Case Studies on Transport Policy 13 (2023) 101039

Contents lists available at ScienceDirect

Case Studies on Transport Policy

journal homepage: www.elsevier.com/locate/cstp

Meta-heuristic aggregate calibration of transport models exploiting data collected in mobility

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A R T I C L E I N F O

Keywords:
Model calibration
Parameter estimation
Metaheuristic optimization
Particle swarm optimization
Transport demand model
Supply road network model
4-stage model
Road link-node delay functions

A B S T R A C T

The wide diffusion of data collected in mobility led to an unprecedented amount of information about people's mobility behavior. While on one hand the availability of big data from multiple sources enables to calibrate complex models with a high number of parameters, on the other hand, the dimension of the problem increases, and computational efficiency becomes an important issue. The paper presents a general methodology for the aggregate calibration of transport system models that exploits data collected in mobility jointly with other data sources within a multi-step optimization procedure based on metaheuristic algorithms. The methodology is applied to two real large-scale case studies in two different contexts. The first concerns the aggregate calibration updating a national strategic 4-step demand model in use in a big European Country; the second deals with the calibration of link and node performance functions implemented in a traffic network model of a town of around 3 million inhabitants. The results demonstrate the effectiveness of the aggregate calibration methodology in significantly improving earlier models' estimations. The results also highlight that the errors are in the same order of magnitude as the intrinsic variation of the data collected in the field.

1. Introduction

Model calibration is the process of estimating the values of model coefficients to obtain the best correspondence between observed and simulated values of the variables. If the model consists of one function or a system of independent functions, usual optimization algorithms such as projected gradient can be applied to solve the calibration problem by minimizing an error function or maximizing the likelihood. However, more complex models require a set of adjustment trial-and-error procedures that can be partially formalized as an optimization problem (Park and Qi, 2005), so that their calibration is a process characterized by significant heuristic components. The latter is the case of transport network models, which imply either the solution of a fixed-point model or the explicit simulation of the transport networks. In practical engineering applications, guidelines require the calibration of transport networks to comply with a set of procedures to ensure the results are consistent with both observed data and the physical and logical characteristics of the system (U.K. Department for Transport, 2020a; Dowling et al., 2004). This is also the case of 4-stage transport models, which imply feedback from lower to upper models, although their different stages are usually calibrated separately (U.K. Department for Transport, 2020b). A sound statistical framework for combining traffic counts with other information sources was provided by Cascetta and Russo (1997). They generalized the most common Origin-Destination (O-D) trip matrix estimation method to the problem of estimating the parameters of pre-specified aggregated travel demand models.

The wide diffusion of Intelligent Transportation Systems applications has made available new sources of information on people's mobility and an unprecedented amount of data on transport system performances, which provided new opportunities for enhanced calibration methods (Shafiei et al., 2018; Ben-Akiva et al., 2012; Fusco et al., 2013; Qin and Mahmassani, 2004). For privacy reasons, these data do not contain personal information on the users' attributes, so a disaggregate calibration of behavioral models is unfeasible, and an aggregated calibration can be applied. In the literature, most applications are based on metaheuristic methods like genetic algorithms (Leal et al., 2020; Ma et al., 2007), particle swarm optimization (Poole and Kotsialos, 2016; Tavassoli, 2020; Mohammadian et al., 2021), and SPSA (Ben-Akiva et al., 2012; Antoniou et al., 2015) to search for the values of model coefficients that best fit traffic or passenger flow counts. Several studies (Hou et al., 2013; Vaze et al., 2009; Zheng and Van Zuylen, 2014) extended this approach to use multiple data sources (including link

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https://doi.org/10.1016/j.cstp.2023.101039
Received 24 October 2022; Received in revised form 29 May 2023; Accepted 14 June 2023
Available online 17 June 2023

counts and point-to-point travel times) for the calibration. The new sources of information can also contribute to providing accessibility measuring, necessary in activity-based applications (Carrese et al., 2019).

Floating Car Data provide information on every single trip and on the positions and speeds of the connected vehicles (Fusco et al., 2016; Isaenko et al., 2017) that can be effectively combined with traffic counts and speeds collected by roadside sensors to improve the level of knowledge of the transport system significantly. In the literature, these sources of information coming from tracking travelers in mobility are often used for a direct estimate of the O-D trip matrices (Cipriani et al., 2014; Rahmani et al., 2017; Carrese et al., 2017; Nigro et al., 2018; Oh et al., 2019; Carrese et al., 2019; Cantelmo and Viti, 2020) or to calibrate the coefficients of microsimulation models (Osorio and Punzo, 2019, among others). Crowdsourcing of mobility data can provide ample opportunities to enhance also the methods for transport model calibration. On the other hand, the use of individual data collected through probe sensors introduces new issues related to the cumbersome computation effort (Mudigonda and Ozbay, 2015) and the need for including the statistical representativeness of the data in the goodness-of-fit function (Fusco et al., 2016).

The paper aims to provide a general methodology for the calibration of transport system models, which exploits the crowdsourcing of data collected in mobility and metaheuristic algorithms' features to optimize non-convex problems through two relevant large-scale case studies. Although most of the literature on the calibration through floating car data refers to microsimulation models, as the recent guidelines by Wunderlich et al. (2019), the methodological implications of using data in mobility are general and can be applied to the whole chain of transport system models. Based on this motivation, the method is illustrated through two big real case studies. One refers to the calibration of a 4-stage demand model used for a national strategic model of a big EU country; the other considers link performance functions and node delay functions of a road network model of a 3-million-inhabitant town, where the demand matrix was derived by application of an activity-based model (Bhat and Koppelman, 2003), in spite of usual limitations small and medium size networks (Acheampong and Silva, 2015). As far as the calibration of the 4-stage model, it is worth mentioning that the real case study described concerns the update of an existing model using new data. Thus, it is presented to highlight how the aggregate calibration procedure is general and can be applied to different models.

With reference to the existing literature, the paper tackles the practical problems arising in large-scale applications that, on one hand, make available huge amounts of mobility data from different sources and, on the other hand, present wide degrees of indeterminateness due to the size of the problem. Concerning these issues, the methodology introduces a specific term in the objective function that considers the statistical significance of the crowdsourcing observations. To reduce the extent of the search of the optimal solution, suitable boundaries are introduced to the feasibility set of the model coefficients. The methodology is general, and the paper demonstrates its practical applicability to very different case studies, ranging from a national multimodal demand model to an urban traffic network model.

This paper is organized as follows. In Section 2 the General Calibration methodology together with the optimization algorithm used are explained. Section 3 presents the specification of the models used in the study cases. Section 4 reports the methodology applied for the aggregated calibration of a demand model, while Section 5 concerns the case of the aggregated calibration of a supply model. The conclusions are addressed in Section 6.

2. General calibration methodology

The aggregate calibration of a transport system model is an efficient method to adjust the model coefficients by using observations of traffic flows or network performance measures at different scales. It is usually formulated as an optimization problem consisting of minimizing an error function with respect to the coefficients of the model, given a set of traffic counts and the corresponding simulated values, also given the values of the variables describing the activity system and the transport network supply model.

The availability of many sources of information from ITS applications enables using even different variables from traffic counts, like speed, density, and travel times, which characterize the transport network performances and exploit observed users' trips as sample estimations of the O-D trip matrices. The heterogeneity of target variables implies that the error function is composed of different terms; furthermore, any type of variable is affected by significantly different estimation and measurement errors. Thus, the error function is formulated as a linear combination of every target variable's error functions multiplied by a coefficient that represents the degree of belief in the estimation of the measure.

The uncertainty in estimating the average flow provided by traffic detectors is due to the measurement error, the unavoidable model error, and the variance of the variable measured. The latter is consequent to the inherent variability of the phenomenon; that is, the variability of travel demand traveling through the detector site over the measurement interval. The information provided by the Floating Car Data is affected by an additional source of uncertainty related to the sampling rate, which, on the one hand, affects the accuracy of the estimate and, on the other hand, is unknown and can be estimated through the average penetration rate of vehicles that serve as probes and provide the requested measures. In this section, the most general components of the methodology –objective function and optimization algorithm– are introduced first; then, the demand and supply models are specified.

2.1. Objective function

The objective function is defined to consider the different sources of uncertainty with their significance, depending on the measurement method and the sampling rate.

Generally, the measured variables used in the objective function are related to the directly observable measures such as flows and speeds. These measures are usually represented by traffic count locations and possibly by FCD. While the count locations provide the estimation based on the total number of vehicles that pass through and therefore represent statistically significant data, the distribution of the floating vehicles is not uniform. Thus, the coefficients of the error function are inversely weighted by the width of their confidence intervals, which depend on the standard deviations of their measures and the number of observations for each element.

For example, the average speed on every link of the network can be estimated as the average value of FCD's point speed measures. Since different numbers of speed measures are collected for different network elements, every element is considered proportionally to its statistical significance. The calibration problem is then formulated as the problem of minimizing the following error function, subject to comply the equilibrium of the transport system.

$$z=\sum_{i=1}^{n}\varphi_i\sum_{k=1}^{n_i}\frac{\sqrt{n_{i,k}}}{\sigma_{i,k}}\left[\widehat{y}_{i,k}(\boldsymbol{\beta})-y^{o}_{i,k}\right]^2 \quad (1)$$

i generic variable under observation$n$ number of variables under observationn_i number of elements of observations for the variable $i\varphi_i$ relative weight introduced to balance and normalize the relative importance of the terms referred to the different variables i in the objective function$k$ generic measurement element of observation for the variable $in_{i,k}$ number of measures for the element k of the variable $i\sigma_{i,k}$ standard deviation of the measures of the element k of the variable i, depending on the measurement methodβ vector of model coefficients$\widehat{y}_{k,i}$ output of the model for the variable i and element $ky^{o}_{k,i}$ measure of the variable i and element k

2.2. Optimization algorithm

The calibration of a transport system model is a well-known non-convex problem, whose size and mathematical properties depend on the specific sub-problem. In the case of estimation of one component of the demand model, the number of coefficients is limited, in the range of few dozens, while it may be much larger in the calibration of supply networks. Metaheuristic optimization algorithms are seen as suitable a solution method for this kind of problem. They provide a general procedure that can be applied to different problems without requiring to changing the mathematical formulation of their steps as it is required, for example, if using a gradient projection algorithm. Among the metaheuristic algorithms, the Particle Swarm Optimization (PSO) algorithm is proposed because, unlike genetic algorithms, it is defined on continuous variables and allows exploring a wide range of the feasibility region (Kennedy and Eberhart, 1995).

PSO algorithm is a stochastic optimization technique inspired by social natural organisms' behavior, like ants and bees, and applies a collective strategy to reach the minimum value of the objective function. Each individual of the swarm moves according to a strategy that combines different rules: random steps and movements toward social and individual previous bests. The system is initialized with a population of random solutions and searches for optima by updating generations. In PSO, the potential solutions, assimilated to particles of a swarm, move through the feasible region of the problem space. Each particle's movement direction and speed depend on a combination of the distance of its current position from its individual best and the current collective best of the swarm (Fig. 1). The iterative searching procedure continues constrained within the boundaries of the feasibility set until maximum iterations, or minimum error criteria is not attained.

The choice of the feasibility region is an essential issue of the procedure. It enables the analyst to impose constraints on the variables that reflect the statistical properties of the variables, the physical nature of the problem, and the engineering aspects to take into account.

In order to compute the fitness value in case of calibration of a transport model by the Particle Swarm Optimization algorithm the transport model has to be run at each step for the algorithm. The highlighted box indicates the implemented run of the transport model in the functional architecture of the Particle Swarm Optimization algorithm reported in Fig. 2.

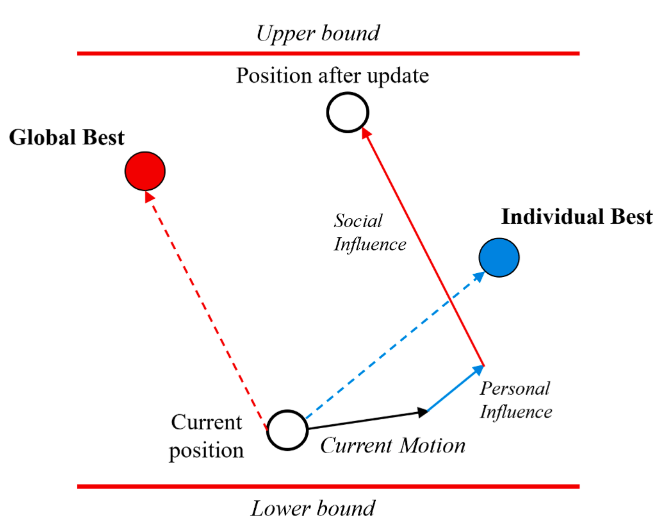

Fig. 1. Movement logic of single particles belonging to the swarm.

3. Model specification

3.1. Transport demand model

The demand model can be synthetically defined as a mathematical relationship between travel-demand flows $\boldsymbol{f}$ and socio-economic variables $\boldsymbol{S}$ related to the activity system and the users and of a vector $\boldsymbol{T}$ of level-of-service attributes of the transportation supply system (Cascetta, 2009).

$$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{S}, \boldsymbol{T}) \tag{2}$$

To be more specific, the classic 4-stage transport demand model is assumed here. It is composed of a sequence of models that sequentially estimate the different components of transport demand: generation, distribution and modal split, and assignment. The generation model provides the average number of trips $f_o^{i,s}$ undertaken from a zone o in a particular period h for purpose s by users of class i given the attributes $\boldsymbol{S_o}$ of the users and the system of activities in zone o. Distribution model provides the fraction $p_{o,d}^{i,s}$ of users of class i that travel from a zone o to a zone d in the period h for purpose s and provides the origin–destination trip matrices for every purpose and class of users; the modal split model estimates, for each origin–destination matrix the modal share $p_{o,d,m}^{i,s}$ for each transport mode m.

$$f_{o,d,m}^{i,s} = f_o^{i,s}(\boldsymbol{S}, \boldsymbol{T}; \boldsymbol{\beta}_o) p_{o,d}^{i,s}(\boldsymbol{S}, \boldsymbol{T}; \boldsymbol{\beta}_d) p_{o,d,m}^{i,s}(\boldsymbol{S}, \boldsymbol{T}; \boldsymbol{\beta}_m) \tag{3}$$

The coefficients vector of every stage of the demand model is denoted by the Greek letter $\boldsymbol{\beta}$ and denoted by the subscript corresponding to the respective demand component. The fourth stage of the model, the path choice model, is not presented here and is introduced in the next subsection related to the supply transport network.

Specification of the different stages of the transport model is not only for clarity but also for defining the multi-step framework of the calibration process. Although simultaneous calibration is possible and even straightforward by applying an aggregated approach, a sequential calibration of every stage enables better control of the process and provides a clearer understanding of the results. The aggregate calibration process of a 4-stage demand model will be exemplified in the study case presented in Section 4.

3.2. Supply network model

The transport network is topologically represented by a graph $\boldsymbol{G}$ composed by a set of links $\boldsymbol{L}$ connected among them at nodes belonging to a set $\boldsymbol{N}$. The circular interaction between the flow vector $\boldsymbol{q}$ and the vector of travel costs $\boldsymbol{c}$ on links and at nodes is simulated by the assignment model, which takes the O-D trip matrix $f_{o,d}$ in input and provides the flow values $\boldsymbol{q}$ that comply with a rational principle that reflects users' choices, depending on the performances of every element of the transport network, modeled as functions of the flows that affect that element of the network.

$$\boldsymbol{q} = \boldsymbol{q}\left(\boldsymbol{c}, f_{o,d}; \boldsymbol{G}\right) \tag{4}$$

$$\boldsymbol{c} = \boldsymbol{c}(\boldsymbol{q}; \boldsymbol{G}, \boldsymbol{\beta}_G) \tag{5}$$

where $\boldsymbol{\beta}_G$ denotes the vector of coefficients of the performance functions associated with links and nodes of the graph.

The relationships between travel times and flows that determine the traffic state on the network are typically different not only for different transport modes but also for links and nodes, as the structure of their interaction is different. Regarding the road networks, link travel time functions on urban networks where overtaking is usually not allowed can be modeled through separable travel time functions. However, node travel time functions depend on the flows arriving at every approach of the junction.

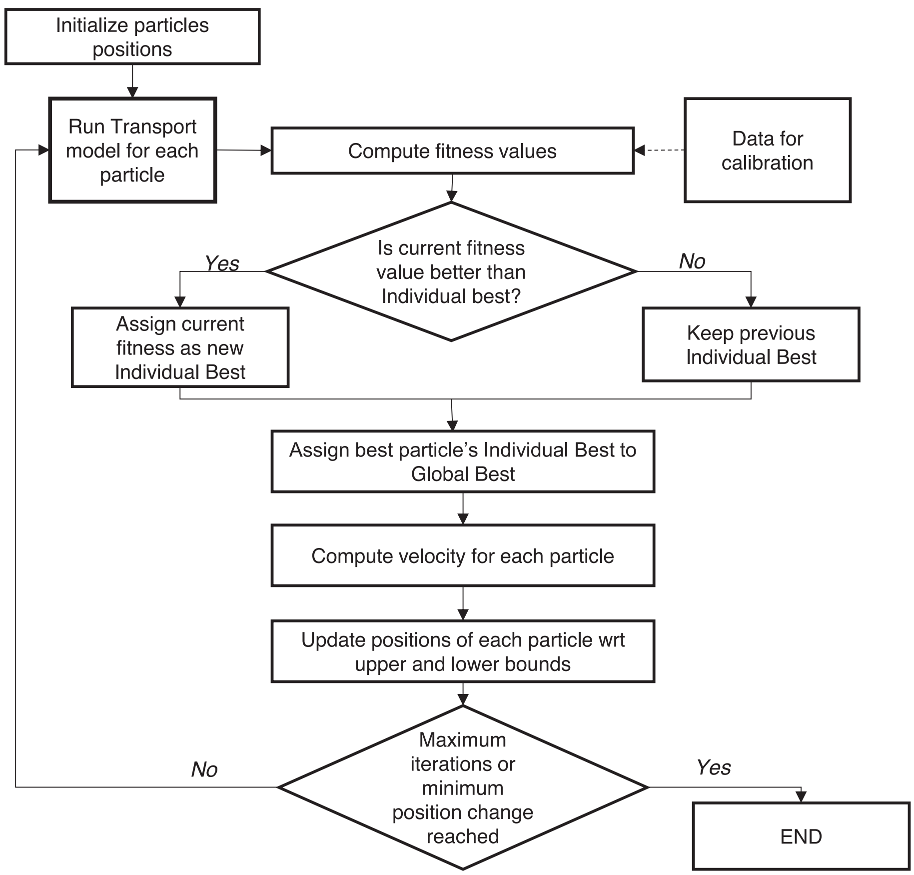

Fig. 2. Functional framework of the Particle Swarm Optimization Algorithm.

The modeling framework is reflected onto the calibration methodology, which preliminarily estimates the basic parameters of the link performance functions based on speed-flow measures and then exploits route travel time measures to estimate more complex interactions that occur at junctions.

Observations of route travel times provide synthetic information on the performances of a sequential set of links and nodes taken by a sample of users. For model calibration, both outputs and inputs must be known for all the network elements. In this case, the inputs are the O-D trip demand matrix, traffic counts on links, and turn movements at junctions, while the outputs are defined as routes. So, the value of the objective function is expressed through a combination of observed and estimated values. These concepts will be clarified with the example provided in the study case presented in Section 5.

4. Case study: aggregate calibration of demand model

The case study presented to exemplify the methodology for the aggregate calibration of a demand model concerns the updating process of a 4-stage model designed for long-term national strategic planning and previously calibrated from specific disaggregate surveys. The demand model was implemented by a sequence of generation, distribution, and modal split models, as presented in Section II. The generation model belongs to the category analysis approach and applies a constant trip rate for each demand segment composed of 9 user classes (based on employment status, age, car availability, and the number of family members) and 4 trip purposes (work/study, holiday, business, and other). Both distribution and modal split models adopted a multinomial logit structure.

The zoning system included 345 zones, which cover the whole territory of the Country, representative of the level of the provinces and major cities disaggregated into smaller zones. The transport modes available for passenger demand included light vehicles, air transport, bus, and train. The road network was modeled by around 230.000 links, while the transit network is composed of approximately 232 000 links. The whole train network is long about 20 000 km. The data used for calibration are described in detail in Section 4.1.

4.1. Data

The following data are available and are taken as reference in the calibration of the demand model:

- Daily O-D matrix for light vehicles, estimated from the O-D trips on a fleet of floating cars, corresponding to around the 4% of the total vehicular fleet of the Country, collected during one month of observations;
- Daily O-D matrix of train passenger flows;
- Aggregated measures of air traffic demand, estimated as the total incoming and the total outcoming passenger flows;
- Traffic counts collected at 1,915 locations.

The modes available for the model included train, airplane and light vehicles. The bus service is not represented in the model, since it represents only a marginal share for the inter-regional movements in the Country. Graph and zoning, as well as the assignment model, are not object of any calibration or modification.

4.2. Calibration framework

Fig. 3 reports a flow chart of the aggregate calibration method applied to demand models.

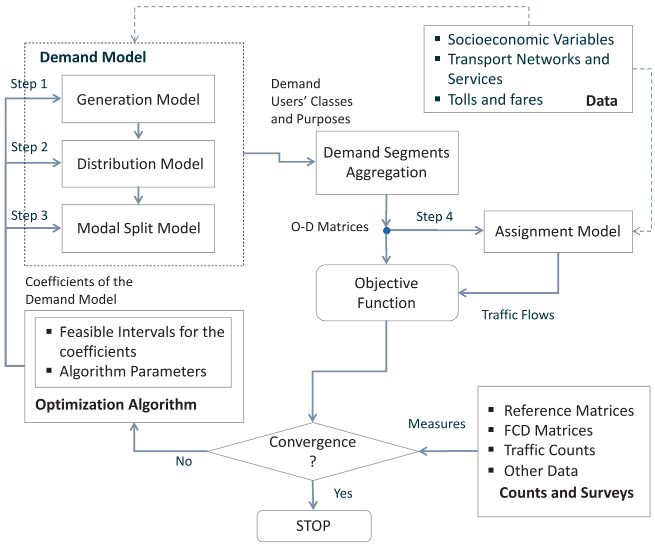

Fig. 3. General framework of the aggregate calibration process of a typical 4-stage demand model.

The method seeks to minimize the deviation between the measured data and the final outputs of a classic 4-stage model –that is, the flows assigned to the network by the assignment model– by correcting the demand model's coefficients, namely generation, distribution, and modal split models. The calibration methodology includes inputting the data related to the activity system and the transport network and services. These data are assumed to be constant and are not modified during the calibration procedure as denoted by the connections with the demand and the assignment models represented by dashed lines in the figure. The outputs of the different stages of the demand model (generation, distribution, and modal split), possibly divided for different demand segments, are finally combined in aggregated O-D matrices at every iteration of the optimization algorithm. These matrices are inputs for the assignment model, which outputs the traffic flows on the transport network.

In general, the application of the calibration method implies running the assignment model for every individual at each successive iteration, assuming fixed the socio-economic data together with road network performances, providing outputs from the model chain of both demand model and assignment model. However, if direct observations of trips at a given spatial disaggregation level are available, the same meta-heuristic optimization procedure can be applied by skipping the error component corresponding to the assigned flows.

The stopping criterion evaluates if the difference between the objective function of the current iteration and the previous iteration is smaller than a fixed threshold. If the threshold is met, the optimization procedure is stopped, while if the difference is significant, the procedure continues with a new iteration by updating coefficients of the demand model.

4.3. Objective function

The following formulation of the objective function is provided for the aggregate calibration of the demand model:

where:

- $f_{o,d,m,s}(\boldsymbol{\beta})$ is a generic component of demand with origin o and destination d for a mode m and a scope s produced by the demand model with a vector of coefficients β;
- $f^o_{o,d,m,s}$ is a generic component of demand with origin o and destination d for a mode m and a scope s obtained by the demand model with updated socio-economic data and network;
- $f_{o,d,m=\overline{m}}(\boldsymbol{\beta})$ is a generic component of demand with origin o and destination d for mode $m=\overline{m}$ (cars) for all the segments of demand produced by the demand model with a vector of coefficients β**,** being $f_{o,d,m=\overline{m}} = \sum_{s=1}^{n_s} f_{o,d,m=\overline{m},s}$;
- $f^0_{o,d,m=\overline{m}}$ is a generic component of demand with origin o and destination d for all segments of demand estimated from data collected in mobility on a specific mode $m=\overline{m}$, through floating car data;
- $q_{h,c}(\boldsymbol{\beta})$ is a traffic flow on link h for vehicle class k, obtained as the result of the assignment model, corresponding to the demand estimated with a vector of coefficients β;
- $q^o_{h,c}$ is the traffic flow observed on a link h for vehicle class k;
- $\lambda_1, \lambda_2, \lambda_3$ are the weights introduced in order to balance different data and normalize the relative importance of the sources;
- $\gamma_{o,d,m=1}$ is the coefficients matrix that expresses the level of confidence of each component of the demand matrix obtained by floating car data (FCD);
- $\gamma_{o,d,m,s}$ is the coefficients matrix that expresses the level of confidence of each component of the demand *o,d* for each mode m for each demand segment s;
- $\gamma_{h,k}$ is the coefficients matrix that expresses the level of confidence of each traffic measure $f^o_{h,k}$;
- n_o, n_d, n_m, n_s are the dimensions of the demand model, being respectively the number of origins, number of destinations, number of transport modes, number of demand segments;
- n_h, n_k are respectively the number of links where measures of traffic flows are available and the number of vehicle classes detected.

The formulation of aggregate demand model calibration (6) is coherent with the general methodology formalized in (1) and also includes equilibrium constraints (4) and (5) through the computation of traffic flows $q_{h,k}(\boldsymbol{\beta})$. The specific case concerned calibration of the passenger demand model, therefore only light vehicles were used for the class of the vehicles.

4.4. Algorithm application

The calibration procedure is carried out by a Python routine that implemented a PSO algorithm, described in Section 2.2.

The initialization of the algorithm consists of the specification of the demand model, characterized by each individual of the population by a vector of coefficients.

$\boldsymbol{\beta} = \{\boldsymbol{\beta}_o, \boldsymbol{\beta}_d, \cdots \boldsymbol{\beta}_m\}_{i,s}$ related to generation model, distribution model, and modal split, respectively, for each of demand segment s and user class i. To adjust the calibration of an existing demand model at the first iteration of the algorithm, the vector of the existing coefficients β is used for the first individual, while other individuals may assume some perturbations in the coefficients. The calibration procedure is carried out by

$$z == \lambda_1 \sum_{o,d,m=\overline{m}}^{n_o,n_d} \gamma_{o,d,\overline{m}} \left(f_{o,d,\overline{m}}(\boldsymbol{\beta}) - f^o_{o,d,\overline{m}} \right)^2 + \lambda_2 \sum_{o,d,m,s=1,1,1,1}^{n_o,n_d,n_m,n_s} \gamma_{o,d,m,s} \left(f_{o,d,m,s}(\boldsymbol{\beta}) - f^o_{o,d,m,s} \right)^2 + \lambda_3 \sum_{h,k=1}^{n_h,n_k} \gamma_{h,k} \left(q_{h,k}(\boldsymbol{\beta}) - q^o_{h,c} \right)^2 \tag{6}$$

a multiple step optimization.

The first step of the calibration process concerns the generation model, based on classification tables. At this stage, the trip rate coefficients of the generation model are subject to optimization while the existing coefficients of the distribution and the modal split models are kept constant. The trip rate coefficients are optimized by the PSO algorithm, considering 9 purposes for 9 demand segments, for a total of 81 coefficients. The error function is the squared sum of the differences between the demand estimated by the generation model for every zone and the corresponding number of trips reported by the marginal total of the reference O-D matrix.

The second step of the calibration process concerns the coefficients of the distribution model, which is an aggregated logit model. Both the calibrated coefficients of the generation model and the old coefficients of the modal split model are kept constant. PSO algorithm updated the coefficients of the following 12 variables:

- two specific attributes that characterize either the trips between the main towns of different provinces or the trips directed to the major metropolitan areas;
- number of workers in the zone;
- logsum variables for every purpose, representing the users' satisfaction in traveling between pairs of zones.

The objective functions used for the first two stages of the optimization process considered the demand matrices directly, without the term relative to observed flows. This enabled running only the demand-side of the model chain, which was less computationally expensive.

The last step of the calibration procedure concerns the modal split model, which is an aggregated logit model. At this stage, the coefficients of the generation and the distribution model obtained at the previous steps are taken as invariant. The optimization process focused on the coefficients of the value of time, which is considered the most relevant variable, segmented for purpose and user class. The objective function consists of two terms: one related to the O-D demand trips matrices, the other to the traffic flows on the road network. Thus, in order to compute the error between the observed and the simulated traffic flows, the transport assignment model has to be run.

4.5. Calibration results

Heat maps in Fig. 4 illustrate the errors for every element of the distribution model, before and after the calibration. Red points represent overestimations, while blue points identify underestimations. Since the zone codes refer to geographical criteria, it is possible to derive some considerations about the errors' structure.

The heat map related to the O-D matrix before the calibration highlights a general overestimation of the demand in the first columns and the first rows of the matrix, which denote the main town areas, and an underestimation of the trips between neighboring zones, which are represented by cells around the principal diagonal. The overestimation of the demand between the main towns before the calibration is due to the specific attributes of the major metropolitan areas that were corrected during the calibration procedure. The calibration procedure concerned the simultaneous calibration of 81 coefficients for the generation model and 12 coefficients for the distribution models and the objective function considered the average root square error computed over around 120,000 of OD pairs. Evaluation of the objective function for each vector of the coefficients required running of the demand models, which took approximately 20 min per run. Considering the extended space of the feasible region and the necessity to limit the computational time the convergence criterium was set to a maximum of 10,000 fitness evaluations. The applied metaheuristic algorithm improved the overall objective function value. Indeed, after the calibration of the distribution model, the errors' structure is more balanced since the errors are more uniformly distributed over the matrix and the RMSEN improved from 0.96 to 0.66.

The optimization procedure's capability to reduce the model structure's bias is evident in Fig. 5, related to the train mode, whose heat maps illustrate the intensity of O-D passenger flows for the reference

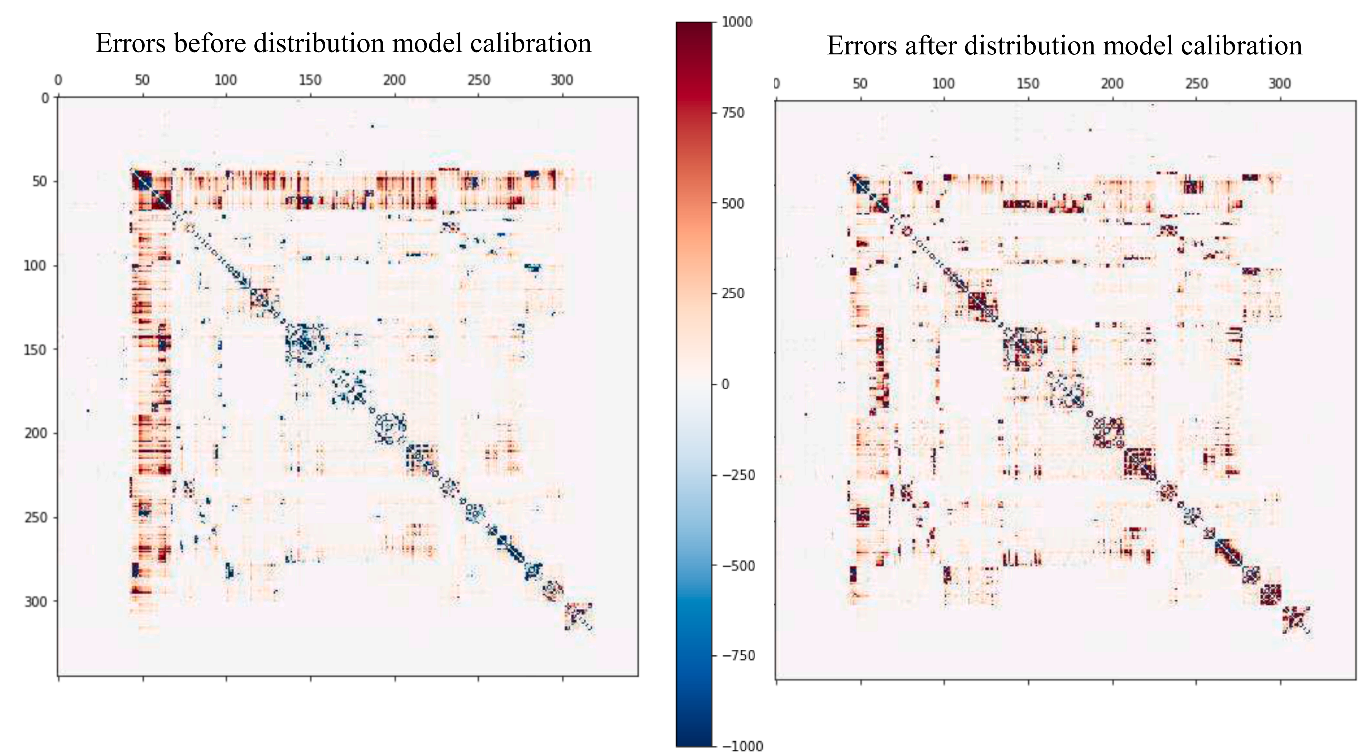

Fig. 4. Heat maps of the distribution model's errors, before (left) and after (right) the calibration.

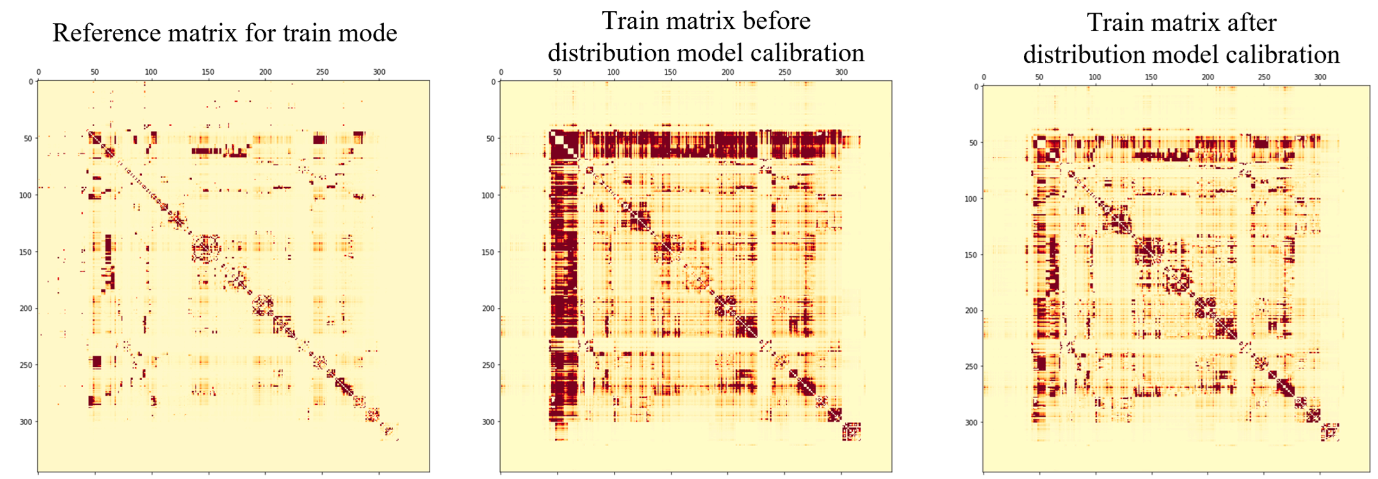

Fig. 5. Heat maps of passenger flows on train mode in the reference matrix and before and after the calibration of the distribution model.

matrix and the model before and after the calibration. The darker bands corresponding to the first columns and the first rows in the heat map of the matrix before calibration, compared to the reference matrix, highlight an evident overestimation of the trips generated and attracted to the main towns and the greater metropolitan areas, as already observed on the level of the distribution model. On the other hand, the matrix obtained after the optimization exhibits a flow structure more similar to the reference matrix, with a lower bias for main towns and metropolitan areas.

It is worth noting that the modal split demand was considered invariant with respect to the existing model at this stage of the calibration procedure. Thus, a general overestimation of the demand for the train mode could not be overcome at this stage of the calibration process.

Results of the calibration of generation and distribution models are reported in Table 1. The mean absolute normalized error (MAEN) reduces by about 11%, while the reduction of the root mean square error normalized (RMSEN) is around 30%. The higher reduction of the root square error indicates that the procedure significantly affects the cells of the demand matrix with higher errors.

The results of the calibration of the modal split model are reported in Table 2. The mean absolute error reduces by around 28% and 51% for car and train modes, respectively; the root mean square error reduces by around 65% and 24%, respectively for car and train modes. For what it concerns the comparison between the traffic counts and the flows assigned on the road network, the root total root square error reduces by 77%, from 94,777 veh/h to 10,655 veh/h, while the reduction of the mean absolute error reduction is about 19%. The coefficient of determination R^2, which relates observed and simulated flows, increases from 0.54 to 0.67, as reported in Fig. 6. Moreover, it is worth noting that the initial overestimation of the model was highly improved by the calibration procedure. Thus, the slope of the regression line of the initial model is greater than 1.7, resulting in a systematic positive bias in the simulated flows. After the calibration procedure, the slope of the regression line is around 1.2, some points still preserve overestimation by the traffic model, but the greatest errors are eliminated by the calibration process.Fig. 7 illustrates the structures of the errors in the

Table 1
Error statistics of the previous model and of the updated model after the calibration of the distribution submodel.

	Previous model before distribution calibration	After distribution model calibration	Difference Absolute	%
RMSEN	0.96	0.66	−0.29	−30.6%
MAEN	1.02	0.91	−0.11	−11.2%

demand matrices observed before and after the modal split model's calibration as heat maps. As already observed for the distribution model, the train passenger matrix before the calibration of the modal split overestimates the values of flows from and to the main towns and the greater metropolitan areas.

The calibration of the modal split model leads to a general improvement of the matrix structure and the underestimation of the demand for those areas.

For what it concerns the car mode, a general underestimation of the demand for the neighboring zones (individuated by the elements near to the main diagonal) was observed before the calibration procedure. The final model reduces the absolute error by 28%, even if some overestimation of the demand for neighboring zones can still be observed.

The structures of errors In the matrices are improved, and the error reduction from the old model is appreciable (the absolute error reduced by 28% and by 51% for car and train modes, respectively). However, the errors remain high (absolute normalized terms are around 0.7 and 1.1 for car and train modes, respectively); nevertheless, they are comparable to the results of the main state-the-art study on the aggregate calibration of a demand model (Cascetta and Russo, 1997), which was referred to a smaller area and a simpler model. The obtained results are also comparable in terms of the improvement gained by the model obtained by Shafiei et al. (2018), where an improvement of 30% was observed in the case of calibration of the demand model based on the observed traffic flows.

In this regard, it is worth noting that the aggregate calibration exploits only aggregate data such as reference matrices and traffic data counts to update an earlier demand model. The coefficients' adjustment was also constrained to feasible intervals that ensure the coefficients of the earlier model are not deranged. Thus, the obtained result must be seen as an improvement of an already existing model rather than a new demand model's calibration.

5. Case study: calibration of road network model

The case study presented to exemplify the aggregate calibration methodology of a supply network model concerns the updating procedure developed and applied to the detailed road network model of a metropolitan area of about 3 million inhabitants. The zoning consists of about 1,800 zones; the graph comprises about 300,000 links and 5,000 nodes.

5.1. Traffic data

The datasets used for the calibration of the road network model mainly come from two different sources:

Table 2
Error statistics of the previous model and of the updated model after the calibration of the modal split submodel.

	Previous model before modal split calibration		After modal split model calibration		Difference			
	Car	Train	Car	Train	Car	%	Train	%
RMSEN	0.94	1.01	0.55	0.88	−0.39	−41%	−0.13	−13%
MAEN	0.95	2.24	0.68	1.10	−0.27	−28%	−1.14	−51%

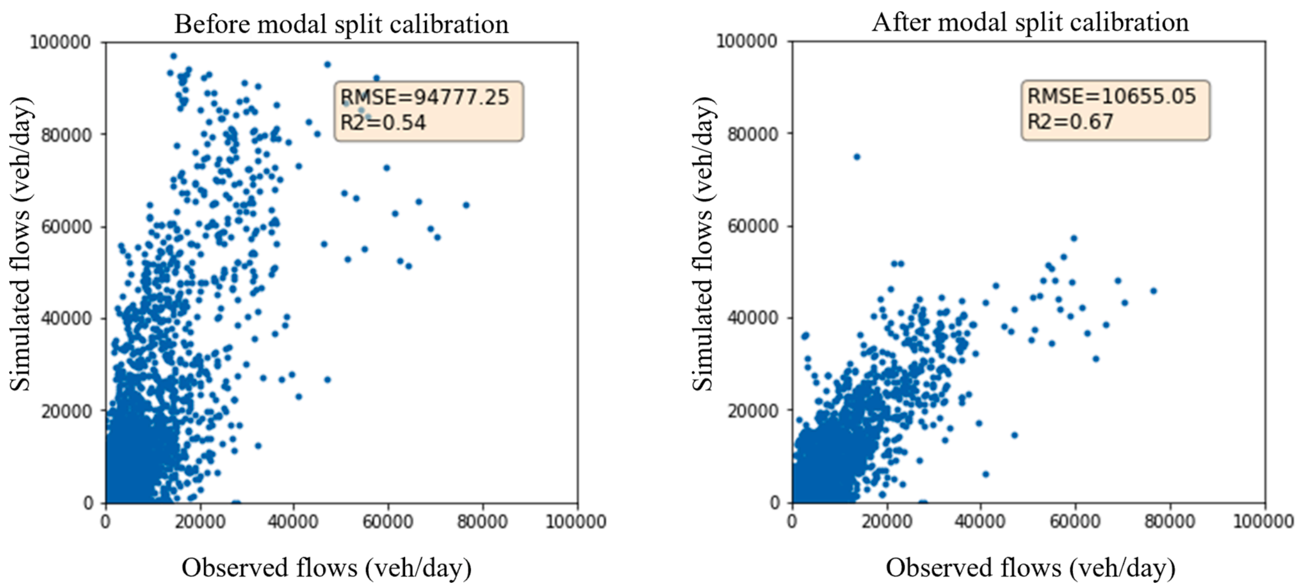

Fig. 6. Simulated and observed link flows before and after the calibration of the modal split demand model.

- Automatic Traffic Counts (ATC), containing a set of 521 pairs of point speed-flow values, collected at 221 locations illustrated;
- Turn Movement Counts (TMC), composed of 1,790 turn movement counts detected at 331 road junctions;
- Travel Time Surveys (TTS), composed of 4157 runs, collected along 75 routes at different time periods (AM, MD, PM), among which 1870 on 71 unidirectional routes are used for calibration, and the remaining are retained for model validation.

5.2. Delay models for links

Delay models for links are represented by the well-known BPR curves (Bureau of Public Roads, 1965), which were first proposed by the USA Bureau of Public Roads and are widely used in traffic assignment models (Mtoi and Moses, 2014).

$$t = t_0\left[1 + \alpha\left(\frac{q}{s}\right)^{\beta}\right] \tag{7}$$

- t_0 : Free flow travel time
- q: traffic flow on the link
- α, β, s: calibration parameters

It is worth underlining that the relationship between travel delays and flows used in macroscopic traffic models is extremely simplified and behaviorally unrealistic, yet it is commonly applied in big-scale traffic demand models. In the link delay functions, the flow can become higher than the capacity while, in-field data measurements, the vehicle flow –by definition– cannot exceed physical capacity. This approximation gives rise to a critical issue while estimating the shape of link delay functions. Namely, the macroscopic static traffic flow models represent the congestion with a functional formulation that cannot be empirically observed and, in turn, cannot be estimated to reproduce the actual traffic speeds. Practitioners usually overcome this and estimate only the hypocritical part only for which the problem does not raise since the observed speed can be expressed with the unique monotonically decreasing function of flow. The hypercritical part (when the flow starts decreasing) is usually neglected, and arbitrary parameterizations are used.

5.3. Delay model for nodes

Delay models for nodes also have a BPR-like function, as reported in equation (6).

$$t_N = \begin{cases} (t_N^o + a) + d\left(\dfrac{Q}{cQ_m} + f\right)^{b} \text{for } Q \leq Q_m \\ (t_N^o + a') + d'\left(\dfrac{Q}{cQ_m} + f'\right)^{b'} \text{for } Q > Q_m \end{cases} \tag{8}$$

- t°_N Constant additional time to cross the node N
- Q Total flow at the node
- Q_m Total capacity of the node
- a, b, c, d, f Calibration parameters

5.4. Calibration methodology

The calibration methodology concerns both link and node delay functions. The objective is to calibrate a detailed road network supply model in which the nodes of the road network are represented with high granularity in terms of control type (yield, stops, signal) and numbers of turn movements and type of turn movements, as depicted in the upper right side of Fig. 8. This detailed representation of the road network entails that the vehicle travel times on the road network depend not only on the link cost function but also on the node cost function. The figure highlights the main elements of the calibration process, which depends on:

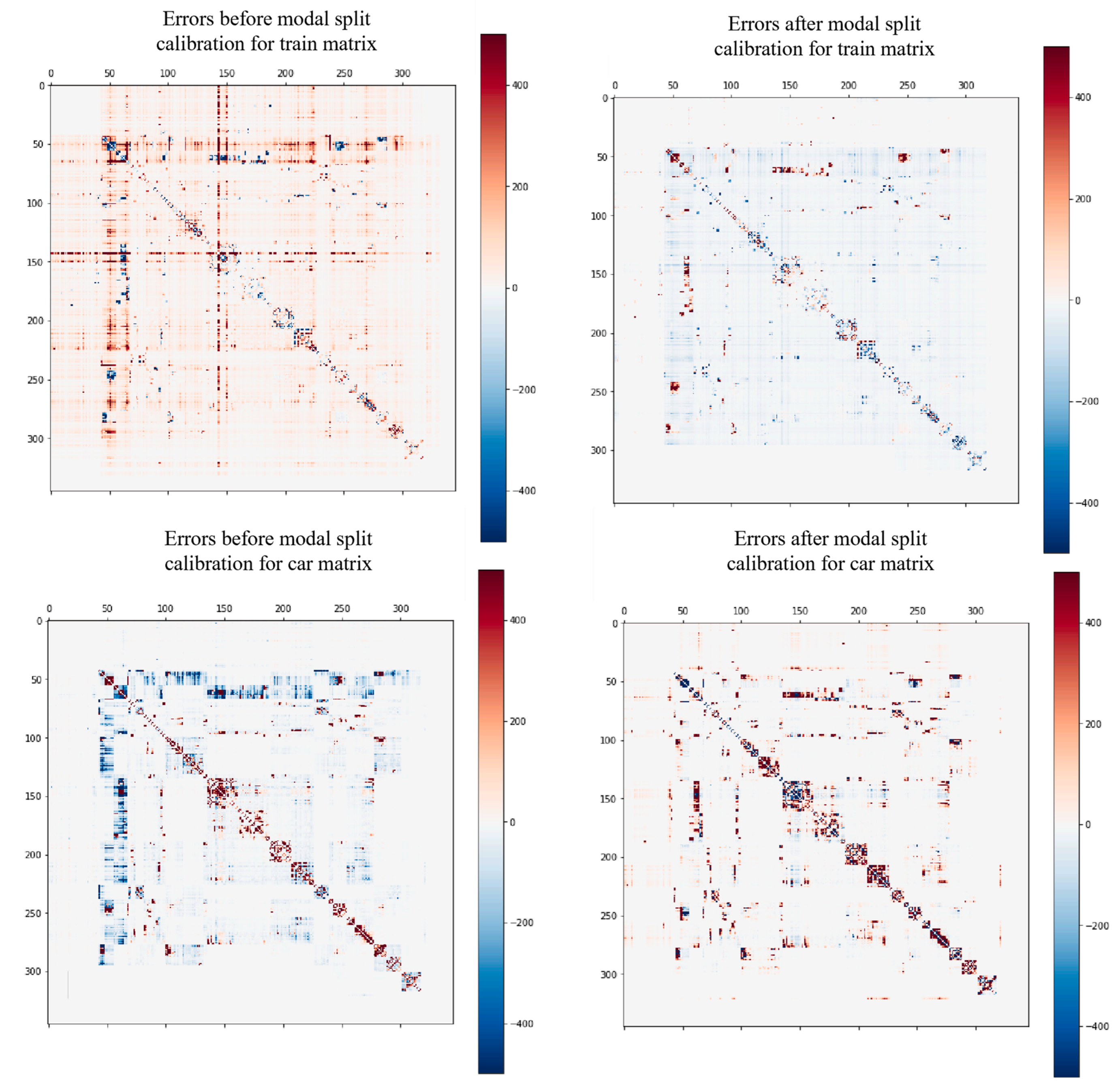

Fig. 7. Heat maps of the errors in car traffic flows and train passenger flows before and after the calibration of the modal split model.

- the road network coding for links and nodes;
- the data available to calibrate the link and node cost functions, consisting of automatic traffic counts, turn movement costs at junctions, and travel time surveys on a defined set of routes.

5.5. Objective function

The link flow-delay functions can be calibrated independently for every road type since both speed and flows are measured at the automated traffic count sections. However, delays at junctions are not available and only aggregate measures of route travel times are available on the surveyed routes. To calibrate the flow-delay functions for nodes, modeled route travel times are computed as the sum of flow-delay functions of links and nodes along the route and therefore compared with the available travel times survey data. After that, the new (sub) optimal main node parameters can be obtained by minimizing the following objective function:

$$z = min \sum_{r' \in R} \left(\widehat{t}_r(\boldsymbol{\varphi}, \boldsymbol{Q}) - t^o_{r'} \right)^2 \rightarrow \boldsymbol{\varphi}_{opt} \tag{9}$$

where t^o_r is the observed route travel time for each run.

The objective has to consider that not all the elements of the routes are fully observed. So, the simulated route travel time can be computed as follows:

$$\widehat{t}_r(\boldsymbol{\varphi}, \boldsymbol{Q}) = \sum_{l \in L_r} t_l(\widehat{q}_l) + \sum_{n \in N_r} t_n(\widehat{q}_n) + \sum_{\tau \in T_r} t^o_\tau + \sum_{m \in M_r} t_m(\boldsymbol{\varphi}, \widehat{Q}_m) + \sum_{m^\circ \in M^\circ{}_r} t_{m^\circ}(\boldsymbol{\varphi}, Q^o_m) \tag{10}$$

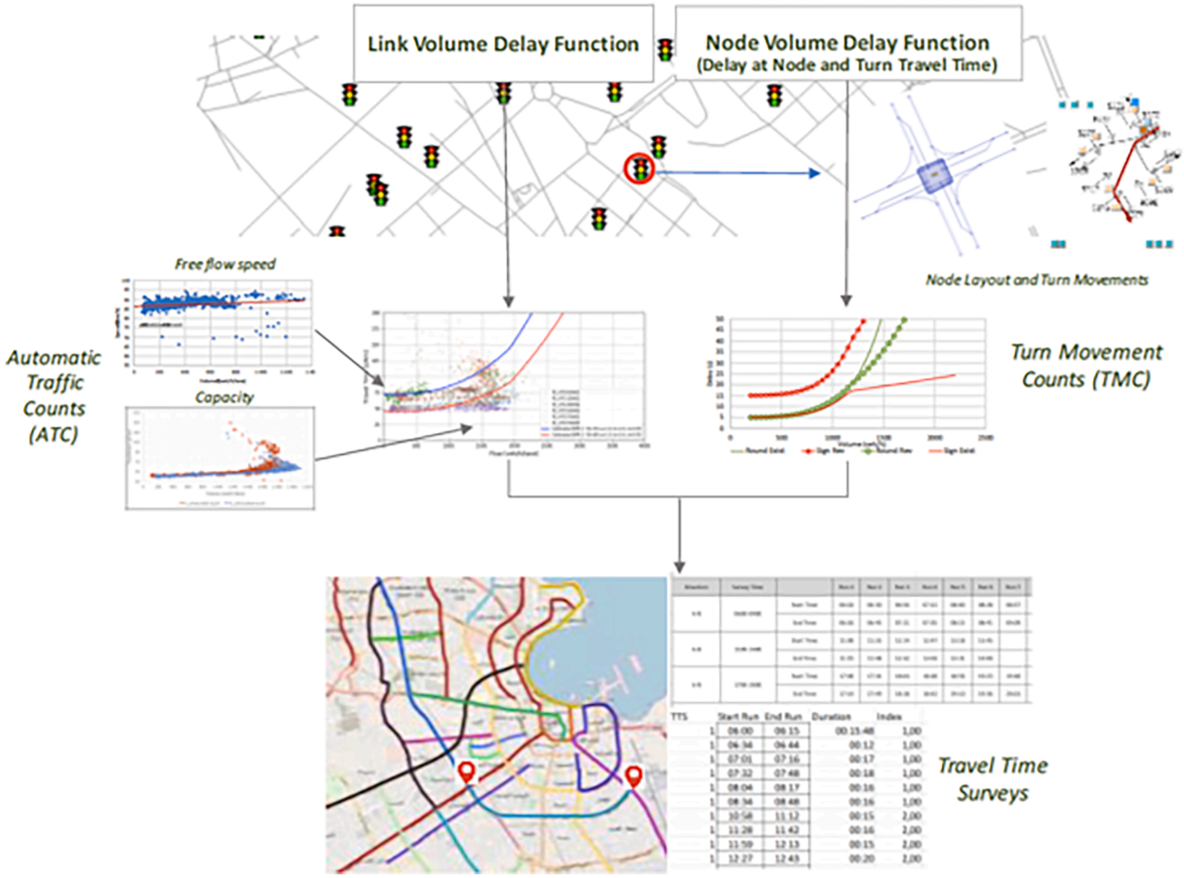

Fig. 8. General framework of the aggregate calibration process of the supply network model.

$\sum_{l\in L_r} t_l(\widehat{q}_l)$ link travel time;
$\sum_{n\in N_r} t_n(\widehat{q}_n)$ node travel time;
$\sum_{\tau\in T_r} t_\tau^o$ delay at turns;
$\sum_{m\in M_r} t_m(\varphi, \widehat{Q}_m)$ travel time at unobserved nodes as a function of simulated flows.
$\sum_{m^\circ\in M^\circ{}_r} t_{m^\circ}(\varphi, Q_m^o)$ estimated travel time at monitored nodes as a function of observed flows.

5.6. Calibration of link flow-delay functions

Before carrying out the parameter estimation, a preliminary data analysis of traffic counts is conducted, for each road type, in three steps:

1. A first analysis concerns the comparison between the observed free-flow speeds and the posted speed values, aimed at classifying road types, in case of biases from posted speeds and the corresponding observed values.
2. A second analysis concerns the estimation of the capacity for every class of homogenous roads.
3. The last step is the selection of the calibration interval, consisting of the speed-flow values that are significant for the calibration of the link delay functions.

About the third point, it is worth noting that only data corresponding to congested but not oversaturated conditions are useful in achieving proper calibration because they correspond to the monotone-increasing part of the link delay functions. Therefore, the calibration process is carried out only on the automated traffic count locations whose observed traffic flow reached values close to the road capacity. It is also worth noting that the oversaturated part of the flow-delay curve, which corresponds to higher demand values than the capacity, cannot be observed on the road because the capacity cannot be exceeded in reality. Thus, oversaturation values have to be pondered carefully by considering a consistent trend of increased travel time over the critical point corresponding to capacity.

Since a one-to-one correspondence between traffic flows and travel times is necessary for complying with the monotone shape of the flow-delay functions, a filtering procedure is applied that excludes all oversaturated measures. The oversaturated region is identified by the travel times that exceed the highest travel time in the saturation region, corresponding to the observed flows in the 95th-100th percentile interval, as illustrated in Fig. 9.

A specific remark is worthy concerning capacity estimation. Capacity

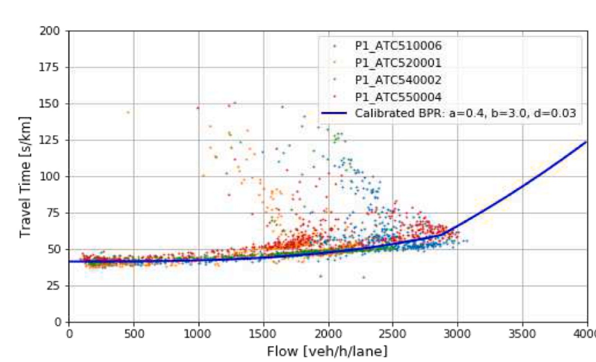

Fig. 9. Traffic flow and travel time measures selected or excluded (orange area) for the calibration of link flow-delay functions. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

is a fundamental parameter of the road assignment models, included in most flow-delay functions as a divisor of the traffic flow. Noticeably, it is the significant parameter for the assignment models with capacity constraints. Despite its importance, capacity is a road attribute that is very difficult to be observed.

In fact, it is the maximum flow that can travel across a road section in standard conditions. To be noted, it is necessary that a sufficient level of demand feeds the road segment. However, capacity can be reached and observed just downstream from the junction with entering traffic if the demand is sufficient. In this case, the point of capacity is relatively easy to be observed as it is the critical point when the speed falls. Even slight disturbance factors near the capacity can affect traffic stability and produce a capacity drop so that both the speed and the flows reduce. On the other hand, there are road segments where the capacity will never be observed because of insufficient demand, like just downstream from an exit ramp of a freeway.

Thus, the capacity is not necessarily coincident with the maximum flow observed on a road section. A typical example taken from real traffic data collected in our test case is illustrated in Fig. 10, which plots travel times and flows observed at two road sections in two directions, Northbound and Southbound. In both cases, an inversion of the flow-delay relationship is evident. However, capacity (about 2,100 veh/h/lane) is observed only in the Northbound direction, which is located just upstream of the on-ramp from the service road; in fact, Southbound flows are measured just downstream of the off-ramp toward the service road, and the maximum flows do not exceed 1,900 veh/h/lane. Nevertheless, the two results are consistent since traffic flows counted on the service road are up to 350 veh/h/lane. These flows include vehicles just exited from the expressway and vehicles that were already on the service road.

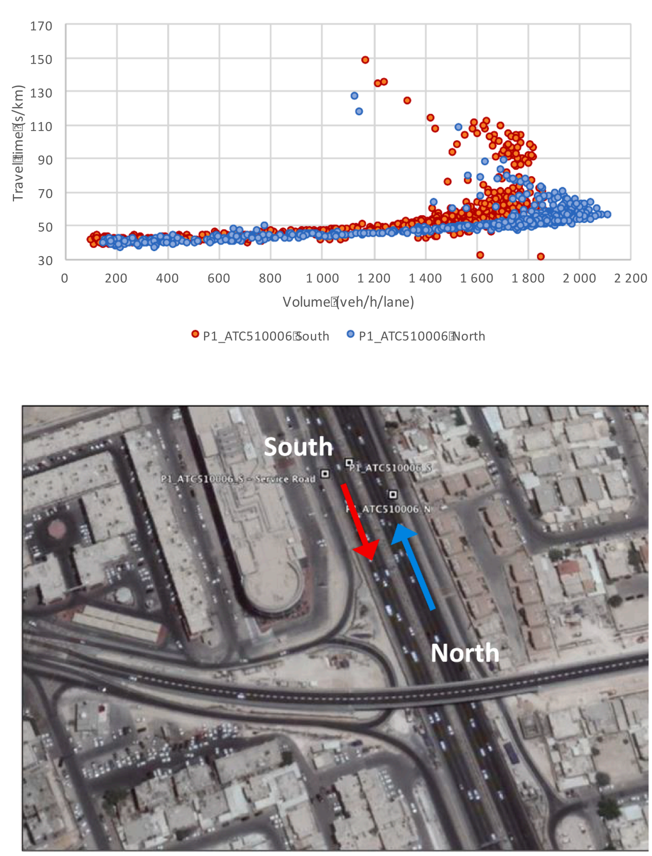

Fig. 10. The plot at the top of the figure depicts the traffic flows and the travel times measured in the two directions of a freeway shown at the bottom.

5.7. Calibration of node flow-delay functions

The methodology used for calibration of node flow-delay functions is necessarily different from that applied to link flow-delay functions since only input values (turn movement counts) are directly observed at a set of junctions. Moreover, the travel times along the set of surveyed routes provide aggregate measures that include the node delays other than link travel times. Since turn movement counts are available only at a subset of nodes, simulated flows are used instead.

In the specific case under study, the optimization process is carried out on 1,870 individual trips, belonging to the 71 routes illustrated in Fig. 11 and containing 208 main nodes. Among them, 80 main nodes contained observed turn movement counts. The simulated flow values are used for the unobserved nodes.

Node delay functions are defined by the equation (8), for signalized junctions and roundabouts, each having seven parameters (a, b, d, f, a', b', f') that need to be optimized, while d' is expressed in function of other parameters in order to ensure curve continuity when the degree of saturation is equal to 1.

The optimization problem for the aggregate calibration of node delay functions is represented by 1870 equations, having the general form (10), with 14 variables, which refer to the delay functions' coefficients (8) for the two types of junctions, signalized and roundabouts. A model-based procedure enhanced by a metaheuristic algorithm for to optimize the control system introduced by Colombaroni et al., 2020 can be applied to optimize the signal settings. To ensure coherence with theoretical and empirical considerations on the road hierarchy, reasonable feasibility intervals are preliminarily determined for the two types of junctions, as illustrated in Fig. 12. Moreover, to improve the calibration process, the initial swarm population includes the existing model's coefficients and reasonable values derived from the technical literature.

Analogously to the calibration of the demand model the procedure for the supply network model was implemented as a Python routine, calling the assignment model for each individual solution at each iteration of the PSO algorithm.

Fig. 13 reports the profile of the objective function during the optimization process. The convergence of the algorithm is relatively fast, and the solution approaches the optimum after about six iterations, achieving a 60% improvement.

5.8. Calibration results

The final results of the optimization are illustrated graphically in Fig. 14, which depicts the correspondence between observed and simulated travel times, and the coefficients of the regression line, which has a slope near one and an intercept near zero.

Table 3 reports the mean absolute errors in travel time estimations for the earlier and the newer model for the different simulation periods. The gains in accuracy are around 40% in AM and MD periods (from 306 to 188 s and from 363 s to 216 s, respectively) and about 60% in the PM period (from 460 to 181 s). In terms of percentage, Table 4 shows that

Fig. 11. Surveyed routes used for Node Flow-Delay Function Calibration.

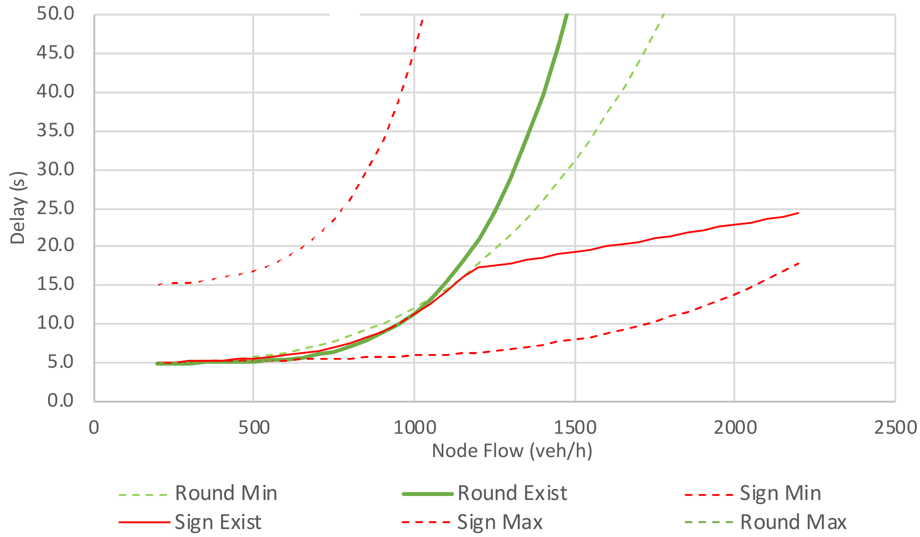

Fig. 12. Boundaries of the feasibility set of the node-delay functions for signalized junctions and roundabouts.

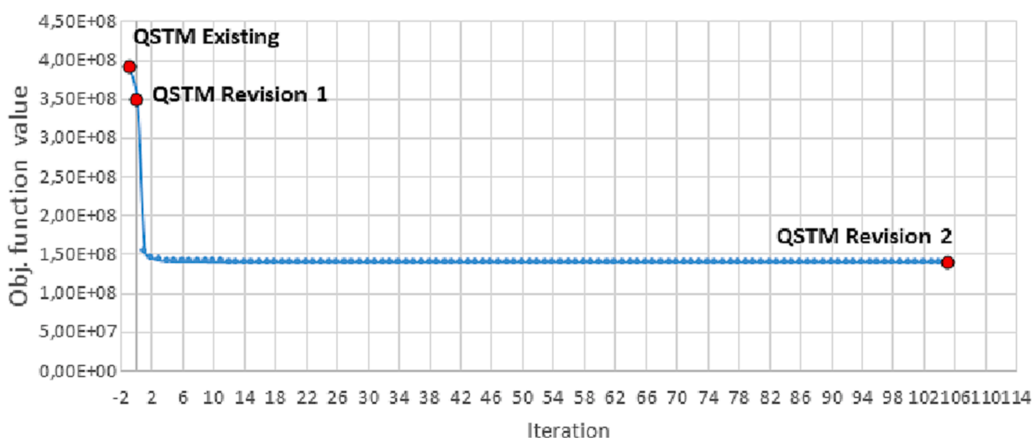

Fig. 13. Convergence of the PSO algorithm applied for the aggregate calibration of node delay function.

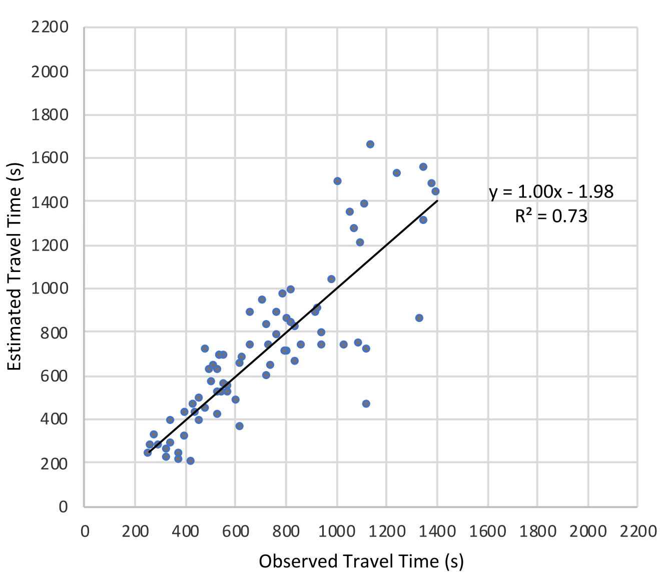

Fig. 14. Comparison between simulated and observed travel times after the aggregate calibration of the network supply model.

Table 3
Error statistics of travel time estimations before and after the aggregate calibration of the network model compared with the observed standard deviation of route travel times.

Period	Mean Absolute Error [s] Previous model	Updated Model	Standard Deviation of Data [s]
AM	306	188	191
MD	363	216	184
PM	460	181	248

Table 4
Relative error statistics of travel time estimations before and after the aggregate calibration of the supply network model compared with the observed coefficient of variation of route travel times.

Period	Mean Absolute Error [%] Previous model	Updated Model	Coefficient of Variation of Data [s]
AM	29%	18%	18%
MD	35%	21%	19%
PM	45%	16%	27%

the updated model has mean absolute percentage errors ranging from 16% and 21% of the observed values. The two tables also report the standard deviation of the data and coefficient of variation of the data collected in the field and highlight that the average errors of the model obtained after the aggregate calibration is in the same order of magnitude as the intrinsic variation of the traffic phenomenon, which ranges from 18% to 27%.

Although it is not possible to make direct comparisons with other studies in the scientific literature due to differences in the size of the networks, in the levels of detail in the representation of mobility, and in the nature of the models, for the sole purpose of providing an indicative idea of the order of magnitude accuracy obtainable in the best examples of calibration of traffic models on large networks, it is worth mentioning one of the most advanced studies on this topic (Ben-Akiva et al., 2012), which introduced a combined calibration method for a dynamic assignment model. When applied to a network consisting of 1,698 nodes and 3,129 connections and demand with 2,927 origin–destination pairs, that method exhibited average square root errors of 0.28 in traffic flows and 0.40 in travel times. In a different study (Antoniou et al., 2009) the calibration of 1129 OD pairs together with the route-choice model and fundamental diagrams led to a 0.1 normalized root mean square error in terms of traffic flows, and 0.09 error in terms of the observed speeds. Another study (Shafiei et al., 2018) that calibrated and validated a dynamic traffic assignment model on a network of 55,719 links obtained similar results in terms of validation by the travel times revealed from Google, that is 0.75 in terms of R^2 and 1.01 for the slope.

The method presented in the paper is applied however to a static assignment model with explicit node delay functions but to a larger network (about 300,000 links and 5,000 nodes, and more than 3 million O-D pairs). The normalized root mean squared errors in travel times ranged from 0.20 (in the mid-day simulation) to 0.24 (in the PM peak period). Such results are not directly comparable but it is comforting to note that the results are of the same order of magnitude and indeed, with all the possible differences, numerically better.

6. Conclusions

The paper has presented a general methodology for the aggregate calibration of transport models that exploits different data sources and is suitable to deal with individual data collected in mobility, like Floating Car Data. The proposed methodology seeks to minimize to deviations between the observed data and the modeled values while considering the significance of different data sources. A metaheuristic algorithm, namely Particle Swarm Optimization (PSO), particularly suitable for optimization on continuous variables and exploring a wide range of the feasibility region is adopted as an optimization technique.

The application of the methodology has been exemplified in two case studies concerning the update of two strategic models with new data collected in mobility: one related to the demand-side, the other the supply-side of the transport system.

The examples show that the optimization methodology is general enough to be applied to these two cases without requiring specific changes in the optimization algorithm except the variables' choice to estimate and the specification of the corresponding objective function.

For the demand model case the optimization methodology was applied on a three – step process. For the calibration of generation and distribution models the reference OD matrices were considered by the procedure, therefore only a partial run of the transport model was necessary to compute the objective function value. The final step of the calibration concerned the modal split and required running an assignment for computing errors based on the observed traffic counts and traffic flows.

The aggregate calibration methodology has provided a significant improvement of the demand model: it reduced the absolute errors by 28% and by 51% for car and train modes, respectively, and corrected the structural bias in the error distribution of the previous model, whose higher errors were concentrated on the principal diagonal and from and to the main towns. The errors nevertheless remain high but are comparable to the results of the main state-the-art study. The difference between the traffic counts and the values of the flows assigned on the road is reduced by 77% in terms or the root mean square errors, while the reduction of the mean absolute error is about 19%.

The second study case concerned application of the calibration methodology for the supply network model. The delay model for the nodes was calibrated by PSO algorithm that optimized 14 coefficients of the model. For the computation of each objective function value the assignment model had to be run in order to obtain the simulated flows that are necessary to compute the simulated travel time.

The results of the calibration of the supply network model also are satisfactory, despite the numerous approximations implied in the formulation of the flow-delay functions implemented in the assignment model, illustrated and commented on in the paper. The regression slope between observed and simulated data is around 1, while the R-squared values is 0.73.

The average absolute errors in the estimation of the travel times range from 18% to 21%, depending on the day period simulated. They are in the same order of magnitude as the intrinsic variation of the data collected in the field and, with the caution needed when comparing different applications, even numerically better than the results of one of the main state-the-art studies.

Other than the effectiveness of the model in fitting the observed mobility data, a major advantage of the aggregate calibration is its effectiveness in exploiting, in a unique coherent framework, the huge amount of mobility data that can be collected with limited economic resources from direct measurements of traffic flows, speeds, and travel times. Thus, it makes it relatively easy and cheap to update existing models with new data collected through the direct observation of the network. This has become a convenient feature since many Countries, Regions, and towns have a consolidated transport system model and a monitoring mobility system, which enables one to easily and frequently update the model.

Also, from a theoretical point of view, the aggregate calibration appears more consistent with the holistic concept of mobility modeling as a unique framework rather than a chain of serial models with possible and rarely applied feedback.

The limitation of the proposed calibration can be seen as an intrinsic feature of the metaheuristic algorithms, which usually require a great number of evaluations of the objective function by running the transport model, which can be costly in terms of computational time in case of real networks with big number of links and nodes. The paper suggested several expedients that can be taken to lighten the computational task. Thus, running only demand part of the transport model for in the first case study and defining suitable feasible regions in the second case study enabled calibration of a great number of coefficients within a reasonable number of iterations.

The application fields of the proposed methodology for aggregate model calibration are broad and cover different types of models, independently of their specific formulation and extent. The crowdsourcing of individual data collected in mobility is today a suitable source of data for updating transport system models and will be even more widely available in the future. These technological advances open even broader perspectives for the application of aggregate calibration based on the crowdsourcing of mobility data.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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