager's expected payoffs as a function of her disclosure choice.

Disclosure Choice and Manager Payoffs

Let $\pi_x\left(\tilde{y}; T_L, T_H\right)$ denote the manager's expected payoff in a strategic complexity equilibrium characterized by the thresholds T_L and T_H, given that the manager observes $\tilde{y}$ and selects $x \in \{S, U, C\}$. Standard arguments imply that a strategic complexity equilibrium exists if and only if, upon observing $\tilde{y} = T_L$, the manager is indifferent between simple and complex uninformative disclosure, and upon observing $\tilde{y} = T_H$, the manager is indifferent between simple and complex informative disclosure. This leads to the following equilibrium conditions:

$$Q_{SC}(T_L,T_H) \equiv \pi_S(T_H;T_L,T_H) - \pi_C(T_H;T_L,T_H) = 0;$$
$$Q_{SU}(T_L,T_H) \equiv \pi_S(T_L;T_L,T_H) - \pi_U(T_L;T_L,T_H) = 0.$$

Note that, if the manager chooses $x = S$, her payoff is a weighted average of investors' beliefs conditional on the disclosure revealing $\tilde{y}$ versus being uninformative:

$$\pi_S(\tilde{y};T_L,T_H) \equiv \rho_S\tilde{y} + (1-\rho_S)\frac{\beta\omega_S\mu + (1-\beta)(F(T_H)-F(T_L))\mathbb{E}(\tilde{y}|\tilde{y}\in[T_L,T_H])}{\beta\omega_S + (1-\beta)(F(T_H)-F(T_L))}.$$

Similarly, if the manager chooses $x = C$, her payoff is a weighted average of the beliefs of sophisticated and unsophisticated investors:

$$\pi_C(\tilde{y};T_L,T_H) \equiv \chi\tilde{y}+(1-\chi)\frac{\beta(1-\omega_S)\mu + (1-\beta)(1-F(T_H)+F(T_L))\mathbb{E}(\tilde{y}|\tilde{y}\notin[T_L,T_H])}{\beta(1-\omega_S)+(1-\beta)(1-F(T_H)+F(T_L))}.$$

Finally, if the manager chooses $x = U$, her payoffs are a weighted average of sophisticated and unsophisticated investors' beliefs, as well as the sophisticated investors' beliefs as a function of whether the disclosure reveals $\tilde{y}$:

$$\begin{aligned}\pi_U(\tilde{y};T_L,T_H) \equiv \chi\left(\rho_U\tilde{y} + (1-\rho_U)\frac{\beta\omega_U\mu + (1-\beta)F(T_L)\mathbb{E}(\tilde{y}|\tilde{y}<T_L)}{\beta\omega_U + (1-\beta)F(T_L)}\right)\\ + (1-\chi)\frac{\beta(1-\omega_S)\mu + (1-\beta)(1-F(T_H)+F(T_L))\mathbb{E}(\tilde{y}|\tilde{y}\notin[T_L,T_H])}{\beta(1-\omega_S)+(1-\beta)(1-F(T_H)+F(T_L))}.\end{aligned}$$

Characterizing Strategic Complexity Equilibria

In the next proposition, we characterize the strategic complexity equilibria that exist in our model.[22]

Proposition 2. *In any strategic complexity equilibrium, the manager never issues complex uninformative disclosure upon observing positive news, i.e.,* $T_L < \mu$. *Moreover,*

(i) *There* ***always exists*** *a strategic complexity equilibrium in which* $T_H < \mu$. *In this equilibrium, the manager selects simple disclosure only when she has negative news,*

[22]This result is a non-trivial extension of the classic arguments used to prove existence of disclosure equilibria. The classic argument involves showing that by varying the disclosure threshold, by continuity, one ultimately finds a point at which the manager on the threshold is indifferent between disclosing and not disclosing. In our setting, this argument does not apply in its standard form because (i) there are two disclosure thresholds, T_L, T_H, and (ii) varying, for example, the threshold T_H, simultaneously affects the inference made from simple and complex informative disclosure. Our model is further distanced from classic disclosure models because unsophisticated investors are unable to distinguish between two forms of complex disclosure $x = C$ and $x = U$. This feature gives rise to the second equilibrium in Proposition 2.

and thus simple disclosure sends a negative signal to investors:

$$\mathbb{E}\left(\tilde{y}|\tilde{x}=S\right)<\mu;\;\; \mathbb{E}\left(\tilde{y}|\tilde{x}\in\{U,C\}\right)>\mu.$$

(ii) *There exists a strategic complexity equilibrium in which* $T_H > \mu$ *if and only if* $\chi < \xi(\rho_S)$*, for an increasing function* $\xi(\cdot)$ *that satisfies* $\xi(\rho_S) \in (\rho_S, 1)$*. In this equilibrium, the manager selects simple disclosure when she has, on average, positive news, and thus simple disclosure sends a positive signal to investors:*

$$\mathbb{E}\left(\tilde{y}|\tilde{x}=S\right)>\mu;\;\; \mathbb{E}\left(\tilde{y}|\tilde{x}\in\{U,C\}\right)<\mu.$$

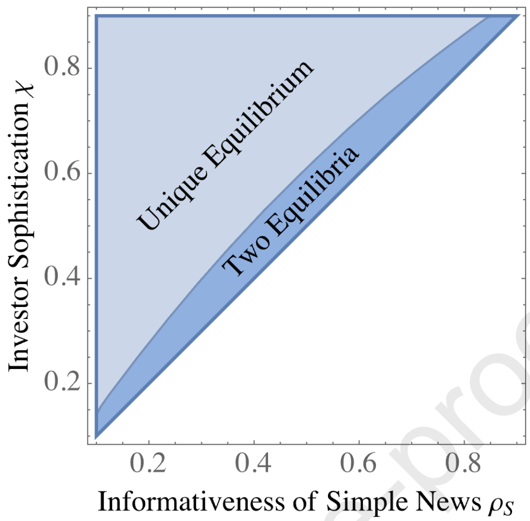
Proposition 2 first establishes that the manager with discretion never obfuscates upon observing positive news, i.e., $T_L < \mu$. By obfuscating, not only would the manager reduce the market reaction to her information, but her disclosure choice would also send a negative signal to investors. The remainder of the proposition states that there can exist two strategic complexity equilibria. Figure 3 demonstrates the conditions for existence of these two types of equilibria in our model. Figure 4 demonstrates the features of these two equilibria, showing that they are differentiated by the range of signals that lead the manager to issue a simple disclosure.

The first equilibrium is robust in that it exists regardless of the model's parameters. In this equilibrium, the manager issues a simple disclosure only when she observes moderately negative news. Thus, simple disclosure sends a negative signal to investors. This may be surprising given the common narrative that managers raise the complexity of their disclosure upon observing negative news. The second equilibrium arises if and only if simple and complex informative disclosure provide similar amounts of information to the average investor, i.e., when χ is sufficiently close to ρ_S. In this equilibrium, the manager issues simple disclosure both when observing moderately positive and moderately negative news. Moreover, simple disclosure sends a *positive* signal to investors.

The first, robust equilibrium can be understood as follows. In such an equilibrium, the manager chooses $x = S$ when she observes moderately negative news to partially temper the reaction to this news. She prefers $x = S$ over $x = U$, as obfuscating sends a negative signal to sophisticated investors, who would recognize that she chose $x = U$. It may initially be surprising that, in this equilibrium, the manager chooses complex informative disclosure when she observes mildly negative news (i.e., when $y \in [T_H, \mu]$). The manager does so in order to avoid the negative inference investors draw from simple disclosure.

The reason why there may also exist a second type of equilibrium in which the reaction

Figure 3: This figure depicts Proposition 2. It shows the conditions under which there is a unique equilibrium that satisfies $T_H < \mu$ versus two equilibria, one with $T_H < \mu$ and one with $T_H > \mu$. In the figure, we let $\tilde{y}$ be uniformly distributed on $[0, 1]$. The parameter values are $\beta = 0.5$, $\omega_S = 0.8$; $\omega_U = 0.1$; $\rho_U = 0.3$.



to simple news is positive (i.e., $T_H > \mu$) is as follows. When ρ_S is close to χ, the average investor reacts almost as strongly to complex informative disclosure as they do to simple disclosure. Thus, when the manager observes positive news, she is only marginally swayed towards complex informative disclosure based upon the increase in the average investor's reaction that it creates. Moreover, in this equilibrium, simple disclosure sends a positive signal to investors as they know the manager chooses $x = S$ when she observes, on average, positive news. This motivates the manager to choose $x = S$ when she observes not only moderately negative news but also moderately positive news. Figure 4 illustrates the two types of equilibria discussed in the proposition.

The following corollary establishes two additional properties of the robust equilibrium of our model.

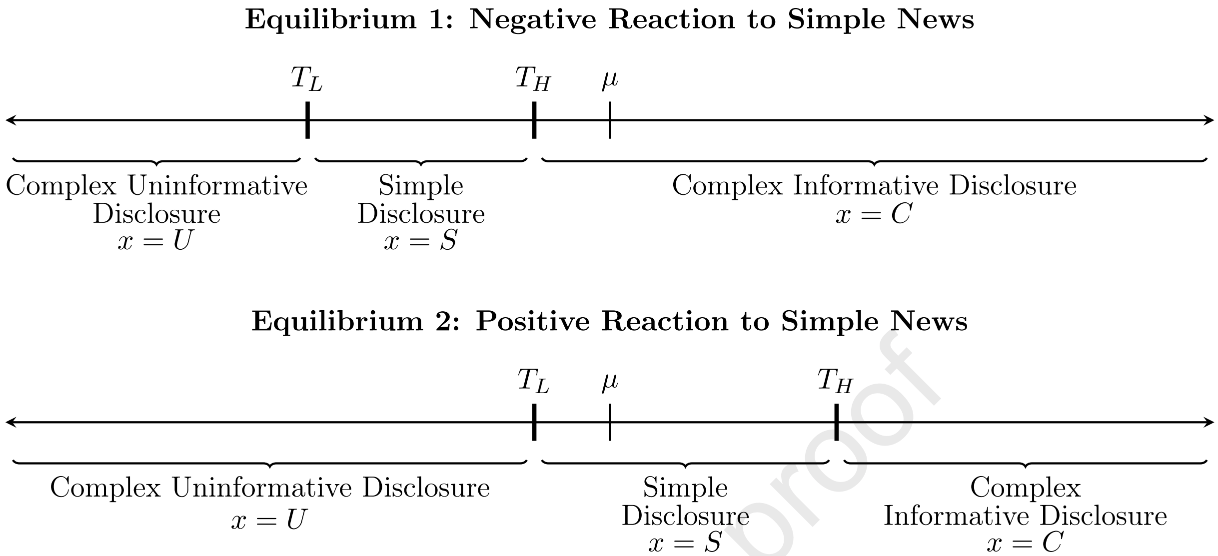
Corollary 1. *Consider the equilibrium in which $T_H < \mu$. In this equilibrium, the expected the price response to simple (complex) disclosure is negative (positive):*

$$\mathbb{E}\left(\tilde{P}|\tilde{x} = S\right) - \mu < 0; \ \ \mathbb{E}\left(\tilde{P}|\tilde{x} \in \{U, C\}\right) - \mu > 0.$$

Moreover, if the firm's value is symmetrically distributed, complex disclosure is more likely to represent information provision rather than obfuscation, i.e.,

$$\Pr\left(\tilde{x} = C|\tilde{x} \in \{U, C\}\right) > \Pr\left(\tilde{x} = U|\tilde{x} \in \{U, C\}\right).$$

Figure 4: This figure depicts the two potential types of equilibria characterized in Proposition 2.



In the equilibrium in which $T_H > \mu$, these results are reversed.

This corollary shows that, in the robust equilibrium of our model, the market reaction to complex news is positive. Moreover, complex disclosure more often represents information provision than it does obfuscation. Thus, this result suggests that, when a firm's investor base is relatively sophisticated (i.e., when χ is significantly higher than ρ), we should expect that complex disclosures are linked with *positive* economic outcomes.

4 Properties of Strategic Complexity Equilibria

Now that we have established the basic features of strategic complexity equilibria, we next explore their properties in more detail. Our goal with these analyses is to develop intuition and generate empirical predictions. We study both types of equilibria. As we will see, the two equilibria make similar predictions along the dimensions we study below. Throughout, we conduct numerical comparative statics by assuming the manager's information $\tilde{y}$ follows a uniform distribution (on $[0, 1]$, without loss of generality).[23]

[23]We have verified that the results hold over a broad range of parameter values as well as for normally distributed $\tilde{y}$. This analysis is available upon request.

4.1 Relative Likelihood of Simple and Complex Disclosure

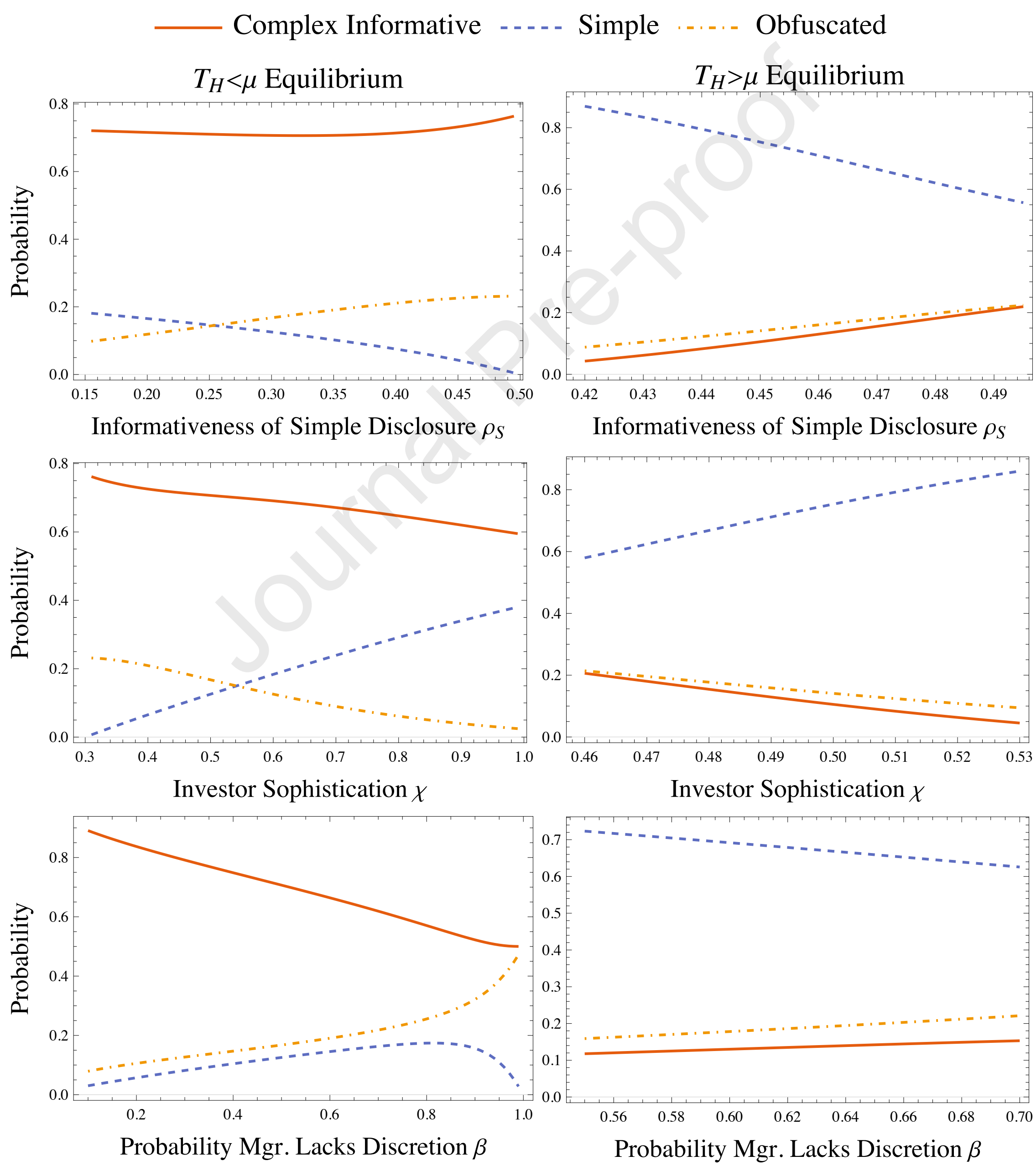
To assess the drivers of a firm's equilibrium disclosure choice, we conduct numerical comparative statics on the equilibrium probabilities of the three types of disclosure in our model. We focus on the informativeness of simple disclosure (ρ_S), the fraction of sophisticated investors (χ), and the probability that the manager does have discretion over the reporting choice (β). Figure 5 illustrates the results.

While the exact relationships between the parameters ρ_S, χ, and β and firms' disclosure choices depend upon the equilibrium under consideration, several key findings are robust to both potential equilibria. First, the lower panel in Figure 5 shows that an increase in the likelihood the manager does not have discretion, β, increases the likelihood that she chooses $x = U$ (but has an ambiguous impact on the likelihood she chooses $x = C$). Intuitively, as β rises, the penalty that sophisticated investors place on a firm that issues complex uninformative disclosure declines, which pushes the manager, when she is on the margin between $x = S$ and $x = U$, towards $x = U$. Furthermore, in the equilibrium in which $T_H < \mu$ ($T_H > \mu$), the manager has negative (positive) news when she is on the margin between $x = S$ and $x = C$. Thus, an increase in β causes investors' inferences from simple disclosure to rise (fall), which implies that the manager is more (less) inclined to choose S over C. Our model therefore predicts that an increase in disclosure regulation that constrains firms to specific forms of disclosure increases obfuscation but has an ambiguous impact on informative complexity.

The middle and lower panels in Figure 5 illustrate two counter-intuitive results. Notably, an increase in the manager's ability to communicate via simple disclosure, ρ_S, *decreases* the likelihood that she chooses simple disclosure in equilibrium. Similarly, an increase in investor sophistication, χ, *decreases* the likelihood that the manager chooses complex disclosure, whether informative or not, in equilibrium. While these relationships hold in any equilibrium, the intuition differs across the equilibria in which $T_H > \mu$ and $T_H < \mu$.

Consider first the equilibrium in which the average response to $x = S$ is negative, i.e., $T_H < \mu$. In such an equilibrium, the positive relationship between ρ_S and complexity stems from the fact that the manager chooses $x = S$ when she possesses negative news. Consequently, the manager dislikes an increase in the response to her news, resulting in less simple disclosure as ρ_S increases. The negative relationship between investor sophistication and complex uninformative disclosure arises because sophisticated investors recognize that such disclosure was likely obfuscated by a manager with discretion and therefore penalize it heavily. Finally, the negative relationship between investor sophistication and complex informative disclosure arises because the manager is on the margin between $x = S$ and

Figure 5: This figure depicts how the probabilities of complex informative, simple, and complex uninformative disclosure vary as functions of the informativeness of simple disclosure, ρ_S, the fraction of investors who are sophisticated, χ, and the probability that the manager does not have discretion over the reporting choice, β, in both types of equilibria presented in Proposition 2. The parameters held constant in the plots are $\beta = 0.5$; $\omega_S = 0.8$; $\omega_U = 0.1$; $\chi = 0.5$; $\rho_U = 0.3$. In the left-hand plots, $\rho_S = 0.3$, while in the right-hand plots, $\rho_S = 0.45$; this ensures the equilibrium in which $T_H > \mu$ exists over the parameter range considered.



$x = C$ when she possesses negative news. Thus, this manager dislikes the fact that, as sophistication rises, so too does the response to complex informative disclosure.[24]

Next, consider the equilibrium in which the response to simple disclosure is positive, i.e., $T_H > \mu$.[25] In this case, a rise in ρ_S has two offsetting effects on the probability that the firm chooses $x = S$. In particular, since the manager is on the margin between $x = S$ and $x = C$ when she possesses positive news, the increase in the reaction to this news caused by a rise in ρ_S pushes her towards $x = S$. On the other hand, since the manager is on the margin between $x = S$ and $x = U$ when she possesses negative news, a rise in ρ_S pushes her towards $x = U$. The latter effect dominates, resulting in an overall lower probability that the manager issues simple disclosure, because $T_H - \mu > \mu - T_L$; given a symmetric distribution such as the uniform, this implies that the manager is more likely to be on the margin between $x = U$ and $x = S$ than between $x = S$ and $x = C$.[26] A similar argument explains why an increase in investor sophistication (χ) has the opposite effect, shifting the firm away from complex disclosure and towards simple disclosure.

4.2 Disclosure Complexity and Belief Dispersion

We next analyze the difference in the average beliefs of sophisticated versus unsophisticated investors, which we refer to as "belief dispersion." Belief dispersion is important as it ostensibly determines the ability of sophisticated investors to earn trading profits at the cost of unsophisticated investors. It is further relevant to understanding the relationship between disclosure complexity and the trading behavior of sophisticated and unsophisticated investors.

Figure 6 demonstrates how average belief dispersion given each type of disclosure depends on the simplicity of the firm's information ρ_S and the sophistication of the firm's investor base χ. Observe first that belief dispersion is always minimized when the firm chooses simple disclosure, as both types of investors possess identical information following simple disclosure. Whether belief dispersion is greater when the firm issues complex informative or complex uninformative disclosure is more subtle. One might posit that complex informative disclosure leads to the greatest dispersion in beliefs because it is very informative to sophisticated investors and uninformative to unsophisticated investors.

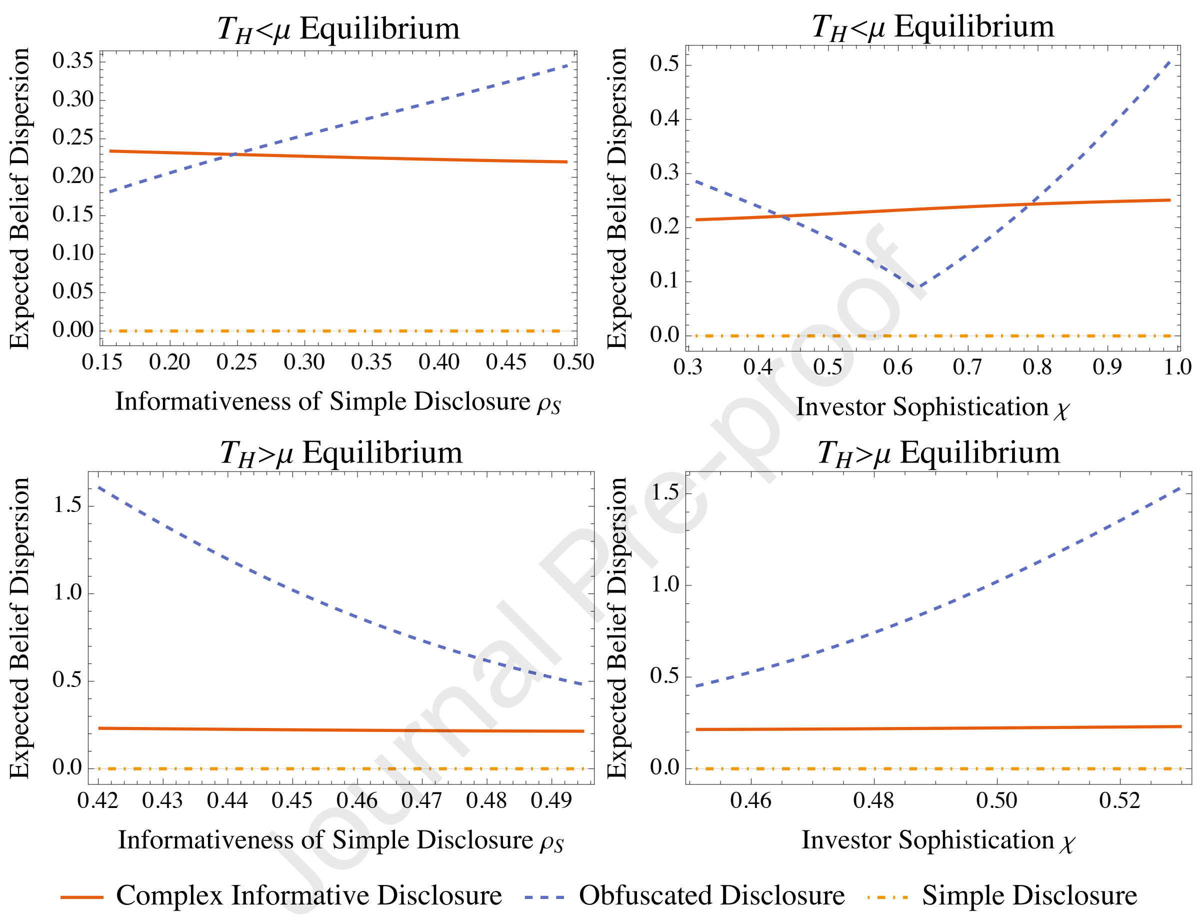
However, an additional, countervailing force is present: only sophisticated investors can

[24] See Bertomeu et al. (2021) for a related point in a traditional voluntary disclosure setting.

[25] Note the parameter ranges analyzed in the right panels of the figure are limited, as an equilibrium with $T_H > \mu$ only exists for a limited range of the parameter space.

[26] In other words, under a symmetric distribution, T_L is closer to the unconditional mean, since $\mathbb{E}(\tilde{y}|x = S) > \mu$ in the equilibrium where $T_H > \mu$. Hence, the threshold-type $y = T_L$ is more common than $y = T_H$.

Figure 6: This figure depicts expected belief dispersion when the firm provides complex uninformative, complex informative, or simple disclosure under the two equilibria established in Proposition 2. The parameters held constant in the plot are: $\beta = 0.5$; $\omega_S = 0.8$; $\omega_U = 0.1$; $\chi = 0.5$; $\rho_U = 0.3$. In the upper plots, $\rho_S = 0.3$, while in the lower plots, $\rho_S = 0.45$; this ensures the equilibrium in which $T_H > \mu$ exists over the parameter range considered.



distinguish a complex uninformative from a complex informative disclosure and recognize that it may be a strategic choice by a manager who observed negative news. As a consequence, Figure 6 shows that in the equilibrium in which $T_H < \mu$, complex uninformative disclosure can lead to the greatest belief dispersion. In this equilibrium, the average firm that chooses to obfuscate possesses very negative news. Thus, given complex uninformative disclosure, sophisticated investors are thus highly pessimistic relative to unsophisticated investors.

4.3 Disclosure Complexity and Price Volatility

We next study the relationship between disclosure complexity and the volatility in prices that the disclosure creates. We define price volatility given a disclosure choice $x \in \{S, C, U\}$ as $Var\left(\tilde{P}\,|x\right)^{1/2}$. Two economic forces drive the relationship between disclosure complexity and price volatility. First, the disclosure's complexity determines the magnitude of the average investor's reaction to the news, which directly affects the amount of variation in prices it creates. Based on this force alone, whether simple or complex disclosure generates more volatility is unclear. While complex informative disclosure is more informative, complex uninformative disclosure is less informative than simple disclosure. Whether, on average, complex disclosure is more informative thus depends upon the relative likelihood that the manager selects $x = U$ versus $x = C$.[27] Second, in equilibrium, independent of the information contained in the disclosure itself, the manager chooses complex disclosure when she observes extremely positive or negative news. This tends to increase the volatility that complex disclosure creates.

In Figure 7, we illustrate the relationship between disclosure complexity and volatility in both classes of equilibria, finding that, independent of the parameters, complex disclosure is associated with significantly more price volatility. Thus, any difference in the informativeness of simple and complex news is ultimately dominated by the fact that the manager selects complex disclosure when her news is more significant. The figure demonstrates an additional unanticipated feature of the model. In particular, price volatility given a simple disclosure can *fall* as simple disclosure becomes more informative. Intuitively, as simple disclosure becomes more informative, in equilibrium, the manager chooses simple disclosure when her information is more moderate, attenuating price volatility.

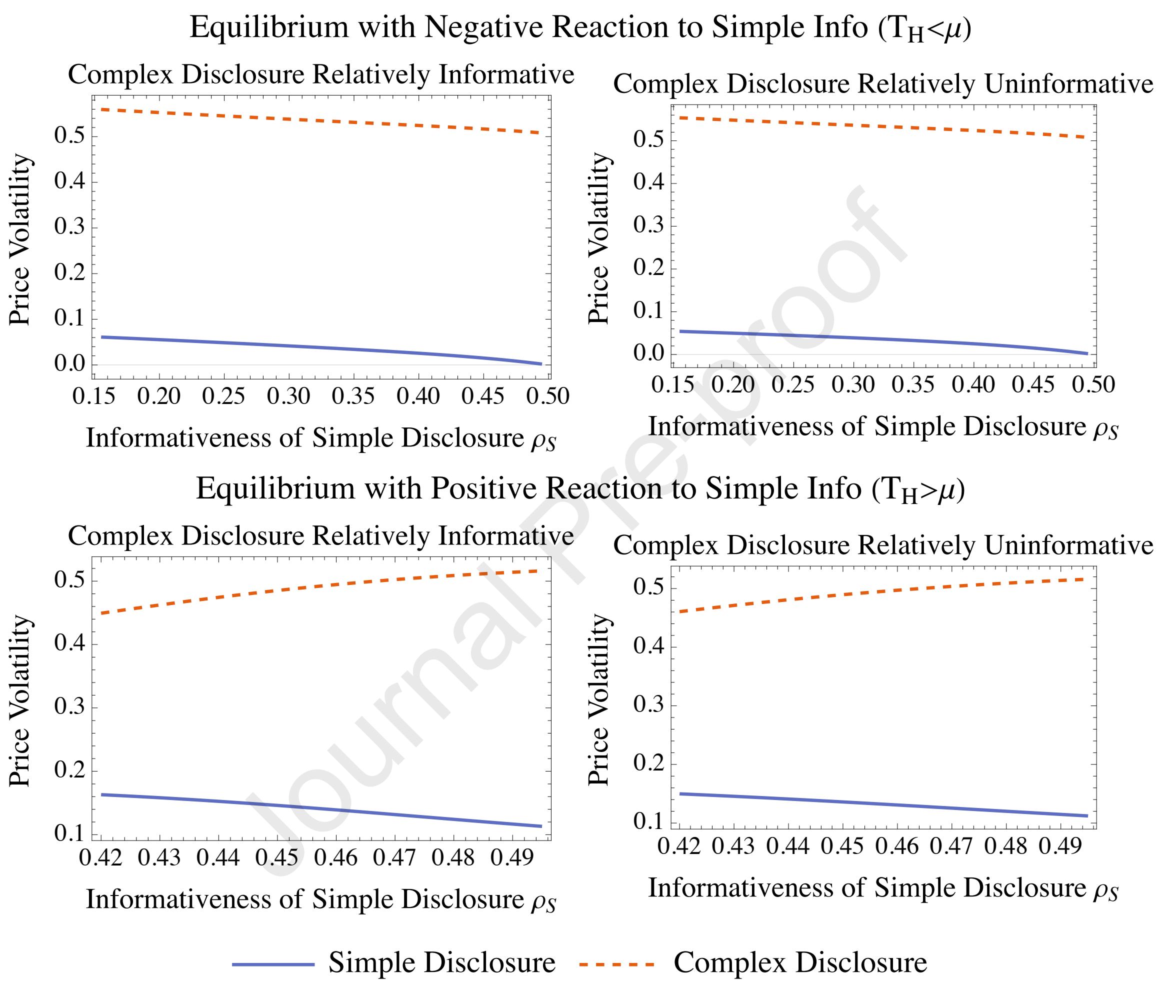
5 Additional Analyses

5.1 Role of Obfuscation

In this section, we examine the role of the manager's ability to strategically obfuscate on the overall information environment. We show that the ability to obfuscate reduces the overall quality of information available to investors. In fact, obfuscation has both direct and indirect

[27] Note that when the distribution of the manager's news is symmetric and the likelihood that the manager has no discretion and must choose $x = U$ and $x = C$ are the same, we can show that, in an equilibrium in which $T_H < \mu$, complex disclosure tends to be more informative than simple disclosure, and vice versa when $T_H > \mu$. This follows directly from the fact that when $T_L < T_H < \mu$, $F(T_L) < 1 - F(T_H)$. That is, among complex firms, a greater fraction selects complex informative disclosure than complex uninformative disclosure.

Figure 7: This figure depicts the volatility of the firm's price conditional on complex and simple news under the two varieties of equilibria. Across all plots, we hold constant the parameters $\beta = 0.5$; $\omega_S = 0.8$; $\omega_U = 0.1$; $\chi = 0.5$. The left-hand plots consider the case in which disclosure under $x = U$ is relatively informative; here, we set $\rho_U = 0.3$. The right-hand plots consider the case in which disclosure under $x = U$ is relatively uninformative; here, we set $\rho_U = 0$.



negative effects on information quality. It directly reduces unsophisticated investors' ability to understand disclosure. Moreover, obfuscation disincentivizes the manager from issuing a complex informative disclosure when she has good news, because investors can perceive this disclosure as potential obfuscation.

Corollary 2. *Suppose that the manager is unable to obfuscate, i.e., when she has discretion, she can only choose* $x \in \{S, C\}$. *Then, there is a unique equilibrium, where there exists a* $T < \mu$ *such that the manager chooses* $x = C$ *when* $y > T$ *and* $x = S$ *when* $y < T$. *The expected amount of information available to investors in this equilibrium is strictly greater than that in the equilibria described in Proposition 2.*

Corollary 2 establishes that the possibility of issuing complex uninformative disclosure results in a strictly worse information environment for the firm.[28] Moreover, from an ex-ante perspective, the manager is not better off by having the option to obfuscate.[29] This result therefore suggests that disclosure regulation may be welfare-enhancing if it can reduce a manager's ability to obfuscate disclosures.

5.2 Unsophisticated Investor Base ($\rho_S > \chi$)

Thus far, we have focused on the case in which complex informative disclosure is more informative to the average investor than simple disclosure, i.e., $\rho_S < \chi$. This is consistent with a firm whose investor base is reasonably sophisticated relative to the information loss caused by a simple disclosure. However, in certain cases, the majority of investors may be unable to process complex information, or information can be simplified with minimal information loss, i.e., we might expect that $\rho_S > \chi$. This may lead the average investor to learn more from simple than from complex informative disclosure. We conclude our analysis by studying the nature of the equilibrium that arises in this case.

In contrast to our previous analysis, when $\rho_S > \chi$, investors react most strongly to simple disclosure. Therefore, in any equilibrium, the manager chooses $x = S$ upon observing sufficiently positive news and $x = U$ upon observing sufficiently negative news. It is less clear whether, in equilibrium, the manager will ever find it optimal to choose $x = C$. The next proposition formalizes the nature of equilibria that arise, showing that the likelihood the manager is constrained to choosing complex uninformative disclosure, $\beta\omega_U$, is pivotal in determining whether she ever chooses $x = C$.

Proposition 3. *Suppose $\rho_S > \chi$. Then, in any equilibrium, there exists a $T_L < T_H < \mu$ such that the manager chooses $x = U$ when she observes $y < T_L$ and $x = S$ when she observes $y > T_H$. Moreover, there exists a $Z \in (0, 1)$ such that the following statements hold.*

(i) *Suppose $\beta\omega_U < Z$. Then, there exists an equilibrium in which, for some $T_L < T_H < \mu$, the manager chooses $x = U$ when she observes $y < T_L$, $x = C$ when she observes $y \in (T_L, T_H)$, and $x = S$ when she observes $y > T_H$.*

[28] The results in Corollary 2 hold for any $\omega_U \in [0, 1)$ and the equilibrium depends only on the sum $\omega_U + \omega_C$ and not the individual probabilities ω_U and ω_C. Intuitively, when the manager chooses $x = C$, sophisticated investors observe y, and so their beliefs do not depend on ω_U or ω_C. Unsophisticated investors do not observe y and know only that the disclosure was complex. Thus, their beliefs depend only on the total probability that no-discretion managers choose complex disclosure, $\omega_U + \omega_C$.

[29] This is due to the fact that, prior to observing private information, the manager's expectation of price is the unconditional mean.

(ii) *Suppose $\beta\omega_U \geq Z$. Then, there exists an equilibrium in which, for some $T < \mu$, the manager chooses $x = U$ when she observes $y < T$, chooses $x = S$ when she observes $y > T$, and never chooses $x = C$.*

The proposition demonstrates that in any equilibrium, the manager chooses $x = S$ when she observes either positive news ($\tilde{y} > \mu$) or mildly negative news, and chooses $x = U$ when she observes sufficiently negative news. As a result, the price reaction to simple disclosure is always positive. Moreover, assuming the distribution of the manager's information y is not heavily skewed, simple disclosure is the manager's most common choice. In sum, the results in this section suggest that, when firms are unable to provide additional information via complex disclosure, there will be a negative relationship between disclosure complexity and firm performance. Thus, in such settings, our model's predictions are consistent with early empirical work on complex disclosure, e.g., Li (2008).

When $\beta\omega_U > Z$, the manager never chooses $x = C$, while when $\beta\omega_U < Z$, there exists an intermediate range of signals that lead her to choose $x = C$ in equilibrium. Intuitively, $\beta\omega_U$ determines the nature of the equilibrium because it determines the discount imposed on complex uninformative disclosure by sophisticated investors. When $\beta\omega_U$ is large, this discount is minor, as $x = U$ is likely to have arisen from a manager without discretion. Thus, when the manager observes negative news, she prefers to diminish the response to this news by choosing $x = U$. In contrast, when $\beta\omega_U$ is small, the discount given $x = U$ is severe. Consequently, the manager prefers $x = C$ over $x = U$ upon observing moderately negative news, despite the fact that the average investor reacts more strongly to this news.

6 Empirical Implications

In this section, we discuss empirical predictions that emerge from our model. Our aim is to provide potential new avenues for future research; as such, many of the predictions discussed below have yet to be investigated in the empirical literature. However, we make connections with the literature when possible.[30]

The key features of disclosure complexity in our model are that complexity (*i*) reduces unsophisticated investors' ability to process the disclosure, and (*ii*) is essential to convey more information. This notion of complexity may reflect either the provision of additional details or the use of technical language. Thus, it is consistent with the common empirical proxies for reporting complexity including disclosure length (e.g., You and Zhang (2009),

[30] We also note that the working paper version of this article includes regression analyses regarding some of the model implications.

Guay et al. (2016), deHaan et al. (2020)), and metrics of linguistic complexity such as complex word count and the fog index (e.g., Li (2008), Miller (2010), Lehavy et al. (2011), Lo et al. (2017)). It is essential to control for non-discretionary complexity in testing our model's predictions, as our predictions regarding complexity pertain to the discretionary component of complexity. Bushee et al. (2018) argue that one can do so by controlling for the complexity of analysts' questions in conference calls.

Our primary analyses focus on a firm that can convey additional information to the market via a complex disclosure, which occurs when its investor base is sophisticated or its economics are such that simplifying the disclosure materially reduces its information content. In this case, the firm's manager issues a complex disclosure when she has either highly positive or negative news, and simple disclosure when she has intermediate news. This leads to the following cross-sectional prediction.

Prediction 1. *The relation between discretionary reporting complexity and managers' private information is U-shaped among firms or industries that have a high degree of sophisticated investors, or in industries where simplifying disclosure leads to considerable information loss.*

Our model suggests that potential proxies for managers' private information include contemporaneous or future returns. The reason is that, in our model, as long as disclosure is partially informative to some market participants, regardless of the manager's disclosure choice, the manager's information is partially conveyed to some investors on the date that the report is released. As a result, these investors partially impound this information into price, and the remaining reaction to the disclosure occurs over time. Figure 1 provides evidence consistent with Prediction 1, though future work may consider testing this prediction with alternative proxies for complexity or firm information, more comprehensive methodologies, or in different settings.[31]

Our model also makes predictions on how the market reacts to disclosures within industries that exhibit this U-shaped pattern. Proposition 2 shows that, when a firm's investor base is sophisticated or simple information entails significant information loss, there is a unique equilibrium in which the market reacts negatively (positively) to simple (complex) disclosure.

Prediction 2. *Among firms or industries with a high level of investor sophistication or in*

[31] We also note that Li (2008) documents a negative linear relationship between return on assets (ROA) and complexity. However, our model makes no predictions regarding the relationship between ROA and complexity. The reason is that ROA is a public, numerical statistic that can be readily observed by investors. Our predictions instead apply to the novel information that managers convey in their textual disclosures.

more complex industries where simple disclosure is less informative, on average, the market responds negatively to simple disclosures and positively to complex disclosures.

Our model further offers predictions on the relative frequency with which firms issue simple and complex disclosures. Section 4.1 shows that a firm is *less* likely to use discretion to simplify its disclosure when doing so leads to a lower loss of information content, such as in less complex industries. Likewise, the frequency of complex (simple) disclosures is decreasing (increasing) in the level of investor sophistication.

Prediction 3. *The mass of firms issuing simple relative to complex disclosure is decreasing as simple disclosures become more informative, and increasing in the level of investor sophistication.*

Finally, our results have implications for belief dispersion among investors and return volatility upon the release of a disclosure. As discussed in Section 4, belief dispersion is always greater for complex disclosure due to some investors not being able to process the disclosure. Likewise, price volatility is higher for complex disclosure as managers with extreme news tend to issue more complex disclosures.

Prediction 4. *Belief dispersion and price volatility are greater for firms that issue complex (informative or uninformative) disclosures than firms that issue simple disclosures.*

Some evidence for the above prediction has been documented in the empirical literature. Miller (2010) finds that investor belief dispersion is greater among firms that issue complex disclosures. Relatedly, Lawrence (2013) documents that unsophisticated investors have a lower information disadvantage among firms that issue simple disclosures. Both findings are consistent with Prediction 4.

As noted previously, our main results rely on the firm's ability to convey additional information to the market via complex disclosure, which is captured by Assumption 2 in the model. If this does not hold, as shown in Section 5.2, the relation between complexity and news is instead monotonic and negative. Thus, our results suggest cross-industry variation in the relation between news and complexity.

In particular, we expect a monotonic relation in industries which have a lower proportion of sophisticated investors, or in industries where information can be conveyed in a simple manner (Proposition 3). These can include, for example, less technical or more established industries which can more easily convey information simply without significant information loss, or industries which do not often experience innovations (e.g., the oil industry). In contrast, industries with rapidly evolving product markets, growth, or high-tech industries may naturally be more complicated, and hence simple disclosure is less effective in conveying complex information in such industries.

7 Alternative Modeling Choices

While central to accounting research and practice, the choice of disclosure complexity has received little attention in the theoretical literature. Moreover, complexity is a multidimensional construct that cannot be captured entirely by a single model. In this section, we discuss the key assumptions underlying our results and outline potential various alternative modeling choices. Our goals in doing so are to clarify the empirical settings that our model applies to and to motivate further theoretical work on the strategic choice of disclosure complexity.

7.1 Dynamic Complexity Choice

Our baseline setting focuses on disclosure complexity choice in a single period, which abstracts from long-term considerations. A natural extension of our baseline model is a repeated-game or dynamic setting in which the manager cares about long-term prices and chooses the complexity of disclosures.[32] We outline such a model below and discuss how long-term considerations can influence our results.

Consider a T-period setting where in each period $t = 1, ..., T$ the manager privately observes an *i.i.d.* signal of the firm's performance for that period, y_t, and then chooses the complexity of disclosure regarding y_t for that period.[33] The firm retains earnings and thus market beliefs over the firm's liquidation value, and price, depend on current and past disclosures. For the same reasons that we discuss in the static setting, we assume that, in each period, the manager lacks discretion over the firm's complexity choice with some probability and must issue one of the three disclosure types. As in Liang et al. (2018), when the manager has discretion, we assume she seeks to maximize the long-term price P_T.[34]

Commitment to a disclosure choice. We first consider the case where a manager with discretion is able to commit to the type of the disclosure $x \in \{S, C, U\}$ prior to observing any private information. In this case, the manager chooses the disclosure complexity to maximize the expected long-term price of the firm evaluated at time 1, $\mathbb{E}_1(P_T)$. Because the manager does not have private information at this point, this expectation reduces to the unconditional mean of y_t, summed over each period. Consequently, the manager is indifferent over

[32] Repeated communication settings have been studied in Sobel (1985), Benabou and Laroque (1992), Stocken (2000), Morris (2001), Einhorn and Ziv (2008), Beyer and Dye (2012), and Liang et al. (2018). Closely related to these are models of dynamic voluntary disclosure, such as Acharya et al. (2011), Guttman et al. (2014), Marinovic and Varas (2016), Aghamolla and An (2021), and Bertomeu et al. (2022).

[33] The arguments that follow do not specifically rely on the private signals being *i.i.d.* over time.

[34] The economic forces we discuss in this section also apply when the manager's objective is to maximize the price of the firm in every period discounted back to the present, i.e., $u_m = \sum_{t=1}^{T} \delta^{t-1} P_t$, where $\delta \in (0, 1)$ is the manager's discount factor.

her complexity choice in this case. However, if investors are risk averse with respect to the firm's idiosyncratic payoffs, then the price will have a risk premium, where this premium is increasing (and the price decreasing) in investor uncertainty (Bertomeu and Cheynel (2015)). Thus, the manager has a strict preference to commit to the complexity choice that is the most informative to investors (which corresponds to complex informative (simple) disclosure when the investor base is sophisticated (unsophisticated)). These arguments demonstrate that the manager's inability to commit to disclosure complexity in our baseline setting is a key driver of our results.

No commitment. We next consider the case where, as in our baseline model, the manager cannot commit to a disclosure choice. Instead, she must choose the level of complexity in each period t after privately observing that period's signal y_t. In this case, the main novel force that arises in a dynamic setting is that investors can learn over time about whether the manager has discretion. A key determinant of this learning, and thus the equilibrium that arises, is how the disclosure choice of no-discretion managers changes over time. For example, suppose that no-discretion managers' complexity choice is fixed over time. This may be the case if these managers are constrained by their contracts to issue a specific type of disclosure in every period. In this case, any shift in a manager's complexity choice will reveal her to have discretion. As a result, following any such change, the disclosure equilibrium in each period will unravel.

More generally, when no-discretion managers' disclosure choice changes over time, investors' ability to learn about managers' discretion from their complexity choice is imperfect. They will revise beliefs about whether the manager has discretion in each period – for instance, if managers who have discretion are more likely to issue complex uninformative disclosure, following a string of such disclosures, investors will find it more likely the manager has discretion. Nevertheless, they will continue to face uncertainty, as it remains possible that this stemmed from a manager without discretion. As a result, rather than unravelling, the equilibrium in each period is likely to resemble that in our baseline model.

Importantly, it is reasonable to expect that, in many scenarios, the disclosure choice that no-discretion managers issue will change over time. For instance, even if the manager is contractually constrained to a specific disclosure policy in each period, the firm may decide to change this policy following shifts in its product market, the complexity of its business model, strategic direction, or the composition of its board. Likewise, if we interpret no-discretion managers as those who have limited ability to communicate complex information, we may expect their disclosure to vary in informativeness over time, as the firm's transactions in any given period vary in how easy they are to communicate. Similarly, managers who are sensitive to regulation and litigation concerns may not always issue complex disclosures.

The reason is that the specific transactions their firms undertake in any given period may be more or less subject to such concerns.

7.2 Rational Attention Choice

We consider a setting in which some investors can process complex disclosure while others cannot. Stated differently, whether an investor processes the disclosure is not a choice but an innate characteristic of the investor. This reflects inherent differences in investor sophistication that are fixed in the short term. For instance, consider asset managers who lack expertise in biotechnology. Such managers will be incapable of processing the details of a biotech firm's disclosure about the state of its clinical trials without devoting a costly, extended period of time to obtaining such expertise, after which point the disclosure is likely to have already made its way into price through other means. As such, one can simply model such managers as incapable of processing the disclosure.

In other cases, whether an investor processes a disclosure will be determined by whether they devote enough effort or attention to doing so in the short-term (e.g., Umar (2022)). This short-term attention decision would introduce an additional set of economic forces into equilibrium complexity choice that may be interesting for future research to analyze. Notably, investors' beliefs about how informative a disclosure is would influence whether they find it worthwhile to process the disclosure. For instance, if investors perceived complex disclosures by a certain class of firms to typically reflect information provision (obfuscation), they would likely find it optimal to process (not process) these disclosures. This, in turn, would affect the magnitude of the price reaction to complex versus simple disclosures, and thus the trade-offs that managers face in deciding on disclosure complexity. Lu (2022) considers an entropy-costs framework for studying attention choice in an exogenous disclosure setting that might be extended to study the interaction between attention costs and disclosure complexity choice.

7.3 Multiple Signals

Because investors' beliefs are ultimately what matter for determining stock prices, we model complexity via its impact on the beliefs of different investors. However, there is opportunity for future work to consider modeling the specific features of qualitative disclosures that determine how costly they are for investors to process, in conjunction with managerial discretion. Multidimensional voluntary disclosure models provide one potential starting point for doing so.[35] In these models, firms have multiple signals and can selectively choose which signals to

[35]See, e.g., Fishman and Hagerty (1990), Shin (2003), Pae (2005), Livdan and Nezlobin (2021), and Aghamolla and An (2022).

disclose. However, the provision of multiple information signals likely increases processing costs and resembles information complexity, a feature that has not been considered in prior studies. In addition, it is possible that managers can obfuscate by mingling uninformative signals together with informative signals.

8 Conclusion

Firm managers have considerable latitude in the level of complexity of their disclosures. The empirical literature has found mixed results concerning the informativeness of complex disclosures, which appear to be not only a means to obfuscate (e.g., Li (2008)), but also necessary to convey more precise information (e.g., Bushee et al. (2018)). In this paper, we develop a parsimonious model to help reconcile these conflicting findings and provide theoretical underpinnings for the notion of complexity in disclosure. Our results show that any equilibrium must take the form of a strategic complexity equilibrium, where both high-performing and low-performing firms complexify information, while intermediate-performing firms issue simple disclosures. Additionally, our results provide conditions under which we expect the market reaction to simple disclosure to be negative, in contrast to the conventional wisdom that bad news is more often complexified.

While framed in terms of financial disclosures, our model and findings apply more broadly to any form of strategic, technical communication. For example, researchers often present results to multiple audiences, only a fraction of whom understand the methods applied. In this case, researchers with unfavorable results may be inclined to present their methods in an obscure manner. At the same time, researchers with favorable results might likewise present in a seemingly obscure manner because doing so enables them to better communicate to the domain experts in the audience. Our results indicate that both patterns of behavior can arise in equilibrium, even when unsophisticated audiences rationally anticipate that researchers with unfavorable results may attempt to mislead them.

References

Acharya, V. V., P. DeMarzo, and I. Kremer (2011): "Endogenous information flows and the clustering of announcements," *American Economic Review*, 101, 2955–2979.

Aghamolla, C. and B.-J. An (2021): "Voluntary disclosure with evolving news," *Journal of Financial Economics*, 140, 21–53.

——— (2022): "Mandatory vs. Voluntary ESG Disclosure, Efficiency, and Real Effects," *Working paper*.

Aghamolla, C., C. Corona, and R. Zheng (2021): "No reliance on guidance: counter-signaling in management forecasts," *The RAND Journal of Economics*, 52, 207–245.

Avdis, E. and S. Banerjee (2019): "Clear and liquid: The interaction of firm disclosure and trader competition," *Available at SSRN 3250467*.

Banerjee, S. and B. Breon-Drish (2021): "Dynamics of Research and Strategic Trading," *The Review of Financial Studies*.

Banerjee, S., I. Marinovic, and K. Smith (2022): "Disclosing to informed traders," *Available at SSRN*.

Benabou, R. and G. Laroque (1992): "Using privileged information to manipulate markets: Insiders, gurus, and credibility," *The Quarterly Journal of Economics*, 107, 921–958.

Bertomeu, J., A. Beyer, and R. A. Dye (2011): "Capital structure, cost of capital, and voluntary disclosures," *The Accounting Review*, 86, 857–886.

Bertomeu, J. and E. Cheynel (2013): "Toward a positive theory of disclosure regulation: In search of institutional foundations," *The Accounting Review*, 88, 789–824.

——— (2015): "Disclosure and the cost of capital: A survey of the theoretical literature," 386–415.

Bertomeu, J., E. Cheynel, and D. Cianciaruso (2021): "Strategic withholding and imprecision in asset measurement," *Journal of Accounting Research*, 59, 1523–1571.

Bertomeu, J., I. Marinovic, S. J. Terry, and F. Varas (2022): "The dynamics of concealment," *Journal of Financial Economics*, 143, 227–246.

Beyer, A. (2009): "Capital market prices, management forecasts, and earnings management," *The Accounting Review*, 84, 1713–1747.

Beyer, A. and R. A. Dye (2012): "Reputation management and the disclosure of earnings forecasts," *Review of Accounting Studies*, 17, 877–912.

Blankespoor, E., E. deHaan, and I. Marinovic (2020): "Disclosure processing costs, investors' information choice, and equity market outcomes: A review," *Journal of Accounting and Economics*, 70, 101344.

Bloomfield, R. (2008): "Discussion of "annual report readability, current earnings, and earnings persistence"," *Journal of Accounting and Economics*, 45, 248–252.

Breon-Drish, B. (2015): "On existence and uniqueness of equilibrium in a class of noisy rational expectations models," *The Review of Economic Studies*, 82, 868–921.

Bushee, B. J., I. D. Gow, and D. J. Taylor (2018): "Linguistic complexity in firm disclosures: obfuscation or information?" *Journal of Accounting Research*, 56, 85–121.

Carlin, B. I. (2009): "Strategic price complexity in retail financial markets," *Journal of Financial Economics*, 91, 278–287.

Carlin, B. I., S. Kogan, and R. Lowery (2013): "Trading complex assets," *The Journal of Finance*, 68, 1937–1960.

Carlin, B. I. and G. Manso (2011): "Obfuscation, learning, and the evolution of investor sophistication," *The Review of Financial Studies*, 24, 754–785.

Chen, Q., C. Corona, and Y. Zhang (2020): "The Usefulness and Endogenous Supply of Disclosure Accessibility," *Available at SSRN 3294745*.

Chen, Q., T. R. Lewis, K. Schipper, and Y. Zhang (2017): "Uniform versus discretionary regimes in reporting information with unverifiable precision and a coordination role," *Journal of Accounting Research*, 55, 153–196.

Chychyla, R., A. J. Leone, and M. Minutti-Meza (2019): "Complexity of financial reporting standards and accounting expertise," *Journal of Accounting and Economics*, 67, 226–253.

Cohen, L., D. Lou, and C. Malloy (2013): "Playing favorites: How firms prevent the revelation of bad news," Tech. rep., National Bureau of Economic Research.

Cohen, L., C. Malloy, and Q. Nguyen (2020): "Lazy prices," *The Journal of Finance*, 75, 1371–1415.

deHaan, E., Y. Song, C. Xie, and C. Zhu (2020): "Disclosure Obfuscation in Mutual Funds," *Available at SSRN 3540215.*

Dewatripont, M. and J. Tirole (2005): "Modes of communication," *Journal of Political Economy*, 113, 1217–1238.

Diamond, D. W. (1985): "Optimal release of information by firms," *The Journal of Finance*, 40, 1071–1094.

Dye, R. A. (1985): "Disclosure of nonproprietary information," *Journal of Accounting Research*, 123–145.

——— (1998): "Investor sophistication and voluntary disclosures," *Review of Accounting Studies*, 3, 261–287.

Dyer, T., M. Lang, and L. Stice-Lawrence (2017): "The evolution of 10-K textual disclosure: Evidence from Latent Dirichlet Allocation," *Journal of Accounting and Economics*, 64, 221–245.

Dzielinski, M., A. F. Wagner, and R. J. Zeckhauser (2016): "In no (un) certain terms: Managerial style in communicating earnings news," *Working paper.*

Einhorn, E. (2018): "Competing information sources," *The Accounting Review*, 93, 151–176.

Einhorn, E. and A. Ziv (2008): "Intertemporal dynamics of corporate voluntary disclosures," *Journal of Accounting Research*, 46, 567–589.

Ellison, G. and A. Wolitzky (2012): "A search cost model of obfuscation," *The RAND Journal of Economics*, 43, 417–441.

Ertugrul, M., J. Lei, J. Qiu, and C. Wan (2017): "Annual report readability, tone ambiguity, and the cost of borrowing," *Journal of Financial and Quantitative Analysis*, 52, 811–836.

Fishman, M. J. and K. M. Hagerty (1990): "The optimal amount of discretion to allow in disclosure," *The Quarterly Journal of Economics*, 105, 427–444.

——— (2003): "Mandatory versus voluntary disclosure in markets with informed and uninformed customers," *Journal of Law, Economics, and Organization*, 19, 45–63.

Goldstein, I., J. Schneemeier, and L. Yang (2020): "Market Feedback: Who Learns What?" *Unpublished working paper.*

Graham, J. R., C. R. Harvey, and S. Rajgopal (2005): "The economic implications of corporate financial reporting," *Journal of Accounting and Economics*, 40, 3–73.

Gu, Y. and T. Wenzel (2014): "Strategic obfuscation and consumer protection policy," *The Journal of Industrial Economics*, 62, 632–660.

Guay, W., D. Samuels, and D. Taylor (2016): "Guiding through the fog: Financial statement complexity and voluntary disclosure," *Journal of Accounting and Economics*, 62, 234–269.

Guttman, I., I. Kremer, and A. Skrzypacz (2014): "Not only what but also when: A theory of dynamic voluntary disclosure," *American Economic Review*, 104, 2400–2420.

Guttman, I. and I. Marinovic (2018): "Debt contracts in the presence of performance manipulation," *Review of Accounting Studies*, 23, 1005–1041.

Hughes, J. S. and S. Pae (2004): "Voluntary disclosure of precision information," *Journal of Accounting and Economics*, 37, 261–289.

Jung, W.-O. and Y. K. Kwon (1988): "Disclosure when the market is unsure of information endowment of managers," *Journal of Accounting Research*, 146–153.

Kielty, P., K. P. Wang, and D. Weng (2022): "Simplifying complex disclosures: Evidence from disclosure regulation in the mortgage markets," *The Accounting Review.*

Kim, C., K. Wang, and L. Zhang (2018): "Readability of 10-K reports and stock price crash risk," *Contemporary Accounting Research.*

Kirschenheiter, M. and N. D. Melumad (2002): "Can "big bath" and earnings smoothing co-exist as equilibrium financial reporting strategies?" *Journal of Accounting Research*, 40, 761–796.

Kumar, P., N. Langberg, and K. Sivaramakrishnan (2016): "Voluntary disclosure with informed trading in the IPO market," *Journal of Accounting Research*, 54, 1365–1394.

Lang, M. and L. Stice-Lawrence (2015): "Textual analysis and international financial reporting: Large sample evidence," *Journal of Accounting and Economics*, 60, 110–135.

Langberg, N. and K. Sivaramakrishnan (2008): "Voluntary disclosures and information production by analysts," *Journal of Accounting and Economics*, 46, 78–100.

Lawrence, A. (2013): "Individual investors and financial disclosure," *Journal of Accounting and Economics*, 56, 130–147.

Lee, C. (2019): "Voluntary Disclosure of a Noisy Signal and Its Precision," Ph.D. thesis, New York University.

Lehavy, R., F. Li, and K. Merkley (2011): "The effect of annual report readability on analyst following and the properties of their earnings forecasts," *The Accounting Review*, 86, 1087–1115.

Li, F. (2008): "Annual report readability, current earnings, and earnings persistence," *Journal of Accounting and Economics*, 45, 221–247.

Li, F. et al. (2010): "Textual analysis of corporate disclosures: A survey of the literature," *Journal of Accounting Literature*, 29, 143–165.

Liang, P. J. and G. Zhang (2019): "On the social value of accounting objectivity in financial stability," *The Accounting Review*, 94, 229–248.

Liang, Y., I. Marinovic, and F. Varas (2018): "The credibility of financial reporting: A reputation-based approach," *The Accounting Review*, 93, 317–333.

Livdan, D. and A. Nezlobin (2021): "Bayesian Pricing of News," *Available at SSRN 3225448*.

Lo, K., F. Ramos, and R. Rogo (2017): "Earnings management and annual report readability," *Journal of Accounting and Economics*, 63, 1–25.

Loughran, T. and B. McDonald (2014): "Measuring readability in financial disclosures," *The Journal of Finance*, 69, 1643–1671.

Lu, J. (2022): "Limited attention: Implications for financial reporting," *Journal of Accounting Research*, 60, 1991–2027.

Marinovic, I. and F. Varas (2016): "No news is good news: Voluntary disclosure in the face of litigation," *The RAND Journal of Economics*, 47, 822–856.

Michaeli, B. (2017): "Divide and inform: Rationing information to facilitate persuasion," *The Accounting Review*, 92, 167–199.

Miller, B. P. (2010): "The effects of reporting complexity on small and large investor trading," *The Accounting Review*, 85, 2107–2143.

Morris, S. (2001): "Political correctness," *Journal of Political Economy*, 109, 231–265.

Myatt, D. P. and C. Wallace (2012): "Endogenous information acquisition in coordination games," *The Review of Economic Studies*, 79, 340–374.

Osterland, A. (2004): "The Best CFOs in America," *Institutional Investor*, February 14, 2004.

Pae, S. (2005): "Selective disclosures in the presence of uncertainty about information endowment," *Journal of Accounting and Economics*, 39, 383–409.

Penno, M. (1996): "Unobservable precision choices in financial reporting," *Journal of Accounting Research*, 34, 141–149.

Petrov, E. (2020): "Voluntary Disclosure and Informed Trading," *Contemporary Accounting Research*, forthcoming.

Securities and Exchange Commission (1998): *A plain English handbook: How to create clear SEC disclosure documents*, Office of Investor Education.

Shin, H. (2003): "Disclosures and asset returns," *Econometrica*, 71, 105–133.

Sobel, J. (1985): "A theory of credibility," *The Review of Economic Studies*, 52, 557–573.

Solomon, D. H. (2012): "Selective publicity and stock prices," *The Journal of Finance*, 67, 599–638.

Stocken, P. C. (2000): "Credibility of voluntary disclosure," *The RAND Journal of Economics*, 359–374.

Subramanyam, K. (1996): "Uncertain precision and price reactions to information," *The Accounting Review*, 207–219.

Teoh, S. H. and C. Y. Hwang (1991): "Nondisclosure and adverse disclosure as signals of firm value," *Review of Financial Studies*, 4, 283–313.

Titman, S. and B. Trueman (1986): "Information quality and the valuation of new issues," *Journal of Accounting and Economics*, 8, 159–172.

Umar, T. (2022): "Complexity aversion when seeking alpha," *Journal of Accounting and Economics*, 73, 101477.

Verrecchia, R. E. (1990): "Information quality and discretionary disclosure," *Journal of Accounting and Economics*, 12, 365–380.

——— (2001): "Essays on disclosure," *Journal of Accounting and Economics*, 32, 97–180.

You, H. and X.-j. Zhang (2009): "Financial reporting complexity and investor underreaction to 10-K information," *Review of Accounting Studies*, 14, 559–586.

Table 1: Glossary of Notation

y	Manager's private information relevant to firm value
$[y_L, y_H]$	Support of y
μ	Mean of y
$f(\cdot)$	Probability density function of y
$F(\cdot)$	Cumulative distribution function of y
χ	Fraction of investors that are sophisticated
Δ_S	Signal observed by all investors following simple disclosure
Δ_C	Signal observed by sophisticated investors following informative complex disclosure
Δ_U	Signal observed by sophisticated investors following complex uninformative disclosure
ρ_S	Probability that simple disclosure reveals y
ρ_U	Probability that complex uninformative disclosure reveals y
β	Probability that the manager does not have discretion over the disclosure policy
ω_S	Fraction of no-discretion managers who issue simple disclosure
ω_C	Fraction of no-discretion managers who issue informative complex disclosure
ω_U	Fraction of no-discretion managers who issue complex uninformative disclosure
P	Price of the firm after disclosure
x	Manager's disclosure choice
T_L	Lower disclosure threshold
T_H	Upper disclosure threshold

Appendix

A Proofs

A.1 Proof of Proposition 1

Conjecture a generic equilibrium in which, upon observing $y \in Y_x$, the manager chooses disclosure type x, where Y_U, Y_S, and Y_C are three disjoint sets (of which some may be empty) with $Y_U \cup Y_S \cup Y_C = [y_L, y_H]$. Let $\pi_U(y)$, $\pi_S(y)$, and $\pi_C(y)$ denote the manager's expected payoffs given each of the respective disclosure choices and let $\lambda(\mathcal{I}) = \int_{\mathcal{I}} f(t)\,dt$ denote the probability that $y \in \mathcal{I}$, for any measurable set $\mathcal{I}$. Note that:

$$\begin{aligned}
\pi_S(y) &= \rho_S y + (1-\rho_S)\frac{\beta\omega_S\mu + (1-\beta)\lambda(Y_S)\mathbb{E}(\tilde{y}|\tilde{y}\in Y_S)}{\beta\omega_S + (1-\beta)\lambda(Y_S)};\\
\pi_C(y) &= \chi y + (1-\chi)\frac{\beta(1-\omega_S)\mu + (1-\beta)\lambda(Y_C\cup Y_U)\mathbb{E}(\tilde{y}|\tilde{y}\in Y_C\cup Y_U)}{\beta(1-\omega_S) + (1-\beta)\lambda(Y_C\cup Y_U)};\\
\pi_U(y) &= \chi\left(\rho_U y + (1-\rho_U)\frac{\beta\omega_U\mu + (1-\beta)\lambda(Y_U)\mathbb{E}(\tilde{y}|\tilde{y}\in Y_U)}{\beta\omega_U + (1-\beta)\lambda(Y_U)}\right)\\
&\quad + (1-\chi)\frac{\beta(1-\omega_S)\mu + (1-\beta)\lambda(Y_C\cup Y_U)\mathbb{E}(\tilde{y}|\tilde{y}\in Y_C\cup Y_U)}{\beta(1-\omega_S) + (1-\beta)\lambda(Y_C\cup Y_U)}.
\end{aligned}$$

Now, in any equilibrium, Y_U cannot be empty. If it were, then one can follow the arguments of Jung and Kwon (1988) to show that, for some $\tau \in [y_H, y_L]$, we have $Y_S = [y_L, \tau]$ and $Y_C = (\tau, y_H]$. But, this implies that the final two terms in $\pi_U(y)$ are weakly greater than μ and the final term in $\pi_S(y)$ is weakly less than μ. Hence, we have:

$$\pi_U(y_L) - \pi_S(y_L) > (\rho_S - \chi)y_L + (1-\chi)\mu - (1-\rho_S)\mu = (\rho_S - \chi)(y_L - \mu) > 0.$$

Thus, by selecting $x = U$, a manager with y in a neighborhood of y_L strictly prefers to deviate from $x = S$ to $x = U$. Following analogous reasoning, it also cannot be the case in any equilibrium that Y_C is empty, or a manager with y in a neighborhood of y_H prefers to deviate from $x = S$ to $x = C$.

Next, because $\rho_S > \chi\rho_U$ and $\chi > \rho_S$, $\pi_S(y) - \pi_U(y)$ and $\pi_C(y) - \pi_S(y)$ are both increasing in y. Therefore, if Y_S is nonempty, $Y_U < Y_S < Y_C$, and if Y_S is empty, $Y_U < Y_C$. Thus, to complete the proof, we need only to show that Y_S cannot be empty in an equilibrium. Suppose by contradiction that there were an equilibrium in which $Y_S = \emptyset$ so that, for some $\tau \in [y_H, y_L]$, we have $Y_U = [y_L, \tau]$ and $Y_C = (\tau, y_H]$. Note that $\tau < \mu$, since, in such an equilibrium, for any $y > \mu$,

$$\pi_C(y) - \pi_U(y) = \chi(1-\rho_U)\left[y - \frac{\beta\omega_U\mu + (1-\beta)F(\tau)\mathbb{E}(\tilde{y}|\tilde{y} < \tau)}{\beta\omega_U + (1-\beta)F(\tau)}\right] > 0.$$

Now, note that, in such an equilibrium, $\mathbb{E}(\tilde{y}|\tilde{y} \in Y_C \cup Y_U) = \mu$, and thus:

$$\pi_C(y) = \chi y + (1-\chi)\mu;\ \pi_S(y) = \rho_S y + (1-\rho_S)\mu.$$

However, since $\tau < \mu$ and $\chi > \rho_S$, this implies that $\pi_C(\tau) < \pi_S(\tau)$ and thus manager type τ wishes to deviate to S.

A.2 Proof of Proposition 2

We first prove that there is no equilibrium in which $T_L > \mu$. Note this would imply:

$$\begin{aligned} & Q_{SU}(T_L, T_H) \\ = \; & (1-\rho_S)\left[\frac{\beta\omega_S\mu + (1-\beta)(F(T_H) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, T_H])}{\beta\omega_S + (1-\beta)(F(T_H) - F(T_L))} - T_L\right] \\ & -\chi(1-\rho_U)\left[\frac{\beta\omega_U\mu + (1-\beta)F(T_L)\mathbb{E}(\tilde{y}|\tilde{y} < T_L)}{\beta\omega_U + (1-\beta)F(T_L)} - T_L\right] \\ & -(1-\chi)\left[\frac{\beta(1-\omega_S)\mu + (1-\beta)(1-F(T_H)+F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, T_H])}{\beta(1-\omega_S) + (1-\beta)(1-F(T_H)+F(T_L))} - T_L\right] > 0, \end{aligned}$$

i.e., the equilibrium condition cannot be satisfied.

Part (i) To begin, we show that, $\forall T_L < \mu$, there exists a unique value $\gamma(T_L) \in (T_L, \mu)$ such that $Q_{SC}(T_L, \gamma(T_L)) = 0$, and that $\gamma(T_L)$ is continuous. Note:

$$\begin{aligned} Q_{SC}(T_L, X) \;=\; & (\rho_S - \chi)X + (1-\rho_S)\frac{\beta\omega_S\mu + (1-\beta)(F(X) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, X])}{\beta\omega_S + (1-\beta)(F(X) - F(T_L))} \\ & -(1-\chi)\frac{\beta(1-\omega_S)\mu + (1-\beta)(1-F(X)+F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, X])}{\beta(1-\omega_S) + (1-\beta)(1-F(X)+F(T_L))}. \end{aligned}$$

We have that:

$$\begin{aligned} & \lim_{X\to\mu} Q_{SC}(T_L, X) \\ = \; & (1-\rho_S)\left[\frac{\beta\omega_S\mu + (1-\beta)(F(\mu) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, \mu])}{\beta\omega_S + (1-\beta)(F(\mu) - F(T_L))} - \mu\right] \\ & -(1-\chi)\left[\frac{\beta(1-\omega_S)\mu + (1-\beta)(1-F(\mu)+F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, \mu])}{\beta(1-\omega_S) + (1-\beta)(1-F(\mu)+F(T_L))} - \mu\right] < 0. \end{aligned}$$

Next, given that $\rho_S < \chi$,

$$\begin{aligned}
&\lim_{X \to T_L} Q_{SC}(T_L, X) \\
= \ & (\rho_S - \chi) T_L + (1-\rho_S) \frac{\beta\omega_S\mu + (1-\beta)(F(T_L) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, T_L])}{\beta\omega_S + (1-\beta)(F(T_L) - F(T_L))} \\
& - (1-\chi) \frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(T_L) + F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, T_L])}{\beta(1-\omega_S) + (1-\beta)(1 - F(T_L) + F(T_L))} \\
= \ & (\rho_S - \chi)(T_L - \mu) > 0.
\end{aligned}$$

The existence of a $\gamma(T_L) \in (T_L, \mu)$ such that $Q_{SC}(T_L, \gamma(T_L)) = 0$ now follows by the intermediate value theorem. Next, in order to show that such a $\gamma(T_L)$ is unique, we show that $Q_{SC}(T_L, \gamma(T_L)) = 0$ implies that $\left\{\frac{\partial}{\partial X} Q_{SC}(T_L, X)\right\}_{X=\gamma(T_L)} < 0$. Notice that we can write:

$$\begin{aligned}
Q_{SC}(T_L, X) &= (1-\rho_S)\left(\frac{\beta\omega_S\mu + (1-\beta)(F(X) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, X])}{\beta\omega_S + (1-\beta)(F(X) - F(T_L))} - X\right) \\
&- (1-\chi)\left(\frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(X) + F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, X])}{\beta(1-\omega_S) + (1-\beta)(1 - F(X) + F(T_L))} - X\right).
\end{aligned}$$

Now, $\forall X \in (T_L, \mu)$, $\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, X]) > \mu$, and thus:

$$\frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(X) + F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, X])}{\beta(1-\omega_S) + (1-\beta)(1 - F(X) + F(T_L))} - X > 0.$$

Thus, $Q_{SC}(T_L, \gamma(T_L)) = 0$ implies that:

$$\begin{aligned}
&\frac{\beta\omega_S\mu + (1-\beta)(F(\gamma(T_L)) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, \gamma(T_L)])}{\beta\omega_S + (1-\beta)(F(\gamma(T_L)) - F(T_L))} - \gamma(T_L) \\
= \ & \frac{1-\chi}{1-\rho_S}\left[\frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(X) + F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, \gamma(T_L)])}{\beta(1-\omega_S) + (1-\beta)(1 - F(\gamma(T_L)) + F(T_L))} - \gamma(T_L)\right] > 0.
\end{aligned}$$

Now, notice that this implies:

$$\begin{aligned}
d_1 \ &\equiv \ \left\{\frac{\partial}{\partial X}\frac{\beta\omega_S\mu + (1-\beta)(F(X) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, X])}{\beta\omega_S + (1-\beta)(F(X) - F(T_L))}\right\}_{X=\gamma(T_L)} \\
&= \ \frac{(1-\beta) f(\gamma(T_L))}{\beta\omega_S + (1-\beta)(F(\gamma(T_L)) - F(T_L))} \times \\
& \quad \left[\gamma(T_L) - \frac{\beta\omega_S\mu + (1-\beta)(F(\gamma(T_L)) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, \gamma(T_L)])}{(\beta\omega_S + (1-\beta)(F(\gamma(T_L)) - F(T_L)))}\right] < 0.
\end{aligned}$$

Moreover,

$$\begin{aligned} d_2 &\equiv \left\{\frac{\partial}{\partial X}\frac{\beta\left(1-\omega_S\right)\mu+\left(1-\beta\right)\left(1-F\left(X\right)+F\left(T_L\right)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\notin\left[T_L,X\right]\right)}{\beta\left(1-\omega_S\right)+\left(1-\beta\right)\left(1-F\left(X\right)+F\left(T_L\right)\right)}\right\}_{X=\gamma(T_L)} \\ &= \left\{\frac{\partial}{\partial X}\frac{\beta\left(1-\omega_S\right)\mu+\left(1-\beta\right)\int_{t\notin[T_L,X]}tf\left(t\right)dt}{\beta\left(1-\omega_S\right)+\left(1-\beta\right)\left(1-F\left(X\right)+F\left(T_L\right)\right)}\right\}_{X=\gamma(T_L)} \\ &= \frac{\left(1-\beta\right)f\left(X\right)}{\beta\left(1-\omega_S\right)+\left(1-\beta\right)\left(1-F\left(\gamma\left(T_L\right)\right)+F\left(T_L\right)\right)}\times \\ &\quad \left[\frac{\beta\left(1-\omega_S\right)\mu+\left(1-\beta\right)\int_{t\notin[T_L,\gamma(T_L)]}tf\left(t\right)dt}{\beta\left(1-\omega_S\right)+\left(1-\beta\right)\left(1-F\left(\gamma\left(T_L\right)\right)+F\left(T_L\right)\right)}-\gamma\left(T_L\right)\right]>0. \end{aligned}$$

Therefore,

$$\left\{\frac{\partial}{\partial X}Q_{SC}\left(T_L,X\right)\right\}_{X=\gamma(T_L)}=\rho_S-\chi+d_1-d_2<0.$$

To see that $\gamma\left(X\right)$ is continuous, note that the implicit function theorem implies that for each T_L, $\gamma\left(T_L\right)$ is the unique solution X to $Q_{SC}\left(T_L,X\right)=0$ in a neighborhood of T_L and is continuous in this neighborhood. Applying this argument pointwise at each point T_L, we have that $\gamma\left(T_L\right)$ is globally continuous.

We next show that there exists an $X<\mu$ such that $Q_{SU}\left(X,\gamma\left(X\right)\right)=0$, which completes the proof of part i. Given that $\gamma\left(X\right)$ is continuous, we have that $Q_{SU}\left(X,\gamma\left(X\right)\right)$ is continuous, and thus need only to find two points less than μ on which $Q_{SU}\left(X,\gamma\left(X\right)\right)$ takes positive and negative values. Note:

$$\begin{aligned} & Q_{SU}\left(X,\gamma\left(X\right)\right) \\ =\; & \rho_S X+\left(1-\rho_S\right)\frac{\beta\omega_S\mu+\left(1-\beta\right)\left(F\left(\gamma\left(X\right)\right)-F\left(X\right)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\in\left[X,\gamma\left(X\right)\right]\right)}{\beta\omega_S+\left(1-\beta\right)\left(F\left(\gamma\left(X\right)\right)-F\left(X\right)\right)} \\ & -\chi\left(\rho_U X+\left(1-\rho_U\right)\frac{\beta\omega_U\mu+\left(1-\beta\right)F\left(X\right)\mathbb{E}\left(\tilde{y}|\tilde{y}<X\right)}{\beta\omega_U+\left(1-\beta\right)F\left(X\right)}\right) \\ & -\left(1-\chi\right)\frac{\beta\left(1-\omega_S\right)\mu+\left(1-\beta\right)\left(1-F\left(\gamma\left(X\right)\right)+F\left(X\right)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\notin\left[X,\gamma\left(X\right)\right]\right)}{\beta\left(1-\omega_S\right)+\left(1-\beta\right)\left(1-F\left(\gamma\left(X\right)\right)+F\left(X\right)\right)}. \end{aligned}$$

Note that, as $\gamma\left(X\right)\in\left(X,\mu\right)$, $\lim_{X\to\mu}\gamma\left(X\right)=\mu$. Therefore, $\lim_{X\to\mu}\mathbb{E}\left(\tilde{y}|\tilde{y}\in\left[X,\gamma\left(X\right)\right]\right)=\mathbb{E}\left(\tilde{y}|\tilde{y}\notin\left[X,\gamma\left(X\right)\right]\right)=\mu$ and:

$$\lim_{X\to\mu}Q_{SU}\left(X,\gamma\left(X\right)\right)=-\chi\left(1-\rho_U\right)\left[\frac{\beta\omega_U\mu+\left(1-\beta\right)F\left(\mu\right)\mathbb{E}\left(\tilde{y}|\tilde{y}<\mu\right)}{\beta\omega_U+\left(1-\beta\right)F\left(\mu\right)}-\mu\right]>0.$$

Next, we have:

$$\lim_{X \to y_L} \frac{\beta \omega_U \mu + (1-\beta) F(X) \mathbb{E}(\tilde{y}|\tilde{y} < X)}{\beta \omega_U + (1-\beta) F(X)} = \frac{\beta \omega_U \mu + \int_{y_L}^{y_L} t f(t) dt}{\beta \omega_U + (1-\beta) \int_{y_L}^{y_L} f(t) dt} = \mu. \tag{3}$$

Thus, substituting, we arrive at:

$$\begin{aligned} & \lim_{X \to y_L} Q_{SU}(X, \gamma(X)) \\ = \; & (\rho_S - \chi \rho_U) y_L + (1-\rho_S) \frac{\beta \omega_S \mu + (1-\beta)(F(\gamma(y_L)) - F(y_L)) \mathbb{E}(\tilde{y}|\tilde{y} \in [y_L, \gamma(y_L)])}{\beta \omega_S + (1-\beta)(F(\gamma(y_L)) - F(y_L))} \\ & -\chi(1-\rho_U)\mu - (1-\chi) \frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(\gamma(y_L)) + F(y_L)) \mathbb{E}(\tilde{y}|\tilde{y} \notin [y_L, \gamma(y_L)])}{\beta(1-\omega_S) + (1-\beta)(1 - F(\gamma(y_L)) + F(y_L))} \\ < \; & (\rho_S - \chi \rho_U) y_L + (1-\rho_S)\mu - \chi(1-\rho_U)\mu - (1-\chi)\mu \\ = \; & (\rho_S - \chi \rho_U)(y_L - \mu) < 0, \end{aligned}$$

where the inequality in the second-to-last line follows from the fact that $\mathbb{E}(\tilde{y}|\tilde{y} \in [y_L, \gamma(y_L)]) < \mu$ and $\mathbb{E}(\tilde{y}|\tilde{y} \notin [y_L, \gamma(y_L)]) > \mu$. This completes the proof.

Part (ii) Note that the equilibrium conditions are equivalent to:

$$Q_{SC}(T_L, T_H) = 0 \tag{4}$$

$$Q_{SU}(T_L, T_H) - Q_{SC}(T_L, T_H) = 0. \tag{5}$$

Let $\delta^* < \mu$ be the unique solution to:

$$\delta^* - \frac{\beta \omega_U \mu + (1-\beta) F(\delta^*) \mathbb{E}(\tilde{y}|\tilde{y} < \delta^*)}{\beta \omega_U + (1-\beta) F(\delta^*)} = 0, \tag{6}$$

i.e., δ^* is the value such that, given $T_L = \delta^*$, the expected firm value conditional on $x = U$ equals δ^*. As equation (6) takes the same form as the equilibrium condition in Jung and Kwon (1988), their arguments ensure its existence and uniqueness.

We proceed by proving two lemmas. The first lemma establishes that, fixing $T_H > \mu$, there is a $T_L \in (y_L, \mu)$ such that equilibrium condition (5) is satisfied. Moreover, this solution is continuous and converges pointwise in T_H to δ^* as ρ_S approaches χ. This, in turn, implies that, for any $T_H > \mu$, the expectation of $\tilde{y}$ given $\tilde{y} \in (\delta(T_H), T_H)$ converges to a well-defined limit as $\rho_S \to \chi^-$, which will be useful in a subsequent step.

Lemma 1. *There exists a unique, continuous function $\delta(T_H) : (\mu, y_H) \to (y_L, \mu)$ such that:*

$$Q_{SU}(\delta(T_H), T_H) - Q_{SC}(\delta(T_H), T_H) = 0.$$

Moreover, as $\rho_S \to \chi^-$, $\delta(T_H)$ converges pointwise in T_H to δ^* and thus:

$$\mathbb{E}\left[\tilde{y}|\tilde{y} \in \left(\delta\left(T_H\right), T_H\right)\right] \to \mathbb{E}\left[\tilde{y}|\tilde{y} \in \left(\delta^*, T_H\right)\right].$$

Proof of Lemma 1. Upon simplifying, we obtain:

$$\begin{aligned}
&Q_{SU}\left(X, T_H\right) - Q_{SC}\left(X, T_H\right) \\
&= \left(\rho_S - \chi\rho_U\right) X + \left(\chi - \rho_S\right) T_H - \chi\left(1 - \rho_U\right) \frac{\beta\omega_U\mu + (1-\beta) F(X) \mathbb{E}\left(\tilde{y}|\tilde{y} < X\right)}{\beta\omega_U + (1-\beta) F(X)}.
\end{aligned}$$

For any $T_H > \mu$, to see that there exists a $\delta(T_H) \in (y_L, \mu)$ such that $Q_{SU}(\delta(T_H), T_H) - Q_{SC}(\delta(T_H), T_H) = 0$, note first that, from (3) the final term in this expression approaches μ as $X \to y_L$. Thus,

$$\begin{aligned}
\lim_{X \to y_L} \left[Q_{SU}\left(X, T_H\right) - Q_{SC}\left(X, T_H\right)\right] &= (\rho_S - \chi\rho_U) y_L + (\chi - \rho_S) T_H - \chi(1-\rho_U)\mu \\
&= (\rho_S - \chi)(y_L - T_H) + \chi(1-\rho_U)(y_L - \mu) < 0.
\end{aligned}$$

Moreover, adding and subtracting $\chi(1-\rho_U)\mu$,

$$\begin{aligned}
&\lim_{X \to \mu^-} \left[Q_{SU}\left(X, T_H\right) - Q_{SC}\left(X, T_H\right)\right] \\
&= \left(\chi - \rho_S\right)\left(T_H - \mu\right) - \chi\left(1 - \rho_U\right)\left[\frac{\beta\omega_U\mu + (1-\beta) F(\mu) \mathbb{E}\left(\tilde{y}|\tilde{y} < \mu\right)}{\beta\omega_U + (1-\beta) F(\mu)} - \mu\right] > 0.
\end{aligned}$$

Thus, we have that $\delta(T_H)$ exists. Next, observe that:

$$\begin{aligned}
&\frac{\partial}{\partial X}\left[Q_{SU}\left(X, T_H\right) - Q_{SC}\left(X, T_H\right)\right] \\
&= \rho_S - \chi\rho_U - \frac{\chi(1-\rho_U)(1-\beta) f(X)}{\beta\omega_U + (1-\beta) F(X)}\left[X - \frac{\beta\omega_U\mu + (1-\beta) F(X) \mathbb{E}\left(\tilde{y}|\tilde{y} < X\right)}{\beta\omega_U + (1-\beta) F(X)}\right].
\end{aligned} \tag{7}$$

Now, note that $Q_{SU}(\delta(T_H), T_H) - Q_{SC}(\delta(T_H), T_H) = 0$ is equivalent to:

$$\chi(1-\rho_U)\left[\delta(T_H) - \frac{\beta\omega_U\mu + (1-\beta) F(\delta(T_H)) \mathbb{E}\left(\tilde{y}|\tilde{y} < \delta(T_H)\right)}{\beta\omega_U + (1-\beta) F(\delta(T_H))}\right] = (\chi - \rho_S)(\delta(T_H) - T_H) < 0,$$

and so:

$$\begin{aligned}
&\left\{\frac{\partial}{\partial X}\left[Q_{SU}\left(X, T_H\right) - Q_{SC}\left(X, T_H\right)\right]\right\}_{X = \delta(T_H)} \\
&= \rho_S - \chi\rho_U + \frac{(1-\beta) f(\delta(T_H))}{\beta\omega_U + (1-\beta) F(\delta(T_H))}(\chi - \rho_S)(T_H - \delta(T_H)) > 0.
\end{aligned} \tag{8}$$

Thus, by the implicit function theorem, we have that $\delta(T_H)$ is the unique, continuous solution to $Q_{SU}(\delta(T_H), T_H) - Q_{SC}(\delta(T_H), T_H)$ in a neighborhood of T_H. This argument applies at every point $T_H \in (\mu, y_H)$, which implies that, for $T_H \in (\mu, y_H)$, $\delta(T_H)$ is the unique, continuous solution to $Q_{SU}(\delta(T_H), T_H) - Q_{SC}(\delta(T_H), T_H) = 0$.

Finally, note that, as $\rho_S \to \chi^-$, the condition $Q_{SU}(X, T_H) - Q_{SC}(X, T_H) = 0$ converges to the condition satisfied by δ^*, i.e., equation (6), regardless of T_H. Note further that, given (8), we can once again apply the implicit function theorem, which tells us that, fixing X, $\delta(X)$ is a continuous function of ρ_S in a neighborhood of χ, and converges to δ^* as $\rho_S \to \chi^-$. Hence, $\delta(X)$ converges pointwise in X to δ^* as $\rho_S \to \chi^-$. □

The next lemma establishes that for the equilibrium conditions to be satisfied with $T_H > \mu$, it must be the case that the expected firm value given that the firm chooses simple disclosure exceeds μ. This both verifies the statement in the proposition and will be useful in a subsequent step in the proof.

Lemma 2. *For any $T_H > \mu$, we have:*

$$Q_{SC}(\delta(T_H), T_H) = 0 \Rightarrow \mathbb{E}(\tilde{y}|\tilde{y} \in [\delta(T_H), T_H]) > \mu.$$

Thus, in any equilibrium with $T_H > \mu$, $\mathbb{E}(\tilde{y}|\tilde{x} = S) > \mu > \mathbb{E}(\tilde{y}|\tilde{x} \in \{U, C\})$.

Proof of Lemma 2. Note first that, by adding and subtracting $(1-\chi)\mu$, we can rewrite $Q_{SC}(\delta(X), X)$ as follows:

$$\begin{aligned} &Q_{SC}(\delta(X), X) \\ &= (1-\chi)\left[\frac{\beta\omega_S\mu + (1-\beta)(F(X) - F(\delta(X)))\mathbb{E}(\tilde{y}|\tilde{y} \in [\delta(X), X])}{\beta\omega_S + (1-\beta)(F(X) - F(\delta(X)))} - \mu\right] \\ &- (1-\chi)\left[\frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(X) + F(\delta(X)))\mathbb{E}(\tilde{y}|\tilde{y} \notin [\delta(X), X])}{\beta(1-\omega_S) + (1-\beta)(1 - F(X) + F(\delta(X)))} - \mu\right] \\ &- (\chi - \rho_S)\left[X - \frac{\beta\omega_S\mu + (1-\beta)(F(X) - F(\delta(X)))\mathbb{E}(\tilde{y}|\tilde{y} \in [\delta(X), X])}{\beta\omega_S + (1-\beta)(F(X) - F(\delta(X)))}\right]. \end{aligned} \tag{9}$$

Now, suppose by contradiction that $\mathbb{E}(\tilde{y}|\tilde{y} \in [\delta(T_H), T_H]) < \mu$. Then, it is straightforward to verify that, for $X > \mu$, each of the three terms in (9) is negative. This contradicts the assumption that $Q_{SC}(\delta(T_H), T_H) = 0$. Finally, because $\mathbb{E}(\tilde{y}|\tilde{x} = S)$ is a weighted average of $\mathbb{E}(\tilde{y}|\tilde{y} \in [\delta(T_H), T_H])$ and μ, we have $\mathbb{E}(\tilde{y}|\tilde{x} = S) > \mu$. In turn, the property $\mu > \mathbb{E}(\tilde{y}|\tilde{x} \in \{U, C\})$ follows by iterated expectations. □

To conclude the proof, we now apply these lemmas to show that there exists a $T_H > \mu$ such that $Q_{SC}(\delta(T_H), T_H) = 0$ if any only if $\chi > \xi(\rho_S)$, for some increasing function $\xi(\cdot)$.

Note first that:

$$\begin{aligned}
&\lim_{X \to y_H} Q_{SC}(\delta(X), X) \\
&= (\rho_S - \chi) y_H + (1-\rho_S) \frac{\beta \omega_S \mu + (1-\beta)(F(y_H) - F(\delta(y_H))) \mathbb{E}(\tilde{y} | \tilde{y} \in [\delta(y_H), y_H])}{\beta \omega_S + (1-\beta)(F(y_H) - F(\delta(y_H)))} \\
&- (1-\chi) \frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(y_H) + F(\delta(y_H))) \mathbb{E}(\tilde{y} | \tilde{y} \notin [\delta(y_H), y_H])}{\beta(1-\omega_S) + (1-\beta)(1 - F(y_H) + F(\delta(y_H)))} \\
&= (1-\rho_S) \left(\frac{\beta \omega_S \mu + (1-\beta)(F(y_H) - F(\delta(y_H))) \mathbb{E}(\tilde{y} | \tilde{y} \in [\delta(y_H), y_H])}{\beta \omega_S + (1-\beta)(F(y_H) - F(\delta(y_H)))} - y_H \right) \\
&- (1-\chi) \left(y_H - \frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(y_H) + F(\delta(y_H))) \mathbb{E}(\tilde{y} | \tilde{y} \notin [\delta(y_H), y_H])}{\beta(1-\omega_S) + (1-\beta)(1 - F(y_H) + F(\delta(y_H)))} \right) < 0
\end{aligned}$$

Now, since $\delta(X)$ is continuous, the intermediate value theorem tells us that there exists a $T_H > \mu$ such that $Q_{SC}(\delta(T_H), T_H) = 0$ if and only if we can find a point $X^* > \mu$ such that $Q_{SC}(\delta(X^*), X^*) \geq 0$. Note by Lemma 2 we can further restrict attention to X^* such that:

$$X^* \in \mathcal{A} \equiv \{X : X \in (\mu, y_H), \mathbb{E}(\tilde{y} | \tilde{y} \in [\delta(X), X]) > \mu\}.$$

We next show the following four results:

Result 1. Fixing ρ_S, for χ sufficiently close to ρ_S, there exists a point $X^* \in \mathcal{A}$ such that $Q_{SC}(\delta(X^*), X^*) > 0$.

Result 2. Fixing ρ_S, for χ sufficiently close to 1, there does not exist a point $X^* \in \mathcal{A}$ such that $Q_{SC}(\delta(X^*), X^*) \geq 0$.

Result 3. $\forall X \in \mathcal{A}$, $Q_{SC}(\delta(X), X)$ strictly decreases in χ.

Result 4. $\forall X \in \mathcal{A}$, $Q_{SC}(\delta(X), X)$ strictly increases in ρ_S.

Together, these results complete the proof: the first three results imply that, fixing ρ_S, X^* exists if and only if χ is sufficiently close to 1, i.e., $\chi > \xi(\rho_S)$, for some function $\xi(\cdot)$. The fourth result implies that $\xi(\cdot)$ is strictly increasing.

Proof of Result 1. Consider $Q_{SC}(\delta(X), X)$ as expressed in (9). The final term in expression (9) approaches zero as $\rho_S \to \chi^-$. Thus, we need only show that, as $\rho_S \to \chi^-$, there exists an X such that the first two terms are positive. Note that, for any $X \in \mathcal{A}$, we have $X > \mathbb{E}(\tilde{y} | \tilde{y} \in [\delta(X), X]) > \mu$, and by iterated expectations, $\mathbb{E}(\tilde{y} | \tilde{y} \notin [\delta(X), X]) < \mu$, which ensures these terms are positive. Now, we must verify that $\mathcal{A}$ is non-empty when $\rho_S \to \chi^-$. Note that, for X sufficiently close to y_H, we have $\mathbb{E}(\tilde{y} | \tilde{y} \in [\delta^*, X]) > \mu$. Choosing such an X

and applying Lemma 1, we have that $\lim_{\rho_S \to \chi^-} \mathbb{E}\left(\tilde{y}|\tilde{y} \in [\delta(X), X]\right) = \mathbb{E}\left(\tilde{y}|\tilde{y} \in [\delta^*, X]\right) > \mu$, so that $X \in \mathcal{A}$ for ρ_S sufficiently close to χ.

Proof of Result 2. We have that:

$$
\begin{aligned}
&\lim_{\chi\to 1} Q_{SC}\left(\delta(X), X\right) \\
&= \lim_{\chi\to 1}\left\{(1-\rho_S)\left(\frac{\beta\omega_S\mu + (1-\beta)\left(F(\delta(X)) - F(X)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\in[\delta(X), X]\right)}{\beta\omega_S + (1-\beta)\left(F(\delta(X)) - F(X)\right)} - X\right)\right. \\
&\left. - (1-\chi)\left(\frac{\beta(1-\omega_S)\mu + (1-\beta)\left(1 - F(\delta(X)) + F(X)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\notin[\delta(X), X]\right)}{\beta(1-\omega_S) + (1-\beta)\left(1 - F(\delta(X)) + F(X)\right)} - X\right)\right\} \\
&\propto \frac{\beta\omega_S\mu + (1-\beta)\left(F(\delta(X)) - F(X)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\in[\delta(X), X]\right)}{\beta\omega_S + (1-\beta)\left(F(\delta(X)) - F(X)\right)} - X.
\end{aligned}
$$

For $X > \mu$, this is negative, and so the equilibrium condition can never be satisfied for χ sufficiently close to 1.

Proof of Results 3 and 4. We have that:

$$
\begin{aligned}
\frac{dQ_{SC}\left(\delta(X), X\right)}{d\rho_S} &= \frac{\partial Q_{SC}\left(\delta(X), X\right)}{\partial\rho_S} + \frac{\partial Q_{SC}\left(\delta(X), X\right)}{\partial\delta(X)}\frac{\partial\delta(X)}{\partial\rho_S}; \\
\frac{dQ_{SC}\left(\delta(X), X\right)}{d\chi} &= \frac{\partial Q_{SC}\left(\delta(X), X\right)}{\partial\chi} + \frac{\partial Q_{SC}\left(\delta(X), X\right)}{\partial\delta(X)}\frac{\partial\delta(X)}{\partial\chi}.
\end{aligned}
\tag{10}
$$

Calculating, we obtain:

$$
\begin{aligned}
\frac{\partial Q_{SC}\left(\delta(X), X\right)}{\partial\rho_S} &= X - \frac{\beta\omega_S\mu + (1-\beta)\left(F(X) - F(\delta(X))\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\in[\delta(X), X]\right)}{\beta\omega_S + (1-\beta)\left(F(X) - F(\delta(X))\right)} > 0 \\
\frac{\partial Q_{SC}\left(\delta(X), X\right)}{\partial\chi} &= \frac{\beta(1-\omega_S)\mu + (1-\beta)\left(1 - F(X) + F(\delta(X))\right)\mathbb{E}\left(\tilde{y}|\tilde{y}\notin[\delta(X), X]\right)}{\beta(1-\omega_S) + (1-\beta)\left(1 - F(X) + F(\delta(X))\right)} - X < 0,
\end{aligned}
\tag{11}
$$

since, for any $X \in \mathcal{A}$, we have $X > \mathbb{E}\left(\tilde{y}|\tilde{y} \in [\delta(X), X]\right) > \mu$, and by iterated expectations, $\mathbb{E}\left(\tilde{y}|\tilde{y} \notin [\delta(X), X]\right) < \mu < X$. Now, writing $\mathbb{E}\left(\tilde{y}|\tilde{y} \notin [\delta(X), X]\right)$ in its integral form yields:

$$
\begin{aligned}
\frac{\partial Q_{SC}\left(\delta(X), X\right)}{\partial\delta(X)} &= \frac{\partial}{\partial\delta(X)}\left(\frac{\beta\omega_S\mu + (1-\beta)\int_{\delta(X)}^{X} tf(t)\,dt}{\beta\omega_S + (1-\beta)\left(F(X) - F(\delta(X))\right)}\right) \\
&\propto \frac{\beta\omega_S\mu + (1-\beta)\int_{\delta(X)}^{X} tf(t)\,dt}{\beta\omega_S + (1-\beta)\left(F(X) - F(\delta(X))\right)} - \delta(X) > 0.
\end{aligned}
\tag{12}
$$

Next, we have that:

$$\frac{\partial\delta\left(X\right)}{\partial\rho_{S}}=-\frac{\frac{\partial}{\partial\rho_{S}}\left[Q_{SC}\left(\delta\left(X\right),X\right)-Q_{SU}\left(\delta\left(X\right),X\right)\right]}{\frac{\partial}{\partial\delta\left(X\right)}\left[Q_{SC}\left(\delta\left(X\right),X\right)-Q_{SU}\left(\delta\left(X\right),X\right)\right]}; \tag{13}$$
$$\frac{\partial\delta\left(X\right)}{\partial\chi}=-\frac{\frac{\partial}{\partial\chi}\left[Q_{SC}\left(\delta\left(X\right),X\right)-Q_{SU}\left(\delta\left(X\right),X\right)\right]}{\frac{\partial}{\partial\delta\left(X\right)}\left[Q_{SC}\left(\delta\left(X\right),X\right)-Q_{SU}\left(\delta\left(X\right),X\right)\right]}.$$

To sign these expressions, first recall from equation (8) that their denominator is negative. Moreover,

$$\frac{\partial\left[Q_{SC}\left(\delta\left(X\right),X\right)-Q_{SU}\left(\delta\left(X\right),X\right)\right]}{\partial\rho_{S}}=\delta\left(X\right)-X<0,$$

and:

$$\frac{\partial}{\partial\chi}\left[Q_{SC}\left(\delta\left(X\right),X\right)-Q_{SU}\left(\delta\left(X\right),X\right)\right] \tag{14}$$
$$=X-\delta\left(X\right)+\left(1-\rho_{U}\right)\left(\delta\left(X\right)-\frac{\beta\omega_{U}\mu+\left(1-\beta\right)F\left(\delta\left(X\right)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}<\delta\left(X\right)\right)}{\beta\omega_{U}+\left(1-\beta\right)F\left(\delta\left(X\right)\right)}\right). \tag{15}$$

Now, the equilibrium condition that $\delta(X)$ satisfies, $Q_{SC}\left(\delta\left(X\right),X\right)-Q_{SU}\left(\delta\left(X\right),X\right)=0$, can be expressed as:

$$\delta\left(X\right)-\frac{\beta\omega_{U}\mu+\left(1-\beta\right)F\left(\delta\left(X\right)\right)\mathbb{E}\left(\tilde{y}|\tilde{y}<\delta\left(X\right)\right)}{\beta\omega_{U}+\left(1-\beta\right)F\left(\delta\left(X\right)\right)}=\frac{\chi-\rho_{S}}{\chi\left(1-\rho_{U}\right)}\left(\delta\left(X\right)-X\right). \tag{16}$$

Combining equations (15) and (16) yields:

$$\frac{\partial}{\partial\chi}\left[Q_{SC}\left(\delta\left(X\right),X\right)-Q_{SU}\left(\delta\left(X\right),X\right)\right]=\frac{\rho_{S}}{\chi}\left(X-\delta\left(X\right)\right)>0.$$

Substituting these signs into the equations in (13) yields that $\frac{\partial\delta(X)}{\partial\rho_{S}}>0$ and $\frac{\partial\delta(X)}{\partial\chi}<0$. Together with (10), (11), and (12), this implies that, for $X\in\mathcal{A}$, $\frac{dQ_{SC}(\delta(X),X)}{d\rho_{S}}>0$ and $\frac{dQ_{SC}(\delta(X),X)}{d\chi}<0$, as desired.

A.3 Proof of Corollary 2

Suppose that the manager chooses $x = C$ for $\tilde{y} \in Y_C$ and $x = S$ for $\tilde{y} \in Y_S$, where $Y_C \cup Y_S = [y_L, y_H]$. Note that the manager's payoff from $x = C$ less that from $x = S$ is:

$$\pi_C(\tilde{y}) - \pi_S(\tilde{y}) = (\chi - \rho_S)\,\tilde{y} + (1-\chi)\,\frac{\beta(1-\omega_S)\,\mu + (1-\beta)\int_{Y_C} tf(t)\,dt}{\beta(1-\omega_S) + (1-\beta)\int_{Y_C} f(t)\,dt} - (1-\rho_S)\,\frac{\beta\omega_S\mu + (1-\beta)\int_{Y_S} tf(t)\,dt}{\beta\omega_S + (1-\beta)\int_{Y_S} f(t)\,dt}.$$

Clearly, $\pi_C(\tilde{y}) - \pi_S(\tilde{y})$ decreases in $\tilde{y}$, which implies any equilibrium is of the threshold type. Note a threshold equilibrium characterized by threshold T must satisfy $\pi_C(T) - \pi_S(T) = 0$. To see that a unique such equilibrium exists, with $T < \mu$, note:

$$\lim_{T\to\mu}[\pi_C(T) - \pi_S(T)] = (1-\chi)\left(\frac{\beta(1-\omega_S)\,\mu + (1-\beta)(1-F(\mu))\mathbb{E}(\tilde{y}|\tilde{y} > \mu)}{\beta(1-\omega_S) + (1-\beta)(1-F(\mu))} - \mu\right) - (1-\rho_S)\left(\frac{\beta\omega_S\mu + (1-\beta)F(\mu)\,\mathbb{E}(\tilde{y}|\tilde{y} < \mu)}{\beta\omega_S + (1-\beta)F(\mu)} - \mu\right) > 0.$$

Furthermore,

$$\lim_{T\to y_L}[\pi_C(T) - \pi_S(T)] = (1-\chi)(\mu - y_L) - (1-\rho_S)(\mu - y_L) = (\rho_S - \chi)(\mu - y_L) > 0.$$

Uniqueness follows as $\pi_C(T) - \pi_S(T)$ can be easily shown to be increasing. To complete the proof, we must show that the information communicated in this equilibrium is strictly greater than that in the equilibria in Proposition 2. To prove this, we show that $T < T_H$, where T_H is the upper threshold in a strategic complexity equilibrium. This completes the proof as it implies that, when the manager cannot obfuscate, she (a) chooses $x = S$ rather than $x = U$ for $y \in (y_L, T_L)$, and (b) chooses $x = C$ rather than $x = S$ for $y \in (T, T_H)$.

To see why we must have $T < T_H$, suppose $T = T_H$. Then, the beliefs of sophisticated (unsophisticated) investors when the manager chooses $x = S$ and the disclosure is uninformative are strictly greater when the manager can obfuscate than when she cannot. The reason is that the selection of signals that lead the manager to choose $x = S$ when she can obfuscate are (T_L, T_H), and when she cannot obfuscate are (y_L, T_H). But, this implies that, when the manager cannot obfuscate, she strictly prefers $x = S$ when $y = T_H$, which implies that we must have $T < T_H$.

A.4 Proof of Proposition 3

Note first that in any equilibrium, since sufficiently positive types always prefer S and sufficiently negative types always prefer U, it must be the case that there exist two possibly equal thresholds, T_L and T_H, such that types $t < T_L$ choose U and types $t > T_H$ choose S. Consequently, any equilibrium must take either the form in (i) or (ii). We next characterize when each of these equilibria exist.

Part (i) In such an equilibrium, we must have that the following two functions are zero, where the first function equals the relative payoffs of S to C of type T_H, and the second equals the relative payoffs of C to U to type T_L:

$$\begin{aligned} Q^d_{SC}(T_H) &\equiv (\rho_S - \chi) T_H + (1-\rho_S) \frac{\beta\omega_S\mu + (1-\beta)(1-F(T_H))\mathbb{E}(\tilde{y}|\tilde{y} > T_H)}{\beta\omega_S + (1-\beta)(1-F(T_H))} \\ &\quad - (1-\chi)\frac{\beta(1-\omega_S)\mu + (1-\beta)F(T_H)\mathbb{E}(\tilde{y}|\tilde{y} < T_H)}{\beta(1-\omega_S) + (1-\beta)F(T_H)}; \\ Q^d_{SU}(T_L) &\equiv \chi(1-\rho_U)\left(T_L - \frac{\beta\omega_U\mu + (1-\beta)F(T_L)\mathbb{E}(\tilde{y}|\tilde{y} < T_L)}{\beta\omega_U + (1-\beta)F(T_L)}\right). \end{aligned}$$

Note that $Q^d_{SC}(T_H) > 0$ for any $T_H \in (\mu, y_H)$, which implies there is no equilibrium in which $T_H > \mu$. Furthermore, note that:

$$\begin{aligned} \lim_{T_H \to \mu} Q^d_{SC}(T_H) = (1-\rho_S) &\frac{\beta\omega_S\mu + (1-\beta)(1-F(\mu))\mathbb{E}(\tilde{y}|\tilde{y} > \mu)}{\beta\omega_S + (1-\beta)(1-F(\mu))} \\ &- (1-\chi)\frac{\beta(1-\omega_S)\mu + (1-\beta)F(T_H)\mathbb{E}(\tilde{y}|\tilde{y} < \mu)}{\beta(1-\omega_S) + (1-\beta)F(\mu)} > 0. \end{aligned}$$

Moreover, $\lim_{T_H \to y_L} Q^d_{SC}(T_H) = (\rho_S - \chi)(y_L - \mu) < 0$. Thus, there is a $T_H < 0$ such that $Q^d_{SC}(T_H) = 0$. To aid in the proof of part (ii), let T^d_H equal the maximum of such zeroes (in (y_L, y_H)), if multiple exist.

Next, note that by the minimum principle of Acharya et al. (2011), there is a unique τ^* such that:

$$Q^d_{SU}(\tau^*) = \tau^* - \frac{\beta\omega_U\mu + (1-\beta)F(\tau^*)\mathbb{E}(\tilde{y}|\tilde{y} < \tau^*)}{\beta\omega_U + (1-\beta)F(\tau^*)} = 0$$

Moreover,

$$\left[\frac{\partial}{\partial\tau}\frac{\beta\omega_U\mu + (1-\beta)F(\tau)\mathbb{E}(\tilde{y}|\tilde{y} < \tau)}{\beta\omega_U + (1-\beta)F(\tau)}\right]_{\tau=\tau^*} = 0. \tag{17}$$

In order for an equilibrium of the type in part (i) to exist, in which $T_L < T_H$, it must be that $\tau^* < T^d_H$. To see why, note if this did not hold, then the only solution to the equilibrium condition that T_L must solve would lie above any solution to the equation that T_H must solve, and so there would be no solution (T_L, T_H) to the equilibrium conditions with $T_L < T_H$. Now, expression (17) implies that $\left[\frac{\partial}{\partial\tau}Q^d_{SU}(\tau)\right]_{\tau=\tau^*} > 0$, and thus:

$$\begin{aligned}\frac{\partial\tau^*}{\partial\beta\omega_U} &\propto -\frac{\partial}{\partial\beta\omega_U}\left[\tau^* - \frac{\beta\omega_U\mu + (1-\beta)F(\tau^*)\mathbb{E}(\tilde{y}|\tilde{y}<\tau^*)}{\beta\omega_U + (1-\beta)F(\tau^*)}\right] \\ &\propto \mu - \frac{\beta\omega_U\mu + (1-\beta)F(\tau^*)\mathbb{E}(\tilde{y}|\tilde{y}<\tau^*)}{\beta\omega_U + (1-\beta)F(\tau^*)} > 0.\end{aligned}$$

Moreover, we have that:

$$\begin{aligned}\lim_{\beta\omega_U\to 1}\left[\tau - \frac{\beta\omega_U\mu + (1-\beta)F(\tau)\mathbb{E}(\tilde{y}|\tilde{y}<\tau)}{\beta\omega_U + (1-\beta)F(\tau)}\right] &= \tau - \mu; \\ \lim_{\beta\omega_U\to 0}\left[\tau - \frac{\beta\omega_U\mu + (1-\beta)F(\tau)\mathbb{E}(\tilde{y}|\tilde{y}<\tau)}{\beta\omega_U + (1-\beta)F(\tau)}\right] &= \tau - \mathbb{E}(\tilde{y}|\tilde{y}<\tau) > 0.\end{aligned}$$

The first equation above tells us that $\lim_{\beta\omega_U\to 1}\tau^* = \mu$. The second equation tells us that, for any $\tau > y_L$, as $\beta\omega_U \to 0$, the equilibrium condition becomes strictly positive. This implies that $\lim_{\beta\omega_U\to 0}\tau^* = y_L$. Combining these facts, we have that $\exists Z \in (0,1)$ such that $\beta\omega_U < Z \implies \tau^* < T^d_H$ and $\beta\omega_U > Z \implies \tau^* > T^d_H$.

Part (ii) For such an equilibrium to exist, we need that the manager is indifferent between S and U upon observing $\tilde{y} = T$:

$$\begin{aligned}0 = \; & Q^s_{SC}(T) \equiv (\rho_S - \chi\rho_U)T + (1-\rho_S)\frac{\beta\omega_S\mu + (1-\beta)(1-F(T))\mathbb{E}(\tilde{y}|\tilde{y}>T)}{\beta\omega_S + (1-\beta)(1-F(T))} \\ & -(1-\chi)\frac{\beta(1-\omega_S)\mu + (1-\beta)F(T)\mathbb{E}(\tilde{y}|\tilde{y}<T)}{\beta(1-\omega_S) + (1-\beta)F(T)} \\ & -\chi(1-\rho_U)\frac{\beta\omega_U\mu + (1-\beta)F(T)\mathbb{E}(\tilde{y}|\tilde{y}<T)}{\beta\omega_U + (1-\beta)F(T)}, \end{aligned} \tag{18}$$

and does not prefer C to U:

$$Q^d_{SU}(T) = \chi(1-\rho_U)\left(T - \frac{\beta\omega_U\mu + (1-\beta)F(T)\mathbb{E}(\tilde{y}|\tilde{y}<T)}{\beta\omega_U + (1-\beta)F(T)}\right) \leq 0. \tag{19}$$

Observe first that each of the three final terms in (18) converge to μ as $T \to y_L$, and thus:

$$\lim_{X\to y_L} Q^s_{SC}(X) = (\rho_S - \chi\rho_U)(y_L - \mu) < 0.$$

Next, observe from the proof of part (i) that condition (19) holds if and only if $T \leq \tau^*$. Together, these results imply that, to complete the proof, we need only show that $Q^s_{SC}(\tau^*) \geq$

0. To see why this will complete the proof, note by the intermediate value theorem, it will yield there is a $\hat{T} \leq \tau^*$ such that $Q^s_{SC}\left(\hat{T}\right) = 0$. Moreover, since $\hat{T} \leq \tau^*$, $\hat{T}$ satisfies the necessary condition for an equilibrium (19).

To characterize when $Q^s_{SC}(\tau^*) \geq 0$, observe that we can write:

$$\begin{aligned} Q^s_{SC}(\tau^*) &= Q^d_{SC}(\tau^*) + Q^d_{SU}(\tau^*) \\ &= Q^d_{SC}(\tau^*), \end{aligned}$$

where the second line follows because $Q^d_{SU}(\tau^*) = 0$. Now, because T^d_H is the largest zero of $Q^d_{SC}(X)$ and because $Q^d_{SC}(X)$ is positive for X sufficiently large, we have that $\tau^* > T^d_H \implies Q^d_{SC}(\tau^*) > 0$. Recall from the proof of part (i), $\beta\omega_U > Z \implies T^d_H < \tau^*$. Combining these facts, we have that $\beta\omega_U > Z \implies Q^d_{SC}(\tau^*) > 0 \implies Q^s_{SC}(\tau^*) > 0$.

B Empirical Methodology

In this section, we discuss the details underlying the construction of Figure 1. We start by calculating $Complex\ Word_{i,t}$ as the number of words with three syllables or more in firm i's 10-Q filing in quarter t for each firm-quarter. Data for this measures comes from the WRDS SEC Analytics Suite database. For each of these three metrics, we then run the following regression:

$$\begin{aligned} Complexity_{i,t} = \beta_0 + \beta_1 Size_{t-1} + \beta_2 MTB_{t-1} + \beta_3 RetVol_{t-1} + \beta_4 Leverage_{t-1} \\ + IndustryFE + YearFE + \varepsilon_{i,t}, \quad (20) \end{aligned}$$

and obtain the residuals, $\varepsilon_{i,t}$, which capture the level of complexity controlling for size, market to book, leverage, volatility, three-digit SIC code industry fixed effects, and year fixed effects. We obtain these controls form Compustat, and we obtain the three-day abnormal return around the 10-Q filling date and over the subsequent quarter from CRSP. In untabulated analyses, we find that the non-monotonic relationship between complexity and contemporaneous and future returns is significant in quadratic regression analyses controlling for industry and year fixed effects, and Bushee et al. (2018)'s measure of non-discretionary complexity based on analyst conference call questions.

C Endogenous Price Function

In this appendix, we extend the analysis to the case in which sophisticated and unsophisticated investors have mean-variance preferences and the firm's price is determined by market clearing. We show that our key results continue to hold in this case. Specifically, we demonstrate that any equilibrium must be a strategic complexity equilibrium and that there always exists a strategic complexity equilibrium in which the price responds negatively to a simple disclosure.

For tractability, we assume further that unsophisticated investors do not learn from price. This is consistent with the information processing constraints that these investors face extending to their ability to interpret the firm's price. We conjecture that our results would continue to hold in a model with learning from price if there is a source of noise in price such as liquidity trade. The key feature of price for our results is that unsophisticated investors cannot perfectly back out the sophisticated investors' information. This should hold as long as there is noise in price.[36]

To do so, we make two minor adjustments to our assumptions. First, it is essential to have some residual source of uncertainty in the firm's cash flows, even if the disclosure is perfect. This ensures that the price is always influenced at least to some extent by the beliefs of unsophisticated investors. The reason is that, when the disclosure is informative, it perfectly reveals $\tilde{y}$. Hence, if $\tilde{y}$ is the only source of uncertainty, sophisticated investors would face no uncertainty given informative disclosure and any deviation of the price from their beliefs would generate risk-free arbitrage. We now denote the firm's cash flows as $\tilde{\theta}$ and assume they satisfy:

$$\tilde{\theta} = \tilde{y} + \tilde{\varepsilon},$$

where the firm disclosure concerns $\tilde{y}$ and $\tilde{\varepsilon}$ is a mean-zero noise term. The distribution of $\tilde{\varepsilon}$ is unimportant for the analysis, subject to having a positive, well-defined variance.

Second, we now assume that $\rho_S > \rho_U$, as opposed to $\rho_S > \chi\rho_U$ in the main text. Intuitively, given obfuscation, we will see that sophisticated investors' beliefs are, in expectation, more strongly impounded into prices than unsophisticated investors. The reason is that these investors face less uncertainty and thus trade more intensely on their information. We therefore impose a stricter standard on the relative informativeness of complex uninformative

[36]Note learning from price is challenging to incorporate in models with voluntary disclosure due to the lack of normality. Banerjee et al. (2022) study a model with voluntary disclosure and learning from price by applying the approach in Breon-Drish (2015). In general, this approach does not apply in our setting, where sophisticated investors' privately observe the disclosure rather than independent signals about cash flows. This implies that sophisticated investors' private information does not, in general, fit into the exponential family, which is necessary to apply Breon-Drish (2015)'s approach.

relative to simple disclosure to ensure the market reacts more weakly to it, in expectation.

We proceed by: (*i*) establishing the firm's equilibrium price and key properties of this price; (*ii*) showing that any equilibrium must be a strategic complexity equilibrium with $T_L < \mu$; and (*iii*) showing that there exists such an equilibrium with $T_H < \mu$.

Equilibrium price

The next lemma characterizes the firm's equilibrium price by solving for investors' optimal demands and applying market clearing.

Lemma 3. *The firm's equilibrium price given the disclosure choice x and the realized disclosure Δ_x satisfies:*

$$P = A(x, \Delta_x)\,\mathbb{E}_N(\tilde{y}) + (1 - A(x, \Delta_x))\,\mathbb{E}_I(\tilde{y}),$$

where $A(x, \Delta_x) \equiv \frac{(1-\chi)\mathbb{V}_I(\tilde{\theta})}{(1-\chi)\mathbb{V}_I(\tilde{\theta})+\chi\mathbb{V}_N(\tilde{\theta})}$. *Moreover,*

(i) $A(S, \Delta_S) = 1 - \chi$;

(ii) $1 - A(C, \Delta_C) > \chi$;

(iii) $A(C, y) = A(U, y)$*; in addition,* $A(C, y) = A(U, y)$ *does not depend upon* y.

Proof. In this case, standard derivations yield that, for $i \in \{I, N\}$, investors' demand functions satisfy:

$$D_i = \frac{\mathbb{E}_i\left(\tilde{\theta}\right) - P}{\gamma\mathbb{V}_i\left(\tilde{\theta}\right)} = \frac{\mathbb{E}_i(\tilde{y}) - P}{\gamma\mathbb{V}_i\left(\tilde{\theta}\right)},$$

and so the market-clearing condition is:

$$\begin{aligned}
0 &= \chi\frac{\mathbb{E}_I(\tilde{y}) - P}{\mathbb{V}_I\left(\tilde{\theta}\right)} + (1-\chi)\frac{\mathbb{E}_N(\tilde{y}) - P}{\mathbb{V}_N\left(\tilde{\theta}\right)} \\
\Leftrightarrow 0 &= \chi\mathbb{V}_N\left(\tilde{\theta}\right)\mathbb{E}_I(\tilde{y}) + (1-\chi)\mathbb{V}_I\left(\tilde{\theta}\right)\mathbb{E}_N(\tilde{y}) - \left(\chi\mathbb{V}_N\left(\tilde{\theta}\right) + (1-\chi)\mathbb{V}_I\left(\tilde{\theta}\right)\right)P \\
\Leftrightarrow P &= \frac{(1-\chi)\mathbb{V}_I\left(\tilde{\theta}\right)\mathbb{E}_N(\tilde{y}) + \chi\mathbb{V}_N\left(\tilde{\theta}\right)\mathbb{E}_I(\tilde{y})}{(1-\chi)\mathbb{V}_I\left(\tilde{\theta}\right) + \chi\mathbb{V}_N\left(\tilde{\theta}\right)}. \qquad (21)
\end{aligned}$$

Property 1 now follows since when $x = S$, $\mathbb{V}_I\left(\tilde{\theta}\right) = \mathbb{V}_N\left(\tilde{\theta}\right)$. To see that property 2 holds, note that regardless of the outcome of the disclosure, when $x = C$, $\mathbb{V}_N\left(\tilde{\theta}\right) = \mathbb{V}_N(\tilde{y}) + \mathbb{V}(\tilde{\varepsilon}) >$

$\mathbb{V}(\tilde{\varepsilon}) = \mathbb{V}_I\left(\tilde{\theta}\right)$. Property 3 follows because given that the firm's disclosure equals y when $x = C$ or $x = U$, we have that $\mathbb{V}_I\left(\tilde{\theta}\right) = \mathbb{V}(\tilde{\varepsilon})$ and $\mathbb{V}_N\left(\tilde{\theta}\right) = \mathbb{V}(\tilde{\varepsilon}) + \mathbb{V}(\tilde{y}|\tilde{x} \in \{U, C\})$. □

Note that $A(\cdot,\cdot)$ captures the relative weight on sophisticated and unsophisticated investors' beliefs. Naturally, $A(\cdot,\cdot)$ depends on the nature of the equilibrium. However, properties (i)-(iii) hold regardless of the equilibrium, and are the key features necessary for the results. Thus, for notational parsimony, we do not explicitly write out the dependence of $A(\cdot,\cdot)$ on the features of equilibrium below.

Uniqueness of strategic complexity equilibria

In this section, we verify that any equilibrium must be a strategic complexity equilibrium with $T_L < \mu$, following similar steps to the proof of Propositions 1 and 2 in the main text. Conjecture a generic equilibrium in which, upon observing $y \in Y_x$, the manager chooses disclosure type x, where Y_U, Y_S, and Y_C are three disjoint sets (of which some may be empty) with $[y_L, y_H] = Y_U \cup Y_S \cup Y_C$. Let $\pi_U(y)$, $\pi_S(y)$, and $\pi_C(y)$ denote the manager's expected payoffs given each of the respective disclosure choices. Note that given simple disclosure, investors' beliefs are aligned regardless of the disclosure, and so:

$$\pi_S(y) = \rho_S y + (1-\rho_S)\,\mathbb{E}(\tilde{y}|\tilde{x} = S).$$

Now, substituting for investors' beliefs given $x = C$ and $x = U$ we obtain:

$$\pi_C(y) = A(C,y)\,\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) + (1 - A(C,y))\,y;$$
$$\pi_U(y) = \begin{array}{c}(1-\rho_U)\left[A(U,\emptyset)\,\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) + (1-A(U,\emptyset))\,\mathbb{E}(\tilde{y}|\tilde{x} = U)\right] \\ +\rho_U\left[A(U,y)\,\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) + (1-A(U,y))\,y\right]\end{array}.$$

As in the main text, in any equilibrium, Y_C and Y_U must be non-empty. The reason is that, if one were to conjecture an equilibrium in which Y_C and/or Y_U were empty, then sufficiently high and low types would prefer C and U, respectively. Moreover, if Y_S is non-empty, $Y_U < Y_S < Y_C$, and if Y_S is empty, $Y_U < Y_C$. To complete the proof, we need only to show that Y_S cannot be empty in an equilibrium. Suppose by contradiction that there were an equilibrium in which $Y_S = \emptyset$. Then, we have $Y_U = [y_L, \tau]$ and $Y_C = (\tau, y_H]$. Note that

$\tau < \mu$, since, in such an equilibrium, for any $y > \mu$,

$$\pi_C(y) - \pi_U(y) = \begin{array}{c} \{A(C,y) - [\rho_U A(U,y) + (1-\rho_U) A(U,\emptyset)]\} \mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) \\ -(1-\rho_U)(1-A(U,\emptyset)) \mathbb{E}(\tilde{y}|x \in U) \\ +(1 - A(C,y) - \rho_U(1-A(U,y))) y \end{array}.$$

Applying property 3 in Lemma 3 and the fact that $\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) = \mu$, this reduces to:

$$\pi_C(y) - \pi_U(y) = (1-\rho_U)\{(1-A(C,y))(y-\mu) - (1-A(U,\emptyset))(\mathbb{E}(\tilde{y}|x \in U) - \mu)\}.$$

Since $\mathbb{E}(\tilde{y}|x \in U) < \mu < y$, this is positive. Next, note that, in such an equilibrium, $\mathbb{E}(\tilde{y}|\tilde{y} \in Y_C \cup Y_U) = \mu$, and thus:

$$\pi_C(y) = (1-A(C,y))y + A(C,y)\mu; \pi_S(y) = \rho_S y + (1-\rho_S)\mu,$$

so that:

$$\pi_C(\tau) - \pi_S(\tau) = (1 - A(C,y) - \rho_S)(\tau - \mu).$$

However, since $\tau < \mu$ and $1 - A(C,y) > \chi > \rho_S$, this implies that $\pi_C(\tau) < \pi_S(\tau)$ and thus manager type τ wishes to deviate to S. We next prove that there is no strategic complexity equilibrium in which $T_L > \mu$. Note this would imply:

$$\begin{aligned} & \pi_S(T_L) - \pi_U(T_L) \\ = & \begin{array}{c} \rho_S T_L + (1-\rho_S)\mathbb{E}(\tilde{y}|\tilde{x}=S) \\ -(1-\rho_U)[A(U,\emptyset)\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) + (1-A(U,\emptyset))\mathbb{E}(\tilde{y}|\tilde{x}=U)] \\ -\rho_U[A(U,y)\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) + (1-A(U,y))T_L] \end{array} \\ = & \begin{array}{c} (1-\rho_S)(\mathbb{E}(\tilde{y}|\tilde{x}=S) - T_L) \\ -(1-\rho_U)[A(U,\emptyset)\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) + (1-A(U,\emptyset))\mathbb{E}(\tilde{y}|\tilde{x}=U) - T_L] \\ -\rho_U[A(U,y)\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) + (1-A(U,y))T_L - T_L] \end{array}, \end{aligned} \tag{22}$$

where the second line adds and subtracts T_L. Now, note that:

$$\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\}) < T_L; \mathbb{E}(\tilde{y}|\tilde{x}=U) < T_L. \tag{23}$$

To see why, note that, if the manager had discretion, the firm's expected values given that $x \in \{U,C\}$ and $x = U$ are $\mathbb{E}(\tilde{y}|\tilde{y} \notin \{T_L, T_H\}) < \mu < T_L$ and $\mathbb{E}(\tilde{y}|\tilde{y} < T_L) < T_L$, respectively. Moreover, if she did not have discretion the firm's expected value is $\mu < T_L$. The result now follows because $\mathbb{E}(\tilde{y}|\tilde{x} \in \{U,C\})$ and $\mathbb{E}(\tilde{y}|\tilde{x} = U)$ are probability weighted averages between these two beliefs. Substituting (23) into (22), we obtain:

$$\begin{aligned}\pi_S(T_L)-\pi_U(T_L) > & \begin{array}{c}(1-\rho_S)\left(\mathbb{E}(\tilde{y}|\tilde{x}=S)-T_L\right)\\ -(1-\rho_U)\left[A(U,\emptyset)T_L+(1-A(U,\emptyset))T_L-T_L\right]\\ -\rho_U\left[A(U,y)T_L+(1-A(U,y))T_L-T_L\right]\end{array}\\ &= (1-\rho_S)\left(\mathbb{E}(\tilde{y}|\tilde{x}=S)-T_L\right)>0.\end{aligned}$$

Hence, managers that observe y marginally less than T_L strictly prefer to deviate to S from U.

Existence of a strategic complexity equilibrium

We now show that a strategic complexity equilibrium always exists by following analogous steps to the proof of Proposition 2. To begin, we show that, $\forall T_L<\mu$, there exists a unique value $\gamma(T_L)\in(T_L,\mu)$ such that $Q_{SC}(T_L,\gamma(T_L))=0$, and that $\gamma(T_L)$ is continuous. Note:

$$\begin{aligned}Q_{SC}(T_L,X) = & \begin{array}{c}\rho_S X+(1-\rho_S)\lim_{T_H\to X}\mathbb{E}(\tilde{y}|\tilde{x}=S)\\ -A(C,X)\lim_{T_H\to X}\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\})-(1-A(C,X))X\end{array}\\ &=(1-\rho_S)\left(\lim_{T_H\to X}\mathbb{E}(\tilde{y}|\tilde{x}=S)-X\right)-A(C,X)\left(\lim_{T_H\to X}\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\})-X\right).\end{aligned} \tag{24}$$

Now, observe that:

$$\lim_{T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}=S)<\mu;$$
$$\lim_{T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\})>\mu.$$

The first property follows because $\mathbb{E}(\tilde{y}|\tilde{x}=S)$ is a weighted average of $\mathbb{E}(\tilde{y}|\tilde{y}\in[T_L,\mu])<\mu$ and μ. The second property follows because $\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\})$ is a weighted average of $\mathbb{E}(\tilde{y}|\tilde{y}\notin[T_L,\mu])>\mu$ and μ. Together, these properties imply that (24) is negative when $X=\mu$. Next, given that $\rho_S<\chi$, and that $\lim_{T_H\to T_L}\mathbb{E}(\tilde{y}|\tilde{x}=S)=\lim_{T_H\to T_L}\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\})=\mu$,

$$\begin{aligned}&\lim_{X\to T_L}Q_{SC}(T_L,X)\\ &=(1-\rho_S)\left(\lim_{T_H\to T_L}\mathbb{E}(\tilde{y}|\tilde{x}=S)-T_L\right)-A(C,T_L)\left(\lim_{T_H\to T_L}\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\})-T_L\right)\\ &=(1-\rho_S-A(C,T_L))(\mu-T_L).\end{aligned}$$

This is positive given that $1-\rho_S > 1-\chi > A(C,y)$. The existence of a $\gamma(T_L) \in (T_L, \mu)$ such that $Q_{SC}(T_L, \gamma(T_L)) = 0$ now follows by the intermediate value theorem. Next, in order to show that such a $\gamma(T_L)$ is unique, we show that $Q_{SC}(T_L, \gamma(T_L)) = 0$ implies that:

$$\left\{\frac{\partial}{\partial X} Q_{SC}(T_L, X)\right\}_{X=\gamma(T_L)} < 0.$$

To see that this holds, notice that we can write:

$$Q_{SC}(T_L, X) = (1-\rho_S)(\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, X]) - X) - A(C,X)(\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, X]) - X).$$

Now, $\forall X \in (T_L, \mu)$, $\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, X]) > \mu$. Thus, $Q_{SC}(T_L, \gamma(T_L)) = 0$ implies that:

$$\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, \gamma(T_L)]) - \gamma(T_L) = \frac{A(C, \gamma(T_L))}{1-\rho_S}(\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, \gamma(T_L)]) - \gamma(T_L)) > 0$$

Now, notice that this implies:

$$\begin{aligned}
d_1 &\equiv \left\{\frac{\partial}{\partial X}\mathbb{E}(\tilde{y}|\tilde{x} = S)\right\}_{X=\gamma(T_L)} \\
&= \left\{\frac{\partial}{\partial X}\frac{\beta\omega_S\mu + (1-\beta)(F(X) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, X])}{\beta\omega_S + (1-\beta)(F(X) - F(T_L))}\right\}_{X=\gamma(T_L)} \\
&= \frac{(1-\beta) f(\gamma(T_L))}{\beta\omega_S + (1-\beta)(F(\gamma(T_L)) - F(T_L))} * \\
&\quad \left[\gamma(T_L) - \frac{\beta\omega_S\mu + (1-\beta)(F(\gamma(T_L)) - F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \in [T_L, \gamma(T_L)])}{(\beta\omega_S + (1-\beta)(F(\gamma(T_L)) - F(T_L)))}\right] < 0.
\end{aligned}$$

Moreover,

$$\begin{aligned}
d_2 &\equiv \left\{\frac{\partial}{\partial X}\mathbb{E}(\tilde{y}|\tilde{x} \in \{U, C\})\right\}_{X=\gamma(T_L)} \\
&= \left\{\frac{\partial}{\partial X}\frac{\beta(1-\omega_S)\mu + (1-\beta)(1 - F(X) + F(T_L))\mathbb{E}(\tilde{y}|\tilde{y} \notin [T_L, X])}{\beta(1-\omega_S) + (1-\beta)(1 - F(X) + F(T_L))}\right\}_{X=\gamma(T_L)} \\
&= \left\{\frac{\partial}{\partial X}\frac{\beta(1-\omega_S)\mu + (1-\beta)\int_{t \notin [T_L, X]} t f(t)\, dt}{\beta(1-\omega_S) + (1-\beta)(1 - F(X) + F(T_L))}\right\}_{X=\gamma(T_L)} \\
&= \frac{(1-\beta) f(\gamma(T_L))}{\beta(1-\omega_S) + (1-\beta)(1 - F(\gamma(T_L)) + F(T_L))} * \\
&\quad \left[\frac{\beta(1-\omega_S)\mu + (1-\beta)\int_{t \notin [T_L, \gamma(T_L)]} t f(t)\, dt}{\beta(1-\omega_S) + (1-\beta)(1 - F(\gamma(T_L)) + F(T_L))} - \gamma(T_L)\right] > 0.
\end{aligned}$$

Therefore, because $A(C, X)$ does not depend on X, we have:

$$\left\{\frac{\partial}{\partial X} Q_{SC}(T_L, X)\right\}_{X=\gamma(T_L)} = \rho_S - (1 - A(C, \gamma(T_L))) + d_1 - d_2 < 0.$$

To see that $\gamma(T_L)$ is continuous, note that the implicit function theorem implies that for each T_L, $\gamma(T_L)$ is the unique solution X to $Q_{SC}(T_L, X) = 0$ in a neighborhood of T_L and is continuous in this neighborhood. Applying this argument pointwise at each point T_L, we have that $\gamma(T_L)$ is globally continuous.

We next show that there exists an $X < \mu$ such that $Q_{SU}(X, \gamma(X)) = 0$, which completes the proof of part i. Given that $\gamma(X)$ is continuous, we have that $Q_{SU}(X, \gamma(X))$ is continuous, and thus need only to find two points less than μ such that $Q_{SU}(X, \gamma(X))$ takes positive and negative values. Note since $\gamma(X) \in (X, \mu)$, $\lim_{X \to \mu} \gamma(X) = \mu$, so:

$$\begin{aligned} &\lim_{X\to\mu} Q_{SU}(X, \gamma(X)) \\ &= \begin{matrix} \rho_S\mu + (1-\rho_S)\lim_{T_L,T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}=S) \\ -(1-\rho_U)\left[A(U,\emptyset)\lim_{T_L,T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\}) + (1-A(U,\emptyset))\lim_{T_L,T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}=U)\right] \\ -\rho_U\left[A(U,X)\lim_{T_L,T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\}) + (1-A(U,X))\mu\right] \end{matrix}. \end{aligned}$$

Now, $\lim_{X\to\mu}\mathbb{E}(\tilde{y}|\tilde{y}\in[X,\gamma(X)]) = \lim_{X\to\mu}\mathbb{E}(\tilde{y}|\tilde{y}\notin[X,\gamma(X)]) = \mu$ and so:

$$\lim_{T_L,T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}=S) = \lim_{T_L,T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}\in\{U,C\}) = \mu.$$

Substituting, we obtain:

$$\begin{aligned} \lim_{X\to\mu} Q_{SU}(X,\gamma(X)) &= \begin{matrix} \rho_S\mu + (1-\rho_S)\mu \\ -(1-\rho_U)\left[A(U,\emptyset)\mu + (1-A(U,\emptyset))\lim_{T_L,T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}=U)\right] \\ -\rho_U\left[A(U,X)\mu + (1-A(U,X))\mu\right] \end{matrix} \\ &= (1-\rho_U)\left[(1-A(U,\emptyset))\left(\mu - \lim_{T_L,T_H\to\mu}\mathbb{E}(\tilde{y}|\tilde{x}=U)\right)\right] > 0. \end{aligned}$$

Moreover, note that:

$$\begin{aligned} \lim_{T_L\to y_L}\mathbb{E}(\tilde{y}|\tilde{x}=U) &= \lim_{T_L\to y_L}\frac{\beta\omega_U\mu + (1-\beta)F(x)\mathbb{E}(\tilde{y}|\tilde{y}<T_L)}{\beta\omega_U + (1-\beta)F(x)} \\ &= \frac{\beta\omega_U\mu + \int_{y_L}^{y_L} tf(t)dt}{\beta\omega_U + (1-\beta)\int_{y_L}^{y_L} f(t)dt} = \mu. \end{aligned}$$

Thus, substituting, we arrive at:

$$\begin{aligned}\lim_{X\to y_L} Q_{SU}\left(X,\gamma\left(X\right)\right) = & \begin{array}{c}\left(\rho_S-\rho_U\left(1-A\left(U,X\right)\right)\right)y_L+\left(1-\rho_S\right)\mathbb{E}\left(\tilde{y}|\tilde{x}=S\right)\\ -\left(1-\rho_U\right)\left[A\left(U,\emptyset\right)\mathbb{E}\left(\tilde{y}|\tilde{x}\in\{U,C\}\right)+\left(1-A\left(U,\emptyset\right)\right)\mu\right]\\ -\rho_U A\left(U,X\right)\mathbb{E}\left(\tilde{y}|\tilde{x}\in\{U,C\}\right)\end{array}\\ & <\left(\rho_S-\rho_U\left(1-A\left(U,X\right)\right)\right)y_L+\left(1-\rho_S\right)\mu-\left(1-\rho_U\left(1-A\left(U,X\right)\right)\right)\mu\\ & =\left(\rho_S-\rho_U\left(1-A\left(U,X\right)\right)\right)\left(y_L-\mu\right)<\mu,\end{aligned}$$

where the inequality in the second-to-last line follows from the fact that $\mathbb{E}\left(\tilde{y}|\tilde{x}=S\right)<\mu<\mathbb{E}\left(\tilde{y}|\tilde{x}\in\{U,C\}\right)$. This completes the proof.