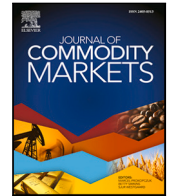


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Journal of Commodity Markets

journal homepage: www.elsevier.com/locate/jcomm

Regular article

Carr and Wu's (2020) framework in the oil ETF option market[☆]Xiaolan Jia^a, Xinfeng Ruan^{b,*}, Jin E. Zhang^{c,*}^a Department of Finance, School of Economics and Business Administration, Chongqing University of Education, Chongqing 400060, China^b Department of Finance, International Business School Suzhou, Xi'an Jiaotong-Liverpool University, Suzhou, 215000, China^c Department of Accountancy and Finance, Otago Business School, University of Otago, Dunedin 9054, New Zealand

ARTICLE INFO

JEL classification:

G13

G12

Keywords:

Crude oil

Risk-neutral covariance

Implied volatility smile

Return predictability

Option pricing

ABSTRACT

This paper studies the information inferred from the Carr and Wu's (2020) formula based on a new option pricing framework in the United States Oil Fund (USO) options. We first document the term structure and dynamics of the risk-neutral variance and covariance rates which lead to a "U"-shaped implied volatility smile with a positive curvature. We then investigate the return predictability of the innovations in the risk-neutral variance and covariance rates (*DRNV* and *DRNC*) and their term structures (*TRNV* and *TRNC*) and find that *DRNC* is a significant and robust predictor to forecast daily, weekly and monthly USO excess returns in both statistical and economic terms based on in-sample and out-of-sample tests.

1. Introduction

This paper studies the information inferred from the (Carr and Wu, 2020) formula based on a new option pricing framework in the United States Oil Fund (USO) option market and examines whether the information is correlated with the future USO excess returns. The no-arbitrage formula links the implied volatility smile to the risk-neutral implied volatility variance and covariance with the security return. The risk-neutral variance and covariance estimates extracted from the formula could contain useful information regarding the future underlying asset returns and provide complementary information for risk management and investment decisions. Our research is the first effort that examines the information content of the risk-neutral estimates in the crude oil market. We also investigate the predictability of the extracted risk-neutral information in forecasting the future crude oil returns.

Compared with the traditional option pricing models (e.g., Black–Scholes model), which links the values of all option contracts to a single reference dynamics specification, the new option pricing framework links the current value of one option contract's implied volatility to current conditional moments of log changes in the security price and the given contract's implied volatility. Under the new framework, the implied volatility surface does not need to start with the full specification of the underlying security price and volatility dynamics. Instead, one can start with analyzing the co-movement structure on the percentage implied volatility changes of contracts across maturity and moneyness levels. This is the main motivation why we use (Carr and Wu, 2020) framework.

Crude oil is the most important commodity in the world and plays a major role in global economic activity. A large literature documents how crude oil prices significantly affect the real economy and financial markets (Hamilton, 1983, 2011; Driesprong et al., 2008; Kilian, 2009; Awartani et al., 2016; Nonejad, 2020). USO tracks the price of near-month West Texas Intermediate (WTI) light, sweet crude oil futures contracts, providing investors with easy access to the oil market.¹ It has become the largest and most liquid

[☆] We are grateful to two anonymous reviewers, Ehud I. Ronn (the associate editor), and Sjur Westgaard (the editor) for helpful comments and suggestions.

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¹ We use USO ETF options partially due to the easy availability of the data from OptionMetrics. The crude oil futures options is also an interesting research topic which is left for further research.

<https://doi.org/10.1016/j.jcomm.2023.100334>

Received 2 June 2021; Received in revised form 9 December 2022; Accepted 3 May 2023

Available online 13 May 2023

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crude oil exchange-traded fund (ETF) traded on the New York Stock Exchange (NYSE) with the inception date of April 10, 2006, with an asset value of over 4.7 billion dollars and an average daily volume of over 5.2 million contracts as of October 5, 2020. As for a tradable asset, studying the future return predictability is important for investors and researchers. Therefore, this paper mainly focuses on the study of return predictability in the USO market.

According to the no-arbitrage formula of the new framework, both the variance and covariance rates determine the shape of the implied volatility smile. The variance rate is the risk-neutral conditional variance rate of the implied volatility percentage change and the covariance rate is the risk-neutral conditional covariance rate of the implied volatility change and the underlying security return. Under commonality assumptions, the market-expected variance and covariance rates can be extracted from the observed shape of the implied volatility smile. There are a few methodologies to quantify the implied volatility smile. [Goncalves and Guidolin \(2006\)](#), [Chalamandaris and Tsekrekos \(2011\)](#) and [Kearney et al. \(2019\)](#) fit the whole implied volatility surface by using functions of moneyness and time to maturity. [Zhang and Xiang \(2008\)](#) and [Soini and Lorentzen \(2019\)](#) use a second-order polynomial to describe the implied volatility–moneyness function and quantify the shape of the implied volatility smile. They all model the implied volatility smile or surface by using the assumed functions rather than functions derived from theories. In contrast, [Carr and Wu \(2020\)](#) derive a no-arbitrage formula from a new pricing theory, which is completely different from others.

In this paper, we first interpolate to construct the floating series of the implied volatility levels on a target maturity-moneyness grid by using the filtered USO option data from May 9, 2007 to June 28, 2019. We document the term structure of the implied volatility smile of USO options, which shows a “U”-shaped implied volatility smile with a positive curvature. On the basis of the no-arbitrage formula of the new framework, we extract the positive variance and negative covariance estimates from the implied volatility smile.

We investigate the USO return predictability of the risk-neutral information inferred from the no-arbitrage formula at daily, weekly and monthly frequencies. Following [Dennis et al. \(2006\)](#), [Ang et al. \(2006\)](#) and [Chang et al. \(2013\)](#), we choose the innovations in the risk-neutral estimates, which are the first differences in variance and covariance estimates ($DRNV$ and $DRNC$) at one-month maturity, as predictors for forecasting the future USO excess returns.² As [Vasquez \(2017\)](#) suggests, the implied volatility term structure conveys information about the future option returns. We are motivated to find whether the term structures of the variance and covariance rates contain information about the future excess returns on USO. We define the difference between the risk-neutral variance rate at one-year maturity and the one at one-month maturity ($TRNV$) and the difference between the risk-neutral covariance rate at one-year maturity and the one at one-month maturity ($TRNC$). Empirically, we find that $DRNC$ is a strong predictor of future USO returns at daily, weekly and monthly horizons based on in-sample and out-of-sample tests. It outperforms existing forecasting variables, including first differences in risk-neutral volatility, skewness and excess kurtosis ($DVOL$, $DSKEW$ and $DKURT$), variance risk premia (VRP) and risk-neutral drift of implied volatility changes (RND). The significant predictive power of $DRNC$ still holds after controlling for crude oil market-specific variables and macroeconomic variables. In addition, $DRNC$ can generate significantly large Sharpe ratio (SR) gains and the certainty equivalent (CE) gains that exceed those provided by other predictors, indicating the economic significance of $DRNC$'s predictive ability. In addition, $TRNC$ and $DRNV$ have some in-sample predictive power at short forecasting horizons.

This paper contributes to the literature that studies the informational content of the implied volatility smile. [Xing et al. \(2010\)](#) show that the shape of the volatility smirk has significant cross-sectional predictive power for future equity returns. [Yan \(2011\)](#) and [Atilgan et al. \(2015\)](#) find a negative relation between the slope of the implied volatility smile and future stock returns. [Han and Li \(2021\)](#) argue that aggregate implied volatility spread is significantly and positively related to future stock market returns at daily, weekly, and monthly to semi-annual horizons.³ These papers highlight the importance of the implied volatility smile on forecasting underlying asset returns. In this paper, we study the new information extracted from the no-arbitrage formula which links the implied volatility smile to the risk-neutral implied volatility variance and covariance with the security return and investigate the return predictability of the extracted risk-neutral information.

Our paper also contributes to the return predictability in the crude oil market. [Yin and Yang \(2016\)](#) investigate the capacity of technical indicators to directly forecast the oil returns and compare their performance with that of macroeconomic variables. [Kang and Pan \(2015\)](#) and [Da Fonseca and Xu \(2017\)](#) examine the return predictability of VRP on oil future returns in the crude oil market. [Ruan and Zhang \(2018\)](#) study the return predictability for the crude oil market by using risk-neutral moments and the differences in them. [Jia et al. \(2021\)](#) use the information from the dynamics of the term structure of implied volatility smile parameters to forecast future oil excess returns. However, our paper differs from the previous literature. We study the oil return predictability by using the risk-neutral variance and covariance estimates from the no-arbitrage formula.

The rest of the paper is organized as follows. Section 2 presents the data and the control variables. Section 3 gives the methodology. Section 4 provides the empirical results, and Section 5 concludes the paper.

² First differencing can remove most of the autocorrelation of time series covariance and variance rates. The daily, weekly and monthly time series of the $DRNC$ and $DRNV$ are more stationary than the original time series.

³ [Xing et al. \(2010\)](#) and [Atilgan et al. \(2015\)](#) use the implied volatility difference between out-of-the-money put options and at-the-money call options to measure the slope of the implied volatility smile which can also be interpreted as the implied volatility spread. [Yan \(2011\)](#) defines the difference between the implied volatilities of one-month-to-expiration put and call options with deltas equal to -0.5 and 0.5 as the slope of the implied volatility smile or the implied volatility spread. [Han and Li \(2021\)](#) calculate the implied volatility spread for each stock as the difference in the implied volatilities for a pair of at-the-money call and put options with 30-day time to maturity.

Table 1
Descriptive statistics for excess returns.

	Mean	Std.dev.	Skewness	Kurtosis	Min	Max
Panel A: USO						
Daily excess returns	-0.0481	2.1630	-0.1934	5.3354	-11.2996	9.1691
Weekly excess returns	-0.2328	4.5758	-0.4291	4.4101	-19.5223	16.6934
Monthly excess returns	-1.0065	9.7959	-0.7543	4.2730	-38.9758	24.0114
Panel B: WTI						
Daily excess returns	-0.0045	2.3875	0.0853	7.6734	-13.0654	16.4097
Weekly excess returns	-0.0238	4.9313	-0.1083	5.6408	-22.7284	23.3456
Monthly excess returns	-0.0942	9.6680	-0.7201	4.5404	-39.5760	26.0165
Panel C: Correlations						
	Daily	Weekly	Monthly			
Correlation	0.9124	0.8684	0.9515			

The table reports the summary statistics and correlations for USO ETF and WTI crude oil excess returns (in percentage) at different horizons (daily, weekly and monthly). Both the USO ETF and WTI crude oil futures data are downloaded from Bloomberg from May 9, 2007 to June 28, 2019. The excess return on USO (WTI) is the return on USO ETF (WTI crude oil futures) in excess of the risk-free rate. The risk-free rate is the one-month Treasury bill rate from the Kenneth R. French Data Library.

2. Data

2.1. USO ETF

USO ETF data are downloaded from Bloomberg for the period from May 9, 2007 to June 28, 2019. USO tracks the price of near-month WTI oil futures contracts.⁴ USO buys WTI oil futures in the nearest monthly contract, rolling to the next month's contract two weeks before expiration. When contracts in the future are priced higher (a situation called contango), USO requires fewer contracts to maintain the ETF's value. Over the long term, the cost of rolling adds up, negatively affecting the performance of the fund. Although the USO performance deviates slightly from the WTI oil prices because of the cost of rolling over oil futures contracts, both have the similar volatility.

We define the USO excess return as the log return on the USO ETF minus the risk-free rate,

$$r_t = \ln(S_t/S_{t-1}) - r_{f_t}, \quad (1)$$

where S_t is the USO ETF price at time t , and r_{f_t} is the one-month Treasury bill rate from the Kenneth R. French Data Library.⁵ The daily USO ETF excess returns and the WTI crude oil excess returns are shown in Fig. 1. We find that they follow the similar pattern. Table 1 reports the summary statistics for USO and WTI crude oil excess returns (in percentage) at different horizons (daily, weekly and monthly). First, the means of both crude oil excess returns at different frequencies are negative, varying from -0.0045% to -1.0065% , even though the average WTI excess returns are very small in absolute value. Second, the standard deviations of the WTI returns and the USO returns are similar, even though the mean of both are different. In other words, USO and WTI have the similar historical volatility. Third, all the skewness of the USO and WTI returns for various time horizons are negative except for the daily WTI excess returns. For example, the skewness of the monthly USO returns is -0.7543 with a negative mean return of -1.0065% , which indicates that the return distribution has a longer left tail in physical measure. Finally, from Panel C, we find that the correlations between the excess returns on USO and WTI at daily, weekly and monthly horizons are high, over 86.84%. Therefore, we consider the WTI excess returns as the alternative market returns to investigate the return predictability of risk-neutral estimates.

2.2. USO ETF options

USO option data are obtained from OptionMetrics. The sample period is from May 9, 2007 to June 28, 2019. We impose several filters on the option data. We first discard options with zero bid price, zero ask price and implied volatility less than 0.01. Then we delete options with less than seven days to expiration. Table 2 reports the trading summary of the USO ETF options by maturity categories after cleaning the data. The statistical variables are mean number of observations, mean daily trading volume, and mean daily open interest. After filtering, we have a total of 1,464,666 observations with a mean number of strikes of 32, a mean daily trading volume of over 78 thousand contracts and a mean daily open interest of over 1.4 million contracts. For the maturity group from 30 days to 90 days, the four statistical variables have the largest values. For maturities less than 90 days, the trading volume and the open interest account for a huge proportion of the total value. That indicates that the shorter the time to maturity, the higher the liquidity. Options investors prefer to trade options for short-term profits.

⁴ After April 17, 2020, USO changed the exposure from holding specifically front-month contracts to holding 80% of its portfolio in front-month contracts and 20% in second-month contracts.

⁵ Kenneth French's website is https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

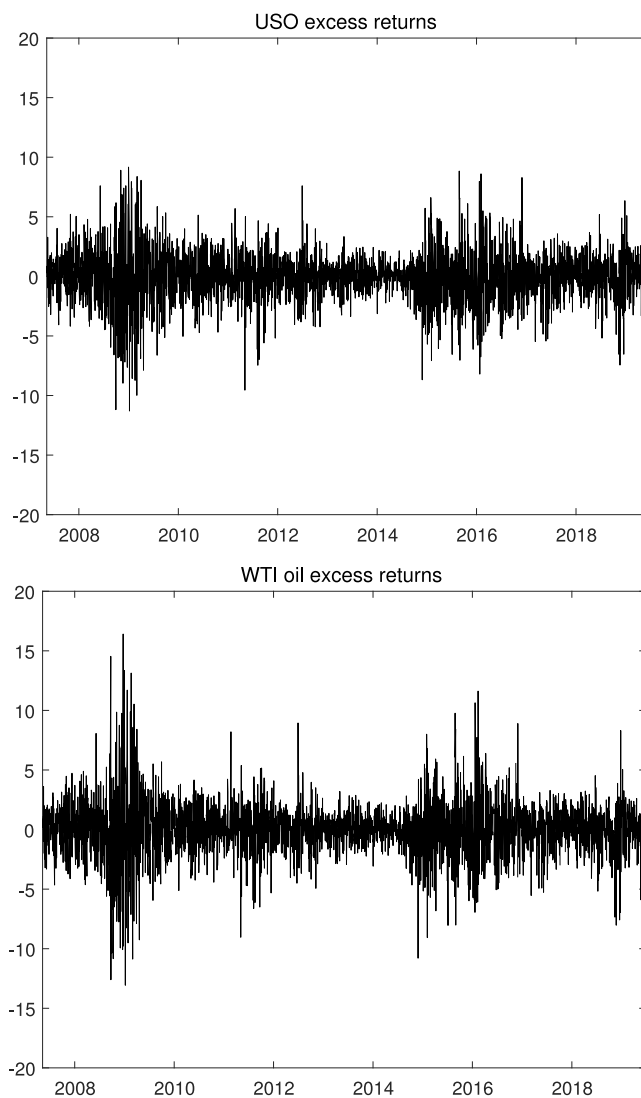


Fig. 1. Daily USO excess returns and WTI oil excess returns.

This figure shows daily time series of the USO ETF excess returns and WTI crude oil futures excess returns from May 9, 2007 to June 28, 2019. All data are downloaded from Bloomberg.

Table 2
Descriptive statistics for daily USO options.

	Overall	< 30	30–90	90–180	180–360	> 360
Number of observations	1,464,666	213,834	432,668	324,876	293,612	199,676
Mean number of strikes	32	25	32	41	38	27
Mean daily trading volume	78,664	32,951	33,668	9,384	5,331	2,341
Mean daily open interest	1,400,951	309,247	476,649	297,943	246,980	146,181

This table shows the number of observations, the mean daily number of strikes, trading volume and open interest of the USO ETF options overall and for each maturity category after cleaning the options data.

2.3. Control predictors

To test whether our main risk-neutral predictor has distinct forecasting ability for future USO returns, we consider a set of control variables, which includes crude oil market-specific variables and macroeconomic variables (Hong and Yogo, 2012; Kang and Pan, 2015).

The crude oil market-specific predictors are described as follows:

1. Crude oil basis (BASIS): Difference between the current crude oil futures price and the spot price. Both prices are obtained from U.S. Energy Information Administration (EIA)'s website.
2. Net short positions of hedgers (NSHORT): Short hedge positions minus long hedge positions, and then divided by total hedge positions (short positions plus long positions), as in Kang and Pan (2015). The short and long positions data are from the Commodity Futures Trading Commission (CFTC)'s Commitments of Traders reports.⁶
3. Open interest growth (OIG): 12-month geometric average of open interest changes, proposed by Hong and Yogo (2012). The open interest data are from CFTC's reports.
4. Historical returns (HRET): 12-month geometric average of crude oil futures returns, as in Kang and Pan (2015). The crude oil futures prices are sourced from EIA's website.
5. Crude oil storage data (STORAGE): The log growth of the U.S. field production of crude oil from EIA's website.
6. Global economic activity index (KI): The index is based on a global index of dry cargo single voyage freight rates, and it is found to significantly influence crude oil prices, proposed by Kilian (2009).⁷
7. USO ETF volatility index (OVX): Market's expectation of 30-day volatility of crude oil prices by applying the VIX methodology to the USO Fund. We consider the log change in *OVX*. The daily *OVX* is downloaded from the website of the Chicago Board Options Exchange (CBOE).

A number of macroeconomic predictors are also included (Bollerslev et al., 2009; Atilgan et al., 2015; Kang and Pan, 2015), which are obtained from the Federal Reserve Bank of St. Louis.

1. Default yield spread (DEF): Difference between the yields on Moody's BAA- and AAA-rated corporate bonds.
2. Term spread (TERM): Difference between the yields on the 10-year Treasury bond and three month Treasury bill.
3. Stochastically detrended risk-free rate (RREL): Yield on the one-month Treasury bill minus its one-year backward moving average.
4. Chicago Fed National Activity Index (CFNAI): A measure of overall economic activity.
5. Industrial Production growth (IP): Log growth in industrial production over the last 12 months, as in Kang and Pan (2015).

Among the above control variables, *OVX*, *DEF*, *TERM* and *RREL* are available at daily, weekly and monthly frequencies. *BASIS* and *NSHORT* are available at weekly and monthly frequencies. *OIG*, *HRET*, *STORAGE*, *KI*, *CFNAI* and *IP* are only available monthly.

Table 3 shows the descriptive statistics for the control predictors at different frequencies. In Panel A, the daily mean of *DEF* is 1.119%, with a standard deviation of 0.510%, and the daily mean of *TERM* is 2.024%, with a standard deviation of 0.914%. Both are larger than the statistical values in Bollerslev et al. (2009). The average *RREL* is negative, which is consistent with the result of Bollerslev et al. (2009). We find that in Panels B and C, the three control variables show statistical values similar to the values in Panel A. Compared with Kang and Pan (2015), our sample averages of *BASIS* and *NSHORT* are higher and the averages of *OIG* and *HRET* are lower. One explanation for this phenomena might be due to the different sample periods. *NSHORT* has a positive mean of 0.157% per month, indicating that commercials tend to take short positions in the crude oil futures market to hedge the price risk. *OIG*, *STORAGE*, *OVX* and *IP* also have positive sample averages, which suggest that open interest, crude oil storage, crude oil volatility and industrial production are growing in general.

3. Methodology

3.1. The no-arbitrage formula based on a new framework

Carr and Wu (2020) develop a new option pricing framework that links the pricing of a security to its daily profit and loss attribution (P&L) without directly referring to the terminal payoffs of the investment. The framework uses the Black–Scholes option pricing formula to attribute the short-term option investment risk to variation in the underlying security price and the option's implied volatility. Taking risk-neutral expectation and demanding no dynamic arbitrage result in an option pricing relation (see Theorem 1 in Appendix A).⁸

Proposition 1 (Carr and Wu, 2020 Formula). According to the Theorem 1 and Assumption 1 in Appendix A, a no-arbitrage formula from the new framework can be arrived at

$$I_t^2 - A_t^2 = 2\gamma_t z_+ + \omega_t^2 z_+ z_-, \quad (2)$$

where I_t is the implied volatility on date t , A_t is the at-the-money implied volatility on date t , ω_t^2 denotes the risk-neutral conditional variance of the implied volatility percentage change, γ_t is the conditional covariance between the implied volatility percentage change and underlying security return, and the terms $z_{\pm} = (k \pm \frac{1}{2} I_t^2 \tau)$, where τ is the time to maturity, represent the convexity-adjusted moneyness of the call under the risk-neutral measure.⁹

⁶ This variable measures the supply and demand imbalance of commercial traders (hedgers) in CFTC's reports.

⁷ Prof. Lutz Kilian's webpage is <https://sites.google.com/site/lkilian2019/research/data-sets>.

⁸ Carr and Wu (2016) also propose a new theoretical framework by directly modeling the near-term implied volatility dynamics and deriving no-arbitrage constraints on the shape of the implied volatility surface.

⁹ When deriving the no-arbitrage formula, the first terms contained μ_t in Eqs. (A.6) and (A.7) are eliminated (see Appendix A). This means that the shape of the implied volatility smile only depends on its variance and covariance with the USO return, not on the risk-neutral rate of implied volatility changes. Therefore, our research focuses more on the risk-neutral variance and covariance estimates.

Table 3
Summary statistics for control predictors.

	Mean	Std.dev.	Skewness	Kurtosis	Min	Max
Panel A: Daily						
OVX	0.000	0.049	0.767	12.270	-0.440	0.425
DEF (%)	1.119	0.510	2.567	10.368	0.530	3.500
TERM (%)	2.024	0.914	-0.202	2.672	-0.350	4.010
RREL (%)	-0.109	0.572	-2.174	8.396	-3.539	0.719
Panel B: Weekly						
BASIS	0.002	0.013	18.182	403.963	-0.038	0.283
NSHORT	0.154	0.093	-0.058	2.053	-0.040	0.352
OVX	-0.003	0.050	1.432	14.538	-0.198	0.425
DEF (%)	1.119	0.511	2.587	10.502	0.540	3.490
TERM (%)	2.017	0.909	-0.181	2.660	-0.290	3.870
RREL (%)	-0.112	0.572	-2.161	8.215	-3.086	0.677
Panel C: Monthly						
BASIS	0.001	0.006	2.951	20.349	-0.019	0.038
NSHORT	0.157	0.094	-0.028	2.065	-0.020	0.352
OIG	0.003	0.012	-0.412	2.320	-0.025	0.027
HRET	-0.001	0.030	-0.417	2.666	-0.066	0.059
STORAGE	0.006	0.030	-1.981	33.965	-0.230	0.176
KI	-4.189	77.092	0.925	3.287	-159.644	190.729
OVX	0.002	0.163	0.456	4.286	-0.394	0.621
DEF (%)	1.121	0.520	2.618	10.593	0.540	3.430
TERM (%)	1.999	0.912	-0.286	2.820	-0.350	3.890
RREL (%)	-0.108	0.567	-2.186	8.389	-2.585	0.597
CFNAI	-0.209	0.560	-2.497	10.764	-2.900	0.550
IP	0.003	0.050	-1.832	6.344	-0.166	0.082

This table shows the mean, standard deviation, skewness, kurtosis, minimum and maximum of daily, weekly and monthly control predictors from May 9, 2007 to June 28, 2019. *OVX* stands for the log change of the USO ETF volatility index, the market's expectation of 30-day volatility of crude oil prices. *DEF* stands for the difference between the yields on Moody's BAA- and AAA-rated corporate bonds. *TERM* stands for the difference between the yields on the 10-year Treasury bond and three-month Treasury bill. *RREL* stands for the yield on the one-month Treasury bill minus its one-year backward moving average. *BASIS* is the spread between futures prices and spot prices. *NSHORT* is calculated as the commercials net short position (short positions minus long positions) as a proportion of commercials total positions (short positions plus long positions). *OIG* is the 12-month geometric average of open interest changes. *HRET* is the 12-month geometric average of crude oil futures returns. *STORAGE* stands for the log growth of the U.S. field production of crude oil. *KI* stands for Kilian's (2009) real global economic activity index from the website of Prof. Lutz Kilian. *CFNAI* is Chicago Fed National Activity Index. *IP* is the log growth in industrial production over the last 12 months.

The local commonality assumption on variance and covariance rates is that the variance and covariance rates of implied volatilities across a range of strikes at the same maturity are the same,

$$\omega_t^2(k) \doteq \omega_t^2, \quad \gamma_t(k) \doteq \gamma_t, \quad (3)$$

for all k within a certain strike range.

Under the local commonality assumption, the risk-neutral variance and covariance rates (ω_t^2, γ_t) can be estimated by performing a cross-sectional regression of the implied variance difference from the at-the-money level $(I_t^2 - A_t^2)$ on the two convexity-adjusted moneyness measures $2z_+$ and z_+z_- .¹⁰

3.2. Construct floating series of implied volatility levels

To estimate the risk-neutral variance and covariance rates, we first construct floating series of implied volatility levels. We define a maturity-moneyness grid (τ, x) , where the time to maturity (τ) is 1, 2, 3, 6 and 12 months and the moneyness is $x = 0, \pm 0.5, \pm 1, \pm 1.5, \pm 2$. The standardized moneyness measure is $x \equiv z_+ / I_t \sqrt{\tau}$.

We estimate the implied volatility level I_t at each interpolation grid point following the Carr and Wu (2020) weighting schemes. Since the out-of-the-money option contract is more liquid, we put more weight on it. For the option contract with an absolute forward delta less than 80%, we define one minus the absolute delta as its weight. For the option contract with an absolute delta

¹⁰ Other methodologies modeling the shape of the implied volatility smile or surface (Goncalves and Guidolin, 2006; Zhang and Xiang, 2008; Soini and Lorentzen, 2019; Kearney et al., 2019) use assumed implied volatility functions and the coefficients of the functions are constants. In contrast, for the no-arbitrage formula shown in PROPOSITION 1, the coefficients (ω_t^2, γ_t) are variables. Only under the local commonality assumption in Eq. (3), can the coefficients be viewed as constants.

Table 4
Summary statistics of at-the-money implied volatility levels.

Maturity	1	2	3	6	12
Mean	0.365	0.362	0.361	0.358	0.355
Std.dev.	0.115	0.110	0.107	0.102	0.098
Min	0.160	0.163	0.165	0.168	0.172
Max	0.855	0.819	0.804	0.780	0.758

The table reports summary statistics for the at-the-money implied volatility levels at maturities of 1, 2, 3, 6, and 12 months in the crude oil market. The statistics include mean, standard deviation, minimum and maximum.

Table 5
Mean implied volatility smile.

Maturity	Moneyness								
	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
1	0.401	0.391	0.381	0.372	0.365	0.361	0.362	0.365	0.370
2	0.399	0.389	0.379	0.369	0.362	0.359	0.359	0.362	0.366
3	0.398	0.388	0.377	0.368	0.361	0.357	0.356	0.359	0.364
6	0.395	0.385	0.375	0.365	0.358	0.353	0.353	0.355	0.359
12	0.392	0.382	0.372	0.363	0.355	0.350	0.349	0.350	0.354

The table shows the sample mean of the implied volatilities across nine moneyness levels at each of the five interpolated maturities in the crude oil market. The maturities are 1, 2, 3, 6 and 12 months and the moneyness are $0, \pm 0.5, \pm 1, \pm 1.5, \pm 2$.

over than 80%, we set its weight at zero. We also weight each contract i based on its distance to the target log time to maturity ($\ln \tau$) and its distance to the target moneyness level (x). Taken together, we define the weight of each contract i as

$$w_i = (1 - |\delta_i|) I_{|\delta_i| < 0.8} \exp\left(-\frac{(x_i - x)^2}{2h_x^2}\right) \exp\left(-\frac{(\ln \tau_i - \ln \tau)^2}{2h_\tau^2}\right), \quad (4)$$

where δ_i denotes the BMS forward delta of the option, and (h_x, h_τ) denote the two bandwidths based on a bivariate Gaussian kernel.¹¹

According to the weight, we interpolate to construct floating series of the implied volatility levels at a target maturity-moneyness grid (τ, x) . Based on the interpolated implied volatility levels, we could perform a cross-sectional regression of the implied variance spread $(I_t^2 - A_t^2)$ on the two convexity-adjusted moneyness measures $[2z_+, z_+ z_-]$ according to Eq. (2) and extract the risk-neutral variance and covariance rates.

4. Empirical analysis

4.1. USO implied volatility smile

Table 4 reports the summary statistics of the interpolated floating at-the-money implied volatility levels at the five maturities. We find that the average implied volatility decreases from 36.5% at the one-month maturity to 35.5% at the one-year maturity as the time to maturity increases. The annualized standard deviation of the at-the-money implied volatility series declines from 11.5% at the one-month maturity to 9.8% at the one-year maturity with increasing maturity, meaning the at-the-money implied volatility tends to mean-revert.

Table 5 reports the sample average of the implied volatility levels at nine selected moneyness levels and five interpolated maturities. At each maturity, the implied volatility level for USO is higher at higher absolute moneyness levels than at lower absolute moneyness levels, showing a “U”-shaped implied volatility smile. From Fig. 2, it is obvious that for different maturities the implied volatility curves are “U”-shaped with a positive curvature. The curve shows more curvature at shorter maturities because the implied volatility at higher strikes is higher.

Panel A of Table 5 in Carr and Wu (2020) gives a downward-sloping implied volatility pattern for the S&P 500, leading to a negative implied volatility skew, which is different to the USO implied volatility pattern in this paper. That is because, in sharp contrast to equities' crash concerns, oil prices are subject to both spikes as well as crashes.

4.2. Risk-neutral estimates

The no-arbitrage formula in Eq. (2) links the implied volatility smile to the conditional risk-neutral variance of the percentage implied volatility change and its covariance with the security return. At each date and maturity, we apply commonality assumptions,¹² perform a cross-sectional regression of the implied variance difference from the at-the-money level on the two moneyness

¹¹ The implied volatility surface from OptionMetrics is constructed by using a similar kernel smoothing technique.

¹² We provide evidence on local and global commonality in the co-movements among the floating implied volatility change series. The results are presented in Figs. B1 and B2 in Appendix B. Fig. B1 shows that the log implied volatility changes for the at-the-money contract are highly correlated with the ones for other contracts within one standard deviation of at-the-money. Fig. B2 shows that the first three principal components can explain over 95% of the common movements of the implied volatility surface by performing principal component analysis on the interpolated implied volatility change series.

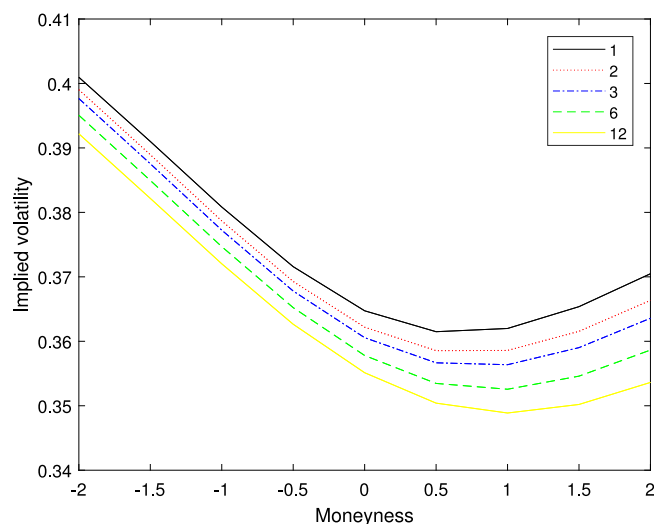


Fig. 2. USO implied volatility smile.

This figure shows USO implied volatility smiles based on interpolated implied volatility levels at nine selected moneyness ($x = 0, \pm 0.5, \pm 1, \pm 1.5, \pm 2$) and five maturities (1, 2, 3, 6 and 12 months).

measures ($2z_+$) and (z_+z_-) and extract the risk-neutral variance ω_t^2 and covariance γ_t . We then examine their information content in predicting the USO excess returns.

According to Eq. (2), a positive curvature will lead to a positive variance rate ω_t , while a negative implied volatility skew is expected to generate a negative covariance rate γ_t . Table 6 reports summary statistics for the term structures of the variance and covariance rates in the USO options market. From Panel A, we can see that the average covariance rates are negative over all maturities. Panel A of Table 5 in Carr and Wu (2020) shows that the maximums of covariance rates for the S&P 500 over all maturities are strictly negative, which confirms the downward-sloping implied volatility pattern. However, the maximums of covariance rates are varying from 0.064 to 0.111, indicating that the implied volatility smiles on USO are “U”-shaped. In Panel B, the average variance rates are positive across all maturities. As the maturity increases, the variance rate decreases, suggesting that the implied volatility smile has more curvature at shorter maturities than at longer maturities. The standard deviation for the variance rate is significantly larger than the one for the covariance rate, indicating that the variance rate varies a lot more than the covariance rate. The autocorrelation estimates of covariance and variance rates show a high time series persistence.

From Table 6, we also observe that the covariance estimates seem not so different from zero according to the mean and standard deviation. We perform a one-sample t-test on the covariance estimate and find that we can reject the null hypothesis that the mean is equal to zero with extremely high t values at the 5% significance level for different maturities.¹³ Therefore, the covariance estimates are statistically different from zero.

Fig. 3 presents the dynamics of variance and covariance rates at one-month maturity from May 9, 2007 to June 28, 2019. The variance rate is positive,¹⁴ while the covariance rate is negative most of time. The negative covariance rate indicates that the stock market index and the implied volatility usually have the opposite movement directions. For option markets, option investors bid up put prices for hedging the downward risk during a market crash. The more negative the market index return is, the more the implied volatility increases, and the more negative the covariance rate becomes (Hibbert et al., 2008). In Fig. 3, we also find that the covariance rate spikes downward at financial crises or oil market-specific events. During the 2008 financial crisis, the covariance rate declined to a relatively low level in mid-2009, later than at the time when the oil price shown in Fig. 1 reached the lowest point of the 2008 financial crisis in early 2009. The covariance rate fell to an all-time low around -0.17 during the European debt crisis in 2011.

Besides variance and covariance rates, we also extract risk-neutral drift of implied volatility changes from the implied variance term structure slope defined by two nearby at-the-money option contracts (see Eq. (A.9) in Appendix A).¹⁵ Table 7 reports the summary statistics for the risk-neutral rate of implied volatility changes. The sample mean of the risk-neutral estimates is negative at each of the five interpolated maturities. The drift estimates vary largely from large negative values to positive values at shorter maturities than larger ones. The daily autocorrelation estimates in the last row show that the risk-neutral drift estimates are persistent, between 0.853 and 0.937.

¹³ For example, for the one-month maturity, the t value is -42.20 . Actually, our sample size is very large (3056), which results in a large t value. We would like to thank the anonymous referees for bringing this to our attention.

¹⁴ We have imposed zero intercept and constrain the regression coefficient ω_t^2 to be positive. Therefore the variance rate is positive.

¹⁵ We first estimate the risk-neutral drift at the midpoint of the two adjacent maturities from the at-the-money implied variance term structure slope defined by the two maturities. We then interpolate these drift estimates to construct the estimates at maturities of 1, 2, 3, 6 and 12 months.

Table 6
Estimates of covariance and variance rates.

Maturity	1	2	3	6	12
Panel A: Covariance rate estimates γ_t					
Mean	-0.030	-0.030	-0.030	-0.027	-0.020
Std.dev.	0.040	0.036	0.033	0.029	0.027
Min	-0.166	-0.151	-0.141	-0.123	-0.109
Max	0.111	0.098	0.086	0.064	0.069
Auto	0.922	0.933	0.936	0.935	0.930
Panel B: Variance rate estimates ω_t^2					
Mean	0.449	0.432	0.416	0.380	0.330
Std.dev.	0.344	0.322	0.305	0.268	0.222
Min	0.000	0.000	0.005	0.004	0.002
Max	1.983	1.892	1.747	1.392	1.144
Auto	0.928	0.936	0.940	0.943	0.942

This table reports summary statistics for the term structures of the covariance and variance rates, which are extracted from the no-arbitrage formula based on a new option pricing framework in the crude oil market. The statistics include mean, standard deviation, minimum, maximum and daily autocorrelation.

Table 7
Estimate of the risk-neutral drift.

Maturity	1	2	3	6	12
Mean	-0.057	-0.048	-0.035	-0.020	-0.012
Std.dev.	0.193	0.158	0.110	0.057	0.032
Min	-2.124	-1.824	-1.343	-0.627	-0.282
Max	0.492	0.394	0.261	0.127	0.068
Auto	0.853	0.868	0.891	0.916	0.937

This table reports summary statistics for the risk-neutral drift of implied volatility changes from the at-the-money implied variance local term structure slope defined by the two nearest maturities. The statistics include mean, standard deviation, minimum, maximum and daily autocorrelation.

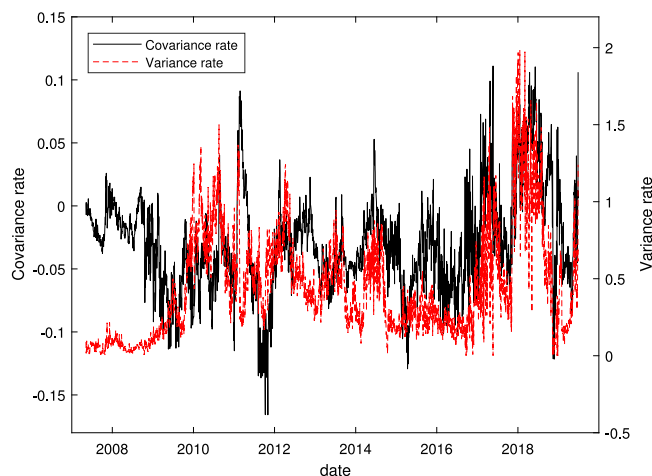


Fig. 3. Daily risk-neutral variance and covariance rates.

This figure shows the daily time series of the risk-neutral variance and covariance rates at one-month maturity. The sample period is from May 9, 2007 to June 28, 2019. The risk-neutral variance and covariance rates are extracted from the no-arbitrage formula based on a new option pricing framework.

Following [Dennis et al. \(2006\)](#), [Ang et al. \(2006\)](#) and [Chang et al. \(2013\)](#), we choose $DRNV$ and $DRNC$ at a one-month maturity, which are good proxies of innovations in the variance and covariance rates, as predictors for forecasting the future excess returns. As [Vasquez \(2017\)](#) suggests, the implied volatility term structure conveys information about the future option returns. We consider $TRNV$ and $TRNC$, the term structures of the variance and covariance rates, as predictors. $TRNV$ is defined as the difference between the risk-neutral variance rate at one-year maturity and the one at one-month maturity and $TRNC$ is the difference between the risk-neutral covariance rate at one-year maturity and the one at one-month maturity. Besides the main predictors of interest related to risk-neutral variance and covariance rates, we also include RND at one-month maturity and other predictors that have been found to show some predictive power for future crude oil returns proposed in the literature. Ones are $DVOL$, $DSKEW$ and $DKURT$ proposed by [Ruan and Zhang \(2018\)](#). The other is VRP , the difference between the expected realized return variance and

Table 8
Summary statistics for key predictors.

Panel A: Summary statistics							
	Mean	Std.dev.	Skewness	Kurtosis	Min	Max	Auto
DRNC	0.0001	0.0341	0.3952	3.9665	-0.0805	0.1211	-0.2536
DRNV	0.0025	0.2228	0.3569	6.0481	-0.7515	0.9776	-0.1978
TRNC	0.0206	0.0354	-0.4676	3.9845	-0.0923	0.1142	0.6565
TRNV	-0.4154	0.3285	-1.3340	5.0244	-1.8120	-0.0178	0.7864
RND	-0.0574	0.1947	-1.7172	7.3919	-1.0126	0.2434	0.4967
DVOL	-0.0003	0.0645	0.5931	10.3976	-0.2408	0.3277	0.1828
DSKEW	0.0035	0.2196	-0.1493	4.5606	-0.7686	0.6982	-0.3030
DKURT	-0.0066	0.2372	0.6341	9.4250	-0.7953	1.1156	-0.4555
VRP	-0.0285	0.0533	-2.5030	12.1834	-0.3035	0.0603	0.2612

Panel B: Correlations												
	DRNC	DRNV	TRNC	TRNV	RND	DVOL	DSKEW	DKURT	VRP	NSHORT	HRET	OVX
DRNC	1.000											
DRNV	0.193	1.000										
TRNC	-0.428	-0.001	1.000									
TRNV	-0.135	-0.319	0.374	1.000								
RND	-0.101	0.131	0.037	-0.306	1.000							
DVOL	-0.009	-0.208	-0.131	0.141	-0.256	1.000						
DSKEW	0.462	0.115	-0.241	-0.041	-0.069	-0.161	1.000					
DKURT	-0.173	-0.104	0.084	0.048	0.005	0.104	-0.581	1.000				
VRP	0.013	-0.008	-0.241	-0.274	0.426	-0.224	-0.013	0.007	1.000			
NSHORT	0.071	0.010	-0.377	-0.363	0.140	-0.053	0.043	-0.009	0.384	1.000		
HRET	0.021	-0.015	-0.353	-0.372	0.360	0.091	-0.003	0.025	0.329	0.088	1.000	
OVX	-0.032	-0.201	-0.070	0.107	-0.419	0.735	-0.102	0.127	-0.174	-0.034	-0.038	1.000

This table gives descriptive statistics for the monthly time series of the predictor variables from May 9, 2007 to June 28, 2019. *DRNC* is the first differences in the covariance rate. *DRNV* is the first differences in the variance rate. *TRNC* is the difference between the risk-neutral covariance rate at one-year maturity and the one at one-month maturity. *TRNV* is the difference between the risk-neutral variance rate at one-year maturity and the one at one-month maturity. *RND* is the risk-neutral drift of implied volatility changes at one-month maturity. *DVOL*, *DSKEW* and *DKURT* are the first differences in risk-neutral volatility, skewness and excess kurtosis. *VRP* is the difference between the expected realized return variance and the risk-neutral expected variance in the crude oil market. Detailed descriptions of other predictors see [Table 3](#).

the risk-neutral expected variance in the crude oil market, used in [Kang and Pan \(2015\)](#). We calculate the *VRP* based on the USO ETF and option data instead of the crude oil futures and option data. We compute the expected realized variance using daily ETF returns rather than the high-frequency intraday returns. We also use *OVX* as a proxy for the risk-neutral expected variance.

Panel A of [Table 8](#) reports the summary statistics for the main predictors and the other return predictors that are used for comparison purposes. We find that both *DRNC* and *DRNV* are weakly serially correlated. For example, *DRNC* has an effective mean of zero (less than 0.0001), a standard deviation of 0.0341, and weak serial correlation (the first-order autocorrelation is -0.2536). In contrast, the risk-neutral covariance estimate at one-month maturity is highly serially autocorrelated with a first-order autocorrelation of 0.922 in [Table 6](#). This indicates that first differencing can remove most of the autocorrelation of time series covariance rate.

Panel B of [Table 8](#) presents the correlations among all return predictors and control predictors.¹⁶ In general, the four main return predictors have relatively low correlations with other variables (below 0.5). This indicates the risk-neutral predictors contain different information from other predictors, and we expect that they could deliver different information about the future USO returns. In addition, it is expected that *DRNC* and *DSKEW* are positively correlated, with an autocorrelation coefficient of 0.462, since both are related to the slope of the implied volatility smile. Interesting, we find that *DVOL* and *OVX* are highly positively correlated, while both predictors are relatively weakly and negatively correlated with *RND*. This is because they are calculated using options with different moneyness: the former use out-of-the-money option contracts and the latter uses at-the-money option contracts.

4.3. USO return predictability

In this subsection, we examine the return predictability on USO ETF by using risk-neutral estimates and verify whether the information content of risk-neutral estimates is correlated with future USO returns. First, we run univariate predictive regressions for each of return predictors. We test the in-sample and out-of-sample predictive power of the predictors at daily, weekly, and monthly forecasting horizons. We choose these horizons following ([Han and Li, 2021](#)). Second, we consider a set of control variables and do multiple predictive regressions to test whether the predictive ability of our risk-neutral variables is not affected by the control predictors. Finally, to evaluate the economic significance of out-of-sample predictability, we construct trading strategies based on return forecasts.

¹⁶ To save space, we only display control predictors that are relatively more closely correlated with our main return predictors. The results for additional predictors are available upon request.

4.3.1. Univariate predictive regression

We use a univariate predictive regression, which is a standard framework for multi-period return predictability, to forecast excess returns at daily, weekly and monthly frequencies,

$$\sum_{h=1}^H \frac{r_{t+h}}{H} \equiv r_{t,t+H} = \alpha + \beta X_t + \epsilon_{t,t+H}, \tag{5}$$

where r_t denotes the USO ETF excess return at time t . X_t denotes a set of predictor variables at time t . H is the forecast horizon. When H is equal to one day, one week or one month, we use nonoverlapping returns. Otherwise, we use overlapping observations. We correct the serial correlation and conditional heteroskedasticity using the Newey–West correction (Newey and West, 1987).

Table 9 reports the results of univariate predictive regressions for the risk-neutral predictors at horizons ranging from daily, weekly to monthly frequency. We demonstrate the estimate of the coefficient as well as adjusted R^2 statistics and the Newey and West (1987) t-statistics for in-sample tests. Obviously, *DRNC* can significantly predict the USO excess returns at daily, weekly and monthly horizons. It outperforms the other predictors at all forecast horizons except for the three-week horizon. In detail, for one-day forecast horizon, the slope coefficient of *DRNC* is 11.97, indicating a 1% increase in *DRNC* is associated with an 11.97% increase in USO excess returns over the following day. The R^2 statistic of *DRNC* is 0.7%. We also find that the t-statistics of *DRNC* decline from 4.13 (one-day forecast horizon) to 0.34 (three-week forecast horizon), and then increase at four-week and one-month forecast horizons. In other words, the predictive ability of *DRNC* becomes weaker as the forecast horizon becomes longer (from one-day to three-week). Then, the predictive power of *DRNC* becomes significant at four-week and one-month forecast horizons. The similar pattern for the change of the forecasting power is shown in Han and Li (2021) who investigate the predictability of the aggregate implied volatility spread in forecasting stock market returns at daily, weekly, and monthly to semi-annual horizons. In their paper, the t-statistic values from the four-day to one-month forecast horizons are 3.07, 2.57, 2.63, 2.22, 1.83 and 2.92, respectively.¹⁷ The predictive power of the implied volatility spread becomes weaker from the four-day to four-week forecast horizons, and then becomes highly significant at the one-month horizon. The one-month adjusted R^2 statistic of *DRNC* is 1.16%, which is much larger than the statistic at other forecast horizons.

The three risk-neutral predictors (*DRNV*, *TRNC* and *TRNV*) show some predictive power in predicting the USO returns. *DRNV* only can predict two-day-, three-day- and four-day-ahead USO excess returns, which means the innovation in risk-neutral variance from the no-arbitrage formula contains less information about the future USO returns than the innovation in risk-neutral covariance. *TRNC* not only has predictive ability at the daily horizon, but also can significantly predict one-week-ahead USO excess returns. Both predictors fail to predict monthly USO excess returns. In contrast, *TRNV* shows poor predictive performance at all the forecasting horizons.

In terms of the other comparative predictors in Table 9, both *DVOL* and *DSKEW* have strong predictive power for USO excess returns at monthly frequency, while *DKURT* fails to predict monthly USO returns. The result is consistent with the findings in Ruan and Zhang (2018). Besides, *DVOL* is still significant at daily horizons. We find that *RND*, the risk-neutral implied volatility change, is statistically significant at the 10% level only at the four-day, two-week, three-week and four-week horizons. Compared with *DVOL*, the different forecasting performance between the two predictors might be due to the fact we mentioned in Section 4.2 that they are constructed based on option contracts with different moneyness. *VRP* exhibits insignificant predictive power across all forecast horizons, which is different from the findings in Kang and Pan (2015) that *VRP* can significantly predict the one-month oil futures returns at the 10% significance level. This might be due to the different forecasting sample periods and the slightly different constructed method for *VRP*.¹⁸

Jia et al. (2021) also use the innovations in parameters that determine the implied volatility smile shape, the first differences of the slope and curvature, to predict the monthly USO excess returns. They find that both predictors do not exhibit significant in-sample predictive performance. However, our *DRNC*, the innovation in risk-neutral covariance rate, can significantly predict future USO excess returns in in-sample tests.

Next, following Campbell and Thompson (2008) and Rapach et al. (2010), we compare the predictive performance of each predictor under the given regression model with the benchmark model that uses recent historical average returns as the forecasts.

We use an expanding window to calculate the R_{OS}^2 statistic. The out-of-sample R^2 is given by

$$R_{OS}^2 = 1 - \frac{\sum_{t=n}^{N-1} (r_{t,t+H} - \hat{r}_{t,t+H|t})^2}{\sum_{t=n}^{N-1} (r_{t,t+H} - \bar{r}_{t,t+H|t})^2}, \tag{6}$$

where H is the forecast horizon, n is the number of observations for the initial forecast, $\hat{r}_{t,t+H|t}$ is the return forecast estimated using all observations except those overlapping with $r_{t,t+H}$ and $\bar{r}_{t,t+H|t}$ is the historical average of excess returns calculated by using all observations except those overlapping with $r_{t,t+H}$. If the R_{OS}^2 is positive, then the predictive regression has a lower average mean-squared forecast error (MSFE) than the historical mean benchmark, indicating that the predictive regression forecast outperforms the benchmark.

¹⁷ The forecast horizons are the four days, one week, two weeks, three weeks, four weeks and one month, respectively.

¹⁸ As mentioned in Section 4.2, we calculate the VRP based on the USO ETF and option data instead of the crude oil futures and option data. We compute the expected realized variance using daily ETF returns rather than the high-frequency intraday returns. We also use the USO ETF volatility index (OVX) as a proxy for the risk-neutral expected variance.

Table 9
Univariate return predictability.

		Daily				Weekly				Monthly
		1-day	2-day	3-day	4-day	1-week	2-week	3-week	4-week	1-month
DRNC	β	11.97***	6.79***	4.24***	2.65**	21.36**	10.49*	1.35	10.49***	39.16***
	t	(4.13)	(3.55)	(3.25)	(2.46)	(2.11)	(1.93)	(0.34)	(3.08)	(2.68)
	$R^2(\%)$	0.70	0.46	0.26	0.12	0.92	0.34	-0.15	0.73	1.16
	$R_{os}^2(\%)$	0.54***	-0.12	0.15***	0.04**	0.19*	-0.41	-0.13	0.75**	1.87**
DRNV	β	-0.23	-0.31**	-0.31***	-0.15*	-1.67	-0.58	-0.20	-0.29	1.42
	t	(-1.00)	(-2.16)	(-2.64)	(-1.75)	(-1.58)	(-0.95)	(-0.41)	(-0.73)	(0.57)
	$R^2(\%)$	-0.01	0.04	0.07	0.00	0.23	-0.07	-0.14	-0.12	-0.60
	$R_{os}^2(\%)$	-0.13	-0.01	0.14***	0.01	-1.22	-0.56	-0.41	-0.37	-1.44
TRNC	β	-5.08***	-3.41**	-2.81*	-2.48	-16.27*	-7.60	-6.01	-6.94	-30.56
	t	(-2.85)	(-2.04)	(-1.79)	(-1.63)	(-1.81)	(-0.96)	(-0.82)	(-0.99)	(-1.05)
	$R^2(\%)$	0.28	0.26	0.27	0.28	0.47	0.11	0.08	0.23	-0.13
	$R_{os}^2(\%)$	0.01**	-0.21	-0.23	-0.39	-0.11	-0.74	-0.98	-1.25	-2.33
TRNV	β	-0.04	-0.04	-0.12	-0.17	-1.05	-1.27	-1.25	-1.15	-2.74
	t	(-0.17)	(-0.22)	(-0.61)	(-0.92)	(-0.95)	(-1.24)	(-1.32)	(-1.24)	(-0.55)
	$R^2(\%)$	-0.03	-0.03	-0.01	0.04	-0.03	0.22	0.36	0.39	-0.49
	$R_{os}^2(\%)$	-0.05	-0.09	-0.09	-0.05	0.03	0.14	0.06	-0.12	-0.89
RND	β	0.31	0.36	0.34	0.36*	1.95	2.22*	2.14*	2.10*	7.36
	t	(1.30)	(1.58)	(1.55)	(1.68)	(1.56)	(1.79)	(1.78)	(1.84)	(1.12)
	$R^2(\%)$	0.04	0.19	0.26	0.40	0.50	1.47	2.00	2.42	1.44
	$R_{os}^2(\%)$	-0.24	-0.51	-1.00	-1.60	-1.58	-2.73	-3.38	-3.50	-12.67
DVOL	β	11.67***	6.15***	4.50***	3.66***	1.65	-0.52	-5.26	-3.57	-58.62***
	t	(3.77)	(3.50)	(3.44)	(2.89)	(0.30)	(-0.10)	(-1.52)	(-1.06)	(-4.82)
	$R^2(\%)$	1.68	0.98	0.79	0.69	-0.19	-0.21	0.50	0.18	12.99
	$R_{os}^2(\%)$	-5.43	-2.29	0.13**	0.34**	8.60	7.04	4.21	-1.98	5.47**
DSKEW	β	-0.27	-0.06	-0.03	-0.02	0.54	0.25	-0.14	0.18	14.12**
	t	(-1.09)	(-0.37)	(-0.25)	(-0.16)	(0.52)	(0.35)	(-0.31)	(0.46)	(2.48)
	$R^2(\%)$	-0.01	-0.04	-0.04	-0.04	-0.16	-0.19	-0.21	-0.20	2.25
	$R_{os}^2(\%)$	-6.94	-2.28	-0.19	0.33	8.61	7.41	5.03	-0.96	1.59**
DKURT	β	0.48*	0.15	0.15	0.10	-0.94	-0.74	0.51	-0.29	-2.70
	t	(1.80)	(1.01)	(1.31)	(1.13)	(-0.93)	(-1.26)	(1.18)	(-0.75)	(-0.27)
	$R^2(\%)$	0.07	-0.02	-0.01	-0.02	-0.07	-0.05	-0.10	-0.17	-0.81
	$R_{os}^2(\%)$	-6.63	-2.18	-0.10	0.41	9.11	7.79	4.80	-0.90	-5.06
VRP	β	-0.30	-0.07	0.05	0.29	3.24	5.67	5.96	5.79	15.05
	t	(-0.23)	(-0.05)	(0.04)	(0.25)	(0.61)	(1.24)	(1.54)	(1.45)	(0.75)
	$R^2(\%)$	-0.03	-0.03	-0.03	-0.01	-0.02	0.65	1.13	1.36	-0.04
	$R_{os}^2(\%)$	-0.07	-0.22	-0.42	-0.87	-3.73	-5.54	-5.06	-6.10	-4.32

This table reports the results of univariate predictive regressions for the risk-neutral predictors at horizons ranging from daily, weekly to monthly frequency from May 9, 2007 to June 28, 2019 in the crude oil market. When the forecast horizon is one day, one week or one month, we use nonoverlapping returns. Otherwise, we use overlapping observations. We correct the serial correlation and conditional heteroskedasticity using the Newey–West correction (Newey and West, 1987). The table presents the estimate of the coefficient β , the Newey and West (1987) t -statistics, in-sample adjusted R^2 statistics and out-of-sample R^2 statistics (R_{OS}^2). *DRNC* is the first differences in the covariance rate. *DRNV* is the first differences in the variance rate. *TRNC* is the difference between the risk-neutral covariance rate at one-year maturity and the one at one-month maturity. *TRNV* is the difference between the risk-neutral variance rate at one-year maturity and the one at one-month maturity. Detailed descriptions of other predictors see Table 8.

*Significance at the 10% level.

**Significance at the 5% level.

***Significance at the 1% level.

We use the Clark and West (2007) adjusted mean squared prediction error statistic to test whether the out-of-sample R^2 statistic is significantly greater than zero.

$$f_{t,t+H} = (r_{t,t+H} - \bar{r}_{t,t+H}|_t)^2 - [(r_{t,t+H} - \hat{r}_{t,t+H}|_t)^2 - (\bar{r}_{t,t+H} - \hat{r}_{t,t+H}|_t)^2]. \quad (7)$$

To test whether $R_{OS}^2 > 0$, we regress this statistic on a constant and provide one-sided p-values for the R_{OS}^2 statistic. We have shown that *DRNC* is a strong predictor that can predict daily, weekly and monthly ahead USO excess returns based on in-sample tests. If *DRNC* is a robust predictor, its out-of-sample R^2 should be significantly greater than zero.

Table 9 also reports the out-of-sample R^2 statistic for various predictors and horizons. Generally, *DRNC* performs better than the other predictors based on the out-of-sample tests, which is consistent with the in-sample results. For daily frequencies (except for the two-day forecast horizon), *DRNC* has a positive R_{OS}^2 statistic, which is significant, according to the Clark and West (2007)

statistic. The R_{OS}^2 for *DRNC* is 0.54% for one-day ahead, 0.15% for three-day ahead and 0.04% for four-day ahead. For weekly horizons, *DRNC* has a positive and significant R_{OS}^2 at one-week and four-week forecast horizons. *DRNC* also shows a significant one-month out-of-sample R^2 of 1.87. The monthly predictive result is superior to the finding in Jia et al. (2021) that the innovation in the slope parameter has no out-of-sample predictive performance. The other three main predictors (*DRNV*, *TRNC* and *TRNV*) exhibit poor out-of-sample predictive performance at most of the forecast horizons. For example, although *DRNV* and *TRNC* have a positive and significant R_{OS}^2 at daily frequency, the values are extremely small. In addition, *DVOL* and *DSKEW* show significant out-of-sample performance at monthly frequency with large and significant R_{OS}^2 , which further confirms their in-sample performance.

In summary, we show evidence that *DRNC* is informative about the future USO returns at daily, weekly and monthly frequencies. We provide robust evidence, from both in-sample and out-of-sample tests, that *DRNC* is a strong predictor of future USO excess returns. The predictive ability for *DRNC* is superior to *DRNV*, the term structure predictors (*TRNV* and *TRNC*) and the other comparative predictors.

4.3.2. Multiple predictive regression

We run the following multiple predictive regressions to verify the predictive power of *DRNC* is still significant after controlling for other predictors described in Section 2.3.

$$r_{t,t+H} = \alpha + \sum_{i=1}^M \beta_i X_{i,t} + \epsilon_{t:t+H}, \tag{8}$$

where if M is equal to 2, it means a bivariate predictive regression using *DRNC* and one of the alternative predictors.

Tables 10 and 11 report the results for multiple predictive regressions on the basis of daily, weekly and monthly time series. We include *BASIS* and *NSHORT* in weekly and monthly regressions and other six control variables (*CFNAI*, *IP*, *OIG*, *HRET*, *STORAGE* and *KI*) in monthly regressions, since they are available in different time series. First, after controlling for the risk-neutral predictors, macroeconomic variables or crude oil market-specific variables, *DRNC* is expected to significantly predict USO excess returns across different horizons. To save space, we only display the one-day, one-week and one-month multivariate predictive results.¹⁹

Second, in general, t-statistics of *DRNC* in the multiple predictive regressions are similar to the corresponding ones in the univariate regressions of Table 9. This indicates that *DRNC* indeed contains some independent information content about the future USO returns that is not affected by other variables, which is also confirmed by the findings that *DRNC* has a relatively low correlation of below 0.5 with other predictors in Table 8.

Finally, *OIG* positively predicts monthly future crude oil returns, consistent with Hong and Yogo (2012). *CFNAI* and *STORAGE* also predict future crude oil returns with a positive sign, consistent with Kang and Pan (2015). In contrast, the signs of *BASIS* and *NSHORT* are opposite of the ones in Hong and Yogo (2012). This might be because we predict crude oil ETF returns rather than crude oil futures returns. And the forecasting period is different as well. However, it does not affect the multivariate regression results that *DRNC* retains its significant return predictive power after controlling for other predictors.

4.3.3. Trading strategies

Following Rapach et al. (2010) and Ferreira and Santa-Clara (2011), to evaluate the economic significance of out-of-sample predictability, we construct trading strategies based on return forecasts, and calculate the CE return for a mean–variance investor who tries to allocate between USO ETF and risk-free assets.

For a mean–variance investor, the optimal weight allocated to USO ETF at the end of the forecast horizon H is

$$\omega_t = \hat{r}_{t,t+H} / \gamma \hat{\sigma}_{t,t+H}^2, \tag{9}$$

where γ is the investor’s coefficient of relative risk aversion, $\hat{r}_{t,t+H}$ is the excess return forecast and $\hat{\sigma}^2$ is the forecast of USO excess return variance. Following the measure of Pyun (2019), we use the square of OVX as a proxy for $\hat{\sigma}^2$.²⁰ In line with Rapach et al. (2016), we set γ to 3 and limit ω_t to -0.5 to 1.5 . The portfolio return at the end of each time horizon is

$$r_{p,t,t+H} = \omega_t r_{t,t+H} + r_{f,t,t+H}, \tag{10}$$

where $r_{t,t+H}$ is the USO excess return and $r_{f,t,t+H}$ is the risk-free rate. The average CE return is

$$CE = \overline{r_p} - 0.5\gamma\sigma^2(rp), \tag{11}$$

where $\overline{r_p}$ is the sample mean of portfolio returns and $\sigma^2(rp)$ is the sample variance of portfolio returns.

The CE gain is the difference between the CE of the strategy based on predictors and the CE of the strategy based on the historical mean of market return. We also calculate the gain in SR for each strategy.

Table 12 reports summary statistics on the performance of the trading strategies, including the annualized portfolio return and the annualized portfolio volatility. Overall, *DRNC* is the only predictor that has a positive portfolio return and portfolio volatility

¹⁹ The results at other forecast frequencies are available upon request.

²⁰ Rapach et al. (2010) and Ferreira and Santa-Clara (2011) use a moving window of past returns to estimate $\hat{\sigma}^2$. In contrast, OVX, which measures the 30-day volatility of crude oil prices, could provide a more accurate forecast.

Table 10
Multivariate return predictability: Daily and weekly time series.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel A: 1-day										
DRNC	11.90*** (4.09)	10.87*** (3.66)	11.98*** (4.13)	11.70*** (3.94)	11.67*** (3.93)	11.71*** (3.94)	11.68*** (4.06)	11.02*** (3.68)		
DRNV	-0.07 (-0.31)							0.07 (0.30)		
TRNC		-3.73** (-2.03)						-0.89 (-0.60)		
TRNV			-0.06 (-0.30)					0.12 (0.80)		
DEF (%)				-0.16 (-1.25)				-0.16 (-1.12)		
TERM (%)					-0.08** (-1.97)			-0.05 (-1.11)		
RREL (%)						0.04 (0.42)		-0.03 (-0.31)		
OVX							1.45 (1.63)	1.59* (1.75)		
Constant	-0.05 (-1.25)	-0.01 (-0.24)	-0.06 (-1.07)	0.14 (1.06)	0.13 (1.46)	-0.04 (-0.95)	-0.05 (-1.25)	0.32** (1.97)		
Observations	3,055	3,055	3,055	3,031	3,032	3,032	3,054	3,030		
Adj. R^2 (%)	0.67	0.84	0.67	0.79	0.77	0.65	0.78	0.84		
Panel B: 1-week										
DRNC	22.57** (2.22)	18.39* (1.78)	21.11** (2.09)	21.37** (2.06)	21.38** (2.06)	21.41** (2.06)	20.79** (2.04)	21.70** (2.14)	22.15** (2.13)	23.81** (1.99)
DRNV	-1.91* (-1.86)									-2.44** (-2.09)
TRNC		-11.43 (-1.26)								3.37 (0.39)
TRNV			-0.96 (-0.88)							-0.96 (-1.11)
DEF (%)				-0.80 (-1.31)						-1.73** (-2.37)
TERM (%)					-0.35 (-1.54)					-0.46* (-1.91)
RREL (%)						0.08 (0.17)				-0.22 (-0.41)
BASIS							29.30*** (3.95)			41.45*** (4.66)
NSHORT								-1.24 (-0.48)		-8.10*** (-2.83)
OVX									-3.65 (-0.91)	-4.65 (-1.14)
Constant	-0.24 (-1.16)	-0.12 (-0.58)	-0.35 (-1.23)	0.71 (1.13)	0.52 (1.12)	-0.18 (-0.98)	-0.29 (-1.41)	-0.05 (-0.11)	-0.25 (-1.22)	3.33*** (2.91)
Observations	629	629	629	620	621	621	629	626	629	617
Adj. R^2 (%)	1.27	1.05	0.87	1.59	1.25	0.78	1.41	0.85	0.92	4.10

This table reports the results for multiple predictive regressions on the basis of daily and weekly time series from May 9, 2007 to June 28, 2019 in the crude oil market. The table presents the estimate of the coefficient, the Newey and West (1987) t -statistics and in-sample adjusted R^2 statistics. The definitions of all predictors are the same as those in Table 3.

*Significance at the 10% level.

**Significance at the 5% level.

***Significance at the 1% level.

across different horizons. For example, the average return is 3.77% at the one-day forecast horizon, 5.28% at the one-week forecast horizon and 2.39% at the one-month forecast horizon for the strategies based on *DRNC*.²¹ If a mean–variance investor rebalances the strategy monthly on the basis of *DRNC*, the volatility of portfolio returns is 10.96%, larger than the volatility from daily and weekly rebalancing.

Table 12 also reports the performance of portfolios constructed on return and volatility forecasts assessed by the SR gain and CE gain. *DRNC* outperforms the other predictors in terms of the two measures. It can generate economically significant profits up to one month ahead for a mean–variance investor. For instance, at a monthly rebalancing frequency, it produces a positive SR gain

²¹ For investors, the trading strategy based on a daily rebalancing is not realistic because of expensive transaction costs.

Table 11
Multivariate return predictability: Monthly time series.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
DRNC	48.99** (2.27)	36.44*** (2.76)	34.92*** (2.64)	38.55*** (2.68)	33.78** (2.40)	39.91*** (2.86)	38.89*** (2.81)	37.95*** (2.64)	43.76*** (2.86)	38.94*** (2.67)	42.16*** (2.74)	63.40*** (3.26)
DRNV	-1.19 (-0.36)											-3.41 (-1.16)
TRNC	26.00 (0.87)											56.69 (1.26)
TRNV	-2.49 (-0.58)											-4.71 (-1.10)
DEF (%)		-3.16 (-0.97)										-3.02 (-0.81)
TERM (%)		-1.43* (-1.94)										-3.92** (-2.41)
RREL (%)		-1.79 (-0.85)										-2.16 (-0.80)
CFNAI			3.80 (1.28)									1.66 (0.73)
IP				18.11 (0.58)								-22.69 (-0.91)
BASIS					274.92 (1.57)							270.29 (1.60)
NSHORT						-3.81 (-0.25)						-4.05 (-0.22)
OIG							207.91 (1.63)					222.65** (2.01)
HRET								14.88 (0.48)				-57.72 (-1.26)
STORAGE									36.90 (1.05)			64.22** (2.48)
KI										0.01 (0.99)		0.04** (2.07)
OVX											-10.42 (-0.80)	-8.45 (-0.65)
Constant	-2.67 (-0.86)	5.16 (1.32)	-0.28 (-0.34)	-1.14 (-1.06)	-1.35 (-1.28)	-0.49 (-0.16)	-2.03 (-1.60)	-1.34 (-1.29)	-1.30 (-1.23)	-1.03 (-0.97)	-1.00 (-0.94)	7.15 (1.15)
Observations	144	144	144	144	144	144	137	137	144	144	144	137
Adj. R^2 (%)	-0.17	3.36	5.24	1.31	3.31	0.59	6.78	0.73	1.73	1.41	0.72	14.08

This table reports the results for multiple predictive regressions on the basis of monthly time series from May 9, 2007 to June 28, 2019 in the crude oil market. The table presents the estimate of the coefficient, the Newey and West (1987) t -statistics and in-sample adjusted R^2 statistics. The definitions of all predictors are the same as those in Table 3.

*Significance at the 10% level.

**Significance at the 5% level.

***Significance at the 1% level.

(0.44) and CE gain (1.98%) relative to investing based on the historical average.²² None of the other predictors can generate positive SR gains and CE gains, which indicates that these predictors cannot show any economic significance of out-of-sample predictability at monthly frequency. In addition, the term structure predictor *TRNC* performs well at shorter horizons, which can produce positive SR and CE gains.

We conclude that *DRNC* predicts daily, weekly and monthly USO excess returns out of sample with economic significance. The trading strategy based on *DRNC* can produce positive and relatively large SR and CE gains over the historical average forecast. However, *DRNV* and two term structure predictors (*TRNV* and *TRNC*) cannot achieve economic forecasting gains across different frequencies (daily, weekly and monthly).

If markets are efficient, then all information is already incorporated into prices, and so there is no way to “beat” the market, because there are no undervalued or overvalued securities available. In other words, investors cannot outperform the market, and that market anomalies should not exist because they will immediately be arbitrated away. Our findings indeed imply a violation of market efficiency in the Oil ETF market. Röscher et al. (2017) document that market efficiency is influenced by financial frictions (e.g., limits to arbitrage) and use some option-based variables to measure the dynamics of market efficiency. Han and Li (2021) explain the predictive power of the implied volatility spread is due to common informed trading in the option market. The return predictability of the *DRNC* might be caused by the market frictions or informed trading.

²² The monthly SP based on the historical average forecast benchmark is -0.27. All SP of the historical benchmark are negative, from -0.72 to -0.22.

Table 12
Trading strategies.

		Daily				Weekly				Monthly
		1-day	2-day	3-day	4-day	1-week	2-week	3-week	4-week	1-month
DRNC	Portfolio return (%)	3.77	2.01	0.85	0.60	5.28	1.14	0.12	1.97	2.39
	Portfolio volatility (%)	2.79	1.58	0.72	0.43	6.92	3.15	0.95	2.18	10.96
	SR gain	1.55	1.51	1.14	0.97	0.92	0.55	-0.03	1.17	0.44
	CE gain (%)	3.26	1.59	0.46	0.21	4.45	0.92	-0.02	1.83	1.98
DRNV	Portfolio return (%)	-0.62	0.29	0.50	0.38	0.55	0.22	-0.19	-0.14	-3.50
	Portfolio volatility (%)	1.29	1.13	0.64	0.39	7.02	3.30	1.53	1.30	8.63
	SR gain	-0.49	0.37	0.67	0.44	0.23	0.26	-0.10	0.01	-0.19
	CE gain (%)	-1.04	-0.11	0.11	-0.01	-0.30	-0.02	-0.36	-0.23	-3.22
TRNC	Portfolio return (%)	2.70	1.58	1.06	0.83	3.33	0.48	0.11	-0.15	-3.73
	Portfolio volatility (%)	2.90	1.50	0.95	0.76	8.38	3.15	2.01	2.02	10.64
	SR gain	1.13	1.27	1.25	1.16	0.56	0.34	0.16	0.18	-0.13
	CE gain (%)	2.18	1.16	0.66	0.43	2.17	0.25	-0.08	-0.27	-4.03
TRNV	Portfolio return (%)	0.36	0.38	0.53	0.56	0.85	0.91	0.80	0.67	-2.98
	Portfolio volatility (%)	0.28	0.20	0.24	0.23	2.05	2.13	2.21	1.67	7.37
	SR gain	-0.10	-0.02	0.80	1.01	0.40	0.55	0.49	0.60	-0.20
	CE gain (%)	-0.03	-0.01	0.14	0.17	0.67	0.77	0.60	0.56	-2.39

This table reports the performance of trading strategies based on return forecasts at daily, weekly and monthly horizons. The sample period is from May 9, 2007 to June 28, 2019. The performance measures are the annualized portfolio return, the annualized portfolio volatility, the annualized SR gain and the annualized CE gain. SR gain is the difference between the SP of forecasts generated by our regression model and the historical average forecast benchmark. CE gain is the difference between the CE of forecasts generated by our regression model and the historical average forecast benchmark. *DRNC* is the first differences in the covariance rate. *DRNV* is the first differences in the variance rate. *TRNC* is the difference between the risk-neutral covariance rate at one-year maturity and the one at one-month maturity. *TRNV* is the difference between the risk-neutral variance rate at one-year maturity and the one at one-month maturity.

The fact that the market is not fully efficient would seem to conflict with “no dynamic arbitrage” assumption in Carr and Wu’s (2020) framework. However, Carr and Wu’s (2020) framework provides a scientific way to extract information (e.g., *DRNC*) from the option price or implied volatility. Bollerslev et al. (2015) construct tail risk premia based on “no dynamic arbitrage” assumption and use it to predict future stock returns. Bardgett et al. (2019) use a standard option pricing model based on “no dynamic arbitrage” assumption to infer the variance risk premium from the S&P 500 and VIX markets and find that the based-based variance risk premium can significantly improve S&P 500 return forecasts. Please note that because financial markets can give rise to risk premiums – either the well-known equity risk premium or its equally valid volatility risk premium – the existence of a variance risk premium does not imply market inefficiency.²³

4.4. Alternative market returns

Our paper uses the USO ETF returns to present the crude oil market returns, and examines the return predictability by using the extracted risk-neutral estimates from the no-arbitrage formula. The *DRNC* has significant predictive ability for forecasting daily, weekly and monthly excess USO returns based on in-sample and out-of-sample tests. To provide a comparison, we conduct the same tests using the returns on WTI oil futures, which are the most liquid energy futures contracts, with an average daily volume of 822 thousand and open interest of 2.1 million contracts as of October 5, 2020.

We study the predictive power of *DRNC* for predicting WTI oil futures returns on different maturities. We use 1, 2, 3 and 4 month futures contracts available from the EIA. Contract 1 means the nearby or front month contracts. Contracts 2–4 mean the successive delivery months following Contract 1. The results for predictability of the excess returns on the WTI oil price are presented in Table 13. M1R, M2R, M3R and M4R represent the crude oil futures returns for 1, 2, 3 and 4 months to maturity contracts.

For in-sample tests, we find that *DRNC* can significantly predicts crude oil futures returns for 1, 2, 3 and 4 months to maturity contracts at daily, weekly and monthly horizons. In general, the regression coefficients, t-statistics and R^2 statistics of *DRNC* is slightly smaller than the ones on the basis of USO return predictability in Table 9. This should be expected since *DRNC* measured from the USO ETF option market should predict the USO ETF returns better than the crude oil futures returns. In addition, the statistical values in Table 13 decrease from M1R to M4R for each forecasting horizon except for the one-day and two-week horizons. For example, for the one-month forecast horizon, the coefficients drop from 37.30 (M1R) to 32.09 (M4R) and the t-statistics values drop from 2.68 (M1R) to 2.34 (M4R). This indicates that our predictor performs better in predicting crude oil futures returns on short maturities than long maturities. It appears that the term structure of crude oil futures prices has some extent influence on the oil futures return predictability.

²³ We thank the associate editor for suggesting this clarification.

Table 13
Oil futures return predictability.

		Daily				Weekly				Monthly
		1-day	2-day	3-day	4-day	1-week	2-week	3-week	4-week	1-month
M1R	β	9.97***	6.60***	4.35***	2.47**	25.07**	10.73*	0.83	10.36***	37.30***
	t	(3.16)	(3.34)	(3.16)	(2.13)	(2.54)	(1.88)	(0.21)	(2.93)	(2.68)
	$R^2(\%)$	0.39	0.36	0.23	0.08	1.11	0.31	-0.16	0.66	1.03
	$R_{os}^2(\%)$	0.40***	-0.20	0.06**	0.00	0.13*	-0.62	-0.09	0.53**	1.65**
M2R	β	9.99***	5.96***	4.03***	2.42**	22.19**	11.19**	0.41	9.53***	35.91**
	t	(3.28)	(3.12)	(3.13)	(2.25)	(2.38)	(2.09)	(0.11)	(2.90)	(2.58)
	$R^2(\%)$	0.45	0.33	0.22	0.09	1.00	0.43	-0.16	0.62	1.03
	$R_{os}^2(\%)$	0.55***	0.01***	0.18***	0.04**	0.31*	-0.71	-0.12	0.50**	1.57**
M3R	β	9.88***	5.68***	3.76***	2.30**	21.17**	11.23**	0.23	8.94***	33.40**
	t	(3.36)	(3.06)	(3.08)	(2.24)	(2.29)	(2.20)	(0.06)	(2.80)	(2.41)
	$R^2(\%)$	0.47	0.32	0.20	0.08	0.97	0.49	-0.16	0.59	0.89
	$R_{os}^2(\%)$	0.63***	0.11***	0.21***	0.08**	0.30*	-0.72	-0.13	0.47**	1.39*
M4R	β	9.74***	5.44***	3.57***	2.19**	20.78**	11.11**	0.09	8.58***	32.09**
	t	(3.40)	(3.01)	(3.04)	(2.22)	(2.28)	(2.24)	(0.02)	(2.75)	(2.34)
	$R^2(\%)$	0.49	0.31	0.19	0.08	0.99	0.51	-0.16	0.57	0.85
	$R_{os}^2(\%)$	0.67***	0.15***	0.22***	0.10	0.32*	-0.69	-0.14	0.44**	1.34*

This table reports the results for predictability of the WTI oil futures returns on different maturities by using the predictor, $DRNC$, at horizons ranging from daily, weekly to monthly frequency. $DRNC$ is the first differences in the covariance rate. The sample periods are from May 9, 2007 to June 28, 2019. M1R, M2R, M3R and M4R represent the crude oil futures returns for 1, 2, 3 and 4 months to maturity contracts. The oil futures contracts data are from EIA. When the forecast horizon is one day, one week or one month, we use nonoverlapping returns. Otherwise, we use overlapping observations.

*Significance at the 10% level.

**Significance at the 5% level.

***Significance at the 1% level.

For out-of-sample tests, $DRNC$ shows the generally similar performance to what it did on the USO return predictability. It has a positive and significant out-of-sample R^2 for the daily frequencies (except for the two-day and four-day forecast horizons), weekly frequencies (except for the two-week and three-week forecast horizons) and monthly frequency. We also observe that $DRNC$ exhibits better out-of-sample predictive power (large and significant R_{OS}^2) for longer maturities at shorter forecasting horizons or for shorter maturities at longer forecasting horizons. For example, for the one-day forecast horizon, the R_{OS}^2 value of $DRNC$ increases from 0.40 (M1R) to 0.67 (M4R). For the one-month forecast horizon, the R_{OS}^2 value of $DRNC$ decreases from 1.65 (M1R) to 1.34 (M4R) and its significance level drops to 10%.

In summary, $DRNC$ can significantly predict crude oil futures returns on different maturities at daily, weekly and monthly horizons. Therefore, our results on USO return predictability are robust to the alternative measure of market returns. Furthermore, the term structure of crude oil futures prices has some extent influence on the oil futures return predictability.

5. Conclusion

In this paper, we study the information extracted from the no-arbitrage (Carr and Wu, 2020) formula based on a new option pricing framework in the USO option market and investigate the predictability of the information in forecasting the future USO returns. The risk-neutral variance and covariance estimates can be obtained from the no-arbitrage formula under the new framework. We document the term structure and dynamics of the risk-neutral estimates which lead to a “U”-shaped implied volatility smile with a positive curvature.

We also investigate the return predictability of the innovations in the risk-neutral estimates and their term structures at daily, weekly and monthly frequencies. First, we run univariate predictive regressions for each of the predictors based on in-sample and out-of-sample tests. $DRNC$ is a strong predictor of future crude oil market returns at various horizons, and outperforms other existing return predictors. Second, we consider a set of control variables, including crude oil market-specific variables and macroeconomic variables, and do multiple predictive regressions to test whether the predictive ability of our risk-neutral variables is not affected by the control predictors. The significant predictive power of $DRNC$ still holds after controlling for other variables. Third, we run out-of-sample trading strategies to assess the economic importance of the different predictors for forecasting excess returns. $DRNC$ can generate significantly large SR gains and CE gains that exceed those provided by other predictors, which confirms the economic significance of $DRNC$'s predictive ability. Finally, we conduct the same tests using the excess returns on the WTI crude oil futures on different maturities. Our result is robust to the alternative market returns. Therefore, $DRNC$ is a significant and robust predictor for predicting daily, weekly and monthly excess returns in both statistical and economic terms. $DRNC$ contains substantial information about future USO returns. In contrast, $DRNV$ and the two term structure predictors ($TRNV$ and $TRNC$) show relatively weak predictive ability for USO excess returns.

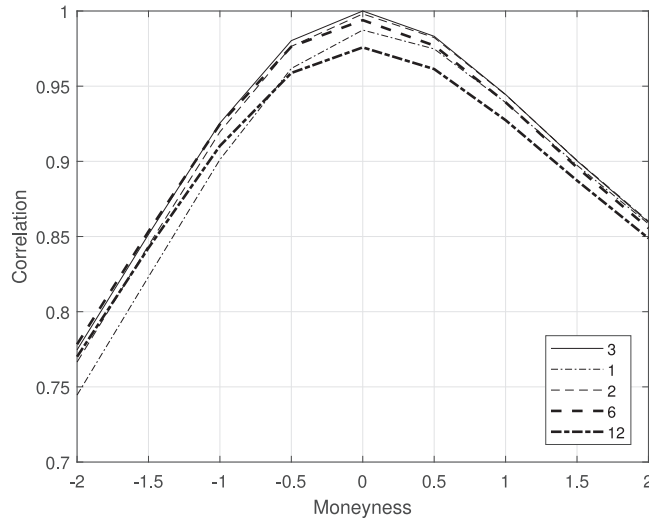


Fig. B1. Implied volatility change correlation with the three-month at-the-money option. The figure presents the cross-correlation estimates of the percentages of the implied volatility change series between the three-month at-the-money option (the reference point) and all other contracts at different maturities and moneyness. The solid line plots the correlation estimates with contracts at the same three-month maturity. The other lines plot the correlation estimates with contracts at other maturities (1 month, 2 months, 6 months and 12 months). The log percentage implied volatility change is defined as $R'_{i,t+1} = \ln(I'_{i,t+1}/I'_i)$, where I'_i is the implied volatility on date t for each option contract i .

CRedit authorship contribution statement

Xiaolan Jia: Methodology, Software, Data curation, Investigation, Validation, Writing – original draft, Writing – review & editing. **Xinfeng Ruan:** Supervision, Conceptualization, Methodology, Investigation, Writing – review & editing. **Jin E. Zhang:** Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no financial conflicts of interests that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A. Proof

The pricing formula for a European call option contract with strike price K and expiry date T is given by

$$B(t, S_t, I_t; K, T) = S_t \left(-\frac{(k - \frac{1}{2} I_t^2 \tau)}{I_t \sqrt{\tau}} \right) - KN \left(-\frac{(k + \frac{1}{2} I_t^2 \tau)}{I_t \sqrt{\tau}} \right), \tag{A.1}$$

where S_t is the underlying security price, I_t is the implied volatility, $\tau = T - t$ is the time to maturity, $k = \ln(K/S_t)$ is the relative strike and the terms $z_{\pm} = (k \pm \frac{1}{2} I_t^2 \tau)$ represent the convexity-adjusted moneyness of the call under the risk-neutral measure.

The instantaneous P&L of the option investment can be attributed to the variation in the calendar time, the underlying security price and the implied volatility.

$$dB = [B_t dt + B_S dS_t + B_I dI_t] + \left[\frac{1}{2} B_{SS} (dS_t)^2 + \frac{1}{2} B_{II} (dI_t)^2 + B_{IS} (dS_t dI_t) \right] + J_t, \tag{A.2}$$

where the partial derivatives are commonly labeled as the option’s theta (B_t), delta (B_S), vega (B_I), gamma (B_{SS}), volga (B_{II}) and vanna (B_{IS}), respectively. The last term J_t is associated with random jumps in the stock price and option implied volatility. When they are purely continuous movements, the last term J_t could be dropped.

The expectation of the option P&L attribution in Eq. (A.2) under the risk-neutral measure Q is divided by the instantaneous investment horizon dt ,

$$\frac{Et[dB]}{dt} = B_t + B_I I_t \mu_t + \frac{1}{2} B_{SS} S_t^2 \sigma_t^2 + \frac{1}{2} B_{II} I_t^2 \omega_t^2 + B_{IS} I_t S_t \gamma_t, \tag{A.3}$$

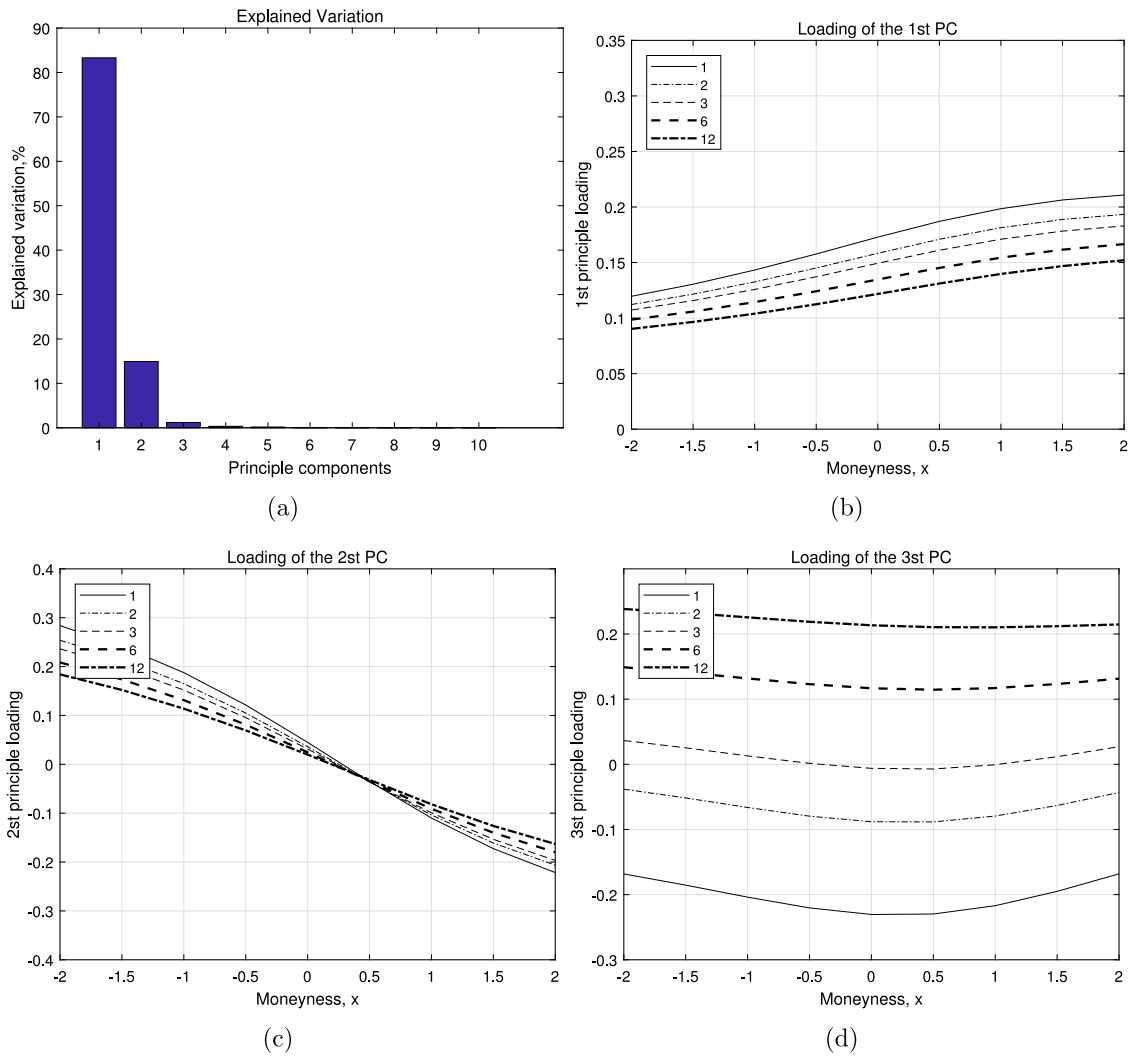


Fig. B2. Principal component analysis on implied volatility movements. This figure presents the principal component analysis on the interpolated implied volatility change series. Panel a uses bar charts to show the explained variation of the top 10 principal components on the 45 interpolated implied volatility change series and Panels b–d plot the loadings of the first, second, and third principal component, respectively, across all moneyness levels and maturities.

where $\mu = E_t \left[\frac{dI_t}{I_t} \right] / dt$, $\sigma_t^2 = E_t \left[\left(\frac{dS_t}{S_t} \right)^2 \right] / dt$, $\omega_t^2 = E_t \left[\left(\frac{dI_t}{I_t} \right)^2 \right] / dt$, $\gamma_t = E_t \left[\left(\frac{dS_t}{S_t}, \frac{dI_t}{I_t} \right) \right] / dt$.

μ_t denotes the annualized risk-neutral expected rate of percentage change in the BMS implied volatility of the option contract. σ_t^2 , ω_t^2 and γ_t denote the time- t conditional variance and covariance rate of the stock return and the implied volatility change.

The zero financing cost assumption and no dynamic arbitrage indicate that the risk-neutral expected return on the option investment is zero, then the pricing relation is

$$-B_t = B_I I_t \mu_t + \frac{1}{2} B_{SS} S_t^2 \sigma_t^2 + \frac{1}{2} B_{II} I_t^2 \omega_t^2 + B_{IS} I_t S_t \gamma_t. \tag{A.4}$$

Under continuous price and implied volatility movements and zero financing costs, no dynamic arbitrage requires that an option must be priced to balance out the option's theta loss with expected gains and losses from the option's vega, gamma, volga and vanna exposures at any point in time.

The BMS theta (B_t), cash vega ($B_I I_t$), cash vanna ($B_{IS} I_t S_t$) and cash volga ($B_{II} I_t^2$) can all be represented in terms of the BMS cash gamma ($B_{SS} S_t^2$).

$$\begin{aligned} B_t &= -\frac{1}{2} I_t^2 B_{SS} S_t^2, & B_I I_t &= I_t^2 \tau B_{SS} S_t^2, \\ B_{IS} I_t S_t &= z_+ B_{SS} S_t^2, & B_{II} I_t^2 &= z_+ z_- B_{SS} S_t^2. \end{aligned} \tag{A.5}$$

Theorem 1 (Carr and Wu, 2020). Assuming continuous price and implied volatility movements, and performing instantaneous P&L attribution on a European option investment based on the BMS pricing equation, a no-arbitrage pricing relation can be arrived at on the basis of Eqs. (A.4) and (A.5)

$$I_t^2 = [2\tau\mu_t I_t^2 + \sigma_t^2] + [2\gamma_t z_+ + \omega_t^2 z_+ z_-], \quad (\text{A.6})$$

where μ_t and ω_t^2 denote the risk-neutral conditional mean and variance of the implied volatility percentage change, σ_t^2 is the conditional variance of the underlying security return and γ_t is the conditional covariance between the implied volatility percentage change and underlying security return.

When $z_+ = k + \frac{1}{2}A_t^2\tau = 0$, the pricing equation for the at-the-money implied volatility is

$$A_t^2 = 2\tau\mu_t A_t^2 + \sigma_t^2. \quad (\text{A.7})$$

Assumption 1. The expected rates of change for at-the-money implied volatilities of nearby maturities are the same,

$$\mu_t(\tau_1) \doteq \mu_t(\tau_2), \quad (\text{A.8})$$

when $|\tau_1 - \tau_2|$ is small.

Under the local commonality assumption, the risk-neutral expected rate of implied volatility changes can be extracted from the at-the-money implied variance slope within this maturity range $[\tau_1, \tau_2]$,

$$\mu_t = \frac{A_t^2(\tau_2) - A_t^2(\tau_1)}{2(A_t^2(\tau_2)\tau_2 - A_t^2(\tau_1)\tau_1)}. \quad (\text{A.9})$$

Assumption 2. The expected rate of implied volatility change scales proportionally with the at-the-money contract

$$\mu_t I_t^2 = \mu_t^A A_t^2. \quad (\text{A.10})$$

Under the assumption, the no-arbitrage (Carr and Wu, 2020) formula can be obtained by subtracting Eq. (A.7) from (A.6)

$$I_t^2 - A_t^2 = 2\gamma_t z_+ + \omega_t^2 z_+ z_-. \quad (\text{A.11})$$

Appendix B. Implied volatility co-movements

See Figs. B1 and B2.

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