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Analysis of parking traffic in Cologne, Germany, based on an extended Macroscopic transport model and parking API data

Peter Lubrich

Federal Highway Research Institute (BASt), Brüderstraße 53, 51427 Bergisch Gladbach, Germany

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ABSTRACT

While macroscopic transport models are recognised as valuable tools for strategic transport planning, parking is rarely explicitly considered in such models. On the other hand, park-search traffic represents a significant portion of traffic in urban areas, according to many studies. Even if there are already some attempts to integrate parking into transport models, they have not found their way into the practice of modellers.

The presented work enhances and combines previous approaches for modelling parking, and proposes a tool for practitioners to analyse common transport problems involving parking. It builds upon existing, even calibrated transport models, using the conventional four-step modeling technique. The approach also explores some emerging, internet-based data sources for transport models. Parking data is accessed from an Application Programming Interface (API) and embedded within the model procedure. Such new data resources resolve some problems from previous model approaches, which see the availability on parking data as problematic. With the combination of a macroscopic transport model and parking data from an API, aspects of parking supply and demand are represented within the model setting, whereas parking choice is calculated via an optimisation of a park-search route.

The conceptual approach is demonstrated in the context of an existing macroscopic transport model for Cologne, Germany. This model is extended and refined to represent parking patterns within the study area. As a result, the spatial distribution and the effects of park-search traffic are explicitly shown, indicating some realistic results. It is also shown that the level of detail and the expressiveness of the existing model is increased.

1. Introduction

The parking space situation and associated problems, especially in cities, have been a topic for traffic planning and traffic management for decades (Hupfer, 2011; Shoup, 2018). The main reason for this problem is the high degree of motorisation, which has increased significantly in the past. The existing parking space supply could not keep up with the associated growing demand for parking space. Other reasons are inefficiencies due to long parking durations and low turnover rates, as well as a parallel, non-integrated supply of parking capacities in on-street and off-street segments (City2.e 2.0, 2017).

This results in, among others, excessive Park-Search Traffic (PST), which is caused by temporal capacity overloads at parking facilities and/or the travellers' lack of knowledge about available parking spaces (Anke and Scholle, 2016; Weinberger et al., 2020).

PST is the subject of many research works on parking behaviour (Kaplan and Bekhor, 2011), as well as on effects of parking management measures (Böhnke, 2005). These topics are often explored via empirical studies as well as models and simulations (Horni et al., 2013). As revealed in the literature review, integration of parking elements into wider transport models seems a promising approach. However, such integrated models have not found way into every day's practice of transport models, due to, among others, lacking data sources about parking (Gu et al., 2021; Schiller, 2004).

The presented work introduces a practical approach for integrating parking patterns in a macroscopic transport model. It shows four innovations: (1) it ensures consistency with a conventional model setting, by building upon common modelling techniques; (2) it allows for a certain level-of-detail, by considering the spatial distribution of parking demand and different types of parking facilities; (3) it explicitly reveals PST effects, such as park-search-based delays, cruising traffic etc; (4) it exploits some emerging data sources within the model environment. The main data input is based on Smart Parking Systems (SPS), in particular, parking data from an Application Programming Interface (API).

The expected benefit of such model enhancement is a higher expressive power of the model, namely the explicit consideration of PST

E-mail address: lubrich@bast.de.

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patterns, and an integrated view on both parking patterns and regular traffic. This approach allows for detailed analyses of the parking situation within a specific study area, and is demonstrated in a real-life setting in Cologne, Germany.

The paper is organised as follows. Section 2 presents a literature review on the evidence of PST, on previous parking models, and on SPS. Section 3 compiles the main features of the proposed approach for an integrated transport model, explaining the conceptual framework and concrete steps to represent parking-related features in the model environment. Section 4 demonstrates the practicability of the model approach in the context of a real-life model study, while Section 5 summarizes the findings, and gives an outlook on open questions and future research prospects.

2. Literature review

2.1. Park-search traffic

Driving to and finding a parking space at the destination often involves park-search traffic (PST). Birkner, 1995 defines PST as 'additional traffic efforts caused by the search for available, acceptable parking spaces'. Such traffic efforts are caused by search routes, which may optionally include excess-route elements. Search routes are located between the first acceptable parking space, as encountered by a traveller, and the final, accepted parking space. Excess-route elements account for the difference between a realised route and the shortest possible route to the chosen parking location. These definitions are illustrated graphically, see Fig. 1.

Based on the definitions above, PST can be distinguished as direct PST, relating to the park-search routes, and as indirect PST, relating to the (optional) excess-route elements. Some of these definitions are reused later in this paper.

Several studies attempt to quantify PST and related effects on a national scale in Germany (APCOA, 2013; Bachleitner, 2017; Cookson and Pishue, 2017; Galster, 2008; Hagen et al., 2021; Rikus et al., 2015). Altogether, these studies indicate park-search times in a range of about 2 to 10 min per trip. However, a comparability cannot be deduced from these studies, as they are based on different methods, definitions and contexts. Lastly, some authors identify biases in such studies, so a comparability and transferability of related results seems challenging (Weinberger et al., 2020).

2.2. Parking models

Phenomena and relationships of the parking system have been explored by the research community by empirical studies, simulations and models (Cao et al., 2019; Khaliq et al., 2019). Goals of parking models are: to quantify parking demand; to analyse choice behaviour of travellers; to assess changes in land use and/or management measures regarding parking choice and parking demand; to reconstruct parking-search patterns; and to assess effects by externalities, e.g., by parking information services (Böhnke, 2005; Hensher and King, 2001; Kaplan and Bekhor, 2011).

Many authors pledge for an explicit interaction of parking traffic with regular traffic, when building transport models. Some researchers present stand-alone macroscopic model frameworks, e.g., via economical optimisation approaches (Arnott and Rowse, 2009) or via transition-matrices for parking-related states (Jakob and Menendez, 2019). Such approaches aggregate groups of travellers into 'pools' or 'families', which are calculated simultaneously for the entire network, thus without a spatial granularity. Some of these approaches apply macroscopic fundamental diagrams (MFD) for the interaction of traffic and parking conditions (Leclercq et al., 2017; Zheng and Geroliminis, 2016). These can well reproduce the dynamics of transport systems (e.g., relating speeds to vehicle accumulations), but again rely on spatial aggregates (sometimes called reservoirs). Such approaches might be applicable for global assessments of parking scenarios but can hardly account for heterogeneously distributed traffic and parking situations. Other modelers address this issue via spatially explicit models, where parking processes are represented on a network-level (Gallo et al., 2011; Leurent and Boujnah, 2014; Levy et al., 2015). However, all these approaches seem to be isolated from more holistic, conventional transport model environments. For example, information on origin-destinations of travellers is often assumed to be exogenous in the above approaches, while such information is integral part of a conventional transport model.

There are some approaches for the analysis of parking in the scope of conventional transport models (Bagloee et al., 2012; Böhnke, 2005; Schiller, 2004). A common way is to refine the model elements of network supply and travel demand, and to enhance the modelling techniques to consider the parking component within the traffic system. Looking at the model procedures, parking is incorporated into the assignment step of the four-step modelling technique.

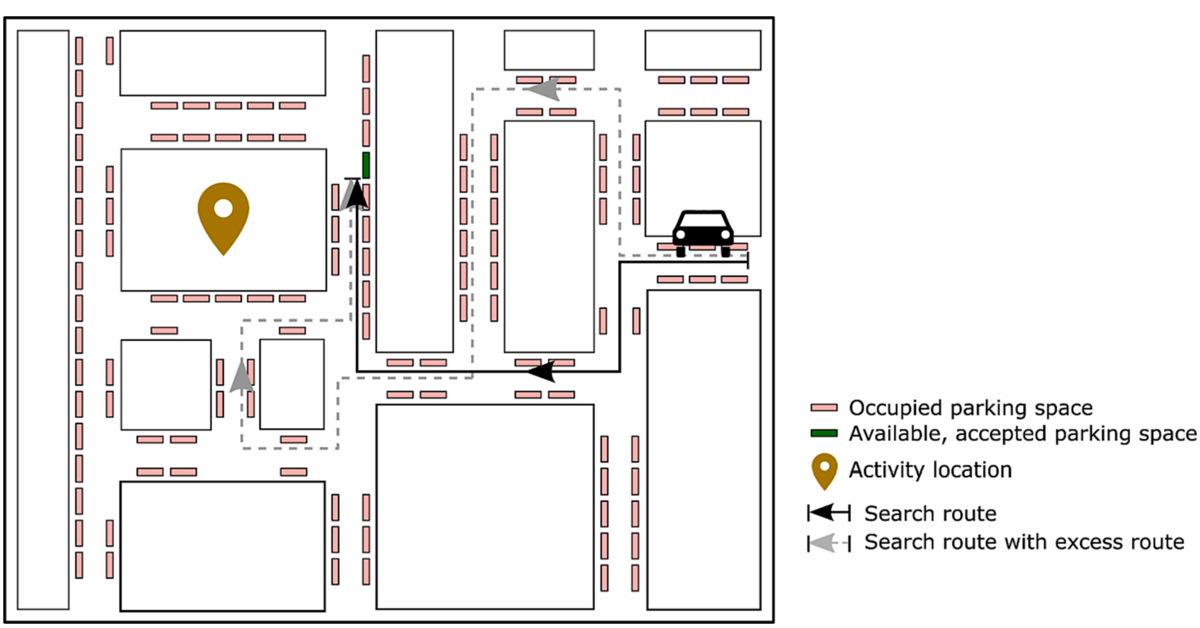

Fig. 1. Search routes and addition routes according to (Birkner, 1995).

Part of the underlying calculations are generalised costs (or disutilities) concerning a parking facility being assessed. Common costs are: travel times, parking fees, park-search times, and access/egress times for parking facilities (Schiller, 2004; Thompson and Richardson, 1998). The parking-related utilities are then integrated into general, network-related utilities. The assignment step returns routes for a given travel demand, based on an assessment of such integrated utilities. Some of the cited model frameworks also contain interactions between travel demand and network supply elements, and/or apply restraint conditions, considering effects of, e.g., parking saturation at individual parking facilities.

Although these integration models show promising ways to consider parking elements within macroscopic transport models, they reveal some deficiencies. They do not explicitly consider PST effects, such as park-search-based delays, cruising traffic etc. Such effects are only implicitly represented, sometimes by additional disutilities saved under specific network elements. This approach does not reveal how PST affects the overall network performance, by adding volumes caused by cruising to the regular network elements. Other deficiencies are found in the modelling process of parking supply. In particular, the representation of parking facilities by given network-model elements, such as traffic zones, seems problematic. Some mechanisms are proposed to account for 'spill-over effects', i.e., the choice of parking facilities at adjacent traffic zones, in case of parking over-saturation within the original traffic zone (Böhnke, 2005). Such mechanisms seem to be rather complicated and hardly replicable.

Another stream of research is found in concepts of route optimisation, by solving individual route queries in road networks via a probabilistic-graph problem. Friedrich et al., 2019 apply such concept to the parking domain. They define the probabilistic graph as the road network attributed with parking probabilities. The objective is to minimise travel times to reach an available parking spot, while minimising walking times from that parking spot, and simultaneously applying a so-called probability mass threshold as a hard constraint. The problem is solved via a set of algorithms. The output is an optimised Park-Search Route (PSR) for a given destination. Hedderich et al., 2018 apply a special algorithm, the A* Algorithm, to solve a similar problem setting, again with an adopted cost function.

The main goal of such PSR-focused models is to identify optimal routes for one individual, searching traveller. Thus, they are rather addressed for traveller services, such as navigation services. In contrast, macroscopic models look at the entire demand market in a study area, consisting of all searching travellers. These models rather address the needs of transport planners and policy makers. In the approach presented in this paper, both domains are integrated, i.e., some aspects of the PSR-focused models are applied to a macroscopic model setting.

As a final aspect, the practicality of parking models is discussed. Researchers point out the heterogeneity of present models in terms of maturity, applications, efforts and benefits (Böhnke, 2005; Gu et al., 2021). Efforts relate to data acquisition, model calibration, and detailed assumptions about the traffic state and traveller behaviour. Benefits relate to the level-of-detail, explanatory and predictive power, practicability, and transferability of the model. Altogether, a practitioner needs to choose a suitable approach, depending on the arguments above, but also on available model skills and tools. This conclusion leads to the question if and how parking models are deployed by model practitioners, at, e.g., consultancies and authorities responsible for transport planning. While regular transport models seem to be a common tool for actors of strategic transport planning (Gertz and Polzin, 2009), there are no signs that models with parking features are similarly often deployed by such actors. A potential reason why parking has not found way into modelling practice is the high effort for parking data collection (Gu et al., 2021; Schiller, 2004).

2.3. Smart parking systems (SPS)

Smart Parking Systems (SPS) aim for an efficient organisation of the parking infrastructure, and for improved traffic performance. SPS are mainly based on emerging information and communication technologies, including new data sources and data-processing algorithms (Ogás et al., 2020).

Lubrich, 2021 presents a taxonomy of SPS-based data, analysing technical, content and functional aspects of such data. Functional aspects reveal potential users being addressed, and services being supported by SPS-based data. While a common target group is found in travellers, e.g., via advanced parking guidance systems, there are also ambitions that SPS might support parking management and planning in the hands of public authorities (Arnd and Cré, 2018). For example, new data sources and technologies can supplement or even replace existing data collection methods, e.g., via manual surveys.

Some SPS-based products expose parking-related data on a commercial basis. They offer processed parking data to third-party services, such as enhanced parking apps, in-vehicle navigation services and similar. In many cases, such data sets can be accessed via internet-based Automated Programming Interfaces (API).

Some model practitioners state that emerging technologies may ease the model-building process. When embedded and interpreted properly into the model setting, emerging technologies might finally enhance the expressiveness of traditional models (Anda et al., 2017). However, there are only few scientific works about the use of emerging data for modelling purposes. Some researchers apply parking-sensor data to build an own parking-model framework (Xiao et al., 2018), while other researchers use other emerging (but non-parking related) data sources to enhance traditional traffic models, e.g., by applying GPS trip data to generate origin–destination matrices (Demissie and Kattan, 2022). There is, however, no evidence on how SPS and SPS-based data can facilitate and ease the modelling of parking.

3. Proposed model approach

3.1. Conceptual framework

The proposed approach incorporates the parking-search sequence, as conducted by travellers, into the framework of a conventional four-step transport model. Such sequence includes the approach to the area of their final destination; the start of the park-search process; the assessment of route and parking options within the search area; the decision to take on an acceptable and available parking spot; the parking of the car; and finally, the walking path to the final destination. This way, the transition-state approach is reused, as proposed by Jakob and Menendez, 2019. Fig. 2 indicates the sequence of such transitions from a traveller perspective and relates them to the components in the model setting.

A central component is a Park-Search Route (PSR) algorithm, representing the decision-process to determine and assess such route. This corresponds to the park-search process, where a traveller examines available parking places along a route and accepts an acceptable and available one. Relevant choice parameters include parking facility-related parameters, such as the current availability and potential parking regulations, and network-related parameters, such as travel times and delays along regular links. These parameters are summarised as utilities in a utility function. After the selection of a PSR, some utilities are updated. In particular, realised PSRs affect traffic loads in the network, whereas traffic loads affect travel times of regular links, being one utility in the PSR assessment. This results in a feedback loop for the utility calculation, as indicated in Fig. 2.

The PSR assessment process is integrated into the traffic-assignment step of the four-step algorithm in a macroscopic transport model. For the proposed approach, an existing model setting with a focus on general traffic, called a Master Model, is utilised. There are two benefits when

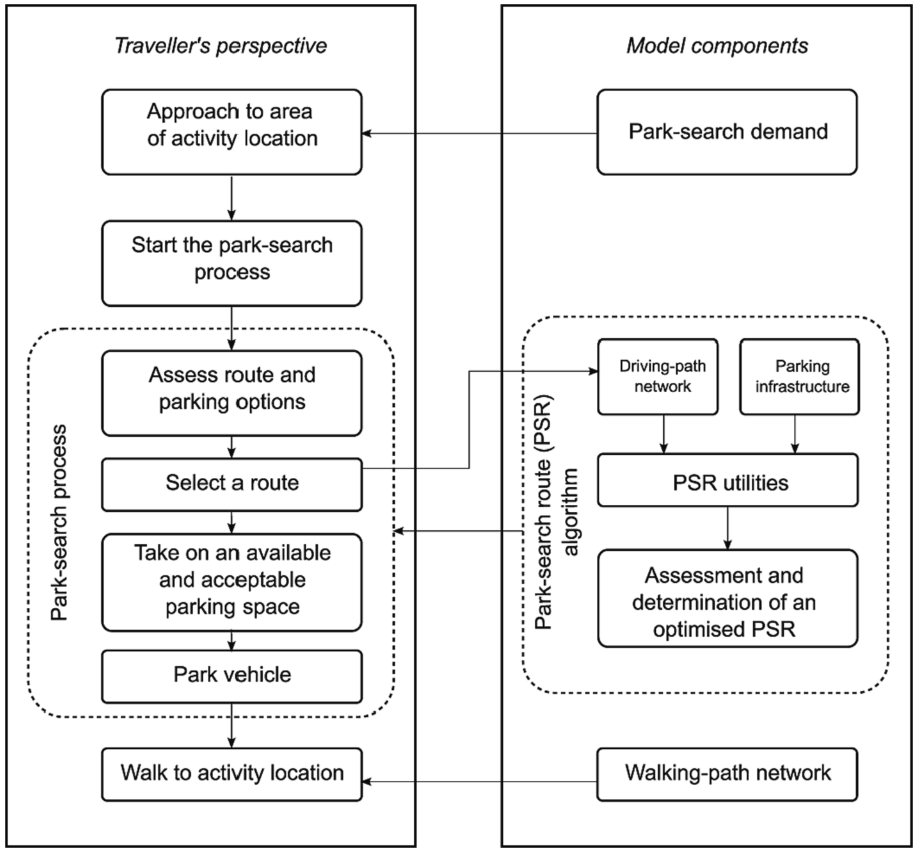

Fig. 2. Parking-related transitions from a traveller perspective and corresponding model components.

building upon an existing model framework. First, only a detailed network model allows for a sufficient representation of parking patterns (Levy et al., 2015). Second, an existing model endogenously provides many attributes needed for the representation of parking patterns, in particular regarding network utilities and the demand for parking. In contrast, other researchers assume that demand parameters are provided exogenously (Gu et al., 2021).

In terms of temporal characteristics, the proposed model approach has time-stationary or ready-state conditions, following traditional mathematical transport models (Cascetta, 2009) and some previous assignment-based parking models (Gallo et al., 2011). Thus, no temporal dynamics are considered, but seem not necessary for the context of this work, namely model-based traffic studies, which usually consider fixed time dimensions, such as a peak hour during a weekday.

3.2. Input data

The main input is a Master Model, being an existing macroscopic transport model for a particular study network. In such model, the transport system is represented by conventional model elements such as links, nodes, traffic zones, and demand matrices. Further, the Master Model also allows for procedures of a conventional four-step algorithm, i.e., traffic generation, distribution, mode choice and assignment. This model is operational, all four steps are executable, and model results are retrievable. When applying a model for a real-life situation, the model is calibrated using empirical data on travel patterns and traffic performance.

A further input is found in parking data from a parking API service. These data contain parking supply parameters, including the location, capacity, type, and regulations of parking facilities; and parking-demand parameters, namely probabilities to find a free parking spot per facility. On-street facilities are represented on a link basis, off-street facilities on a point basis. It is, however, noted that API data only provide parameter on public parking facilities. Private facilities are out of scope and cannot be explicitly considered in this model approach. The implemented API parking data act as an exogenous input model parameter.

Another external data source is a land use or building cadastre. This usually contains the type of land uses or buildings (e.g., as residential), their locations (e.g., via coordinates) and their intensities (e.g., via gross floor area). Such data are easy to obtain, e.g., from municipal Geographic Information Systems (GIS), and are integrated to the network model as an extra layer.

3.3. Modification of the master model

As a baseline, individual parking facilities are represented as part of the network model. To do so, the traffic-zone structure is decoupled from the structure of parking facilities, to avoid some problems with such relationships, as explained in the literature review. This approach allows for many-to-many relationships between parking facilities and traffic zones. This way, for example, one parking facility could accommodate trip ends related to several traffic zones.

First, the existing traffic zones from the Master Model are declared as Master Zones. Second, additional traffic zones are introduced, called Parking Zones. They act as granular representation of physical parking spaces and their characteristics, including types, capacities and usage restraints. Parking types represent the supply segment, such as on-street vs off-street parking. Usage restraints are used to control access by user types and trip purposes. Practically, each Parking Zone represents a uniform group of parking spaces. This might be a group of on-street spots along a road link, or individual off-street parking facilities.

To ensure consistency with the Master Model, the Master Zones and the Parking Zones are coupled via so-called Portal Zones. The Portal Zones act as transition points between PST and regular traffic. They are placed according to the modeler's knowledge of local conditions. Usually, they should be placed along major arterials leading into the study area, and at the boundaries of a potential park-search area. A typical placement of Parking Zones and Portal Zones in the network is shown in Fig. 3.

Both Portal Zones and Parking Zones are accommodated in the network model via some auxiliary model elements, such as auxiliary nodes. This is a common approach when granularising model networks and not explained further.

Similar to previous approaches (Bagloee et al., 2012), park-search demand is extracted from a Master Model's demand matrix. Park-search demand represents the number of travellers who look for parking around their activity locations, within the modelled time period and within the study area. This demand relates to longer-term activities requiring parking, such as residential, commercial or leisure activities.

Looking at the four-step algorithm, the traffic-assignment step is modified for trips related to the park-search demand. This is done by disaggregating trip ends via special Park-Search Route (PSR) algorithms, as explained below. In contrast, the remaining demand portions and their assignment procedures remain unchanged. This remaining demand includes, e.g., through traffic; residents and commuters with own or reserved parking; delivery traffic; and other activities not involving PST.

3.4. Calculation of park-search demand

Park-search demand is distributed and quantified via origin–destination pairs. Each pair connects the PSR starting point and the activity location. The PSR starting point is located at a Portal Zone, so the first step is to calculate park-search demand at the level of a Portal Zone as follows:

$$d_{i,j,k} = d_{j,k} * \frac{t_{i,j,k}}{\sum_i t_{i,j,k}} \quad (1)$$

with:

i = Index of Portal Zone.
j = Index of Master Zone.
k = Index of Demand Segment.
d = Park-Search Demand.
t = Destination Traffic Demand.

$d_{i,j,k}$ represents the demand of travellers of a demand segment k, that pass by at the Portal Zone i, and need to find a parking somewhere within Master Zone j.

Next, $d_{i,j,k}$ is disaggregated in relation to activity locations within Master Zone j. This is done via specific Parking Zones, called Ex-Ante Parking Zones. These zones represent the most-favourable parking facility next to the activity location. Thus, the spatial disaggregation of $d_{i,j,k}$ is based on the spatial intensity and distribution of activity locations, similar to previous spatially-explicit approaches (Levy et al., 2015). In particular, activity parameters per building are determined., e.g., gross floor areas from a land use or building cadastre. Then, each building is assigned to an adjacent Ex-Ante Parking Zone via spatial, e.g., nearest-neighbour analyses. As a result, each Ex-Ante Parking Zone is attributed with activity intensities of all adjacent buildings:

$$a_{j,l,m} = \sum_n a_{j,l,m,n} \quad (2)$$

with:

l = Index of Ex-Ante Parking Zone.
m = Index of Activity Location Type.
n = Index of Building.
a = Activity Intensity.

Different types of activity locations are applied to account for different demand segments. This way, e.g., a demand segment 'workers' might be related to activity locations of type 'office' and 'retail'. As a result, the disaggregated $d_{i,j,k,l}$ is calculated as follows:

$$d_{i,j,k,l} = d_{i,j,k} * \frac{\sum_m a_{j,l,m}}{\sum_{l,m} a_{j,l,m}} \quad (3)$$

This type of demand disaggregation is allowed as long as the trip generation in the Master Model is lineally correlated to activity-location intensities, which is a common technique for trip-generation calculations (Institute of Transportation Engineers, 2017). Eventually, the $d_{i,j,k,l}$ values are saved in park-search demand matrices. A graphical example for such demand disaggregation is shown later in the results section, see Fig. 5.

3.5. Determination of Park-Search routes (PSR)

For each value of the park-search demand matrix, the goal is to calculate which route is taken and which parking facility is chosen. To do so, a set of possible PSR is assessed. From this set, an optimal PSR is selected which justifies the following conditions: the PSR can be realised on an existing network of road links and nodes; the disutility of the PSR, depending on the traffic and parking situation, is minimised; and the probability to find a suitable and available parking spot along the PSR is above a certain threshold.

In this context, another type of Parking Zones is introduced: the Ex-Post Parking Zones, representing the chosen parking facilities after the search process. This can be equal to the Ex-Ante Parking Zone, when this Parking Zone is suitable and available for the traveller, but it can be also some other Parking Zone en-route to the Ex-Ante Parking Zone.

The conceptualisation of the PSR is shown in Fig. 4, indicating how a

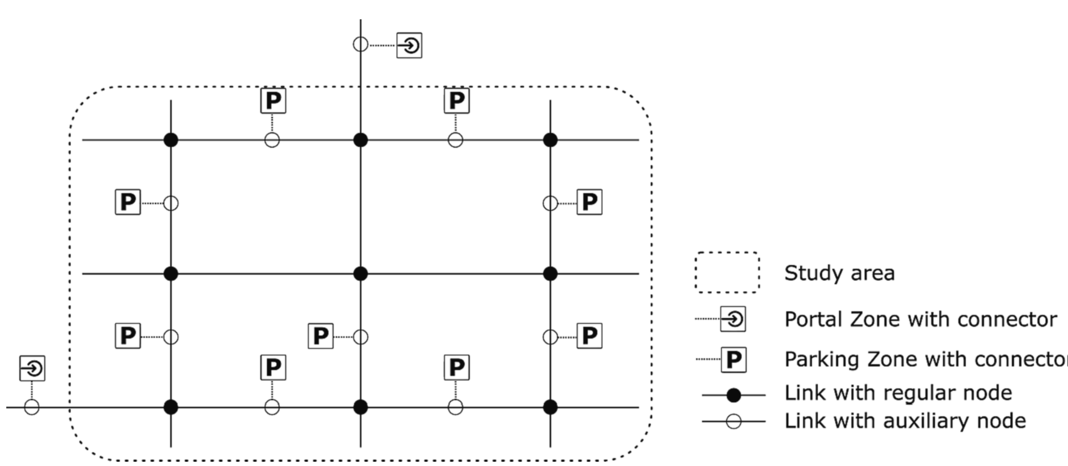

Fig. 3. Representation of traffic zone structure in the proposed model approach.

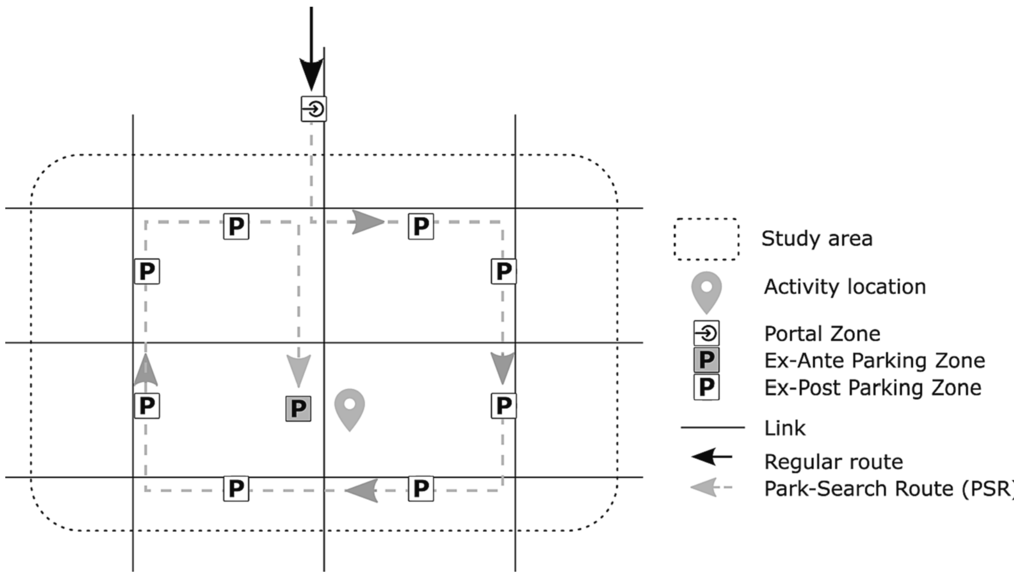

Fig. 4. Conceptualisation of a Park-Search Route (PSR).

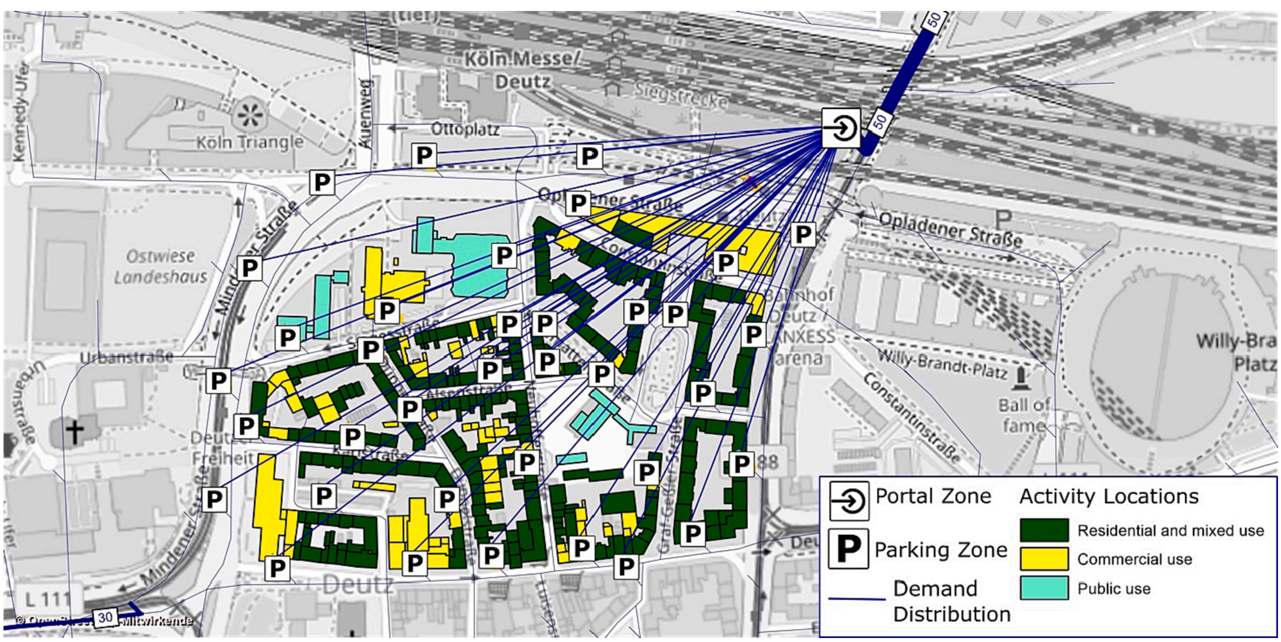

Fig. 5. Example of the demand disaggregation for one Master Zone (Background map. © OpenStreetMap Contributors).

PSR starts at a Portal Zone, continues via several Ex-Post Parking Zones, and eventually ends at the Ex-Ante Parking Zone.

Note that each PSR is bound to an Ex-Ante Parking Zone, because the park-search demand matrices are bound to fixed destinations. The PSR concept, however, allows for some disaggregation of trip ends along a PSR, according to the availability and acceptance of parking options along this route. Each PSR is calculated for a group of travellers (corresponding to the value of the park-search demand matrix). A portion of this group selects a first available parking spot, another portion selects a second, downstream parking spot, etc. Eventually, each traveller within this group will have an individual cost. The optimisation of the cumulated costs for the entire group is part of the PSR algorithm.

The PSR algorithm aims to find and assess park-search routes for a certain origin–destination pair and for a certain user type. A central element of the PSR calculations is a utility function for processes related to parking choice and route choice. Some utility functions and algorithms from previous, PSR-focused model approaches were tested (Friedrich et al., 2019; Hedderich et al., 2018). For reasons of consistence with the Master Model, the approach by Friedrich et al., 2019 was found suitable, but some modifications were required. First, a cost component for the parking fee $c(e_i)$ is added to the cost function. This monetary cost is divided by the Value of time (VOT) to ensure a temporal dimension throughout all cost components. The resulting cost function combines utilities about driving times, walking times and parking fees:

$$c(PSR) = \lambda \sum_{p=1}^{r} t(e_p)^* \left(\prod_{q=1}^{p-1} \left(1 - p(e_q)\right) \right) + \mu \sum_{p=1}^{r} p(e_p)^* d(e_p)^* \left(\prod_{q=1}^{p-1} \left(1 - p(e_q)\right) \right) + \sum_{p=1}^{r} p(e_p)^* \frac{f(e_p)}{VOT}^* \left(\prod_{q=1}^{p-1} \left(1 - p(e_q)\right) \right) \quad (4)$$

with:

$c(PSR)$ = Generalised cost of one PSR.
p,q = Indices of link.
r = Number of links on PSR.
$t(e_p)$ = Driving time on link.p
$p(e_p)$ = Parking probability on link.p
$d(e_p)$ = Walking time from link p to activity location.
$f(e_p)$ = Parking fee for parking on link.p
VOT = Value of time.
λ, μ = Sensitivity parameters.

The threshold to find an available parking spot along the PSR is defined via a Parking Probability Mass. This measure serves as a hard constraint within the algorithm, enforcing some success to find a parking spot along the assessed PSR. It is calculated as the complement of the probability of not finding a parking spot at any of the relevant parking options:

$$p(PSR) = 1 - \prod_{p=1}^{r} \left(1 - p(e_p)\right) \tag{5}$$

with:

$p(PSR)$ = Parking Probability Mass for one PSR $p(e_p)$ = Parking probability on link.p

The pseudo code to determine an optimised PSR is shown next:

```
1   initial route ← route from Shortest Path Search;
2   current route ← initial route;
3   while probability mass (current route) < probability mass threshold do
4       best route ← empty route;
5       best ratio ← 0;
5       foreach outgoing link from current route do
6           if ratio > best ratio then
7               best ratio ← ratio;
8               best route ← current route expanded with link;
9           if c(current route expanded with link) < c(initial route) then
10              initial route ← current route expanded with link;
11      current route ← best route;
12  return current route;
```

with:

$$ratio = \frac{p(currentroute\ \text{expanded with}\ link) - p(initialroute)}{c(currentroute\ \text{expanded with}\ link) - c(initialroute)}$$

The algorithm is modified from the Branch & Bound approach by Friedrich et al., 2019. The shortest-path search is applied to define the initial route (step 1), while Friedrich et al., 2019 take an empty route as the initial route and let the park-search process start at the destination node. This is considered unrealistic at least for travellers who are familiar with the parking situation at the destination, so the proposed algorithm initiates the search process already when entering the vicinity of the activity location, corresponding to previous research on park-search strategies (Polak and Axhausen, 1990).

The shortest-path search is calculated for the route from the Portal Zone to the Ex-Ante Parking Zone. It is usually the route with the least impedance, e.g., based on the lowest driving time. This ensures consistency with the Master Model, which uses the same impedances for the assignment of regular traffic.

The algorithm then checks the probability mass threshold. If the threshold is not met, an expansion of the initial route is executed (step 3). For this expansion, outgoing links, starting at the nodes along the initial route, are assessed (step 5). At this stage, only feasible links are considered, i.e., outgoing links which cannot be entered by a vehicle are excluded. For travellers who start the search process at the destination, the algorithm can be easily adapted by expanding the route only at the very last link before the destination.

For each assessed outgoing link, a *ratio* is calculated describing the local benefit of extending the route (step 6). Here, a gain of the parking probabilities p is weighted against a gain on costs c. The expanded route with the best ratio is returned. If the probability mass threshold is still not reached, the expansion is repeated. A graphical example how an extended route looks like, in comparison to the initial one, is shown later in the results section, see Fig. 6.

Note that the *ratio* compares the cost gain by a route expansion to the initial route. However, if an expansion iteration identifies a route which has a lower cost c than the initial route, the currently expanded route is considered the new initial route (step 9). Otherwise, the denominator of the ratio would be negative.

3.6. Route assignment

The PSR calculations are repeated for the entire park-search demand, resulting in an assignment result for this travel demand segment. Parallelly, routes from travel demand outside the PSR scope, e.g., through traffic, are assigned using the assignment procedures of the Master Model. To consider the feedback loop in Fig. 1, where the resulting traffic-load situation affects the utility calculation, the PSR calculations are repeated until a certain convergence in the traffic-load situation was achieved. This allows for an equilibrium state, as done for conventional traffic assignment problems. As the convergence criterion, travel times along network links are recommended. Practically, values of a skim matrix, containing travel times between all zones, should be compared for subsequent iterations. If these values are below an acceptable threshold, a convergence is reached.

4. Results

4.1. Study goals and study parameters

The goal of the study is to reveal PST and its effects in a real-life setting, based on the model framework presented above. The revealed effect parameters are also compared with results reported in other studies. Another goal is to prove that the proposed model approach can be applied under real-world conditions and that its algorithm provides realistic results.

The study area is part of the city centre of Cologne, Germany, in particular the Deutz district. The study area is characterised as mixed-used, urban and dense; and accommodates about 13,000 residents. There are mostly block-perimeter building structures, with a range of public and on-street parking but limited private and off-street parking. Parking availability is commonly reported as scarce, due to competing parking demand by residents, visitors and commuters. To address this situation, the City has introduced strict regulations for on-street parking in most parts of the district, via tariffs, maximum parking durations and residential-only permits. Such regulations were coded within the PSR algorithms. Parking tariffs were directly applied within the utility function. Further, the allowance of parking spaces for specific demand segments was considered during the assessment process. For example, parking spaces with maximum durations below eight hours were excluded for commuters.

The Master Model originates from the City's traffic administration and is used for the local traffic impact studies etc. The temporal scope of the model corresponds to the weekday p.m. peak hour. Regarding the study area, the Master Model encompasses seven traffic zones, which in total attract a destination-travel demand of about 1,040 vehicles per hour. This travel demand is differentiated into three demand segments: residents, visitors and commuters, whereas residents represent the biggest portion. As a macroscopic model tool, the VISUM software was applied (PTV AG, 2020). The approach, however, is portable to other software.

As an exogenous data source, the parking data API by INRIX, Inc. (INRIX, Inc., n.d.) was accessed. This API offers on-street and off-street data about parking supply and demand for various North American and European cities. The data can be accessed for the current situation and previous time periods. First, static data on the parking supply was recorded, covering in total 3,300 on-street parking spots and 3,900 off-

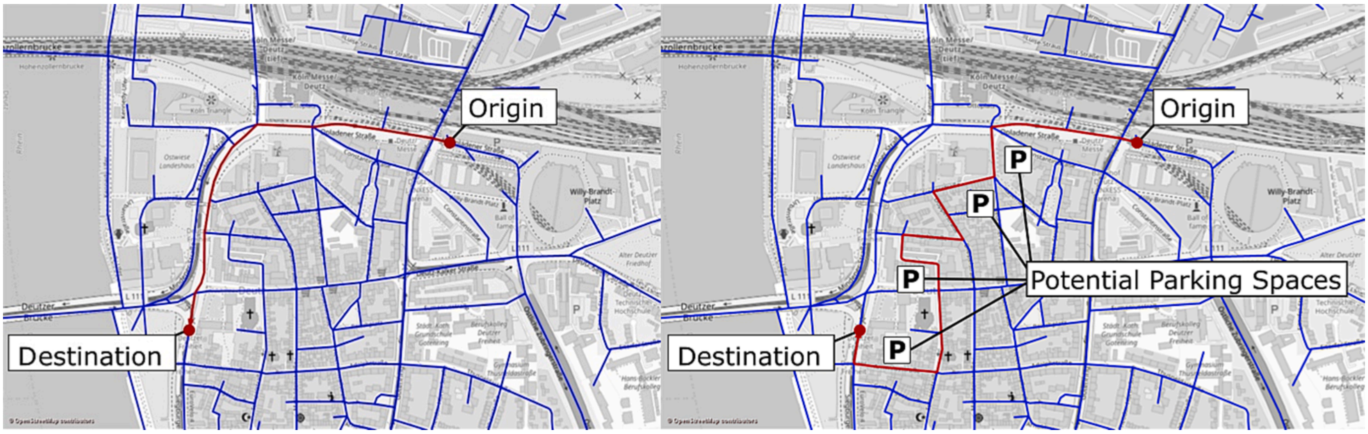

Fig. 6. Example of route assignment with conventional shortest-path search (left); and based on PSR calculations (right) (Background map. © OpenStreetMap Contributors).

street parking spots. Then, parking availability data was recorded for all days in the year 2020, each for the same peak hour as applied for the Master Model. From this time series, all working and non-holiday days were filtered and averaged, analogously to common rules to calculate annual average daily traffic (AADT) volumes. In the end, such averaged data was mapped and integrated into the Master Model via additional attributes of individual network links. As another exogenous data source, a building cadastre via was accessed the City's Open Data portal.

Regarding model network modifications, 160 Parking Zones and twelve Portal Zones were added, as well as three park-search demand matrices, with about 1,000 demand values in total. These demand values are based on the disaggregation of demand from the Master Zones to Parking Zones, as explained in section 3.4. Fig. 5 shows this disaggregation process for one single Master Zone. Incoming traffic, here arriving at the upper-right corner with 50 vehicles per hour, is distributed to numerous Parkin Zones within the Master Zone, depending on parking and activity locations.

For the three demand segments, local parking regulations were applied. For example: residents can use certain parking spaces for free with a residential permit; visitors and commuters need to pay fees; and commuters look for long-term parking only. In addition, specific traveller preferences could have been applied, e.g., by setting different sensitivity parameters for walking and driving per demand segment. This was, however, neglected, due to a lack of empirical data on sensitivity.

As input for the cost function within the algorithm, some local attributes were applied, including VOT and sensitivity parameters. Finally, the probability mass threshold for the PSR algorithm was set to 0.95. The PSR algorithms were executed via special scripts using Python programming language within the model tool environment. The Python scripts are available at the author upon request.

4.2. Model outcomes

First, the character of routes, as determined by the PSR algorithms, was analysed and compared to conventional route calculations by the Master Model. Fig. 6 shows one example of such comparison. On the left-hand side, the route is assigned to an arterial road with the lowest driving time. However, this route barely offers any parking options, so it is unrealistic that at a least a familiar traveller would pick this route. On the right-hand side, the PSR calculation returns an alternative route, which is longer and slower, but, on the other hand, has more potentials in parking spaces. From an intuitive perspective, the latter case seems to better match reality. This example also reveals how the expansion mechanism, as introduced in the PSR algorithm in section 3.5, works. The alternative route corresponds to the expanded route, as selected by the PSR algorithm.

Other resulting route patterns based on the PSR calculations imply some interesting effects, including circling, passing by at links several times, or moving 'back and forwards' along a link. Such patterns clearly reveal excess-route elements, as defined in section 2.1, and were observed in the study case for many PSR. Also, a disaggregation of trip ends along a PSR was observed, which is part of the PSR conceptualisation, see section 3.5. This means that some travellers traverse the entire PSR, while others find a parking space en-route of a PSR. Altogether, the observed PSR effects correspond to intuition of real-life park-search patterns. These can be reproduced via experimental behavioural studies, such as game-based approaches (Fulman et al., 2020), but are barely feasible via assignment techniques in conventional transport models. The capability to reproduce various PSR patterns within a conventional transport model is considered an innovation of this approach.

With all PSR assigned to the network, a variety of analyses was conducted. First, various performance parameters were calculated to determine the amounts and effects of PST in the studied network. Fig. 7 shows the share of PST, in comparison to all traffic, per link in the network model.

Accordingly, PST is dominant in parts of the network which have a rather access function within the study area, including most of the minor roads, with shares up to 90 %. In contrast, PST is less prominent on arterial roads, with shares up to 20 %. For the share of PST on total traffic, one empirical reference is given from a previous study in central Stuttgart, Germany, indicating a value of 15 % for one analysed Friday (Hampshire and Shoup, 2018). The same study cites an average share of PST of 34 %, based on further international studies, however indicating potential biases with such studies.

Next, specific PST effects were analysed. First, average driving times were determined when searching for parking. This parameter, calculated as the averaged PSR travel times between Portal Zones and Ex-Post Zones, was determined to 136 s. Looking back at reference values for German cities, as cited in the literature review, this value is at the lower boundary of the reported range of 2 to 10 min per trip. However, a further discussion of these values seems not fruitful, due to potential inconsistencies and biases with such absolute parameter, as discussed in the literature review.

Instead, alternative PST effect parameters were analysed which explore excess-route elements due to parking search. Suitable parameters are found in previous studies looking at extra travel efforts during the final segment of a trip, in comparison to a shortest possible route (Hagen et al., 2021; Weinberger et al., 2020). A parameter called 'PST effect on the driving distance' was calculated as the difference of the realised driving distance (including searching for parking) and a

Fig. 7. Model outcomes - Share of PST on Total Traffic [%] (Background map. © OpenStreetMap Contributors).

theoretical journey distance. The latter one ignores any PST patterns, and is, for example, calculated by many navigation services as the predicted journey distance. In the model setting, this parameter was determined by PSR distances between Portal Zones and Ex-Post Zones, minus the distances of direct routes between Portal Zones and Ex-Ante Parking Zones. The resulting mean additional driving distance was calculated to 223 m. A similar parameter was determined in an empirical study for Californian cities in the range of 550 to 650 m (Weinberger et al., 2020). This difference might be explained by different characteristics and parking pressures in the corresponding study areas.

When disaggregating this mean number into classes of additional distances, a portion of trips reveals a negative additional distance, see Fig. 8. This is the case when a parking spot is selected upstream of the activity location, due to decisions made in the park-search process. This case results in a shorter trip length compared to a complete trip until the activity location. This interesting phenomenon has been foreseen by other researchers (Hagen et al., 2021; Weinberger et al., 2020), and is also detected for the study area.

The 'PST effect on the driving distance' can be also extrapolated into a 'PST effect on traffic efforts', by multiplying the number of park-searching vehicles with the gain in distances, as experienced be these vehicles. The resulting additional traffic effort is +184 km*veh, or +29 %. For the interpretation of these numbers, please note they are related to trip segments within the study area. In the presented case, most of the

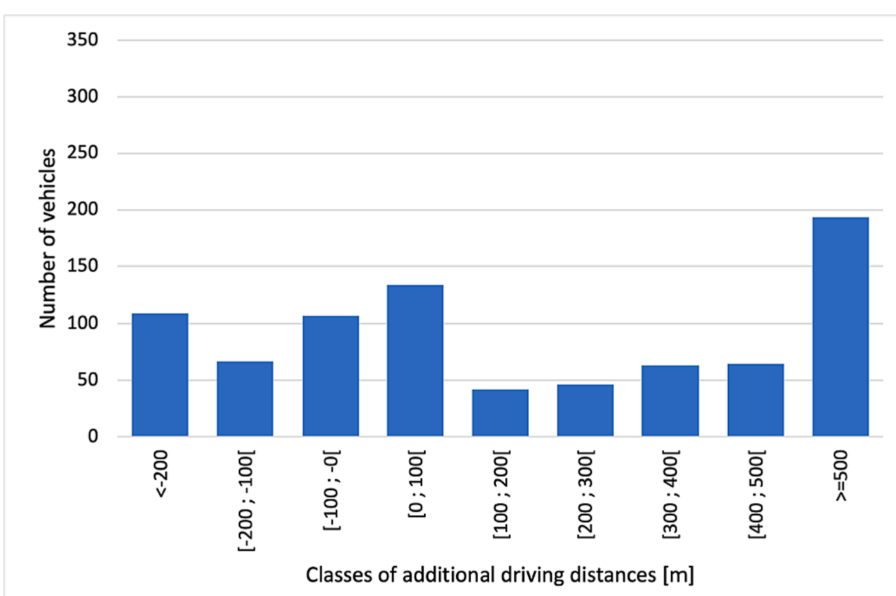

Fig. 8. Model outcomes – Frequency distribution of additional driving distances.

park-searching trips start outside the study area. A more comprehensive view on the increase in travel efforts would involve total traffic efforts, e.g., looking at an entire city. In the study area (a district of a city), park-search traffic is highly dominant, so the percentage increase is quite significant with 29 %, while it is likely to be smaller, when looking at a broader study area. As a reference, in a recent empirical study for Californian cities, the percentage increase is less than 1 % of travel efforts by private cars (Weinberger et al., 2020).

Another effect parameter was introduced, called the 'indirect PST effect on the driving distance'. This allows for the distinguishing of direct vs indirect PST, as defined by Birkner, 1995 and discussed in the literature review. Here, any extra route elements are considered which are used to explore parking spots along the route but are not part of the direct route to the chosen parking spot. This parameter was calculated as the difference of the realised driving distance (including searching for parking) and the direct route to the chosen parking spot. In the model setting, this parameter was determined by PSR distances between Portal Zones and Ex-Post Zones, minus the distances of direct routes between Portal Zones and Ex-Post Parking Zones. The resulting mean additional distance is about 290 m.

The 'indirect PST effect on the driving distance' can be also extrapolated into a 'indirect PST effect on traffic efforts', with an analogous calculation as explained above. The resulting additional traffic effort is +239 km*veh, or +42 %. This measure is highly relevant as it reveals saving potential in terms of traffic efforts, environmental impacts etc. The measure can be also used to assess potential effects of improved parking space management such as parking-guidance systems.

Next, the walking distances are calculated, as the remaining element of the trip chain. The resulting mean walking distance is 139 m. As a reference, an empirical study for employees in a business area of Tel Aviv, Israel, reveals a similar value with an average of 134 m (Levy et al., 2015).

Next, the quality of the model results was checked, represented by the traffic-load situation in the network. For this, vehicle-turn volumes were analysed at all allowed turns within the study area and compared for the states before and after the model modification. Fig. 9 shows the frequency distribution of such turn volumes by volume classes. The left-hand side (before the modification) reveals a big portion of turns with zero volumes and a flat distribution of the other volume classes. This might be due to inability of the Master Model to represent trip ends in a granular manner. In contrast, the right-hand side (after the modification) indicates a far lower prevalence of turns with zero volumes, and a descending curve of the other volume classes. This is explained by park-search routes, which are spread over most network elements based on the assignment procedures. This also implies a more realistic traffic load situation and is another success factor for the proposed approach.

Some sensitivity analyses were conducted to test the model's reactiveness to changing conditions. These analyses look at different scenarios regarding the parking probabilities: scenario no. 1 with status-quo probabilities; scenario no. 2 with probabilities reduced by 20 %; and scenario no. 3 with probabilities increased by 20 %. Table 1 compiles the output parameters for these scenarios.

All parameters about PST effects are as expected: the share of PSR traffic on all traffic is increasing when probabilities are decreased, and vice versa. The same happens for additional traffic efforts caused by PSR as well as the walking distances. Note that the percentages of increase and decrease in terms of PST-effect parameters are much higher than the 20 % of increase and decrease in terms of parking probability. This implies there is a nonlinear correlation between such parameters. Such non-linearities were already shown by previous studies (Millard-Ball et al., 2014).

In addition, the share of non-successful PSR searches in the algorithm is analysed. A non-successful search happens when the PSR calculation requires too many iterations to meet the prescribed probability-mass threshold, leading to an abortion of the calculation. In this situation, the parking saturation is too high to find a suitable PSR with the proposed algorithms. In the model setting, travellers follow a sub-optimal PSR. The remaining travellers, who could not park at available spaces along this route, finalise their trip at their destination. In reality, it is assumed that travellers will opt for solutions outside the model scope, e.g., for illegal parking. This is prevalent in scenario no. 2, where the parking scarcity results in a highly increased share of non-successful PSR searches.

Overall, the resulting parameters from the three scenarios imply that the model reacts correctly to changing conditions.

5. Conclusion, discussion and outlook

The major contribution of the presented work is the integration of parking aspects into a conventional transport model set-up, resulting in a significant increase in level-of-detail regarding parking-search and parking-choice behaviour. It has been shown that the park-search process can be explicitly modelled, including elements of driving; looking for and choosing a parking spot; and walking to the activity location. Some model refinements have been proposed to granularly represent parking supply and demand. Some new modelling concepts have been applied, e.g., a decoupling of destination choice from parking-facility choice, resolving some ambiguities shown in previous model approaches. Lastly, it has been shown that trip ends can be granularised within in the assignment procedure, by adding the choice of a parking spot into the assignment step of the four-step algorithm.

Upon demonstration in a real-life setting, the model results look quite promising. The traffic load situation in the network model seems more realistic than in the state before the model modifications, as park-

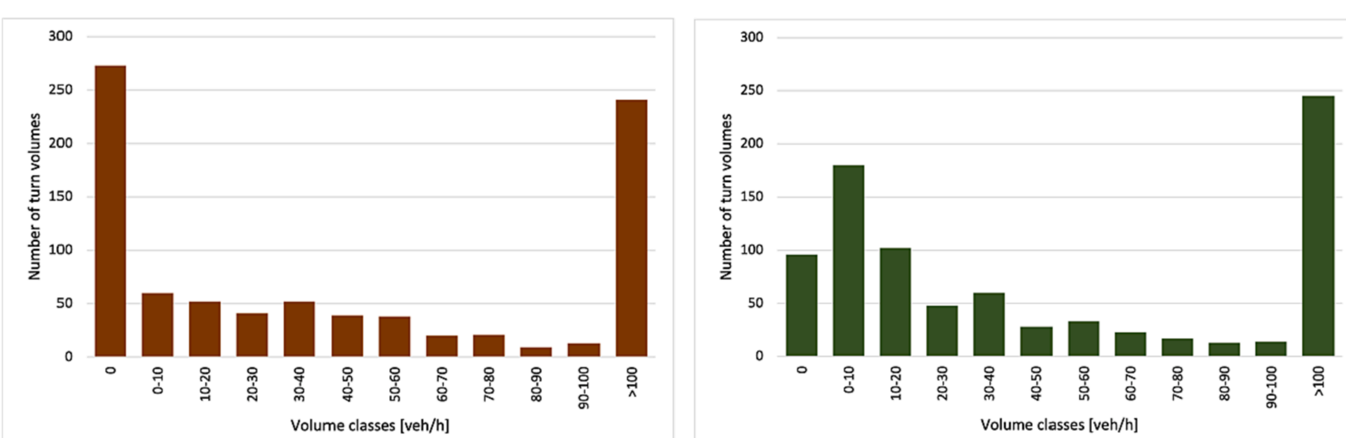

Fig. 9. Model outcomes - Frequency distribution of turn volumes in model network before model modifications (left); and after model modifications (right).

Table 1
Scenario analysis with variations of parking probabilities.

Parameters			Scenario 1: Parking Probabilities Status Quo	Scenario 2: Parking Probabilities-20 %	Scenario 3: Parking Probabilities +20 %
Traffic Efforts	Regular traffic	total [km*veh]	6946.8	6874.8	6891.2
	PSR traffic	total [km*veh]	756.7	876.0	681.6
	Share PSR traffic on all traffic	share [%]	9.8 %	11.3 %	9.0 %
PST effect	Additional driving distance	avg [m]	223	391	110
	Additional traffic efforts	total [km*veh]	184.4	323.0	91.6
		total [%]	29 %	51 %	15 %
Indirect PST effect	Additional driving distance	avg [m]	290	451	191
	Additional traffic efforts	total [km*veh]	239.4	372.7	158.4
		total [%]	42 %	64 %	28 %
Walking distances	avg [m]	139	163	125	
Non-successful PSR searches	share [%]	4.7 %	10.2 %	2.6 %	

search traffic efforts are now visible along most of the network. In addition, various parameters regarding amounts and effects of PST are retrievable for the study area. The PST effect parameters reveal plausible results and correspond mostly to previous empirical studies.

One special feature is the exploitation of parking API data as an exogenous data source. This resolves some data availability issues for parking modelling, as recognised by other researchers. Benefits of this data source are found in their high temporal and spatial resolution; their ability to cover large parts of traffic networks; and the easier access to their data, comparing to manual data collection. This way, the network model is significantly upgraded with detailed parking information.

It has been found that integration of API data about parking supply is fairly straightforward. It can be directly incorporated to describe the parking infrastructure as part of the network model. However, API data about parking demand has to be interpreted correctly when considered with the model environment. The applied API provides probabilities to find a parking spot at a specific facility. Parking probabilities are considered a relevant metric for parking behaviour, as opposed to occupancy levels (Millard-Ball et al., 2014). However, the use of exogenous parking-probability information can be seen problematic for some model practitioners. First, these probabilities are assumed to match reality, i.e., to represent the outcome of real parking supply–demand interactions in the study area. Thus, validation and quality of such data is an important prerequisite, which needs to be assured by the API data provider. Second, and more importantly, the exogenous parking availabilities do not allow for assessing scenarios, in particular measures of parking space management. Such scenarios would assume changes in the parking supply–demand interactions, and eventually in the parking probabilities. Explicit modelling of parking supply–demand interactions, as done by other parking modelers, is not covered in the proposed approach due to incomplete information about parking demand: the initial occupancy levels (at the beginning of the model study time) are unknown; the probabilities to find a parking spot are non-reactive and fixed.

However, an approximation of status-quo parking patterns in the network, as the major objective of the work, is still feasible. The API-based parking probabilities can at least imply the spatial distribution of parking demand and correlating effects. In future work, concepts are recommended to consider the relationship between parking supply and demand more explicitly. This could be accomplished by a closer exchange with the API data providers, by coupling the methods applied by those providers with the methods used in transport models. In such setting, parking probabilities could be endogenously calculated by the model, and then compared to or calibrated with the exogenous API data.

In this context, further features could be added to the model, such as temporal dynamics, occupancy fluctuations and capacity restraints. This will enable explicit interactions between elements of parking supply and parking demand, and eventually allow for scenario analyses, which are fundamental for many transport modelers.

Further challenges were found in the calibration of the model framework. Even if some of the independent variables within the algorithm can be calibrated via empirical or stated- preference studies, a validation of the resulting park-search routes seems challenging. Approaches for empirical validations of such routes were proposed (Hagen et al., 2021; Montini et al., 2012), but these only show a portion of real-life conditions. Instead, it is recommended to apply proxy variables for the calibration, e.g., link and turn volumes in minor network elements, where PST is assumed to be dominant.

Future work may advance the proposed model approach, by adding some of the features applied by other parking modelers. Among others, parking-location choice could be enhanced via discrete choice modelling, looking into the selection or a denial of a found spot along the PSR. Additional factors such a risk-taking could be considered this way. Further, a probabilistic distribution mechanism for selecting a PSR could be added via, e.g., a Multinomial Logit Model. Such a model would deal with a set of possible PSRs for each value of the park-search demand matrices, whereas the proposed approach only produces one single PSR. However, such features have been already explored in previous works, and could be easily added to the approach.

Another area of potential differentiation is found in the travellers' information state about the parking situation, and their familiarity with the study area. The model approach could be easily refined to consider this effect. E.g., unfamiliar travellers could be extracted from the park-search demand matrices and attributed with adapted PSR algorithms.

Still, an important conclusion is that a reasonable, model-based representation of parking patterns is feasible for urban environments, even if such environments are far more complex than 'isolated' parking environments like off-street parking garages.

Lastly, it has been shown that the proposed model modifications fit smoothly into a conventional, macroscopic model setting. The new data sources and additional model elements can be integrated easily, and do not interfere with the model structure before the modifications. Such additions allow for a 'one model for all' concept, with regular and parking traffic considered in one model environment. This would enable more options for model-based analyses by urban transport analysts and planners.

CRediT authorship contribution statement

Peter Lubrich: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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