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Preference estimation from point allocation experiments

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ABSTRACT

Point allocation experiments are widely used in the social sciences. In these experiments, survey respondents distribute a fixed total number of points across a fixed number of alternatives. This paper reviews the different perspectives in the literature about what respondents do when they distribute points across options. We find three main alternative interpretations in the literature, each having different implications for empirical work. We connect these interpretations to models of utility maximization that account for point and budget constraints and investigate the role of budget constraints in more detail. We show how these constraints impact the regression specifications for point allocation experiments that are commonly used in the literature. We also show how a formulation of a taste for variety as entropy that had been previously used to analyse market shares can fruitfully be applied to choice behaviour in point allocation experiments.

1. Introduction

Point allocation experiments have been used in various scientific fields for many years. As far as we know, the approach was used first in the field of psychology where subjects rate the intensity of stimuli (Comrey, 1950; Guilford, 1954; Metfessel, 1947). Later, marketing studies have asked respondents to distribute chips or tokens across options to indicate their preferences for these options (e.g. Silk and Urban, 1978). The method has also been used in health economics, asking respondents to distribute donor livers across patients (e.g. Ubel and Loewenstein, 1996), or a fixed budget across patients (e.g. Schwappach, 2003) or across health programmes (e.g. Skedgel et al., 2015). The point allocation approach is closely related to asking respondents how likely they would be to choose a given brand (e.g. Juster, 1966) or to experiments where respondents assign subjective likelihoods to choice alternatives (e.g. Blass et al., 2010).

The reason for asking respondents to distribute points, tokens or shares in experiments is that it provides richer and more nuanced information about the respondents' preferences than the widely used discrete choice experiments where respondents indicate their preferred option, and possibly their least preferred option (Marley and Louviere, 2005). But it is not obvious what information exactly can be obtained from point allocation experiments.

In this paper, we first provide an overview of the assumptions made in various parts of the social science literature about what respondents do when they distribute points across alternatives. We find three main approaches: interval theory, ratio theory, and a log-ratio model associated with the elicitation of choice probabilities. We link these approaches to existing empirical applications. We then

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investigate the possibility of making these approaches consistent with utility-maximizing behaviour. The specification of a value or direct utility function and of constraints that impact the decision can be helpful when policies are valued or when researchers want to learn more about the determinants of choice. In particular, we show how to deal with the marginal utility of private income or public budget and to allow for heterogeneity therein. We also show that a model previously used to explain market shares, that models taste for variety as entropy (Anderson et al., 1988) can also be used to analyse the respondents' decisions in a point allocation experiment.

The contribution of this paper is very practical and might prove useful for teaching purposes, experimental developers and gives input for the dialogue between economists and choice modelers in the social sciences.

2. Literature review

This section reviews papers where respondent n allocates a number Q points to rate J options, where each of these options is characterized by a vector of attributes X_{nj}. For simplicity it is assumed that the number of points Q and the number of alternatives J are equal for all respondents. Let us define q_{nj} as the number of points allocated to alternative j, under the condition that $\sum_j q_{nj} = Q$. This type of experiment is sometimes called 'constant sum paired comparison' (Moore and Lehmann, 1989; Skedgel et al., 2015; Skedgel and Regier, 2015).[1] Most applications ask respondents to allocate points or the like between two alternatives. Comrey (1950) argues that limiting the task to two alternatives makes answering easier for respondents. Guilford (1954) and Moore and Lehmann (1989) compare asking respondents to rate two or more alternatives at a time, and do not find systematic differences in answers. Some studies in marketing do ask respondents to distribute choice probabilities across a set of brands (e.g. Reibstein, 1975) or to report the likelihood of buying one specific product or type of product (e.g. Juster, 1966).

For the empirical analysis of point allocation experiments, researchers need to make assumptions about the thought process of the respondents when they allocate the points across options. Hauser and Shugan (1980) present an overview of possible assumptions about what respondents are doing when they allocate points. An important question is whether one thinks that points convey meaningful information about the intensity of respondents' preferences, i.e. some cardinal measure of utility, or whether one thinks that points only convey ordinal information. We use the classification of Hauser and Shugan (1980) to structure our literature review, starting with the ordinal interpretation of the point allocation and then discussing the cardinal interpretation. For each possible set of assumptions, we discuss their meaning and which empirical studies are based, explicitly or implicitly, on these assumptions.[2]

2.1. *Point allocation as ordinal information*

The most limited information one can derive from a point allocation is that more points attributed to an option indicate that it is preferred. This is consistent with an idea of utility that is purely **ordinal** and entirely **deterministic** (Marschak, 1950; Neumann and Morgenstern, 1947), and with the assumption that respondents' answers also satisfy these properties. Preferences can then be inferred using conjoint analysis (Krantz and Tversky, 1971). However, the answers given by respondents are bound to contain errors and to violate the axioms that have to be satisfied to apply conjoint analysis. One answer to this problem is to introduce randomness in preferences (**stochastic theory**).[3] If one is willing to attribute more information value to the points allocated, one can then assume that the points allocated convey information about the relative likelihood of choosing the options. Let us denote $P_n(j > k)$ the probability that individual n chooses/prefers option j over option k. Hauser and Shugan (1980) show for a pairwise comparison of options and a sequential allocation of points, the process is stationary. When preferences are transitive, each individual point allocation is Bernoulli, and the maximum-likelihood estimator for $P_n(j > k)$ is equal to the number of points of an alternative divided by the total:

$$P_n(j > k) = \frac{q_{nj}}{q_{nj} + q_{nk}} = \frac{q_{nj}}{Q}.$$

Another way to interpret the point allocation is to apply random utility maximization. A stochastic term is added to the utility function, that can reflect mistakes made by the respondents, measurement errors by the researcher, or characteristics of the alternatives that are not observed by the researcher but do matter for the respondents' choice. In the special case of the logit additive random utility model, which assumes logistically distributed error terms which enter the utility function additively, this becomes (McFadden, 1974):

[1] This term is also used in studies that ask survey respondents to distribute points across attributes of a product to measure their relative importance in consumer choice (Netzer and Srinivasan, 2011; Ujjwal and Bandyopadhyaya, 2021; Zafri et al., 2021; Zwahlen et al., 1996). In this paper, we focus on the literature that asks respondents to distribute points across alternatives.

[2] Note that the literature that we review here assumes that respondents are able to give answers that reflect their internal states accurately, up to some measurement error that is treated as random noise in empirical applications. This needs not be the case in reality. For instance, Regenwetter et al. (2019) argue that respondents may have "thresholds below which they do not distinguish differences between any pair" of alternatives, and build a decision-theoretic model of how answers to rating scales are generated using semiorders. Regenwetter and Marley (2001) discuss the relationship between a probability distribution over semiorders and certain types of random utility functions. The decision-theoretic study of answers to point allocation experiments is an interesting avenue for further research.

[3] See Batley (2008) for a discussion of the origins of the Random Utility Model.

$$P_n(U_{nj} > U_{nk}) = \frac{e^{V_{nj}}}{e^{V_{nj}} + e^{V_{nk}}}$$

where $V_{nj}(X_{nj}, \beta_n)$ is the deterministic part of the conditional additive random utility function $U_{nj} = V_{nj} + \varepsilon_{nj}$. This interpretation of point allocation is not applied so much in the discrete choice literature, probably because it in fact reduces information that is collected in a cardinal way to ordinal discrete choice information, which does not seem very efficient. Nevertheless, data from a point allocation experiment can always be transformed to ordinal choice data. An example is Linley and Hughes (2013), who transform budget allocation between two patient groups into an indicator of whether either one of both groups or none is favoured.

2.2. Asking for choice probabilities

Some studies do ask respondents to compare alternatives by indicating the probability that they would choose each alternative. Early applications in the field of marketing include Byrnes (1964), Ferber and Piskie (1965), Juster (1966), Axelrod (1968), Haley (1970), Reibstein (1975), Granbois and Summers (1975). Such probabilities can be interpreted as individual market shares when repeated choices are made. Blass et al. (2010) describe the rationale for asking for choice probabilities. According to them, alternatives presented in choice experiments are incomplete scenarios: not all relevant information is available to the respondent when making a choice, which introduces some uncertainty. By asking respondents to state choice probabilities, they are allowed to express that "resolvable uncertainty": the elicited choice probability q_{nj} is the subjective probability that person n places on the event that the realisations of ε_{nj} will make option j optimal. The choice probability for alternative j is given by:

$$q_{nj} = \frac{e^{V_{nj}}}{\sum_{j=1}^{J} e^{V_{nj}}}$$

which yields the following estimation equation for linear in attributes V_{nj}:

$$\ln\left[\frac{q_{nj}}{q_{n1}}\right] = (X_{nj} - X_{n1})\beta_n + u_{nj} \tag{1}$$

where the alternative $j = 1$ can be chosen arbitrarily and u_{nj} is an error term that can capture measurement errors or unobserved preferences. Blass et al. (2010) suggest that preference parameters can best be estimated using median regression, because that method is insensitive to the way probabilities equal to 0 or 1 are treated, as at these points log-odds equal to plus or minus infinity.

This method has been used to study preferences for electricity reliability (Blass et al., 2010), for land-use scenarios (Shoyama et al., 2013), for political candidates (Delavande and Manski, 2015), for electric power from different sources (Morita and Managi, 2015), for workplace attributes (Wiswall and Zafar, 2018), for long-term care insurance products (Boyer et al., 2020), and for migration (Koşar et al., 2021).

Note that taking the log of the ratio of a quantity ("part") to a reference quantity is one of the methods used in compositional analysis, the analysis of data that sum to a constant for individual cases. This is called the "additive log-ratios" method (Aitchison, 1982; Smithson and Broomell, 2022). Smithson and Broomell (2022) explain that this method "maps data from the simplex to an unrestricted Euclidean space" (p. 4).

2.3. Point allocation as cardinal information

The literature in the previous sections does not assume that respondents are able to give meaningful information about the magnitude, or intensity, of their preferences. If one is willing to make this assumption, one can use the point allocation as an indicator of preference in the cardinal sense. Hauser and Shugan (1980) distinguish two possible ways of doing so: interval theory and ratio theory.

2.3.1. Interval theory

According to **interval theory**, respondents allocate points in such a way that the difference between the points allocated reflects the intensity of preference. Therefore:

$$V_{nj} - V_{nk} = q_{nj} - q_{nk} \tag{2}$$

This relates to Shapley (1975), who derives axioms implying the existence of such a cardinal utility function. Shapley (1975) defines V_{nj} as unique up to an order-preserving linear transformation. In other words, assuming cardinal utility means that any positive affine transformation of the number of points allocated to an alternative is a valid representation of the utility derived from this alternative: $V_{nj} = \gamma_n + \alpha_n * q_{nj}$ (see also Sen, 1986, p. 1113). If so, $V_{nj} - V_{nk} = \alpha_n * (q_{nj} - q_{nk})$. In that sense, it is more precise to say that the difference between the points allocated is proportional to the difference in utilities.

Hauser and Shugan (1980) show that interval theory, together with "evaluative independence", implies a representation of utility that is additive in the attributes multiplied by their parameters. The assumption of "evaluative independence" is equal to the well-known assumption of "independence of irrelevant alternatives" for multinomial logit models: respondents' answers only depend upon the attributes varying in the pair considered. Hauser and Shugan (1980) assume a utility function that takes the form: $V_{nj} = \sum_i \beta_{ni} X_{nij}$. One can therefore estimate the utility function using the following equation:

$$q_{nj} - q_{nk} = \beta_n \left(X_{nj} - X_{nk} \right) + \varepsilon_{njk} \tag{3}$$

Positive affine transformations of the utility function will result in a scaling of the parameter vectors β and ε: $q_{nj} - q_{nk} = \frac{1}{\alpha_n} \beta_n (X_{nj} - X_{nk}) + \frac{1}{\alpha_n} \varepsilon_{njk}$. Because α_n cannot be identified, it is left out of equations like (3) and (4) in the discussion by Hauser and Shugan (1980) and in empirical applications. Here, ε is usually assumed to represent measurement error.

This estimation approach is the one followed in the health economics literature where respondents allocate donor livers across patients with different characteristics (Chan et al., 2006; Ratcliffe, 2000; Ubel and Loewenstein, 1996), or budget or votes across patient groups or health programmes (Richardson et al., 2018; Schwappach, 2003; Schwappach and Strasmann, 2006; Skedgel et al., 2015; Skedgel and Regier, 2015). The earlier articles are not explicit about the underlying behavioural model (Schwappach, 2003; Schwappach and Strasmann, 2006). Skedgel and Regier (2015, p.157) state: "*the difference in budget shares can be interpreted as proportional to the difference in latent utility between the underlying alternatives*", thereby explicitly adhering to interval theory. Skedgel et al. (2015, p.1232) and Richardson et al. (2018, p.799) make very similar statements.[4] Note that the possible values of the difference $q_{nj} - q_{nk}$ are restricted by the total number of points Q. Observed differences with the values Q or $-Q$ may be censored observations. For this reason, Skedgel and Regier (2015) and Skedgel et al. (2015) use a double-censored Tobit model to estimate equation (3).

2.3.2. Ratio theory

According to **ratio theory**, respondents allocate points in such a way that the ratio between them indicates the intensity of preferences:

$$\frac{q_{nj}}{q_{nk}} = \frac{V_{nj}}{V_{nk}} \tag{4}$$

This idea has been developed by Torgerson (1958) in the context of psychological experiments in which subjects were asked to rate the intensity of stimuli. This theory has not been used much in marketing or economics, but there are a few exceptions. In the field of marketing, Silk and Urban (1978) derive preference values for brands from constant-sum paired comparison data using Torgerson's model, and use these preference values to predict purchase behaviour. In health economics, Ubel and Loewenstein (1996) conduct not only an analysis based on differences in allocated points, but also regress the ratio of allocated livers between two groups of patients on the ratio of survival probabilities for these two groups. Hauser and Shugan (1980) show that ratio theory, together with evaluative independence, implies a utility function in which the attributes enter multiplicatively: $V_{nj} = \prod_i \beta_{ni} X_{nij}$. This model implies the following regression equation:

$$\ln \left[\frac{q_{nj}}{q_{nk}} \right] = \ln \left(V_{nj} \right) - \ln \left(V_{nk} \right) + \varepsilon_{nj} \tag{5}$$

and preference parameters can be estimated by discretization of the attributes. Consistent with this, Moore and Lehmann (1989) estimate preference parameters for brands by assuming that the difference between these preference parameters is equal to the log of the ratio of intentions to purchase measured by a constant-sum comparison task.

3. Utility maximization models for point allocation experiments

In this section, we first model the allocation of points in a constant-sum paired comparison experiment as an optimization process with constraints. This makes the role of prices and budget constraints explicit, and as such, uncovers implicit assumptions made about their role in point allocation experiments. Second, we introduce a model that uses previous insights on representative agent models to analyse product differentiation (Anderson et al., 1988). These models reformulate aggregated discrete choice models as the allocation of consumption shares to different products due to a taste for variety formulated as entropy. We apply the model to individual choices where respondents allocate shares or subjective probabilities to different alternatives in an experiment. This neatly fits the experimental setup of point allocation experiments where a cardinal number is measured instead of a discrete choice.

3.1. Point allocation as an optimization process

The models in section 2 see the point allocation as reflecting the relative preferences of respondents over different alternatives, and possibly the intensity of these preferences, up to some measurement error. However, the allocation of points itself can be interpreted as the result of an optimization process with a constraint on the total number of points. Indeed, in many applications, the allocation of points is presented as a choice situation. In marketing studies, respondents are asked how likely they would be to buy a given product. In health economics, respondents are asked to imagine how they would distribute public budget if they were a government official. In that choice process, the respondent can be viewed as optimizing a latent direct utility function that is a function of the number of points

[4] It is interesting to note that Richardson et al. (2018, p. 801) discuss a way to translate votes into value units that seems in line with ratio theory.

attributed to the alternatives. In this setting, the points represent demand rather than approximations of utility.

We use shorthand notation $V_{nj} = V_{nj}(\beta_n, X_{nj}, \varepsilon_{nj})$ for the systematic element of the utility function. The random component in the conditional systematic utility function can be interpreted as perception error, measurement error or random preference. Direct utility for consumption of the bundle of J goods is defined as a function H(.) of q_{nj}, the number of points allocated to each alternative and V_{nj}, the utility generated by each alternative: $U_n = \sum_{j=1}^{J} H\left(q_{nj}, V_{nj}\right)$. We discuss different possible forms for H(.) below. This utility is maximized subject to the constraint $\sum_{j=1}^{J} q_{nj} = Q$.

3.1.1. A utility maximization model that parallels interval theory

To illustrate the importance of formulating the thought process as utility maximization, let us formulate a model that is consistent with the estimation equation yielded by interval theory. This model can be obtained by assuming that $H(q_{nj}, V_{nj}) = q_{nj} V_{nj} - \frac{1}{2} {q_{nj}}^2$. In other words: respondents derive utility from the sum of the alternatives, weighted by the number of points allocated to them, but points allocated to one specific alternative yield decreasing returns. Then, direct utility takes the form:

$$U_n = \sum_{j=1}^{J} \left(q_{nj} V_{nj} - \frac{1}{2} {q_{nj}}^2 \right)$$

To maximize this utility subject to the constraint that $\sum_{j=1}^{J} q_{nj} = Q$, we formulate the following Lagrangian equation:

$$L_n = \sum_{j=1}^{J} \left(q_{nj} V_{nj} - \frac{1}{2} {q_{nj}}^2 \right) + \lambda \left(Q - \sum_{j=1}^{J} q_{nj} \right).$$

Since we are looking for the optimal values of q_{nk}, we take the first derivatives of the Lagrangian with respect to q_{nk} and the Lagrangian multiplier λ, and set them equal to 0 to obtain an optimum. This yields the following first-order conditions:

$$\frac{\partial L_n}{\partial q_{nk}} = V_{nk} - q_{nk} - \lambda = 0.$$

$$\frac{\partial L_n}{\partial \lambda} = Q - \sum_{j=1}^{J} q_{nj} = 0.$$

Solving the first-order conditions and using the constraint that $\sum_{j=1}^{J} q_{nj} = Q$ leads to the following optimal ${q_{nk}}^*$:

$$ {q_{nk}}^* = V_{nk} - \lambda^* = V_{nk} - \frac{\sum_{j=1}^{J} V_{nj} - Q}{J}. \tag{6}$$

This ${q_{nk}}^*$ can be interpreted as the demand for points for alternative k. The first term is equal to the systematic part of the utility function. The second term is equal to the Lagrangian multiplier in the optimum. It shows with how much the optimal or indirect utility increases when the respondent receives an additional point for distribution. Optimal demand is increasing with rate $\frac{1}{J}$ in the total number of points and is decreasing in the number of alternatives. Demand increases with rate $1 - \frac{1}{J}$ in the own conditional systematic utility V_{nk} and decreases with rate $\frac{1}{J}$ in the conditional systematic utilities of the other alternatives V_{nj}. The second term is equal for all alternatives and therefore the difference in points for alternatives k and j is equal to the difference in systematic utility:

$$ {q_{nj}}^* - {q_{nk}}^* = V_{nj} - V_{nk} \tag{7}$$

This shows that the interval model can be interpreted as measuring the difference in the marginal utilities for the linear term in the direct utility function which is equal to the difference in the systematic part of the utility function.

3.1.2. The role of prices and budgets

The formulation as a utility maximization problem under constraints is particularly interesting when it comes to studying the roles of prices and budgets.

In many point allocation experiments, prices are not an explicit part of the optimization problem. Some studies ask respondents to distribute budget between groups, but do not include any attributes that would depend on the budget allocation (Richardson et al., 2018a; Richardson et al., 2018; Schwappach, 2003). This is practically equivalent to a case where the prices do not differ between groups. Linley and Hughes (2013), Skedgel and Regier (2015) and Skedgel et al. (2015), who ask respondents to distribute budget between two patient groups, do allow the costs of treatment to differ between both groups. They allow the number of patients treated in two groups to be dependent on the allocation of budget between those two groups, and this is explicitly shown to respondents. However, the total budget to be distributed is fixed in the experiment, so that a preference for a larger number of patients to be treated can either indicate a preference for cost-effectiveness, or a preference for treating more patients regardless of the costs, as recognized explicitly by Skedgel et al. (2015). Richardson et al. (2012) and Richardson et al. (2018b) ask respondents to distribute a fixed budget between two patients with different conditions, so that one euro does not yield the same increase in life expectancy for one patient as for the other. In that way, they measure only a preference for cost-effectiveness. Presenting both quantities (e.g. number of patients that can be treated, QALYs that can be gained) and prices (e.g. price of a QALY) as attributes can be a way to disentangle a preference for quantities (patients, QALYs) and a preference for cost-effectiveness. In some experiments, prices are presented as attributes of the alternatives (Schwappach and Strasmann 2006).

As soon as respondents are assumed to have preferences about prices, this implies that there is a potential role for a budget constraint. Respondents care about prices because there is not unlimited money, and they care about alternative uses of money, be it for their private consumption (in marketing research) or for public goals (in health economics). To model the importance of private consumption, we introduce an outside private good in the utility function. Let us denote it z_{priv} and p_{priv} its price. Similarly, to model the importance of alternative uses of public money, we introduce an outside public good in the utility function. Let us denote it z_{pub} and b_{pub} its price. To express the fact that there is not unlimited money to be spent, let us denote Y_n the private income of respondent n, and B_n the public budget that respondent n can allocate. For simplicity, we assume additively separable utility, i.e. no complementarities or substitutions between the different alternatives and the public and private outside goods. The maximization problem therefore becomes:

$$U_n = \sum_{j=1}^{J} H(q_{nj}, V_{nj}) + \mathrm{f}(z_{priv}) + \mathrm{g}(z_{pub}),$$

subject to:

$$\sum_{j=1}^{J} q_{nj} = Q$$

$$Y_n = \sum_{j=1}^{J} p_{nj} q_{nj} + p_{priv} z_{priv}$$

$$B_n = \sum_{j=1}^{J} b_{nj} q_{nj} + b_{pub} z_{pub}$$

If we assume $H(q_{nj}, V_{nj}) = q_{nj} V_{nj} - \frac{1}{2} q_{nj}{}^2$, as in 3.1.1., then in the optimum, one can substitute the budget constraints into the utility function, and use the first-order conditions for utility maximization to obtain the difference in demand shares:

$$q_{nj}{}^* - q_{nk}{}^* = V_{nj} - V_{nk} - \frac{\partial f / \partial z_{priv}}{p_{priv}} (p_{nj} - p_{nk}) - \frac{\partial g / \partial z_{pub}}{b_{pub}} (b_{nj} - b_{nk}). \tag{8}$$

The first term in this equation is the difference in conditional systematic utility. For experiments with private and governmental expenditures, two additional terms enter the equation: the second and third terms show that the differences in prices of the private and public outside goods are multiplied with a coefficient that is the ratio of the marginal utility of the outside good and the price of the outside good. Without further assumptions, the marginal utilities of the outside goods may endogenously depend on all the conditional systematic utilities and all the prices of the alternatives. This can be seen from the private budget constraint in the case of the private good: $z_{priv} = \frac{Y_n - \sum_{j=1}^{J} p_{nj} q_{nj}}{p_{priv}}$. A similar reasoning applies for the public good. This means that one cannot estimate the coefficients on the price differences without further functional form assumptions.

There are two ways to come back to a tractable regression equation that is the same as the one following from interval theory. The first is an experimental design where prices of pairs of alternatives are assumed to be equal (a bit like in Schwappach, 2003). One can then interpret the difference in allocated points as the difference in the systematic part of the utility function, because the last two terms of equation (8) disappear. Second, one can assume that the marginal utility of income is close to 0, i.e. that $\partial f / \partial z_{priv} \approx 0$ and/or $\partial g / \partial z_{pub} \approx 0$. In this case, the model then might give a reasonable approximation of the interval model, but the assumption goes against the very idea of introducing the budget constraint in the first place.

However, the studies that introduce price as an attribute in the experiments do estimate a price coefficient (e.g. Schwappach and Strasmann, 2006). It is therefore interesting to uncover the assumptions that underlie such estimations. It turns out that estimating a price coefficient in a model consistent with interval theory amounts to assuming that *f(.)* and *g(.)* take a linear form: $\mathrm{f}(z_{priv}) = \beta_{priv} z_{priv}$ and $\mathrm{f}(z_{pub}) = \beta_{pub} z_{pub}$. If we make this assumption together with the assumption that $H(q_{nj}, V_{nj}) = q_{nj} V_{nj} - \frac{1}{2} q_{nj}{}^2$, then substituting the budget constraints into the utility function and maximizing with respect to q_{nj} yields:

$$q_{nj}{}^* - q_{nk}{}^* = V_{nj} - V_{nk} - \frac{\beta_{priv}}{p_{priv}} (p_{nj} - p_{nk}) - \frac{\beta_{pub}}{b_{pub}} (b_{nj} - b_{nk}) \tag{9}$$

The estimated marginal utilities of private income and public budget are the ratios of the marginal utilities of the outside private and private goods to the prices of the outside private and private goods, respectively. It is possible to imagine that these marginal utilities are heterogeneous in the sample studied and therefore one might employ methods to allow for heterogeneity in these coefficients in the estimations.

If one wants to relax the assumption that the utility function is linear in the outside goods, one can for instance assume that *f(.)* and *g(.)* take the *ln(.)* form, for instance $\mathrm{f}(z_{priv}) = \beta_{priv} \ln(z_{priv})$ and $\mathrm{f}(z_{pub}) = \beta_{pub} \ln(z_{pub})$. In this case, we obtain:

$$q_{nj}^{\ *} - q_{nk}^{\ *} = V_{nj} - V_{nk} - \beta_{priv} \frac{p_{nj} - p_{nk}}{Y_n - \sum_{j=1}^{J} p_{nj} q_{nj}^{\ *}} - \beta_{pub} \frac{b_{nj} - b_{nk}}{B_n - \sum_{j=1}^{J} b_{nj} q_{nj}^{\ *}} \tag{10}$$

The marginal utilities of private income and public budget are then proportional to the reciprocals of the remaining income (or public budget) after consumption of the chosen bundle. When income (or public budget) is higher, the marginal utility of income (or public budget) is lower, and price differences are less relevant for the respondent. In the presence of information on private income of the respondents and on the public budget available, one can rescale the price differences as in the equation above to obtain unbiased estimates of all parameters.

The question remains whether the ln(.) assumption for f(.) and g(.) is valid. One can test this by assuming Box-Cox transformations with homogeneous curvature parameters ρ and κ respectively in the population. The specification of the dependent variable then becomes:

$$q_{nj}^{\ *} - q_{nk}^{\ *} = V_{nj} - V_{nk} - \left(\frac{Y_n - \sum_{j=1}^{J} p_{nj} q_{nj}^{\ *}}{p_{priv}} \right)^{\rho} \beta_{priv} \frac{p_{nj} - p_{nk}}{Y_n - \sum_{j=1}^{J} p_{nj} q_{nj}^{\ *}} - \left(\frac{Y_n - \sum_{j=1}^{J} b_{nj} q_{nj}^{\ *}}{b_{pub}} \right)^{\kappa} \beta_{pub} \frac{b_{nj} - b_{nk}}{Y_n - \sum_{j=1}^{J} b_{nj} q_{nj}^{\ *}}. \tag{11}$$

One can use a grid search on (ρ, κ) to see which assumptions are most appropriate. Models of choice with budget transfers in experiments are analysed by Dekker et al. (2019), and have been applied in various contexts (Mouter et al., 2017, 2021). These budget transfers can help to identify the relationship between the marginal utility of income and the marginal utility of public budget.

We are well aware that the simple utility maximization framework presented here is often not a realistic representation of human behaviour. In principle, V_{nj} can be modified to include a number of elements that would make it more realistic, such as reference points, or subjective probabilities in line with cumulative prospect theory (Tversky and Kahneman, 1992). As such, it can take context and framing effects into account to some extent. However, it is important to note that context or framing effects related to the price attributes are much more difficult to include in this model. For instance, if respondents react differently to costs of public policy stated in terms of total amounts per year or average costs per beneficiary or per taxpayer, it will be challenging to reflect this in the formulation of the outside public good in the utility function.

3.2. A behavioural model with a taste for variety

3.2.1. Modelling a taste for variety

The literature review in section 2 shows that the points allocated by respondents to alternatives are either assumed to reflect cardinal utility, or interpreted as choice probabilities that incorporate “resolvable uncertainty”. Another plausible reason for spreading points or probabilities across different options is a taste for variety when repeated choices are made. Reibstein (1975) and Silk and Urban (1978) already mention variety-seeking as a possible reason to ask respondents what percent of the time they would choose a given brand, or how likely they would be to choose a given brand, respectively. They do not, however, formalize this idea in more detail. This is the aim of this section.

If we want to model taste for variety in a behavioural model, direct utility for consumption of the bundle of J goods can be written as (see Anderson et al., 1988 for a model with point and budget constraints):

$$U_n = \sum_{j=1}^{J} q_{nj} V_{nj} - \sum_{j=1}^{J} q_{nj} \ln\left[q_{nj}\right]. \tag{12}$$

The first part of this utility function is related to the systematic utility of the alternatives. This systematic utility V_{nj} is multiplied with the allocated share. The second term is the Shannon entropy and captures a taste for variety. Point allocations closer to equal shares receive a higher entropy bonus than allocations where all points are allocated to one alternative. The size of the systematic part of the utility determines the relative size of the entropy in direct utility. Again, it is assumed that the individual allocates points optimally subject to the point constraint.

The entropy term in the utility function is a way of expressing why the individual does not wish to concentrate all the points on the alternative with the highest V_{nj}. We call this a taste for variety, but it can also be seen as a taste for novelty, or curiosity. It therefore also relates to the work of Daniel Berlyne who shows that experimental subjects have a preference for stimuli that provide more variety (or more entropy) (e.g. Berlyne, 1957).[5] In a similar sense, the formulation of the model also shares analytical similarities with reinforcement learning and its application to bandit problems (Sutton and Barto, 2018), in particular the trade-off between exploitation of successful behaviours and exploration of alternative, new behaviours, that might yield even higher rewards. The first term in equation

[5] Note that in our model, we do not include anything about the information value or entropy of the alternatives across which the points are distributed. This could, however, be made part of the utility function if relevant in applications.

(12) can be seen as akin to exploitation, while the second term is related to exploration.[6] Relatedly, the inclusion of the entropy term in our model also offers opportunities to model search and satisficing behaviour (Artinger et al., 2022; Simon, 1955, 1956). The application of our model to dynamic problems is an interesting avenue for further research.

Alternatively, the same formulation of the utility function could be used to express a preference for spreading resources equally across goals or patients. Skedgel et al. (2015, p. 1238) argue that point allocation experiments may be useful because respondents "may be likely to have protected values or rights-based preferences over which they may be reluctant to make or accept absolute trade-offs". Richardson and co-authors repeatedly used constant-sum paired comparisons to show that respondents have a preference for spreading resources equally across patients over maximizing QALYs (Richardson et al., 2012; Richardson et al., 2018b; Richardson et al., 2018). The formulation of the utility that we propose here can be used to formalize such a preference for equity.

Formulating the Lagrangian and solving the first-order conditions leads to logit expressions for optimal demands that are a function of the systematic utilities only (Anderson et al., 1988):

$$q_{nk}{}^{*} = \frac{e^{V_{nk}}}{\sum_{j=1}^{J} e^{V_{nj}}}.$$

The log-ratio of the points can be used in a linear regression model as it gives the difference between the systematic utility of the alternatives:

$$\ln\left[\frac{q_{nj}{}^{*}}{q_{nk}{}^{*}}\right] = V_{nj} - V_{nk}. \tag{13}$$

The scale of the systematic utility functions is incorporated in the size of the estimated coefficients for the different attributes and the scale of the error term. If the attributes enter V_{nj} linearly, this yields the following regression equation:

$$\ln\left[\frac{q_{nj}{}^{*}}{q_{nk}{}^{*}}\right] = \left(X_{nj} - X_{nk}\right)\beta_n + u_{nj} \tag{14}$$

where u_{nj} is an error term that captures measurement error and unobserved preferences. It is interesting to note that this regression equation is the same as the one derived by Blass et al. (2010). While they derive this regression equation using "resolvable uncertainty" and i.i.d. extreme-value distributed unknowns, our model derives it from the maximization of a direct utility function with a taste for variety formulated as entropy. Just like the model of Blass et al. (2010), our model is consistent with an ordinal interpretation of utility and still allows to use the cardinal information given by the respondents to measure the intensity of preference for attributes. More complicated extensions of direct utility functions with nested structures are given by Verboven (1996) and Fosgerau and De Palma (2016). Swait and Marley (2013) discuss other uses of entropy to model variety-seeking and conceptualize probabilistic choice.

3.2.2. Link with ratio theory

The model we use to formalize a taste for variety can also lead to the same regression estimation as ratio theory. If we assume $V_{nj} = \ln[W_{nj}]$, this results in:

$$\ln\left[\frac{q_{nj}{}^{*}}{q_{nk}{}^{*}}\right] = \ln\left[W_{nj}\right] - \ln\left[W_{nk}\right] \tag{15}$$

This is the estimation equation that Hauser and Shugan (1980) show to be consistent with ratio theory. Note that utility, in this setting, is no longer separable in the attributes of an alternative.

3.2.3. The role of prices and budgets

The role of prices and budgets is similar to the role demonstrated above in the model consistent with interval theory. When private prices or governmental prices are part of the experiment, these enter the optimality conditions resulting in:

$$\ln\left[\frac{q_{nj}{}^{*}}{q_{nk}{}^{*}}\right] = V_{nj} - V_{nk} - \frac{\partial f / \partial z_{priv}}{p_{priv}}\left(p_{nj} - p_{nk}\right) - \frac{\partial g / \partial z_{pub}}{b_{pub}}\left(b_{nj} - b_{nk}\right). \tag{16}$$

The marginal (direct) utilities of the private and public outside goods may depend on all systematic utilities and all prices leading to endogeneity issues.

When one assumes linear outside private goods and linear outside governmental goods with unit marginal utilities and unit prices, these multipliers are equal to 1 (Anderson et al., 1988). For a linear value of outside good consumption, one can normalize these regression coefficients to −1, or rewrite the dependent variable to include the difference in prices and budgets:

[6] Note that our model lacks the dynamic aspect of reinforcement learning, since we study only one-shot decisions in this paper. In particular, the entropy term in our model, which would stand for the value of exploration, is not at all related to the expected value of the alternatives or the number of time steps left in the experiment. That seems like an undesirable feature from the point of view of reinforcement learning.

$$\ln\left[\frac{q_{nj}{}^{*}}{q_{nk}{}^{*}}\right] + (p_{nj} - p_{nk}) + (b_{nj} - b_{nk}) = V_{nj} - V_{nk}.$$

If we assume linear outside goods, but neither unit marginal utilities nor unit prices, then price coefficients can again be estimated that are the ratios of the marginal utilities and the prices of the outside goods:

$$\ln\left[\frac{q_{nj}{}^{*}}{q_{nk}{}^{*}}\right] = V_{nj} - V_{nk} - \frac{\beta_{priv}}{p_{priv}}(p_{nj} - p_{nk}) - \frac{\beta_{pub}}{b_{pub}}(b_{nj} - b_{nk}) \tag{17}$$

For a specification with logged utilities of the outside goods, this can be rewritten as:

$$\ln\left[\frac{q_{nj}{}^{*}}{q_{nk}{}^{*}}\right] = V_{nj} - V_{nk} - \beta_{priv}\frac{p_{nj} - p_{nk}}{Y_n - \sum_{j=1}^{J} p_{nj}q_{nj}{}^{*}} - \beta_{pub}\frac{b_{nj} - b_{nk}}{B_n - \sum_{j=1}^{J} b_{nj}q_{nj}{}^{*}}. \tag{18}$$

Again Box-Cox parameters can be added to test the assumptions on the marginal utility of income and public budget.

3.2.4. Link with asking for choice probabilities

It is interesting to discuss how this relates to the work of Blass et al. (2010) and their followers, since they derive the estimation equation (1), which is the same as (14), starting from a discrete choice framework and asking respondents to report choice probabilities.

Let us therefore start from a discrete choice framework, with the following direct utility function:

$$U_n = \sum_{j=1}^{J} y_{nj}V_{nj} + f(z_{priv})$$

For the sake of brevity, we only discuss the case with a private outside good here, but the argument is the same with a public outside good. V_{nj} is defined as above, but y_{nj} is an indicator variable taking value 1 if alternative j is chosen and 0 if not. We introduce the outside good z_{priv} because we are interested in the role of prices and budgets. Let us define p_{nj} as the price of alternative j for individual n, p_{priv} as the price of the outside good, and Y_n as the disposable income of individual n. The following budget constraint has to be respected:

$$Y_n = \sum_{j=1}^{J} y_{nj}p_{nj} + p_{priv}z_{priv}$$

For simplicity, let us start by assuming that the outside good enters the utility function linearly: $U_n = \sum_{j=1}^{J} y_{nj}V_{nj} + \beta_{priv}z_{priv}$.

Here β_{priv} is the preference parameter for the outside good. When a single alternative i is chosen, substituting the budget constraint into conditional direct utility yields:

$$V_{ni} + \beta_{priv}\frac{Y_n - p_{ni}}{p_{priv}}.$$

Alternative i is thus chosen over alternative j if:

$$V_{ni} + \beta_{priv}\frac{Y_n - p_{ni}}{p_{priv}} > V_{nj} + \beta_{priv}\frac{Y_n - p_{nj}}{p_{priv}}, \forall i \neq j,$$

$$V_{ni} - \frac{\beta_{priv}}{p_{priv}}p_{ni} > V_{nj} - \frac{\beta_{priv}}{p_{priv}}p_{nj}, \forall i \neq j. \tag{19}$$

Assuming a random linear conditional utility $V_{ni} = \beta_n X_{ni} + \varepsilon_{ni}$, results in

$$\beta_n X_{ni} - \frac{\beta_{priv}}{p_{priv}}p_{ni} + \varepsilon_{ni} > \beta_n X_{nj} - \frac{\beta_{priv}}{p_{priv}}p_{nj} + \varepsilon_{nj}, \forall i \neq j,$$

If one assumes that the random terms are extreme value Type I distributed, the difference in the random terms has a logistic distribution and the probability takes the conditional logit form. The price coefficient can be estimated, and interpreted as the ratio of the marginal direct utility of the outside good and the price of the outside good. Note that the disposable income of the individual does not play a role here. This is consistent with the approach followed by Blass et al. (2010) when they estimate a coefficient for the price of electricity. They do allow for heterogeneity in this price coefficient by income groups.

If we assume that the external good enters the utility function in a log-linear way, we arrive at the result that, to estimate the marginal utility of income, one should include the term $\ln[Y_n - p_{ni}]$ in the regression equation. Due to discrete choice, it is no longer necessary to include the consumption and prices of the other goods, as in the $\frac{p_{nj} - p_{nk}}{Y_n - \sum_{j=1}^{J} p_{nj}q_{nj}{}^{*}}$ term following form the "taste for variety" model in equation (18)above.

Table 1
Overview of models discussed.

	Theory of point allocation	Regression equation	Direct utility function leading to the same regression equation when optimized
Interval theory	Difference between the points allocated reflects the intensity of preference: $U_{nj} - U_{nk} = q_{nj} - q_{nk}$	$q_{nj} - q_{nk} = \beta_n(X_{nj} - X_{nk}) + \varepsilon_{nj}$ (e.g. Ratcliffe 2000; Schwappach 2003, Skedgel et al., 2015; Skedgel and Regier, 2015)	$U_n = \sum_{j=1}^{J}\left(q_{nj}V_{nj} - \frac{1}{2}q_{nj}^2\right)$
Ratio theory	Ratio between the points allocated reflects the intensity of preference. It implies that attributes enter utility multiplicatively (Hauser and Shugan 1980): $\frac{q_{nj}}{q_{nk}} = \frac{V_{nj}}{V_{nk}} = \frac{\prod_i x_{inj}\beta_{ni}}{\prod_i x_{ink}\beta_{ni}}$	$\ln\left[\frac{q_{nj}}{q_{nk}}\right] = \ln(V_{nj}) - \ln(V_{nk}) + \varepsilon_{nj}$ which can be estimated with discrete attributes, or by estimating only a constant for each alternative (as in Moore and Lehmann 1989).	$U_n = \sum_{j=1}^{J} q_{nj}\ln(V_{nj}) - \sum_{j=1}^{J} q_{nj} \bullet \ln\left[q_{nj}\right]$.
Eliciting choice probabilities	The elicited choice probability q_{nj} is the subjective probability that person n places on the event that the realisations of the unknowns will make option j optimal (Blass et al., 2010): $q_{nj} = \frac{e^{V_{nj}}}{\sum_{j=1}^{J} e^{V_{nj}}}$	$\ln\left[\frac{q_{nj}}{q_{nk}}\right] = (X_{nj} - X_{nk})\beta_n + \varepsilon_{nj}$ (e.g. Blass et al., 2010)	V_{nj} can take any form (as long as it is additive in the attributes).
Taste for diversity	The points are allocated so as to maximize a utility function with a taste for diversity modelled as entropy: $U_n = \sum_{j=1}^{J} q_{nj}V_{nj} - \sum_{j=1}^{J} q_{nj}\ln\left[q_{nj}\right]$ (this paper, inspired by Anderson et al., 1988)	$\ln\left[\frac{q_{nj}}{q_{nk}}\right] = (X_{nj} - X_{nk})\beta_n + \varepsilon_{nj}$	$U_n = \sum_{j=1}^{J} q_{nj}V_{nj} - \sum_{j=1}^{J} q_{nj}\ln\left[q_{nj}\right]$.

3.3. Estimation

Standard OLS or median regression can be used where linear and non-linear impacts of the attributes can be included using polynomial terms (Blass et al., 2010). The error term in this regression can be interpreted as the difference in random systematic utility between the status quo and the selected alternative or as misperception or measurement error. One can decide on the basis of theory and model statistics such as AIC, BIC or adjusted R^2 which specification of the direct utility function is most appropriate. In case the researcher has reliable prior knowledge about the way specific attributes should impact the distribution of points across alternatives, Bayesian inference may offer additional possibilities for parameter estimation and model selection (Etz and Vandekerckhove, 2018).

As the mean of the difference in systematic utility might differ over alternatives, it is useful to include alternative-specific fixed effects. These fixed effects capture the intrinsic preferences for each of the alternatives relative to the status quo alternative. To allow for preference heterogeneity in the attributes one can employ random effects regression with a stochastic distribution on β_n or individual fixed effects. For a detailed discussion of the assumptions behind these methods we refer to Wooldridge (2010). Iterative procedures can be used for determining the curvature of the outside good consumption.

As an alternative to median regression, generalized linear models can also be used. Since the dependent variable is the log of the ratio of two parts of a composition, with a reference part as the denominator, the additive log-ratios method (Aitchison, 1982; Smithson and Broomell, 2022) seems very well suited to our purposes.

An issue that needs to be dealt with when estimating models based on point allocation data is the fact that some alternatives may be given zero points. This forms a practical problem when $\ln\left[\frac{q_{nj}^*}{q_{nk}^*}\right]$ is the dependent variable, as the log ratio becomes undefined if one of the q's is equal to zero. Blass et al. (2010) suggest replacing zeros by a very small positive number, and applying median regression rather than OLS to make the results robust to this transformation of the data. This approach has been adopted by their followers. Smithson and Broomell (2022) discuss a number of alternative options, including re-scaling the whole data to avoid values equal to 0, various methods to replace the zeros, and estimating a hurdle model that accounts for the fact that observations equal to 0 may be generated in a way different from strictly positive observations.

4. Conclusion

Various strands of the literature in psychology, marketing and economics let respondents allocate points across alternatives, with very different motivations for doing so. We have briefly reviewed these motivations for three cases: interval theory, ratio theory, and a log-ratio model derived from the elicitation of choice probabilities, each suggesting a different specification of the dependent variable in point allocation regressions (interval, ratio, log-ratio of points).

Some authors assume that the allocation of points directly reflects cardinal utility derived from the alternatives. Others suggest that choice probabilities reported by respondents are driven by "resolvable uncertainty", i.e. the beliefs of the respondents about the chances that a given option will turn out to be optimal once the factors unknown at the time of the experiment become known.

We have introduced behavioural models for point allocation experiments and have shown potential behavioural underpinnings for the regression equations that are estimated in the literature, thereby making explicit which assumptions are made in the different parts of the literature, mainly about the way income enters the utility function. In particular, we have shown that a utility function with a taste for variety formulated as entropy that had been used previously to model market shares (Anderson et al., 1988) can also be used to model point allocation by one individual. These results might be useful for the conversation between economists and choice modellers in the social sciences. Table 1 offers an overview of the models we discussed.

One of the advantages of point allocation experiments is that for the analysis of choices one can employ linear regression techniques without further assumptions on aggregation. We provided extensions to the literature to provide more detail on the marginal utilities of public budget and private income. We showed how to allow for heterogeneity in the marginal utility of income while keeping tractable specifications of the regression equation. The curvature can be tested by employing iterative regression procedures with fixed Box-Cox parameters.

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CRediT authorship contribution statement

Marion Collewet: Conceptualization, Methodology, Formal analysis, Writing – review & editing. **Paul Koster:** Conceptualization, Methodology, Formal analysis, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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