



Be ambitious or lower your expectation: Goals as optimal reference points

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ABSTRACT

When people undertake a task for themselves or the organization in which they work, they often set the target outcome of the task as their goal. We suggest that people take this goal as their reference point around which they evaluate their performance as a gain or loss. We also suggest that people derive utility from their goal as it can give consumption and/or signaling values. Incorporating these ideas into a simple model in which an agent optimally chooses her goal and effort, we show that the goal and effort chosen by the agent are complementary and that both high and low goals can be optimal for her. These results are consistent with the fact that some people set high goals and work hard, while others set low goals and settle down.

1. Introduction

When people undertake a task for themselves or the organization in which they work, they often set the target outcome of the task as their goal. Should people set their goal high? There exists conflicting advice on this inquiry. One can say that people should set a high goal because then they will work hard to accomplish it. “Boys, be ambitious,” an advice by William S. Clark (1826–1886) for his students, is still famous in Japan and inscribed on the monument stone in the campus of Hokkaido University (where Clark was the president). On the contrary, one can say that people should set a low goal because otherwise they will disappoint themselves when they fail. “Lowering your expectations” is one of the oldest tips to be happy.

Whether people follow this advice or not, as long as they set a goal, it is at the center of people’s choices. There are questions to ask: how does a goal affect the utility of people and their performance? More fundamentally, why do people set a goal? What roles does a goal play? In this paper, we provide a simple formal framework to answer these questions. We suggest that people set a goal and take it as their reference point around which they evaluate their performance as a gain or loss. We also suggest that people derive utility directly from the goal they set. People can have the consumption value of goal. People can be proud of or satisfy themselves by setting and pursuing a high goal. It is similar to the consumption value of self-confidence in Benabou and Tirole (2002) where confident people derive utility from thinking well of themselves. People can have the signaling value of goal as well. Pursuing a high goal can signal that they have ability to accomplish it. It is again similar to the signaling value of self-confidence in Benabou and Tirole (2002) where

confident people can easily convince others of their ability.

As examples, we can think of the cases where high school students set target universities when they study and scholars set target journals when they research and write papers. When pursuing these goals, they may feel happy by thinking of the universities they want to attend or the journals they want to publish. At the end, depending on whether or not they achieve their goals, they feel happy or unhappy. Thus goals give people consumption values and serve as reference points, which are independent of other people’s choices and outcomes.

We formalize this idea in a simple model where an agent (she) who has a reference-dependent preference with loss aversion undertakes a task. The agent first chooses a certain level of the outcome of the task as her goal, which becomes her reference point. The agent then makes an effort, which is either high or low, to complete the task. The agent’s effort determines the probability of the outcome being high. In our formal framework, we focus on the consumption value of goal for simplicity so that the agent derives utility directly from the goal she sets. The agent also derives utility from the outcome of the task she achieves and the deviation of the outcome from the goal (as a gain or loss).

Using this model, we analyze the agent’s choices of goal and effort. In the base model with binary outcomes (and therefore binary goals), we first show that the agent makes a high effort if she set a high goal and a low effort otherwise. Under a high goal, the agent may have a reference-dependent loss, not a gain, because the outcome cannot be higher than the goal. The probability of having the loss decreases with the agent’s effort because a higher effort makes the outcome less likely to be low. Thus, under a high goal, the agent with loss aversion makes a high effort in order to reduce the expected loss. A high goal motivates the agent to

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work hard. In contrast, under a low goal, the agent may have a reference-dependent gain, not a loss. As this gain is not as important as the loss of the same size for an agent with loss aversion, she makes a low effort. Having shown this complementarity between goal and effort, we next show that both high and low goals can be optimal for the agent. In particular, the agent sets a high goal if the consumption value of goal is high (or the cost of effort is low) and a low goal otherwise. Finally, we generalize the model to continuous outcome and goal to show that the results are robust. A higher goal gives a higher consumption value and motivation but a larger reference-dependent loss. In contrast, a lower goal gives a smaller reference-dependent loss but a lower consumption value and motivation. The agent optimally sets the level of goal to balance this tradeoff. As a result, the agent chooses a high goal with high effort or a low goal with low effort at the optimum. Some agents set an ambitious goal and work hard, while others set an ordinary goal and settle down.

Our paper is related to the growing literature on behavioral economics with reference-dependent preference. We suggest that people set goals as their reference points, which in turn affect their effort. In this regard, Wu et al. (2008) is closest to our paper as it provides a model in which an agent's goal as a reference point determines her effort. However, the goal is exogenously given to the agent in their model. Similarly, the goal is exogenously imposed by a manager in Gomez-Minambres (2012) and Corignet et al. (2015). There are a few papers in which an agent chooses a goal as her reference point. However, they do not consider the agent's effort choice. For instance, Hsiaw (2010, 2013) studies the agent's goal setting in the context of an optimal stopping problem over an infinite time horizon. Koch and Nafziger (2011, 2021) study the agent's goal setting in the context of a self-control problem. Brookins, Goerg, and Kube (2017) study the impact of a self-chosen goal on effort like our paper. But it is just an experimental study. Our paper can be seen as the one providing a formal model that can explain the experimental evidence in Brookins et al. (2017).

Our paper studies optimal reference points. Since Kahneman and Tversky (1979), the reference point has been one of the most important, but still unsettled topics in the prospect theory and its applications. While the reference point differs across the contexts of the studies, it has mostly been exogenous, or has not been a choice variable to agents in the literature.¹ Exceptions are Sarver (2012), Thakral and To (2020), Wikman (2020) in addition to the papers about self-chosen goals discussed above. They suggest that an agent's choice of anticipation forms an optimal reference point in a trade-off between loss aversion and anticipatory utility. They cast their models into the study of the agent's risk attitude in dynamic environments.

We study the provision of effort by an agent with reference-dependent preference. There are several papers related to ours in this regard such as effort provision by taxi drivers in Camerer et al (1997), bicycle messengers in Fehr and Goette (2007), professional golfers in Pope and Schweitzer (2011), income tax preparers in Rees-Jones (2018). These papers show that an agent makes a high effort in the loss domain and a low effort in the gain domain as in our paper. Notably some studies take target earnings or wages as reference points as in our paper.

¹ For instance, the reference point is initial wealth in gambles or risky choices (Kahneman and Tversky (1979) and Rabin (2000)), initial endowments in the studies of endowment effects (Knetsch (1989), Kahneman et al. (1990)), prior financial statements in financial investment (Benartzi and Thaler (1995)), purchase prices in house sales (Genesove and Mayer (2001)), and so on. All these reference points are exogenous. Koszegi and Rabin (2006, 2007) relax this pure exogeneity and suggest the agent's expectation on outcome as a reference point. As the agent's choice determines the outcome, their reference point is somewhat endogenous. Many papers have followed this modeling of reference point since then. However, as they require that the agent's expectation be rational, the agent cannot freely choose her expectation or reference point to maximize her utility.

Thakral and To (2021) suggest that the reference point for New York City taxi drivers is adjusted over time based on their previous earnings. Nevertheless, it is not a choice variable. The taxi drivers do not choose one or some of their previous earnings as their reference point.

2. Model

An agent (she) carries out a task for herself or the organization to which she belongs. To complete the task, the agent makes an effort $a \in \{0, 1\}$ at a cost of $C(a)$, which is normalized to 0 for $a = 0$ and $c > 0$ for $a = 1$. We can interpret $a = 0$ as low effort and $a = 1$ as high effort.

The outcome of the task is y . We first study the case of discrete outcome: $y \in \{y_H, y_L\}$, where $\Delta y \equiv y_H - y_L > 0$. We then move to the case of continuous outcome: $y \in Y = [0, \bar{y}]$. Here we present the base model of discrete outcome. In this model, whether the outcome is high ($y = y_H$) or low ($y = y_L$) depends on the agent's effort such that a high effort makes the outcome more likely to be high: $p_a \equiv \text{Prob}(y = y_H|a) \in (0, 1)$, where $\Delta p \equiv p_1 - p_0 > 0$. Let y_a be the expected outcome when the agent makes effort a : $y_a \equiv E[y|a] = p_a y_H + (1 - p_a) y_L$.

Before making an effort, the agent sets her target or goal g for the task. The agent chooses one of the outcomes as her goal: $g \in \{y_H, y_L\}$.

Let $U(g, a)$ be the agent's expected utility:

$$U(g, a) \equiv y_a + \eta[p_a(y_H - g) - \lambda(1 - p_a)(g - y_L)] + \theta g - C(a), \tag{1}$$

where $\eta > 0$, $\lambda > 1$, and $\theta \geq 0$. The agent has a standard utility from the outcome of the task (the first term in (1)). In addition, the agent has a reference-dependent utility where the reference point is the goal she sets (the second term in (1)). The agent takes it as a gain if the outcome is higher than her goal and a loss otherwise. Specifically, the agent will have a gain of $y_H - g$ with probability p_a and a loss of $g - y_L$ with probability $(1 - p_a)$. As in Koszegi and Rabin (2006), η is a parameter measuring the relative importance of reference dependence and λ is a loss aversion parameter. With $\lambda > 1$, the impact of the loss on her utility is larger than the gain of the same size. The agent has a utility from the goal she sets (the third term in (1)). The agent can be proud of herself or feel self-satisfaction by pursuing a high goal. That is, the goal gives the agent a consumption value. θ is a parameter capturing the relative importance of the consumption value of goal g . Finally, the agent has a disutility of effort (the fourth term in (1)).

3. Analysis

We characterize the agent's choices of goal and effort in this chapter. We will show that the goal and effort chosen by the agent are complementary and that both high and low goals can be optimal for her.

3.1. Discrete outcome and goal

Before analyzing the model of discrete outcome, we first present the results on the agent's optimal choices of effort (a^*) and goal (g^*) in Fig. 1.

There are four different cases at the optimum depending on c and θ . The agent chooses a low goal and high effort for small values of c and θ , and a high goal and low effort for large values of c and θ . She chooses a high goal and high effort or a low goal and low effort for intermediate values of c and θ . Below we show how to get these results.

We first solve the agent's choice of effort for the goal she set. Given $g \in \{y_H, y_L\}$, the agent chooses $a \in \{0, 1\}$ to maximize $U(g, a)$:

$$\max_a y_a + \eta[p_a(y_H - g) - \lambda(1 - p_a)(g - y_L)] + \theta g - C(a). \tag{2}$$

We solve this problem for $g = y_H$ first and then for $g = y_L$.

For $g = y_H$, the agent chooses $a = 1$ if

$$\begin{aligned} & U(g = y_H, a = 1) > U(g = y_H, a = 0) \\ \Leftrightarrow y_1 - \eta\lambda(1 - p_1)\Delta y - c > y_0 - \eta\lambda(1 - p_0)\Delta y \\ & \Leftrightarrow c < c_H \equiv (1 + \eta\lambda)\Delta p\Delta y. \end{aligned} \tag{3}$$

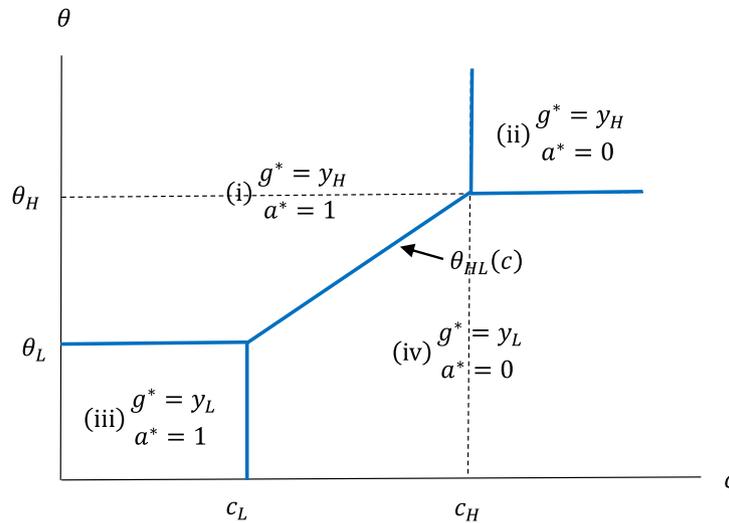


Fig. 1. The optimal choices of goal and effort.

The agent chooses $a = 0$ if $c > c_H$ and she is indifferent between $a = 1$ and $a = 0$ if $c = c_H$.

Compared to the low effort ($a = 0$), the high effort ($a = 1$) increases the expected outcome by $y_1 - y_0 = \Delta p \Delta y$, but it inflicts a cost of c to the agent. Given that the agent set a high goal, she will not have a reference-dependent gain because the outcome cannot be higher than the goal, which is her reference point ($y \leq g = y_H$). The agent will have a loss of $-y_L = \Delta y$ if the outcome is low. The agent is less likely to have the loss if she makes a higher effort. So compared to the low effort, the high effort decreases the expected loss by $\Delta p \Delta y$ whose impact on the agent's utility is $\eta \lambda \Delta p \Delta y$. Thus the agent makes a high effort if its benefits from the increase in the expected outcome ($\Delta p \Delta y$) and the decrease in the expected loss ($\eta \lambda \Delta p \Delta y$) are larger than its cost (c).

Similarly, for $g = y_L$, the agent chooses $a = 1$ if

$$\begin{aligned}
 &U(g = y_L, a = 1) > U(g = y_L, a = 0) \\
 &\Leftrightarrow y_1 + \eta p_1 \Delta y - c > y_0 + \eta p_0 \Delta y \\
 &\Leftrightarrow c < c_L \equiv (1 + \eta) \Delta p \Delta y.
 \end{aligned} \tag{4}$$

The agent chooses $a = 0$ if $c > c_L$ and she is indifferent between $a = 1$ and $a = 0$ if $c = c_L$.

Again, compared to the low effort, the high effort increases the expected outcome by $\Delta p \Delta y$ at a cost of c . Given that the agent set a low goal, she will not have a reference-dependent loss. She will have a gain of $y_H - g = \Delta y$ if the outcome is high. The agent is more likely to have the gain if she makes a higher effort. So compared to the low effort, the high effort increases the expected gain by $\Delta p \Delta y$ whose impact on the agent's utility is $\eta \Delta p \Delta y$. Thus the agent makes a high effort if its benefits from the increase in the expected outcome ($\Delta p \Delta y$) and the increase in the expected gain ($\eta \Delta p \Delta y$) are larger than its cost (c).

Note that the benefits of the high effort under the high goal are larger than under the low goal (i.e., $c_H > c_L$) because the decrease in the expected loss under the high goal ($\eta \lambda \Delta p \Delta y$) is larger than the increase in the expected gain under the low goal ($\eta \Delta p \Delta y$) due to loss aversion ($\lambda > 1$). We can then summarize the results on the agent's effort choice as in the following lemma.

Lemma 1. *The agent's choice of effort a^* is as follows:*

$$a^* = 1 \quad \forall g \in \{y_H, y_L\} \text{ if } c < c_L$$

$$a^* = \begin{cases} 1 & \text{for } g = y_H \\ 0 \text{ or } 1 & \text{for } g = y_L \end{cases} \text{ if } c = c_L$$

$$a^* = \begin{cases} 1 & \text{for } g = y_H \\ 0 & \text{for } g = y_L \end{cases} \text{ if } c_L < c < c_H$$

$$a^* = \begin{cases} 0 \text{ or } 1 & \text{for } g = y_H \\ 0 & \text{for } g = y_L \end{cases} \text{ if } c = c_H$$

$$a^* = 0 \quad \forall g \in \{y_H, y_L\} \text{ if } c > c_H$$

Regardless of the goal she set, the agent makes a high effort for low costs of effort ($c < c_L$), and a low effort for high costs of effort ($c > c_H$). For intermediate costs of effort ($c_L < c < c_H$), the agent makes a high effort if she set a high goal and a low effort otherwise.

Anticipating this choice of effort a^* , the agent chooses $g \in \{y_H, y_L\}$ to maximize $U(g, a)$:

$$\max_g y_a + \eta [p_a (y_H - g) - \lambda (1 - p_a) (g - y_L)] + \theta g - C(a) \quad \text{s.t. } a = a^* \tag{5}$$

We solve this problem according to the size of c : (i) $c < c_L$, (ii) $c > c_H$, (iii) $c_L < c < c_H$, (iv) $c = c_L$, (v) $c = c_H$.

For $c < c_L$, recall that the agent will always make a high effort ($a^* = 1$). From (5), the agent chooses $g = y_H$ if

$$\begin{aligned}
 &U(g = y_H, a = 1) > U(g = y_L, a = 1) \\
 &\Leftrightarrow y_1 - \eta \lambda (1 - p_1) \Delta y + \theta y_H - c > y_0 + \eta p_0 \Delta y + \theta y_L - c \\
 &\Leftrightarrow \theta > \theta_L \equiv \eta [p_1 + \lambda (1 - p_1)].
 \end{aligned} \tag{6}$$

The agent chooses $g = y_L$ if $\theta < \theta_L$ and she is indifferent between $g = y_H$ and $g = y_L$ if $\theta = \theta_L$.

As the agent will always make a high effort, her choice of goal affects neither the expected outcome nor the cost of effort. However, it affects the reference-dependent utility as the goal becomes the reference point. The agent can have a reference-dependent loss if she sets a high goal ($g = y_H$) and a gain if she sets a low goal ($g = y_L$). Compared to the low goal, the high goal turns the gain into the loss, which decreases the expected reference-dependent utility by $\eta [p_1 + \lambda (1 - p_1)] \Delta y$. Of course, compared to the low goal, the high goal increases the agent's consumption value of goal by $\theta \Delta y$. Thus the agent sets a high goal if the increase in the consumption value of goal ($\theta \Delta y$) is larger than the decrease in the expected reference-dependent utility ($\eta [p_1 + \lambda (1 - p_1)] \Delta y$).

Similarly, for $c > c_H$, as $a^* = 0$ for all $g \in \{y_H, y_L\}$, the agent chooses $g = y_H$ if

$$\begin{aligned}
 &U(g = y_H, a = 0) > U(g = y_L, a = 0) \\
 &\Leftrightarrow y_0 - \eta \lambda (1 - p_0) \Delta y + \theta y_H > y_0 + \eta p_0 \Delta y + \theta y_L \\
 &\Leftrightarrow \theta > \theta_H \equiv \eta [p_0 + \lambda (1 - p_0)].
 \end{aligned} \tag{7}$$

The agent chooses $g = y_L$ if $\theta < \theta_H$ and she is indifferent between $g = y_H$ and $g = y_L$ if $\theta = \theta_H$.

Again, if the agent sets a high goal, she can have a reference-

dependent loss instead of a gain, but enjoy a higher consumption value of goal. Thus the agent's choice of goal is qualitatively the same as in the case where $c < c_L$ discussed above, except that the decrease in the expected reference-dependent utility is now valued at the low effort. As $p_0 < p_1$, the decrease in the expected reference-dependent utility valued at the low effort $(\eta[p_0 + \lambda(1 - p_0)]\Delta y)$ is larger than that valued at the high effort $(\eta[p_1 + \lambda(1 - p_1)]\Delta y)$, indicating that $\theta_H > \theta_L$.

For $c_L < c < c_H$, recall that $a^* = 1$ if $g = y_H$ and $a^* = 0$ if $g = y_L$. With this effort decision, the agent chooses $g = y_H$ if

$$\begin{aligned} & U(g = y_H, a = 1) > U(g = y_L, a = 0) \\ \Leftrightarrow & y_1 - \eta\lambda(1 - p_1)\Delta y + \theta y_H - c > y_0 + \eta p_0 \Delta y + \theta y_L \\ \Leftrightarrow & \theta > \theta_{HL}(c) \equiv \eta[p_0 + \lambda(1 - p_1)] + \frac{c}{\Delta y} - \Delta p. \end{aligned} \tag{8}$$

The agent chooses $g = y_L$ if $\theta < \theta_{HL}(c)$ and she is indifferent between $g = y_H$ and $g = y_L$ if $\theta = \theta_{HL}(c)$.

The agent will make a high effort if she sets a high goal, and a low effort otherwise. Compared to the low goal, the high goal increases the expected outcome at a cost of c . Of course, with the high goal, the agent can have a reference-dependent loss instead of a gain, but enjoy a higher consumption value of goal.

For $c = c_L$, recall that $a^* = 1$ if $g = y_H$ and $a^* = 0$ if $g = y_L$. If the agent chooses $a^* = 1$ for $g = y_L$, her choice of goal is the same as the one in the case of $c < c_L$. If the agent chooses $a^* = 0$ for $g = y_L$, her choice of goal is the same as the one in the case of $c_L < c < c_H$.

Finally, for $c = c_H$, recall that $a^* = 0$ or 1 if $g = y_H$ and $a^* = 0$ if $g = y_L$. If the agent chooses $a^* = 0$ for $g = y_L$, her choice of goal is the same as the one in the case of $c > c_H$. If the agent chooses $a^* = 1$ for $g = y_L$, her choice of goal is the same as the one in the case of $c_L < c < c_H$.

With $\theta_H > \theta_L$ as explained above, note that $\theta_{HL}(c)$ linearly increases from θ_L to θ_H as c increases from c_L to c_H . We can get $\theta_{HL}^{-1}(\theta) = (\theta + \Delta p)\Delta y - \eta[p_0 + \lambda(1 - p_1)]\Delta y$. We can then summarize the results on the agent's choice of goal as in the following lemma.

Lemma 2. *The agent's choice of goal g^* is as follows:*

$$g^* = y_L \quad \forall c > 0 \text{ if } \theta < \theta_L$$

$$g^* = \begin{cases} y_H \text{ or } y_L \text{ for } c \leq c_L & \text{if } \theta = \theta_L \\ y_L \text{ for } c > c_L \end{cases}$$

$$g^* = \begin{cases} y_H \text{ for } c < \theta_{HL}^{-1}(\theta) \\ y_H \text{ or } y_L \text{ for } c = \theta_{HL}^{-1}(\theta) & \text{if } \theta_L < \theta < \theta_H \\ y_L \text{ for } c > \theta_{HL}^{-1}(\theta) \end{cases}$$

$$g^* = \begin{cases} y_H \text{ for } c < c_H & \text{if } \theta = \theta_H \\ y_H \text{ or } y_L \text{ for } c \geq c_H \end{cases}$$

$$g^* = y_H \quad \forall c > 0 \text{ if } \theta > \theta_H.$$

Regardless of the costs of effort and therefore the choices of effort (recall from Lemma 1 that the agent's choice of effort depends on the cost of effort), the agent sets a low goal if the consumption value of goal is small ($\theta < \theta_L$) and a high goal if it is large ($\theta > \theta_H$). For the intermediate consumption values of goal ($\theta_L < \theta < \theta_H$), the agent sets a high goal if the cost of effort is small (so that she will make a high effort) and a low goal otherwise.

From Lemmas 1 and 2, we can gather the agent's optimal choices of goal and effort together and illustrate them in Fig. 1. There are four different cases at the optimum: (i) $g^* = y_H, a^* = 1$, (ii) $g^* = y_H, a^* = 0$, (iii) $g^* = y_L, a^* = 1$, (iv) $g^* = y_L, a^* = 0$. In terms of the agent's choices of goal and effort, it appears that they are complementary in cases (i) and (iv), and substitutional in cases (ii) and (iii). However, from Lemmas 1 and 2, we can see that there is no substitution between goal and effort. In case (ii), the agent sets a high goal but make a low effort. The

agent sets a high goal regardless of her choice of effort because the consumption value of goal is so high. The agent makes a low effort regardless of her choice of goal because the cost of effort is high. In case (iii), the agent sets a low goal but makes a high effort. The agent sets a low goal regardless of her choice of effort because the consumption value of goal is so low. The agent makes a high effort regardless her choice of goal because the cost of effort is so low. Thus there is no interaction between goal and effort if the consumption value of goal is so high ($\theta > \theta_H$) or so low ($\theta < \theta_L$), or if the cost of effort is so high ($c > c_H$) or so low ($c < c_L$).

However, there is an interaction between goal and effort for intermediate values of goal ($\theta_L < \theta < \theta_H$) and intermediate costs of effort ($c_L < c < c_H$). For these values and costs, the agent sets a different level of goal depending on the effort she will choose later. To put in the other way, the agent makes a different level of effort depending on the goal she chose earlier. In particular, the agent sets a high goal and makes a high effort in case (i), and a low goal and a low effort in case (iv).

Proposition 1. *For $\theta_L < \theta < \theta_H$ and $c_L < c < c_H$, the goal and effort chosen by the agent are complementary. In particular, the agent sets a high goal ($g^* = y_H$) and makes a high effort ($a^* = 1$) if $\theta > \theta_{HL}(c)$ (equivalently if $c < \theta_{HL}^{-1}(\theta)$), and she sets a low goal ($g^* = y_L$) and makes a low effort ($a^* = 0$) if $\theta < \theta_{HL}(c)$ (equivalently if $c > \theta_{HL}^{-1}(\theta)$). The agent chooses either high goal - high effort or low goal - low effort if $\theta = \theta_{HL}(c)$ (equivalently if $c = \theta_{HL}^{-1}(\theta)$).*

If the agent sets a high goal, as this goal becomes her reference point, she will have a reference-dependent loss when the outcome turns out to be low. To reduce the probability of having the loss, the agent has an incentive to make a high effort. The high goal motivates the agent to work hard. However, this motivational impact of the high goal is not enough for the agent if the consumption value of goal is low (i.e., if $\theta < \theta_{HL}(c)$). In this case, the agent settles for a low goal. With the low goal, she will not have a reference-dependent loss and she may have a gain if the outcome turns out to be high. Of course, the low goal demotivates the agent because she will not have any loss even if she makes a low effort.

3.2. Continuous outcome and goal

We have shown that the goal and effort chosen by the agent are complementary and that both high and low goals can be optimal for her. One may wonder if these results stem from the model of binary outcomes. As the outcome is either high or low and therefore the goal is also either high or low, the agent has only either a gain or a loss, not both.

To check the robustness of the results, we extend the base model into continuous outcome. The outcome of the task is now $y \in Y = [0, \bar{y}]$ with a cumulative distribution function of $F(y|a)$ and its probability density function of $f(y|a)$. As standard, we assume first-order stochastic dominance for the distribution function such that $F(y|a = 0) > F(y|a = 1) \forall y \in (0, \bar{y})$. Recall that in the base model, the result of the complementarity between goal and effort arises for intermediate values of θ and c . To show this complementarity, the analysis here focuses on intermediate values of θ and c : $\eta < \theta < \eta\lambda$ and $(1 + \eta)(y_1 - y_0) < c < (1 + \eta\lambda)(y_1 - y_0)$, where $y_a = \int_0^{\bar{y}} yf(y|a)dy$ for $a \in \{0, 1\}$.

Let $V(g, a)$ be the agent's expected utility "relevant" for her effort decision: $V(g, a) \equiv U(g, a) - \theta g$. As the agent already chose her goal, she can ignore the consumption value of goal (θg) when she makes effort decision. Given $g \in Y$, the agent chooses $a \in \{0, 1\}$ to maximize $V(g, a)$:

$$\max_a y_a + \eta \left[\int_g^{\bar{y}} (y - g)f(y|a)dy - \lambda \int_0^g (g - y)f(y|a)dy \right] - C(a). \tag{9}$$

Note that the level of goal chosen by the agent affects her decision on

effort through its impact on her reference-dependent utility. To see this, we check the shape of $V(g, a)$ with respect to g . $V(g, a)$ decreases with g because she is more likely to have a reference-dependent loss (and she will have a larger loss) instead of a gain if she set a higher goal. More importantly, it decreases faster if the agent makes a lower effort because it makes the loss more likely to occur. For the intermediate values of c , we can show that $V(g, a = 1)$ and $V(g, a = 0)$ cross at a single point as in Fig. 2 (see the proof of Lemma 3 in the appendix).

$V(g, a = 0)$ is larger for small values of g , whereas $V(g, a = 1)$ is larger for large values of g . We can then summarize the agent's choice of effort as in the following lemma.

Lemma 3. For $(1 + \eta)(y_1 - y_0) < c < (1 + \eta\lambda)(y_1 - y_0)$, there exists $\hat{y} \in (0, \bar{y})$ such that the agent's choice of effort a^* is as follows:

$$a^* = \begin{cases} 1 & \text{for } g \geq \hat{y} \\ 0 & \text{for } g < \hat{y} \end{cases}$$

where \hat{y} is defined such that $V(g = \hat{y}, a = 1) - V(g = \hat{y}, a = 0) \equiv 0$.

Proof: In the appendix.

The agent makes a high effort if she set a goal higher than the threshold ($g \geq \hat{y}$) and a low effort otherwise ($g < \hat{y}$). The agent is more likely have a reference-dependent loss (and a larger loss) if she set a higher goal. A high goal motivates the agent to work hard in order to reduce the chances of having a reference-dependent loss (and the size of the loss).

Anticipating this choice of effort a^* , the agent chooses $g \in Y$ to maximize $U(g, a)$:

$$\max_g y_a + \eta \left[\int_g^{\bar{y}} (y - g)f(y|a)dy - \lambda \int_0^g (g - y)f(y|a)dy \right] + \theta g - C(a) \text{ s.t. } a = a^* \tag{10}$$

Ignoring the constraint ($a = a^*$), the first-order condition for (10) is $-\eta[1 + (\lambda - 1)F(g|a)] + \theta = 0$. (11)

For $\eta < \theta < \eta\lambda$, we have an interior solution, defined as $g_a \in Y$. It is the goal that maximizes the agent's ex-ante utility for $a \in \{0, 1\}$. Taking the constraint ($a = a^* \in \{0, 1\}$) into account, we can summarize the agent's choice of goal in the following lemma.

Lemma 4. For $\eta < \theta < \eta\lambda$, the agent's choice of goal g^* is as follows:

$$g^* = \begin{cases} g_0 & \text{for } a = 0 \\ g_1 & \text{for } a = 1 \end{cases}$$

where g_a is defined such that $\eta[1 + (\lambda - 1)F(g_a|a)] \equiv \theta$.

Proof: In the appendix.

As $F(g|a)$ is increasing and first-order stochastic dominant ($F(g|a = 0) > F(g|a = 1)$), from the first-order condition in (11), the utility maximizing goal is higher if the agent makes a high effort:

$$g_0 < g_1. \tag{12}$$

We can then show that the goal and effort chosen by the agent are complementary.

Proposition 2. With continuous outcome, for $\eta < \theta < \eta\lambda$ and $(1 + \eta)(y_1 - y_0) < c < (1 + \eta\lambda)(y_1 - y_0)$, the goal and effort chosen by the agent are complementary such that she sets a high goal ($g^* = g_1 > \hat{y}$) and makes a high effort ($a^* = 1$), or she sets a low goal ($g^* = g_0 < \hat{y}$) and makes a low effort ($a^* = 0$). In particular, $g^* = g_1$ and $a^* = 1$ if $\hat{y} \leq g_0$, and $g^* = g_0$ and $a^* = 0$ if $\hat{y} > g_1$. If $g_0 < \hat{y} \leq g_1$, $g^* = g_1$ and $a^* = 1$ for $c \leq \hat{c}$, and $g^* = g_0$ and $a^* = 0$ for $c > \hat{c}$, where $\hat{c} \equiv u(g = g_1, a = 1) - u(g = g_0, a = 0)$ and $u(g, a) \equiv U(g, a) - C(a)$.

Proof: In the appendix.

With continuous outcome and goal, the agent will have a gain or a loss depending on whether the outcome is higher or lower than the goal. The agent will be more likely to have a loss than a gain (and a loss larger than a gain) if she set a higher goal. Thus a higher goal motivates the agent to work hard in order to reduce the expected reference-dependent loss. One can advise the agent to "be ambitious" because then she will work hard to accomplish the goal. In doing so, the agent can reduce the loss that she will feel when she fails.

In contrast, a lower goal demotivates the agent as it makes the expected reference-dependent loss smaller. Nevertheless, a low goal can still be optimal because the agent is more likely to end up the gain domain. That is, the agent "lowers her expectation" and settles for a low goal.

Finally, we briefly look at the case of continuous effort to show that the result of the complementarity between goal and effort can continue to hold. With continuous effort $a \in [0, \infty)$ and continuous goal $g \in [0, \bar{y}]$, we can show that $\frac{\partial a^*}{\partial g} > 0$ if the agent is sufficiently loss-averse.

The agent chooses $a \in [0, \infty)$ to maximize $V(g, a)$:

$$\max_a \int_0^{\bar{y}} yf(y|a)dy + \eta \left[\int_g^{\bar{y}} (y - g)f(y|a)dy - \lambda \int_0^g (g - y)f(y|a)dy \right] - C(a). \tag{13}$$

The optimal effort a^* satisfies the first-order condition for this problem:

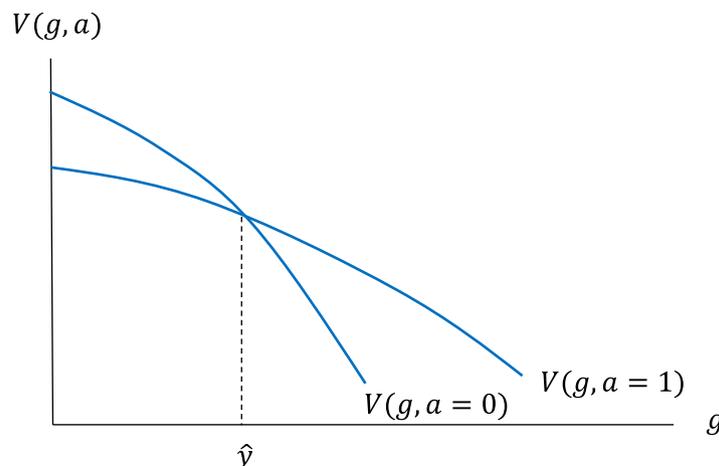


Fig. 2. $V(g, a)$ for intermediate values of c .

$$\int_0^{\bar{y}} y f_a(y|a^*) dy + \eta \left[\int_g^{\bar{y}} (y-g) f_a(y|a^*) dy - \lambda \int_0^g (g-y) f_a(y|a^*) dy \right] - C'(a^*) = 0. \tag{14}$$

From this, using the envelope theorem gives

$$\frac{\partial a^*}{\partial g} = -\eta [1 + (\lambda - 1) F_a(g|a^*)]. \tag{15}$$

From this, we can see that $\frac{\partial a^*}{\partial g} > 0$ if $\lambda > 1 - \frac{1}{F_a(g|a^*)}$, where $F_a(g|a^*) < 0$ due to the first-order stochastic dominance. It can lead to the result of the complementarity between goal and effort at the optimum.

4. Conclusion

When undertaking a task, people often set the target outcome of the task as their goal. In this paper, we suggest that people take this goal as their reference point around which they evaluate the realized outcome as a gain or loss. We also suggest that people derive utility directly from

the goal they set as it can give consumption and/or signaling values. We introduce a simple formal model in which an agent optimally chooses her goal and effort for her task. We show that the goal and effort chosen by the agent are complementary and that both low and high goals can be optimal for the agent.

There can be several extensions and applications of our model. We focus on the consumption value of goal when modeling the utility derived from the goal. We can extend our model to incorporate the signaling value of goal. For instance, we can make the outcome of a task dependent on an agent’s ability. Then the agent may set a high goal as it can signal her ability to others. Our model has a single player who determines both goal and effort. We can extend our model into principal-agent frameworks. For instance, we can introduce a setting where an agent makes an effort to undertake a task for the organization in which she works, and the principal sets the target outcome.

Data availability

No data was used for the research described in the article.

Appendix

A1. Proof of Lemma 3

The agent’s utility relevant for her effort decision is

$$V(g, a) = y_a + \eta \left[\int_g^{\bar{y}} (y-g) f(y|a) dy - \lambda \int_0^g (g-y) f(y|a) dy \right] - C(a). \tag{A.1}$$

As we restrict c such that $(1 + \eta)(y_1 - y_0) < c < (1 + \eta\lambda)(y_1 - y_0)$, we have

$$\begin{cases} V(g, a = 1) > V(g, a = 0) \text{ for } g = \bar{y} \\ V(g, a = 1) < V(g, a = 0) \text{ for } g = 0. \end{cases} \tag{A.2}$$

Taking the first and second derivatives of $V(g, a)$ with respect to g gives

$$\frac{\partial V}{\partial g} = -\eta [1 + (\lambda - 1) F(g|a)] < 0, \tag{A.3}$$

$$\frac{\partial^2 V}{\partial g^2} = -\eta(\lambda - 1) f(g|a) < 0. \tag{A.4}$$

With the first-order stochastic dominance of $F(g|a)$,

$$\left| \frac{\partial V(g, a = 0)}{\partial g} \right| > \left| \frac{\partial V(g, a = 1)}{\partial g} \right|. \tag{A.5}$$

(A.3) and (A.4) indicate that $V(g, a)$ monotonically decreases with g , and (A.5) indicates that the slope of $V(g, a)$ is steeper for $a = 0$ than for $a = 1$. Then with (A.2) there exists $\hat{y} \in (0, \bar{y})$ such that

$$\begin{cases} V(g, a = 1) \geq V(g, a = 0) \text{ for } g \geq \hat{y} \\ V(g, a = 1) < V(g, a = 0) \text{ for } g < \hat{y}. \end{cases} \tag{A.6}$$

where \hat{y} is defined such that $V(g = \hat{y}, a = 1) = V(g = \hat{y}, a = 0)$. Assuming that the agent prefers $a = 1$ to $a = 0$ when she is indifferent, (A.6) gives

$$a^* = \begin{cases} 1 \text{ for } g \geq \hat{y} \\ 0 \text{ for } g < \hat{y}. \end{cases}$$

A2. Proof of Lemma 4

The agent’s utility when she sets her goal is

$$U(g, a) = y_a + \eta \left[\int_g^{\bar{y}} (y-g) f(y|a) dy - \lambda \int_0^g (g-y) f(y|a) dy \right] + \theta g - C(a). \tag{A.7}$$

Taking the first and second derivative of $U(g, a)$ with respect to g gives

$$\frac{\partial U}{\partial g} = -\eta[1 + (\lambda - 1)F(g|a)] + \theta \begin{matrix} > \\ < \end{matrix} 0 \tag{A.8}$$

$$\frac{\partial^2 U}{\partial g^2} = -\eta(\lambda - 1)f(g|a) < 0. \tag{A.9}$$

As we restrict θ such that $\eta < \theta < \eta\lambda$, from (A.8) we have

$$\begin{cases} \frac{\partial U(g, a)}{\partial g} < 0 \text{ for } g = \bar{y} \\ \frac{\partial U(g, a)}{\partial g} > 0 \text{ for } g = 0. \end{cases} \tag{A.10}$$

With (A.9) and (A.10), there is a unique $g \in Y$ for $a \in \{0, 1\}$ that maximizes (A.7), defined as g_a :

$$-\eta[1 + (\lambda - 1)F(g_a|a)] + \theta \equiv 0. \tag{A.11}$$

Then

$$g^* = \begin{cases} g_0 \text{ for } a = 0 \\ g_1 \text{ for } a = 1. \end{cases}$$

A3. Proof of Proposition 2

Due to the first-order stochastic dominance of $F(g|a)$, from (A.11) we have

$$g_0 < g_1. \tag{A.12}$$

As $U(g, a) = V(g, a) + \theta g$, from (A.6) we have

$$\begin{cases} U(g, a = 1) \geq U(g, a = 0) \text{ for } g \geq \hat{y} \\ U(g, a = 1) < U(g, a = 0) \text{ for } g < \hat{y}. \end{cases} \tag{A.13}$$

For $\hat{y} > g_1$, $a^* = 0$ from (A.6) and $g^* = g_0$ from (A.13). For $\hat{y} \leq g_0$, $a^* = 1$ from (A.6) and $g^* = g_1$ from (A.13). For $g_0 < \hat{y} \leq g_1$, $g^* = g_0$ and $a^* = 0$ if $U(g = g_1, a = 1) < U(g = g_0, a = 0)$, and $g^* = g_1$ and $a^* = 1$ otherwise. As $U(g = g_1, a = 1) - U(g = g_0, a = 0)$ linearly decreases with c , there exists $\hat{c} \in R$ such that $g^* = g_0$, $a^* = 0$ for $c > \hat{c}$ and $g^* = g_1$, $a^* = 1$ for $c \leq \hat{c}$ if $g_0 < \hat{y} \leq g_1$, where $\hat{c} \equiv u(g = g_1, a = 1) - u(g = g_0, a = 0)$ and $u(g, a) \equiv U(g, a) - C(a)$. we can then summarize the agent's choices as follow:

$$\begin{cases} g^* = g_0, a^* = 0 \text{ if } \hat{y} > g_1 \\ g^* = g_0, a^* = 0 \text{ for } c > \hat{c} \text{ and } g^* = g_1, a^* = 1 \text{ for } c \leq \hat{c} \text{ if } g_0 < \hat{y} \leq g_1 \\ g^* = g_1, a^* = 1 \text{ if } \hat{y} \leq g_0 \end{cases}$$

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