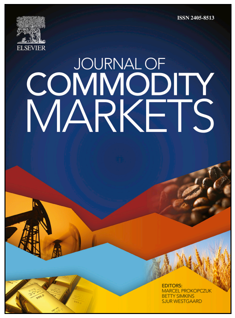

Estimation of value at risk for copper

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Abstract

We analyze various types of models for Value at Risk (VaR) forecasts for daily copper returns. The period of the analysis is from 4 January 2000 to 14 January 2021 including 5,290 daily closing prices. The models considered are GARCH-type models, the Generalized Autoregressive Score model, the Dynamic Quantile Regression model, and the Conditional Autoregressive Value at Risk model specifications. The best model is selected using the Model Confidence Set approach. This approach provides a superior set of models by testing the null hypothesis of equal predictive ability. The findings suggest that the EGARCH model outperforms the rest of the models for the copper commodity under investigation.

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JEL - Classification: C46; C58; G15; F31

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1. Introduction

Over the last few years, commodity markets have experienced significant growth in liquidity and an influx of investors attracted to commodities exclusively as investments (i.e., assets/securities rather than "real assets"), and not as a way to support "real" economic activity through risk hedging (Vivian and Wohar, 2012). While as someone can reasonably expect, the attention rapidly focused on whether the sharp rise of more speculative traders in commodities has triggered large price movements. In light of this, Stout (2011) noted that in the case where speculators disagree then a distortion in market prices is possible, which in turn, will increase the risks of the trader. Thus, one can argue that a better understanding of the nature of variations in the prices of commodities is crucial for a wide range of economic agents such as producers, consumers, governments, and investors. Giot and Laurent (2003) by focusing on commodity markets of aluminum, copper, and nickel, among others pointed out that short-term price swings which come from picks and drop in demand and/or supply result in time frames of extremely volatile behavior and clustering. This type of supply and demand imbalance can arise from many economic factors such as business cycles, major political events, and/or the behavior of investors who are involved in short-term speculation. Another factor may be rooted in the fact that the development of the world market is not so far adept in anticipating international fluctuations in demand. Turning now our attention to the long run, the evolution of prices gives insight into the world's economic health state, thus showing potential bubbles and stagnation periods. For example, fluctuations in copper prices can be viewed as the first sign of global economic performance. This is why several researchers have turned their attention to the influence of commodity price fluctuations and commodity futures returns on the major economic indicators and their role in transmitting inflation and inducing macroeconomic adjustments (see e.g., Labys and Maizels, 1993; Gorton and Rouwenhorst, 2006; Creti et al., 2013; Black et al., 2014; Olson et al., 2014; Jacobsen et al., 2019; Liu et al., 2020; Bannigidadmath and Narayan, 2021; Iyke and Ho, 2021). Additionally, considering that commodity constitutes the main aspect of inflation rate indices, the issue raised is also of relevance for policymakers who adapt their targets in terms of future trends in prices. Except for their uses in industrial settings as raw materials, commodities are widely traded in the markets for hedging and trading. Futures and options contracts are extensively used by trading and financial firms so that they can offset their positions against bear markets. Krehbiel and Adkins (2005) remarked that owing to such high volatility and a risky environment, the urge on the part of practitioners, corporations, and public institutions to get protection against market risk has become imperative.

Our focal point is copper for a variety of reasons. Except for silver, copper has the highest electrical and thermal conductivity of all-natural elements. Not only is it almost one hundred times cheaper than silver but it is also considerably resistant to corrosion, thus rendering copper the most selected metal for all electrical and electronic applications both in the construction industry and for wider industrial uses. Its additional advantage lies in its antibacterial properties which have led to copper being widely used in medical equipment. Considering the significant role that the construction, telecommunication, transportation, and medical sectors play in a modern economy, copper's fluctuations can be viewed as the first sign of global economic performance. In fact, the relatively inelastic supply of copper owing to long technical and resource-based lags in the expansion of production leads to it displaying a prompt reaction to global demand cyclicality, and mostly to demand changes from its largest consumer, namely China. Hence, any kind of change in copper demand manifests itself in movements in copper prices and is regarded by investors as an indicator of changes in global production.

In recent years, a large number of papers suggest useful models for measuring and quantifying market risk. A market risk metric can be viewed as a degree of the level of uncertainty in the future value of an asset or portfolio (see Alexander, 2009). One of the most popular and widely used metrics in asset and risk management is the "Value at Risk" (or VaR hereafter). The VaR measure can be employed to quantify the maximum loss of a position occurring with a given probability over a given period of time. This quantification is of the essence for asset managers and portfolio risk managers, especially when it comes to the design strategies targeted at evading unanticipated great losses. While standard measures of market risk, such as the standard deviation of asset returns, perform well for usual market conditions, the VaR is better fitted for unusual market phases (see Longin, 2000; Bali, 2000, Gkillas and Longin, 2019, among others). In this framework, risk measurement arises as a fundamental tool component of portfolio risk management in defining the proper modeling approach required to quantify commodity price risk exposure. Therefore, VaR predictions enable asset managers and portfolio risk managers to measure their exposure to significant unexpected losses and, hence, moderate the overall riskiness of the market. Lately, this matter has been greatly focused on, with academics and practitioners getting increasingly attracted to it. The existing empirical literature has broadly applied methods to measure and forecast the VaR: for commodities (Giot, 2003; Giot and Laurent, 2003; Chkili et al., 2014; Dolatabadi et al., 2018; Apergis et al., 2020), energy commodities (Pilipovic, 1998; Alizadeh et al., 2008; Fan et al., 2008; Hung et al., 2008; Marimoutou et al., 2009; Aloui and Mabrouk, 2010) and financial markets (Pownall and Koedijk, 1999; Gerlach et al., 2012; Chen and Gerlach, 2013; Youssef et al., 2015; Degiannakis and Potamia, 2017). In this study, we extend the existing scattering literature on VaR forecasts on the copper commodity by examining the accuracy of the estimated VaR by a variety of univariate econometric models employed considering GARCH models (GARCH, Log-GARCH, EGARCH, and GJR-GARCH), Generalized Autoregressive Score (GAS), Conditional Autoregressive Value at Risk (CAViaR) and the Dynamic Quantile Regression (DQR).

Against this backdrop, we proceed to a VaR forecasting exercise and we provide an analytical comparison among some of the most widely used univariate econometric models employed in the existing empirical literature on commodities, whereas many of them have never been used for the case of the copper commodity. Therefore, we contribute to the scattering commodity literature by evaluating the ability of various models to forecast copper tail risk. The intent is to find the model that predicts VaR accurately and replicates the stylized facts encountered in the copper time series. Furthermore, in order to evaluate competing VaR forecasts models and capture the superior model across a range of different VaR forecasts models, we apply the Model Confidence Set (MCS) procedure for the first time in copper for VaR forecasting (see also Maréchal, 2021 for the use the MCS method in copper's realized volatility forecasting). The MCS procedure can be viewed as the confidence interval of a specific parameter as it contains the best prediction model at a certain level of probability. The models considered are GARCH, Log-GARCH, EGARCH, GJR-GARCH, Generalized Autoregressive Score (GAS), Dynamic Quantile Regression (DQR), and the Conditional Autoregressive Value at Risk (CAViaR). The underlying goal is to identify which of the models above forecasts VaR in precision and verify the stylized facts observed in a position in copper. As we can see, we base our analysis on two well-founded models commonly used in the existing empirical literature, namely GARCH-type model specifications having different error term distributions and the CAViaR-type model specifications. The GARCH (Log-GARCH) model was introduced by Bollerslev (1986) (Geweke, 1986), and its most simple form is obtained if we assume an ARMA model for the error variance. The CAViaR model was proposed by Engle and Manganelli (2004) in order to model the quantile of the asset return distribution. The CAViaR model is in fact a quantile regression with an autoregressive formulation straight to the quantile. Additionally, the Exponential GARCH (EGARCH) of Nelson (1991) and the GJR-GARCH model of Glosten et al. (1993) are included so as to take into account the asymmetric impact of negative shocks on volatility. Besides this, we apply Generalized Autoregressive Score (GAS) models following Creal et al. (2013) and Baig et al.(2021). We also make use of a more recently introduced dynamic quantile regression (DQR) model which can consider the dynamic time-varying nature of the time series. The GAS models, which were developed by Creal et al. (2013), have turned out to be an alternative to GARCH models with regard to volatility and VaR modeling. Crucially, these models boast the updating mechanism of the parameters throughout time by means of the scaled score log-likelihood. In order to represent the time-varying volatility of the asset prices and gain insights into the commodity's past and future price behavior, we apply the GAS model which includes GARCH models and has the edge in that it exploits the complete density of the returns instead of the first and second-order moments. These models were employed for the estimation of VaR in asset returns by Kuester et al. (2006), Abad et al. (2013), Ardia et al. (2016), Bernardi and Catania (2016), and Baig et al.(2021) for stocks and FX rates by Lucas and Zhang (2016), for energy markets by Laporta et al. (2018) yet never, to the best of the authors' knowledge, for the copper commodity. Since VaR estimation remains the main objective of the study, the dynamic quantile regression (DQR) model is also employed. This regression allows us to model the quantiles of the copper returns without having to assume any parametric assumption on the error term. For the DQR model, the regression parameters are time-varying as they are allowed to evolve over time following a first-order stochastic process. This model has the potential to validate key stylized facts of a time series and to estimate the VaR metric including the case of a high confidence interval, namely in those conditions in which an underestimation of market risk can end up being highly pricey and may encompass possibly large losses.

From a practical point of view, it is vital to construct models which are able to forecast VaRs effectively in that they are of essence only if they estimate upcoming risks in precision. This is why it is of relevance to assess the viability of the VaR estimates by conducting some targeted statistical tests. In the framework of risk management, backtesting (i.e., the practice for assessing the performance of a forecast-risk model would have been done ex-post) is the most common test procedure, as it has been used in the work of Jorion (2006), Alexander (2009), Christoffersen

(2009), Cont et al. (2010), Emmer et al. (2015), McNeil et al. (2015), Roccioletti (2015) and Nodle and Ziegel (2017), among many others. Regarding market participants, the degree of risk can be the function of either a fall or a rise in the price of the copper commodity, based on their position (see Giot and Laurent, 2003). As concerns short positions now, the level of risk derives from an increase in the price of copper, whilst concerning long positions, the level of risk emerges from a fall in the price of copper. In this practical analysis, we are particularly interested in long positions on the copper market where practitioners are aware of tail risk on the left tail of the return distribution of copper. In our case, the findings, reveal that the EGARCH representations outperform all other models considered at 95% and 99% confidence levels.

The remaining paper is structured as follows: Section 2 provides more information about the role of copper in the modern era and reviews some relevant literature. Section 3 provides the data and some basic statistics of the empirical analysis. Section 4 presents the model used for measuring market risk in copper. Section 5 deals with the backtesting and model confidence set analyses. Section 6 discusses the empirical results, while Section 7 concludes the paper.

2. The role of copper in the modern era

Throughout the last century, there has been a rise in the "industrial" demand for refined copper from 500,000 metric tons to over 19 million metric tons. Given that our progressively smaller world keeps witnessing population growth parallel to increasing expansion, there will be an exponential increase in the demand for copper. The remarkable escalation in the need for copper will indeed lead to the rising demand for new mines and processing plants since the upgrading and expansion of existing facilities are further observed. With the gradual growth of the premises and infrastructure of the so-called "less developed" areas, it is not striking that copper stands out as the most dependable "building block" paving the way to the construction of a new civilization and the improvement in the average standards of living. The latest Chinese construction boom serves as an illustrative case of this phenomenon. Remarkably enough, in 2020, the price of copper rocketed by more than 36%. This unexpected upturn in the midst of the global pandemic found many traders unprepared. Still, this situation can be attributed to a variety of reasons. To begin with, despite the downturn of the majority of economies, in 2020, China's economy thrived. Having recovered in the second quarter, its economy kept on growing for the rest of the year. This boom is of the essence, in that China is the world's biggest purchaser of copper. Also, there has been a reduction in production in most countries, such as Chile and Peru, since the countries were faced with the pandemic. In fact, in 2020, a drop in the overall quantity of mined copper by more than 1%. Likewise, there has been considerable ease in the US dollar in the same year. The dollar index fell by more than 9%, rendering 2020 one of its worst years since 2017. Copper and the US dollar have a negative relation since copper is traded in the currency. Eventually, the institutional demand for copper was widely attested in 2020, which contributed to a further escalation of its price (Nguyen et al., 2020).

The effect of the US dollar on commodities can be specified as follows: Historically, it is observed that when the US dollar appreciates, commodities become more expensive than other nondollar currencies. Additionally, the result of such an effect negatively influences demand. On the other hand, when the US dollar depreciates, commodity prices in other currencies tend to drop, thus leading to an increase in demand. Considering the impact of the US dollar on commodity prices described above, it is easily understood that normally, there is an inverse relationship between the US dollar and commodity prices (Nguyen et al., 2020). The value of the dollar has an impact on commodity prices for a number of reasons. The main reason the value of the dollar influences commodities prices is that raw materials are priced and exchanged in US dollars. In fact, the dollar is the benchmark pricing mechanism for most commodities. Considering that the dollar tends to be the most stable foreign exchange instrument, it is common that most countries to hold dollars as reserve assets. Another important reason is that commodities are often viewed as investment assets traded worldwide. Indeed, commodity markets have attracted the attention of international investors not only as a "safe haven" during periods of turmoil but also as alternative investment assets in portfolios with other financial assets (Baur and McDermott, 2010). Oil and gold are the most widely traded commodities that are widely regarded as the most popular economic indicators. All the above, lead us to the financialization process of commodity markets such as oil, gold, and the US dollar along with other stock prices that have the past years acquired further diversification properties (Zhu et al., 2020; Lombardi et al., 2014). The related literature has highlighted the correlation between each of these assets and the further correlation of commodities to the global business cycle. In that framework, the price dynamics of all these assets are an important indicator of market expectations on the future state of the world economy (Arfaoui and Rejeb, 2017). However, empirical findings in this context, indicate that the pairwise relations between oil–US dollar and gold–US dollar became weaker in longer time frames, suggesting that short-term correlations are much higher in recent years (Zhu et al., 2020). More specifically, when analyzing the dynamic interdependences, it is observed that the crude oil market is mostly dominated by the US dollar while on the other hand, gold leads the US dollar in the short run. In periods of financial crisis, interdependences between oil–US dollar and gold–US dollar is accelerated, and as a result the original relationship between these assets changes (Lai et al., 2016). Another important aspect when analyzing commodities is that commodity prices are often considered as indicators of inflation mainly following two basic channels. More specifically, leading indicators often exhibit measurable economic changes that precede the economic changes in the economy as a whole. This is due to the fact that commodity prices tend to respond quickly to general economic shocks such as increases in demand. Additionally, changes in prices many times reflect systemic shocks, for instance, hurricanes which can decimate the supply of agricultural products and subsequently increase supply costs. In the above example, overall prices would have increased, and inflation would be present.

The strongest case for commodity prices as a leading indicator of expected inflation is that commodities respond quickly to widespread economic and information shocks. Global commodity prices have witnessed wild swings recently, with large increases prior to the global financial crisis followed by declines after the crash (Hegerty, 2016). As a result, commodity price volatility has caused increased uncertainty caused often regarded as a major threat to economic development. This can further cause adverse macroeconomic consequences, in particular in episodes of prolonged or excessive price fluctuations (Jin et al., 2022). As already mentioned, the increased financialization has encouraged financial investors to consider commodity as a distinct financial asset, that faces the worst impact of external shocks further observed on the volatility of commodity prices (Jin et al., 2022; Ordu-Akkaya and Soytas, 2020). In this context, the related literature notes that specific energy and metal USD-based commodities are more volatile in response to exogeneous shocks as measured by shocks in important financial indices representing the stock market, real economy, and foreign exchange markets (Jin et al., 2022).

Following the Economist (2011), which uses a slightly wider categorization of emerging economies than that made use of by the IMF reports, 60% of the world's energy, 65% of all copper, and 75% of all steel are consumed by emerging economies. 55% of the world's oil is used by emerging economies. Still, per capita, oil consumption corresponds to less than 1/5 of the consumption in developed economies. Oil constitutes a chief fuel source made use of across the whole world but the greatest rises in oil consumption are anticipated to result from emerging economies Copper is a major industrial commodity showing a co-movement with the business cycle acted as an efficient barometer of the economic well-being of a region (for co-movements on commodities see, Zaremba et al., 2021). Additionally, copper is one of the key commodities in the financial market, which is due to the fact that copper is used in several major fields in the economy. For instance, on account of its high conductivity rate, it is generally employed in the electrical field. The major electricity grids of the vast majority of countries use copper. Moreover, lately, the continuing change from gasoline cars to electric vehicles has resulted in more demand for copper. This is attributed to the fact that a normal electric car, such as Tesla, needs more copper than a standard fuel car. Consequently, financial participants commonly regard copper as a sign of the health of the world economy. In addition, the price has a tendency to increase when there is growth in the global economy. Likewise, it tends to drop when shocks to the world economy are observed. In terms of the supply, the greatest part of the world's copper originates from countries, including Chile, Peru, China, US, Congo, Australia, and Zambia. Escondida in Chile is the greatest mine. The world's top producer and exporter of copper are Chile, in which copper plays an important role. Considering the high sensitivity to copper price changes, the government of Chile rendered copper the principal benchmark for the country's structural budget rule, which was enacted in 2000, while the country made to decrease the exposure of Chile's GDP oscillations to copper price fluctuations (Spilimbergo, 2002). Accessing precise predictions of the VaR of copper is therefore vital both from Chile's point of view and from a more global viewpoint on the use of copper inputs in production and economic development.

3. Data and descriptive statistics

This section gives a presentation of the data description and descriptive statistics of the copper commodity. In particular, our dataset is made up of daily prices of copper and covers the period from 4 January 2000 to 14 January 2021 incorporating various market phases such as booms and crashes. Therefore, our sample includes 5,290 daily prices. We compute the daily returns as follows:

$$r_t = \log(p_t) - \log(p_{t-1}) \tag{1}$$

where r_t refers to daily returns of copper, p_t is for the daily closing price in day t.

Also, we separate the dataset into two parts: one part which includes 1,600 observations namely the in-sample period, and the other part which includes 3,690 observations namely the out-of-sample period. The out-of-sample period will be our testbed to investigate the performances of inspected models presented in the next section. Table 1 presents the descriptive statistics where we witness that the mean corresponds to 244.8824 and the median is equal to 269.4000, with maximum and minimum being equal to 462.8500 and 60.6000, respectively. The std. deviation is equal to 105.7756; skewness is negative, accounting for -0.3674, which suggests that their distributions are skewed to the left and kurtosis stands for 1.9509, which suggests that the distribution of copper series displays platykurtic creating fewer and less extreme outliers. In Addition, we test the normality with the use of the popular and widely used Jarque-Bera test under the null hypothesis of normality in copper return series.

4. Model specification

In this section, in a bid to estimate VaR, we take into account a set of models which can identify the outstanding properties of this kind of series.

4.1. VaR forecasts

We use GARCH models with some error distributions which allow for the consideration of skewness and heavy tails. In addition, in order to account for the asymmetric effect of positive and negative shocks on the volatility, we apply Log-GARCH, EGARCH, and GJR-GARCH models. In order to capture the dynamic time-varying features of the real indefinite data which generate the process of the copper series, we also encompass the GAS model with some different error distributions which can capture kurtosis and asymmetric effects. These kinds of models include GARCH models and have the edge in that they can exploit the total density of returns and not just the first and second-order moments. This condition, which has not had any application to copper so far, facilitates a sounder representation of the asset price volatility which varies over time and the comprehension of the past and future prices. In an effort to provide a direct model of the quantile of returns, which falls within the major goals of this study, we take into account the four CAViaR model specifications (see Engle and Manganelli, 2004) (for applications of CAViaR models in other commodities see, Xiliang and Xi, 2009; Füss et al., 2010; Ratuszny, 2016; other studies see, Huang et al., 2009; Gkillas et al., 2020; Peng 2021). Moreover, we apply a dynamic quantile regression approach, namely DQR whose dynamic nature is illustrated by a first-order stochastic process on the parameters. Therefore, DQR adjusts to quick fluctuations in prices, while at the same time, it covers high volatility conditions and accounts for the stylized facts. Regarding the models mentioned above, we investigate their specifications and characteristics. We define r_t as the return at time t, where $t = 1,2, \ldots, T$. We determine that $\mathcal{F}_{t-1}$ is the informative set at time $t - 1$ which includes past information.

4.1.1 GARCH and log-GARCH model

Bollerslev (1986) (Geweke, 1986) was the first to introduce the GARCH (Log-GARCH) model. As the model in question at modeling time-varying volatility, has had several successful applications in a great number of fields. As Hansen and Lunde (2005) argued, the simplest version of the GARCH model is the GARCH(1,1) specification. This version is most prevalent in less parsimonious fits in most of its applications. The GARCH(1,1) model is given by the following expression:

$$r_t = \mu + \varepsilon_t,$$

$$\varepsilon_t = \sigma_t d_t, \tag{2}$$

$$d_t | F_{t-1} \sim D_{\Theta}(0,1)$$

and

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{3a}$$

$$\sigma_t^2 = \omega + \alpha \ln \varepsilon_{t-1}^2 + \beta \ln \sigma_{t-1}^2 \text{ (for Log-GARCH)} \tag{3b}$$

$$\omega > 0 \tag{4}$$

$$a \geq 0 \text{ and } b \geq 0 \tag{5}$$

$$a + \beta < 1 \tag{6}$$

where σ_t denotes the time-varying standard deviation. In (2) is determined the time-varying link of the conditional variance σ_t^2. Since past values of σ_t^2 are related to its present values, GARCH models can offer better treatment of clustering and high volatility of the series. What is more, Conditions (4) – (6) must be satisfied to guarantee weak stationary and nonnegative σ_t^2. The differences between the GARCH models concern the specification of the volatility dynamic σ_t^2. The innovation parameter d_t given above is assumed to follow a normal distribution $D_\Theta(0,1)$ with 0 mean and variance equal to 1. Θ denotes the vector of parameters ruling the behavior of the distribution. The vector of the parameters $\Pi = (\mu, \omega, \alpha, \beta, \Theta)$ is estimated by MLE. Here, following Laporta et al. (2018), we take into account four different distributions $D_\Theta(0,1)$ for the innovation term d_t. These are: (i) the normal, (ii) the skew-normal, (iii) the student-t, and (v) the skew-student-t distribution.

4.1.2 EGARCH model

The assumption underlying GARCH models is that σ_t^2 responds symmetrically to past shocks ε_{t-1}^2, whereas, as the existing literature suggests the violation of that restriction, specifically asset returns display a negative correlation with changes in return volatility. Given that σ_t^2 has an asymmetric response to positive and negative news, Nelson (1991) put forward the Exponential GARCH (EGARCH). For this GARCH model specification, the log conditional variance is given as follows:

$$\ln(\sigma_t^2) = \omega + \alpha d_{t-1} + \beta \ln(\sigma_{t-1}^2) + g(d_t), \tag{7}$$

$$g(d_t) = \gamma(|d_{t-1}| - E[|d_{t-1}|]) \tag{8}$$

where the parameter a captures the sign effect, whereas the parameter γ captures the size effect. The term $g(d_t)$ stands for the magnitude effect. By construction of process, $g(k_t) = ad_{t-1}\gamma(|d_{t-1}| - E[|d_{t-1}|])$ enables the conditional variance process to have an asymmetric response to increases and drops in prices. The two parameters of $g(k_t)$ are ad_{t-1} and $\gamma(|d_{t-1}| - E[|d_{t-1}|])$, which both have zero mean. In range $0 < d_{t-1} < +\infty$, $g(k_t)$ is linear and the slope stands for $a + \gamma$, while in the range $-\infty < d_{t-1} \leq 0$, $g(k_t)$ is linear and the slope corresponds to $a - \gamma$, indicating that the volatility process responds asymmetrically to positive and negative shocks (rises and falls in commodity prices). The parameter vector $\Gamma = (\mu, \omega, \alpha, \beta, \Theta)$ in Equation (7), in addition to Equation (2), is obtained by MLE. We also take into account four different distributions, that is, (i) the Normal, (ii) the skew-normal, (iii) the Student-t, and (iv) the skew-Student-t distribution.

4.1.3. GJR-GARCH model

The GJR-GARCH model of Glosten et al. (1993) accounts for asymmetries responding to the conditional variance through the inclusion of a dummy variable (indicator function) in its equation. The GJR-GARCH(1,1) is given below:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \eta \boldsymbol{I}_{\{\varepsilon_{t-1}<0\}}\varepsilon_{t-1}^2, \tag{9}$$

where $\boldsymbol{I}_{(\varepsilon_{t-1})} = \begin{cases} 1, if\ \varepsilon_{t-1} < 0 \\ 0,\ if\ \varepsilon_{t-1} > 0 \end{cases}$. The parameter vector $\Psi = (\mu, \omega, \alpha, \beta, \Theta)$ estimates in Equation (9), along with Equation (2) (see sub-subsection 3.1.1), are obtained again by MLE. In the present study, we also take into account three distributions presented as (i) the Normal, (ii) the skew-normal, (iii) the Student-t.

4.1.4. GAS model

Creal et al. (2013) proposed the Generalized Autoregressive Score (GAS) model. The main key factor of the GAS model is the use of updating model parameters over time through the score of the log-likelihood function (see, Blasques et al., 2014; Fonseca and Cribari-Neto, 2018; Bernardi and Catania, 2019). A major benefit of this model is that it allows us to exploit full likelihood information. Thus, by considering a scaled (local density) score step as a driving mechanism, the time-varying parameter automatically decreases its one-step-ahead forecast error at the current observation concerning the current value of the parameters. This new type of model has several attractive features as it gives a unified and consistent context for introducing time-varying parameters in an extensive range of nonlinear specifications. Creal et al. (2013) noted that *"since the GAS model is based on the score, it exploits the complete density structure rather than means and higher moments only"*. In formal terms, the GAS model assumes that r_t is generated as follows:

$$r_t \sim p(r_t | f_t, \mathcal{F}_{t-1}; \theta_t) \tag{10}$$

and θ_t is given by the following autoregressive equation,

$$\theta_{t+1} = \omega + As_t + B\theta_t \tag{11}$$

where $\theta_t \in \Theta \subseteq R^p$ corresponds to a vector of time-varying parameters, ω denotes a vector of constants, and A and B denote coefficients matrices of proper dimension. In the above equation,

θ_t derives from two components. These are the vector s_t and the autoregressive factor θ_t. The former vector is given by the following:

$$s_t = S_t\theta_t \nabla_t, \text{ where } \nabla_t = \frac{\partial \ln f(r_t;\theta_t)}{\partial \theta_t} \tag{12}$$

where $S_t\theta_t$ corresponds to a $p \times p$ positive definite scaling matrix function at time t, and $\nabla_t(r_t, \theta_t)$ denotes the score of the density function of $f(r_t, \theta_t)$assessed at θ_t. Creal et al. (2013) showed that setting the scaling matrix $S_t\theta_t$ to a power c of the inverse of the information matrix of θ is usually preferred:

$$S_t\theta_t = I_t(\theta_t)^{-\gamma} \tag{13}$$

and

$$I_t\theta_t = \mathbb{E}_{t-1}[\nabla_t \nabla_t{}'] \tag{14}$$

where the expectation $\mathbb{E}_{t-1}$ stands for the expectation with respect to $(r_t|f_t, \mathcal{F}_{t-1}; \theta_t)$, and the parameter γ takings value in $\left\{0, \frac{1}{2}, 1\right\}$. In this study, we define $\gamma = 1$ suggested by Creal et al. (2013). In order to ensure weak stationarity of the process in (11), eigenvalues of B must lie within the unit circle. This suggests that $(\boldsymbol{I} - B)^{-1}\omega$ accounts for the unconditional mean of θ_t, where $\boldsymbol{I}$ denotes the identity matrix. We obtain the parameter estimates of vector $\Sigma = (\omega, A, B)$ through MLE expressing the maximization problem as follows

$$\hat{\theta} = \arg\max_{\theta} \sum_{t=1}^{n} l_t \tag{15}$$

where $l_t = p(r_t|f_t, \mathcal{F}_{t-1}; \theta_t)$ for a realization of r_t.

In the same way as in the GARCH model, by coupling Equations (10) and (11), defining a GAS(1,1) model specification. In the remainder of this work, we will consider GAS(1,1) under the assumption of four different conditional distributions. These are: (i) the Normal, (ii) the skew-normal, (iii) the Student-t, and (iv) the skew-Student-t distribution.

4.1.5. CAViaR model

Engle and Manganelli (2004) introduced the Conditional Autoregressive Value at Risk (CAViaR). Contrary to GARCH and GAS models, the quantile of the return distribution is directly modeled by the CAViaR using Koenker and Bassett’s (1978) standard quantile regression approach. An autoregressive formulation is used directly to the quantile by this model. In particular, let $f_t(\boldsymbol{\beta})$ stand for the τ-th conditional quantile at time t of the asset return distribution conditional on $\mathcal{F}_{t-1}$. Let $\boldsymbol{x}_t$ stand for the matrix of

$$r_t = f_t(\beta) + \varepsilon_\tau,\ q_\tau\left(\varepsilon_{\tau,t}\middle|F_t\right) = 0 \tag{16}$$

and

$$f_t(\boldsymbol{\beta}) = g(\boldsymbol{x}_{t-1}, \boldsymbol{\beta}_\tau) \tag{17}$$

where $t = 1, \dots, T$. $q_\tau\left(\varepsilon_{\tau,t}\middle|\mathcal{F}_t\right)$ corresponds to the τ-th quantile of $\varepsilon_{\tau,t}$ of conditional on the information set at time t. Additionally, the functional form $g(\cdot)$ connects $f_t(\beta)$ with noticeable variables falling into $\mathcal{F}_{t-1}$. In its representation $\boldsymbol{x}_{t-j}$, may consist of predetermined variables or lagged exogenous information or endogenous information (i.e., past copper returns). A generic CAViaR specification could be followed (Engle and Manganelli, 2004),

$$f_t(\boldsymbol{\beta}) = \beta_0 + \sum_{\iota=1}^{m} \boldsymbol{\beta}_i f_{t-i}(\beta) + \sum_{j=1}^{n} \boldsymbol{\beta}_j l\left(\boldsymbol{x}_{t-j}\right) \tag{18}$$

where $p = m + n + 1$ corresponds to the dimension of vector $\boldsymbol{\beta}$ and l stands for the function of a finite number of lagged values of observables. Last, $\boldsymbol{\beta}_i f_{t-i}(\beta)$ $i = 1, \dots, q$ is an autoregressive part in which the quantile changes in time. In this paper, we solely use endogenous information based on lagged copper returns. Following Engle and Manganelli (2004) we used the following four forms of the CAViaR model specifications:

(i) Adaptive:

$$f_t(\beta) = f_{t-1}(\beta) + \beta\{[1 + \exp(G[r_{t-1} - f_{t-1}(\beta)])]^{-1} - \tau\}, \tag{19}$$

where $G \in \mathbb{R}^+$ is some positive finite number. Following Engle and Manganelli's (2004), we set $G = 10$.

(ii) Symmetric absolute value:

$$f_t(\boldsymbol{\beta}) = \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + \beta_3|r_{t-1}|. \tag{20}$$

(iii) Asymmetric slope:

$$f_t(\boldsymbol{\beta}) = \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + \beta_3(r_{t-1})^+ + \beta_4(r_{t-1})^-, \tag{21}$$

where $(r_t)^- = -min(r_t, 0)$ and $(r_t)^+ = max(r_t, 0)$ correspond to the negative and positive parts of r_t, respectively. Asymmetric slope CAViaR model asymmetric response r_{t-1}, thus, those allow negative and positive returns to have a different impact on VaR.

The final specification is called Indirect GARCH(1,1) which is modeled under the assumption that the underlying data process follows a true GARCH(1,1) with an i.i.d. error distribution, given by:

(iv) Indirect GARCH(1,1):

$$f_t(\boldsymbol{\beta}) = (\beta_1 + \beta_2 f_{t-1}^2(\boldsymbol{\beta}) + \beta_3 r_{t-1}^2)^{1/2}. \tag{22}$$

Symmetric absolute value and Indirect GARCH(1,1) CAViaR models respond symmetrically to r_{t-1}. For any fixed q-th quantile, the quantile regression is employed to estimate the vector of parameters $\boldsymbol{\beta}$ in Equations (19)–(22), thereby minimizing the quantile version of a loss function suggested by Koenker and Bassett (1978).

4.1.6. DQR model

We also take into account another regression model (DQR) which has the ability to capture the dynamic nature of the copper returns under consideration. This model was suggested by Xiong and Tian (2014) and applied by Laporta et al. (2018) and Baig et al.(2021), among others (for the dynamic panel quantile regression model see Harding et al., 2020). As for the DQR model, its parameters are allowed to vary over time under the hypothesis of a first-order stationary stochastic process. The DQR model is given as follows:

$$r_t = \boldsymbol{x}_{t-1}' \beta_{\tau,t} + \varepsilon_{\tau t} \tag{23}$$

and

$$\beta_{\tau,t} = \omega_\beta + \varphi \beta_{\tau,t-1} + h\big(r_{t-1}, \gamma_\beta\big), \tag{24}$$

for $t = 2, 3, \ldots, T$ where $\tau \in (0,1)$ and $\boldsymbol{x}_t = \big(1, x_{1,t}, x_{2,t}, \ldots, x_{p,t}\big)'$ is the set of covariates. In Equation (23), it is assumed that the τ-th quantile of $\varepsilon_{\tau t}$ is equal to zero, and therefore, $q_t\big(r_t \big| \boldsymbol{x}_{t-1}, \beta_{\tau,t}\big) = \boldsymbol{x}_{t-1}' \beta_{\tau,t}$. In Equation (24), $\omega_\beta \in \mathbb{R}^{p+1}$ and φ denotes a square diagonal matrix (i.e., a symmetric matrix, so this can also be called the symmetric diagonal matrix) including the autoregressive parameters, eigenvalues of which are supposed to lie within the unit circle to maintain stationarity. Furthermore, the function $h: \mathbb{R}^{p+2} \to \mathbb{R}^{p+1}$ serves as a forcing or running variable, which in turn it may be dependent on the vector of parameters γ_β; in this study, in line with Laporta et al. (2018) we take that $h(r_{t-1}, \gamma_\beta) = \gamma_\beta |r_{t-1}|$. It is also assumed that $\beta_{\tau,1} = (\boldsymbol{I}_{p+1} - \varphi)^{-1}\omega_\beta$, where $\boldsymbol{I}_{p+1}$ corresponds to the identity matrix of dimension $p+1$. The independent variables $\boldsymbol{x}_{t-1}$ may encompass exogenous variables in addition to past copper returns. Here, we assume that $\boldsymbol{x}_{t-1} = r_{t-1}$, where r_{t-1} denotes past copper returns.

5. Backtesting

Value at Risk (VaR) has turned out to be one of the most commonly used risk measurement metrics among asset managers and portfolio risk managers. Therefore, the investigated of models which can predict the VaR precisely is of great significance in that their usefulness lies in their potential to predict future risks with accuracy. This is why the evaluation of the quality of the VaR estimates by conducting a set of targeted tests is of relevance. Backtesting is considered to be the most frequently employed technique that is commonly implemented to validate the accuracy of the VaR forecast. This method is based on quantitative tests which inspect the model performance with regard to precision and efficiency in terms of a determined criterion.

In this work, we assess the forecasting performance of copper returns of each model considered by employing the following tests: (i) the Actual over Expected (denoted as AE) exceedance ratio, (ii) the Unconditional Coverage (denoted as UC) test proposed by Kupiec (1995), (iii) the Conditional Coverage (denoted as CC) test suggested by Christoffersen (1998), (iv) and the Dynamic Quantile (denoted as DQ) test proposed by Engle and Manganelli (2004). The latter test can be regarded as an overall goodness-of-fit test for the VaR estimations. The AE ratio identifies the actual number of times that the returns have been in excess of the estimated VaR over the expected VaR violations. For example, since daily VaR estimates are computed at a t-confidence level, a percentage of violations of $100(1-t)\%$ would be anticipated. More specifically, in order to obtain better VaR estimations, the ratio must tend to one. The UC test is a likelihood ratio test, where the null hypothesis states that the unconditional probability of a violation is equal to $1-t$. The CC test constructs a likelihood ratio test in which the observed violations must be independently distributed. These tests are asymptotically distributed following a χ^2 distribution with one degree of freedom for the UC test and two degrees of freedom for the CC test. The DQ test is based on a regression-based method. In particular, we set up a linear form of a regression model, and for this model, we consider the sequence of violations as the response variable, whereas past violations and/or any other variables are considered as the explanatory variables. This test statistic is also distributed following a χ^2 distribution with q degrees of freedom (i.e., the number of lagged violations). In this work, the DQ test uses both VaR forecasts and lagged violations at lag q equal to four.

Conducting the backtesting technique does not necessarily mean that a specific model estimate outperforms the rest of the other models considered. This is why in an attempt to provide a more thorough analysis with respect to the best-fitting model, the Model Confidence Set (MCS) process is employed in the subsection below.

6.1. Model confidence set

We follow the superior set of models (SSM) deriving models from various competing ones. Being first put forward by Hansen et al. (2011), the MCS method proposes a criterion to differentiate between an initial set of models C^0 of dimension m and a set of best models C^* of dimension c^*, in which $c^* \leq c$. The MCS generates the SSM $\hat{C}^*_{1-\delta}$, which constitutes a subset that includes the best models (with a given confidence level $1-\delta$). The MCS process is performed in the following steps: (i) stipulating the initial set of models C^0; (ii) conducting a test of equal predictive ability (EPA test hereafter) with regard to a defined loss function for each model in C^0; and (iii) in the case where the null hypothesis of the EPA test for a particular set of models is not rejected, terminating the method and handing over $\hat{C}^*_{1-\delta}$; otherwise, excluding the worst model by employing a defined elimination rule and returning to step (ii). Following Hansen et al. (2011), we present formally the MCS procedure when the loss function under consideration is associated with quantiles. We assume that $\widehat{VaR}^q_{i,t}$ is the VaR estimate of model i at time t at level q. The $\ell_{i,t}$ (which denoted the loss function) related to the *i*-th model is given as follows:

$$\ell_{i,t} = \ell\left(y_t, \widehat{VaR}^q_{i,t}\right). \tag{25}$$

At this point, an asymmetric loss function is considered, namely:

$$\ell\left(r_t, \widehat{VaR}^q_{i,t}\right) = \begin{cases} (\tau - 1)\left(r_t, \widehat{VaR}^q_{i,t}\right) if\ r_t < \widehat{VaR}^q_{i,t} \\ \tau\left(r_t, \widehat{VaR}^q_{i,t}\right) if\ r_t \geq \widehat{VaR}^q_{i,t} \end{cases} \tag{26}$$

This loss function corresponds to the quantile loss function. It is this very loss that penalizes heavily negative returns which are in excess of $\widehat{VaR}^q_{i,t}$. To build the EPA statistic, the $d_{ij,t}$ is defined as the loss differential of model i in comparison with model j, whereas $d_{i,t}$ is defined as the average loss differential between model i and all other compelling models in the generic C set of them at time t. Hence:

$$s_{ij,t} = \ell_{i,t} - \ell_{j,t} \text{ for all } i, j \in M \tag{27}$$

and

$$s_{i\cdot,t} = \frac{1}{m-1} \sum_{j \in M} s_{ij,t} \tag{28}$$

By intuition, there is a preference for the alternative i over j when $s_{ij,t} < 0$. The EPA null and the alternative hypothesis of the test are assumed to have the following forms:

$$H_0 \colon \mathbb{E}[s_{i\cdot}] = 0 \text{ for all } i \in C \text{ and } H_1 \colon \mathbb{E}[s_{i\cdot}] \neq 0, \text{ for some } i \in C. \tag{29}$$

To test H_0 against H_1 in Equation (29), Hansen et al. (2011) suggested a t-statistic, namely:

$$t_{i,\cdot} = \frac{\bar{s}_{i,\cdot}}{\sqrt{\widehat{var}(\bar{s}_{i,\cdot})}} \text{ for all } i \in C \tag{30}$$

where $\bar{s}_{i,\cdot} = \frac{1}{m-1} \sum_{j \in M} \bar{s}_{ij}$ denotes the sample loss of model i (which is assumed to be relative to the average losses across any other model j), while $\bar{s}_{ij} = \frac{1}{n} \sum_{j \in M} \bar{s}_{ij,t}$ measures the average loss between model i and j. Also, $\widehat{var}\left(\bar{s}_{i,\cdot}\right)$ corresponds to the bootstrap estimate of $var(\bar{s}_i)$. Lastly, the EPA hypothesis in Equation (29) is tested by applying:

$$T_{max,M} = \max_{i \in M} t_{i,\cdot} \tag{31}$$

The MCS technique calculates the statistic in the equation above. Again, a large value of $t_{i,\cdot}$,·shows that the estimates of a specific model, say model i are distant from actual realizations compared to those of any other model $j \in M$, and consequently, the i-th model may be discarded from M. Following Hansen et al. (2011), a coherent elimination rule with the statistic in Eq. (31) is the following:

$$E_M = \arg\max_{i \in M} \frac{\bar{s}_i}{\sqrt{\widehat{\text{var}}\ \bar{s}_{i,\cdot}}} = \arg\max_{i \in M} t_{i,\cdot} \tag{32}$$

The elimination rule in Equation (31) dismisses the model which is conducive to increasing t_i the most. If a specific model i is found to have the highest standardized excess loss (relative to the average across all other models in C), it will be the worst model. At every single iteration, if the null hypothesis given in Equation (29) is rejected at a specific fixed level of significance denoted by δ, then, the elimination rule given in Equation (32) invalidates the worst model. At this point, the procedure recomputes the statistic in Equation (30) for all $i, j \in C \subset C^0$. The iterations cease only when the null hypothesis of EPA in Equation (29) is not rejected, and eventually set the SSM $\hat{C}^*_{1-\delta}$.

6. Results

In this section, we first present the tail risk results as different VaR forecasts considering (i) GARCH, (ii) Log-GARCH, (iii) EGARCH, (iv) GJR-GARCH, (v) GAS, (vi) CAViaR, and (vii) DQR in several distributions of returns taking into account the normal, skew-normal, student-t and skew-student-t distribution. Second, we present the MCS results in order to reveal the "superior set of models" for the copper commodity series.

6.1. Tail risk results

Concerning copper time series, we apply all the models given in Section 4 by providing tail risks forecast results. Such evidence is crucial for commodity risk managers to assess the exposure to large, unexpected losses and, therefore, to mitigate the overall riskiness of the copper market. Under this framework, a better tail risk analysis turns out to be a necessary factor of risk evaluation and management in verifying the proper modeling basis needed to measure copper price risk exposure. To gain knowledge on those models, we make a comparison among the models considered individually with regard to the AIC, SIC, and Shibata criteria. The values of these criteria are available upon request.

As the findings indicate, it is inferred that the GARCH-N and EGARCH are reliable concerning the conclusions drawn about the copper commodity. On the other hand, after noticing the quantile models, we deduce that the CAViaR-Sym model stands out on the basis of all three information criteria at 95%. Nevertheless, it is remarkable that there is no association between such model selection criteria and the VaR forecasting ability, which means that their overall performance may not be regarded as sufficient in case the assessment of risk arises as to the major concern. Therefore, in this study, we make use of the backtesting procedure which is more apposite for the validation of the VaR precision prediction of a model. In simple words, backtesting assesses the "accuracy" of the estimated models considered in this study by "counting" the number of actual price movements that are higher than the estimated value of the tail risk (i.e., VaR) and assessing this with the theoretically expected number of excesses for a given probability level. Obviously, the closer the empirically observed number of excesses is to the theoretically expected number, the better the model is for forecast tail risk exposure.

More analytically, within this framework, the estimation of each model is conducted on the first 1,600 daily copper returns and subsequently, the method is rolled forward by the addition of the next day and the drop of the most distant day. In the present study, we apply an iterative procedure to compute the 1-day-ahead VaR beginning at the in-sample period, namely the first 1,600 observations. Next comes the estimation and recording of the VaR predicted for subsequent assessment. At the consecutive iterations, we roll forward the estimation sample for the inclusion of one extra day and the drop of the more distant day in order to ensure that the estimation window does not change in terms of size. We then re-estimate the model at each iteration and record and forecast the VaR. The procedure is iterated to the extent that we attain the inclusion of all days, that is, until the 5,290th observation. In this study, we deal with the VaR at two confidence levels, namely 95% and 99%. Table 2 provides the backtesting results for the out-of-sample VaR.

At a 95% confidence level, it becomes evident that all GARCH and EGARCH specifications are valid for the copper commodity. In other words, this means that this family of models is suitable for catching volatility clustering effects, as they failed only for a number of exclusions. It also showcases that in the case of the assumption of the Gaussian distribution, the GARCH model is indeed able to consider high kurtosis limited to the series effectively. Actually, GARCH-N models pass the LRuc and the LRcc tests at the 95% confidence level for the copper commodity. Remarkably enough, EGARCH models can guarantee adequate results for the cases in which they do not the DQ test. All models, namely GARCH, Log-GARCH, EGARCH, GJRGARH, CAViaR, and GAS, do not pass the DQ test, except for the GAS-skew-T model exclusively, which do not pass all three tests invariably. The DQR although can take into account some well-known stylized facts that exist in the traditional financial markets and predict tail risk estimates even for very high confidence levels (i.e., in extreme cases where a miscalculation of risk can be exceedingly damaging and may entail possibly extreme losses and bankruptcy) do not pass the DQ test. On the other hand, for the copper market normal distribution seems to accommodate the return distribution behavior much more efficiently than the Student-t and skew-Student-t distributions, which shows the symmetry of the empirical distribution of returns and its somewhat high kurtosis. However, in case the empirical distribution displays a pronounced leptokurtic feature, it is the Student-t and the skew-Student-t exclusively, which can fit the data properly, yet this is not always the case. This matter is non-existing when VaR is estimated through EGARCH and GJR-GARCH or when the quantile is modeled directly by implementing the CAViaR-Sym model. The AE ratio of EGARCH varies in a range much closer to one: it oscillates between 0.935 and 1.000, which permits a more precise risk evaluation. This shows that the leverage effect is significant in the copper market and the model fitting is significantly improved by adding the asymmetric effect parameters. Such evidence point to the ability of asymmetric GARCH-type models to generate better results. Lastly, CAViaR-Sym provides a direct as well as dynamic modeling of the quantiles of the distribution of the returns which may be more rational in a quantile estimation problem.

As a final remark, it is also worth mentioning that all of the models under examination do not pass successfully the backtesting analysis at a 99% confidence level. But it should be noted that this evidence of an accurate VaR model is mainly based on hit sequences that satisfy both unconditional coverage and independence properties. This is one of the main reasons that we proceed with the MCS procedure. However, there is some evidence that in order to better forecast tail risk based on a VaR forecast model for the copper market is a combination of the models considered int this study and a Pareto distribution of residuals based on extreme value theory, similar to that used in the peaks-over-threshold method is promising. But this is an idea for future research.

6.2. MCS procedure results

In order to evaluate competing VaR forecast models and capture the superior model across a range of different VaR forecast models for the copper market, we also employ the MCS method. The MCS can be seen as the confidence interval of a specific parameter as it contains the best model for the forecast at a given probability level. On of the main advantages of this method is that recognizes data limitations, such that uninformative data produce an MCS with many models, while informative data produce an MCS with only a few models. The MCS method does not suppose that a specific model is the correct model, but it can be used for comparison of more general objects, and not only for model comparison. As we said before, the method is based on a sequence of tests that allows the creation of a set of ’superior’ models under the null hypothesis of equal predictive ability is not rejected at a certain confidence level. The statistic tests are calculated for an arbitrary loss function, meaning that we could test models on various aspects, for example, punctual forecasts.

Setting out from the initial set C^0 in which all the specifications of the models are accumulated, we obtain the SSM, that is $\hat{C}^*_{1-\delta}$, by employing the MCS procedure presented in Section 6.1. Table 3 reports the $\hat{C}^*_{1-\delta}$ for copper returns at 95% and 99% confidence levels. In each entry, we report the ranking of a specific model within $\hat{C}^*_{1-\delta}$. This ranking encompasses the likelihood of eliminating a model according to the t-statistics. As we can see, it is obvious that the dimension of $\hat{C}^*_{1-\delta}$ commonly varies from 6 to 11, which is suggestive of the equal predictive ability that several models display in VaR forecasting. Looking at this more thoroughly, it becomes apparent that the performances of GAS models are dubious on account of the fact that they are frequently dismissed by the MCS method exhibiting poor results at both confidence levels considered (i.e., 95% and 99%). In addition, the GJR-GARCH model specification, which commonly falls into the $\hat{C}^*_{1-\delta}$ set and rank better than GAS models, have a poor performance both at 95% and 99% confidence level. Rather, there is clear evidence that EGARCH model specifications do generate satisfactory results for the copper commodity series and for both confidence levels. In comparison with GARCH, GJR-GARCH, GAS, and CAViaR models, their performance is noteworthy, as they always rank at least at the top end of $\hat{C}^*_{1-\delta}$. Therefore, the MCS method verifies that EGARCH models slightly outperform the other models considered here (i.e., GARCH, Log-GARCH, GJR-GARCH, GAS, CAViaR, and DQR models). It is noteworthy that models which have undergone successful backtesting occupy the top positions in $\hat{C}^*_{1-\delta}$. This bears confirming evidence of the good performance that the asymmetric GARCH-type models based on different selection criteria. In other words, this means that this family of models is suitable for catching volatility clustering effects in the copper market, as they failed only for a number of exclusions. It also showcases that under the assumption of the normal distribution, the GARCH-type models are capable to take into account kurtosis limited to the time series successfully.

7. Conclusion

Copper is one of the key commodities in the commodity market, which is due to the fact that it is used in several major fields in the economy. For example, on account of its high conductivity rate, it is generally employed in the electrical field. But copper prices can be highly volatile and actually can undergo extreme volatility conditions. Such conditions always represent a painful experience for investors in the commodity market as they can experience great losses. Thus, commodity market participants operating within the copper market can hedge commodity positions via derivatives transactions. Nevertheless, the intended results can be attained after the implementation of the hedging strategy only in case it is accompanied by a precise evaluation of tail risk. In this context, risk measurement appears to be an integral part of risk management with respect to the determination of the suitable modeling framework needed for the quantification of risk exposure in the copper market.

In this study, we proceed to the measurement of risk exposure through VaR estimation, and we examine the performances of various models for the copper commodity. The VaR metric is one of the most popular and widely used metrics in asset and risk management as it quantifies the maximum loss of a position occurring with a given probability over a given period of time. Specifically, we employ well-known models in the existing empirical literature. The models considered are GARCH, Log-GARCH, EGARCH, GJR-GARCH, Generalized Autoregressive Score (GAS), Dynamic Quantile Regression (DQR), and the Conditional Autoregressive Value at Risk (CAViaR). The underlying goal is to identify which of the models above forecasts VaR in precision and verify the stylized facts observed in a position in copper. For each model under examination, we carry out the backtesting analysis, and then the predictive potential of VaR forecasting has been tested by applying the model confidence set method. Our dataset is made up of daily prices of copper and covers the period from 4 January 2000 to 14 January 2021 including 5,290 daily prices and incorporating various market phases such as booms and crashes. The empirical results give insights into the edge that the asymmetric GARCH type models, namely the EGARCH, have over the remaining models with regard to backtesting and the MCS procedure, especially at a 95% confidence level.

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Tables

Table 1. Descriptive statistics for cryptocurrencies

	Copper price
Mean	244.8824
Median	269.4000
Maximum	462.8500
Minimum	60.6000
Std. Dev.	105.7756
Skewness	-0.3674
Kurtosis	1.9509
Jarque-Bera	361.3713***
Probability	[0.0000]
Observations	5286

Note: This table reports descriptive statistics for the price series of the copper commodity. The null hypothesis that the jump series are normally distributed is tested by the Jarque-Bera test. ***, **, * refers to a rejection of the null hypothesis that the data is normally distributed, at 1, 5, and 10%, respectively.

Table 2. Copper backtesting results

	Mean	SD	LRuc	LRcc	AE	DQ	Mean	SD	LRuc	LRcc	AE	DQ
	95% confidence level						**99% confidence level**					
GARCH - N	-0.0290	0.0113	0.0063	0.0626	1.0057	15.6043^{***}	-0.0408	0.0160	148.1485^{***}	148.3427^{***}	0.2609	115.8024^{***}
GARCH – skew - N	-0.0287	0.0114	0.0237	0.0645	1.0111	14.5184^{***}	-0.0404	0.0163	142.7053^{***}	142.8411^{***}	0.2718	112.2961^{***}
GARCH - T	-0.0284	0.0114	0.5683	0.6310	1.0546	14.9687^{***}	-0.0450	0.0170	198.4810^{***}	199.6284^{***}	0.1740	139.8807^{***}
GARCH – skew - T	-0.0284	0.0116	0.6858	0.7322	1.0601	16.0958^{***}	-0.0448	0.0173	194.9293^{***}	195.9863^{***}	0.1794	138.0975^{***}
Log-GARCH - N	-0.2708	0.0150	0.8123	1.3566	0.9943	14.6438^{***}	-0.0444	0.0150	203.5449^{***}	155.1034^{***}	0.1644	117.0449^{***}
Log-GARCH – skew - N	-0.2718	0.0110	0.8091	0.9952	0.9852	14.6076^{***}	-0.4510	0.1874	195.6029^{***}	156.4190^{***}	0.1617	115.2693^{***}
Log-GARCH - T	-0.2753	0.0107	0.8176	1.4474	0.944	14.5099^{***}	-0.0430	0.1524	196.7801^{***}	153.4101^{***}	0.1593	127.4302^{***}
Log-GARCH – skew - T	-0.2801	0.0110	0.8237	1.0925	0.9855	15.6012^{***}	-0.0425	0.0146	195.5530^{***}	148.5024^{***}	0.1607	144.2397^{***}
EGARCH - N	-0.0290	0.0105	0.8345	1.3239	0.9350	16.1539^{***}	-0.0408	0.0147	150.9330^{***}	151.1608^{***}	0.2555	118.0962^{***}
EGARCH – skew - N	-0.0287	0.0106	0.8345	1.9181	0.9350	13.9361^{***}	-0.0404	0.0151	148.1485^{***}	148.3427^{***}	0.2609	117.2203^{***}
EGARCH - T	-0.0284	0.0108	0.0000	0.3672	1.0003	12.7092^{**}	-0.0446	0.0159	205.7758^{***}	207.1195^{***}	0.1631	144.2506^{***}
EGARCH – skew - T	-0.0284	0.0109	0.0218	0.4813	0.9894	12.9807^{**}	-0.0446	0.0163	202.0959^{***}	203.3387^{***}	0.1685	142.4807^{***}
GJRGARCH - N	-0.0290	0.0108	0.6994	1.1398	0.9405	15.8536^{***}	-0.0408	0.0153	142.7053^{***}	142.8411^{***}	0.2718	113.1574^{***}
GJRGARCH – skew - N	-0.0288	0.0110	0.2798	1.0168	0.9622	11.9514^{**}	-0.0404	0.0157	142.7053^{***}	142.8411^{***}	0.2718	113.3329^{***}
GJRGARCH - T	-0.0283	0.0110	0.0218	0.1395	0.9894	11.5902^{**}	-0.0447	0.0163	209.5229^{***}	210.9732^{***}	0.1577	146.3622^{***}
GAS – N	-0.0305	0.0120	0.8345	0.9897	0.9350	26.8123^{***}	-0.0430	0.0170	137.4243^{***}	137.5127^{***}	0.2827	110.1105^{***}
GAS – skew - N	-0.0306	0.0189	0.8143	2.0475	1.0655	60.1517^{***}	-0.0430	0.0264	79.4762^{***}	80.3336^{***}	0.4295	103.2039^{***}
GAS - T	-0.0290	0.0120	0.0471	0.2961	1.0166	17.9861^{***}	-0.0449	0.0186	168.5791^{***}	169.0738^{***}	0.2229	131.0629^{***}
GAS – skew - T	-0.0289	0.0131	6.2792^{**}	6.3786^{**}	1.1851	48.9469^{***}	-0.0457	0.0200	132.2995^{***}	132.3510^{***}	0.2936	107.3856^{***}
CAViaR – Adap	-0.0283	0.0073	0.3658	0.3659	1.0438	35.1322^{***}	-0.1132	0.1875	148.1485^{***}	150.1258^{***}	0.2609	117.8228^{***}
CAViaR – Sym	-0.0284	0.0118	0.0501	0.5344	0.9840	8.8030^{*}	-0.0431	0.0158	174.8441^{***}	175.4552^{***}	0.2120	127.1499^{***}
CAViaR – Asym	-0.0290	0.0109	0.2073	0.6033	1.0329	17.4151^{***}	-0.0444	0.0155	145.4062^{***}	145.5698^{***}	0.2664	115.0943^{***}
CAViaR - IG	-0.0290	0.0107	0.0501	0.1932	0.9840	16.7849^{***}	-0.0427	0.0142	145.4062^{***}	145.5698^{***}	0.2664	115.1366^{***}
DQR	-0.0294	0.0116	0.0218	2.3382	0.9894	21.8541^{***}	-0.0432	0.0145	140.0449^{***}	140.1557^{***}	0.2772	111.6036^{***}

Note: This table represents the backtesting results of the copper commodity both for 95% and 99% confidence levels. AE refers to the Actual over Expected exceedance ratio, UC refers to the Unconditional Coverage test, CC refers to the Conditional Coverage test and DQ refers to the Dynamic Quantile test. The critical values of the LRuc, LRcc, and DQ are 2.71, 4.61, and 7.78 for 90% confidence level; 3.84, 5.99, and 9.49 for 95% confidence level; 6.64, 9.21, and 13.28 for 99% confidence level, respectively. *, **, and *** denote that the null hypothesis is rejected at the 10%, 5%, and 1% significance level.

Table 3. Copper MCS procedure results

	Ranking	
	Panel A	**Panel B**
	95% confidence level	99% confidence level
GARCH - N	11	13
GARCH – skew - N	-	9
GARCH - T	-	12
GARCH – skew - T	-	10
Log-GARCH - N	-	17
Log-GARCH – skew - N	-	-
Log-GARCH - T	-	-
Log-GARCH – skew - T	-	-
EGARCH - N	6	1
EGARCH – skew - N	4	2
EGARCH - T	2	4
EGARCH – skew - T	1	3
GJRGARCH - N	8	8
GJRGARCH – skew - N	5	6
GJRGARCH - T	7	7
GAS – N	-	-
GAS – skew - N	-	-
GAS - T	-	14
GAS – skew - T	-	-
CAViaR – Adap	-	-
CAViaR – Sym	3	5
CAViaR – Asym	9	16
CAViaR - IG	10	15
DQR	-	11

Note: This table represents the superior set of models provided by the Model Confidence Set (MCS) procedure for the copper commodity. Panel A refers to a 95% confidence level (5% 1-day VaR) and Panel B refers to a 99% confidence level (1% 1-day VaR).

The authors of the paper listed below declare that they have no conflicting interest or no competing interests.

Estimation of Value at Risk for copper

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