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Revisiting the Silver Crisis[☆]

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ABSTRACT

This paper examines the Silver Crisis of the late 1970s, which resulted in a $150 million lawsuit against the Hunt Brothers. In August 1988, the Hunt Brothers were found guilty by a jury of conspiracy, manipulation, monopolization, racketeering and fraud. Using a behavioural model, we aim to quantify the extent of manipulation in the silver market during the 1970s and the 1980s, with a specific focus on the period leading up to the Silver Crisis. Our behavioural model takes account of the role of fundamentals, manipulation and speculation. Our results indicate very little evidence of manipulation in the silver market in the run up to the Silver Crisis. Both fundamentals and speculation dominate the silver market during our sample, with speculation particularly important in the latter half of the 1970s. The distinction between manipulation and speculation is critical. While manipulation forces prices away from their fundamental value, speculation does not. Speculators certainly aim to take advantage of price changes but the actions are fully rational and consistent with the fundamental value of silver.

1. Introduction

Manipulation and markets are rarely too far from sharing the headlines. During the Spring of 2022, the focus was on nickel and the potential role of manipulation. Market prices reached record highs (three month nickel prices reached $48,048 per tonne on the 10 March 2022 at the London Metal Exchange), while trading was suspended at the London Metal Exchange, as well as the Shanghai Futures Exchange. Add to that the reference to the dominant market players such as ‘Big Shot’ and a reflection on one of the most important crises in commodity markets, and the subsequent high profile court case, will be to the fore.[1] The Hunt Brothers and the Silver Crisis of 1979/80 is regularly cited as the classic case of market manipulation. However, our contention is that the classic case may be somewhat of a stretch. For example Williams (1995) indicates that the, ‘... lack of any identification of the Hunts alleged manipulation profoundly affected the trial, especially the experts testimony’, (see page 96). For such a noteworthy event, the extent of the academic literature addressing this particular crisis is quite limited and as far as the authors are aware, there is no formal empirical identification of manipulation for the Silver Crisis in the literature. In his survey, Pirrong (2017) raises doubts

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[1] The Financial Times, “Nickel ‘Big Shot’ shows metals market desperately needs overhaul”, 11 March 2022, https://www.ft.com/content/867c53b5-3016-4008-9c48-308a9dea09a3.

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on the existence of manipulation, while Kleit (2018) draws similar conclusions.[2] In this study we empirically examine the extent of manipulation of silver prices during what became known as the Silver Crisis.[3] Our understanding is that this is the first known empirical evaluation of manipulation during the Silver Crisis.[4]

The Hunt Brothers were found guilty, along with a number of their associates, of manipulating the price of silver during late 1979 and into early 1980.[5] The plaintiff in the case, Minpeco, were awarded over $130 million in damages.[6] Williams (1995) provides a detailed and forensic account of the trial and the approaches adopted by both the plaintiff and the defence.[7] Our aim in this study is to quantify the extent of manipulation in the silver market, with a specific focus on the period leading up to the Silver Crisis. Although considerable coverage has appeared in both academic circles and in the popular press since the historic price highs of January 1980 and the subsequent trial, to our knowledge this is the first formal quantification of silver market manipulation and speculation during this volatile period. In addition to our quantification of manipulation versus speculation, we also address whether the manipulation was an isolated incident or whether manipulation became pervasive into the 1980's.

In order to quantify the role of manipulation in the silver market, we examine the role of a combined set of actors or investors, using a behavioural finance model. We distinguish between long-term investors, namely those focusing on fundamentals, and short-term investors. We focus on two types of short-term investors, *Speculators* and *Manipulators*.[8] We consider the case of short-term investors operating on legitimate grounds, who attempt to benefit from price changes (*Speculators*) and those that actively engage in pushing prices away from their fundamental value (*Manipulators*). Gorton and Rouwenhorst (2006) have highlighted that commodity futures allow producers to hedge their exposure to uncertain price movements in the future. In return for providing this insurance, the investors (or *Speculators*) in commodity futures are paid a compensation for bearing this risk. This transfer of risk from producers to *Speculators* reflects the socially beneficial role being played by *Speculators*. However, with *Manipulators* pushing prices away from their fundamental value, prices do not reflect the proper information signal to consumers and producers. Both *Speculators* and *Manipulators* take a short-term investment horizon, however the *Speculators* act rationally, while the *Manipulators* take a contrarian position. We also examine the case of agents that are only interested in long-run fundamentals (*Fundamentalists*). Rather than examining each case separately, we simultaneously combine all three agents in relation to silver price movements.

Our results provide consistent evidence that the price of silver was not driven by manipulation in the latter half of the 1970s, the early 1980's or at any point during our 1970/1980 sample. While *Manipulators* were present, their role in influencing silver prices was small and was always less than that played by *Speculators*. During the 1970s and the subsequent Silver Crisis, we find that *Speculators* dominated silver prices. After Spring 1980, fundamentals returned to drive silver prices, with limited roles for both speculation and manipulation. Our results indicate that peak speculative activity occurred in 1978 and 1979, in advance of the period price highs (21st January 1980 - $50). Our results also emphasize the role of speculation during the mid to late 1970's and the limited role of manipulative activity during this particular period. At its peak in 1978–1979, there are regular periods where silver prices are dominated by *Speculators*.

Overall we find considerable evidence that the period linked to the Hunt Brothers activities is very much consistent with silver prices being driven by *Speculators* rather than *Manipulators*. Critically, while *Manipulators* push prices away from their fundamental value (and so are socially harmful), *Speculators* play a socially beneficial role by pushing prices towards their fundamental value. Our behavioural model casts doubt on the conclusion that the Silver Crisis was an episode driven by manipulation and emphasizes the importance of distinguishing the alternative forms of short-term trading activity. On a methodological level, we find consistent evidence that silver prices are far better explained by simultaneously taking account of all three actors (*Fundamentalists*, *Speculators* and *Manipulators*) using our behavioural model. In addition, our empirical analysis indicates that silver prices are much more influenced by rational actors, *Fundamentalists* and *Speculators*, with very little role for *Manipulators* during the two decade period analysed.

[2] Allen et al. (2006) also addresses the silver crisis along with 13 other incidences of manipulation during the period 1863–1980. The authors find that these incidences led to greater market volatility and had implications for other assets.

[3] See for example, The New York Times, "Trial of Hunt Brothers In Silver Case Begins", 25 February 1988, https://www.nytimes.com/1988/02/25/business/trial-of-hunt-brothers-in-silver-case-begins.html?auth=login-google1tap&login=google1tap.

[4] Silver represents a precious metal, along with gold, platinum and palladium. Post the Silver Crisis, silver prices remained relatively stable. Figuerola-Ferretti and Gilbert (2001) highlight the implications of the Silver Crisis on the overall market and in particular on the increase in price volatility in the early 1980's. Including data for both the 1980's and the 1990's the authors find no evidence of any increase in silver price volatility, with the exception of the Silver Crisis. Notable recent exceptions, to the overall market stability, are the considerable rise and subsequent fall in silver prices during the global financial crisis. See Figuerola-Ferretti and McCrorie (2016).

[5] The Federal court case of the Hunt Brothers (Bunker, Herbert and Lamar) was held in 1988.

[6] In August 1988, the Hunt Brothers were found guilty by a jury of conspiracy, manipulation, monopolization, racketeering and fraud. In addition to the lawsuit taken by Minpeco, the Commodity Futures Trading Commission's (CFTC) Division of Enforcement also took a lawsuit against the Hunt Brothers. Again the claim was that manipulation in the silver market took place. The trial was delayed as a result of the Minpeco lawsuit and eventually concluded in December 1989. Nelson Bunker Hunt and William Herbert Hunt were ordered to pay fines of $10 million each, as well as a ban on trading in the American commodity markets.

[7] Jeffrey Williams was an expert witness for the defence in the trial.

[8] At the heart of our classical fundamental-based model, lies the rational asset pricing model (RAPM). The RAPM indicates that commodities are valued based on their *cash flows* (convenience yield) that are likely to be generated, see Bredin et al. (2018). We refer to this price as the price determined by the long-run actors, the Fundamentalists.

2. The silver crisis and forms of manipulation

In August 1988, the Hunt Brothers (Bunker, Herbert and Lamar) were found guilty by a jury of conspiracy, manipulation, monopolization, racketeering and fraud. The US Securities and Exchange Commission (SEC) describes manipulation as an intentional or wilful conduct designed to deceive or defraud investors by controlling or artificially affecting the price of securities. The CFTC defines manipulation (Rule 180) as the specific intent to effect a price or price trend that does not reflect legitimate forces of supply and demand. Clearly defining the *legitimate forces of supply and demand* is problematic. The difficulties in relation to a suitable definition of manipulation have been highlighted in the literature, e.g. see, Fischel and Ross (1991), Markham (1991), Pirrong (1994) and most recently Fletcher (2020).[9]

We will formally use a behavioural model to empirically determine the extent of manipulation (using the CFTC definition of manipulation) and as a result formally quantify the extent of manipulation in 1979 and 1980, versus price determination by two rational alternatives, fundamentals and speculation. Our choice of silver, and the period associated with the Hunt case, can be considered a 'controlled' experiment, as we are examining a period where a court has ruled that price manipulation has taken place. Evidence consistent with the court ruling would find results in favour of manipulation during the period in the run up to the Silver Crisis.

Williams (1995) highlights four manipulative schemes: a 'corner' or a 'squeeze', a 'rumour manipulation', an 'investor-interest manipulation' and finally the 'price effect manipulation' where the trader buys (or sells) the commodity in such a quantity to influence a price.[10] The key criteria here is that the trader knows the long-run value (fundamental value) and that their actions can affect the current price. Williams (1995) defines a 'corner' or a 'squeeze', which relies on both a considerable long futures position and a simultaneous holding any locally available inventory.[11] The corner occurs when the long indicates a wish to take delivery. With a shortage in the spot market, the short will need to find alternatives in the spot market or buy the futures at a premium.[12] The 'rumour manipulation' relies on an "informed" insider spreading a rumour of a shortage to provide an incentive to take a long position. Thirdly, the 'investor-interest manipulation' is where actions and statements indicate a desire to hold commodity. Critically these actions by the manipulator are temporary. In the court case, Minpeco vs. Hunt, a fourth form of manipulation was developed. The price-effect occurs when based on their long-run expectation, a trader buys (or sells) in order to influence the current market price. Rather than specify the form of manipulation, as in Williams (1995) and Pirrong (2017), we adopt the CFTC definition. While our approach represents a low bar, we are prioritizing the identification of manipulation. We are also sensitive to the reality that any Federal or regulatory trial is likely to adopt the CFTC definition. We feel our approach is the most appropriate to evaluate and compare the Hunt trial outcomes from 1988. In the next section we describe our behavioural model and how we formally quantify manipulation.

3. Behavioural model & methodological approach

3.1. A model of fundamentals

In the rational asset pricing model (RAPM), the silver spot price (P_t) is the rational expectation of future 'payoffs' associated with holding silver:[13]

$$P_t = \sum_{i=1}^{\infty} \delta_t^i E_t \psi_{t+i} \tag{1}$$

The term $\delta_t^i = \frac{1}{1+\mu_t}$, where the discount rate μ_t represents the expected return required by the marginal investor at time t to hold silver for one period. The one-period payoff paid at time $t+i$ is denoted by ψ_{t+i} and is defined as the value of the overall benefit that accrues to the holder of silver, rather than the owner of the futures contract, in the period between t and $t+i$.[14] More concretely, it represents the benefits associated with holding inventory, e.g. maintaining a smooth production process and so avoiding disruption to the flow of goods being produced, net of any storage cost and/or any other cost of carry except funding costs at the risk-free rate. Following Pindyck (1993), we refer to this measure of net benefit as the marginal convenience yield or, more simply, the convenience yield. Drawing on Pindyck (1993), the market expectation of the silver convenience yield can be extracted from the prices of spot and/or futures contracts for two successive maturities.[15]

[9] The consensus points to a confusing and contradictory nature of the legislation. The current regulations on manipulation have a micro-prudential focus (harm associated with the institution, investor, or asset). An alternative setting would broaden the scope to a macro-prudential focus, with an emphasis on minimizing manipulation and reducing systemic risk. See Fletcher (2020). Pirrong (1994) concludes that, "In its current state, the law is less a deterrent to manipulators than an invitation to them". See Pirrong (1994), page 1013.

[10] See Pirrong (2017) for an excellent recent survey of manipulation in commodity markets.

[11] A consistent description is provided in Pirrong (2017).

[12] In legal terms McDermott (1979) has referred to the corner as a 'hindrance', with one party effectively putting in place a hindrance for the other party to fulfil the contract.

[13] For the remainder of the paper, we will refer to the spot price of silver as the *price*.

[14] See Pindyck (2001). Silver is an extremely versatile precious metal, being an essential input for solar panels, the automotive industry as well as brazing and soldering.

[15] Formal derivation is provided in Appendix A.

The RAPM in (1) can thus be interpreted as a reduced form dynamic demand-supply model of silver prices. We define the silver rate of return as:

$$q_t = (P_{t+1} - P_t + \psi_t)/P_t \tag{2}$$

which can be re-written purely in terms of the convenience yield and the percentage net basis (PNB)[16]:

$$\begin{aligned} q_t &= \psi_t y_t/\psi_{t-1} + \psi_t y_t/(\psi_{t-1} y_{t+1}) - 1 \\ &= y_t \frac{\psi_t}{\psi_{t-1}} + \frac{\psi_t}{\psi_{t-1}} \frac{y_t}{y_{t+1}} - 1 \end{aligned} \tag{3}$$

It will be convenient, following Pindyck (1993), to linearize the right hand-side of (3) around $\psi = \overline{\psi}$ and $y = \overline{y}$, where the variables on the right-hand side are long run (unconditional) means of the variables on the left-hand side.[17] Defining $\beta = 1/(1+\overline{y})$ and the normalized variables $y'_t = \frac{y_t}{\bar{y}}$ and $\psi'_t = \frac{\psi_t}{\bar{\psi}}$, then (3) can be solved for y'_t as follows:[18]

$$y'_t \approx \beta y'_{t+1} + \beta q_t - \Delta\psi'_t \tag{4}$$

The solution for this difference equation that rules out an explosive behaviour of y'_t is:

$$y'_t \approx \sum_{j=0}^{\infty} \beta^j (\beta q_{t+j} - \Delta\psi'_{t+j}) \tag{5}$$

This solution implies that the normalized PNB approximates the discounted present value of future commodity returns in excess of the change in the normalized convenience yield, where the discount factor is the coefficient β. As noted by Pindyck (1993), drawing on Campbell and Shiller (1989), (5) is an approximate accounting identity with no economic content, other than ruling out an explosive behaviour of y'_t.

We now take the conditional expectation of both sides of this accounting identity. We have:

$$y_t'^F \approx \sum_{j=0}^{\infty} \beta^j E_t(\beta q_{t+j} - \Delta\psi'_{t+j}) \tag{6}$$

The approximate equality in (6) can be seen as describing the normalized PNB when the price is set by a rational agent with a long-run horizon. We therefore refer to it as the *Fundamentals (F)* model of the PNB, which is consistent with RAPM under the linearization described in Appendix B.

3.2. Quantification of manipulation & speculation

Besides the long-term solution for y'_t, that represents the view of a long horizon investor, we can also obtain a short horizon (one period) alternative. The short-run (one-period rather than infinite horizon) counterpart of (5)[19]:

$$y_t \approx q_t - \frac{\bar{y}}{\bar{\psi}} \Delta\psi_t - \frac{\Delta P_{t+1}}{P_t}$$

Now, rewriting the above in terms of the normalized variables $y'_t = \frac{y_t}{\bar{y}}$ and $\psi'_t = \frac{\psi_t}{\bar{\psi}}$ leads to

$$y'_t \approx \frac{1}{\bar{y}} q_t - \Delta\psi'_t - \frac{1}{\bar{y}} \frac{\Delta P_{t+1}}{P_t} \tag{7}$$

Like (5), (7) is an approximate accounting identity with no economic content. Thus, as was the case for the *Fundamentals* model of the PNB (implied by the RAPM), we impose rational expectations by taking the conditional expectation of both sides of (7). Hence, using the fact that $\beta = 1/(1+\overline{y})$, we obtain

$$\begin{aligned} y_t'^S &\approx \frac{\beta}{1-\beta} E_t(q_t) - E_t(\Delta\psi'_t) - \frac{\beta}{1-\beta} E_t\left(\frac{\Delta P_{t+1}}{P_t}\right) \\ &= \frac{\beta}{1-\beta}\left(E_t(q_t) - E_t\left(\frac{\Delta P_{t+1}}{P_t}\right)\right) - E_t(\Delta\psi'_t) \end{aligned} \tag{8}$$

The approximate equality in (8) is consistent with the *Fundamentals* model, with the exception that the price is now set by a rational agent with a *short-run* horizon. We therefore refer to it as the *Speculators (S)* model of the PNB.

In the model in which investors are the *Fundamentals*, namely (6), the normalized PNB is a weighted average, with geometrically declining weights, of the expected return adjusted for the one period expected change in the convenience yield. Instead, in the model

[16] The PNB is defined as, $y_t = \psi_{t-1}/P_t$. In the commodity pricing framework proposed by Pindyck (1993), the PNB plays a consistent role to the dividend-price ratio in equity pricing according to Campbell and Shiller (1989).

[17] Formal derivation is provided in Appendix B.

[18] Since $\beta = 1/(1+\overline{y})$ and, therefore, $1 + 1/\overline{y} = 1/(\beta\overline{y})$, we have that $q_t \approx y_t(1+1/\overline{y}) + \Delta\psi_t(1+\overline{y})/\overline{\psi} - y_{t+1}/\overline{y} = y_t/(\beta/\overline{y}) + \Delta\psi_t/(\beta\overline{\psi}) - y_{t+1}/\overline{y} = y'_t/\beta + \Delta\psi'_t/\beta - y'_{t+1}$. Hence, multiplying through by β, we have $\beta q_t \approx y'_t + \Delta\psi'_t - \beta y'_{t+1}$. Rearranging terms, we obtain (4). For commodities, the $\overline{y}$ is 1% (or less), so β the smoothing parameter is less than or close to one. See Pindyck (1993).

[19] Formal derivation is provided in Appendix B.

with *Speculators* (S), namely (8), the normalized PNB is proportional to the difference between expected return and the expected rate of change of the commodity price, adjusted for the one period expected change in the normalized convenience yield. Therefore, while expected changes in the convenience yield and rates of return play a role in both the short-run and long-run model, the expected rate of price change (capital gain) plays a direct role (and with a larger weight) only in the short-run model.

Finally, we also examine the case of manipulation (M). In this case, silver prices are influenced by a short-horizon irrational contrarian, an agent whose expectation of next period price changes, $E_t^M(\Delta P_{t+1})$, is the opposite to the rational expectation:

$$E_t^M(\Delta P_{t+1}) = -E_t(\Delta P_{t+1}) \tag{9}$$

The normalized PNB when prices are set by this agent is thus

$$\begin{aligned} y_t'^M &\approx \frac{\beta}{1-\beta}E_t(q_t) - E_t(\Delta\psi_t') + \frac{\beta}{1-\beta}E_t\left(\frac{\Delta P_{t+1}}{P_t}\right) \\ &= \frac{\beta}{1-\beta}\left(E_t(q_t) + E_t\left(\frac{\Delta P_{t+1}}{P_t}\right)\right) - E_t(\Delta\psi_t') \end{aligned} \tag{10}$$

We refer to (10) as the *Manipulators (M)* model of the PNB. The PNB in this case is proportional to the sum of the expected return and the expected rate of change of the commodity price, rather than their difference as in the case of the model (*S*) with the rational short-horizon marginal agent, adjusted for the one period expected change in the normalized convenience yield.

Following Brock and Hommes (1998) and, more recently, Lof (2015), we aggregate the three agents in a heterogeneous agent model in which the PNB, y_t^{ha}, is determined as follows;

$$y_t^{ha} = G_t^F y_t'^F + G_t^S y_t'^S + G_t^M y'^M \tag{11}$$

Here, the term G^j represents the fraction of the PNB determined at the margin by each agent (*Fundamentalists*, *Speculators* and *Manipulators*). The fraction assigned to each agent increases when the agent's prediction outperform the others', and the fractions sum to unity, as described by the following multinomial logit model;

$$\begin{aligned} G_t^j &= \frac{exp(\omega^j U_t^j)}{\sum_k exp(\omega^k U_t^k)} \quad j,k \in F,S,M \\ where\ U_t^j &= -(y_{t-1}'^j - y_{t-1}')^2 \quad j \in F,S,M \end{aligned} \tag{12}$$

The ω^j parameters represent the willingness of agents to switch their strategy. A higher ω^j indicates greater willingness to switch. The term U_t^j represents a measure of fit, defined as the difference between the theoretical and the actual normalized PNB. Following the original recommendation of Hong et al. (2007) and the subsequent application by Lof (2015), we allow each ω^j to vary depending on the agent.

In terms of formal tests we examine two particular hypotheses. Our first hypothesis test will test the dominance of manipulation during the second half of the 1970's into the early 1980's and across our overall 1970/1980's sample. We examine whether manipulation dominated, as would be consistent given the trial outcome, during the Silver Crisis. Using our behavioural model, we can formally determine the dominance of rational focused agents (Speculators and Fundamentalists) versus irrational agents (Manipulators).

Hypothesis 1: Manipulation was the dominant strategy in advance and during the Silver Crisis.

H1a: *Manipulators* dominated the combined role played by *Fundamentalists* and *Speculators*.
H1b: *Manipulators* dominated the role played by *Speculators*.

Our second hypothesis test considers an alternative scenario during the 1970s and the subsequent Silver Crisis. We consider the case that rather than manipulation being the dominant activity during the crisis and pre-crisis, that it was in fact speculation. The narrative around the court case was that manipulation was proven to exist and that this was an isolated incident. We test whether the silver market activity was consistent with speculation during the 1970s and that this activity was much more prevalent than acknowledged. Evidence consistent with our stated hypothesis would report dominant levels of speculation during and including the Silver Crisis.

Hypothesis 2: Speculation was the dominant strategy in advance and during the Silver Crisis.

H2a: *Speculators* dominated the combined role played by *Fundamentalists* and *Manipulators*.
H2b: *Speculators* dominated the role played by *Fundamentalists*.

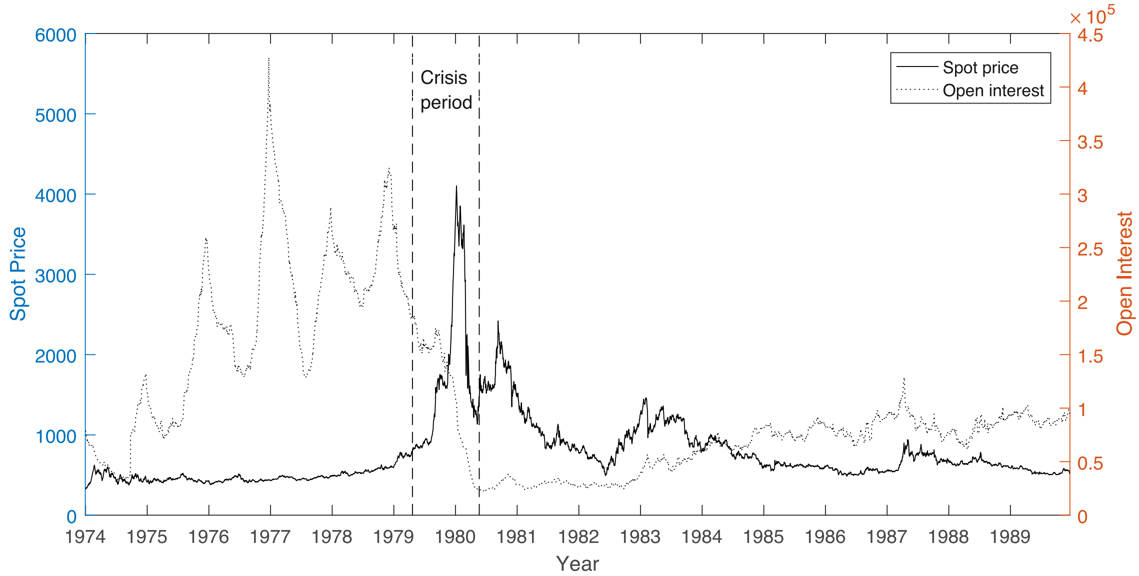

Fig. 1. Evolution of price and open interest silver market 1974–1990. The figure plots the temporal evolution of silver spot prices by the solid line (values on the left axis) and silver open interests by the dotted line (values in hundreds of thousands on the right axis) for the full sample period 1974 to 1990. The period between May 1979 and April 1980 (associated with the Silver Crisis) is represented between the two vertical dashed lines.

4. Data & empirical results

4.1. Data

Our study examines the market for silver during the 1970's and the 1980's. The market prices for silver trading on the COMEX exchange, with delivery months in January, February, March, April, May, July, September, December are considered.[20] Data is monthly and sourced via the Commodity Research Bureau (CRB). All reported data is for futures prices on the first Wednesday of each month, Tuesday prices were adopted when the Wednesday price is not available. The proxy for the spot price (P_t) is the spot (front month) futures contract, i.e. the contract expiring in month t. The proxy for the futures price (F_t) will then be the next to expire futures contract. Thus the spot and futures prices represent the exact same good and the time intervals between the two delivery dates is known.[21]

Silver prices (solid line) and open interest (broken line), representing gross positions, are reported in Fig. 1.[22] The figure clearly displays a distinct difference in open interest in the 1970s versus the 1980s. The series of spikes in open interest are large and peak prior to 1979, with a considerable stabilization occurring from 1981 onward. The series of spikes are in line with the gradual, but consistent, accumulation of silver inventory by the Hunt Brothers. In terms of prices, the significant event is clearly the Silver Crisis of 1979/80, with no evidence of price spikes of this nature either before or after.

4.2. Behaviour model results

In Fig. 2, we plot the relationship between the actual PNB (solid line) and the theoretical PNB (broken line) using a purely fundamentals based model, for silver during the 1970s and 1980s, with the vertical line representing the Silver Crisis (May 1979–April 1980).[23] In the figure, we also report two measures of fit between the theoretical and actual PNB, namely the correlation between the two series and their variance ratio. The correlation coefficient and the variance ratio represent an indication of the long-run and short-run proximity, respectively, between the actual PNB and the PNB determined by our fundamental pricing model.

[20] A complication that can exist when examining commodities, is that a 'spot' (or futures) contract may not trade in every month for all commodities. For those months when a 'spot' contract does not trade, the standard approach is to use a linear interpolation to determine the 'spot' contract price. See Pindyck (1993).

[21] In order to calculate the convenience yield, a consistent measure of spot and futures price is required. Our choice of spot price (spot futures) and futures price (next to expire futures) guarantee a consistent price is adopted and so is appropriate for calculating the convenience yield.

[22] Open interest is the total of all futures contracts entered into and not yet offset by a transaction, by delivery, by exercise, etc. See https://www.cftc.gov/MarketReports/CommitmentsofTraders/ExplanatoryNotes/index.htm.

[23] The theoretical PNB is calculated using a forecasting system of equations based on a vector autoregression (VAR) model in which the endogenous variables are the (normalized) PNB and convenience yield, under cross-equation restrictions given by Eqs. (1), (6), (8), and (10) and the assumption that, consistent with rational expectations, the one-period expected return is equal to the one-period discount rate μ.

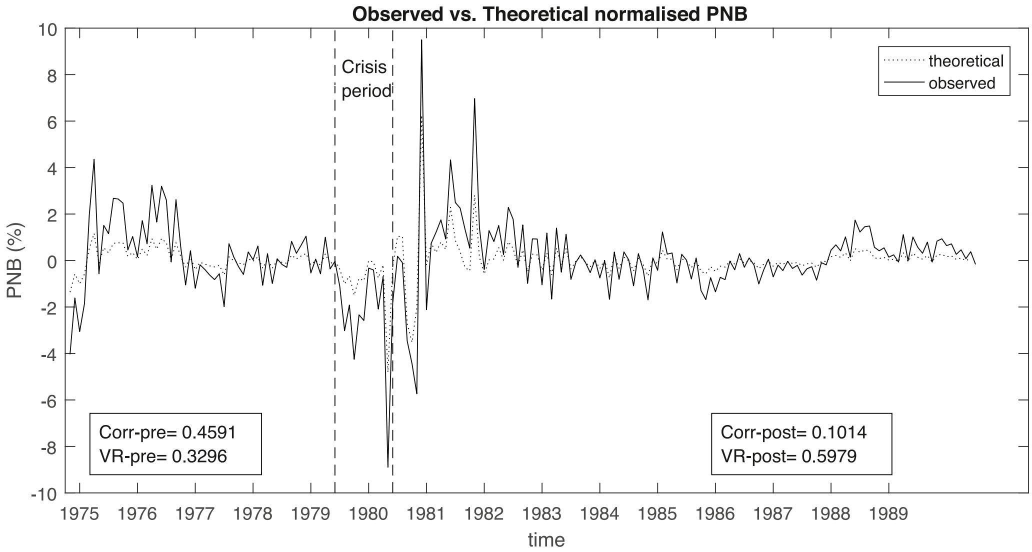

Fig. 2. Silver market — Fundamentalists. The figure presents the observed normalized percentage net basis (solid line) and the predicted normalized percentage net basis (dotted line) for silver over the period 1974 to 1990. The predicted normalized percentage basis is obtained using the expectations of the fundamentalist agent. The period between May 1979 and April 1980 (representing the Silver Crisis) is represented between the two vertical dashed lines. The bottom left box displays the correlation and the variance ratio between the observed and the predicted normalized percentage net basis for the period prior to May 1979 (before the Silver Crisis) and the bottom right box displays the correlation and the variance ratio between the observed and the predicted normalized percentage net basis for the period post April 1980 (after the Silver Crisis).

Both the variance ratios and the correlation coefficients are below 0.6, for the pre and post Silver Crisis period. As expected, the Silver Crisis is clearly a period where the fundamentals based model breaks down. Although silver prices would appear to be relatively aligned with movements in fundamentals, there is clearly some missing information which is reflected in the imperfect correlations/variance ratios between actual and theoretical PNB.

In Fig. 3, we present our behavioural model, which now augments the *Fundamentalists* (highlighted above) with both *Manipulators* and *Speculators*. Our metrics of model accuracy increase considerably, with the correlation coefficient and the variance ratio consistently above 0.74 for the overall sample and above 0.83 for the post Silver Crisis sample. On balance, these results support the behavioural multi-agent setting, suggesting it can explain both long-run and short-run behaviour of silver prices better than the classical fundamentals only model. The outcome of the Hunt trial and the subsequent literature might point to the dominant role played by *Manipulators* during the Silver Crisis. We now examine in detail the role played by *Manipulators* throughout the 1970s/1980s, with a particular emphasis on the Silver Crisis.

4.3. Manipulation or Speculation?

While the statistical performance of the behavioural model is certainly superior to the alternatives, the results presented to date do not formally indicate the dominance of any one actor in relation to the silver market. We now turn to the specific weights associated with *Manipulators*, *Speculators* and *Fundamentalists*. The weights are presented in Fig. 4 and what is immediately obvious is the very low weights associated with *Manipulators*. While there are certainly periods when the weights rise, they are never above 50% and certainly not during the late 1970s, in the run up to the Silver Crisis. We see a very different picture, when we address the case of *Speculators*. There is a steady rise in the weights from 1977, a peak in 1979, with a subsequent decline. The weights associated with *Speculators* is certainly not inconsistent with the events in the run up to the Silver Crisis and the subsequent crisis. Finally, we also plot the role of *Fundamentalists*, which interact considerably with their short-term equivalent, *Speculators*. When Speculation reached a peak in 1979, the market was effectively replaced by *Fundamentalists*. This remained the dominant driver of silver market interactions until mid 1981. Both *Speculators* and *Fundamentalists* play a consistent and dominant role throughout the 1970s and 1980s. Most importantly, there is no evidence of any significant role being played by *Manipulators* during this period.

The results presented in Fig. 4 point towards a significant and consistent role for both speculation and fundamentals. Our graphical results are very much at odds with the trial outcome, but perfectly consistent within the narrative of the relatively limited literature on the Silver Crisis. To shed further light on this, we formally test our hypothesis (H1a) that *Manipulators* dominates the combined role of *Speculators* and *Fundamentalists*. We test our hypothesis on the full sample of data, as well as a series of sub-samples including the Silver Crisis and both the pre and post crisis. Table 1 reports consistent evidence that the share associated with *Manipulators* does not dominate the combined role of *Speculators* and *Fundamentalists* (H1a). Importantly this includes an analysis for both the crisis and the pre-crisis. It may be argued that we have set the bar too high, in terms of the potential role associated with *Manipulators* on the eve of the Silver Crisis. In Table 1, we also examine the case where *Manipulators* are compared to *Speculators*,

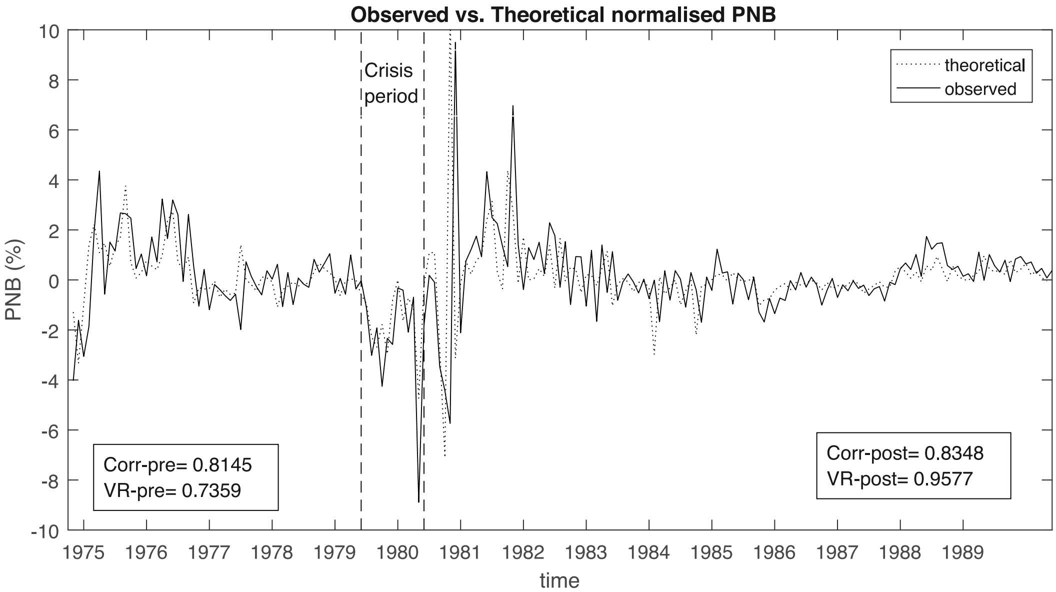

Fig. 3. Silver market — Behavioural model. The figure presents the observed normalized percentage net basis (solid line) and the predicted normalized percentage net basis (dotted line) for silver over the period 1974 to 1990. The predicted normalized percentage basis is obtained using the expectations of the behavioural model including three agents: fundamentalists, speculators and manipulators. The period between May 1979 and April 1980 (representing the Silver Crisis) is represented between the two vertical dashed lines. The bottom left box displays the correlation and the variance ratio between the observed and the predicted normalized percentage net basis for the period prior to May 1979 (before the Silver Crisis) and the bottom right box displays the correlation and the variance ratio between the observed and the predicted normalized percentage net basis for the period post April 1980 (after the Silver Crisis).

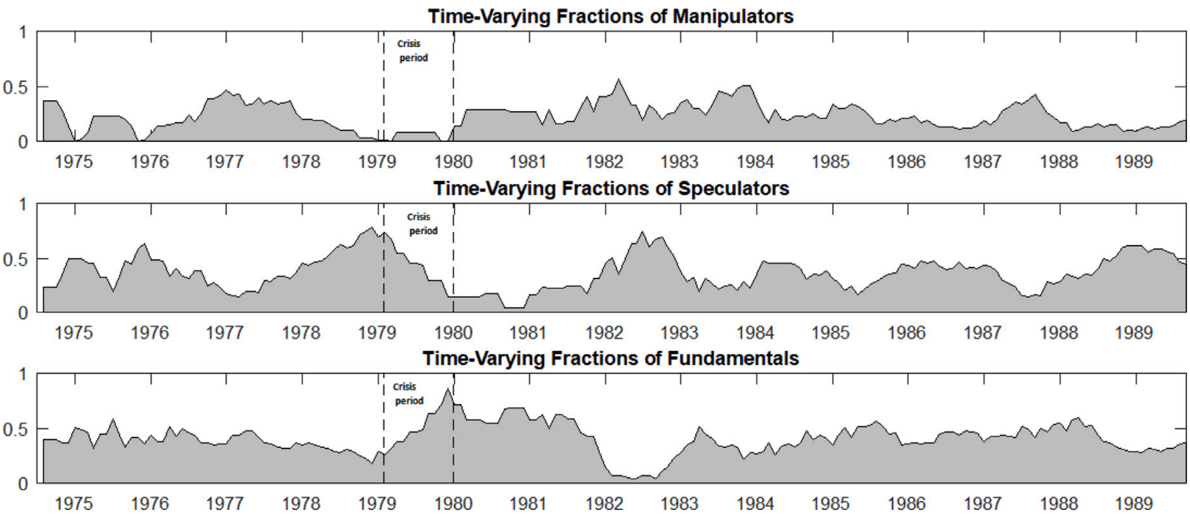

Fig. 4. Silver market — Behavioural model weights. The figure shows the weights of each agent in the behavioural model for silver over the period from 1974 to 1990. The top plot shows results for manipulators. The middle figure represents the weights for the speculators. The bottom plot displays the results for fundamentalists. The period between May 1979 and April 1980 (representing the Silver Crisis) is represented between the two vertical dashed lines.

again for the full and sub-samples (H1b). Again the dominance of *Manipulators* is consistently rejected in all cases. Importantly this rejection includes the pre and crisis period.

Our second hypothesis test, considers an alternative scenario during the 1970s and the subsequent Silver Crisis. Rather than manipulation being the dominant activity during the crisis and pre-crisis, we formally test whether speculation dominated. We examine whether *Speculators* dominated a combination of *Fundamentalists* and *Manipulators* (H2a) and whether *Speculators* dominated *Fundamentalists* (H2b). In Table 1, we report results indicating that speculation did not dominate a combination of fundamentals and manipulation (H2a). A similar set of results is found when we compare speculation and fundamentals (H2b). However this is only the case for the post crisis period. For the pre-crisis and crisis period we report evidence that *Speculators* did in fact dominate

Table 1
Test equality means for different subsample periods.

Statistic (p-value)	Period Pre-May 1979	May 1979-April 1980	Period Post-April 1980	Full sample excluding May 1979–April 1980	Full sample
$H_{1a}: \mu_M > \mu_{F\&S}$	−22.3148**	−47.9984**	−36.7307**	−42.2852**	−43.7268**
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$H_{1b}: \mu_M > \mu_S$	−6.5221**	−6.2794**	−5.9892***	−8.6496**	−9.7386***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$H_{2a}: \mu_S > \mu_{F\&M}$	−6.2301**	−2.1223*	−14.3816**	−15.2214**	−15.1617**
	(0.0000)	(0.0227)	(0.0000)	(0.0000)	(0.0000)
$H_{2b}: \mu_S > \mu_F$	0.9101	−1.4143	−2.5613**	−1.7793*	−2.1211*
	(0.8176)	(0.0856)	(0.0055)	(0.0380)	(0.0173)

Top row of this table shows the statistic and the p-values of one-tailed equality mean tests between the manipulator weights and the sum of speculators and fundamental weights from the behavioural model for silver. The second row of this table shows the statistic and the p-values of one-tailed equality mean tests between the manipulator weights and the speculators weights from the behavioural model for silver. The third row of this table shows the statistic and the p-values of one-tailed equality mean tests between the speculator weights and the sum of the fundamentalists and manipulator weights from the behavioural model for silver. The final row of this table shows the statistic and the p-values of one-tailed equality mean tests between the speculator weights and the fundamentalists weights from the behavioural model for silver. Different periods are analysed in columns where the first column analyses the period prior to May 1979, the second column shows the statistics for the period between May 1979–April 1980 (associated with the Hunt brother's case) and the third column shows the evidence for the period post April 1980. The last two columns display the results for the full sample period 1974 to 1990 excluding the observations between May 1979 and April 1980 and for the full sample period, respectively. **, * represents rejection of the null hypothesis at 1% and 5% significance level.

Fundamentalists. Although the narrative around the court case was that manipulation was proven to exist and that this was an isolated incident, our results show that it was in fact speculation that was the dominant factor and this included the crisis and pre-crisis period.

4.4. Discussion and result implication

The outcome of the 1988 trial was that the Hunt brothers (along with other associates) were found guilty of manipulating the silver market during 1979 and 1980 and fined over $130 million. Our study does not attempt to formally test the judgement in the trial. However, what is beyond doubt is that the extent of silver investments by the Hunt Brothers in the late 1970s was significant. Silver futures alone controlled by the Hunt's increased from 25 million troy ounces in January 1979 to 50 million troy ounces in the summer of 1979. Mark Cymrot's (attorney for Minpeco) opening statement at the trial highlighted the Hunt's total silver investments of $15 billion.[24] Given the size of the overall Hunt Brothers silver exposure and the subsequent outcomes of both (Federal and CFTC) trials, an overall market level analysis is a reasonable setting to examine the Silver Crisis episode.

Our empirical analysis examines the Silver Crisis and the decades either side of this significant event. In our behavioural model, we consider the case of short-term silver investors operating on legitimate grounds, who attempt to benefit from price changes (*Speculators*) and those that actively engage in pushing prices away from their fundamental value (*Manipulators*). Both take a short-term horizon, however critically the *Speculators* act rationally, while the *Manipulators* take a contrarian focus. Gorton and Rouwenhorst (2006) have highlighted that commodity futures allow producers to hedge their exposure to uncertain price movements in the future. In return for providing this insurance, the investors (or *Speculators* in our model) in commodity futures are paid a compensation for bearing this risk. This transfer of risk from producers to *Speculators* reflects the socially beneficial role being played by *Speculators*. However, with *Manipulators* pushing prices away from their fundamental value, prices do not reflect the proper information signal to consumers and producers. The distinction between manipulation and speculation, in terms of risk sharing and rationality, is key from an academic perspective, but more importantly from a policy and regulatory point of view. Our behavioural model provides very consistent evidence of the small role being played by manipulation during the Silver Crisis, with a much greater role for speculation.

How relevant is the Silver Crisis of over 40 years ago? We would argue extremely relevant. The flood of funds into commodities, via commodity futures, which has been referred to as the financialization of commodities, is the primary reason. What happens in commodity markets is critical for the stability of our financial system. The financialization process represents a structural change in commodity market participation and, combined with the price increase during the commodity super cycle, leads to concerns over the potential role of speculation and manipulation. Formal evidence of the increased role of financialization since 2005 has been provided by Tang and Xiong (2012) and Bhardwaj et al. (2015). Further evidence is provided by Büyükşahin and Harris (2011), Büyükşahin and Robe (2013) and Silvennoinen and Thorp (2013), who show that the return correlation between commodities and conventional financial assets (stocks) has turned positive and statistically significant since 2008, after being negative and significant in previous years. Cheng and Xiong (2014) highlight the implications of financialization in relation to risk sharing.[25] The risk sharing

[24] See, The New York Times, "Trial of Hunt Brothers In Silver Case Begins", 25 February 1988, https://www.nytimes.com/1988/02/25/business/trial-of-hunt-brothers-in-silver-case-begins.html?auth=login-google1tap&login=google1tap.

[25] The authors also examine the case of information frictions and the potential impact on price discovery.

implications may be positive when we consider financial investors as providers of liquidity. However, it might also be the case that these investors are consumers of liquidity and so risk sharing may be reduced. In addition to the risk sharing considerations, there is also the question over the changing influence over price as a result of the process of financialization. There is, however, considerable evidence in favour of fundamentals driving recent movements in commodity prices. For the case of a broad range of commodities, Sanders and Irwin (2011) and Sanders et al. (2010) find evidence against speculative effects. While Büyükşahin and Harris (2011) adopt CFTC data for the case of crude oil and find consistent results.[26] Bredin et al. (2022) also examine the case of food price movements and indicate that price movements due to speculation and manipulation are relevant, but small. The primary determinant of food price movements is fundamentals.[27]

Combining the implications of commodity financialization and the lack of clarity regarding manipulation regulation, along with our findings on the Silver Crisis point to inevitable uncertainties that lie ahead. Instances such as the price spikes in nickel during the Spring of 2022 point to worrying instances of manipulative behaviour, with a less than clear path for regulatory intervention. What our results point to, is that actually proving manipulation can be problematic, even when the focus was the events scrutinized by a long running, and very public, trial. Our conclusion is that a clear distinction between manipulation and speculation is required. The popular press may easily confound speculation and manipulation, but using our behavioural model we show that the theoretical and empirical implications are poles apart.

5. Conclusions

We examine the Silver Crisis of the late 1970s. During this period silver prices swung dramatically, from $8 in Spring 1979, to the peak of $50 (21 January 1980) to the subsequent fall to $10 (27 March 1980). In 1988, the Hunt Brothers were found guilty, along with a number of their associates, of manipulating the price of silver during this period and led to a fine in excess of $130 million. Media coverage of the case was significant, as was the CFTC civil complaint which took place once the Federal court case was concluded. That case, the biggest market manipulation case taken by the CFTC up to that point, led to a $10 million fine for both Nelson Bunker Hunt and William Herbert Hunt, and a trading ban from American commodity markets for the Hunt Brothers. The Federal court judgement, along with the CFTC settlement has meant that the Silver Crisis and manipulation are permanently intertwined.

Using a behavioural finance model, and for the first time (to the best of our knowledge), we formally examine the role of manipulation in the silver market, including the Silver Crisis period. We distinguish between long-term investors, namely those focusing on fundamentals, and short-term investors. Our specification of short-term investors, *Speculators* and *Manipulators*, are most relevant for the Silver Crisis period. In particular, *Speculators* in our behavioural model act in a fully rational manner and make no attempt to push prices away from their fundamental value. On the contrary, *Manipulators* behave in an irrational manner and actively engage in pushing prices away from their fundamental value. Besides being consistent with economic theory, our interpretation of *Speculators* and *Manipulators* in our behaviour model is very much in the spirit of the discussions during the course of the Federal trial.

Our results provide consistent evidence that the price of silver was not driven by manipulation in the latter half of the 1970s, the early 1980's or at any point during our 1970/1980 sample. While we do find evidence of the presence of *Manipulators*, their role in the run-up and during the Silver Crisis was small. We do find that *Speculators* played a dominant role during this period. Peak speculative activity occurred in 1978 and 1979, in advance of the period highs in prices (21st January 1980 - $50). If the Hunt Brothers were in fact dominant during this period, we can characterize them as *Speculators*. This distinction between *Speculators* and *Manipulators* is critical. While *Manipulators* push prices away from their fundamental value (and so are socially harmful), *Speculators* play a socially beneficial role by pushing prices towards their fundamental value. Our behavioural model casts doubt on the conclusion that the Silver Crisis was an episode driven by manipulation and emphasizes the importance of distinguishing alternative forms of short-term trading activity.

CRediT authorship contribution statement

Don Bredin: Conceptualization, Writing – original draft, Writing – review & editing, Project administration, Funding acquisition. **Valerio Potì:** Conceptualization, Supervision, Writing – review & editing, Visualization, Funding acquisition. **Enrique Salvador:** Data curation, Software, Calculation, Writing – review & editing, Visualization, Funding acquisition.

[26] CFTC data is only available from 1986 onward. Therefore, it has not been possible to analyse the Silver Crisis using CFTC data. See, https://www.cftc.gov/MarketReports/CommitmentsofTraders/HistoricalCompressed/index.htm.

[27] In addition Bredin et al. (2022) compared the results from a behavioural model (similar to that presented here) with those using CFTC data, using Commitment of Traders' (COT) reports for a series of food based commodities for the period 1990–2017. In Bredin et al. (2022), the shares of Manipulators, Speculators and Fundamentals are all determined using the COT CFTC data. The Fundamentals based traders are defined as the percentage of trades opened exclusively by 'commercial' traders. Speculators are defined where the long (short) position of the trade is held by a commercial agent and the short (long) position is held by a 'non-commercial' trader. Finally, Manipulators are defined as the percentage of trades opened exclusively by 'non-commercial' traders. The authors find results using the CFTC data that are consistent with the results found using the behavioural model.

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Appendix A. The convenience yield

The convenience yield for silver (and all commodities) is unobservable but can be estimated via a no-arbitrage relationship between spot prices and futures prices. Following Pindyck (1993), assume that one unit of the commodity is held today at time t and the current spot price is P_t. Holding the asset between today (t) and some future date (T) will pay the following payoff (stochastic):

$$\psi_{t,T} + P_{t+T} - P_t \tag{13}$$

where $\psi_{t,T}$ is the convenience yield of holding the commodity, net of storage costs, for the period between time t and time T. Assume further that, at the same time (t), one also takes a short position in a futures contract with delivery at time $t+T$. Denoting by $F_{t,T}$ the futures trading price at time t, the payoff (also stochastic) of this position will equal the following:[28]

$$F_{t,T} - P_{t+T} \tag{14}$$

The total payoff of the combined position (long spot and short the futures) will be non-stochastic and equal to the following:

$$\psi_{t,T} + F_{t,T} - P_t \tag{15}$$

Since this payoff is non-stochastic, the combined position must earn the risk-free rate of return on the only outlay of cash, namely P_t. Therefore, it must earn $r_{t,T}P_t$, where $r_{t,T}$ denotes the risk-free rate between t and T. Therefore, if we rule out arbitrage opportunities, the expression in Eq. (15) must equal $r_{t,T}P_t$ and, therefore, we have:

$$\psi_{t,T} + F_{t,T} - P_t = r_{t,T}P_t \tag{16}$$

or, equivalently,

$$\psi_{t,T} = P_t(1 + r_{t,T}) - F_{t,T} \tag{17}$$

This no-arbitrage relation provides a convenient way to estimate the otherwise unobservable convenience yield. It can be equivalently rewritten, using continuous compounding, as follows:

$$\psi_{t,T} \ = \ P_t e^{r(t,T)(T-t)} - F_{t,T} \tag{18}$$

Here, $r(t,T)$ is the continuously compounded risk-free interest rate between time t and T. For convenience, we shall henceforth work with one-period versions of $F_{t,T}$ and $\psi_{t,T}$, namely $F_{t,1}$ and $\psi_{t,1}$, which we shall denote as ψ_t and F_t, respectively.

Appendix B. Rates of returns

Long-run view

Following Pindyck (1993), we linearize the right hand-side of (3) around the point that minimizes the sum of the squared approximation errors. In (ψ_{t-1}, y_t) space, this is the point with coordinates $\overline{\psi}$ and $\overline{y}$. To do so, we first linearize (3) around ψ_{t-1} and

[28] The no-arbitrage relation defined below strictly only holds for forward contracts. However, given differences in prices are small between commodity forwards and futures, we adopt a consistent approach to the literature and adopt futures (see Chow et al. (2000) and Alquist et al. (2014)).

y_t and then we evaluate this linearization at $\psi_{t-1} = \overline{\psi}$ and $y_t = \overline{y}$:

$$\begin{aligned}
q_t &= y_t \frac{\psi_t}{\psi_{t-1}} + \frac{\psi_t}{\psi_{t-1}} \frac{y_t}{y_{t+1}} - 1 \\
&\approx \left(y_t \frac{\psi_{t-1}}{\psi_{t-1}} + \frac{\psi_{t-1}}{\psi_{t-1}} \frac{y_t}{y_t} - 1 \right) \\
&+ \left(y_t \frac{1}{\psi_{t-1}} \Delta\psi_t + 0 \times \frac{\psi_{t-1}}{\psi_{t-1}} \Delta y_{t+1} \right) \\
&+ \left(\frac{1}{\psi_{t-1}} \frac{y_t}{y_t} \Delta\psi_t - \frac{\psi_{t-1}}{\psi_{t-1}} \frac{y_t}{y_t^2} \Delta y_{t+1} \right) \\
&= y_t + 1 - 1 + \frac{y_t}{\psi_{t-1}} \Delta\psi_t + \frac{\Delta\psi_t}{\psi_{t-1}} - \frac{\Delta y_{t+1}}{y_t} \\
&= y_t + y_t \frac{\Delta\psi_t}{\psi_{t-1}} + \frac{\Delta\psi_t}{\psi_{t-1}} - \frac{\Delta y_{t+1}}{y_t} \\
&= y_t + y_t \frac{\Delta\psi_t}{\psi_{t-1}} + \frac{\Delta\psi_t}{\psi_{t-1}} - \frac{y_{t+1}}{y_t} + \frac{y_t}{y_t} \\
&= y_t + \frac{y_t}{y_t} + y_t \frac{\Delta\psi_t}{\psi_{t-1}} + \frac{\Delta\psi_t}{\psi_{t-1}} - \frac{y_{t+1}}{y_t} \\
&\approx y_t + \frac{y_t}{\bar{y}} + \bar{y} \frac{\Delta\psi_t}{\bar{\psi}} + \frac{\Delta\psi_t}{\bar{\psi}} - \frac{y_{t+1}}{\bar{y}} \\
&= y_t \left(1 + \frac{1}{\bar{y}} \right) + \frac{\Delta\psi_t}{\bar{\psi}} (\bar{y} + 1) - \frac{y_{t+1}}{\bar{y}} \\
&= y_t \left(1 + \frac{1}{\bar{y}} \right) + \frac{\Delta\psi_t (1 + \bar{y})}{\bar{\psi}} - \frac{y_{t+1}}{\bar{y}}
\end{aligned}$$

That is:

$$q_t \approx y_t (1 + 1/\overline{y}) + \Delta\psi_t (1 + \overline{y})/\overline{\psi} - y_{t+1}/\overline{y} \tag{19}$$

Short-run view

Again we take (3) as our starting point and write

$$\begin{aligned}
q_t &= \frac{\psi_t}{\psi_{t-1}} y_t + \frac{\psi_t}{\psi_{t-1}} \frac{y_t}{y_{t+1}} - 1 \\
&= \frac{\psi_t}{\psi_{t-1}} y_t + \frac{\psi_t}{\psi_{t-1}} \frac{\psi_{t-1}/P_t}{\psi_t/P_{t+1}} - 1 \\
&= \frac{\psi_t}{\psi_{t-1}} y_t + \frac{P_{t+1}}{P_t} - 1 \\
&\approx \frac{\psi_t}{\psi_{t-1}} y_t + \frac{\Delta P_{t+1}}{P_t}
\end{aligned} \tag{20}$$

We now linearize $\frac{\psi_t}{\psi_{t-1}} y_t$ around $\bar{y}$ and $\bar{\psi}$:

$$\begin{aligned}
q_t &\approx \bar{y} \left(1 + \frac{\Delta\psi_t}{\bar{\psi}} \right) + \frac{\bar{\psi}}{\bar{\psi}} (y_t - \bar{y}) + \frac{\Delta P_{t+1}}{P_t} \\
&= \bar{y} + \bar{y} \frac{\Delta\psi_t}{\bar{\psi}} + y_t - \bar{y} + \frac{\Delta P_{t+1}}{P_t} \\
&= \frac{\bar{y}}{\bar{\psi}} \Delta\psi_t + y_t + \frac{\Delta P_{t+1}}{P_t}
\end{aligned} \tag{21}$$

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