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The relative pricing of WTI and Brent crude oil futures: Expectations or risk premia?[☆]

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ABSTRACT

This paper studies the spread of Brent-WTI futures prices using a no-arbitrage term structure model with one common and two latent idiosyncratic risk factors. We document more negative risk premia for WTI than for Brent, and the differences are more pronounced at longer maturities. The expectation of future spot price dominates the risk premium in determining the term structure of Brent-WTI futures spread, especially at short maturities. The common risk premia in both markets are negative and similar, while their corresponding idiosyncratic risk premia have opposite signs. The common risk prices of WTI and Brent are generally related to the US crude commercial stock, inflation, economic uncertainty, and hedging pressure; however, idiosyncratic risk prices are more related to their corresponding local production, short rate, and the term structure factors. The variance decomposition indicates that the idiosyncratic factors account for a considerable part at longer forecast horizons in both markets.

1. Introduction

Crude oil is an essential commodity and dominates many aspects of global economics and politics. There are two major benchmarks for world oil prices, West Texas Intermediate (WTI henceforth) crude oil and Brent crude oil, which are both light and sweet.[2] WTI refers to oil extracted from wells in the US and sent via pipeline to Cushing, Oklahoma. The supplies are land-locked, and it is relatively expensive to ship to certain parts of the globe. Brent refers to oil from fields in the North Sea. Because the supply is water-borne, it is easier to transport to distant locations. As of November 2020, crude oil made up 43.72% of the entire Standard and Poor Goldman Sachs Commodity Index (S&P GSCI) in terms of dollar value, of which WTI and Brent accounted for 25.31% and 18.41%, respectively.

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2 The sulfur content of crude oil is one of the measures that determine how easy (less costly) to refine crude oil into other products, and its level is referred to as how sweet or sour the crude is. The American Petroleum Institute (API) gravity is commonly used to measure how heavy or light a petroleum liquid is compared to water. Crude oil with high API and low sulfur content is referred to as light and sweet, which is appropriate to be refined into various useful components such as gasoline, diesel fuel, and jet fuel.

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Understanding the dynamic differences between the WTI and Brent crude oilprices has become increasingly of interest to both academics and practitioners in the past decade.[3] As shown in Fig. 1, before 2011, Brent and WTI futures prices tracked with each other closely, with Brent typically trading at a slight discount to WTI.[4] Such a discount reflects a consensus view that there exists transportation bottlenecks near Cushing, Oklahoma, the physical delivery hub for WTI crude oil futures (U.S. Energy Information Administration, EIA henceforth, 2012). In early 2011, this long-standing relationship began to change, and since then, WTI crude oil has been priced at a persistent discount to Brent. This price spread kept widening and peaked at about $25 in late 2011. Since 2013, along with the worldwide decline in crude oil prices, the spread became narrower.

In this paper, we propose a no-arbitrage term structure model to examine the dynamic price relationship between WTI and Brent futures. This model allows us to investigate the dynamics of expected future oil spot prices and risk premia, and to examine their contributions in explaining the term structure of price spreads between WTI and Brent futures.[5] Our model features three risk factors – one common risk factor that summarizes the information driving both WTI and Brent futures prices simultaneously and two latent idiosyncratic risk factors capturing the information that is distinct in each market, and provides analytical solutions to the pricing of oil futures. We study the model-implied risk prices for each factor and investigate their relationship with observed economic variables.

We estimate our model using weekly WTI and Brent futures prices with maturities within one year from January 2010 to February 2016. Several interesting results emerge from the model estimation. First, we find that both markets exhibit time-varying risk premia, which are predominantly negative throughout the sample period across all maturities. The negative risk premia are consistent with the existing literature about financialization in the commodity futures markets[6] and the fact that both markets are on average in contango during our sample.[7] Second, our model implies that both expected future spot price and risk premium are in general higher for Brent than for WTI. However, the differences in expectations are on average larger than the differences in risk premia and have higher correlations with the Brent-WTI futures spread, suggesting that the expected future spot price plays a more important role in determining the Brent-WTI spread. Third, the expected future spot price contributes more to the term structure of futures price spread at short maturities. At the long end of the term structure, we observe that the differences in risk premia between the two markets become more important in explaining the futures price spreads. Overall, for individual crude oil term structure modeling, our findings suggest that we need a time-varying term structure that reflects both risk premia and the market's forecast of oil prices.

We next examine the pricing of common and idiosyncratic risks in WTI and Brent futures. We observe that all risk factors carry significant risk premia in both markets. This finding implies that, to explain futures prices and risk premia of individual crude oil futures, we need state variables extracted from both the overall world crude oil market and the individual crude oil (e.g., WTI- or Brent-specific) market. Specifically, the prices of the common risk factor are negative and similar for WTI and Brent, while the prices of the two idiosyncratic risk factors in the two markets have opposite signs and are weakly correlated. Further investigation reveals that the common risk premia in both markets are generally related to the US crude commercial stock, inflation, the economic uncertainty index introduced by Jurado et al. (2015), and the hedging pressure in both crude oil markets. Besides, the WTI and Brent idiosyncratic factors are related to the measures capturing the shape of the oil futures curve. The slope factor or basis (curvature or relative basis) correlates most with the first (second) idiosyncratic factor in its corresponding oil market. Both the WTI and Brent idiosyncratic risk prices are negatively correlated with the slope factor in their own market, consistent with the theory of storage in the futures markets that a positive slope (long-term futures prices are higher than short-term futures prices) implies a contango market and negative risk premia (e.g., Working, 1933, 1949; Kaldor, 1939; Kilian and Murphy, 2014). The risk prices of the two WTI idiosyncratic factors are both negatively correlated with the 3-month US Treasury rate. The risk price of the first Brent idiosyncratic factor is negatively correlated with the UK and Norway crude production. However, the WTI risk prices have much smaller and insignificant correlations with the production in Norway and UK, consistent with the literature showing that WTI plays a leading role in the global crude oil market (e.g., Klein, 2018; Liu et al., 2018).

Finally, we perform a variance decomposition for the total forecast variances of WTI and Brent futures into the percentage contributions stemming from the three state variables across all maturities and forecast horizons. We find that in both markets, the common factor dominates the movements in futures prices across the term structure spectrum for all forecast horizons, accounting for approximately 93% (88%) of the variation for WTI (Brent) on average. Meanwhile, the contribution of the two idiosyncratic factors combined increases sharply with the length of the forecast horizon for both WTI and Brent futures. Furthermore, the variance decomposition reveals that the idiosyncratic factors are relatively more important for Brent futures in determining futures prices than for WTI futures across almost all maturities and forecast horizons.

Our paper contributes to the large body of literature studying the WTI and Brent price spread.[8] These studies tend to focus on identifying structural breaks or examining the long-term integration in crude oil prices and relate the results to globalization

[3] See, for example, Büyükşahin et al. (2013), Chen et al. (2015), Liu et al. (2018), Caporin et al. (2019), Luong et al. (2019), and Geyer-Klingeberg and Rathgeber (2021).

[4] The prices of contacts with other maturities exhibit very similar patterns.

[5] Futures prices can be decomposed into a forecast of future spot price and an expected risk premium (Fama and French, 1987; Gorton and Rouwenhorst, 2006).

[6] A large number of empirical studies document the existence of negative risk premia in the commodity futures markets, in particular the oil market, e.g., Hamilton and Wu (2014, 2015), Li (2018), Heath (2019), Singleton (2014) and Brunetti et al. (2016).

[7] For example, Gorton and Rouwenhorst (2006) implies that in a contango futures market, futures prices have to decrease to converge to the future spot price, leading to a negative risk premium.

[8] See, for example, Geyer-Klingeberg and Rathgeber (2021), Chen et al. (2015), Büyükşahin et al. (2013), Caporin et al. (2019), Liu et al. (2018), and Luong et al. (2019).

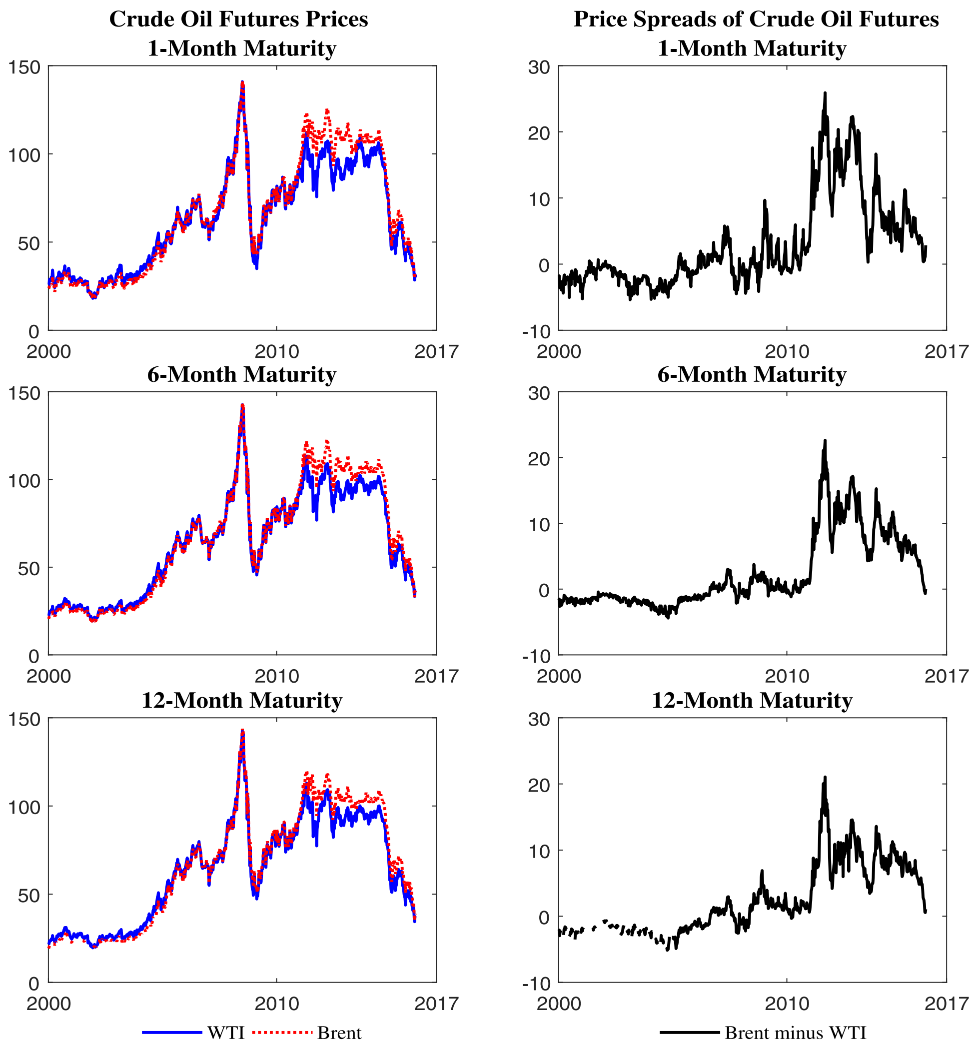

Fig. 1. WTI and Brent crude oil futures prices and price spreads. Notes to Figure: This figure plots the weekly futures prices with 1-month, 6-month, and 12-month maturities on the left panels. The solid line (blue) is for the WTI futures price, and the dotted line (red) is for the Brent futures price. We also plot the weekly price spreads between Brent and WTI crude oil futures (the solid line in black) on the right panels. The sample period is from 2000:01 to 2016:02. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

or regionalization of the crude oil market.[9] Furthermore, previous studies have suggested a set of Brent-WTI determinants, such as convenience yield, inventories, transportation costs, and open interest.[10] A number of studies attribute the price differences between WTI and Brent to the increase in oil production in the US, due to the shale oil revolution, the infrastructure development from limited capacity to an expanding number of pipelines, and the policy changes such as removing the ban of exporting crude oil, etc. (e.g., Fattouh, 2010; Luong et al., 2019; Caporin et al., 2019; Liu et al., 2015; Scheitrum et al., 2018). However, these studies mainly focus on the Brent-WTI price differences in the spot market or the nearest to mature futures. We adopt a no-arbitrage term structure model to investigate the term structure of WTI and Brent futures spreads. This model also allows us to decouple the expected future spot price and risk premium from the current futures price. In such a way, we are able to identify the impact of each component on the Brent-WTI futures spread.

Additionally, our work is related to the literature on modeling commodity futures prices as affine functions of state variables (Gibson and Schwartz, 1990; Schwartz, 1997; Casassus and Collin-Dufresne, 2005; Casassus et al., 2013; Hamilton and Wu, 2014; Chiang et al., 2015; Heath, 2019). It is also related to the literature studying the WTI futures prices and returns using the Nelson and Siegel (1987) term structure factors (GrØnborg and Lunde, 2016; Bredin et al., 2021). However, none of these studies focuses

[9] See Caro et al. (2020), Klein (2018), Mastroeni et al. (2021), Mann and Sephton (2016), and Liao et al. (2014).

[10] See Geyer-Klingeberg and Rathgeber (2021), Milonas and Henker (2001), Büyükşahin et al. (2013), Giulietti et al. (2014), Mensi et al. (2014), and Schmidbauer and Rösch (2012).

on the price differences between the two oil markets. We distinguish the common factor from the idiosyncratic factors in model specification and estimation to capture both the commonalities and the distinctness in WTI and Brent futures. Moreover, the choice of state variables in the affine term structure model is crucial for determining the model's implications on spot price expectations and risk premia. This framework allows us to analyze the extent to which oil futures are globally and locally determined.

The paper proceeds as follows: Section 2 presents the term structure model with common and idiosyncratic factors, and describes the estimation method. Section 3 presents the data. Section 4 reports model estimation results. Section 5 discusses the economic implications of the model, and Section 6 concludes.

2. Model specification and estimation

2.1. The model

In this section, we describe the pricing model for the term structure of crude oil futures in the two markets. The log spot price in market i denoted by s_t^i is assumed to follow an affine function of the risk factors

$$\begin{aligned} s_t^i &= \delta_0^i + \delta_X^i X_t + \delta_Y^i Y_t^i, \\ &= \delta_0^i + \delta_1^i Z_t^i, \end{aligned} \tag{1}$$

where X_t, the $N \times 1$ vector of risk factors, is common to both the WTI and Brent markets, Y_t^i, the $M \times 1$ vector of risk factors, is specific to market i. δ_0^i is a scalar, δ_X^i is a $1 \times N$ vector, and δ_Y^i is a $1 \times M$ vector. $\delta_1^i = [\delta_X^i, \delta_Y^i]$ is a $1 \times (N+M)$ vector. $Z_t^i = \left[X_t', Y_t^{i'}\right]'$ represents the $(N+M) \times 1$ vector of risk factors. Given the low dimensional structure of the crude oil futures prices, we adopt a model with three factors (see, for example, Schwartz, 1997; Casassus and Collin-Dufresne, 2005).[11] We take the common risk factor X_t as the first principal component of the observed WTI and Brent log futures prices. Y_t^i, the 2×1 idiosyncratic risk factors in market i, are latent. We assume Z_t^i follows a first-order Gaussian vector autoregression (VAR) under the physical P and the risk-neutral Q measures

$$Z_t^i = K_0^{P,i} + K_1^{P,i} Z_{t-1}^i + \Sigma^i \varepsilon_t^i, \tag{2}$$

$$Z_t^i = K_0^{Q,i} + K_1^{Q,i} Z_{t-1}^i + \Sigma^i \varepsilon_t^i, \tag{3}$$

where $K_0^{P,i}$, $K_0^{Q,i}$, and ε_t^i are 3×1 vectors, $K_1^{P,i}$ and $K_1^{Q,i}$ are 3×3 matrices. ε_t^i is assumed to be distributed $N(0, I_3)$. Σ^i is a 3×3 lower triangular matrix, and $\Sigma^i \Sigma^{i'}$ is the variance–covariance matrix.

Given the dynamics in Eqs. (1) and (3), the price of a futures contract in market i with h-period maturity is given by[12]

$$F_t^{h,i} = \exp(A_h^i + B_h^i Z_t^i), \tag{4}$$

where the scalar A_h^i and the 1×3 vector, B_h^i are functions of the parameters under the Q-dynamics, $\Theta^{Q,i} = \{K_0^{Q,i}, K_1^{Q,i}, \delta_0^i, \delta_1^i, \Sigma^i\}$, through a set of recursive equations

$$A_h^i = A_{h-1}^i + B_{h-1}^i K_0^{Q,i} + \frac{1}{2} B_{h-1}^i \Sigma^i \Sigma^{i'} B_{h-1}^{i'}, \tag{5}$$

$$B_h^i = B_{h-1}^i K_1^{Q,i}, \tag{6}$$

where $A_0^i = \delta_0^i$, and $B_0^i = \delta_1^i$. The derivation of the recursive relations is provided in Appendix A. The model-implied term structure of log futures prices in market i is therefore affine in the state variables Z_t^i

$$f_t^i = A^i(\Theta^{Q,i}) + B^i(\Theta^{Q,i}) Z_t^i, \tag{7}$$

where f_t^i is a $H^i \times 1$ vector of log futures prices. H^i denotes the total number of available maturities for each market in the sample. A_h^i is the hth element in the $H^i \times 1$ vector $A^i(\Theta^{Q,i})$, and B_h^i is the hth row in the $H^i \times 3$ matrix $B^i(\Theta^{Q,i})$.

To link the physical and risk-neutral measures, we specify the pricing kernel in market i to take the form

$$m_{t+1}^i = \exp\left(-s_t^i - \frac{1}{2} \lambda_t^{i'} \lambda_t^i - \lambda_t^{i'} \varepsilon_{t+1}^i\right), \tag{8}$$

where λ_t^i is a 3×1 vector. We follow the term structure literature and assume that the time-varying prices of risks in market i evolve with the risk factors

$$\lambda_t^i = \lambda_0^i + \lambda_1^i Z_t^i, \tag{9}$$

[11] The data in the crude oil futures market, as well as the data from other commodity markets, has a structure that resembles the data from the fixed income markets. The stylized facts of the Treasury yield curve dynamics also characterize the crude oil futures data set. Both yield and oil futures curves are on average upward sloping. The yields and oil futures prices at different maturities are all very persistent, and exhibit skewness and excess kurtosis. More importantly, only a small number of factors are needed to explain the variation of the whole term structure in both markets. Therefore models from the fixed income literature are often applied to analyze the commodity futures contracts.

[12] See Duffie and Kan (1996), Gibson and Schwartz (1990), Schwartz (1997), Schwartz and Smith (2000), Casassus et al. (2013), and Hamilton and Wu (2014).

where λ_0^i is a 3×1 vector and λ_1^i is a 3×3 matrix. This is the essentially affine risk price specification as in the Treasury bond literature (Duffee, 2002; Dai and Singleton, 2002; Cheridito et al., 2007). The risk prices λ_t^i measure the additional expected return required per unit of risk in each of the shocks in ε_t^i. In a Gaussian model, the only source of time-variation in risk premia are changes in the market prices of risk. The P- and Q-parameters in Eqs. (2) and (3) are therefore related as follows

$$K_0^{Q,i} = K_0^{P,i} - \Sigma^i \lambda_0^i, \tag{10}$$

$$K_1^{Q,i} = K_1^{P,i} - \Sigma^i \lambda_1^i. \tag{11}$$

According to Eq. (1), the expected h-period ahead spot price in market i, $E_t\left[s_{t+h}^i\right]$, is given by

$$\begin{aligned} E_t\left[s_{t+h}^i\right] &= E_t[\delta_0^i + \delta_1^i Z_{t+h}^i], \\ &= \delta_0^i + \delta_1^i E_t\left[Z_{t+h}^i\right], \end{aligned} \tag{12}$$

where $E_t\left[Z_{t+h}^i\right]$ is a function of parameters under P, $\Theta^{P,i} = \{K_0^{P,i}, K_1^{P,i}\}$, given $Z_t^i = \left[X_t', Y_t^{i\prime}\right]'$ and horizon h

$$E_t\left[Z_{t+h}^i\right] = (I_3 + K_1^{P,i} + \cdots + (K_1^{P,i})^{h-1})K_0^{P,i} + (K_1^{P,i})^h Z_t^i. \tag{13}$$

The idiosyncratic factors Y_t^i are inferred from the term structure of futures prices in market i. Following Hamilton and Wu (2014), we define the risk premia on h-period futures contract in market i as

$$rp_t^{h,i} = \widetilde{f}_t^{h,i} - f_t^{h,i}, \tag{14}$$

where $f_t^{h,i}$ is the model-implied log futures price for h-period contract in market i, given by Eq. (7). $\widetilde{f}_t^{h,i}$ is the model-implied log futures price for h -period contract in market i when there is no compensation for risk. λ_0^i and λ_1^i in Eqs. (10) and (11) would be zeros if there was no compensation for risk. Therefore $\widetilde{f}_t^{h,i}$ can be computed using Eq. (7), where $A^i(\Theta^{Q,i})$ and $B^i(\Theta^{Q,i})$ is obtained from the recursions in Eqs. (5) and (6) with $K_0^{Q,i} = K_0^{P,i}$ and $K_1^{Q,i} = K_1^{P,i}$. Using Eqs. (12) and (14), we characterize the dynamic behavior of spot price expectations and risk premia in WTI and Brent markets.

2.2. Estimation method

The idiosyncratic risk factors Y_t^i are latent for both markets. We use the Kalman filter to filter the latent idiosyncratic factors.[13] Filtering is a natural approach in our setup, because the latent variables have affine dynamics and thus the analytic expressions of the first two moments of the conditional density are available. The Kalman filter corresponds to maximum likelihood when the latent variables are Gaussian and the error is also normally distributed. Harvey (1991) and Hamilton (1994) are classic references for the maximum likelihood estimation in conjunction with the Kalman filter approach.

In our setup, the state propagation equation is Gaussian

$$Y_t^i = K_{0Y}^{P,i} + K_{1Y}^{P,i} Z_{t-1}^i + \Sigma_Y^i \varepsilon_t^i, \tag{15}$$

where $K_{0Y}^{P,i}$, the 2×1 vector, are the last two elements in $K_0^{P,i}$. $K_{1Y}^{P,i}$ and Σ_Y^i, the 2×3 matrices, are the last two rows in $K_1^{P,i}$ and Σ^i respectively. To estimate the model, we assume that all log futures prices in market i are measured with errors. The observed common state variable X_t is also measured with errors. The measurement equation thus is given by

$$\begin{pmatrix} f_t^i \\ X_t \end{pmatrix} = \begin{pmatrix} A^i + B^i K_0^{P,i} \\ K_{0X}^{P,i} \end{pmatrix} + \begin{pmatrix} B^i K_1^{P,i} \\ K_{1X}^{P,i} \end{pmatrix} Z_{t-1}^i + e_t^i \tag{16}$$

where $K_{0X}^{P,i}$, the scalar, is the first element in $K_0^{P,i}$. $K_{1X}^{P,i}$, the 1×3 vector, is the first row in $K_1^{P,i}$. e_t^i is a $(H^i + N) \times 1$ vector of measurement errors that is assumed to be $i.i.d.$ normal. Recall that H^i denotes the total number of available maturities for each market in the sample. N, the number of common state variable, equals to one in our model. We apply the Kalman filter to the state–space representation of our model and estimate the model parameters and filter the idiosyncratic risk factors via maximum likelihood. The estimation strategy has been widely used in the term structure modeling of interest rates (see, for example, Christoffersen et al., 2014; Cieslak and Povala, 2016; Bikbov and Chernov, 2010; Duffee, 2011) and also in the pricing of commodity derivatives (Schwartz, 1997; Trolle and Schwartz, 2009; Casassus et al., 2013; Schwartz and Smith, 2000; Nielsen and Schwartz, 2004; Liu and Tang, 2011). Appendix B provides more detailed information on the maximum likelihood estimation with Kalman filter algorithm.

[13] Several estimation methodologies have been proposed in the literature to identify the latent variables, such as the efficient method of moments (Gallant and Tauchen, 1996), the exact inversion likelihood approach (Chen and Scott, 1993; Pearson and Sun, 1994), and the Kalman filter method. Duffee and Stanton (2012) compare these methods and conclude that the Kalman filter has superior finite-sample properties.

Table 1
Summary statistics.

Panel A: Log WTI futures price							
	Central Moments				Autocorrelation		
	Mean	St.Dev	Skewness	Kurtosis	AC(1)	AC(12)	AC(30)
1 month	4.3925	0.2929	−1.4677	4.3025	0.9698	0.6968	0.4017
2 month	4.3990	0.2852	−1.4629	4.2530	0.9707	0.7016	0.4073
3 month	4.4045	0.2776	−1.4611	4.2163	0.9715	0.7062	0.4123
4 month	4.4087	0.2707	−1.4615	4.1969	0.9721	0.7101	0.4169
5 month	4.4119	0.2643	−1.4645	4.1969	0.9724	0.7128	0.4205
6 month	4.4141	0.2586	−1.4696	4.2121	0.9726	0.7147	0.4233
9 month	4.4155	0.2536	−1.4746	4.2342	0.9727	0.7161	0.4257
12 month	4.4165	0.2490	−1.4793	4.2597	0.9727	0.7174	0.4280
Panel B: Log Brent futures price							
	Central Moments				Autocorrelation		
	Mean	St.Dev	Skewness	Kurtosis	AC(1)	AC(12)	AC(30)
1 month	4.4903	0.3068	−1.3643	4.1574	0.9742	0.7173	0.4380
2 month	4.4923	0.2991	−1.3732	4.1961	0.9740	0.7154	0.4350
3 month	4.4943	0.2916	−1.3792	4.2240	0.9738	0.7149	0.4332
4 month	4.4960	0.2844	−1.3854	4.2531	0.9736	0.7145	0.4318
5 month	4.4975	0.2777	−1.3917	4.2820	0.9735	0.7142	0.4307
6 month	4.4986	0.2715	−1.3991	4.3144	0.9734	0.7139	0.4299
9 month	4.4998	0.2608	−1.4233	4.4306	0.9724	0.7133	0.4282
12 month	4.5004	0.2462	−1.4468	4.5287	0.9720	0.7126	0.4249

Notes to Table: This table presents the summary statistics for the log WTI futures prices (Panel A) and the log Brent futures prices (Panel B). We use the closing prices of WTI and Brent futures on Wednesday of each week for 1-month to 6-month, 9-month, and 12-month maturities. We present the sample mean, standard deviation, skewness, kurtosis, and autocorrelations for each maturity. The sample period is from 2010:01 to 2016:02.

3. Data

We obtain the WTI crude oil futures data from Chicago Mercantile Exchange (CME) and the Brent crude oil futures data from Genesis Financial Technologies.[14] Crude oil futures contracts are more liquid at short maturities. To study the term structure of crude oil futures market, we follow Trolle and Schwartz (2009) and first keep the six nearest monthly futures contracts (M1–M6) with 15 and more days to expiration. For crude oil futures contracts with maturities beyond six months, liquidity is concentrated in the contracts expiring in March, June, September, and December; therefore, we also keep the first two futures contracts with maturities beyond six months and expire in either March, June, September, or December (Q1 and Q2). We use the crude oil futures end of day (EOD) settlement prices on Wednesdays from January 2000 to February 2016.[15]

The left panel of Fig. 1 plots the WTI and Brent futures prices with 1-month (M1), 6-month (M6), and 12-month (Q2) maturities.[16] Note that for long-maturity Brent futures, we have missing observations due to the illiquid market at the beginning of the sample. We observe a close correspondence between the two indicators of the world oil markets for all maturities, especially before 2010. The price spread before 2010 is relatively small compared to the later sample, with Brent being lower than WTI most of the time. The two crude oil markets began to diverge at the end of 2010. The difference continued until about the end of our sample. The deviation was more pronounced in the period from 2011 to 2013: Brent was more expensive than WTI for all maturities. Both WTI and Brent futures prices began to decrease at the end of 2014 due to an oversupply of petroleum compared to demand, largely driven by the growth of US crude oil production (Baumeister and Kilian, 2016).

The right panel of Fig. 1 plots the spread of the Brent-WTI futures prices for the three different maturities.[17] We observe that the Brent-WTI spread widened to as much as \$25 per barrel by the end of 2011 for short maturity (1-month). For long maturity (12-month), the spread was about \$21 per barrel by the end of 2011. For short maturity, Brent futures price on average is about \$9 per barrel higher than WTI futures price, and it is about \$7 per barrel higher for long maturity. The literature has confirmed a major structural breakpoint of the price spread between WTI and Brent in 2010 (e.g., Geyer-Klingeberg and Rathgeber, 2021; Chen et al., 2015; Büyükşahin et al., 2013; Caporin et al., 2019). We therefore focus on the sample since 2010 in our estimation to analyze the differences in market expectations and risk premia between the two markets.

Table 1 presents the summary statistics of the log WTI and Brent futures prices for all eight maturities from January 2010 to February 2016. On average, the term structures of log WTI and Brent futures prices are slightly upward sloped, and the volatilities

[14] The Brent crude oil futures are traded on the Intercontinental Exchange (ICE).

[15] One crude oil futures contract is based on 1000 barrels of crude oil. Our data ends in February 2016 because that is the time when we purchased the data.

[16] For ease of exposition, we only present results for three different maturities. The results for all eight different maturities are reported in the internet appendix, Figure IA1.

[17] The results for all eight different maturities are reported in the internet appendix, Figure IA2.

Table 2
Parameter estimates.

Panel A: Log WTI futures price										
K_0^P	K_1^P				K_0^Q	K_1^Q				
−0.0573	0.9999	−0.2458	0.0491		0.0318	0.9973	−0.0564	−1.0222		
(−0.8526)	(37.5157)	(−0.4554)	(0.0129)		(0.9523)	(7.1555)	(−0.0166)	(−0.1862)		
−0.0369	0.0013	0.9502	−0.4699		0.0131	−0.0004	0.9867	−0.8146		
(−9.3924)	(0.8454)	(30.1526)	(−2.1061)		(0.5023)	(−0.0895)	(9.3206)	(−4.7659)		
−0.0030	0.0003	−0.0013	0.8861		0.0126	−0.0001	−0.0047	0.4759		
(−5.8861)	(1.2448)	(−0.3132)	(30.6485)		(0.6857)	(−0.0483)	(−0.0624)	(3.9703)		
λ_0	λ_1			$\lambda_0 + \lambda_1 \overline{Z}$	δ_0	δ_1		$\Sigma\Sigma' \times 1e2$		
−0.5096	0.0149	−1.0825	6.1223	−0.0986	−0.3676	0.4039		3.0618	−0.0357	−0.0528
(−0.1509)	(1.9548)	(−1.9533)	(6.4116)		(−0.0355)	(2.363)		(2.4071)	(−0.5139)	(−7.5191)
−4.5478	0.1546	−3.4509	31.8574	0.3553		0.9705		−0.0357	0.0130	0.0020
(−1.7439)	(1.1871)	(−3.2551)	(9.2589)			(5.6785)		(−0.5139)	(2.9963)	(5.0998)
−2.7015	0.0590	1.0696	92.2891	−0.1841		0.3188		−0.0528	0.0020	0.0028
(−1.4697)	(0.9586)	(0.7152)	(38.4499)			(2.7984)		(−7.5191)	(5.0998)	(39.012)
Panel B: Log Brent futures price										
K_0^P	K_1^P				K_0^Q	K_1^Q				
0.2568	0.9781	−0.7469	0.3081		0.0045	0.9993	0.0655	−0.0530		
(7.5292)	(28.3681)	(−1.0556)	(3.2293)		(0.2261)	(5.9765)	(1.2725)	(−0.1133)		
0.0418	−0.0005	0.6250	−0.0272		0.0018	−0.0004	0.9997	0.0583		
(5.8429)	(−0.6642)	(42.0605)	(−0.2714)		(0.5689)	(−0.0286)	(5.3893)	(0.0856)		
0.0000	0.0001	−0.0058	0.9346		0.0010	−0.0002	0.0097	0.9999		
(−0.0114)	(0.2826)	(−0.7315)	(17.525)		(0.0482)	(−0.1649)	(1.2852)	(2.7183)		
λ_0	λ_1			$\lambda_0 + \lambda_1 \overline{Z}$	δ_0	δ_1		$\Sigma\Sigma' \times 1e2$		
2.3486	−0.1969	−7.5646	3.3619	−0.1018	−0.0238	0.5451		1.1533	0.0139	−0.0011
(1.0773)	(−1.7559)	(−6.7581)	(3.1604)		(−0.0478)	(3.8653)		(4.2872)	(3.293)	(−0.4488)
4.2880	0.0202	−42.3269	−10.4192	−0.2045		−1.1582		0.0139	0.0076	0.0008
(2.6307)	(1.3747)	(−21.1258)	(−13.3372)			(−4.1061)		(3.293)	(64.0631)	(18.547)
−1.5907	0.0929	8.2761	−17.3040	0.1811		1.2369		−0.0011	0.0008	0.0011
(−2.3896)	(0.681)	(21.4965)	(−4.6368)			(2.5299)		(−0.4488)	(18.547)	(32.538)

Notes to Table: This table presents the estimated parameters (t-statistics are in parentheses) for the no-arbitrage term structure model with common and idiosyncratic risk factors using weekly WTI futures prices (Panel A) and Brent futures prices (Panel B), respectively. We use futures with 1-month to 6-month, 9-month, and 12-month maturities in the estimation. The sample period is from 2010:01 to 2016:02.

of log WTI and Brent futures prices are relatively higher for short maturities. The upward-sloped term structure implies that the crude oil futures markets are on average in contango.[18] The log WTI and Brent futures prices both exhibit negative skewness and excess kurtosis for all maturities. The autocorrelations indicate that log WTI and Brent futures prices are persistent for all maturities. The log WTI futures prices are slightly more persistent at the long-end of the term structure, while the log Brent futures prices are slightly more persistent at the short-end of the term structure.

4. Estimation results

In this section, we first report the estimated parameters. Subsequently, we discuss the time-series properties of the filtered WTI and Brent risk factors. Finally, we examine the model's in-sample performance for WTI and Brent futures.

4.1. Parameter estimates

Table 2 reports the estimated parameters using the term structure of WTI futures prices (Panel A) and the term structure of Brent futures prices (Panel B). The table also reports the corresponding t-statistics computed using the maximum likelihood standard errors. All diagonal terms in the feedback matrices K_1^P and K_1^Q are statistically significant. The off-diagonal terms are generally small and statistically insignificant except for the $(2,3)$ entries in K_1^P and K_1^Q for WTI and the $(1,3)$ entry in K_1^P for Brent. These results suggest that the second WTI idiosyncratic factor significantly predicts the first WTI idiosyncratic factor with a negative sign and the second Brent idiosyncratic factor significantly predicts the common factor with a positive sign. All elements in δ_1 are statistically significant, suggesting that both common and idiosyncratic factors have a significant effect on the log spot price of WTI and Brent.

The common factor is highly persistent under both P- and Q-measures for the WTI and Brent futures markets, which implies that the common factor should have a long-run effect on the price structure between WTI and Brent. The idiosyncratic factors are

[18] Following the literature on commodities, e.g., Litzenberger and Rabinowitz (1995) and Trolle and Schwartz (2009), the futures market is in backwardation if the discounted futures prices are below the current spot price, while the futures market is in contango if the discounted futures prices are above the current spot price.

Table 3
Unconditional correlations between model-implied idiosyncratic factors and term structure factors.

Term structure factors	Idiosyncratic factors			
	1st WTI	2nd WTI	1st Brent	2nd Brent
WTI slope (Basis)	−0.42***	−0.06	0.22***	0.02
Brent slope (Basis)	0.00	0.12**	0.33***	−0.07
WTI curvature (Relative basis)	0.07	0.55***	0.11*	−0.08
Brent curvature (Relative basis)	−0.06	−0.01	−0.12**	0.23***
WTI basis-momentum	−0.20***	−0.46***	0.05	0.08
Brent basis-momentum	−0.25***	0.02	0.24***	−0.23***

Notes to Table: This table presents the unconditional contemporaneous correlations between the model-implied idiosyncratic factors and the term structure factors for both WTI and Brent markets. Following Boons and Prado (2019), we define slope or basis using the ratio of the first- and second-nearby futures prices minus one. Curvature or relative basis is computed as the difference between the basis, which is based on the first- and second-nearby futures prices, and a longer-term basis that is based on the second- and third-nearby futures contracts. Basis-momentum is defined as the difference between momentum in a first- and second-nearby futures contracts, where momentum is computed based on the weekly excess return of the first-nearby futures contract during the past four weeks. The sample period is from 2010:01 to 2016:02.

Table 4
In-sample fits of log futures prices.

Panel A: RMSEs		
	Log WTI futures price	Log Brent futures price
1 month	2.09	8.81
2 month	20.39	11.47
3 month	5.52	6.61
4 month	12.65	10.14
5 month	12.77	9.21
6 month	8.24	20.83
9 month	5.95	25.56
12 month	15.70	11.43
Avg.	10.41	13.01
Panel B: MSEs		
	Log WTI futures price	Log Brent futures price
1 month	0.0004	0.0078
2 month	0.0416	0.0132
3 month	0.0030	0.0044
4 month	0.0160	0.0103
5 month	0.0163	0.0085
6 month	0.0068	0.0434
9 month	0.0035	0.0653
12 month	0.0246	0.0131
Avg.	0.0140	0.0207
Panel C: MAEs		
	Log WTI futures price	Log Brent futures price
1 month	8.88	6.38
2 month	16.86	7.29
3 month	4.91	5.80
4 month	6.54	9.27
5 month	8.43	6.25
6 month	7.05	13.50
9 month	4.13	18.22
12 month	9.69	12.03
Avg.	8.31	9.84

Notes to Table: This table presents the in-sample RMSEs (Panel A), MSEs (Panel B) and MAEs (Panel C) of the log futures prices for 1-month to 6-month, 9-month, and 12-month maturities. We estimate the no-arbitrage term structure model with common and idiosyncratic factors using weekly WTI prices and Brent prices, respectively. The last row in the table shows the averages across all maturities. All numbers are reported in basis points. The sample period is from 2010:01 to 2016:02.

less persistent than the common factor under P-measure for both WTI and Brent. The first WTI factor is more persistent than the second one under both P- and Q-measures. But the first Brent factor is less persistent than the second one under both measures. Another interesting observation is that all three state variables are highly persistent under Q-measure for Brent futures. Since risk premia in no-arbitrage models are intimately connected to the difference between the physical and risk-neutral drifts, the difference

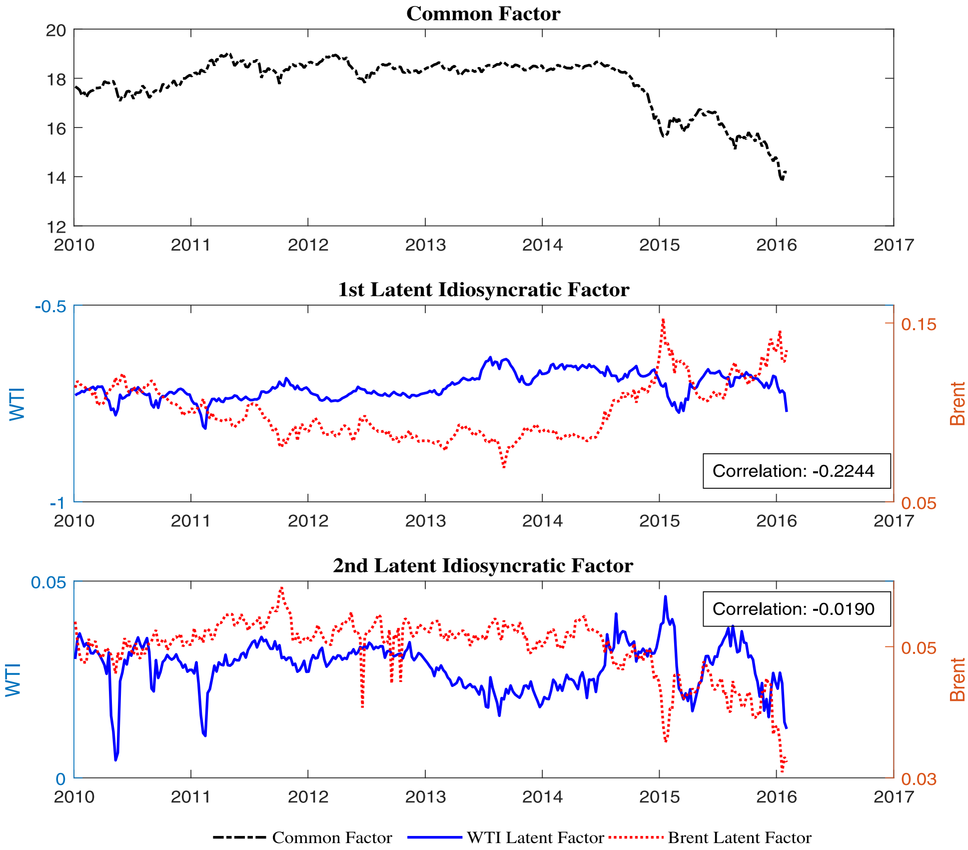

Fig. 2. Model-implied factors. Notes to Figure: This figure plots the common factor and the latent idiosyncratic factors implied from the estimations using WTI futures prices and Brent futures prices, respectively. The dash-dotted line (black) in the top panel represents the common factor of WTI and Brent. In the other two panels, the solid line (blue) represents the WTI factors, and the dotted line (red) represents the Brent factors. The numbers in the text-box show the unconditional correlation between the WTI and Brent factors. The sample period is from 2010:01 to 2016:02. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

in persistence under the two measures is informative of time-varying risk premia embedded in the crude oil futures, which we take on in-depth in Section (5.2).

4.2. Latent idiosyncratic risk factors

Our model incorporates one common factor and two latent idiosyncratic factors for each crude oil futures market. Fig. 2 plots the time series of the filtered WTI and Brent factors together with the common factor. The top panel plots the common factor, which is the first principal component of the observed WTI and Brent log futures prices. The middle and bottom panels plot the first and second idiosyncratic factors, respectively. The unconditional correlations between the filtered WTI and Brent factors are negative: −0.2244 for the first latent factor and −0.0190 for the second latent factor. The idiosyncratic risk factors are much less correlated between the two markets than the common risk factor, especially for the second idiosyncratic factor. After control for the level of the world's crude oil futures prices, we filter the risk factors that are in general weakly correlated between WTI and Brent futures markets.

Fig. 3 plots the term structure of the factor loadings for the two markets. The common factor by design can be well characterized as the level of the futures prices in both markets. The first idiosyncratic factor can be interpreted as capturing the slope of the term structure of futures prices in the corresponding market, and the second idiosyncratic factor exhibits a curvature effect, especially in the Brent futures market. To further assess the relevance of the idiosyncratic factors with the shape of the term structure of futures prices, we follow the literature (e.g., Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006; Szymanowska et al., 2014; Boons and Prado, 2019; Gu et al., 2021) to construct the slope and curvature factors of the futures curve. We define the slope (basis) using the ratio of the first- and second-nearby futures prices in our sample minus one, and the curvature (relative basis) as the difference between the short-term basis, which is calculated using the first- and second-nearby futures prices, and a longer-term basis that uses the second- and third-nearby futures contracts in our sample. We also consider the basis-momentum of the futures curve as proposed by Boons and Prado (2019), which has been shown to be related to the slope and curvature of the

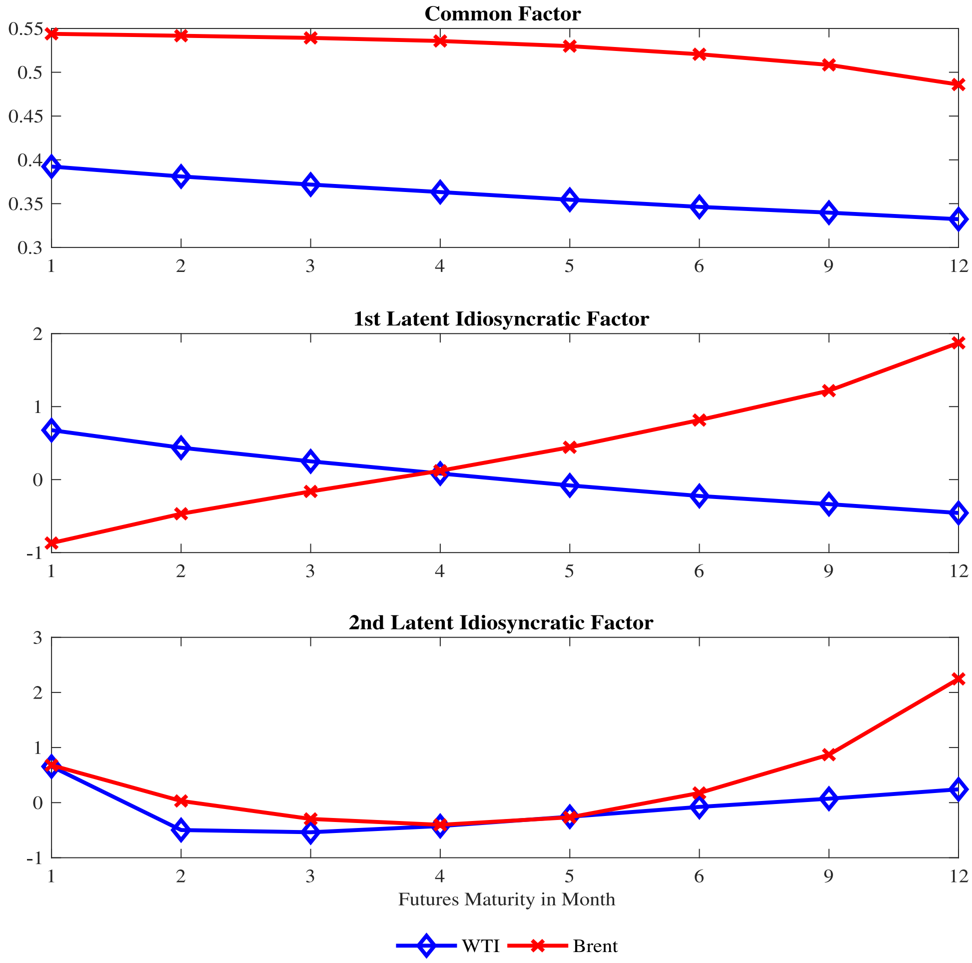

Fig. 3. The loadings of factors. Notes to Figure: This figure plots the loadings of the common factor and the latent idiosyncratic factors implied from the estimations using WTI futures prices and Brent futures prices, respectively. The diamond line (blue) represents the loadings of the WTI factors, and the cross line (red) represents the loadings of the Brent factors. We plot the results for futures with 1-month to 6-month, 9-month, and 12-month maturities. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

futures term structure. We compute basis-momentum as the difference between momentum in the first- and second-nearby futures contracts, where momentum is computed based on the weekly excess return of the first-nearby futures contract during the past four weeks.

Table 3 reports the unconditional correlations between the latent idiosyncratic factors and the measures capturing the shape of the futures curve for the two markets. We find that the WTI (Brent) slope factor or basis has the largest and significant correlation with the first WTI (Brent) factor. Specifically, the correlation between the WTI (Brent) basis and the first WTI (Brent) factor is −0.42 (0.33). It is not surprising that the signs of the correlations for the two markets are different. Recall that the first WTI and Brent factors are negatively correlated with each other as shown in Fig. 2. The curvature factor correlates the most with the second idiosyncratic factor in its corresponding market. For example, the correlation between the WTI (Brent) curvature factor and the second WTI (Brent) factor is 0.55 (0.23). Additionally, both idiosyncratic factors are significantly correlated with the basis-momentum in the corresponding oil market, implying that the idiosyncratic factors may capture both the slope and curvature effect of the futures term structure.

4.3. Model performance

While our main objective is to study the risk premia structure in the two markets, it is also important to verify that our model adequately captures the stylized facts in the data. We therefore examine the model's performance in this section. Panel A of Table 4 reports the in-sample root mean squared errors (RMSEs) of the term structure of log futures prices for WTI and Brent. The average RMSE across eight maturities is about 10 basis points for WTI and 13 basis points for Brent. The average log WTI and Brent futures price across maturities is about \$4.4 in our sample. Therefore the in-sample pricing error on average is about 0.02% (0.001/4.4)

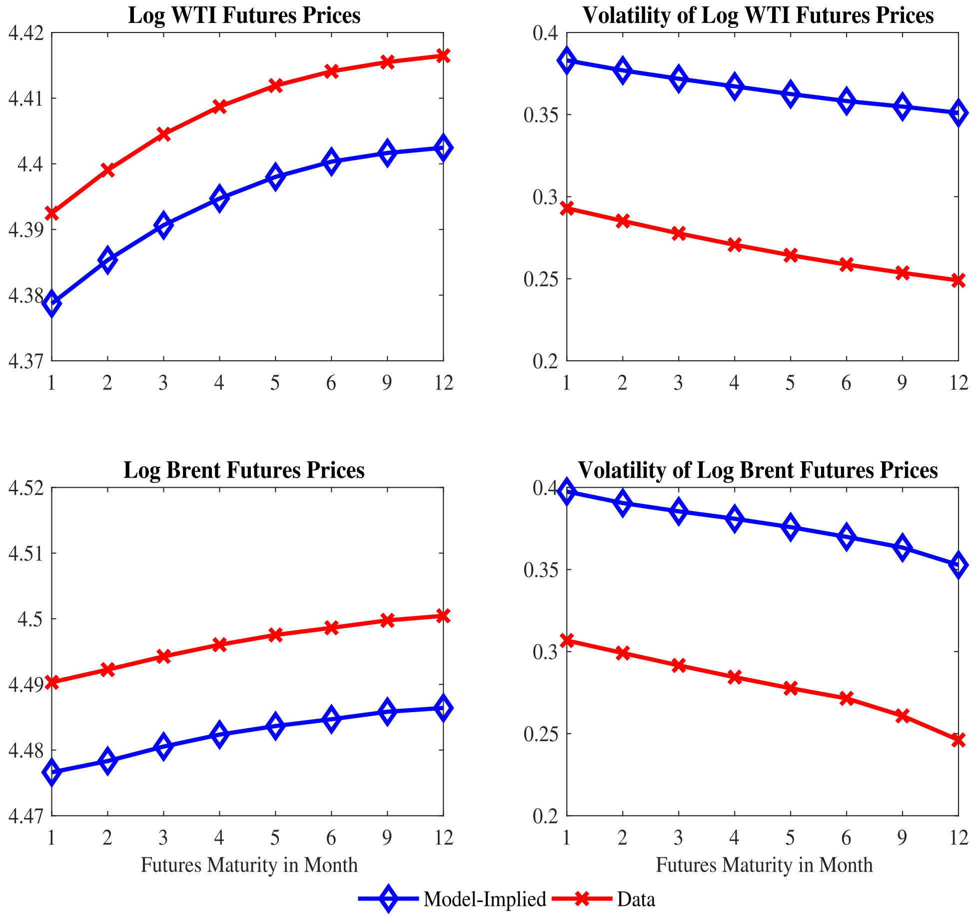

Fig. 4. Term structure of model-implied futures prices and volatilities. Notes to Figure: This figure plots the model-implied log futures prices and the price volatilities together with the counterparts from the data. The diamond line (blue) represents the model-implied expected price and volatility, and the cross line (red) represents the mean and volatility of the data. The top two panels are for WTI, and the bottom two panels are for Brent. We plot the results for futures with 1-month to 6-month, 9-month and 12-month maturities. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

for WTI and 0.03% (0.0013/4.4) for Brent. For comparison purposes, we also include the in-sample mean squared errors (MSEs) and the mean absolute errors (MAEs) in Panels B and C of Table 4. The results based on the MSEs are consistent with the results based on the RMSEs. The average MAE across eight maturities is about 8 basis points for WTI and 10 basis points for Brent. The results are similar in magnitude to the results based on the RMSEs. Fig. 4 shows that the model with common and idiosyncratic factors is capable of replicating both the upward pattern of the log futures prices as well as the downward slope in price volatilities across maturities. Furthermore, we investigate the out-of-sample performance of the model. We compute the out-of-sample root mean squared forecasting error (RMSFE) based on the observed and the model-predicted futures prices at different forecast horizons for all eight maturities used in our estimation and report the results in Table IA1 in the internet appendix. We find that the out-of-sample fit of the model is similar for WTI and Brent futures at the short forecast horizons (1-, 4-, and 13-week). While at long forecast horizons (26-, 39-, and 52-week), the model leads to better forecasts for WTI futures. The model's forecasting performance in general is better at short forecast horizons than at long forecast horizons, especially for Brent futures.

5. Economic implications

In this section, we discuss the economic implications of the model with common and idiosyncratic factors for WTI and Brent futures. We first investigate the model-implied spot price expectations and risk premia and, in particular, their impacts on the term structure of price spreads between WTI and Brent futures. Next, we present the model-implied risk prices of common and idiosyncratic factors and also the economic interpretation of the risk prices in the two markets. Finally, we document the relative contributions of the common and idiosyncratic factors to the forecast variance of the futures prices.

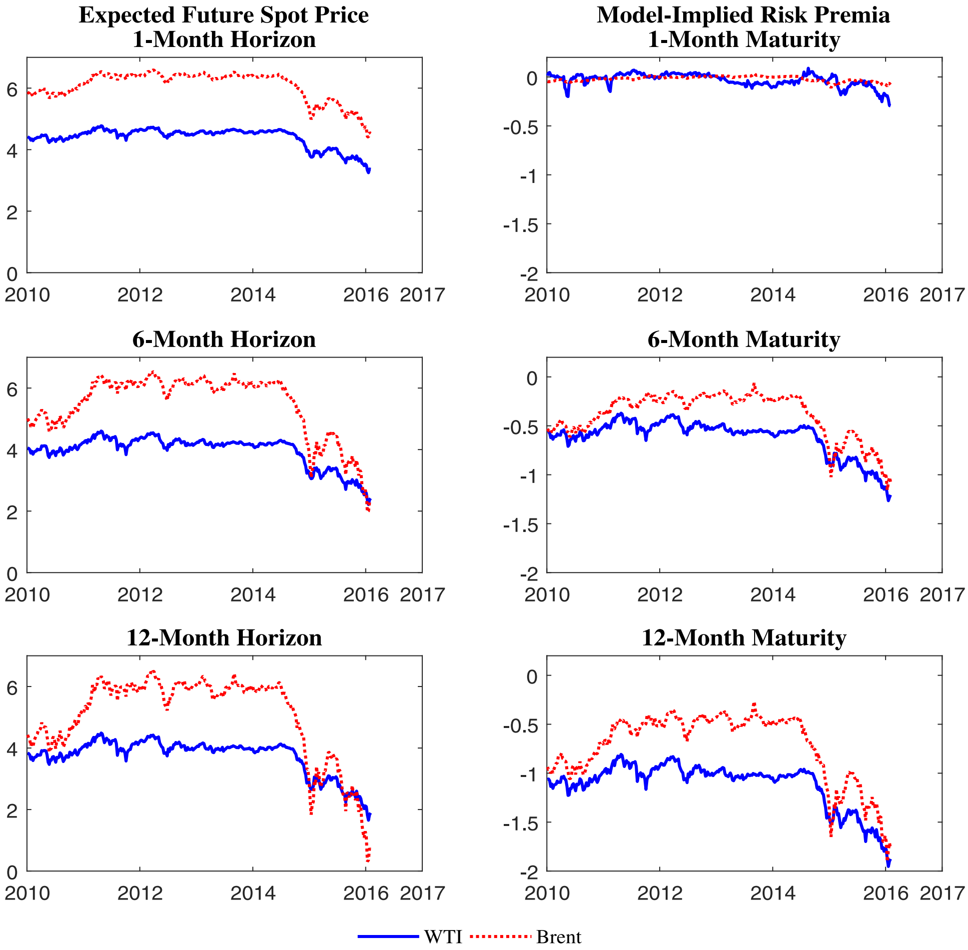

Fig. 5. Expected future spot price and risk premia. Notes to Figure: This figure plots the expected future log spot price implied from the estimations using WTI futures prices and Brent futures prices, respectively, on the left panels. We plot the expected spot price in 1-month, 6-month, and 12-month horizons. In the right panels, we plot the model-implied risk premia for WTI futures prices and Brent futures prices, respectively. The risk premium is defined as the difference between the model-implied log futures price and the log futures price if there was no compensation for risk. We plot the risk premia for futures with 1-month, 6-month, and 12-month maturities. The solid line (blue) is for the WTI futures price, and the dotted line (red) is for the Brent futures price. The sample period is from 2010:01 to 2016:02. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5.1. Expectations and risk premia

The left panel of Fig. 5 plots the time series of the model-implied expectations of the future spot price at 1-month, 6-month, and 12-month horizons for the two markets.[19] Note that the expectations in the two markets are highly correlated with each other. The unconditional correlation of the expectations in the two markets is 0.96 on average across horizons. The expectations of future spot prices also closely comove with the term structure of futures prices in both markets, as shown in Fig. 1. Consistent with the movement in the whole term structure of WTI and Brent futures prices, the expectations of future spot prices in both markets begin to decrease at the end of 2014.

We observe that the expected log spot Brent prices are almost always higher than the expected log spot WTI prices, except for the contracts with longer maturities (e.g., 6-month and 12-month) at the very end of our sample. On average, over time and across all maturities, the expected log spot Brent price is about 1.53 higher than the expected log spot WTI price. After accounting for the log transform, the average difference in the expected spot price between the two markets is about \$5 in our sample.

The right panel of Fig. 5 plots the time series of the model-implied risk premia for the WTI and Brent futures with 1-month, 6-month, and 12-month maturities.[20] For the nearest maturity, the risk premia of WTI and Brent futures are on average very close to zero (−0.03 for WTI and −0.02 for Brent). However, the WTI risk premia are more volatile than the Brent risk premia. The risk

[19] The results for all eight different horizons are reported in the internet appendix, Figure IA3.

[20] The results for all eight different maturities are reported in the internet appendix, Figure IA4.

premia in both futures markets decrease with maturity. For contracts with 6-month and 12-month maturities, both WTI and Brent risk premia are uniformly negative in our sample. More importantly, we find that the risk premia of WTI futures are almost always smaller than that of Brent futures for these contracts. The average risk premium for 12-month WTI futures and Brent futures are −1.12 and −0.73, respectively. On average, over time and across all maturities, the WTI risk premium is about −0.45, and the Brent risk premium is about −0.29. Furthermore, the risk premia in the two markets become to share more dynamics at long maturity than short maturity. For the nearest maturity, the unconditional correlation of the model-implied risk premia between the two markets is 0.35. While for the 12-month futures contract, the unconditional correlation of the risk premia between the two markets is 0.92.

The theory of normal backwardation postulated that short hedgers outnumber long hedgers and the long speculators get a positive risk premium for taking the price risk (Keynes, 1923, 1930). Therefore, if the commodity futures market is in backwardation, there should be a positive risk premium, while when the market is in contango, we expect a negative risk premium. Recall that in Table 1, the upward-sloped term structure shows that the crude oil futures markets during our sample should be, on average, in contango. This observation implies that the risk premium in the two crude oil markets should be negative. Our model estimation verifies that both markets exhibit negative and time-varying risk premia during the sample. Besides, in Table 1, we observe that the difference between the 12-month and 1-month futures prices for WTI is about two times larger than that for Brent, suggesting a much lower risk premium in the WTI market. This hypothesis is confirmed by the small and mostly negative WTI risk premia implied by our model.

Financialization has been an important and ongoing topic in the commodity markets. The "so-called" financialization in the commodity markets refers to the rise of financial trading activities in the commodity markets, especially index-based investments, since the mid-2000s. Commodity futures and other commodity-based financial instruments have become popular investment vehicles among financial investors, such as hedge funds, index funds, as well as individual investors (e.g., Tang and Xiong, 2012; Henderson et al., 2015; Basak and Pavlova, 2016). Financialization in crude oil futures markets can explain the negative risk premia for WTI and Brent reported in Fig. 5. First, the literature regarding the financialization in the commodity markets documents that the increasing number of financial speculators in the commodity markets is related to the change of the signs in risk premium since financialization. Financial traders who invest in the commodity markets for portfolio diversification purposes may require a lower or even negative risk premium (see, for example, Hamilton and Wu, 2014, 2015; Li, 2018; Singleton, 2014; Brunetti et al., 2016).

Second, Fig. 5 also shows that the average risk premium in the WTI futures market is larger in magnitude (more negative) than that in the Brent futures market, suggesting larger net long speculative positions in the WTI market. The higher buying pressure for WTI may result from the rapid growth of commodity-index investments, such as ETFs, taking long-only positions in a basket of commodity futures with preset weights with the weight on WTI being typically larger than that on Brent.[21] It is interesting to note that Tudor and Anghel (2021) investigate the overall efficiency of WTI and Brent futures markets using technical analysis and conclude that the financialization process within the WTI market is more intense than that for Brent.

Following Gorton and Rouwenhorst (2006), futures price can be inferred by the difference between the expected future spot price and risk premium. Therefore the difference in futures prices depends on the differences in the two components. We observe that both expected future spot price and risk premium are in general higher for Brent than for WTI. More importantly, the differences in both expectations and risk premia seem to be high from 2011 to 2013, during which we observe a more pronounced deviation between WTI and Brent futures prices, especially for longer maturities. However, the differences in expectations on average are larger than those in risk premia, suggesting that the price spreads between Brent and WTI futures are mainly driven by the expectations in the spot market. Our results are consistent with the literature showing that WTI spot price is lower than Brent spot price due to US oil supply shock (e.g., Luong et al., 2019; Caporin et al., 2019). Furthermore, our model implies that the gradual decline of futures prices since the end of 2014 in both markets can be mainly attributed to the decreases in expectations of the future spot price.

Moreover, we find that the difference between Brent and WTI expected future spot prices on average decreases with maturity, while the difference between Brent and WTI risk premia on average increases with maturity. For example, the largest average difference in expectations between the two markets (1.68) is at the nearest maturity of the term structure, whereas the largest average difference in risk premia (0.39) is at the longest maturity. This result seems to suggest that the role of the two components in explaining the Brent-WTI spreads are not the same across the term structure.

To further investigate the contribution of expectations and risk premia to the term structure of price spreads between the two futures markets, we present the unconditional correlations between the Brent-WTI futures spread and the differences in the two components (the difference in the expectation of future spot prices and the difference in the risk premia) for all maturities in Table 5. We find that, on average, the unconditional correlation between the Brent-WTI spread and the difference in expectations of future spot price is larger than that between the Brent-WTI spread and the difference in risk premia. For the differences in expectations, the unconditional correlations decrease with maturity. While for the differences in risk premia, the unconditional correlations increase with maturity except for the longest maturity. More importantly, we observe that the differences in expectations comove more with the price spread than the differences in risk premia at very short maturities. While the differences in risk premia are more correlated with the price spread than the differences in expectations at longer maturities (9-month and 12-month). We therefore further conclude that the expected future spot price plays a more important role in determining the term structure of Brent-WTI price spreads at the short-end of the term structure. The differences in risk premia become more important in explaining the price spreads at longer maturities. Our findings suggest that we need a time-varying term structure which reflects both risk premia and market's forecast of oil prices when modeling crude oil term structure.

[21] As of November 2020, the dollar-value weight of WTI in S&P GSCI is 25.31%, whereas Brent makes up 18.41% of the index. See https://www.spglobal.com/spdji/en/documents/indexnews/announcements/20201112-1255559/1255559_spgsci2021cpwindexannouncement.pdf.

Table 5
Unconditional correlations with Brent-WTI futures spread.

	Differences in expectations	Differences in risk premia
1 month	0.60	−0.09
2 month	0.54	−0.02
3 month	0.52	0.12
4 month	0.51	0.26
5 month	0.50	0.39
6 month	0.50	0.48
9 month	0.39	0.47
12 month	0.18	0.31
Avg. of Abs. value	0.47	0.27

Notes to Table: This table presents the unconditional correlations between Brent-WTI price spread and the differences in expectations of future log spot price and the differences in risk premia between the Brent and WTI markets, respectively. We use the estimates of Brent minus the estimates of WTI. The risk premium is defined as the difference between the model-implied log futures price and the log futures price if there was no compensation for risk. We report the results for futures with 1-month to 6-month, 9-month, and 12-month maturities. The last row in the table shows the averages of the absolute unconditional correlations across all maturities. The sample period is from 2010:01 to 2016:02.

5.2. Risk prices of factors

We now study the risk-price structure of the common and idiosyncratic factors in the two markets. The differences between the P- and Q-measures, or the implied characterization of market prices of risks, are reported in Table 2. The absence of arbitrage requires the consistency of the time-series dynamics of futures prices with their cross-sectional behavior, allowing for a risk adjustment. The risk-price parameters λ_0 and λ_1 determine this risk adjustment and the behavior of the risk premia. As specified in Eq. (9), λ_t captures the extra expected returns required by investors for each extra unit in the standard deviation of the factors. For the common and idiosyncratic risk factors, if the corresponding element in λ_0 or any element in the corresponding row of λ_1 is statistically significant, the exposure to this risk factor is priced. We observe that both common and idiosyncratic risks are priced in the WTI and Brent markets, implying that investors in both markets worry about (and require risk compensation for) not only the shocks to the common trend of the world crude oil futures market but also the shocks to the idiosyncratic factors. To explain futures prices and risk premia of individual crude oil futures, we need state variables extracted from both the overall world crude oil market and the individual crude oil (e.g., WTI- or Brent-specific) market.

The first row of λ_1 governs the market prices of common risk. All entries in the first row of λ_1 are statistically significant for the two markets, suggesting that not only the common factor but also the idiosyncratic factors are important for forecasting the risk premia associated with exposures to the common factor (level of the observed WTI and Brent log futures prices). The second and third rows of λ_1 determine the time-varying premia for idiosyncratic risks. We observe that only the idiosyncratic factors have statistically significant effects on the associated risk premia in both markets.

Fig. 6 plots the time-varying prices of risks λ_t for the two markets, as in (9). The top panel is for the common risk. We find that the common risk premia in the two markets are positively correlated. The unconditional correlation between the common risk premia in the two markets is 0.1609 during our sample. However, the Brent common risk premia are more volatile than the WTI common risk premia. We also report the average risk prices $\lambda_0 + \lambda_1 \overline{Z}$, where $\overline{Z}$ vector is the average value of the risk factors over the sample, in Table 2. $\lambda_0 + \lambda_1 \overline{Z}$ is essentially the average of the time series in Fig. 6. The average common risk premium of WTI (−0.0986) is slightly larger than that of Brent (−0.1018). Both WTI and Brent have negative common risk premia on average through the sample, consistent with the summary statistics documented in Table 1 that both markets are on average in contango.

The middle and bottom panels of Fig. 6 plot the prices of risks for the two idiosyncratic factors of WTI and Brent. The unconditional correlation between the first idiosyncratic risk price of WTI and that of Brent is negative (−0.2082), while the unconditional correlation between the second idiosyncratic risk price of WTI and that of Brent is very small and positive (0.0618). As shown in Table 2, the average risk price for the first WTI factor is positive (0.3553), while the average risk price for the first Brent factor is negative (−0.2045). The second WTI risk prices are negative most of the time. The average risk premium of Brent futures associated with exposures to the second idiosyncratic factor (0.1811) is much higher than that of WTI (−0.1841). These findings suggest that the risk-price structures of factors in the two markets are quite different, especially for the idiosyncratic factors.

We further explore what information drives the variation in risk prices for WTI and Brent using correlation analysis. To this end, we collect the data on a set of observables. Specifically, we obtain the short rate (3-month constant maturity Treasury rate), inflation (annual growth rate of CPI in the US and Euro area), industrial production growth in the US from the Federal Reserve Economic Data.[22] We download the 12-month ahead macroeconomic uncertainty index constructed by Jurado et al. (hereafter, JLN, 2015) from the author's website.[23][24] We also obtain the US crude commercial stock (the weekly US ending inventory excluding strategic

[22] https://fred.stlouisfed.org.

[23] https://www.sydneyludvigson.com/macro-and-financial-uncertainty-indexes.

[24] We acknowledge that several alternative uncertainty measures have been proposed in the literature, such as the stock market implied volatility of Bloom (2009), the survey-based uncertainty measure of Bachmann et al. (2013), and news-based uncertainty indices by Baker et al. (2016) and Husted et al. (2020).

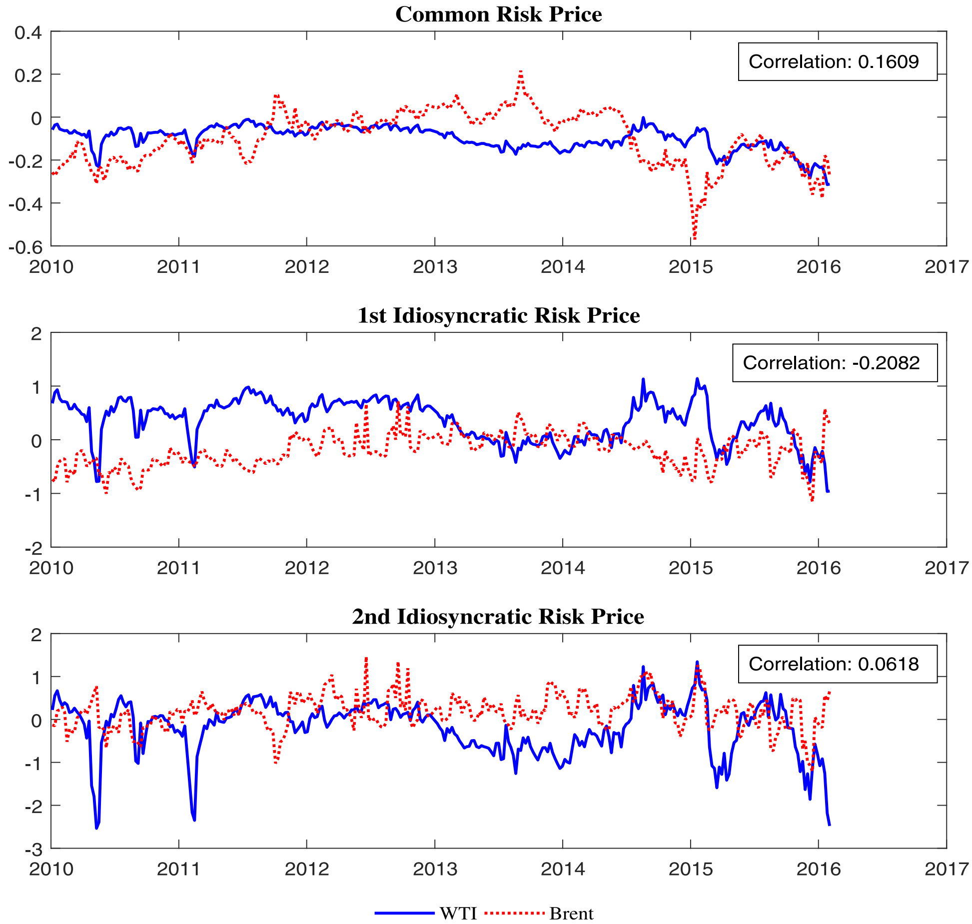

Fig. 6. The risk prices of factors. Notes to Figure: This figure plots the time-varying price of risk $\lambda_0 + \lambda_1 Z_t$ implied from the estimations using WTI futures prices and Brent futures prices, respectively. We plot the results for both the common factor and the latent idiosyncratic factors. The solid line (blue) is for the estimation using WTI futures prices, and the dotted line (red) is for the estimation using Brent futures prices. The numbers in the text-box show the unconditional correlation between the risk prices of factors implied from the estimations using WTI futures prices and Brent futures prices. The sample period is from 2010:01 to 2016:02. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

petroleum reserve), the US pipeline added capacity (measured in barrels per day), and the crude oil production in UK and Norway (including lease condensate production) from the EIA website.[25] Finally, we use Producer/Merchant/Processor/User position data to proxy for commercial traders' net hedging position and obtain the data from the disaggregated futures-only Commitments of Traders (COT) reports.[26] The US crude commercial stock and traders' position data is available at a weekly frequency. The US pipeline added capacity is quarterly, and all the other observables are available at a monthly frequency.

Table 6 presents the unconditional correlations between these observables and the risk prices of factors for the two markets. Panel A reports the correlations with the risk prices of the common factor in the two markets. We find that the US crude commercial stock, inflation in the US and Euro area, and economic uncertainty are all significantly correlated with the risk prices of the common factor in the two markets. Specifically, the common risk prices are all positively correlated with inflation, both in the US and Euro area.

We adopt the JLN measure for several reasons. First, the index is constructed from the common volatility of the unforecastable components of a comprehensive set of 279 macroeconomic variables, and thereby capturing whether the economy has become more or less forecastable. In contrast, the measures based on stock market volatilities reflect whether the economy has become more or less volatile, and tend to have a relatively narrow focus on financial markets. Second, Lahiri and Sheng (2010) argue that survey-based uncertainty measures are more likely to reflect heterogeneity among survey respondents rather than uncertainty. Third, as pointed out by JLN, news-based uncertainty measures, though free of model estimation, tend to suffer from excess volatility and lower persistence. In unreported results, we calculate the unconditional correlations between our model-implied common risk factor and the alternative uncertainty measures and find the results are qualitatively similar.

[25] https://www.eia.gov.

[26] For WTI, we obtain the COT reports at: https://www.cftc.gov/MarketReports/CommitmentsofTraders/index.htm; For Brent, they are at: https://www.theice.com/marketdata/reports/122. The COT reports for Brent crude oil became available in January 2011. To be consistent, we use the same sample for the WTI COT reports.

Table 6
Unconditional correlations between factor risk prices and observables.

Panel A: Risk prices of common factor		
Variables	WTI	Brent
US crude commercial stock	−0.68***	−0.34***
US inflation	0.57***	0.36***
Inflation for Euro area	0.62***	0.46***
US industrial production growth	0.60***	0.07
Economic uncertainty	−0.50***	−0.55***
US pipeline added capacity	0.26	0.53**
WTI commercial net position	0.13**	0.57***
Brent commercial net position	0.28***	0.50***
WTI volatility	−0.46***	−0.57***
Brent volatility	−0.47***	−0.59***

Panel B: Risk prices of idiosyncratic factors				
Variables	1st WTI	2nd WTI	1st Brent	2nd Brent
3-month Treasury rate	−0.39***	−0.52***	−0.08	−0.09
UK crude production	0.03	−0.14	−0.39***	−0.20*
Norway crude production	0.16	0.01	−0.41***	−0.10
WTI slope (Basis)	−0.11**	−0.17***	−0.35***	−0.21***
Brent slope (Basis)	−0.11*	0.01	−0.44***	−0.19***
WTI curvature (Relative basis)	0.50***	0.56***	−0.03	0.08
Brent curvature (Relative basis)	0.02	−0.01	−0.02	−0.19***
WTI basis-momentum	−0.35***	−0.46***	−0.05	−0.04
Brent basis-momentum	0.08	0.00	−0.05	0.21***

Notes to Table: This table presents the unconditional contemporaneous correlations between the observed variables and the risk prices of factors $\lambda_0 + \lambda_1 Z_t$ for both WTI and Brent markets. Panel A is for the risk prices of common factor, and Panel B is for the risk prices of idiosyncratic factors of WTI and Brent. The macro data is obtained from the Federal Reserve Economic Data. We consider 3-month constant maturity Treasury rate, inflation (annual growth rate of CPI in the US), inflation based on the consumer prices for the Euro area, industrial production growth in the US, and a factor-based estimate of economic uncertainty developed by Jurado et al. (2015). We also obtain the US crude commercial stock, US pipeline added capacity, and the crude oil production in UK and Norway from the EIA website. We measure hedging pressure using commercial traders' net positions as in Prokopczuk and Wu (2013). We use Producer/Merchant/Processor/User position data to proxy for commercial traders' positions and obtain them from the disaggregated futures-only Commitments of Traders (COT) reports. Oil volatility is measured by the historical volatility of the nearest maturity crude oil futures returns. We compute monthly historical volatility using daily futures returns. Following Boons and Prado (2019), we define slope or basis using the ratio of the first- and second-nearby futures prices minus one. Curvature or relative basis is computed as the difference between the basis, which is based on the first- and second-nearby futures prices, and a longer-term basis that is based on the second- and third-nearby futures contracts. Basis-momentum is defined as the difference between momentum in the first- and second-nearby futures contracts, where momentum is computed based on the weekly excess return of the first-nearby futures contract during the past four weeks. The US crude commercial stock, traders' position data, basis, curvature, and basis-momentum are available at a weekly frequency, and the US pipeline added capacity is available at a quarterly frequency. All the other variables are available at a monthly frequency. The COT reports for Brent crude oil became available in January 2011. To be consistent, we use the same sample for the WTI COT reports. For all the other variables, the sample period is from 2010:01 to 2016:02.

These findings are consistent with the large literature developed on the relationship between oil prices and the macroeconomy.[27] Furthermore, the common risk prices in both markets are negatively correlated with the economic uncertainty index. It suggests that the common crude oil risk premia will be lower in a highly uncertain market state. This observation is consistent with Dew-Becker et al. (2020) that portfolios exposed to the realization of large shocks to fundamentals have historically earned negative premia. The industrial production growth in the US has a significantly positive correspondence with the WTI common risk price. This is as expected given that crude oil is a fundamental resource for economic growth.

The literature has documented that the US experienced a strong positive supply shock due to technological advancements such as hydraulic fracturing and horizontal drilling; at the same time, it also experienced an inventory buildup given the infrastructure development in recent years (e.g., Liu et al., 2015; Scheitrum et al., 2018; Caporin et al., 2019). We measure the US crude oil production using US crude commercial stock and find that its correlations with the common risk prices of WTI and Brent are both significantly negative (−0.68 and −0.34), implying the world crude oil prices are closely linked to the US crude oil supplies. The higher the US crude oil production, the lower the crude oil risk premia. We use the US pipeline added capacities to measure the crude oil infrastructure capacity and find that it is positively correlated with the common factor of WTI and Brent. The correlation coefficient for US pipeline added capacity is especially large (0.53) for the Brent crude oil common risk price, suggesting that the crude oil technology development in the US in recent years has a large positive impact on world oil prices.

Following Prokopczuk and Wu (2013), we measure hedging pressure using commercial traders' net positions, $(L - S)/(L + S)$, where $L(S)$ represents traders' long (short) position, respectively. In Panel A of Table 6, we see that both WTI and Brent commercial

[27] See, for example, Hamilton (2008), Barsky and Kilian (2004), and Gronwald (2008).

Table 7
Variance decompositions (in percentages).

Panel A: Log WTI futures price					
Maturity	Forecast horizon	Common factor	1st latent	2nd latent	Total latent
1 month					
	13 weeks	99.73	0.24	0.03	0.27
	26 weeks	98.65	0.47	0.88	1.35
	39 weeks	95.47	0.70	3.83	4.53
	52 weeks	91.09	0.85	8.06	8.91
2 month					
	13 weeks	99.75	0.16	0.09	0.25
	26 weeks	98.35	0.48	1.17	1.65
	39 weeks	94.87	0.71	4.43	5.13
	52 weeks	90.32	0.86	8.82	9.68
3 month					
	13 weeks	99.71	0.20	0.10	0.29
	26 weeks	97.88	0.53	1.58	2.12
	39 weeks	94.00	0.76	5.24	6.00
	52 weeks	89.24	0.90	9.86	10.76
4 month					
	13 weeks	99.58	0.27	0.16	0.42
	26 weeks	97.27	0.60	2.12	2.73
	39 weeks	93.00	0.82	6.18	7.00
	52 weeks	88.02	0.96	11.02	11.98
5 month					
	13 weeks	99.31	0.36	0.33	0.69
	26 weeks	96.45	0.69	2.86	3.55
	39 weeks	91.75	0.90	7.35	8.25
	52 weeks	86.59	1.02	12.39	13.41
6 month					
	13 weeks	98.90	0.48	0.61	1.10
	26 weeks	95.46	0.79	3.74	4.54
	39 weeks	90.39	0.98	8.63	9.61
	52 weeks	85.07	1.09	13.84	14.93
9 month					
	13 weeks	98.45	0.60	0.95	1.55
	26 weeks	94.51	0.89	4.60	5.49
	39 weeks	89.14	1.06	9.81	10.86
	52 weeks	83.71	1.16	15.13	16.29
12 month					
	13 weeks	97.79	0.76	1.45	2.21
	26 weeks	93.25	1.01	5.74	6.75
	39 weeks	87.55	1.15	11.30	12.45
	52 weeks	82.04	1.23	16.72	17.96

(*continued on next page*)

traders' net hedging positions are positively correlated with their corresponding common risk prices, implying that higher net short hedging pressure from commercial traders leads to lower risk premia in both markets. This result may seem contradictory to the traditional view of the hedging pressure hypothesis, which posits that speculators should earn a positive risk premium by taking long positions in backwardated futures contracts on which commercial hedgers are net short (e.g., Keynes, 1930; Hicks, 1939; Hirshleifer, 1988). However, such a result is consistent with recent significant changes in the commodity futures markets due to financialization, particularly the rapid growth of index-based investing in commodity markets since the mid-2000s. These funds, by design, constantly take long-only positions in commodity futures and offer investors exposure to the alternative asset class and diversification benefits beyond traditional assets, such as equities and bonds. Therefore, they are willing to absorb the net short pressure from commercial traders without risk compensation. In addition, the buying pressure from commodity index funds may overwhelm the selling pressure from producers or even shift the risk reward from the long side to the short side of the futures contract, i.e., negative risk premium (e.g., Masters, 2008; Basak and Pavlova, 2016; Hamilton and Wu, 2014, 2015). As a result, risk premia do not necessarily initiate from the commercial traders (hedgers) but instead possibly from speculators. Thus, our finding of a positive correlation between hedging pressure and risk premium must be interpreted with caution in light of financialization. For example, Brunetti and Reiffen's (2014) model implies that the cost of hedging for commercial traders falls as aggregate commodity index traders' positions increase.

We compute monthly historical volatility for the nearest maturity futures contract using daily futures returns. The monthly historical volatilities for Brent and WTI are negatively correlated with their corresponding common risk prices, consistent with the results of JLN (2015) economic uncertainty measure reported in Table 6 since volatilities are often regarded as uncertainty measures (Bloom, 2009).

The correlations of the observables with the idiosyncratic risk prices in the two markets are reported in Panel B of Table 6. We find that the 3 -month Treasury rate significantly corresponds to the WTI risk prices but not the Brent risk prices. The negative signs

Table 7 (*continued*).

Panel B: Log Brent futures price					
Maturity	Forecast horizon	Common factor	1st latent	2nd latent	Total latent
1 month					
	13 weeks	91.71	3.15	5.14	8.29
	26 weeks	87.53	2.78	9.69	12.47
	39 weeks	84.75	2.53	12.72	15.25
	52 weeks	83.06	2.38	14.56	16.94
2 month					
	13 weeks	94.51	3.02	2.47	5.49
	26 weeks	90.33	2.84	6.83	9.67
	39 weeks	87.33	2.60	10.07	12.67
	52 weeks	85.45	2.46	12.10	14.55
3 month					
	13 weeks	95.55	2.84	1.60	4.45
	26 weeks	91.53	2.80	5.67	8.47
	39 weeks	88.48	2.59	8.92	11.52
	52 weeks	86.53	2.45	11.02	13.47
4 month					
	13 weeks	96.02	2.62	1.36	3.98
	26 weeks	92.03	2.70	5.27	7.97
	39 weeks	88.98	2.52	8.51	11.02
	52 weeks	87.00	2.39	10.61	13.00
5 month					
	13 weeks	96.14	2.36	1.51	3.86
	26 weeks	91.99	2.51	5.50	8.01
	39 weeks	88.91	2.37	8.73	11.09
	52 weeks	86.94	2.25	10.81	13.06
6 month					
	13 weeks	95.36	2.16	2.48	4.64
	26 weeks	90.94	2.31	6.75	9.06
	39 weeks	87.88	2.19	9.93	12.12
	52 weeks	85.98	2.09	11.93	14.02
9 month					
	13 weeks	92.60	2.26	5.15	7.40
	26 weeks	88.22	2.24	9.54	11.78
	39 weeks	85.40	2.11	12.49	14.60
	52 weeks	83.71	2.01	14.28	16.29
12 month					
	13 weeks	82.82	3.35	13.84	17.18
	26 weeks	79.83	2.69	17.48	20.17
	39 weeks	78.04	2.42	19.53	21.96
	52 weeks	77.00	2.29	20.71	23.00

Notes to Table: This table presents the contributions of common and latent idiosyncratic factors to the forecast variance of futures prices for different maturities and forecast horizons. Panel A is for the WTI price, and Panel B is for the Brent price. All numbers are in percentages.

suggest that the lower the local borrowing rate, the higher consumption of crude oil, and the higher risk premia in the crude oil market. These findings are related to the studies on the monetary policy responses to oil price changes (e.g., Bernanke et al., 1997; Barsky and Kilian, 2001; Kilian, 2010; Kilian and Lewis, 2011). Moreover, the correlations between the productions near the North Sea and the risk price of the first Brent factor are significantly negative (−0.39 for UK and −0.41 for Norway), while the correlations with the risk price of the first WTI factor are not significant. Together with the results in Panel A that the US commercial stock is significantly linked to the common risk prices in both markets, these findings further confirm that the WTI crude market has a more extensive impact on the global crude oil price than Brent. This observation is consistent with the literature documenting the leading effect of WTI over Brent (e.g., Klein, 2018; Liu et al., 2018).

We also report the correlations of the idiosyncratic risk prices with the term structure factors, such as slope (basis), curvature (relative basis), and basis momentum, in Panel B of Table 6. The WTI slope significantly correlates with the risk prices of the idiosyncratic factors in the two markets, while the Brent basis correspondences more with the idiosyncratic risk prices of Brent. The theory of storage states that the slope or basis, adjusted for the carrying costs, depends upon the stocks held in inventory. A positive slope corresponds to higher futures prices and implies high stocks in the commodity; therefore, it means a contango futures market and lower risk premia (e.g., Working, 1933, 1949; Kaldor, 1939; Kilian and Murphy, 2014). This is in line with our finding that the correlations between the slope and the risk prices are significantly negative. Gu et al. (2021) claim that the curvature is a more precise measure of convenience yield in commodity markets. Boons and Prado (2019) show that the basis-momentum is a commodity return predictor related to the slope and curvature of the futures term structure. We find that the curvature and basis-momentum factors of one market are significantly related to the idiosyncratic risk prices in only the corresponding oil market; The WTI (Brent) curvature and basis-momentum have no impact on the idiosyncratic risk prices of Brent (WTI).

Overall, we find that the oil risk factors in both markets carry significant risk premia. On average, during 2010–2016, the common risk prices are negative and similar in the two markets. However, the idiosyncratic risk prices, on average, have opposite signs in the two markets. The risk prices of the common factor in the two markets are generally related to the US crude commercial stock, inflation, and economic uncertainty. The Treasury rate significantly corresponds to the WTI idiosyncratic risk prices but not the Brent idiosyncratic risk prices. The Brent idiosyncratic risk price is significantly related to the productions near the North Sea, while the WTI idiosyncratic risk prices have no significant correlations with the North Sea productions. Moreover, the term structure factors are significantly related to the idiosyncratic risk prices in the corresponding oil market.

5.3. Variance decompositions

Table 7 presents the variance decomposition for the WTI (Panel A) and Brent (Panel B) futures prices. We present the variance decomposition for futures with different maturities and various forecast horizons. We decompose the total variance for each maturity and forecast horizon into the proportion explained by the observed common factor and the two latent idiosyncratic factors. The last column in both panels contains the total contribution of the two idiosyncratic factors to the forecast variance for different forecast horizons and futures maturities.

Panel A of Table 7 shows that the proportion of the variance accounted for by the common factor is decreasing with maturity and forecast horizon for WTI futures. The proportions of the variance accounted for by the second WTI factor are increasing with maturity and forecast horizon. Also, the explanatory power of the first WTI factor increases as the forecast horizon increases for all maturity futures. The common factor explains a large proportion of total variance for all maturities and forecast horizons. To illustrate this, the largest contribution of the two WTI factors is 17.96% for the 12-month WTI futures and the 52-week ahead forecast. This means 17.96% of the 52-week ahead forecast variance of the 12-month WTI futures is explained by the two WTI factors and 82.04% by the common factor of the world crude oil futures market.

Panel B of Table 7 shows that the explanatory power of the common factor decreases as the forecast horizon increases for the Brent futures with all maturities. But the explanatory power of the common factor is a hump-shaped function of maturity at all forecast horizons. The common factor explains a larger proportion of forecast variance for medium-maturity Brent futures (4 -month and 5-month maturities). For example, the common factor explains 96.14% of the 13-week ahead forecast variance of the 5-month Brent futures, while it explains 77.00% of the 52-week ahead forecast variance of the 12-month Brent futures. The first Brent factor explains slightly more at short forecast horizons (13-week and 26-week horizons) than long forecast horizons (39-week and 52-week horizons) for all maturities. The explanatory power of the second Brent factor is also a hump-shaped function of maturity at all forecast horizons, and it increases as the forecast horizon increases for all maturities. The largest contribution of the two Brent factors is 23.00%, for the 12-month Brent futures and the 52-week ahead forecast.

The common factor is the most important determinant of variation in the term structure of futures prices, especially for WTI futures. The idiosyncratic factors account for a considerable part of the forecast variance at longer forecast horizons for all maturity WTI and Brent futures. But the idiosyncratic factors are relatively more important for Brent futures than for WTI futures for almost all maturities and forecast horizons. Additionally, the second idiosyncratic factor appears to be relatively more important than the first one for most maturities and forecast horizon combinations in both markets.

6. Conclusion

We study the price spread between Brent and WTI futures using a reduced form no-arbitrage model with common and idiosyncratic state variables. We estimate our model using the Kalman filter to identify the idiosyncratic factors in WTI and Brent markets, respectively. The model does a very good job in fitting the term structure of WTI and Brent futures prices.

We examine the model-based expectations of future spot price and risk premia and find that both expected future spot price and risk premium are in general higher for Brent than for WTI. The price spread between Brent and WTI futures is driven by both the differences in expectations of future spot price and the differences in risk premia between the two markets. However, the differences in expectations and their correlations with the Brent-WTI futures spread are on average larger than those for risk premia, suggesting that the price spreads between Brent and WTI futures are mainly driven by the expectations in the spot market, especially for short maturity. The differences in risk premia become more important in determining the price spreads at longer maturities.

Our model implies that both markets in general exhibit negative and time-varying risk premia during the sample. However, the risk-price structure of the common and idiosyncratic factors are different in the two markets. The common risk prices are on average negative in both markets, while the risk prices of idiosyncratic factors generally have opposite signs and weak correlations in the two markets. The common risk prices in both markets are significantly correlated with the US crude commercial stock, inflation, and economic uncertainty. The WTI idiosyncratic risk prices significantly correspond to the US short rate. The first Brent idiosyncratic risk price is significantly related to the crude oil productions in Norway and UK; however, the WTI idiosyncratic risk prices have much lower and insignificant correlations with the Norway and UK crude oil production. Both WTI and Brent idiosyncratic risk prices are negatively correlated with the slope or basis of the corresponding oil futures curve.

The variance decomposition suggests that the common factor explains most of the variation in the term structure of futures prices for all forecast horizons in both markets. The idiosyncratic factors account for a considerable part of the forecast variance at longer forecast horizons for all maturity WTI and Brent futures. Comparing the results for the idiosyncratic factors between the two markets, we find that the idiosyncratic factors are relatively more important for Brent futures than for WTI futures for almost all maturities and forecast horizons.

CRediT authorship contribution statement

Xin Gao: Conceptualization, Methodology, Data curation, Software, Formal analysis, Writing – review & editing. **Bingxin Li:** Conceptualization, Methodology, Data curation, Software, Formal analysis, Writing – original draft, Writing – review & editing. **Rui Liu:** Conceptualization, Methodology, Data curation, Software, Formal analysis, Writing – original draft, Writing – review & editing.

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Appendix A. Affine pricing formula

To derive the recursions in Eqs. (5) and (6), we express the h-maturity futures prices under the risk neutral measure: futures prices are martingales (see Cox et al., 1981). For notational simplicity, we drop the superscript i and write

$$\begin{aligned} F_t^h &= E_t^Q[F_{t+1}^{h-1}] \\ &= E_t^Q\left[\exp\left(A_{h-1} + B_{h-1}Z_t\right)\right] \\ &= E_t^Q\left[\exp\left(A_{h-1} + B_{h-1}\left(K_0^Q + K_1^Q Z_t + \varepsilon_{t+1}\right)\right)\right] \\ &= \exp\left(A_{h-1} + B_{h-1}K_0^Q + B_{h-1}K_1^Q Z_t\right) E_t^Q\left[\exp\left(B_{h-1}\varepsilon_{t+1}\right)\right] \\ &= \exp\left(A_{h-1} + B_{h-1}K_0^Q + B_{h-1}K_1^Q Z_t + \frac{1}{2}B_{h-1}\Sigma\Sigma' B_{h-1}'\right). \end{aligned} \tag{A.1}$$

Mapping coefficients results in the recursion relations in Eqs. (5) and (6).

Appendix B. Maximum likelihood estimation with Kalman filter

We estimate the P- and Q-parameters simultaneously by applying the Kalman filter to the state–space representation in Eqs. (15) and (16). For notational simplicity, we drop the superscript i in the following Kalman filter algorithm. The contemporaneous forecast of the idiosyncratic state vector and its corresponding covariance matrix are denoted by $Y_{t|t}$ and $P_{t|t}$. The Kalman filter algorithm works as follows at any time t:

1. Given $Y_{t|t}$ and $P_{t|t}$, compute the one-period ahead forecast of the idiosyncratic state vector and its corresponding covariance matrix[28]

$$\begin{aligned} Y_{t+1|t} &= K_{0Y}^P + K_{1Y}^P Z_{t|t}, \\ &= K_{0Y}^P + K_{1YX}^P X_t + K_{1YY}^P Y_{t|t}, \end{aligned} \tag{B.1}$$

$$P_{t+1|t} = K_{1YY}^{P\prime} P_{t|t} K_{1YY}^P + \Sigma_Y \Sigma_Y', \tag{B.2}$$

   where K_{1YX}^P, the 2×1 vector, is the first column in K_{1Y}^P that is defined in Eq. (15). K_{1YY}^P, the 2×2 matrix, are the second and third columns in K_{1Y}^P.

2. Compute the one-period ahead forecast of the futures prices and the common factor

$$\begin{pmatrix} f_{t+1|t} \\ X_{t+2|t} \end{pmatrix} = \begin{pmatrix} A \\ K_{0X}^P \end{pmatrix} + \begin{pmatrix} B_X & B_Y \\ K_{1XX}^P & K_{1XY}^P \end{pmatrix} \begin{pmatrix} X_{t+1} \\ Y_{t+1|t} \end{pmatrix}, \tag{B.3}$$

   and the corresponding covariance matrix

$$V_{t+1|t} = \underbrace{\begin{pmatrix} B_Y \\ K_{1XY}^P \end{pmatrix}}_{\overline{B}_Y} P_{t+1|t}' \underbrace{\begin{pmatrix} B_Y \\ K_{1XY}^P \end{pmatrix}'}_{\overline{B}_Y'} + R, \tag{B.4}$$

   where K_{1XX}^P, the scalar, is the first element in K_{1X}^P that is defined in Eq. (16). K_{1XY}^P, the 1×2 vector, are the second and third elements in K_{1X}^P. B_X is the first column in the $H \times 3$ matrix B that is defined in Eq. (7), and B_Y are the second and third columns in B. Recall that H denotes the total number of available maturities in the sample. R is a $(H+N) \times (H+N)$ diagonal matrix. N, the number of common state variable, equals to one in our model. The diagonal terms are the same, as denoted by σ_e^2. We assume that the pricing errors of futures with different maturities and also of the common factor have equal variance σ_e^2.

[28] The unconditional two first moments are used in the first step of recursion.

3. Compute the forecast error

$$e_t = \begin{pmatrix} f_{t+1} & - f_{t+1|t} \\ X_{t+2} - X_{t+2|t} \end{pmatrix}. \tag{B.5}$$

4. Update the contemporaneous forecast of the idiosyncratic state vector and its corresponding covariance matrix

$$Y_{t+1|t+1} = Y_{t+1|t} + P_{t+1|t}\overline{B}_Y^{'} V_{t+1|t}^{-1} e_t, \tag{B.6}$$

$$P_{t+1|t+1} = P_{t+1|t} - P_{t+1|t}\overline{B}_Y^{'} V_{t+1|t}^{-1} \overline{B}_Y P_{t+1|t}. \tag{B.7}$$

5. Return to the first step.

The conditional likelihood function is

$$\Gamma(f_t, X_t | f_{t-1}, X_{t-1}\Theta) = \Gamma(f_t, X_t | Z_t, \Theta^Q, \Theta^P, \sigma_e^2) \times \Gamma(Z_t | Z_{t-1}, \Theta^P), \tag{B.8}$$

where $\Theta = \{\Theta^Q, \Theta^P, \sigma_e^2\}$. The logarithm of the first term in Eq. (B.8) is given by

$$\log \Gamma(f_t, X_t | Z_t, \Theta^Q, \Theta^P, \sigma_e^2) = \text{constant} - 0.5(H + N)\log(\sigma_e^2) - 0.5\frac{\|e_t\|^2}{\sigma_e^2}. \tag{B.9}$$

$\|e_t\|$ denotes the Euclidean norm of the vector of measurement errors. The second term of Eq. (B.8) is the likelihood capturing the time-series dynamics of the common and idiosyncratic risk factors. It corresponds to the likelihood of a conditionally Gaussian VAR process. The logarithm is given by

$$\log \Gamma(Z_t | Z_{t-1}, \Theta^P) = \text{constant} - 0.5\log(\det(\Sigma\Sigma')) - 0.5\left\|\Sigma^{-1}(Z_t - K_0^P - K_1^P Z_{t-1})\right\|^2. \tag{B.10}$$

Appendix C. Additional results

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jcomm.2022.100274.

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