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## Full length article

# Are banks risk-averse or risk-neutral investors?

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#### 1. Introduction

In this study, we focus on the asset allocation problems for banking companies. Specifically, we consider the risk-averse and risk-neutral behavior of banks as utility maximization problems and apply non-linear and linear utilities to them, respectively. Using calibration, we then investigate the optimal asset allocation for the risk-averse or risk-neutral bank that fits the actual bank asset holding. For calibration, as an example, we utilize the financial statements of large Japanese banking companies.

Bank behavior has traditionally been examined in some studies previously. Kane and Malkiel (1965) applied Markowitz's mean-variance portfolio selection problem to address how a bank determines its asset allocation with deposit variability. Ishii (1971) extended Kane and Kane and Malkiel's (1965) model taking into account credit creation. Jeitschko and Jeung (2005) considered the compensation maximization problem (i.e., linear utility maximization problem) for the bank manager to verify the effect of capital on the bank's asset allocation. Mallick (2019) considered the optimal asset allocation problems for the riskneutral bank to illustrate how bank regulation influences the bank's asset allocation.

Meanwhile, the bank's optimization problems have been applied to analyze the effects of the monetary policy and banking regulations on the macroeconomy and banks' financial state. Kahane (1977), Koehn and Santomero (1980), Kim and Santomero

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## ABSTRACT

In this study, we verify how to construct the utility maximization problem for banks in their asset allocation. We consider two utility maximization problems for the risk-averse and risk-neutral banks to determine the optimal lending ratios to represent banks' optimal asset allocation. We apply the meanvariance utility for the risk-averse problem and impose the risk constraint for the risk-neutral problem to obtain the optimal solution. In order to validate the model, we investigate how the optimal lending ratios derived from the two models fit the actual bank lending ratio through calibration. Statistical tests for the calibration results do not indicate a significant difference in the model fitting between the risk-averse and risk-neutral models. Hence, this enables us to use both models to describe banks' asset allocation.

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(1988), Keeley and Frederick (1990), Rochet (1992), Cuoco and Liu (2006), and Halaj (2013) considered the utility maximization problems of risk-averse banks to examine how the capital requirement effects bank's asset risk. Greenwald and Stiglitz (1989) also applied the utility maximization problem of the risk-averse bank to verify the effects of monetary policy on the bank's lending and the macroeconomy. Lizarazo (2013) used the utility maximization problem of the risk-averse investor to describe the bond markets in emerging economies. Aside from financial markets, Merzifonluoglu (2015) applied the utility maximization problem of a risk-averse firm to construct the optimal supply-chain management in the context of enterprise risk management. Furlong and Keeley (1989), Gennotte and Pyle (1990), and Chiba (2020) considered net worth maximization problems for riskneutral banks and analyzed financial stabilities under the capital requirement stipulation or with deposit insurance. Fischer (1983), Greenwald and Stiglitz (1989), Hartley and Walsh (1991), Jacques (2008), Chami and Cosimano (2010), Wang (2013), and Goel et al. (2020) considered the bank profit maximization problems by employing production, costs, and loan-demand functions.

In the context of equilibrium pricing, Brunnermeier and Pedersen (2009) used the utility maximization problems of the risk-averse investors to discuss the liquidity problem during a financial crisis. Danielsson et al. (2009) and Shin (2010) considered the optimization problems for risk-neutral banks with risk constraints to describe the asset amplification. Arellano (2008), Yue (2010), and Takino (2016) used utility maximization problems for both risk-averse and risk-neutral investors to derive the equilibrium prices for the defaultable contingent claims under the risk-neutral probability space.

While the utility maximization problems for risk-averse and risk-neutral banks have been applied to various analyses in financial economics as mentioned above, Takino and Ishinagi (2022) examined how the framework of the mean-variance utility maximization describes the actual bank behavior. They derived the bank optimal lending ratios via the mean-variance utility maximization problems with and without the internalized balance sheet model. Further, they demonstrated that the utility maximization problem with the internalized balance sheet model boosts the accuracy of fitting the theoretical lending ratio to the actual lending ratio. By contrast, Kim (2014) and Cohn et al. (2015) illustrated that risk-aversion is countercyclical, that is, it weakens during a boom period. Therefore, whether the model fitting accuracy for the risk-averse bank is time-varying and depends on the business cycle also needs to be considered. Aside from the risk-aversion perspective, Jokipii and Milne (2008), Shim (2013), and Saadaoui (2014) demonstrated that the bank capital buffer is countercyclical, that is, the bank capital buffer decreases (increases) during a boom (downturn). This is because the bank tends to exhaust its capital during a boom as the capital is reduced. The risk constraint (which corresponds to the capital constraint in this study) is binding when we solve the optimization problem for the risk-neutral bank. Hence, the optimization problem for the risk-neutral bank might accurately describe the actual behavior of the bank rather than the risk-averse model. Moreover, we incorporate derivatives into the bank's asset allocation problem unlike most previous studies that have never considered derivatives. As the current value of derivatives listed on the balance sheet is calculated by the risk-neutral pricing approach (e.g., Black–Scholes model), modeling the bank's behavior as a risk-neutral agent is consistent with asset pricing. Hence, in this study, we address which of the two models, risk-averse or risk-neutral, accurately describes the bank's actual behavior.

We solve the optimal lending ratios for the risk-averse and risk-neutral banks as the bank asset allocation through the utility maximization problems. The lending ratio is defined as the proportion of lending to the total asset. As regards the risk-averse bank problem, we apply the classical mean-variance utility maximization problem to the bank asset portfolio return. By contrast, for the risk-neutral problem, we consider the expected return maximization problem (i.e., linear utility maximization problem) for the bank asset portfolio under the risk-neutral probability measure by posing the risk constraint which is convertible with the capital adequacy requirements for banks by the Bank of International Settlement, to obtain the optimal solution. We consider the risk-neutral problem under the risk-neutral probability space to maintain the consistency of asset values in the bank balance sheet with the risk-neutral pricing (e.g., for derivatives). This construction leads all asset returns to be equivalent to the riskfree return and reduces the type of parameters to be estimated. As an example, we then, calibrate the model parameters from the actual financial statements of large Japanese banking companies for each year from 2000 to 2019 and simultaneously calculate the estimation errors. The estimation error is assigned to the accuracy of the model fitting, that is, the smaller the estimation error, the more accurate the model fitting. However, from the calibration results, we cannot distinguish which of the two models, the riskaverse or risk-neutral, is more accurate. Thus, we conduct the *t*-test for the estimation error. The result of the *t*-test does not indicate any significant difference between the errors for the riskaverse and risk-neutral models. This implies that one can use both the utility maximization problems for the risk-averse and risk-neutral banks to model the behavior of a bank in its asset allocation. However, the risk-neutral model applied in this study is more efficient as the types of parameters for the risk-neutral model are lower than those for the risk-averse model. Finally,

we test whether the accuracy of the model fitting depends on the business cycle to comprehend how the bank's behavior in its asset allocation is related to the business cycle. As an example of describing the Japanese business cycle, we use the composite index (CI) lagging index. The simple regression analyses do not show a relationship between the accuracy of the model fitting and the business cycle.

The rest of the paper is organized as follows: In Section 2, we introduce the balance sheet model, which enables us to express the monetary amount of each asset held by the bank in terms of the lending ratio and reduce three assets to one asset. We also define a stochastic model for each asset. In Section 3, we consider the optimal asset allocation problem for the risk-averse bank and solve the optimal lending ratio. In Section 4, we present the optimal asset allocation problems for the risk-neutral bank in two ways and solve the optimal lending ratios. The two models comprise the expected return of the asset portfolio with linear and non-linear risk constraints. In Section 5, we provide numerical results. We calibrate the model parameters for each model and examine how each model fits the actual data. Moreover, we perform the regression analysis to verify how our models relate to the business cycle. Finally, in Section 6, we summarize the study.

#### 2. Model

## 2.1. Balance sheet model

We introduce a balance sheet model where the items on the liability side of the balance sheet are presented in terms of the assets. This balance sheet model, proposed by Ishii (1971), improves the accuracy of the model fitting, as demonstrated by Takino and Ishinagi (2022).

The bank's (current) balance sheet is represented by

$$C_r + L + B + F = D_g + N + e,$$
 (2.1)

where  $C_r$  denotes cash, L is the monetary amount of lending, B represents the monetary amount invested into securities, F is the net asset amount of derivatives, defined as the difference in derivatives on the asset and the liability side (hereafter derivatives),  $D_g$  denotes the total deposit, N is the borrowed money at interbank markets, and e represents capital. From Ishii (1971), we suppose that

$$N = \lambda_N L$$
,

and

$$C_r = rD_g$$

where  $\lambda_N$  and r are constants. The lending ratio l is defined as the proportion of the monetary amount of lending to the sum of lending and securities

$$l = \frac{L}{L+B}$$

Considering derivatives as risky assets is natural. However, we omit it from the scope of a risky asset because the amount of derivatives F is relatively small.

We suppose that the total deposit  $D_g$  is given by

$$D_g = D_p + L$$

where  $D_p$  is the primary deposit and D is the secondary deposit defined based on Ishii (1971) as

$$D=k_LL+k_BB.$$

Then, the primary deposit is given by

$$D_p = D_g - (k_L L + k_B B),$$

where  $k_i > 0$  (j = L, B) is regarded as the coefficient of the credit creation. We also define the amount of derivatives by the linear function of the lending and primary deposit as

$$F = \lambda_F (X + D_p), \tag{2.2}$$

where  $\lambda_F$  is a constant. Sinkey and Carter (2000) showed that the amount of derivatives held by banks depend on the banks' total assets. Instefjord (2005) considered the model in which the derivatives position of the bank is determined according to the amount of lending. Hirtle (2009), Vuillemey (2015), and Piiroinen (2017) illustrated that the amounts of derivatives and lending amounts are interactively connected. Moreover, Bartram et al. (2009) and Piiroinen (2017) demonstrated that the amount of derivatives held by financial and non-financial companies are affected by the amount of liabilities. Eq. (2.2) reflects these facts.

Substituting the definitions of  $C_r$ , F,  $D_g$ , and N into (2.1) yields

$$L + B + \lambda_F (L + D_p) = (1 - r)D_p + (1 - r)k_L L + (1 - r)k_B B + \lambda_N L + e.$$
(2.3)

We rewrite (2.3) as

$$M_1 L + M_2 B = M_3, (2.4)$$

where

 $M_1 = 1 + \lambda_F - (1 - r)k_L - \lambda_N,$  $M_2 = 1 + \lambda_F - (1 - r)k_B$  $M_3 = (1 - \lambda_F - r)D_p + e.$ 

Substituting the definition of l into (2.4), we have

$$X = \frac{1}{M_4 l + M_5},$$
(2.5)

where

 $M_4=\frac{M_1-M_2}{M_3},$  $M_5 = \frac{M_2}{M_3}$ 

Therefore, from (2.5), all assets are represented in terms of l, and the intrinsic deposit  $D_p$  and capital *e* are only given, that is,

$$L = \frac{l}{M_4 l + M_5},$$
 (2.6)

$$B = \frac{1-l}{M_4 l + M_5}.$$
 (2.7)

#### 2.2. Stochastic model for assets

 $\rho_{LB}\sigma_L\sigma_B$ 

 $\rho_{LF}\sigma_L\sigma_F$   $\rho_{BF}\sigma_B\sigma_F$ 

 $\sigma_B^2$ 

We consider one period model, that is, "present" and "future", under a real probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . We denote the gross return per unit of lending by  $R_L$ , the gross return per unit of securities by  $R_B$ , and the gross return per unit of derivatives by  $R_F$ . Then, the value of the asset j (j = L, B, F) at the future time is

#### $R_i \times j$ .

We suppose that  $R_i$  (j = L, B, F) is a random variable under the above probability space and denote its expectation and standard deviation under  $\mathbb{P}$  by  $E_i$  and  $\sigma_i$  (j = L, B, F), respectively. We represent the correlation coefficient of  $R_i$  and  $R_j$  (i, j = L, B, F, $i \neq j$ ) by  $\rho_{ij}$ .

We set 
$$\mathbf{R}^{\top} = (R_L, R_B, R_F), \mathbf{E}^{\top} = (E_L, E_B, E_F), \mathbf{D}^{\top} = (L, B, F), \text{ and}$$
  

$$\mathbf{E} = \begin{pmatrix} \sigma_L^2 & \rho_{LB}\sigma_L\sigma_B & \rho_{LF}\sigma_L\sigma_F \\ \rho_{LB}\sigma_L\sigma_B & \sigma_R^2 & \rho_{BF}\sigma_B\sigma_F \end{pmatrix}.$$

Then, the future value  $V_g$  of the asset portfolio **D** is

$$V_{\sigma} = \mathbf{D}^{\top} \mathbf{R}, \qquad (2.8)$$

the expected future value of the asset portfolio **D** is

$$E[V_g] = \mathbf{D}^\top \mathbf{E},\tag{2.9}$$

and the variance of the future value of the asset portfolio **D** is

$$Var[V_g] = \mathbf{D}^{\top} \Sigma \mathbf{D}. \tag{2.10}$$

We assume that cash earns the risk-free return and denote the gross return per unit of cash holding by  $R_{\rm C}(> 1)$ . Moreover, we assume that there exists a risk-neutral probability measure  $\mathbb{O}$ equivalent to  $\mathbb{P}$  to consider the risk-neutral bank's problem. Thus, under  $\mathbb{O}$ , the gross returns of all assets are  $R_C$ . Then, the expected future value of the asset portfolio **D** under  $\mathbb{O}$  is

$$E^{\mathbb{Q}}[V_g] = \mathbf{D}^{\top} \mathbf{E}_{F_g}$$

where  $\mathbf{E}_{F}^{\top} = (R_{C}, R_{C}, R_{C})$ . We also suppose that the variance of the future value of the asset portfolio **D** under  $\mathbb{O}$  is

$$Var^{Q}[V_g] = Var[V_g],$$

without loss of generality.<sup>1</sup>

## 3. Optimal asset allocation for a risk-averse bank

In this section, we consider the optimal asset allocation problem for the risk-averse bank (hereafter, RA model).

We assume that the risk-averse bank has a mean-variance utility and determine the asset allocation to maximize the utility for the future value of the portfolio. Then, the optimization problem is formulated by

$$\max_{\mathbf{D}} \mathbf{D}^{\mathsf{T}} \mathbf{E} - \frac{1}{2} K \mathbf{D}^{\mathsf{T}} \boldsymbol{\Sigma} \mathbf{D}, \tag{3.1}$$

where K is the risk-aversion parameter. From the balance sheet model denoted by (2.6) and (2.7), we reduce the optimization problem in (3.1) with respect to **D** to that with respect to the lending ratio *l*. That is,

$$\max_{l} E_g - \frac{1}{2} K Var_g, \tag{3.2}$$

where

$$E_g = \frac{1}{M_4 l + M_5} \{ (E_L - E_B + \lambda_F E_F) l + E_B \} + \lambda_F E_F D_p,$$
  

$$Var_g = \left(\frac{1}{M_4 l + M_5}\right)^2 (U_1 l^2 + 2U_2 l + \sigma_B^2) + \sigma_F^2 \lambda_F^2 D_p^2$$
  

$$+ 2\rho_{BF} \sigma_B \sigma_F \lambda_F D_p M_5,$$

with

$$U_{1} = \sigma_{L}^{2} + \sigma_{B}^{2} + \sigma_{F}^{2}\lambda_{F}^{2} - 2\rho_{LB}\sigma_{L}\sigma_{B} + 2\rho_{LF}\sigma_{L}\sigma_{F}\lambda_{F} - 2\rho_{BF}\sigma_{B}\sigma_{F}\lambda_{F} + 2(\rho_{LF}\sigma_{L}\sigma_{F}\lambda_{F} - \rho_{BF}\sigma_{B}\sigma_{F}\lambda_{F} + \sigma_{F}^{2}\lambda_{F}^{2})D_{p}M_{4}, U_{2} = \rho_{LB}\sigma_{L}\sigma_{B} + \rho_{BF}\sigma_{B}\sigma_{F}\lambda_{F} - \sigma_{B}^{2} + (\rho_{LF}\sigma_{L}\sigma_{F}\lambda_{F} - \rho_{BF}\sigma_{B}\sigma_{F}\lambda_{F} + \sigma_{F}^{2}\lambda_{F}^{2})D_{p}M_{5} + \rho_{BF}\sigma_{B}\sigma_{F}\lambda_{F}D_{p}M_{4}.$$

From the first-order-condition (FOC) for the optimization, we have

$$\frac{\partial E_g}{\partial l} - \frac{1}{2} K \frac{\partial Var_g}{\partial l} = 0.$$
(3.3)

<sup>&</sup>lt;sup>1</sup> We essentially suppose that each asset price follows the Black-Scholes model. For such a model, the variance does not change even if changing the measure is changed.

## This leads to

 $\zeta_1 l + \zeta_2 = 0,$ 

## where

$$\begin{split} \zeta_1 = & (E_L - E_B + \lambda_F E_F) M_4 M_5 - E_B M_4^2 - K(U_1 M_5 - U_2 M_4), \\ \zeta_2 = & (E_L - E_B + \lambda_F E_F) M_5^2 - E_B M_4 M_5 - K(U_2 M_5 - \sigma_B^2 M_4). \end{split}$$

Hence, the optimal lending ratio *l*\* is

$$l^* = -\frac{\zeta_2}{\zeta_1}.$$
 (3.4)

#### 4. Optimal asset allocation for a risk-neutral bank

In this section, we consider the optimal asset allocation problem for the risk-neutral bank under the risk-neutral probability measure  $\mathbb{Q}$ .

As the utility function of the risk-neutral investor is represented by the linear function, the risk-neutral bank determines the optimal asset portfolio as the one that maximizes the expected future value of the portfolio, that is,

$$\max_{\mathbf{D}} \mathbf{D}^{\mathsf{T}} \mathbf{E}_{F}. \tag{4.1}$$

As regards the optimization problem denoted by (4.1), we cannot ascertain a unique inner solution because the objective function of (4.1) is linear in the portfolio **D**. As we assume that the value of  $R_C$  is larger than one, investing its money into lending or securities or derivatives unlimitedly is optimal for the bank. To avoid this, Danielsson et al. (2009) and Shin (2010) considered the (linear) utility maximization problem with a risk constraint. In this study, we apply their models to find the optimal asset portfolio for the risk-neutral bank.

**Remark 4.1.** Under the risk-neutral measure  $\mathbb{Q}$ , the returns of all assets coincide with the risk-free return. This vanishes the bank's incentive to manage its asset portfolio because the bank cannot earn the return exceeding the risk-free return from the asset portfolio. However, the framework under the risk-neutral measure is consistent with the perspective that values of all assets (especially derivatives) are priced in the risk-neutral pricing approach. Therefore, we consider the optimal asset allocation problems for the risk-neutral bank together with the risk-neutral measure  $\mathbb{Q}$ .

#### 4.1. Linear risk constraint

We first introduce the risk constraint,

$$L+B+F-e \leq L(R_C-v_L\sigma_L)+B(R_C-v_B\sigma_B)+F(R_C-v_F\sigma_F), \quad (4.2)$$

based on Shin (2010). The left-hand side of (4.2) is the debt value and the right-hand side is the admissible minimum value of the asset portfolio as  $v_j$  (j = L, B, F) indicates that possible loss the bank anticipates for risky assets. Note that the constraint in (4.2) implicitly assumes that the returns on assets are perfectly positively correlated (see Remark 4.3). Hence, the constraint is regarded as the linear risk constraint in **D**. We consider the constraint with correlations between returns on assets in Section 4.2.

Then, the utility maximization problem for a risk-neutral bank with the linear risk constraint (hereafter RN model) is

$$\max_{\mathbf{D}} \quad \mathbf{D}^{\top} \mathbf{E}_{F}, \\ \text{subject to} \quad L + B + F - e \leq L(R_{C} - v_{L}\sigma_{L}) + B(R_{C} - v_{B}\sigma_{B}) \\ + F(R_{C} - v_{F}\sigma_{F}).$$

## From (4.3), the constraint can be rewritten as

$$\mu_L L + \mu_B B + \mu_F F \le e,$$
(4.4)
where  $\mu_j = 1 - R_C + v_j \sigma_j$   $(j = L, B, F)$ . We assume that
 $\mu_j > 0,$ 

for j = L, B, F.

**Remark 4.2.** By rewriting the risk constraint (4.2) as (4.4), one can easily find that the risk constraint (4.2) is convertible with the capital adequacy requirements for banks by the Bank of International Settlement (hereafter BIS rule). We denote the risk weight for asset *j* by  $\omega_j \times 100\%$  (> 0) for j = L, B, F and the capital adequacy ratio by  $\eta$  (> 0). Then, the BIS rule is formulated by

$$\frac{e}{\omega_L L + \omega_B B + \omega_F F} \ge \eta.$$
(4.5)

From (4.5), we have

$$\eta(\omega_L L + \omega_B B + \omega_F F) \le e. \tag{4.6}$$

Thus, by comparing (4.4) and (4.6), one can observe that  $\mu_j$  in (4.4) assumes the role of  $\eta \omega_j$  in (4.6) for each asset *j*. In fact, for example,  $\omega_j$  and  $\sigma_j$  increase when the risk for each asset deteriorates during a bust. This tightens both constraints (4.4) and (4.6).

The optimization problem (4.3) is solved as follows. Since we have assumed that  $\mu_j > 0$  for each asset *j*, the optimal lending ratio  $l^*$  is a solution of

$$\mu_L L + \mu_B B + \mu_F F = e. \tag{4.7}$$

Substituting (2.2), (2.6), and (2.7) into (4.7), and then solving the equation with respect to l, we have an optimal lending ratio  $l^*$  as

$$l^{*} = \frac{(e - \lambda_{F} \mu_{F} D_{p}) M_{5} - \mu_{B}}{\mu_{L} - \mu_{B} + \lambda_{F} \mu_{F} - (e - \lambda_{F} \mu_{F}) M_{4}}.$$
(4.8)

## 4.2. Non-linear constraint

Next, following Danielsson et al. (2009), we introduce a risk constraint considering the correlation between the assets' returns as

$$v\sqrt{\mathbf{D}^{\top}\mathbf{\Sigma}\mathbf{D}} \le e,\tag{4.9}$$

for a non-negative constant v. (4.9) means that the portfolio risk multiplied by the constant v does not exceed the bank's capital e. Then, the inverse of v is regarded as the risk-appetite coefficient. For example, if the constraint (4.9) eases as v decreases, then the bank is able to take more risks on its asset portfolio. That is, the decrease in v increases the bank's tolerance for risk. The correlations between the asset returns with less than or equal to one are reflected in  $\mathbf{D}^{\top} \Sigma \mathbf{D}$  unlike in the case of (4.2). Moreover, (4.9) is regarded as a non-linear constraint because  $\mathbf{D}^{\top} \Sigma \mathbf{D}$  is non-linear in  $\mathbf{D}$ . From Danielsson et al. (2009), the risk constraint (4.9) whose risk is measured by the standard deviation is the consequence of imposing the Value-at-Risk restriction on the bank. Aside from Danielsson et al. (2009), Mallick (2019) and Goel et al. (2020) considered the bank's optimization problem with the risk constraints.

**Remark 4.3.** The constraint (4.9) is also convertible with the BIS rule. When  $\rho_{LB} = \rho_{LF} = 1$ , it holds

$$v\sqrt{\mathbf{D}^{\top}\Sigma\mathbf{D}} = v(\sigma_{L}L + \sigma_{B}B + \sigma_{F}F).$$
(4.10)

The right hand side in (4.10) coincides with the left hand side in (4.4) as  $\mu_i = v\sigma_i$  (j = L, B, F). Thus, as discussed in Remark 4.2,

(4.3)

#### Table 1

Model Parameters. "RA" means the risk-aversion model, "RN" corresponds to the risk-neutral model with the linear risk constraint, and "RN+" relates to the risk-neutral model with the non-linear risk constraint. " $\sqrt{}$ " indicates the corresponding parameter is used in the model and "-" is otherwise.

Parameter	RA	RN	RN+
k <sub>L</sub>	$\checkmark$	$\checkmark$	$\checkmark$
$k_B$			$\checkmark$
EL	$\checkmark$	-	-
E <sub>B</sub>	$\checkmark$	-	-
$E_F$	$\checkmark$	-	-
R <sub>C</sub>	-	$\checkmark$	$\checkmark$
$\sigma_L$	$\checkmark$	$\checkmark$	$\checkmark$
$\sigma_B$	$\checkmark$	$\checkmark$	$\checkmark$
$\sigma_F$		$\checkmark$	
$ ho_{LB}$		-	
$ ho_{LF}$		-	
$\rho_{BF}$		-	$\checkmark$
K	$\checkmark$	-	-
$v_L$	-		-
$v_B$	-		-
$v_F$	-	$\checkmark$	- ,
υ	-	-	$\checkmark$

 $v\sigma_j$  assumes the role of  $\eta\omega_j$  in the BIS rule (4.6) for each j (j = L, B, F). However, because  $\sqrt{\mathbf{D}^\top \Sigma \mathbf{D}}$  is the risk amount of the bank asset portfolio taking into account correlations between assets less than or equal to one, the constraint (4.9) enables the bank to take more risks, that is,

$$v\sqrt{\mathbf{D}^{\top}\mathbf{\Sigma}\mathbf{D}} \leq v(\sigma_{L}L + \sigma_{B}B + \sigma_{F}F).$$

Finally, from (4.10), the linear risk constraint of (4.2) is regarded as the risk constraint where all assets are perfectly positively correlated.

The optimization problem for the risk-neutral bank with the non-linear constraint (hereafter RN+ model) is formulated by

$$\max_{\mathbf{D}} \quad \mathbf{D}^{\top} \mathbf{E}_{F},$$
  
subject to  $v \sqrt{\mathbf{D}^{\top} \Sigma \mathbf{D}} \le e.$  (4.11)

The optimal solution for (4.11) is given by

$$l^* = -\frac{\zeta_4}{\zeta_3},\tag{4.12}$$

where

$$\zeta_3 = R_C(\lambda_F M_4 M_5 - M_4^2) - 2 \frac{v\sqrt{\xi}}{e} (M_5 U_1 - M_4 U_2)$$

and

$$\zeta_4 = R_C(\lambda_F M_5^2 - M_4 M_5) - 2 \frac{v\sqrt{\xi}}{e} (M_5 U_2 - \sigma_B^2 M_4).$$

The derivation of (4.12) is presented in Appendix.

**Remark 4.4.** As the standard deviation of the future portfolio value is used as its risk in the non-linear risk constraint, the FOC in (A.8) for the risk-neutral bank's problem with a non-linear constraint is the same as the FOC in (3.3) for the risk-averse bank's problem if  $R_C = E_j$  (j = L, B, F) and  $K = \frac{2v\sqrt{\xi}}{e}$ . This leads to the same form of the optimal lending ratio represented by (4.12) as that for the risk-averse bank (i.e. (3.4)). In fact,  $\zeta_3$  and  $\zeta_4$  correspond to  $\zeta_1$  and  $\zeta_2$ , respectively.

## 5. Calibration result

In this section, we calibrate all optimal lending policies  $l^*$  using actual financial statements of Japanese banking companies and verify how the model describes the actual act.

Table 2

Number of banks *n* used in calibration for

each year.				
Year	п			
2000	9			
2001	7			
2002-2004	6			
2005-2012	5			
2013-2021	4			

## 5.1. Data and calibration procedure

We use the balance sheets of large Japanese banking companies between 2000 and 2019 as sample data. These banking companies have reported derivatives on their balance sheets. The definition of a large Japanese banking company follows that of the Japanese Bankers Association. As large Japanese banks have repeatedly merged from the beginning of the 2000s, the number of the sample banking companies in the calibration differs by year. Japan has had nine large banks in 2000, seven in 2001, six during 2002–2004, five during 2005–2012, and four large banks since 2013. The variables in our study are assigned to the balance sheet items as follows:

L =Call loans + Loans and bills discounted;

- B = Securities;
- F = Derivatives on the asset side

Derivatives on the liability side;

 $D_{g}$  = Deposits + Negotiable certificates of deposit;

N =Debentures + Call money + Commercial papers

+ Borrowed money

- + Bonds payable + Bonds with share acquisition rights;
- $C_r$  = Cash and due from banks;

e = Total net assets.

Recall, the actual lending ratio  $\hat{l}$  is calculated by

 $\hat{l} = \frac{L}{L+B}$ 

In calibration, we treat the model parameters as the common values among all banks for each year and identify the model parameters by year. The parameters to be calibrated for each model are presented in Table 1, in which the parameter used in each model is marked by  $\sqrt{}$ . The number of parameters used in the RN and RN+ models is the least among the three models.

The calibration is performed as follows:

- 1. The actual lending ratio  $\hat{l}$  for each bank is calculated from its balance sheet;
- The (model) optimal lending ratio l\* for each bank is computed under the initial given parameters for the three models;
- 3. Using the Excel solver, the parameters to minimize the squared relative error between  $l^*$  and  $\hat{l}$  for the actual lending ratio is determined, that is,

Error := 
$$\sum_{p=1}^{n} \left( \frac{l_p^* - \hat{l}_p}{\hat{l}_p} \right)^2 \rightarrow \text{minimize}_{\Pi},$$
 (5.1)

where *n* is the number of banks and  $\Pi$  means the set of parameters. Simultaneously, the value of Error is calculated in this step. The accuracy of the model fitting is measured by Error in (5.1). That is, the lower the Error, the higher the accuracy of the model fitting. Note that we use a relative difference in the objective function to eliminate the

Accuracy	ccuracy in the model fitting for each model and year. The table shows the value of Error as represented by (5.1).						
FY	2000	2001	2002	2003	2004	2005	2006
RA	$1.35 \times 10^{-2}$	$2.72  imes 10^{-4}$	$4.44  imes 10^{-9}$	$2.53  imes 10^{-9}$	$3.70 \times 10^{-3}$	$2.90 \times 10^{-10}$	$3.20  imes 10^{-12}$
RN	$1.37 \times 10^{-3}$	$2.77 \times 10^{-4}$	$2.10 \times 10^{-2}$	$8.48 \times 10^{-3}$	$1.35 \times 10^{-2}$	$2.98 \times 10^{-2}$	$8.38 \times 10^{-3}$
RN+	$2.57 \times 10^{-2}$	$5.34  imes 10^{-4}$	$6.39 \times 10^{-7}$	$1.53 \times 10^{-2}$	$1.02 \times 10^{-2}$	$7.17 \times 10^{-3}$	$4.03 \times 10^{-10}$
FY	2007	2008	2009	2010	2011	2012	2013
RA	$5.79  imes 10^{-4}$	$9.68  imes 10^{-9}$	$2.31 \times 10^{-9}$	$8.84  imes 10^{-10}$	$5.69  imes 10^{-9}$	$7.64 imes10^{-6}$	$1.25 \times 10^{-12}$
RN	$9.54  imes 10^{-3}$	$4.25 \times 10^{-3}$	$7.71 \times 10^{-3}$	$3.61 \times 10^{-3}$	$9.29  imes 10^{-3}$	$8.98 \times 10^{-3}$	$2.67 \times 10^{-3}$
RN+	$4.34\times10^{-11}$	$7.30  imes 10^{-8}$	$1.48 \times 10^{-12}$	$3.21 \times 10^{-9}$	$6.99  imes 10^{-9}$	$6.65 \times 10^{-9}$	$6.21 \times 10^{-12}$
FY	2014	2015	2016	2017	r	2018	2019
RA	$2.70  imes 10^{-10}$	$8.92 \times 10^{-1}$	-8 7.46 × 1	10 <sup>-8</sup> 1.89	$\times 10^{-9}$	$5.94  imes 10^{-13}$	$1.44 \times 10^{-12}$
RN	$1.04 \times 10^{-3}$	$1.19 \times 10^{-1}$	- <sup>3</sup> 1.03 × 1	10 <sup>-4</sup> 1.07	$\times 10^{-3}$	$2.07 \times 10^{-3}$	$3.25  imes 10^{-3}$
RN+	$3.22  imes 10^{-10}$	5.73 × 10⁻	-4 5.64 × 1	10 <sup>-8</sup> 1.84	$\times 10^{-10}$	$3.74 \times 10^{-10}$	$7.52 \times 10^{-11}$

Table 4

Results of *t*-Test comparing the RA and RN models.

	RA	RN
Mean	$9.05  imes 10^{-4}$	$6.88 \times 10^{-3}$
Variance	$9.54  imes 10^{-6}$	$5.71 \times 10^{-5}$
Observations	20	20
Hypothesized Mean Difference	0	
Degree of Freedom	25	
t Stat	-3.2742	
$P(T \le t)$ one-tail	0.0015	
t Critical one-tail	1.7081	
$P(T \le t)$ two-tail	0.0031	
t Critical two-tail	2.0595	

#### Table 5

Results of *t*-Test comparing the RN+ and RN models.

	RN+	RN
Mean	$2.98 \times 10^{-3}$	$6.88 \times 10^{-3}$
Variance	$4.61 \times 10^{-5}$	$5.71 \times 10^{-5}$
Observations	20	20
Hypothesized Mean Difference	0	
Degree of Freedom	38	
t Stat	-1.7180	
$P(T \le t)$ one-tail	0.0470	
t Critical one-tail	1.6860	
$P(T \le t)$ two-tail	0.0939	
t Critical two-tail	2.0244	

#### Table 6

Results of t-Test comparing the RA and RN+ models.

	RA	RN+
Mean	$9.05 \times 10^{-4}$	$2.98 \times 10^{-3}$
Variance	$9.54 imes10^{-6}$	$4.61 \times 10^{-5}$
Observations	20	20
Hypothesized Mean Difference	0	
Degree of Freedom	27	
t Stat	-1.2426	
$P(T \le t)$ one-tail	0.1123	
t Critical one-tail	1.7033	
$P(T \le t)$ two-tail	0.2247	
t Critical two-tail	2.0518	

influence of the size of the actual lending ratio. Moreover, the number of banks n used in calibration differs according to the year, as guided in the first part of this subsection. The value of n is summarized in Table 2.

## 5.2. Result

The calibration results are illustrated in Table 3. The table presents the Error for each model and year instead of the estimated parameter values. As evident, the Error of the RN model is the largest among the three models for each year. That is, the

RN model is the most subordinated among the three models. By contrast, in comparing the Error of the RA and RN+ models, the Error values for each model vary in size. This prevents us from distinguishing which of the two, RA or RN+ model, more closely describes the actual bank behavior in its asset allocation. Hence, we conduct the *t*-test for data in Table 3.

The results of the *t*-test are illustrated in Tables 4–6. Tables 4 and 5 demonstrate the results of the *t*-test comparing the RA and RN models and the RN+ and RN models, respectively, and reflect that the P-values (one and two-tail) are less than 0.10. Thus, the Error of the RN model, as expected, differs significantly from that of the RA and RN+ models.

Table 6 shows the result of the *t*-test comparing the RA and RN+ models and indicates that both the P-values of one-tail and two-tail are over 0.10. Hence, the difference between the RA and RN+ models is not significant at any significance level, while the mean of the RN+ is lower than that of the RA. The result is not surprising because the optimal lending ratios for the RA and RN+ models are similar, as pointed out in Remark 4.4. Although the asset returns differ between both models, *v* in the RN+ model plays the role of risk-aversion *K* in the RA model.

The results of the *t*-test in Table 6 are also as expected. We first introduced the utility maximization problem for the risk-neutral bank with a linear risk constraint (i.e. RN model). Then, we demonstrated that the accuracy of the model fitting for the RN model is relatively poorer than that for the risk-averse model (i.e. RA model). Using the internalized balance sheet model presented in Section 2, the risk-averse model boosts the fitting accuracy. However, the result of *t*-test shows that the RN+ model describes the actual behavior of the banking company as accurately as the RA model. Moreover, the number of parameters in the risk-neutral models is less than that in the risk-averse model. Hence, the proposed optimization model for the risk-neutral bank is efficient in the estimation of parameters.

#### 5.3. Connection with business cycle

As the final analysis, we address the effect of the business cycle on the bank's behavior. The previous studies have empirically demonstrated that the bank behaviors (e.g. lending, risk-aversion, capital buffer) are related to the business cycle, as highlighted in Section 1. In this section, we investigate how the accuracy of the model fitting for each model depends on the business cycle by constructing simple regression models. We use the composite index (CI) lagging index<sup>2</sup> for 2000–2019 released by the Cabinet Office, Government of Japan, as the business cycle data in Japan. We first obtain the monthly CI lagging index data for 2000–2019.

<sup>&</sup>lt;sup>2</sup> Cabinet Office, Government of Japan, https://www.esri.cao.go.jp/en/stat/di/ di-e.html

#### Table 7

	-									
The an	nual avera	age compos	site index	CI, lagging	; index).					
FY	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
CI	89.93	89.25	88.03	93.13	98.86	101.41	105.92	109.28	102.91	87.08
FY	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
CI	89.12	91.86	93.03	95.18	100.22	99.87	99.81	103.08	103.99	103.16

#### Table 8

Business cycle and accuracy in the model fitting.

Variable	ACC <sup>RA</sup>	ACC <sup>RN+</sup>
CI	$-1.04 \times 10^{-4}$	$-2.45 \times 10^{-4}$
	(0.3386)	(0.3053)

The value of () means P-value.

\*: significance at 10%.

\*\*: significance at 5%.

\*\*\*: significance at 1%.

#### Table 9

Business cycle and the difference in accuracies.

Variable	Diff	Diff
CI	$-1.25 \times 10^{-4}$ (0.4361)	$\begin{array}{c} 1.41 \times 10^{-4} \\ (0.3803) \end{array}$

The value of () means *P*-value.

\*: significance at 10%.

\*\*: significance at 5%.

\*\*\*: significance at 1%.

And then, we assign the average of the monthly index values for each fiscal year to the annual CI lagging index. The data is shown in Table 7.

We first examine the following regression model,

$$ACC_t^i = \alpha^i + \beta^i \times CI_t + \epsilon, \tag{5.2}$$

where  $ACC_t^i$  is the value of Error, represented by (5.1), at time t for model i (i = RA, RN+) as listed in Table 3, Cl<sub>t</sub> is the CI lagging index at time t, and  $\epsilon$  is the white noise. (5.2) implies that the accuracy of the model fitting for the model i turns superior during a boom (bust) if  $\beta^i$  is negative (positive). The result, presented in Table 8, indicates that the coefficients of Cl<sub>t</sub> are not statistically significant at any level for both models. That is, we cannot find any relationship between the business cycle and accuracy of the model fitting for each model.

Next, we examine whether the difference in the accuracy between the RA and RN+ models depends on the business cycle. This test verifies the more accurate model in terms of model fitting for the business cycle. To test this, we examine the following regression model,

$$\operatorname{Diff}_{t} = \alpha + \beta \times \operatorname{CI}_{t} + \epsilon, \tag{5.3}$$

where  $\text{Diff}_t$  is the difference in Errors for the two models at time t, defined by

$$\text{Diff}_t = \text{ACC}_t^{\text{RA}} - \text{ACC}_t^{\text{RN+}}$$

and  $|\text{Diff}|_t$  is its absolute value at time *t*. (5.3) implies that the increase in CI leads to the increase in ACC<sup>RA</sup> or decrease in ACC<sup>RN+</sup> if  $\beta$  is positive. That is, the RN+ model is superior to the RA model in the model fitting during a boom if  $\beta$  is positive. Furthermore, we examine the following regression model,

$$|\text{Diff}|_t = \alpha + \beta \times \text{Cl}_t + \epsilon. \tag{5.4}$$

This illustrates how accurate are the model fitting for the two models according to the business cycle. In fact, (5.4) indicates that the increase in CI decreases (increases) |Diff| if  $\beta$  is positive (negative). That is, if  $\beta$  is positive (negative), the difference in

the model fitnesses for both models reduces during a boom (negative).

The result presented in Table 9 shows that the coefficient of  $Cl_t$  is not statistically significant at any level for the regression models depicted by (5.3) and (5.4). Hence, the business cycle does not indicate which risk-averse or risk-neutral model more correctly describes the actual bank behavior in our framework.

#### 6. Summary

In this study, we addressed whether banks behave as riskaverse or risk-neutral investors in their asset allocation. To this end, we examined how the bank's optimal lending ratio accurately fits the actual lending ratio through the estimation of parameters (i.e. calibration). We used a sample of Japanese banking companies between 2000 and 2019 for the calibration process. We considered the mean-variance utility maximization problem as the risk-averse bank problem and the linear utility maximization problem as the risk-neutral bank problem. For the latter, we added two risk constraints, the linear and non-linear risk constraints. Then, we solved the optimal lending ratio as the bank asset allocation for each model.

The calibration results demonstrated that the risk-neutral model under the linear risk constraint is the most inferior in terms of the model fitting accuracy. However, incorporating the non-linear risk constraint into the linear utility maximization problem led to no significant difference in the accuracy of the model fitting between the risk-averse and risk-neutral models under the risk-neutral probability space. This is the main contribution of this study, that is, we proposed a risk-neutral model to describe bank behavior as accurately as the risk-averse model. Moreover, as the type of the parameters in the risk-neutral model is less than that in the risk-averse model, our risk-neutral model is more efficient from the viewpoint of the model estimation.

The result is interpreted as follows. Since the standard deviation is used in the non-linear risk constraint, adding the nonlinear risk constraint to the linear utility maximization problem effectively makes the risk-neutral bank model convertible with the risk-averse model with the mean-variance utility. This boosts the accuracy of the model fitting for the risk-neutral model. Thus, if one wishes to describe the bank's behavior for its asset allocation, she/he should set the non-linear optimization problem. And then, she/he can use the utility maximization problems for both the risk-averse and the risk-neutral banks to model the behavior of a bank in its asset allocation.

Finally, we examined how the accuracy of the model fitting is related to the business cycle. We did not find any significant relationship between the accuracy of the model fitting and the business cycle for both models. The numerical results in our study are obtained for Japanese banking companies that only account for the derivatives on their balance sheets. Meanwhile, for example, Shim (2013) demonstrated that the bank capital buffer is negatively correlated with the business cycle for the US bank data. Therefore, we expect the proposed models in our study to capture the business cycle for other countries' cases. Moreover, our study did not distinguish between the quality of the loans, that is, the performing and non-performing loans. From Messai and Jouini (2013), the non-performing loan (NPL) held by the bank is negatively correlated with the growth rate of the (real) GDP. That is, the amount of the NPL is countercyclical. Therefore, by applying our models to both the performing and non-performing loans, we expect that the business cycle explains the accuracy of the model fitting in our study. This means that our study contributes to showing how the bank's behavior in its asset allocation is affected by the business cycle.

#### **CRediT** authorship contribution statement

Kazuhiro Takino: Conceptualization, Methodology, Software. Yoshikazu Ishinagi: Supervision, Preparation of balance sheets.

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#### Appendix. Derivation of optimal lending ratio for RN+ model

The Lagrangian for the problem (4.11) is

1

$$\mathcal{L} = \mathbf{D}^{\mathsf{T}} \mathbf{E}_{F} + \eta \left( e - v \sqrt{\mathbf{D}^{\mathsf{T}} \boldsymbol{\Sigma} \mathbf{D}} \right), \tag{A.1}$$

where n is the Lagrange multiplier. From the FOC for the optimal **D**, we have

$$\mathbf{E}_{F} - \eta v (\mathbf{D}^{\top} \boldsymbol{\Sigma} \mathbf{D})^{-1/2} \boldsymbol{\Sigma} \mathbf{D} = \mathbf{0}.$$
(A.2)

As the bank is willing to have risk assets unlimitedly as long as  $R_{\rm C} > 1$ , the risk constraint represented in (4.9) holds in equality, that is,

$$v\sqrt{\mathbf{D}^{\top}\Sigma\mathbf{D}} = e. \tag{A.3}$$

Plugging (A.3) into (A.2), we obtain

$$\mathbf{D} = \frac{e}{\eta v^2} \mathbf{\Sigma}^{-1} \mathbf{E}_F. \tag{A.4}$$

Substituting (A.4) into (A.3), we obtain

$$\eta = \frac{\sqrt{\xi}}{v},\tag{A.5}$$

as derived by Danielsson et al. (2009), where  $\xi = \mathbf{E}_{\mathrm{F}}^{\top} \Sigma^{-1} \mathbf{E}_{\mathrm{F}}$ ,

$$\mathbf{E}_{F}^{\top} \boldsymbol{\Sigma}^{-1} \mathbf{E}_{F} = \frac{K_{C}^{2}}{M} \{ \sigma_{L}^{2} \sigma_{B}^{2} + \sigma_{L}^{2} \sigma_{F}^{2} + \sigma_{B}^{2} \sigma_{F}^{2} - 2\rho_{LF} \sigma_{L} \sigma_{B}^{2} \sigma_{F} - 2\rho_{BF} \sigma_{L}^{2} \sigma_{B} \sigma_{F} - (\rho_{LB} \sigma_{L} \sigma_{B})^{2} - (\rho_{LF} \sigma_{L} \sigma_{F})^{2} - (\rho_{BF} \sigma_{B} \sigma_{F})^{2} + 2\rho_{LB} \rho_{LF} \sigma_{L}^{2} \sigma_{B} \sigma_{F} + 2\rho_{LB} \rho_{BF} \sigma_{L} \sigma_{B}^{2} \sigma_{F} + 2\rho_{LF} \rho_{BF} \sigma_{L} \sigma_{B} \sigma_{F}^{2} \},$$

 $M = (1 + 2\rho_{LB}\rho_{LF}\rho_{BF} - (\rho_{LB}^2 + \rho_{LF}^2 + \rho_{BF}^2))(\sigma_L\sigma_B\sigma_F)^2$ , and we assume  $M \neq 0$ .

Now, we provide the optimal lending ratio  $l^*$ . Substituting (2.2), (2.6), and (2.7) into (A.1), the Lagrangian (A.1) is

$$\mathcal{L} = E_g^{\mathbb{Q}} + \eta \left( e - v \sqrt{Var_g^{\mathbb{Q}}} \right), \tag{A.6}$$

where

$$E_g^Q := E^Q[R_g] = \frac{\lambda_F R_C l + R_C}{M_4 l + M_5} + \lambda_F R_C D_p$$

$$Var_{g}^{Q} := Var^{Q}[R_{g}] = \left(\frac{1}{M_{4}l + M_{5}}\right)^{2} (U_{1}l^{2} + 2U_{2}l + \sigma_{B}^{2}) + \sigma_{F}^{2}\lambda_{F}^{2}D_{p}^{2}.$$

Eq. (A.6) is a function of *l*. The FOC for the optimal *l* is

$$\frac{\partial E_g^Q}{\partial l} - \frac{\eta v}{\sqrt{Var_g^Q}} \frac{\partial Var_g^Q}{\partial l} = 0.$$
(A.7)

Plugging (A.3) into (A.7), then (A.7) can be expressed as

$$\frac{\partial E_g^Q}{\partial l} - \frac{\eta v^2}{e} \frac{\partial Var_g^Q}{\partial l} = 0.$$
(A.8)

As we have obtained  $\eta$  in (A.5), substituting it into (A.8) yields

$$\frac{\partial E_g^Q}{\partial l} - \frac{v\sqrt{\xi}}{e} \frac{\partial Var_g^Q}{\partial l} = 0.$$
(A.9)
From

$$\frac{\partial E_g^Q}{\partial l} = \frac{\lambda_F R_C}{M_4 l + M_5} - \frac{(\lambda_F R_C l + R_C)M_4}{(M_4 l + M_5)^2}$$

and

ζ:

$$\frac{\partial Var_g^Q}{\partial l} = \frac{2(U_1l + U_2)}{(M_4l + M_5)^2} - \frac{2(U_1l^2 + 2U_2l + \sigma_B^2)M_4}{(M_4l + M_5)^3},$$
  
A.9) yields

$$_{3}l+\zeta_{4}=0, \tag{A.10}$$

where

$$\zeta_3 = R_C(\lambda_F M_4 M_5 - M_4^2) - 2 \frac{v\sqrt{\xi}}{e} (M_5 U_1 - M_4 U_2)$$

$$\zeta_4 = R_C(\lambda_F M_5^2 - M_4 M_5) - 2 \frac{v\sqrt{\xi}}{e} (M_5 U_2 - \sigma_B^2 M_4)$$

From (A.10), we obtain the optimal lending ratio

$$l^* = -\frac{\zeta_4}{\zeta_3}.$$

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