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A General Equilibrium Model of Dynamic Loss Aversion

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Abstract

A multi-self model of dynamic loss aversion is developed to explore consumption dynamics in a calibrated general equilibrium for consumers whose utility depends on gain-loss feelings relative to their reference expectations. The model is versatile enough not only to fit the consumption data without resorting to any other mechanism than the preference, but also to reconcile consumption smoothing motivation with the nonmonotonic lifecycle consumption in data. This paper demonstrates that the key mechanism of the flexibility comes from the dual value function for gains and losses, which makes the nonmonotonic profile feasible, as well as the model's descriptive property by which any plan of non-standard consumption behaviors is incorporated into the optimization procedure. Moreover, using a modified model in which the reference dependent consumers are restricted in their ability to foresee their future income flow, this paper provides an insight into loss aversion over a lifecycle implied from the income flow. When the model is extended to an environment under uncertainty, wherein the key mechanisms are well preserved, a negative saving is observed for a small income uncertainty, implying the precautionary-saving motive is compromised by overconsumption intention.

JEL Classification: D91, D11, E21

Keywords: loss aversion, intertemporal gain-loss utility, belief updating, consumer behaviors, general equilibrium

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1 Introduction

In this paper, I study a model of dynamic reference dependence and demonstrate how the model breaks monotonicity in lifecycle consumption and is configured to fit the US consumption data based only on the consumer preference. Reference dependent utility, grounded on the signature idea that *losses loom larger than gains* (Kahneman and Tversky, 1979) or loss aversion, has received much attention because of its applicability in many fields as reviewed with details in DellaVigna (2009) and O'Donoghue and Sprenger (2018).

This adaptability is thought to come not only from the two different evaluations for gains and losses, together with the decision weights, but also from the descriptive property of the theory itself.[1] With the focus on consumption and saving, reference dependent utility can rationalize seemingly irrational behaviors, such as over-consumption or even under-consumption, which are not supported by the canonical economic theory.

In fact, the descriptive model can rationalize any behavior as long as the individual has a strong psychological intention to do so and the choice is consistent with his intention. In respect to lifecycle consumption, this implies that any nonmonotonic consumption profile in the data, which is generally thought to be at odds against the standard theory,[2] can be rationalized by the reference dependent preference. The agent with the preference is regarded as a rational deviator who is willing to execute personal plans which are based on rational beliefs about ex ante optimal, and consumes more or less than the ex ante to fulfill his personal well-being such as the preferred personal equilibrium in Koszegi and Rabin (2009).

In the model, I assume the agent's utility has two preference components following Koszegi and Rabin (2009). One is the usual consumption utility and the other is the reference dependent utility, so that the model takes into account both the absolute and the contrast elements of utility. Individuals with the preference derive utility from comparison to the reference utility, being a gain or a loss to it, with more weight on loss (i.e. loss aversion). The reference status (the utility in expectation) is made through a forward-looking mechanism, as in Koszegi and Rabin (2009) who posit rational expectations for the reference points.

An agent with the preference has an intertemporal choice each period to either follow the ex ante optimal consumption plan, or to deviate for greater or less consumption for the present, creating a different outcome in the future, as demonstrated by Park (2019). If the agent has high loss aversion, he feels that the loss utility is greater than the gain utility and will not deviate. However, if the agent has low loss aversion, he is willing to deviate from the ex ante optimal, deriving gain-loss utility from intertemporal comparison to the reference utility.

[1] Prospect Theory.

[2] The intertemporal optimality $\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{1}{1/(1+r)}$ implies that intertemporal growth in consumption is monotonic with the usual assumption on standard utility function: $u' > 0$, $u'' < 0$.

This gain-loss utility is purely psychological, and the agent only feels the utility at which period he deviates, since he is comparing his new contemporaneous utility to what he would have consumed now if he had not deviated, as well as comparing his prospective utility to what he would have consumed in the future.

With the baseline model, which derives the lifecycle consumption profile for the loss-averse agents, this paper explores the general equilibrium implication related to the consumption data. Utilizing the flexibility of the model, I demonstrate how the nonmonotonic consumption profile is obtained, by restructuring the Euler equation, in both the static and the dynamic environments of loss aversion. The key feature lies in the two opposite value functions for gains and losses, which make the consumption growth rate jump around the reference points. In the case of static loss aversion, by which consumers keep their initial degree of loss aversion and do not reoptimize, the nonmonotonicity is obtained from the very intention of the agent who would have a nonmonotonic consumption series. In other words, there is a consistent consumption path which maximizes a deviator's lifecycle utility in the direction of the deviator's intention.

A path-breaking point of the model is that it internalizes this intention and allows any maximization procedure as the agent wishes. An example of nonmonotonic consumption such as the consumption hump is obtained when the deviator has an intention of {lower, higher, lower} than the ex ante smooth consumption[3] along his three-period life {young, middle, old}. Then the deviator expects a *contemporaneous* loss utility for the first period, while he expects *prospective* gain and loss utilities for the subsequent periods. For this static agent there will be no gain-loss utilities in the second and third periods for which the expectation is met, because the rational agent does not forget where he is from or what his path was.

For the dynamic model, however, I assume that an agent does not swing between the types of spending behavior such as an overconsumer or a natural born saver, unless he is a standard agent. As in the static case, if the agent's loss aversion is low enough then he can opt for more (overconsumer) or less (saver) than the ex ante consumption rule. With the dynamic model, the nonmonotonicity is observed when the agent reoptimizes over time according to his time-varying loss aversion. For example, if an agent's initially low degree of aversion rises over a lifecycle, specifically in later life, then an inverse U-shape of consumption stream is tracked; in case of an overconsumer, due to his concern about the daunting low consumption after retirement, while for an under-consumer, due to her moderation of the propensity to save as she ages. Intuitively, as the agent's available lifespan decreases, the prospective consequences draw closer to the present, and the loss aversion deters the agent from over- or under-using the current resources. This paper finds that certain patterns of dynamic loss

[3]However it may not be entirely reasonable to assume that individuals will have such an intention. The dynamic model in Section 4 does not rely on this assumption. Furthermore, Section 2 demonstrates that even the static model can generate such outcome in aggregate under reasonable assumptions.

aversion, as well as a well-planned static model can replicate the US consumption data in a well-calibrated general equilibrium. Unlike in Pagel (2017) whose model of reference dependence generates a hump-shaped consumption profile in partial equilibrium, this result in general equilibrium is obtained solely based on the preference.

Furthermore, the dynamic model is modified to analyze how income can interact with loss aversion by altering the agent's perception of income. If the agent is able to plan only for the near future, then the resulting consumption becomes more tied to the available income. By letting income stream function as a bridge between consumption behavior and loss aversion, the modified model is able to obtain the characterization of the underlying loss aversion parameters of consumers.

Finally, I analyze the intertemporal loss aversion under uncertainty, constructing a model which considers loss aversion that stems from both time and the state of the world. Using a simplified version of the uncertainty model, I first show that the nonmonotonic property is well preserved in the case of uncertainty. Then I show that with the preference, the precautionary-saving motive is compromised by overconsumption intention, yielding a negative saving when the income uncertainty is small. Also the model derives the general equilibrium interest rate parameterized by the uncertainty level.

Modeling the reference dependent preference in general equilibrium is as follows. First, I develop a baseline lifecycle model of reference dependent agents who may deviate from the ex ante optimal for their personal wellbeing. Section 2 solves the model with static loss aversion and demonstrates how the nonmonotonicity of the consumption pattern is obtained in the intertemporal choice with the preference. In Section 3, I study the consumption dynamics of those who have time-varying degrees of loss aversion. Lifecycle consumption profiles resembling the data are provided for overconsumers, as well as for savers. If heterogeneity is introduced, a hybrid model improves the overall fit to the consumption data. In Section 4, I introduce a modified model to derive the implication of the income flow on the loss aversion parameter. Finally, Section 5 analyzes the intertemporal loss aversion under uncertainty.

1.1 Literature review

Originated from Prospect Theory (1979, 1992),[4] the reference dependence of utility has been applied to diverse fields such as auction (Lange and Ratan, 2010; Banerji and Gupta, 2014; Rosato et al., 2019), household finance (Benartzi and Thaler, 1995; Barberis et al. 2001, 2006; Barberis and Xiong, 2012), housing market (Genesove and Mayer, 2001), endowment effect (Knetsch and Wong, 2009; Ericson and Fuster, 2011), labor supply (Camerer et al. 1997; Fehr and Goette, 2007; Farber, 2008), poverty (Gunther and Maier, 2014), contract (Herweg and Schmidt, 2014), job search (DellaVigna et al., 2017), sports (Pope and Schweitzer, 2011; Allen et al., 2017), and consumption choice (Karle et al.,

[4] Kahneman and Tversky (1979) and Tversky and Kahneman (1992).

2015).

Regarding consumption choice under the preference, Karle et al. (2015) perform an experiment[5] to show that participants with high loss aversion are more likely to choose the goods (sandwiches in their experiment) of lower price rather than that of high prices, in line with theoretical predictions about the effect of loss aversion on consumption behavior. The application of loss aversion on consumption choice is sparsely found, especially in experiments, and even rarer on intertemporal choice like consumption over a lifecycle. The intertemporal model in Park (2019) examines the dynamic implication of loss aversion for an individual who is in a specific intertemporal wealth position and explores the consistent consumption−saving strategies which must be credible over time.

One complaint about reference-dependent models is that the reference point is not naturally defined, and is selected arbitrarily by the researcher (Pesendorfer, 2006). The problem has not gone unnoticed and several candidates for the reference points have been proposed, ranging from aspiration to social comparison, other than the status quo. One successful approach is expectation, as initially sought by Bell (1985) in line of disappointment theory. Koszegi and Rabin (2006) also discuss this problem and propose a model of reference dependent preference based on expectation, from comparison of the outcomes against one's recent beliefs about the outcomes, which could help solve this shortcoming. The expectation-based reference-dependence is well supported by several experiment studies including Ericson and Fuster (2011) and Banerji and Gupta (2014), as well as Abeler et al. (2011) whose work is qualitatively reviewed by Gneezy et al. (2017) with a refined setting with added elements and experiments.

The expectation based preferences can explain many noted features of consumption dynamics in lifecycle literature, like the consumption profile (Pagel, 2017; current paper) and precautionary saving (Koszegi and Rabin, 2009; Park, 2016; Pagel, 2017; current paper). The contribution of this paper over Pagel (2017) is not only about the general equilibrium result calibrated for the US consumption data instead of the partial equilibrium result in Pagel (2017), but also demonstrating that the empirical finding can be obtained by the preference alone, without resorting to any other mechanism, such as precautionary saving against uncertainty.

By providing richer intertemporal dynamics, this paper explains many salient facts regarding consumption in a well-calibrated general equilibrium (Hansen and Imrohoroglu, 2006) similar to the US data. By targeting a calibrated general equilibrium, the result of the model in this paper should closely resemble the actual economy. Of course, not only the model presented in this paper, but many other alternative models can account for the consumption data. In

[5] Loss aversion on consumption choice is one of the fields that the model's implication is not yet tested in terms of lab or field data. Literature shows that the experimental evidence of reference dependence with loss aversion is mostly about monetary gambles in the lab or field. However, as in the case of charitable giving, where money donations and goods donations are considered identical towards total charitable donations, the implication from the monetary result can be applied to consumption choice, if we interpret the lottery in terms of the uncertainty towards future uncertain income.

fact, mortality risk, uninsurable income risk, durable goods, and choice between consumption and leisure have been proposed to explain the data; and more recently time-inconsistent expectations and time-inconsistent preferences, as well as other behavioral approaches. Related to this, Park and Feigenbaum (2018) discuss in depth evaluation of the models, providing the detailed explanation about how a mechanism can account for the salient facts in consumption data.[6]

With the preference, the marked nonmonotonicity in consumption is generated from the different (opposite) gain-loss evaluations, which make the model flexible enough to rationalize the anomalies in intertemporal choice behaviors (Loewenstein and Prelec, 1992) such as overconsumption or broadly time-inconsistent choice, which topic has been extensively discussed in behavioral economics only with the framework of discounting factors i.e. hyperbolic or quasi-hyperbolic consumers (Laibson, 1997; Harris and Laibson, 2001; Hertzberg, 2016).

The landmark features of the model are made possible because the model is built around reference dependent utility, based on prospect theory (Kahneman and Tversky, 1979), instead of reference dependent consumption, the latter called habit formation being well-established in macroeconomics but has not yet directly explained the discrepancy between the hump-shaped consumption data over a lifecycle and the monotonic consumption in standard economic theory. There have been long-standing literatures of external habit and more recently, of internal habit. These models are based on an adaptive evolution of reference points and thus are backward looking. Examples of this type of reference dependence include: Carroll et al., (2000); Christiano et al. (2005); Chetty and Szeidl (2016).

Turning to the models of non-traditional preferences for consumption−savings dynamics, there are hyperbolic discounting models by O'Donoghue and Rabin (1999) and Laibson (1997). Hyperbolic discounting model, or present-biased preference, formalizes the evidence that people tend to value the immediate utilities differently from distant future ones and thus the preference induces time inconsistent optimization. The empirical implication of these works is that the existence of the commitment device like illiquid asset (Laibson) or cost (O'Donoghue and Rabin) play a role that can produce a consumption profile which tracks the income flows. The short term planning approach by Caliendo and Aadland (2007), as well as Park and Feigenbaum (2018) belongs to the time inconsistent preferences too.

Regarding the featured consumption profile which has been a central issue in lifecycle models, several literatures should be addressed. Borrowing constraint (Deaton,1991; Feigenbaum, 2009), mortality risk (Feigenbaum, 2008a; Hansen and Imrohoroglu, 2008), consumption and leisure substitutability (Heckman, 1974; Bullard and Feigenbaum, 2007), income uncertainty and precautionary savings (Hubbard et al.,1995; Carroll, 1997, 2009; Gourinchas and Parker, 2002; Feigenbaum, 2008b) are the main causes that can produce a hump-shaped con-

[6] They point out that "each of these definitely exist in reality, and they all must presumably play some role in explaining consumption over the lifecycle, though we may not be able to include every one of them in a useful model."

sumption series with the standard model. Other mechanisms to generate a hump-like consumption series are household size effect (Attanasio et al., 1999) and consumer durables (Fernandez-Villaverde and Krueger, 2009). In the latter they show how the interaction between durable and nondurable consumption may work to explain the hump in a model where durable goods serve as collateral for loans.

1.2 Modeling reference-dependent preference

This section introduces a couple of short definitions about the reference dependent preference. Following the model presented by Koszegi and Rabin (2006, 2009), the reference dependent utility is defined by,

$$U(c|r) \equiv u(c) + v(c|r) \tag{1}$$

where $u(c)$ is classical consumption utility, which is increasing, concave or quasi-concave, and differentiable. The second term $v(c|r)$ is additively separable gain-loss utility related to deviation from a reference point r. The gain-loss utility $v(c|r)$ satisfies the following, which is originally presented in Bowman et al. (1999) :

- A0 Continuous, twice differentiable except at $c = 0$, and $v(0) = 0$
- A1 Strictly increasing
- A2 If $y > z > 0$, then $v(y) + v(-y) < v(z) + v(-z)$
- A3 $v''(c) \leqslant 0$ for $c > 0$ and $v''(c) \geqslant 0$ for $c < 0$
- A4 $lim_{c\to 0+} \frac{v'(-c)}{v'(c)} \equiv \lambda > 1$

A2 implies that losses are assigned more weight than gains of the same size. The value function for loss grows steeper than for the gains. By A3, it is assumed that the utility function is concave in positive valuation and convex in negative valuation, holding diminishing sensitivity in both valuations. A4 sets the magnitude of loss aversion.

Using the consumption utility $u(c)$, the gain-loss utility is specified by

$$v(c|r) = v(u(c) - u(r)) \tag{2}$$

In the spirit of Koszegi and Rabin (2009) who use the linear gain-loss utilities,[7] the reference dependent utility of the agent who has a functional form of CRRA for the consumption utility is,

$$u(c, z) = \frac{c^{1-\gamma}}{1-\gamma} + \eta\mu(z) \tag{3}$$

[7] Other candidates for the gain-loss functions are power functions and exponential functions.

where $\mu(z)$ is a gain-loss utility as a function of deviation in outcome from a reference point, i.e. $z = u(c) - u(r) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{r^{1-\gamma}}{1-\gamma}$. Also, η is the weight of gain-loss utility relative to consumption utility. If η is very small, then the gain-loss effect is negligible. The gain-loss utility $\mu(z)$ is defined by

$$\mu(z) = \left\{ \begin{array}{cc} z & \text{if } z \geqslant 0 \\ \lambda z & \text{if } z < 0 \end{array} \right\} \tag{4}$$

in which $\lambda > 1$ is the coefficient of loss aversion. In the dynamic model of intertemporal decision making, it is necessary to modify the gain-loss utility to incorporate an agent's temporal psychological weight on the gain-loss utilities over time. It is natural to posit that the weighting is likely to decay as the effect fades. For simplicity, I assume that the time $t -$ initial strength of the concern for loss relative to gain is $\omega_t(t) \equiv \omega_t > 0$. I further assume that the decay follows the standard time discounting $\beta > 0$. In summary, all the preference parameters satisfy: $\{\gamma > 0, \beta > 0, \eta > 0, \omega_t > 0, \lambda > 1\}$. Beyond this, modeling stochastic reference dependence is introduced in Section 5 (Loss aversion under uncertainty).

2 Reference dependence and nonmonotonic consumption

This section first demonstrates how the model of intertemporal loss aversion can generate a nonmonotonic consumption profile, which cannot be directly obtained from the standard economic model on consumption−saving. Because optimality implies that consumers equate their intertemporal marginal rate of substitution to the current and future price ratio, which is given by the interest rate, the consumption profile of any frictionless intertemporal model displays monotonically increasing or decreasing consumption over time. This result often contradicts the data which in general exhibits a hump-shaped profile over a lifecycle.

The monotonicity can be easily broken with a reference-dependence model, as will be mathematically explored, largely because the gains and losses are evaluated with different value functions. Although the main contribution of the current paper is in developing a model by which a type of *dynamic* reference dependence is employed to match the consumption data, the nonmonotonicity can be obtained even with a simpler framework such as *static* reference dependence, in which consumers continue to hold their initial degree of aversion to loss and follow their initial plan of nonmonotonic consumption profile.

Moreover, the descriptive[8] property of prospect theory entails that description is indispensable to the construction and execution of model. Prospect

[8]This point is where the model here is different from the standard mathematical model (Pasquariello, 2014). According to Friedenthal et al. (2012), the descriptive models are "generally not built in a manner that directly supports simulation, animation or execution, but they can be checked for consistency and adherence to the rules of the language, and the logical relationships can be reasoned about."

theory allows the model to be versatile enough to generate an outcome of our interests in consumption−saving. By exploring the intertemporal consistency in a general setting, I will prove that any intended plan can be realized as a fully optimized solution.

If the agent lives up to T lifetime periods, and if his preference regarding loss aversion stays the same[9] throughout the entire planning horizon, then the agent's lifecycle problem is to maximize

$$u(c|c^*) = \sum_{t=0}^{T} \beta^t \{u(c_t) + \eta I_t [u(c_t) - u(c_t^*)]^+ + \eta\omega\lambda(1 - I_t)[u(c_t) - u(c_t^*)]^-\} \quad (5)$$

subject to

$$c_t + b_{t+1} = y_t + Rb_t \quad (6)$$

$$b_0 = 0, b_{T+1} = 0 \quad (7)$$

$$I_t \in \{\ 0\ ,\ 1\ \} \quad (8)$$

where c_t is consumption and y_t is labor income each period $t = 0, 1, 2, ..., T$, while b_{t+1} is saving or bond holding demanded at t for the next period $t + 1$. The agent makes financial income or debt Rb_t by the market gross interest rate $R = 1 + r$. On top of the standard consumption utility $u(c_t)$, the agent with reference-dependent preference derives gain-loss utilities: $[u(c_t) - u(c_t^*)]^+$ for gain and $\omega\lambda[u(c_t) - u(c_t^*)]^-$ for loss, where $\{c_t^*\}_0^T$ is the reference consumption which is believed optimal ex ante to a rational consumer. Because of its static property, the psychological weight ω stays constant over time. Given the usual value of loss aversion $\lambda = 2$ in the literature, the psychologically weighted loss aversion $\omega\lambda$ is determined directly by ω. The parameter η is the weight of gain-loss utility relative to the consumption utility.

The index I_t in Eq. 8 is *binary* and summarizes the *intention* of the agent regarding his consumption plan. In the case of two-period consumption−saving, if the agent has an intention of over-consumption in the first period which then lowers the consumption in the next period $\{c_0 > c_0^*,\ c_1 < c_1^*\}$, then the maximization problem is given by the binary index $\{I\} = \{1, 0\}$, while it is $\{I\} = \{0, 1\}$ for the opposite plan of initial under-consumption $\{c_0 < c_0^*,\ c_1 > c_1^*\}$ for a saver. Likewise, if the agent has a plan of $\{c_0 > c_0^*,\ c_1 > c_1^*, ...\ c_{T-1} > c_{T-1}^*,\ c_T < c_T^*\}$, then this plan is described by $I_t = 1\ \forall t = 0, ..., T - 1$, and $I_t = 0$ at $t = T$. Thus, $\{I\} = \{1, 1, 1, ..., 1, 0\}$. Let us assume CRRA for the period utility:

$$u(c_t) = \begin{bmatrix} \frac{c_t^{1-\gamma}}{1-\gamma} & \text{if} \ \ \gamma \neq 1 \\ \ln c_t & \text{if} \ \ \gamma = 1 \end{bmatrix} \quad (9)$$

Over the lifecycle the agent will not deviate from $\{c^*\}_{t=0}^T$ if his loss feeling

[9] In terms of dynamic reference dependence, the subsequent selves do not change their temporal preference from the initial self.

is grave, such as $\lambda = 2$, $\omega = 0.8$ so that $\omega\lambda > 1$, at any period of his life.[10] Likewise, if $\omega\lambda < 1$, from $\lambda = 2$, $\omega = 0.4$, it is always profitable[11] for the agent to deviate from the ax ante optimal consumption path $\{c^*\}_{t=0}^{T}$. For such a deviator at t, the following optimality condition should be satisfied:

$$c_t^{-\gamma} + \eta I_t c_t^{-\gamma} + \eta\omega\lambda(1 - I_t)c_t^{-\gamma} = R\beta[c_{t+1}^{-\gamma} + \eta I_{t+1}c_{t+1}^{-\gamma} + \eta\omega\lambda(1 - I_{t+1})c_{t+1}^{-\gamma}] \quad (10)$$

Note that the second term and third term in the LHS are the marginal utilities from the contemporaneous gain and loss feelings, respectively, while in RHS those are from prospective gain and loss feelings. This first order condition directly displays the nonmonotonicity of consumption over time. For any adjacent two periods, t and $t+1$, when the gain-loss index remains the same, i.e. either $I_t = I_{t+1} = 0$ or $I_t = I_{t+1} = 1$, there is no difference of the model from the canonical lifecycle model, and the (gross) consumption growth rate is $c_{t+1}/c_t = (R\beta)^{1/\gamma}$. However, if $\{I_t,\, I_{t+1}\} = \{1,\, 0\}$, then $c_{t+1}/c_t < (R\beta)^{1/\gamma}$ for a deviator $\omega\lambda < 1$, because

$$c_t^{-\gamma} = (R\beta)\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)c_{t+1}^{-\gamma} \quad (11)$$

Likewise, if $\{I_t,\, I_{t+1}\} = \{0,\, 1\}$, then $c_{t+1}/c_t > (R\beta)^{1/\gamma}$ because

$$c_t^{-\gamma} = (R\beta)\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)c_{t+1}^{-\gamma} \quad (12)$$

Over a lifecycle, therefore, a monotonic growth rate of consumption always breaks for a deviator, regardless of whether the agent is an overconsumer or

[10] In the 2-period case, the consumer feels a *contemporaneous gain* utility if he consumes more than the suggested ex ante optimal. As a result, his consumption is lowered next period, and this yields a *prospective loss* utility relative to the reference point. If the contemporaneous gain utility is greater than the prospective loss utility, then he chooses not to follow the ex ante optimal but to deviate for more consumption this period. If, however, the prospective loss utility is greater than the contemporaneous gain utility, then he adheres to the ex ante optimal. The consumer's greater concern for the loss (high loss aversion) deters him from over-consuming. This analysis can apply to the opposite case (a natural born saver): she may have a *contemporaneous loss* utility if she consumes less than the ex ante. As a result, her consumption is elevated next period, and this gives her a *prospective gain* utility. If her contemporaneous loss feeling is greater than the prospective gain, then she does not reduce her consumption but follow the ex ante consumption rule. If, however, her prospective gain is more than the contemporaneous loss utility, then she is willing to reduce her consumption and save now more than planned. In this case, the high loss aversion deters the consumer from under-consuming. By both stories, it is clear that when the decision maker's loss aversion is high, the agent does not deviate from the ex ante optimal solution. But if the consumer cares more about the gain because of low loss aversion, then the consumer deviates for personal well-being in either direction of more or less than the ex ante optimal.

[11] Bsed on the 2-period lifetime resource constraint $c_1 = y_1 + R(y_0 - c_0)$, the marginal utility is $\frac{du}{dc_0} = (1+\eta)c_0^{-\gamma} - \beta R(1+\eta\omega\lambda)[y_0 + R(y_0 - c_0)]^{-\gamma}$. If the derivative is evaluated at the optimal consumption $c_0^* = \frac{y_0 + y_1R^{-1}}{1+(\beta R)^{1/\gamma}R^{-1}}$, then $\frac{du}{dc_0^*} = \eta(1-\omega\lambda)\left(\frac{y_0 + y_1R^{-1}}{1+(\beta R)^{1/\gamma}R^{-1}}\right)^{-\gamma} = \eta(1-\omega\lambda)c_0^{*-\gamma}\left\{\begin{array}{ll} \leqslant 0 & if \;\; \omega\lambda \geqslant 1 \\ > 0 & if \;\; \omega\lambda < 1 \end{array}\right\}$.

a saver. In fact, the model can produce any shape of a profile if the agent's *intended plan itself displays the shape.* An agent could likely have such plans when his natural type of spending is not fixed as an overconsumer or a saver. An agent may combine the two and produce an inverse U-shape consumption stream over time.

For example, the model can generate the well-known hump-shaped lifecycle consumption with no other mechanism but the preference. With a three period model which can generate the minimum nonmonotonic consumption path, the hump is obtained when $\omega\lambda < 1$ for a consumer who intends to have a hump-like consumption profile $\{c_0 < c_0^*,\ c_1 > c_1^*,\ c_2 < c_2^*\}$. When deviation is desirable $(\omega\lambda < 1)$ as in the example of $\lambda = 2$, $\omega = 0.4$, the consistent consumption is $\left\{c_0,\ \ c_0\left(\frac{R}{\phi}\right)\nu,\ c_0\left(\frac{R}{\phi}\right)^2\right\}$, in which $\frac{R}{\phi} = (R\beta)^{1/\gamma}$ and $\nu = \left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}$, with $c_0 = \dfrac{y_0 + y_1R^{-1} + y_2R^{-2}}{1+\phi^{-1}\nu+\phi^{-2}}$. This consumption profile produces a hump even in the simplest environment, such as $\beta = 1$ and $R = 1$, under which the standard model generates a flat consumption profile. The reference-dependence consumption under this condition is $\left\{\dfrac{y_0+y_1+y_2}{1+\nu+1}, \dfrac{\nu(y_0+y_1+y_2)}{1+\nu+1}, \dfrac{y_0+y_1+y_2}{1+\nu+1}\right\}$. Thus it is clear that $c_0 < c_1 > c_2$ for all values of the risk aversion parameter $\gamma > 0$, because $\nu > 1$ when $\omega\lambda < 1$. This result is remarkable because the profile is not dependent on any assumption about either income, time preference, or the magnitude of interest rates. Only the loss aversion plays a role to produce a hump.

If the agent does not change his natural type of spending i.e. his type does not oscillate between an overconsumer and a saver within the planning horizon, then a closed form solution for the consistent consumption is obtained in a very general form.

Proposition 1 *Assume that the agent has an intention of overconsumption for the first* $1+\tau$ *periods* $\{c : [c_t > c_t^*]_0^\tau$ *and* $[c_t < c_t^*]_{\tau+1}^T\}$. *Then the agent will end up with fulfilling the plan if* $\omega\lambda < 1$ *and the consistent consumption is* $\{c\} = \left(\frac{R}{\phi}\right)^t c_0$ *for* $t = 0, 1, 2, ..., \tau$ *and* $\{c\} = \mu\left(\frac{R}{\phi}\right)^s c_0$ *for* $s = \tau+1, ..., T$, *where* $c_0 = \left(\sum_{t=0}^{\tau}\left(\dfrac{1}{\phi}\right)^t + \mu\sum_{s=\tau+1}^{T}\left(\dfrac{1}{\phi}\right)^s\right)^{-1}\sum_{t=0}^{T}\frac{y_t}{R^t}$, *and* $\mu = \left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}$.

This proposition implies that for a rational deviator, the consistent consumption is in fact what the agent desires to achieve; for an overconsumer, a consumption point larger than the ex ante point or the optimal consumption point to the standard economic agent. Recall that all of the preference parameters defined in Section 1.2 have positive values: $\{\gamma > 0,\ \beta > 0,\ \eta > 0,\ \omega > 0,\ \lambda > 1\}$. For a deviator, because $\omega\lambda < 1$ and thus $\mu = \left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma} < 1$, it is true that

$$c_0 = \left(\sum_{t=0}^{\tau} \left(\frac{1}{\phi}\right)^t + \mu \sum_{s=\tau+1}^{T} \left(\frac{1}{\phi}\right)^s \right)^{-1} \sum_{t=0}^{T} \frac{y_t}{R^t} \tag{13}$$

$$> \left(\sum_{t=0}^{\tau} \left(\frac{1}{\phi}\right)^t + \sum_{s=\tau+1}^{T} \left(\frac{1}{\phi}\right)^s \right)^{-1} \sum_{t=0}^{T} \frac{y_t}{R^t} = c_0^* \tag{14}$$

Therefore, $c_0 > c_0^*$ when $\omega\lambda < 1$. The same logic applies to the opposite case of an initial under-consumption, and $c_0 < c_0^*$ for a saver when $\omega\lambda < 1$.

Proposition 2 *(Saver's case) Assume that the agent has an intention of under-consumption for the first* $1 + \tau$ *periods* $\{c : [c_t < c_t^*]_0^\tau$ *and* $[c_t > c_t^*]_{\tau+1}^T\}$. *Then the agent will end up with fulfilling the plan if* $\omega\lambda < 1$ *and the consistent consumption is* $\{c\} = \left(\frac{R}{\phi}\right)^t c_0$ *for* $t = 0, 1, 2, ..., \tau$ *and* $\{c\} = \nu \left(\frac{R}{\phi}\right)^s c_0$ *for* $s = \tau + 1, ..., T$, *where* $c_0 = \left(\sum_{t=0}^{\tau} \left(\frac{1}{\phi}\right)^t + \nu \sum_{s=\tau+1}^{T} \left(\frac{1}{\phi}\right)^s \right)^{-1} \sum_{t=0}^{T} \frac{y_t}{R^t}$, *and* $\nu = \left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}$.

When the income profile is deterministic, the discrete index for the gain-loss utility grants two-phase, thus nonmonotonic overall, lifecycle consumption profile with a drift. From the perspective of lifecycle consumption, a common scenario may be that consumers overconsume during their working years, and as a result would have lower consumption during the retirement years. Figure 1 shows this and the consumer has a plan of overconsumption up to age 65, i.e. $I_t = 1$ for $t + 25 = 25, ..., 65$, and $I_t = 0$ for $t + 25 = 66, ..., 80$. The overconsumer enjoys only a small increase in early consumption over the standard and as a result he has to undergo substantial decrease in later consumption after retirement. Figure 2 shows the opposite. This figure is from a saver's consistent consumption and the saver enjoys high consumption in retirement years at a small cost of consumption reduction during the working years.

The consumption series can be a personal equilibrium for the deviators. However, a deviator may find a better plan to fulfill his over- or under-consumption intention. With many potentially more fulfilling plans in the multi-period optimization, it is natural to assume that consumers will gravitate a plan with the highest utility. At the collective level, although each agent consumes according to his personal equilibrium, the overall consensus of these plans is the one that derives the highest total utility. With respect to this, Park (2019) finds that the highest utility is derived when the agent stops his over- or under- consumption at the halfway point of the entire planning horizon. That is, the overall utility is maximized when[12] $\tau^* = \frac{T+1}{2}$.

[12] If the total number of periods $(T+1)$ is an odd number, then the highest utility is obtained at $\tau^{**} = \frac{T}{2}$.

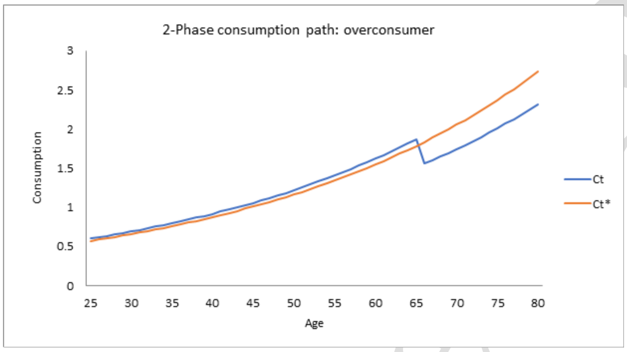

Figure 1: Two-phase consumption path (C_t) for an overconsumer. $R = 1.035$, $\beta = 0.98$, $\gamma = 0.5$, and $\lambda = 2$, $\omega = 0.4$.

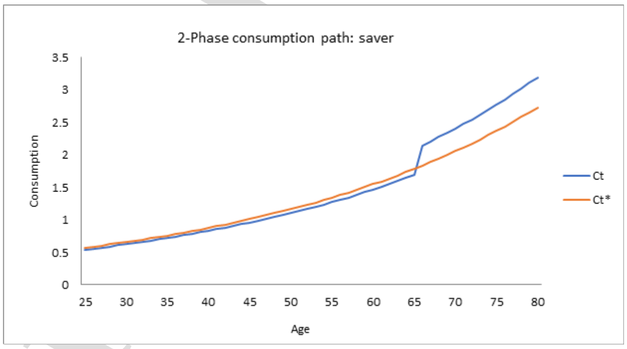

Figure 2: Two-phase consumption path (C_t) for a saver. $R = 1.035$, $\beta = 0.98$, $\gamma = 0.5$, and $\lambda = 2$, $\omega = 0.4$.

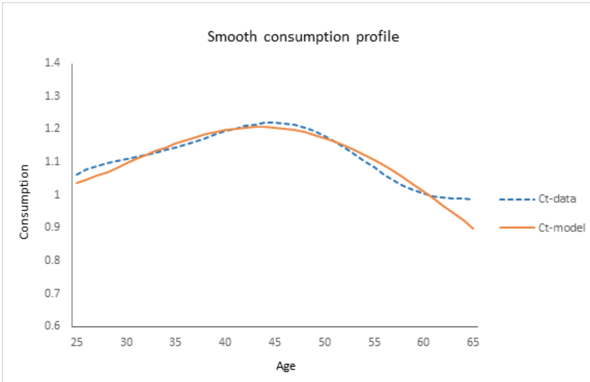

Figure 3: Lifecycle consumption profile with a non-discrete index and the data (Gourinchas and Parker, 2002). $R = 1.035$, $\beta = 0.96$, $\gamma = 1$, and $\lambda = 2$, $\omega = 0.4$.

As shown in the figures, the static model tends to generate a graph with kinks because of the discrete index. The kink arises at the point when the agent reverses his consumption behavior so that there is a shift in the value function with respect to the gain-loss utilities. A non-discrete index $\widetilde{I}_t$ can be constructed as a function of *income* level, given a deterministic income stream:

$$\widetilde{I}_t \equiv \frac{y_t - y_{\min}}{y_{\max} - y_0} \tag{15}$$

with initial income y_0 at age 25. This simple specification provides a continuous function which has its range between 0 and 1. Thus over a lifecycle, it gives the ratio of the agent's willingness to consume compared to the reference points. At the individual level, this implies that the likelihood of the overconsumption increases with higher income. At the aggregate level, the portion of the population who consumes $c_t > c^*$ is directly related to the relative income position. Based on this index, Figure 3 demonstrates the featured consumption hump over a lifecycle in the static environment, when both income profile and consumption profile are obtained from Gourinchas and Parker (2002), which will be utilized for any simulation exercise in this paper. In Section 4, I introduce a dynamic model in which a smooth consumption hump is generated even with the discrete index. For this purpose, I define:[13]

Definition 1 *A consumption hump for a T-period model is a consumption profile* $\{c_t\}_{t=0}^T$ *that satisfies*

i) There is a consumption peak at time $\widehat{t} \in (0, T)$

[13]This term is defined in a strong sense, although it can be defined in a weaker sense, under which it is allowed to have any wiggles over the horizon and there can be many local peaks.

ii) Consumption is monotonically increasing up to $\widehat{t}$
iii) Consumption is monotonically decreasing beyond $\widehat{t}$

3 A general equilibrium model of multi-self dynamic agents

Based on the lifecycle model of loss aversion in Section 2, in this section I study a dynamic model in which the subsequent selves of the agent reoptimize with time-varying degrees of loss-aversion and explore its general equilibrium implication for an economy populated by such agents. Whether they are overconsumers or savers, their subsequent selves should reoptimize whenever they find that the consumption point assigned by the previous self is suboptimal. If all of the subsequent selves keep rebalancing to fulfil their temporal preference, then only the first consumption point is realized from each projected consumption stream, making the actual profile a series of the initial consumption set by each self.

3.1 Environment

Time is discrete and denoted by index τ. Each period a generation of identical cohorts is born in a stationary economy. Each agent (consumer as well as worker) who is indexed by age t, is assumed to live up to T. During working periods, the agent is endowed with one unit of labor productivity, measured in efficiency units, which is supplied inelastically. There is no borrowing constraint, and the agent can borrow and lend freely under the market determined interest rate. There is no government, and there exists a single good which can be either consumed or saved. Finally, retirement occurs exogenously at $t = T_w + 1$ where $T_w < T$, and the agent lives on saving during the retirement years.

3.2 Consumers in multiple selves

The consumer initially plans for lifetime $T + 1$ periods with his taste of ω_0. A deviator (overconsumer) is assumed to consume more than the ex ante optimal for the first half of whatever given lifetime. At any age t if the consumer changes his mind, i.e. the degree of loss aversion, the current self re-plans for the remaining periods and maximizes for $\tau = t, t + 1, ..., T$,

$$u_t(c_t, c_{t+1}, ..., c_T|c^*) = \tag{16}$$

$$\sum_{\tau=t}^{T} \beta^{\tau\text{-}t} \left\{ \frac{c_\tau^{1\text{-}\gamma}(t)}{1-\gamma} + \eta I_\tau \left(\frac{c_\tau^{1\text{-}\gamma}(t)}{1-\gamma} - \frac{c_\tau^{*1\text{-}\gamma}(t)}{1-\gamma} \right)^+ + \eta \omega_\tau(t) \lambda (1\text{-}I_\tau) \left(\frac{c_\tau^{1\text{-}\gamma}(t)}{1-\gamma} - \frac{c_\tau^{*1\text{-}\gamma}(t)}{1-\gamma} \right)^- \right\}$$

subject to

$$c_\tau(t) + b_{\tau+1}(t) = we_t + Rb_\tau(t) \tag{17}$$

$$b_0(0) = 0,\ b_t(t) = given,\ b_{T+1}(t) = 0 \tag{18}$$

$$I_\tau \in \{\ 0\ ,\ 1\ \} \tag{19}$$

Other than the planning time index (t), the notations are the same as those in Section 2 so that $c_\tau(t)$ is consumption planned at t for time τ and $b_{\tau+1}(t)$ is bond demanded at τ for the next period, indexed by the planning time t. The consumer has a lifetime stream of productivity so that he supplies e_t efficiency units of labor at age t to production and earns labor income of we_t, where w is the market determined real wage rate which is assumed to be stationary. The psychological weight on loss is represented by $\omega_\tau(t)$, which is assumed to be constant *within* the planning periods $T-\tau+1$ of each self. Therefore, $\omega_\tau(t)=\omega_t$.

Solving the optimization problem suggests the first conjecture that if the consumer and all of his subsequent selves have high loss aversion $\omega_t\lambda \geqslant 1$ *for all t*, then no deviation occurs: the consistent consumption profile is not different from the optimal solution to the standard model and the model should produce a monotonic consumption series. If the consumer's weight on loss is low at any period $\omega_t\lambda < 1$, then deviation will occur. The solution to the above maximization problem yields a consumption profile for the planning horizon $T-t+1$ starting from t. Initially the consumer intends to follow the planned consistent consumption by consuming $c_t(t)$ at t. At $t+1$, however, the second self realizes that the planned consumption for $t+1$, which is $c_{t+1}(t)$, is now no longer desirable based on his new loss aversion, and thus has to replan to incorporate this. If each of the subsequent selves has their own loss aversion, then the consumer ends up with following only the initial points of all the planned consumption path, thus being an envelope over the entire periods. Therefore, the realized lifecycle consumption is

$$\{c_t\}_{t=0}^{T} = \{c_0(0), c_1(1), c_2(2), ..., c_T(T)\}. \tag{20}$$

And the realized lifecycle bond demand is

$$\{b_{t+1}\}_{t=0}^{T} = \{b_1(0), b_2(1), b_3(2), ..., b_T(T-1)\} \tag{21}$$
$$b_0 = 0,\ b_{T+1} = 0.$$

The two series will constitute the consumer side of the general equilibrium. The next figure shows the realized lifecycle consumption and bond demand of the dynamic overconsumer whose subsequent selves reoptimize following their new degree of loss aversion ω_t. For simulation, I propose the following simple linear weight function for the age-dependent loss aversion by incorporating the assumption that consumers are born at age 25, which corresponds to the model age of 0.

$$\omega_t : \left\{ \begin{array}{lll} \omega_t = K - 0.01(t-25) & \text{for} & 25 \leqslant \ age \leqslant 45 \\ \omega_t = \omega_{t-1} + 0.01 & \text{for} & 45 < \ age \leqslant 65 \end{array} \right\} \tag{22}$$

where K is a constant between 0 to 0.5.[14] The consumer's loss aversion keeps increasing after the age of 45. This weight function is specified from the US

[14]Because I set $\eta = 1, \lambda = 2$ for the simulation exercises, the deviator's preference $\omega_t\lambda < 1$ implies $\omega_t < 0.5$. Also, K is assumed to be chosen at a value which secures $0 < \omega_t \leqslant 0.5$ throughout the planning horizon, unless ω_t is adjusted to meet the range.

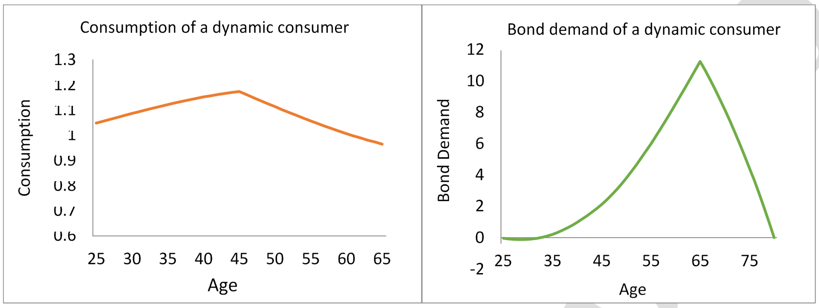

Figure 4: Consumption and bond demand. The loss aversion parameters are $\eta = 1, \lambda = 2$, and the weight function ω_t defined in Eq. 22. Other parameters are $R = 1.035$, $\gamma = 0.5$, and $\beta = 0.967$.

retirement survey (Retirement Confidence Survey, 2007) which documents that many people start planning their retirement around age 45. This implies that from this age consumers are likely to curb their overconsumption behavior and plan for the future. As the overconsumers age, their consumption behavior is likely to converge to that of standard agents, although an overconsumer would not completely change his inherent spending type. Based on this observation, the maximum value of the acceptable ω_t is 0.5, and the overconsumer's weight function is set to have its ceiling at this value. In Figure 4, the lifecycle consumption and the corresponding bond demand are obtained with $K = 0.5$. With the K value, the range of ω_t will be $0.3 \leqslant \omega_t \leqslant 0.49$ for the agent's life before retirement, and $\omega_t = 0.5$ during the retirement years.

Although the hump-shaped consumption profile in Figure 4 is derived from the specific weight function (Eq. 22), a more general class of simple weight functions, such as Eq. 23, can also produce a profile similar to the data.

$$\omega_t : \quad \{\omega_t = \omega_{t-1} - 0.01 \text{ for } 25 \leqslant age \leqslant 65\} \tag{23}$$

Likewise, given an initial value such as $\omega_0 = 0.5$, the range of ω_t of the agent is $0.1 \leqslant \omega_t \leqslant 0.5$ for the agent's life before retirement, and it is negligible and approaching to zero if the same weight function applies afterwards.[15] Figure 5 displays a consumption series when all other parameters are set at the same values as in Figure 4.

[15]Similarly, because $\omega_t > 0$, the minimum value of the acceptable ω_t is $\varepsilon > 0$, such as 0.01 and the agent's weight function will remain at this value once the value reaches at this point during the retirement years.

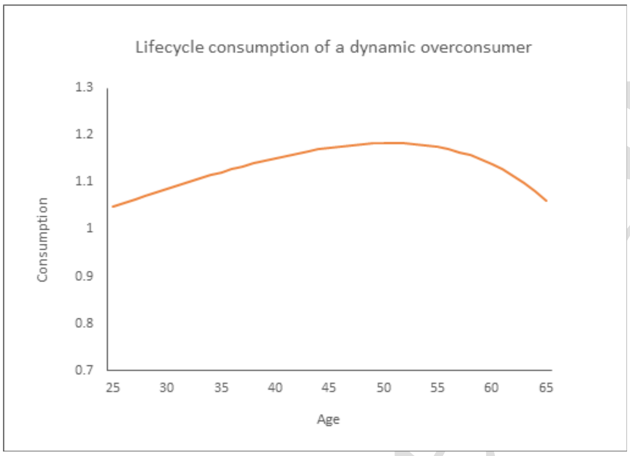

Figure 5: Consumption from a general weight function. The loss aversion parameters are $\eta = 1, \lambda = 2$, and $\omega_t = \omega_{t-1} - 0.01$ with $\omega_0 = 0.5$. Other parameters are $R = 1.035$, $\gamma = 0.5$, and $\beta = 0.967$.

3.3 An overlapping generations general equilibrium

In the general equilibrium where $(T + 1)$ types of cohorts from each age group coexist at any time, the consumers are dynamic agents who have loss aversion low enough to make them deviate from the ex ante optimal consumption. Moreover the consumers can change the degree of their loss aversion over age, while keeping over- or under-consumption up to the midpoint of each planning horizon because this gives them the highest utility among all the alternative strategies available to them.

3.3.1 Production and equilibrium

The stationary economy assumes that there is a continuum of identical perfectly competitive firms whose production function is given by $F(K, N) = K^{\alpha} N^{1-\alpha}$. Thus the marginal productivity w.r.t. capital and labor:

$$F_K = \alpha \left(\frac{K}{N}\right)^{\alpha-1} \quad \textit{and} \quad F_N = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha} \tag{24}$$

Then the competitive equilibrium for this economy is defined as follows.

Definition 2 *A competitive equilibrium in this stationary economy is an allocation* $\{c_t\}_{t=0}^T$*, a set of bond demands* $\{b_{t+1}\}_{t=0}^T$*, an interest rate* R *and a wage rate* w *such that given* R *and* w*, the following conditions are satisfied: (i)* $\{c_t\}_{t=0}^T$ *and* $\{b_{t+1}\}_{t=0}^T$ *solve the consumer's optimization problem, (ii) Factors* *are paid out their marginal productivity,* $w = F_N$ *and* $R - 1 = F_K - \delta$*, (iii) Labor market and bond market clear, i.e.* $N = \sum_{t=0}^{T} e_t$ *and* $K = \sum_{t=0}^{T} b_t$*.*

First, the aggregate consumption is obtained by summing up the consumption from all age groups. Likewise, the aggregate bond demand is obtained by summing up the bond demands of all cohorts. The last market clearing condition specifies that the bond demand among the consumers cancel out in the aggregate and that the net demand for bonds should be equated with the total capital stock. In addition, the aggregate labor supply obtained from adding up over each age group should be equated with the aggregate labor demand. Then, by substituting the marginal productivity of capital into the market clearing condition, the equilibrium gross interest rate R is determined by

$$\sum_{t=0}^{T} b_t(R) = \left(\frac{R - 1 + \delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \sum_{t=0}^{T} e_t. \tag{25}$$

Thus,

$$w(R) = (1-\alpha)\left(\frac{R - 1 + \delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \tag{26}$$

To obtain a well-designed calibration result for the general equilibrium, I intend to match three standard US macroeconomic targets: the interest rate (R), capital-output ratio (K/Y), and consumption output ratio (C/Y). Following Rios-Rull (1996), I set 2.94 as a target value for capital-output ratio and 0.748 for the target ratio of consumption to output. Following McGrattan and Prescott (2000), I set the target of real interest rate at 3.5% ($R = 1.035$), which is similar to Gourinchas and Parker's (2002) estimation of 3.44%. The model assumes that agents are born at age 25 (model age $t = 0$) and are assured to live for 55 years to age 80, when the agents will leave no bequest for the next generation. In the context of the lifecycle profile, the mean consumption profile in Gourinchas and Parker's (2002) estimation and its septic polynomial fit by Feigenbaum (2008a) is adopted. According to this estimation, the ratio of peak consumption to initial consumption $C^{\max}/C_0$ is 1.1476. For the income (productivity) schedule, I also use Feigenbaum (2008a)'s quadratic fit to the income data of Gourinchas and Parker (2002). The income schedule is laid out in terms of efficiency as follows:

$$e_t = 1 + 0.0181t + 0.000817t^2 - 0.000051t^3 + 0.000000536t^4 \tag{27}$$

The next two figures, Figure 6 and Figure 7, display the general equilibrium result for two γ values, $\gamma = 0.5$ and $\gamma = 0.9$. A dynamic model in which consumers deviate for more consumption in their early life due to low loss aversion ($\omega_t\lambda < 1$) can reproduce a hump-shaped consumption profile which closely tracts the data in a well-calibrated general equilibrium. In Figure 6, three loss aversion profiles are proposed with the same risk aversion parameter ($\gamma = 0.5$),

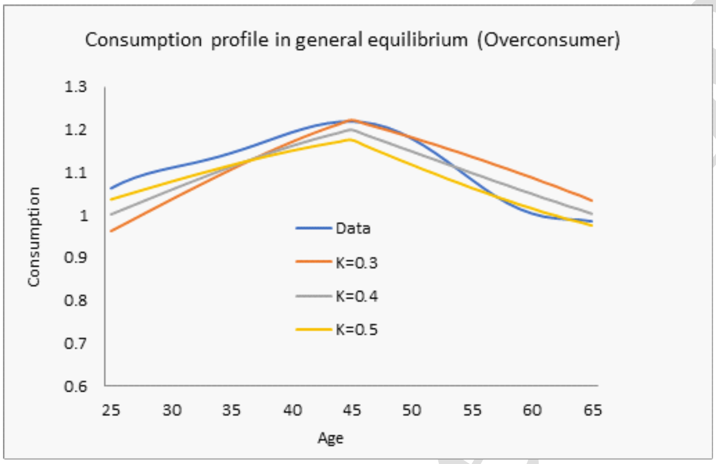

Figure 6: General equilibrium result with overconsumers and data (Gourinchas and Parker, 2002). Parameters are $\eta = 1$, $\lambda = 2$, and $\gamma = 0.5$. The weight function is $\omega_t = 0.5 - 0.01(t-25)$ for $t \leqslant 45$ and $\omega_t = \omega_{t-1} + 0.01$ for $45 < t \leqslant 65$.

or the inverse of the intertemporal elasticity of substitution, which is 2.[16] Using the same weight function ω_t specified in Eq. 22, I set $K = 0.5$ for Profile A, $K = 0.4$ for Profile B and $K = 0.3$ for Profile C.[17] Among these, the best quantitative fit to the data in terms of mean squared error is Profile A (mse: 0.0014). Regarding the ratio of peak consumption to initial consumption, Profile A also gives the best match (1.1340) to the data (1.1476). Any value of risk aversion bigger than $\gamma = 0.5$ does not improve the overall fit. For $\gamma = 0.9$, in Figure 7, the best profile is $K = 0.4$ in terms of quantitative fit (mse: 0.0022) and $K = 0.3$ in terms of the consumption ratio.

3.3.2 Equilibrium with both savers and overconsumers

The general equilibrium can be explored for the model with another type of the consumer who is not an overconsumer but a saver. The consumer thus deviates for more saving (under-consumption) early in the life when $\omega_t \lambda < 1$. Again, the consumer consumes less than the ex ante optimal for the first half of her

[16] In the model with CRRA, the kernel for the intertemporal MRS, i.e. $\frac{1}{\phi} = \frac{(R\beta)^{1/\gamma}}{R}$ has three components R, β, γ which are not separately identified, implying that given R we can have a specific γ if we are allowed to adjust β in general equilibrium. The relatively low values of γ are directly derived from the calibration method that keeps β from exceeding 1, which is not uncommon in the lifecycle literature.

[17] The range of the weight function over the life from 25 to 65 years old is $0.2 \leqslant \omega_t \leqslant 0.4$ for $K = 0.4$ (B), and $0.1 \leqslant \omega_t \leqslant 0.3$ for $K = 0.3$ (C).

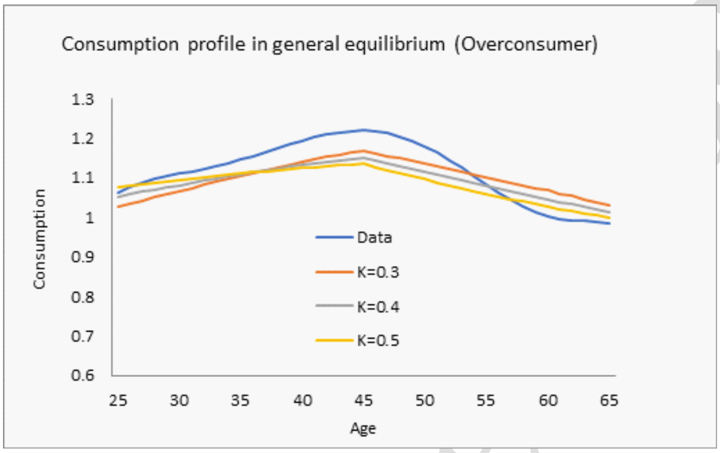

Figure 7: General equilibrium result with overconsumers and data (Gourinchas and Parker, 2002). Parameters are $\eta = 1$, $\lambda = 2$, and $\gamma = 0.9$. The weight function is $\omega_t = 0.5 - 0.01(t-25)$ for $t \leqslant 45$ and $\omega_t = \omega_{t-1} + 0.01$ for $45 < t \leqslant 65$.

planning horizon. Thus the consumer expects loss feelings for the beginning half of the periods and gain feelings for the other half of the periods. At any age t, if the consumer changes her degree of loss aversion, she needs to reoptimize starting from t for the remaining life. All other economic assumptions remain the same as in the environment of overconsumers.

For a simulation exercise, I propose a very simple weight function for the saver as follows:

$$\omega_t : \quad \{\omega_t = K + 0.01(t-25) \quad \text{for} \quad 25 \leqslant t \leqslant 65\ \} \tag{28}$$

where K is a constant term. This specification exhibits a linearly increasing weight function over the whole lifetime, indicating that the saver's propensity to save due to low loss aversion moderates as the consumer ages. For example, when $K = 0.2$, the saver's loss aversion exceeds 0.5 starting from age 55, implying her consumption behavior converges to that of the standard agents from this age.[18] Even with this simplest version of the parameter, the model produces a significant hump in the general equilibrium. Figure 8 summarizes the result. In the figure, K is set to 0.2. The risk aversion parameters γ are 0.5 and 0.3. By all of the measurement criteria, the profile with $\gamma = 0.5$ fits best (mse: 0.0031). With the simple weight function, a risk aversion parameter higher than 0.5 does not improve the overall fit and the hump-shaped figure disappears when γ is very high. Unlike with the equilibrium of the overconsumer, the saver's consumption peak in this exercise usually comes later in life than the data. If

[18] The range of the weight function is $0.2 \leqslant \omega_t \leqslant 0.5$ for the saver.

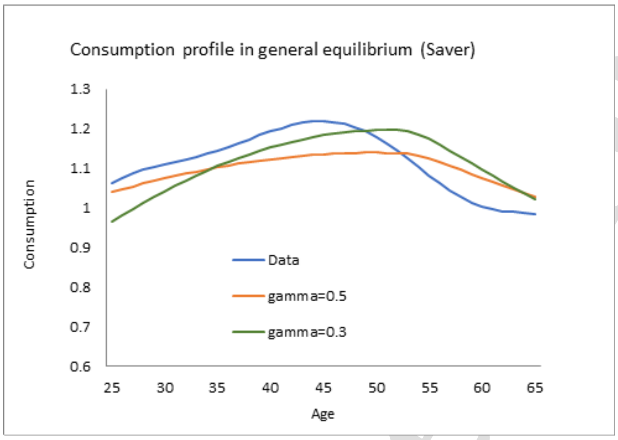

Figure 8: General equilibrium result with savers and data (Gourinchas and Parker, 2002). $\eta = 1, \lambda = 2$ and $K = 0.2$. The weight function is $\omega_t = K + 0.01(t - 25)$ for $25 \leqslant t \leqslant 80$.

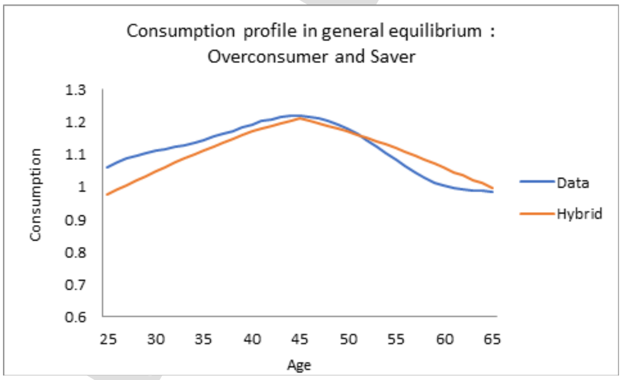

Figure 9: General equilibrium result with both savers and overconsumers with a ratio of 0.2 for the savers. Data is from Gourinchas and Parker (2002).Parameters are $\eta = 1, \lambda = 2$, and for the saver $(K = 0.2, \gamma = 0.9)$ and the overconsumer $(K = 0.5, \gamma = 0.3)$. The weight functions are: $\omega_t = 0.5 - 0.01(t - 25)$ for $t \leqslant 45$ and $\omega_t = \omega_{t-1} + 0.01$ for $45 < t \leqslant 65$ for the overconsumer, and $\omega_t = K + 0.01(t - 25)$ for the saver.

both savers and overconsumers coexist in the economy, then they form a hybrid model [80% of overconsumers : 20% of savers] for which the result of general equilibrium is available in Figure 9. The weight functions remain the same as before with the constant term of $K(saver) = 0.2$ and $K(overconsumer) = 0.5$. Introducing heterogeneity of consumers improves the overall fit and the result is quite robust to alternative parameter values.

4 Comparison with bounded rationality model

Up to this point the dynamic overconsumers or savers are equipped with a certain pattern of loss aversion parameter $\{\omega_t\}$ to generate a consumption profile. Given the lack of concrete experimental data regarding loss aversion preferences so far, posing any assumption on $\{\omega_t\}$, regardless of plausibility, would be considered incomplete. Leaving this parameter a free variable while recontextualizing the model's constraints would allow us to gain some valuable understanding on loss aversion. By exploring the structure of the income flow, which is known to be inversely U-shaped over a lifecycle, with a peak coming later than that of the consumption as in Figure 10, the model finds the conditions by which the loss aversion parameter supports the lifecycle consumption in the data.

Some researchers have focused on the relationship between consumption and income, and argued that contrary to the canonical lifecycle consumption theory, the two may co-move and that consumption is the agent's projection of perceived income at hand. As examined in many behavioral models, the lifecycle consumption can track the income profile when the consumption is closely tied to the current income. In Park and Feigenbaum (2018), the consumers are not fully rational and optimize period by period as new information for a limited planning horizon is revealed, and thus their consumption profile is more strongly dependent on the current income level.

In this section I address the implication of this short term planning when the consumer also has the reference dependent preference and intends to deviate for over- or under-consumption. Specifically I focus on the theoretic premise of this modified model with regard to the consumption data and examine how the new condition interacts with loss aversion.

4.1 A reference$-$dependence model of $S-$period updating

The short horizon model assumes that the agent foresees and optimizes only up to S, instead of the full lifecycle horizon T. Thus the equations in Eq. 16, 17, 18 should be modified accordingly. With this version, the weight ω_t is not assumed to follow a predetermined pattern but works together with income for the dynamic agent reoptimizing his consumption schedule.

A deviating agent at age t plans for the coming $S + 1$ periods based on his reference utility $u(c_t^*, c_{t+1}^*, ..., c_{t+S}^*)$, and will face a new maximization problem each year as his future income unveils over time. Because his loss aversion is left as a free variable, the driving factor of the agent's reoptimization is the

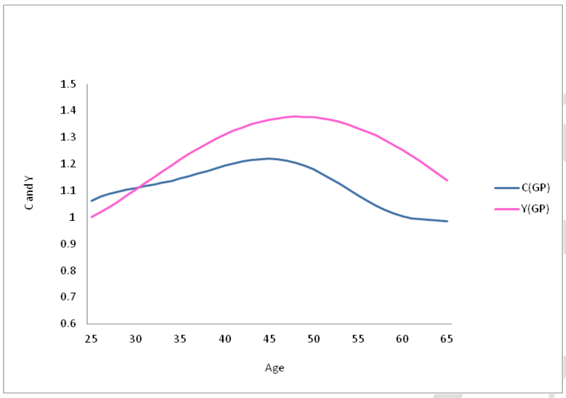

Figure 10: Consumption (C) and income (Y) from data. Estimation by Gourinchas and Parker (Econometrica, 2002).

new information regarding his income. The reference point at which the agent's belief is formed, is now the ex ante optimal solution to the updated maximization problem with the new income occurrence in view. It is still true that the realized consumption only tracks the initial consumption of each planned path, although with this modification the length of the planned path is shorter. As the planning horizon shrinks, the series is forced to more directly reflect the income flow, and the model is able to reconcile the consumption smoothing motivation with the nonmonotonic income flow.

With the modification, I first derive the theoretic implication of the short horizon, by constructing a dynamic model of deviation in which the agent's loss aversion is an *endogenous* variable, with his consumption reflecting the newly revealed income within a planning horizon. The subsequent selves of the agent here reoptimize with respect to not only the temporal loss aversion but also the new income appearance. By maintaining a cohesive intertemporal structure between the consumption profile and income, the model establishes the optimization rule as a direct relationship of loss aversion to income.

Then, the next subsection displays a set of simulation results, from three different plans of consumption behavior, for comparison in terms of both utility and profile shape. For this purpose I introduce a static deviating environment, under which the agent's descriptive plan for deviation directly accompanies the short planning horizon, and his consumption profile evolves over time as the agent accommodates his intention with regard to income flow.

First, to explore the implication of the income updating on the dynamic model, I derive a closed form solution to a simple model in which the agent lives four lifetime periods while he is solving only for *two periods* with the periodic update of income. The rebalanced consistent consumption for an *overconsumer* is

$$\left\{\frac{y_0+\frac{y_1}{R}}{1+\frac{1}{\phi}\mu_0},\ \frac{y_1+Rb_1+\frac{y_2}{R}}{1+\frac{1}{\phi}\mu_1},\ \frac{y_2+Rb_2+\frac{y_3}{R}}{1+\frac{1}{\phi}\mu_2},\ \frac{R}{\phi}\mu_2\left(\frac{y_2+Rb_2+\frac{y_3}{R}}{1+\frac{1}{\phi}\mu_2}\right)\right\} \tag{29}$$

$$\mu_s=\left(\frac{1+\eta\omega_s\lambda}{1+\eta}\right)^{1/\gamma} \tag{30}$$

The last term in the bracket follows from the fact that the agent at the last period has no choice but to consume whatever left for him because no further income is to be realized. Recursively solving for the bond demand and substituting into y_t+Rb_t for $t=0,1,2$, will pin down the entire consumption profile. The bond demands are

$$b_1=\frac{y_0\left(\frac{1}{\phi}\mu_0\right)-\frac{y_1}{R}}{1+\left(\frac{1}{\phi}\mu_0\right)} \tag{31}$$

$$b_2=\frac{R^2y_0\left(\frac{1}{\phi}\mu_0\right)\left(\frac{1}{\phi}\mu_1\right)+Ry_1\left(\frac{1}{\phi}\mu_0\right)\left(\frac{1}{\phi}\mu_1\right)-y_2\left(\frac{1}{\phi}\mu_0\right)}{R\left(\frac{1}{\phi}\mu_0\right)\left(\frac{1}{\phi}\mu_1\right)} \tag{32}$$

Then the consistent consumption $\{c_0, c_1, c_2, c_3\}$ is obtained by substituting the bond demands into the consumption equations. The consumption is[19]

$$c_0=\frac{Ry_0+y_1}{R\left(1+\frac{1}{\phi}\mu_0\right)} \tag{33}$$

$$c_1=\frac{\left(R^2y_0+Ry_1\right)\left(\frac{1}{\phi}\mu_0\right)+y_2\left(1+\frac{1}{\phi}\mu_0\right)}{R\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)} \tag{34}$$

$$c_2=\frac{(Ry_0+y_1)\left(\frac{R}{\phi}\mu_0\right)\left(\frac{R}{\phi}\mu_1\right)+y_2\left(1+\frac{1}{\phi}\mu_0\right)\left(\frac{R}{\phi}\mu_1\right)+y_3\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)}{R\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)\left(1+\frac{1}{\phi}\mu_2\right)} \tag{35}$$

$$c_3=\left(\frac{R}{\phi}\mu_2\right)\left(\frac{(Ry_0+y_1)\left(\frac{R}{\phi}\mu_0\right)\left(\frac{R}{\phi}\mu_1\right)+y_2\left(1+\frac{1}{\phi}\mu_0\right)\left(\frac{R}{\phi}\mu_1\right)+y_3\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)}{R\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)\left(1+\frac{1}{\phi}\mu_2\right)}\right) \tag{36}$$

When the agent is not deviating at any time ($\mu_0=\mu_1=\mu_2=1$), the consumption profile collapses into a simpler version. This is the case where the agent's loss aversion is high enough throughout his life that the agent does not

[19] One can find how the realized consumption is related to the intended one. Thus, for example, $c_1^1=\frac{\left(R^2y_0+Ry_1\right)\left(\frac{1}{\phi}\mu_0\right)+y_2\left(1+\frac{1}{\phi}\mu_0\right)}{R\left(1+\frac{1}{\phi}\mu_0\right)\left(1+\frac{1}{\phi}\mu_1\right)}=\frac{\frac{R}{\phi}\mu_0}{\left(1+\frac{1}{\phi}\mu_1\right)}\left(c_1^0+y_2+\frac{y_2}{R}\right)$

deviate from each of the reoptimized paths of the $S-$period maximization problem with updating. Regarding the featured lifecycle consumption, the dynamic model provides the following propositions.

Proposition 3 *Given R and β, if the agent's dynamic loss aversion $\{\mu_0, \mu_1, \mu_2\}$ and income stream $\{y_0, y_1, y_2, y_3\}$ jointly satisfy that $\left(1 - \frac{R}{\phi}\mu_0 + \frac{1}{\phi}\mu_1\right)(Ry_0 + y_1) < y_2(1 + \frac{1}{\phi}\mu_0)$, then the consistent consumption is increasing initially, i.e. $c_0 < c_1$.*

Although the proof in Appendix examines the condition according to the value of β relative to $1/R$, for an implication, let us simplify the condition by setting $\beta = 1/R$ to get

$$\left(1 - \frac{R}{R}\mu_0 + \frac{1}{R}\mu_1\right)(Ry_0 + y_1) < y_2 + \frac{y_2}{R}\mu_0 \tag{37}$$

Furthermore, if the agent is not deviating, the inequality returns to

$$y_0 + y_1 R^{-1} < y_2 + y_2 R^{-1} \tag{38}$$

which condition is always satisfied with an increasing income profile up to $t = 2$. Thus, the increasing property of the consumption at early stage of life $c_0 < c_1$ is obtained with an increasing income profile for a standard agent. But this condition may not be satisfied if y_2 is much smaller than y_1, although Eq. 37 can be still satisfied if μ_0 is very small. Therefore, the proposition suggests that with the reference-dependent agent, it is possible to have an increasing consumption profile even when the income is not increasing.

Proposition 4 *Given R and β, if the agent's dynamic loss aversion $\{\mu_0, \mu_1, \mu_2\}$ and income stream $\{y_0, y_1, y_2, y_3\}$ jointly satisfy that, either the relative loss aversion parameter (μ_0, μ_1) or the last income is sufficiently small, i.e., $\left(\frac{1}{\phi}\mu_0\left[R^2 y_0 + Ry_1 + y_2\right] + y_2\right)\left(1 + \frac{1}{\phi}\mu_2 - \frac{R}{\phi}\mu_1\right) > y_3\left(1 + \frac{1}{\phi}\mu_0\right)\left(1 + \frac{1}{\phi}\mu_1\right)$, then the consistent consumption is decreasing in later life, i.e. $c_1 > c_2$. The sufficient condition is $y_3 = 0$ with $y_2 > 0$.*

Again when $\beta = 1/R$, the condition yields

$$\left(\mu_0\left[Ry_0 + y_1 + \frac{y_2}{R}\right] + y_2\right)\left(1 + \frac{1}{R}\mu_2 - \mu_1\right) > y_3\left(1 + \frac{1}{R}\mu_0\right)\left(1 + \frac{1}{R}\mu_1\right) \tag{39}$$

If the agent is not deviating, then

$$\left(Ry_0 + y_1 + \frac{y_2}{R} + y_2\right)\left(\frac{1}{R}\right) > y_3\left(1 + \frac{1}{R}\right)^2 \tag{40}$$

For a standard agent with income updating, the income property is the sole determinant of the inequality and a significant reduction in income at the end of life is necessary to obtain the decreasing property. However, with the reference-dependent agent, it is no longer a necessary condition. Even though the last income is not very small, $y_T = y_3 \neq \varepsilon$, it is possible to get the result if (μ_0, μ_1) is sufficiently small. From the hump-shaped consumption data, it may be inferred that overconsumption among young agents can lead to lower consumption than their income level in later years due to the accumulated debt. Finally, combining both conditions yields the consumption hump.

Proposition 5 *If the agent's dynamic loss aversion $\{\mu_0, \mu_1, \mu_2\}$ and income stream $\{y_0, y_1, y_2, y_3\}$ jointly satisfy above two conditions, then the consumption profile of the agent produces a hump.*

An example[20] can be constructed to demonstrate how a dynamic loss aversion with income updating can generate a hump-shaped consumption profile even from a flat income series when the two conditions on loss aversion are satisfied.

4.1.1 Lifecycle result with $S-$period updating

This subsection further probes the model and provides utility comparison in addition to the lifecycle result, when the agents keep their initial loss aversion but still optimize period by period. Although loss aversion remains constant, the static deviator needs to reoptimize because of the income updates. Figure 11 and Figure 12 show the lifecycle profiles of the two different types of deviators from the updating model. The first is from the optimization of overconsumers with a planning horizon of 10 years, while the latter displays the case of savers with 20 years. As the figures indicate, a saver can reliably generate a hump, while it is generally more difficult for an overconsumer to display the desired profile.

With this modification toward bounded rationality, the midpoint rule may not necessarily still bestow the highest utility to the deviators among all the possible strategies. The resulting utility of each deviator with a different intention will vary by each plan. For this, I examine three distinct plans of deviation for the overconsumer, as well as the saver.

As demonstrated in Table 1, the *ex post* utility ranking is not preserved with income updating for the deviating agents. The table summarizes the result for an overconsumer whose loss aversion is $\omega_t \lambda = 0.3 \times 2 = 0.6$. As before, a set $\{I\}_0^S$ of the binary index $I_t \in \{0, 1\}$ represents a plan for the overconsumption. The simulation exercise is obtained with $\gamma = 0.9$ and the planning horizon is 10.

The table shows that the midpoint rule does not provide the highest consumption utility ex post regardless of βR condition. Instead, the highest utility

[20] *Suppose* $\{y_0, y_1, y_2, y_3\} = \{1, 1, 1, 1\}$. *Then the consumption profile of a reference dependent agent who is forward-looking only for two periods and has* $1/R$ *as his time preference, and* $\{\mu_0 = 0.8,\ \mu_1 = 0.4,\ \mu_2 = 1\}$ *as his loss aversion parameter is characterized by* $\{c_0,\ c_1,\ c_2, c_3\} = \left\{\frac{1+R}{0.8+R}, \frac{(0.8+R)+0.4R(R+1)}{(0.8+R)(0.4+R)}, \frac{(0.8+R)(0.4+R)+0.4(0.8+R)R+(0.4)(0.8)RR(1+R)}{(0.8+R)(0.4+R)(1+R)}, c_2\right\}$. *Using the standard interest rate* $R_{ann} = 1.035$, *and per length* $\tau = 15$ *year, this yields* $\{c_0,\ c_1,\ c_2,\ c_3\} = \{1.081,\ 1.179,\ 0.669,\ 0.669\}$. *Clearly,* $c_0 < c_1 > c_2$ *and the hump is achieved because of the preference structure.*

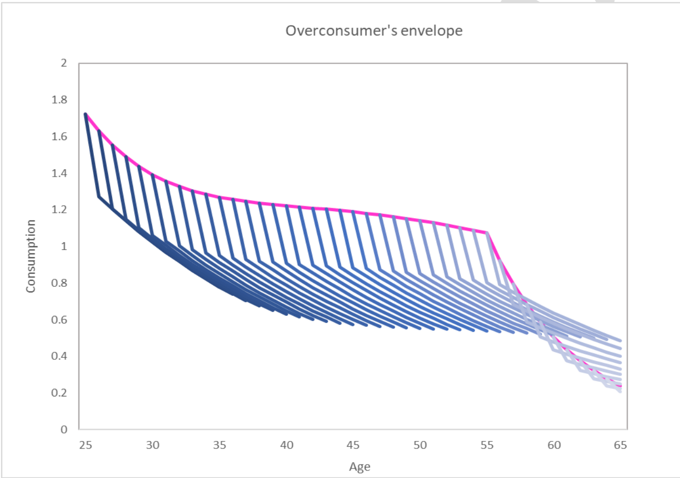

Figure 11: Realized lifecycle consumption as the envelope of planned consumption series. The floating top curve (in pink color) represents a series of the realized consumption, each point of which comes from the first element of the individual planned series (blue). The planning horizon is 10 years and the agent deviates for overconsumption: $\gamma = 0.9$, $R = 1.035$, $\beta = 0.96$ and $\lambda = 2$, $\omega = 0.3$.

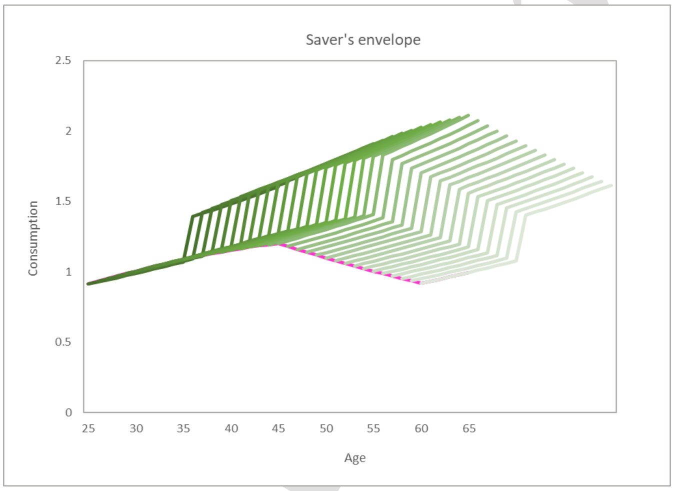

Figure 12: Realized lifecycle consumption as the envelope of planned consumption series. The underlying concave curve (pink) represents a series of the realized consumption, each point of which comes from the first element of the individual planned series (green). The planning horizon is 20 years and the agent deviates for under-consumption (saver): $\gamma = 0.9$, $R = 1.035$, $\beta = 0.98$ and $\lambda = 2$, $\omega = 0.3$.

is obtained with the plan of $\{I_t\} = \{1,1,1,1,1,1,1,1,1,0\}$ when $\beta R \geqslant 1$, while the order is reversed when βR is lower than 1. This is because with the first plan in which the agent spreads his overconsumption intention over many years, lower consumption coupled with higher savings when young will pay off under the environment of $\beta R \geqslant 1$.

[21]Table 1: Overconsumer

	Initial Plan	Utility	Rank	Profile
$\beta R > 1$	$(1,1,1,1,1,1,1,1,1,0)$	338.38	1	Hump
	$(1,1,1,1,1,0,0,0,0,0)$	336.91	2	Mild Hump
	$(1,0,0,0,0,0,0,0,0,0)$	335.32	3	No Hump
$\beta R = 1$	$(1,1,1,1,1,1,1,1,1,0)$	253.36	1	Hump
	$(1,1,1,1,1,0,0,0,0,0)$	252.75	2	Mild Hump
	$(1,0,0,0,0,0,0,0,0,0)$	251.99	3	No Hump
$\beta R < 1$	$(1,1,1,1,1,1,1,1,1,0)$	102.36	3	No Hump
	$(1,1,1,1,1,0,0,0,0,0)$	102.54	2	No Hump
	$(1,0,0,0,0,0,0,0,0,0)$	102.77	1	No Hump

The fifth column displays the shape of the consumption profile. When $\beta R > 1$, i.e. the combined value of the market interest rate and time preference is high enough, many plans can induce a hump. In fact, with the plan of $\{I_t\} = \{1,1,1,1,1,1,1,1,1,0\}$, the overconsumer is able to produce the most prominent consumption hump. Because the plan implies that the consumer spreads the resource over longer periods, he does not spend excessively in each of the first nine periods. Thus the consumption envelope stays low, yielding more wealth accumulation later. However, as the interest rate is getting lower relative to consumer's time preference, the mechanism that induces a hump becomes weaker. Thus when $\beta R < 1$, the humps are completely gone and the utility ranking reverses.

Let us turn to the saver's case. By the same logic as with the overconsumer, the plan of $\{I_t\} = \{0,1,1,1,1,1,1,1,1,1\}$ gives the highest utility ex post for most of the parameter values. Likewise, the plan $\{I_t\} = \{0,0,0,0,0,0,0,0,0,1\}$ gives the lowest utility ex post, as shown in Table 2. When $\beta R > 1$ all plans produce a hump. And the hump is more apparent with the plan of $\{I_t\} = \{0,1,1,1,1,1,1,1,1,1\}$, because the plan that suggests an initial heavy saving to afford higher consumption for the rest of the nine periods results in a very low consumption in early life. The static saver would save in the same manner for the remaining planning periods when she reoptimizes with the updated income. Thus overall, the consumer in this case ends up with a path of increasing consumption before it turns lower in vision of retirement.

[21]For Table 1 and 2: given $R = 1.035$, the values of β are: $\beta = 0.98$; $\beta = 1/R = 0.966$; $\beta = 0.9$ for the three specifications of βR condition. The ex post consumption utility is measured by $\sum_{t=0}^{T} \beta u(c_t)$ with model time $T = 55$, which corresponds to the actual age 80.

With the case of $\{I_t\} = \{0,0,0,0,0,0,0,0,0,1\}$, the consumer spreads his savings over longer periods, and she does not need to undergo substantial saving at the first period. As such, the saver's saving at the first period would not be much in each reoptimization. Thus over the life, less wealth accumulation occurs.

In summary, for a saver, the hump is obtained with $\beta R = 1$ and even with $\beta R < 1$, although the hump intensity is weaker. However, when $\beta R << 1$[22] the ranking is reversed and the plan $\{I_t\} = \{0,0,0,0,0,0,0,0,0,1\}$ provides the best consumption outcome.

Table 2: Saver

	Initial Plan	Utility	Rank	Profile
$\beta R > 1$	$(0,1,1,1,1,1,1,1,1,1)$	340.83	1	Hump
	$(0,0,0,0,0,1,1,1,1,1)$	340.08	2	Hump
	$(0,0,0,0,0,0,0,0,0,1)$	338.89	3	Hump
$\beta R = 1$	$(0,1,1,1,1,1,1,1,1,1)$	254.20	1	Hump
	$(0,0,0,0,0,1,1,1,1,1)$	253.97	2	Hump
	$(0,0,0,0,0,0,0,0,0,1)$	253.55	3	Hump
$\beta R << 1$	$(0,1,1,1,1,1,1,1,1,1)$	101.67	3	Hump
	$(0,0,0,0,0,1,1,1,1,1)$	102.03	2	Hump
	$(0,0,0,0,0,0,0,0,0,1)$	102.29	1	Hump

Finally, under the scheme of the midpoint rule, if the preference of the agent stays constant over the life, i.e. $\omega_t\lambda = \omega\lambda$, then a hybrid model of the two types of consumers with $\omega\lambda < 1$ can reproduce the lifecycle consumption data in a well-calibrated general equilibrium. The result in Figure 13 comes from a hybrid environment of savers and overconsumers, with a ratio of 0.4 for the savers.

5 Loss aversion under uncertainty

This section analyzes the intertemporal loss aversion under uncertainty. In the deterministic environment in Section 2, the agent experiences either gain or loss utility relative to the reference at each point in time. However, in the uncertainty environment, it is necessary to consider one more layer of reference dependence within each time period, namely the gain-loss feelings that arise over different states of the world.

For dynamic loss aversion under uncertainty, it is desirable to have the model analyzed in terms of both (A) stochastic loss aversion (state) and (B) intertemporal loss aversion (time). Regarding stochastic loss aversion, Koszegi and Rabin (2006) introduce a static model of expectations-based reference dependence, allowing for both stochastic consumption and stochastic reference points to be

[22] Given $R = 1.035$, the result is obtained with $\beta = 0.9$ for $\beta R << 1$, while $\beta = 0.96$ for $\beta R < 1$.

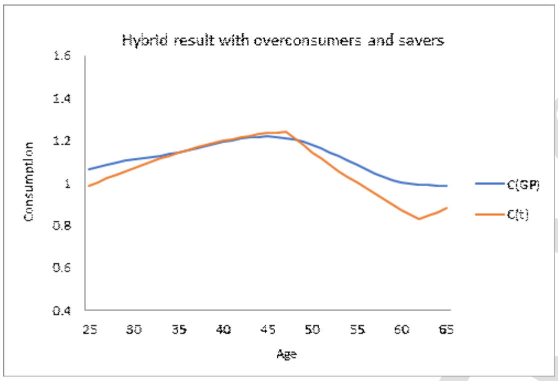

Figure 13: General equilibrium result with a hybrid environment, with a ratio of 0.4 for the savers: $\lambda = 2$, $\omega = 0.3$, $\gamma = 0.5$, and $S = 18$.

added to the riskless model. Given a stochastic reference point $r \in Y$ distributed according to $G(y)$, as well as the stochastic outcome $c \in X$ from $F(x)$, the expected utility of c is

$$U(c|r) = \int \int u(x|y) dG(y) dF(x) \tag{41}$$

In a state-independent stochastic world on which Koszegi and Rabin (2006, 2009) build their model, a consumer derives gain-loss utility from the comparison of every possible stochastic outcome against all possible states of the reference point.

$$EU^{indep} = \int_{-\infty}^{\infty} u(x) dF_X(x) \tag{42}$$

$$+ \int_{-\infty}^{\infty} \eta \left(\int_{-\infty}^{x} (u(x) - u(y)) dF_Y(y) + \lambda \int_{x}^{\infty} (u(x) - u(y)) dF_Y(y) \right) dF_X(x)$$

where $F_X(x) = \mathbb{P}[X \leq x]$ and $F_Y(y) = \mathbb{P}[Y \leq y]$ are the cumulative distribution functions (c.d.f.) of X and Y, respectively. The second term shows both gain utility $u(x) - u(y) > 0$ and loss utility $u(x) - u(y) < 0$. This type of expectation follows disappointment theory (Bell, 1985; Gul, 1991) and is also shown in Cillo and Delquie (2006).

However, a more relevant expectation is provided by state-dependent reference, originating from regret theory (Loomes and Sugden, 1982), which is adopted by Sugden (2003) and Giorgi and Posty (2011). This specification assumes that the consumer evaluates outcomes relative only to the same state, and thus losses are only experienced if they happen in the same state.

$$EU^{dep} = \int_{-\infty}^{\infty} u(x) f_X(x) dx \tag{43}$$

$$+\eta \int_{-\infty}^{\infty} \left(\int_{-\infty}^{x} (u(x) - u(y)) f_{X,Y}(x,y)dy + \lambda \int_{x}^{\infty} (u(x) - u(y)) f_{X,Y}(x,y)dy \right) dx$$

where $f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x_t \partial y}$ is the joint density from the joint c.d.f. $F_{X,Y}(x,y) = P[X_t \leq x; Y \leq y]$, while $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$ is the marginal density of X from the joint density. Park (2016), by introducing a consumption−saving model under the two schemes, compares the two to show that when there is positive state-dependency, for each level of uncertainty the state-independent model generates a greater consumption−and thus lower saving−behavior than the state-dependent one does, relative to the standard model.

Less explored is intertemporal loss aversion, which is related to the dynamic modeling of reference dependence, a branch of which used to be popular in models of habit formation or backward-looking reference dependence shown in Chetty and Szeidl (2016). Intertemporal loss aversion can be constructed both with and without uncertainty as the current paper will demonstrate. With uncertainty, Koszegi and Rabin (2009) extend their static uncertainty model into a dynamic setting by which the prospective gain-loss utility is evaluated through the entire stream of future consumption, under the assumption that the agent makes an ordered (percentile-wise) comparison between his previous (rational) beliefs $F_{t-1,\tau}$ and new (rational) beliefs $F_{t,\tau}$ about future consumption.[23] Related to this comparison, Pagel (2019) argues that separated comparison−separating realized uncertainty from the future uncertainty−would perform better in economic models, because this method yields tractable equilibria when there is intertemporal dependence in beliefs.

The lifecycle model in Pagel (2017) is built on the loss aversion over the states of the world, specifically state-independent, simultaneously incorporating the dynamic setting via a similar way as shown in the model of this section. Pagel (2017) produces several lifecycle results similar to the data, including a hump-shaped consumption stream over the lifecycle in partial equilibrium. However, the model solution in Pagel (2017)'s is based on exponential utility functions and has limitations in its application.[24] Furthermore, the hump-shaped consumption profile is obtained partially from the precautionary saving motivation against income uncertainty.

The model of intertemporal loss aversion in Section 2 can be extended to the full level of dynamic setting incorporating the loss aversion over different states of the world, being simultaneously constructed in terms of both state and time. Let S be the state-space and let $\mathcal{X}$ be the collection of feasible prospects $X_i : S \to \mathbb{R}^S$ and $\mathbb{P}_i[E]$ for the probability that event $E \subset \Omega_E$ occurs. Likewise let Z be the time-space and let $\mathcal{T}$ be the collection of feasible

[23] They propose, for $\tau = t, ..., T$, $u_t = m(c_t) + \sum_{\tau=t}^{T} \varphi_{t,\tau} n(F_{t,\tau} \mid F_{t-1,\tau})$ where $F_{t-1,\tau}$ represents fixed beliefs, inherited from the last period and $F_{t,\tau}$ represents new beliefs that the agent forms. $\varphi_{\tau,\tau} \geqslant \varphi_{\tau-1,\tau} \geqslant ... \geqslant \varphi_{0,\tau} \geqslant 0$ are weights on gain-loss utilities.

[24] Precautionary savings have been well studied in the CRRA class because its marginal utility MU is convex ($u'''(c) > 0$). An increase in uncertainty raises the expected value of the MU, which raises a consumer's motivation to save. Both CRRA and CARA exhibit this property, while in the case of the exponential utility, it is known that uncertainty may lead to negative initial consumption.

prospects $X_t : Z \to \mathbb{R}^Z$ and $\mathbb{P}_t[F]$ for the probability that event $F \subset \Omega_F$ occurs. Let us first consider the case that the consumer experiences gain-loss utility for each possible outcome relative to all other possible outcomes, i.e. state-independent reference points at time t, and which is assumed to be discrete for the model. The lifetime utility of the agent who has a series of non-negative prospect $\{X_{it} : x_{it} \geqslant 0\}_{t=0}^T \in \mathcal{X} \cap \mathcal{T}$ conditioned on a series of non-negative reference points $\{Y_{jt} : y_{jt} \geqslant 0\}_{t=0}^T \in \mathcal{X} \cap \mathcal{T}$ is given by[25]

$$EU^{indep} = \tag{44}$$

$$\sum_{t=0}^{T} \beta^t \left(\int_0^\infty u(x_{it}) dF_{X_i}(x_i) + \eta \int_0^\infty [I_t G_t^{indep} + \omega_t \lambda (1 - I_t) L_t^{indep}] dF_X(x_t) \right)$$

where the temporal state-independent gain G_t^{indep} utility[26]

$$G_t^{indep} = \begin{bmatrix} \int_0^{x_t} \left[\int_0^\infty \int_0^\infty (u(x_{it})\text{-}u(y_{jt})) dF_{Xi} dF_{Yj} \right] dF_Y(y_t) & \text{if } u(x_{it})\text{-}u(y_{jt}) > 0 \\ \lambda \int_0^{x_t} \left[\int_0^\infty \int_0^\infty (u(x_{it})\text{-}u(y_{jt})) dF_{Xi} dF_{Yj} \right] dF_Y(y_t) & \text{if } u(x_{it})\text{-}u(y_{jt}) < 0 \end{bmatrix} \tag{45}$$

and the temporal state-independent loss L_t^{indep} utility[27]

$$L_t^{indep} = \begin{bmatrix} \int_{x_t}^\infty \left[\int_0^\infty \int_0^\infty (u(x_{it})\text{-}u(y_{jt})) dF_{Xi} dF_{Yj} \right] dF_Y(y_t) & \text{if } u(x_{it})\text{-}u(y_{jt}) > 0 \\ \lambda \int_{x_t}^\infty \left[\int_0^\infty \int_0^\infty (u(x_{it})\text{-}u(y_{jt})) dF_{Xi} dF_{Yj} \right] dF_Y(y_t) & \text{if } u(x_{it})\text{-}u(y_{jt}) < 0 \end{bmatrix} \tag{46}$$

in which $F_X(x_{it}) = \mathbb{P}[X_t \leq x_t, X_i \leq x_i]$ and $F_Y(y_{jt}) = \mathbb{P}[Y_t \leq y_t, Y_j \leq y_j]$ are cumulative distribution functions (c.d.f.) of X_t and Y_t, respectively, at state i and j. Note that both the temporal gain and loss utilities contain both gain and loss utilities over different states of the world. $F_X(x_t)$ and $F_Y(y_t)$ are the marginal c.d.f.s obtained by summing up over all states of the world in $X_i \leq x_i$ and $Y_j \leq y_j$, respectively.

Alternatively, however, the agent may experience gain-loss utilities not for each possible outcome relative to all, but rather for those within the same state. In the case of state-dependence, it is necessary to modify the expected utility using joint distributions. For jointly continuous random variables $\{X_{it}, Y_{jt}\}_{t=0}^T$, the state-dependent expected utility will have

$$EU^{dep} = \tag{47}$$

$$\sum_{t=0}^{T} \beta^t \left(\int_0^\infty u(x_{it}) f_X(x_{it}) dx_i + \eta \int_0^\infty [I_t G_t^{dep} + \omega_t \lambda (1 - I_t) L_t^{dep}] f_{X,Y}(x_t, y_t) dy_t dx_t \right)$$

[25] This expression is the full-blown state-independent loss aversion counterpart to Eq. (5) in Section 2: $\sum_{t=0}^T \beta^t \{Eu(c_t) + \eta I_t [u(c_t) - u(c_t^*)]^+ + \eta \omega \lambda (1 - I_t)[u(c_t) - u(c_t^*)]^-\}$

[26] The intertemporal gain utility occurs when $x_t > y_t$.

[27] The intertemporal loss utility occurs when $x_t < y_t$.

with the temporal state-dependent gain G_t^{dep} and loss L_t^{dep} utilities

$$G_t^{dep} = \begin{bmatrix} \int_0^{x_t} \left[\int_0^{\infty}\int_0^{\infty}(u(x_{it})\text{-}u(y_{jt}))f_{X,Y}(x_i,y_j)dx_i dy_j\right] f_Y(y_t)dy_t \text{ if } u(x_{it})\text{-}u(y_{jt}) > 0 \\ \lambda\int_0^{x_t} \left[\int_0^{\infty}\int_0^{\infty}(u(x_{it})\text{-}u(y_{jt}))f_{X,Y}(x_i,y_j)dx_i dy_j\right] f_Y(y_t)dy_t \text{ if } u(x_{it})\text{-}u(y_{jt}) < 0 \end{bmatrix} \tag{48}$$

$$L_t^{dep} = \begin{bmatrix} \int_{x_t}^{\infty} \left[\int_0^{\infty}\int_0^{\infty}(u(x_{it})\text{-}u(y_{jt}))f_{X,Y}(x_i,y_j)dx_i dy_j\right] f_Y(y_t)dy_t \text{ if } u(x_{it})\text{-}u(y_{jt}) > 0 \\ \lambda\int_{x_t}^{\infty} \left[\int_0^{\infty}\int_0^{\infty}(u(x_{it})\text{-}u(y_{jt}))f_{X,Y}(x_i,y_j)dx_i dy_j\right] f_Y(y_t)dy_t \text{ if } u(x_{it})\text{-}u(y_{jt}) < 0 \end{bmatrix} \tag{49}$$

where $f_{X,Y}(x_t,y_t) = \frac{\partial^2 F_{X,Y}(x_t,y_t)}{\partial x_t \partial y_t}$ is the joint density from the joint c.d.f. $F_{X,Y}(x_t,y_t) = P[X_t \leq x_t; Y_t \leq y_t]$. Likewise, $f_{X,Y}(x_i,y_j) = \frac{\partial^2 F_{X,Y}(x_i,y_j)}{\partial x_i \partial y_j}$ is the joint density from the joint c.d.f. $F_{X,Y}(x_i,y_j) = P[X_i \leq x_i; Y_j \leq y_j]$. Furthermore, $f_X = \int f_{X,Y}(x,y)dy$ and $f_Y = \int f_{X,Y}(x,y)dx$ are marginal densities.

5.1 Revisiting the nonmonotonicity under uncertainty

Because of the innate complexity of the modeling, a tractable model fully incorporating both time and state seems implausible. If one of the two channels of reference dependence can be closed, a simplified version of the uncertainty is modeled with the perspective of overconsumption discussed in Section 2. Utilizing the analysis in the previous section, let us assume that the consumer will have only the intertemporal loss aversion but faces uncertainty in his income stream. Also, assume that his loss aversion parameter stays the same over his lifecycle, $\omega_t\lambda = \omega\lambda$. Thus, the consumer maximizes

$$Eu(c|c^*) = E_0\sum_{t=0}^{T}\beta^t\{u(c_t) + \eta I_t[u(c_t) - u(c_t^*)]^+ + \eta\omega\lambda(1-I_t)[u(c_t) - u(c_t^*)]^-\} \tag{50}$$

subject to

$$c_t + b_{t+1} = \widetilde{y}_t + Rb_t \tag{51}$$

$$b_0 = 0, b_{T+1} = 0 \tag{52}$$

where $\widetilde{y}_t$ is uncertain income at time t, and $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$ as well as

$$u(c_t^*) = \arg\max E_t\sum_{s=t}^{T}\beta^{s-t}\left(\frac{c_s^{1-\gamma}}{1-\gamma}\right) \tag{53}$$

subject to the lifecycle budget constraint Eq. 51, as well as $b_t = given$, and $b_{T+1} = 0$. With the consumption−saving model in Section 2, the intertemporal optimization under uncertainty implies that for any adjacent two periods the expected marginal utilities $\{Eu'(c_t),\ Eu'(c_{t+1})\}$ should be equated: for the intertemporal MRS between t and $t+1$, the expectation at time τ is

$$E_\tau[c_t^{-\gamma} + \eta I_t c_t^{-\gamma} + \eta\omega\lambda(1-I_t)c_t^{-\gamma}] = R\beta E_\tau[c_{t+1}^{-\gamma} + \eta I_{t+1}c_{t+1}^{-\gamma} + \eta\omega\lambda(1-I_{t+1})c_{t+1}^{-\gamma}] \tag{54}$$

The second term and third term in the LHS are the time τ expectation of marginal utilities of the contemporaneous gain and loss, while RHS are those of the prospective gain and loss. Here, the expectation operator E_τ turns out to be independent from $E_{\tau+1}$, in a similar vein as in Pagel (2019). The non-monotonicity of consumption discussed in Section 2 can be explained once again under uncertainty. Rewrite Eq. 54. to get

$$E_\tau c_t^{-\gamma} = (R\beta)\left(\frac{1+\eta I_{t+1}+\eta\omega\lambda(1-I_{t+1})}{1+\eta I_t+\eta\omega\lambda(1-I_t)}\right)E_\tau c_{t+1}^{-\gamma} \tag{55}$$

For any adjacent two periods, t and $t+1$, when the gain-loss index remains the same, i.e. either $I_t = I_{t+1} = 0$ or $I_t = I_{t+1} = 1$, there is no difference in the model from the standard one under uncertainty, for which the intertemporal optimality requires

$$E_\tau\left(\frac{1}{c_t^\gamma}\right) = (R\beta)\,E_\tau\left(\frac{1}{c_{t+1}^\gamma}\right) \tag{56}$$

because Eq. 55 returns to $E_\tau c_t^{-\gamma} = (R\beta)\left(\frac{1+\eta}{1+\eta}\right)E_\tau c_{t+1}^{-\gamma}$ when $I_t = I_{t+1} = 1$, and $E_\tau c_t^{-\gamma} = (R\beta)\left(\frac{1+\eta\omega\lambda}{1+\eta\omega\lambda}\right)E_\tau c_{t+1}^{-\gamma}$ when $I_t = I_{t+1} = 0$. However, when $\{I_t, I_{t+1}\} = \{1, 0\}$, because the optimality requires

$$E_\tau c_t^{-\gamma} = (R\beta)\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)E_t c_{t+1}^{-\gamma} \tag{57}$$

it is true that $E_\tau\left(\frac{1}{c_t^\gamma}\right) < (R\beta)\,E_\tau\left(\frac{1}{c_{t+1}^\gamma}\right)$ for an overconsumer with $\omega\lambda < 1$. Likewise, when $\{I_t, I_{t+1}\} = \{0, 1\}$, then $E_\tau\left(\frac{1}{c_t^\gamma}\right) > (R\beta)\,E_\tau\left(\frac{1}{c_{t+1}^\gamma}\right)$ for a saver with $\omega\lambda < 1$, because

$$E_\tau c_t^{-\gamma} = (R\beta)\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)E_\tau c_{t+1}^{-\gamma} \tag{58}$$

Thus, when $\omega\lambda < 1$, the condition implies either $E_\tau c_t^{-\gamma} > (R\beta)\,E_\tau c_{t+1}^{-\gamma}$ or $E_\tau c_t^{-\gamma} < (R\beta)\,E_\tau c_{t+1}^{-\gamma}$ over a lifecycle. Therefore, monotonicity, in terms of time τ expectation, can break.

5.2 Consumption and saving under uncertainty

Assume that the agent with the reference dependent preference lives for two periods, t and $t+1$, and that he faces uncertainty at $t+1$ so that with probability p his income is high $y_{t+1}^h = \overline{y} + \varepsilon_{t+1}$ and with $1-p$ it is low $y_{t+1}^l = \overline{y} - \varepsilon_{t+1}$. At t, it is certain and thus fixed $y_t = \overline{y}$. With the environment, the budget constraint is

$$c_t + b_{t+1} = y_t \tag{59}$$

$$c_{t+1} = \widetilde{y_{t+1}} + Rb_{t+1} \tag{60}$$

From Eq. 57, the optimality condition for an overconsumer at $\tau = t$ is

$$\frac{1}{c_t^\gamma} = (R\beta)\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)\left(p\left(\frac{1}{\left[y_{t+1}^h + R(y_t - c_t)\right]^\gamma}\right) + (1-p)\left(\frac{1}{\left[y_{t+1}^l + R(y_t - c_t)\right]^\gamma}\right)\right) \tag{61}$$

because the expected marginal utility is

$$E_t\left(\frac{1}{c_{t+1}^\gamma}\right) = p\left(\frac{1}{\left[y_{t+1}^h + R(y_t - c_t)\right]^\gamma}\right) + (1-p)\left(\frac{1}{\left[y_{t+1}^l + R(y_t - c_t)\right]^\gamma}\right) \tag{62}$$

Likewise, the optimality condition for a saver is

$$\frac{1}{c_t^\gamma} = (R\beta)\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)\left(p\left(\frac{1}{\left[y_{t+1}^h + R(y_t - c_t)\right]^\gamma}\right) + (1-p)\left(\frac{1}{\left[y_{t+1}^l + R(y_t - c_t)\right]^\gamma}\right)\right) \tag{63}$$

The optimality condition of the standard model is obtained by setting $\omega\lambda = 1$. To derive closed form solutions, let us assume $\gamma = 1$ in the CRRA utility function, i.e. $u(c) = \ln c$. Solving the model generates the optimal consumption at the first period, which is

$$c_t = \frac{(1+R)(2+\beta\mu)}{2(1+\beta\mu)R}\overline{y} - \frac{\sqrt{(1+R)^2\beta^2\mu^2\overline{y}^2 + 4(\beta\mu+1)\varepsilon_{t+1}^2}}{2(1+\beta\mu)R} \tag{64}$$

where $\mu = \frac{1+\eta\omega\lambda}{1+\eta}$. The consumption at time t decreases when the future uncertainty parameter ε_{t+1} increases. However, given the value of ε_{t+1}, the agent with reference dependent preference $\omega\lambda < 1$ consumes more than the standard agent ($\omega\lambda = 1$). This is because, if $\omega\lambda < 1$ then $\mu < 1$ and given $\overline{y}$, the reference dependent consumer (RDP consumer hereafter) has $\frac{(1+R)(2+\beta\mu)}{2(1+\beta\mu)R}$ $> \frac{(1+R)(2+\beta)}{2(1+\beta)R}$, together with $\frac{\sqrt{(1+R)^2\beta^2\mu^2\overline{y}^2+4(\beta\mu+1)\varepsilon^2}}{2(1+\beta\mu)R} < \frac{\sqrt{(1+R)^2\beta^2\overline{y}^2+4(\beta+1)\varepsilon^2}}{2(1+\beta)R}$, where the RHS in both inequalities represents the case of the standard model ($\mu = 1$). The time t bond demand or saving for the next period $t+1$ is $b_{t+1} = y_t - c_t$:

$$b_{t+1} = -\frac{2-\beta\mu(R-1)}{2(1+\beta\mu)R}\overline{y} + \frac{\sqrt{(1+R)^2\beta^2\mu^2\overline{y}^2 + 4(\beta\mu+1)\varepsilon_{t+1}^2}}{2(1+\beta\mu)R} \tag{65}$$

which increases as the uncertainty parameter ε_{t+1} increases. Whenever there is uncertainty, the individual consumes less at the first period and saves more for the second period, i.e. *precautionary saving increases.*[28] However, with the RDP consumer who has a low loss aversion, the precautionary-saving motive can be compromised by his overconsumption intention, and his saving can be

[28] Given a level of cash on hand (the sum of current income and financial income $y_t + Rb_t$), a higher income uncertainty increases the precautionary saving.

negative if the income uncertainty is modest, i.e. the consumer may borrow $b_{t+1} < 0$ for a small ε_{t+1}. The same analysis is applied to the saver's case:

$$E_\tau c_t^{-\gamma} = (R\beta)\left(\frac{1+\eta}{1+\eta\omega\lambda}\right) E_\tau c_{t+1}^{-\gamma} \tag{66}$$

Let $\nu = \frac{1+\eta}{1+\eta\omega\lambda}$, then the saver's consumption, as well as bond demand, is

$$c_t = \frac{(1+R)(2+\beta\nu)}{2(1+\beta\nu)R}\overline{y} - \frac{\sqrt{(1+R)^2\beta^2\nu^2\overline{y}^2 + 4(\beta\nu+1)\varepsilon_{t+1}^2}}{2(1+\beta\nu)R} \tag{67}$$

$$b_{t+1} = -\frac{2-\beta\nu(R-1)}{2(1+\beta\nu)R}\overline{y} + \frac{\sqrt{(1+R)^2\beta^2\nu^2\overline{y}^2 + 4(\beta\nu+1)\varepsilon_{t+1}^2}}{2(1+\beta\nu)R} \tag{68}$$

Because $\omega\lambda < 1$, the saver's parameter is now $\nu > 1$ and there is a greater chance that $b_{t+1} = -\frac{2-\beta\nu(R-1)}{2(1+\beta\nu)R}\overline{y} + \frac{\sqrt{(1+R)^2\beta^2\nu^2\overline{y}^2+4(\beta\nu+1)\varepsilon_{t+1}^2}}{2(1+\beta\nu)R} > 0$[29] i.e. her saving is positive.

5.3 Borrowers and lenders in equilibrium

According to the result in Eq. 65 and Eq. 68, the consumer with $\{I_t, I_{t+1}\} = \{1, 0\}$ would not save more than the consumer with $\{I_t, I_{t+1}\} = \{0, 1\}$ would save. This section derives equilibrium interest rates under uncertainty when the economy is populated by the two types of consumers, i.e. overconsumers and savers, who are otherwise identical. The equilibrium interest rate for the borrowing−lending market is obtained by the condition that the net supply of bonds is equal to zero: bond prices clear the market at equilibrium.

$$\Lambda_{\text{borrower}} \sum b_{t+1}^{\text{borrower}}(R) + \Lambda_{\text{lender}} \sum b_{t+1}^{\text{lender}}(R) = 0 \tag{69}$$

where $0 < \Lambda < 1$ is the population share of each group. From the bond demands obtained in Eq. 65 and Eq. 68, the borrower's and lender's bond demands are

$$b_{t+1}^{-} = -\frac{2-\beta\frac{1+\omega\lambda}{2}(R-1)}{2(1+\beta\frac{1+\omega\lambda}{2})R}\overline{y} + \frac{\sqrt{(1+R)^2(\beta\frac{1+\omega\lambda}{2})^2\overline{y}^2 + 4(\beta\frac{1+\omega\lambda}{2}+1)\varepsilon^2}}{2(1+\beta\frac{1+\omega\lambda}{2})R} \tag{70}$$

$$b_{t+1}^{+} = -\frac{2-\beta\frac{2}{1+\omega\lambda}(R-1)}{2(1+\beta\frac{2}{1+\omega\lambda})R}\overline{y} + \frac{\sqrt{(1+R)^2(\beta\frac{2}{1+\omega\lambda})^2\overline{y}^2 + 4(\beta\frac{2}{1+\omega\lambda}+1)\varepsilon^2}}{2(1+\beta\frac{2}{1+\omega\lambda})R} \tag{71}$$

Figure 14 shows the general equilibrium result for the economy. The uncertainty specification is set to around $\varepsilon = 0.25$ to secure a positive net interest rate. Also Λ is adjusted accordingly to set an equilibrium given a value of $\omega\lambda$. A result from the numerical solution is obtained with $\overline{y} = 1$, $\beta = 1/R$, $\lambda = 2$,

[29] With $R = 1$ and $\beta = 1/R$, it is easly seen that $\frac{1}{1+\nu}\overline{y} < \frac{\sqrt{\nu^2\overline{y}^2+(\nu+1)\varepsilon_{t+1}^2}}{1+\nu}$, when $\nu > 1$.

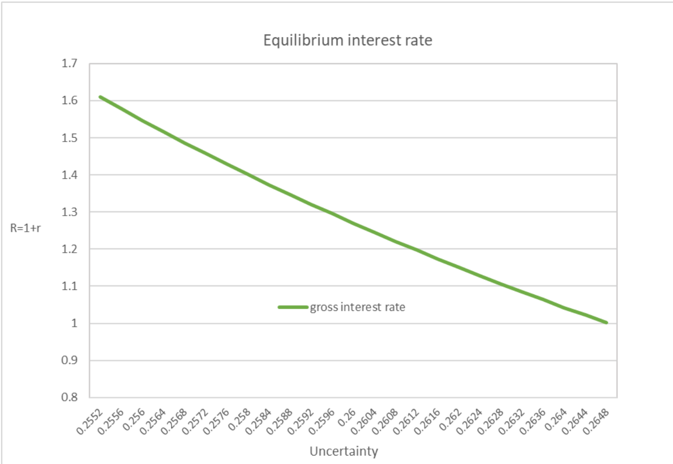

Figure 14: Uncertainty and equilibrium interest rate. The result is obtained with $\overline{y} = 1$, $\beta = 1/R$, $\lambda = 2$, $\omega = 0.3$ and $\Lambda_{\text{borrower}} : \Lambda_{\text{lender}} = 2 : 1$.

$\omega = 0.3$, and $\Lambda_{\text{borrower}} : \Lambda_{\text{lender}} = 2 : 1$. With the exercise, I find that given the symmetric parameter value $\omega\lambda = 0.3 * 2 = 0.6$ for both agents, an equal distribution of the lenders and borrowers would not generate a positive net interest rate when the agents face uncertain income in the second period.

6 Conclusion

By developing a model of dynamic reference dependence, this paper explores a general equilibrium implication for intertemporal optimization and examines its applicability to explain US consumption data. Given an intensity of aversion to loss, the agents of the model form beliefs regarding the ex ante optimal consumption, relative to which they feel intertemporal gains or losses whenever they intend to change their consumption paths. The standard agent is loss averse with respect to the belief-dependent reference point. Only those who have low loss aversion with respect to the reference utility would deviate for more or less than the ex ante optimal, feasible consumption stream.

This paper rigorously demonstrates how the reference dependent preferences can explain many noted features of consumption dynamics in lifecycle theories. The model rationalizes the actions of the deviators, and the resulting economy formed by these individuals is able to replicate the real economy in a calibrated general equilibrium. One contribution of this research is that it constructs a descriptive model under which the agent's intentions are incorporated into the optimization itself, allowing any plan to be executed and solved as a rational choice.

Moreover, this paper compares the baseline model with the short horizon structure in which the consumers are also reference dependent. The agent's income stream is incorporated to consumption, and as the primary driver of reoptimization, it provides a framework to gain understanding on the nature of loss aversion with respect to consumption. From a macroeconomic perspective, this is because the reference-dependent agent who is expecting age-dependent average lifecycle income stream should form expectations regarding his future consumption, but the ex ante consumption path is only made from the income realization into foreseeable future. This gives greater insight on how the news about the distribution of future income would generate multiple layers of consumption dynamics.

Finally, this paper introduces a model of intertemporal loss aversion under uncertainty, and analyzes the precautionary saving behavior for the deviators and finds the equilibrium interest rate for an economy of both lenders and borrowers who are affected by the loss aversion.

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8 Appendix

Proof of proposition 1 The maximization problem of the agent with $\omega\lambda < 1$, who plans to over-consume for the first $1+\tau$ periods $\{c : [c_t > c_t^*]_0^\tau$ and $[c_t < c_t^*]_\tau^T\}$ is $\max u(c|c^*) = \beta^t \sum_{t=0}^{\tau} \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right)\right]$ $+\beta^s \sum_{s=\tau+1}^{T} \left[\frac{c_s^{1-\gamma}}{1-\gamma} + \eta\omega\lambda\left(\frac{c_s^{1-\gamma}}{1-\gamma} - \frac{c_s^{*1-\gamma}}{1-\gamma}\right)\right]$. The optimality condition $c_t^{-\gamma} + \eta I_t c_t^{-\gamma} + \eta\omega\lambda(1-I_t)c_t^{-\gamma} = R\beta[c_{t+1}^{-\gamma} + \eta I_{t+1} c_{t+1}^{-\gamma} + \eta\omega\lambda(1-I_{t+1})c_{t+1}^{-\gamma}]$ requires $\frac{(1+\eta)c_t^{-\gamma}}{(1+\eta)c_{t+1}^{-\gamma}} = R\beta$ for $t = 0, 1, 2, ..., \tau$ and $t = \tau+1, ..., T$, but $\frac{(1+\eta)c_t^{-\gamma}}{(1+\eta\omega\lambda)c_{t+1}^{-\gamma}} = R\beta$ at $t = \tau$. From the budget constraint, this yields $\{c\} = \{\left[\left(\frac{R}{\phi}\right)^t c_0\right]_0^\tau, \left[\left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma}\left(\frac{R}{\phi}\right)^t c_0\right]_{\tau+1}^T\}$, where $c_0 = \sum_{t=0}^{T} \frac{y_t}{R^t} / \left(\sum_{t=0}^{\tau}\left(\frac{1}{\phi}\right)^t + \left(\frac{1+\eta\omega\lambda}{1+\eta}\right)^{1/\gamma} \sum_{s=\tau+1}^{T}\left(\frac{1}{\phi}\right)^s\right)$.

Proof of proposition 2 (saver's case) The maximization problem of the agent with $\omega\lambda < 1$, who plans to under-consume for the first $1+\tau$ periods $\{c : [c_t < c_t^*]_0^\tau$ and $[c_t > c_t^*]_\tau^T\ \}$ is $\max u(c|c^*) = \beta^t \sum_{t=0}^{\tau} \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \eta\omega\lambda\left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{c_t^{*1-\gamma}}{1-\gamma}\right)\right]$ $+\beta^s \sum_{s=\tau+1}^{T} \left[\frac{c_s^{1-\gamma}}{1-\gamma} + \eta\left(\frac{c_s^{1-\gamma}}{1-\gamma} - \frac{c_s^{*1-\gamma}}{1-\gamma}\right)\right]$. The optimality condition $c_t^{-\gamma} + \eta I_t c_t^{-\gamma} + \eta\omega\lambda(1-I_t)c_t^{-\gamma} = R\beta[c_{t+1}^{-\gamma} + \eta I_{t+1}c_{t+1}^{-\gamma} + \eta\omega\lambda(1-I_{t+1})c_{t+1}^{-\gamma}]$ requires $\frac{(1+\eta)c_t^{-\gamma}}{(1+\eta)c_{t+1}^{-\gamma}} = R\beta$ for $t = 0, 1, 2, ..., \tau$ and $t = \tau+1, ..., T$, but $\frac{(1+\eta\omega\lambda)c_t^{-\gamma}}{(1+\eta)c_{t+1}^{-\gamma}} = R\beta$ at $t = \tau$. From the budget constraint, this yields $\{c\} = \{\left[\left(\frac{R}{\phi}\right)^t c_0\right]_0^\tau, \left[\left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}\left(\frac{R}{\phi}\right)^t c_0\right]_{\tau+1}^T\}$, where $c_0 = \sum_{t=0}^{T} \frac{y_t}{R^t} / \left(\sum_{t=0}^{\tau}\left(\frac{1}{\phi}\right)^t + \left(\frac{1+\eta}{1+\eta\omega\lambda}\right)^{1/\gamma}\sum_{s=\tau+1}^{T}\left(\frac{1}{\phi}\right)^s\right)$.

Proof of proposition 3 First, suppose that $\beta = 1/R$. Because $\frac{R}{\phi} = 1$, the condition reduces to $(1-\mu_0+\mu_1)(Ry_0+y_1) < y_2+\frac{y_2}{R}\mu_0$, and the first two consumptions are $c_0 = \frac{Ry_0+y_1}{R+\mu_0}$ and $c_1 = \frac{(Ry_0+y_1)\mu_0+\left(y_2+\frac{y_2}{R}\mu_0\right)}{(R+\mu_0)\left(1+\frac{1}{R}\mu_1\right)}$. Thus, if the condition is satisfied then $\left(1-\mu_0+\frac{1}{R}\mu_1\right)(Ry_0+y_1) < y_2+\frac{y_2}{R}\mu_0$ or $\left(1+\frac{1}{R}\mu_1\right)(Ry_0+y_1) < y_2+\frac{y_2}{R}\mu_0+(Ry_0+y_1)\mu_0$. Therefore $c_0 < c_1$. Second, consider the case $\beta < 1/R$. Because $\frac{R}{\phi} < 1$, it is satisfied that $\left(1-\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < \left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0)$. Therefore $c_0 < c_1$. Third, consider the case $\beta > 1/R$. Because $\frac{R}{\phi} > 1$, it is satisfied that either $\left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < \frac{1}{\phi}\left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0)$ or $\left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0) < \frac{1}{\phi}(Ry_0+y_1)$. In either case, it is true that $\left(1-\frac{R}{\phi}\mu_0+\frac{1}{\phi}\mu_1\right)(Ry_0+y_1) < y_2(1+\frac{1}{\phi}\mu_0)$. Therefore $c_0 < c_1$.

Proof of proposition 4 Suppose $\beta = 1/R$. Then the inequality condition reduces to $\left(\mu_0\left[Ry_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right) > y_3\left(1+\frac{1}{R}\mu_0\right)\left(1+\frac{1}{R}\mu_1\right)$. If (μ_0, μ_1) is sufficiently small: $\left(1+\frac{1}{R}\mu_0\right)\left(1+\frac{1}{R}\mu_1\right) < (\ y_3)^{-1}\left(\mu_0\left[Ry_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right)$ or y_3 is sufficiently small: $y_3 < (1+\frac{1}{R}\mu_0)^{-1}(1+\frac{1}{R}\mu_1)^{-1}\left(\mu_0\left[Ry_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right)$, then $c_1 > c_2$. Because with $y_2 > 0$ it is always true that $\left(\mu_0\left[Ry_0+y_1+\frac{y_2}{R}\right]+y_2\right)\left(1+\frac{1}{R}\mu_2-\mu_1\right) > 0$, since $0 \leqslant \mu_s \leqslant 1$. Thus $\varepsilon = 0$ is the sufficient condition for $c_1 > c_2$. Similarly, if either (μ_0, μ_1) or y_3 is sufficiently small, then $c_1 > c_2$ for all choices of β.

Proof of proposition 5 By the above two propositions, Proposition 3 and 4, it is clear that combining both conditions yields $c_0 < c_1 > c_2$. Thus according to the definition 1, the consumption hump is achieved for $\{c_0, c_1, c_2, c_3\}$ with c_1 the consumption peak, and c_3 denoting consumption in the residual phase.

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Highlights

- Dynamic decision makers reoptimize their lifecycle plan in accordance with their changing degree of loss aversion.
- Find a non-monotonic consumption profile derived purely from consumer preference.
- Rationalize certain behaviors such as overconsumption, as well as under consumption, through belief updating.
- Demonstrate how loss aversion connects to lifecycle income flow through restricting foresight of future income.
- Analyze the model under uncertainty for both intertemporal and stochastic loss aversion.