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journal homepage: www.elsevier.com/locate/jmonecoThe signalling channel of negative interest rates[☆]Oliver de Groot^{a,*}, Alexander Haas^{b,c}^a University of Liverpool & CEPR, Chatham Rd, L69 7ZH, UK^b University of Oxford, Manor Rd, OX1 3UQ, UK^c DIW Berlin, Germany

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ABSTRACT

Negative policy rates can convince markets that deposit rates will remain lower-for-longer, even when current deposit rates are constrained by zero. This is the signalling channel of negative interest rates. We analyse the optimality and effectiveness of negative rates in the context of this novel transmission channel. In a stylized model, we prove two necessary conditions for optimality: time-consistency and a preference for policy smoothing. In an estimated model, we show the signalling channel dominates banks' costly interest margin channel. However, the effectiveness of negative rates depends sensitively on the degree of policy inertia, level of reserves, and ZLB duration.

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1. Introduction

In recent years, negative interest rates have become an additional policy tool for some central banks while others have kept policy rates positive, despite a need for further monetary easing. In the euro area, the deposit facility rate—paid on bank reserves at the European Central Bank (ECB)—and the EONIA interbank market rate turned negative in June 2014 (Fig. 1(a)).¹ In response, average household deposit rates declined but remained positive with the fraction of deposits paying zero interest rising but virtually no banks passing the negative reserve rate on to household depositors (Fig. 1(b)). At the

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¹ The ECB subsequently lowered its deposit facility rate to -0.5% in Sep-2019. The Swiss National Bank reached a low of -1.25 to -0.25% for its 3-month LIBOR target range in Jan-2015. Denmark's Nationalbank set its certificates of deposit rate to a low of -0.75% in Feb-2015. The Bank of Japan reached a low of -0.1% for its short-term policy interest rate in Jan-2016. Finally, the Swedish Riksbank lowered its deposit rate to -1.25% and its repo rate to -0.5% in Feb-2016.

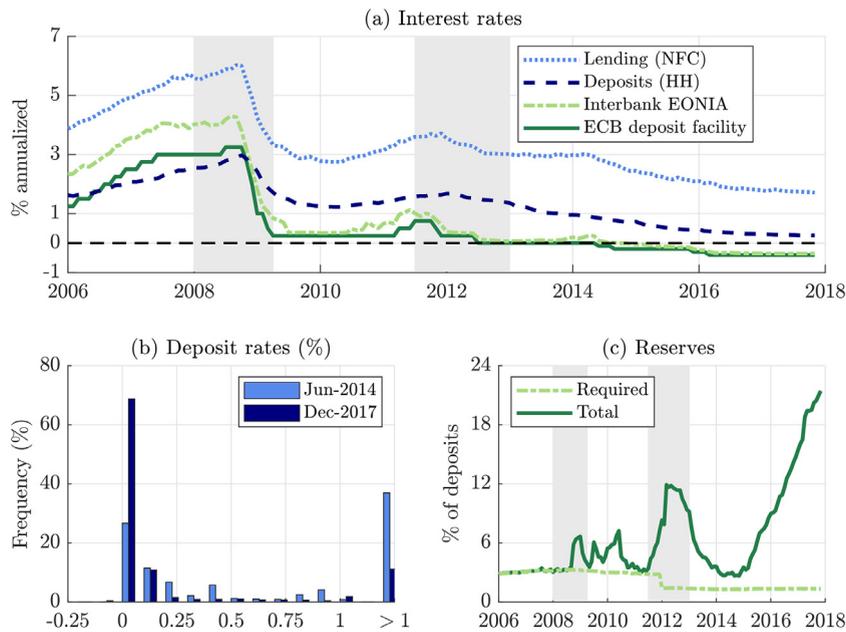


Fig. 1. Interest rates and reserves in the euro area. *Note:* In (a) NFC and HH denote non-financial corporation and household composites, respectively; EONIA is the euro area overnight interbank market rate. In (b) deposit rates are on outstanding amounts as reported by individual banks, plotted as a fraction of total deposits in each bucket. In (c) deposits are HH and NFC deposits; excess reserves are total reserves minus required reserves. Source: ECB.

same time, and despite negative interest margins, banks started to accumulate massive excess reserves with total reserves rising to over 20% of deposits by 2018 (Fig. 1(c)).²

This raises four questions: i) Given that banks do not (or cannot) pass on negative rates to households, what is the transmission channel through which they operate? ii) What are the consequences of a negative rate policy for the banking system and credit creation if a large fraction of banks' assets are reserves that earn a negative return? iii) When both direct and indirect effects are accounted for, are negative rates effective in raising aggregate demand? iv) Under what conditions should negative rates be in an optimal policymaker toolkit? This paper studies the interplay of a contractionary bank interest margin channel and a novel expansionary *signalling* channel to address these questions.

Our first contribution is to analytically explore this signalling channel by which negative reserve rates can be expansionary, even when deposit rates are constrained. We build a stylized model in which banks hold central bank reserves and monetary policy can set a negative reserve rate, but—in line with empirical evidence—household deposit rates are constrained by a zero lower bound (ZLB). Away from the ZLB, the behaviour of the model is isomorphic to a model without reserves. However, when the central bank sets a negative reserve rate, the deposit rate, that enters the household Euler equation, remains at zero, creating a wedge between the return on reserves and the cost of bank funding (deposits). All else equal, a negative reserve rate acts like a contractionary bank net worth shock (“the costly interest margin channel of negative interest rates”), where the shock is scaled by the total amount of reserves. Credit spreads widen, raising lending rates vis-a-vis deposit rates, and depress investment demand. This direct, or *intra-temporal*, effect of negative rates has been a key criticism of negative rate policies by commercial banks.

However, our stylized model shows there can also be substantial expansionary indirect, or *inter-temporal*, effects of negative rates that are less directly ascribable to the policy. In particular, we highlight the role negative rates can play in signalling future policy (“the signalling channel of negative interest rates”). This signalling channel—analogue to [Bhattarai et al. \(2022\)](#)'s signalling theory of quantitative easing (QE)—is active when a time-consistent policymaker with a preference for smoothing interest rates uses a negative rate policy as a credible commitment device to keep deposit rates lower-for-longer.

The mechanism underlying the signalling channel is straightforward: When deposit rates are constrained by zero, they will not rise until the reserve rate turns positive. In an environment where the central bank only adjusts rates gradually—due to a preference for smoothing policy—moving the reserve rate into negative territory increases the distance (and the time taken) for it to turn positive again. Thus, a negative reserve rate signals lower-for-longer deposit rates. This signalling is distinct from forward guidance, which is an “open mouth” commitment about future interest rates. In reality, open mouth policy is not always credible. We show—using gradualism as a commitment device—negative rates can create credible for-

² While banks' deposit creation and lending determine the reserve-to-deposit ratio and required-to-excess reserve split, the ECB's liquidity and asset purchase programs caused the growth of total reserves.

ward guidance. Signalling, in our context, therefore derives from time-consistency and smoothing rather than imperfect information as in [Melosi \(2017\)](#).

Given the trade-off between the costly interest margin channel and the signalling channel we use the stylized model to study optimal policy and analytically prove two conditions for the optimal policymaker to include negative rates in its toolkit: i) it sets time-consistent discretionary policy (i.e., cannot commit to future promises) and ii) it has an intrinsic preference for policy smoothing. Under these conditions, a negative reserve rate can act as a tangible signal of maintaining lower future deposit rates. In contrast, a policymaker that can commit to future promises does not use negative rates as it can generate a credible path of deposit rates without incurring the costs of negative rates via the interest margin channel. Equally, a discretionary policymaker without a smoothing preference has no ability to signal and so negative rates generate only a direct cost to banks.

Our second contribution is to study the trade-off between the signalling and costly interest margin channels quantitatively. We develop a medium-scale version of the stylized model, substitute optimal policy for an inertial Taylor-type rule, and estimate the key structural parameters of the model. When monetary policy is described by an inertial rule, setting a negative policy rate signals lower-for-longer deposit rates, depressing post-ZLB deposit rates and potentially extending the overall ZLB duration. Even with current deposit rates unchanged, this generates an expansionary intertemporal aggregate demand effect.

In our estimated model, we show quantitatively that the contractionary intratemporal effect of negative rates via the costly interest margin channel is more than offset by the expansionary intertemporal aggregate demand effect via the signalling channel. In general equilibrium, asset values increase and banks benefit from capital gains. This reverses the fall in net worth due to the costly interest margin channel, compresses credit spreads (lowering lending rates), and boosts investment demand. We illustrate this with a novel decomposition of bank profits, highlighting the role of capital gains. We also show the effectiveness of negative rates (relative to standard policy) depends on three key factors: i) more policy inertia strengthens the expansionary signalling channel, ii) more reserves in the banking system magnifies the costly interest margin channel, and iii) a longer expected ZLB duration depresses the expansionary signalling channel and magnifies the costly interest margin channel. Finally, we show our results are robust to changes in the size of capital gains and not a product of the new-Keynesian forward guidance puzzle.

Literature. A growing empirical literature studies the transmission and effectiveness of negative rates. [Eisenschmidt and Smets \(2018\)](#) document that—consistent with our model—euro area banks did not lower household deposits rates below zero. They find negative rates eased financial conditions and created modest credit growth, despite some adverse effects on bank balance sheets. Recent evidence in [Altavilla et al. \(2022\)](#) suggests negative rates did not inhibit the monetary transmission to firm deposit rates. Regarding bank profitability, [Altavilla et al. \(2018\)](#) estimate the impact on bank balance sheets, identify a costly interest margin channel, and find—in line with our paper—that overall negative rates substantially increased banks' asset and equity values. [Heider et al. \(2019\)](#) and [Demiralp et al. \(2021\)](#) show that banks adjust both lending quantity and risk profile in response to negative rates. [Girotti et al. \(2021\)](#) also find evidence for bank portfolio rebalancing and argue negative rates flatten the middle of the corporate loan yield curve. While in our model financial frictions and negative rates determine the volume and not the type of credit extended by banks, we also find that negative rates mainly impact the middle of the yield curve. Thus, a negative rate policy can be seen as a complement to QE (which affects the long end of the yield curve) even if—as our model suggests—negative interest rates are less effective in the presence of QE (through the rise in reserves).

In the theoretical literature, [Ulate \(2021a,b\)](#) investigates monopoly power in the banking sector and shows negative rates are expansionary as monopolistic profit margins allow banks to partly pass-through negative reserve rates. In our model, the banking sector is competitive which suggests our estimates of the expansionary effects of negative rates via the signalling channel—also implicitly active in [Ulate \(2021b\)](#)—may be conservative. [Brunnermeier et al. \(2022\)](#) build a model with partial deposit rate pass-through and show that there can exist a (time-varying) “reversal rate” below which further cuts in the policy rate are contractionary for lending. While this reversal rate can theoretically be positive, in a quantitative model the authors estimate it to be around -1% in the euro area.³ Abstracting from signalling and a partial rate pass-through, [Eggertsson et al. \(2022\)](#) find negative rates are, at best, ineffective, and at worst contractionary, depending on the parameterization of bank intermediation costs. In our model, the contractionary interest margin channel microfounds this notion. [Onofri et al. \(2021\)](#) allow households to use non-deposit savings vehicles and banks to use non-deposit funding sources. This feature is key for negative rates to be expansionary in their model.⁴ [Sims and Wu \(2021a,b\)](#) study several unconventional monetary policy measures—including negative rates—in a unified framework. Compared to this literature, our paper is the first to characterize optimal policy in a negative rate environment.⁵ Further, we explicitly model and thoroughly investigate the trade-off between a novel signalling and a costly interest margin channel. However, there are other potential channels through which negative rates may operate, which we and the literature have not yet studied in detail.

³ [Darracq Pariès et al. \(2020\)](#) study macroprudential policy in an environment with a reversal rate.

⁴ In [Onofri et al. \(2021\)](#), the policy rule is inertial in the notional rather than the actual policy rate, rendering the signalling channel inactive. We provide further details on this in [Section 3.1](#).

⁵ [Rognlie \(2016\)](#) studies optimal policy in a model without a banking sector where negative rates can raise aggregate demand but inefficiently subsidize paper currency.

Balloch et al. (2022) provide a valuable overview, including search-for-yield effects (from portfolio choice), exchange rate effects (in an open economy setting), and wealth effects (from long-dated assets).

Our paper also contributes to the literature on how to make forward guidance credible. Woodford (2003) shows that, in the absence of commitment, the delegation of policy to a policymaker with an interest rate smoothing objective can be welfare improving. Nakata and Schmidt (2019) demonstrate delegation is even more valuable with occasional episodes at the ZLB.⁶ We take this a step further and show delegating to a policymaker with a smoothing preference introduces a new (welfare improving) policy tool—negative rates. In the context of QE, Bhattarai et al. (2022) find QE is effective as the government commits to honour outstanding debt, enabling the discretionary policymaker to generate a credible signal of low future interest rates. Our two papers are highly complementary. Our instrument is negative rates and the commitment device is policy smoothing whereas their instrument is QE and the commitment device is outstanding debt obligations.

A future question is whether the policy smoothing preference (required for signalling) might arise, not intrinsically from delegation, but extrinsically from a feature of the economy. Stein and Sunderam (2018) show policy gradualism under discretion is optimal if the central bank i) has private information and ii) is averse to bond market volatility. However, they do not microfound the policymaker's aversion to the latter. McKay and Wieland (2021) build a model of lumpy durable consumption demand. In their model, a monetary easing increases durable consumption demand today at the expense of tomorrow, forcing policy to remain accommodative for longer. Although beyond the scope of this paper, this feature may give rise to an extrinsic preference for policy smoothing and hence generate a signalling channel of negative interest rates without delegation.

The paper proceeds with Section 2 presenting a stylized model and optimality conditions for negative rates. Section 3 presents a quantitative model and results on the strength of the signalling channel and the effectiveness of negative rates. Section 4 concludes.

2. Stylized model and optimal policy

This section sets up a stylized model to qualitatively illustrate the signalling channel of negative interest rates. We show the model can be reduced to a 3-equation new-Keynesian model with an endogenous demand shifter in the IS equation resulting from negative rates and present analytical and numerical results regarding the optimality of negative rates.

2.1. Set up

The model consists of households, banks, firms, and a central bank. Two types of households—savers and borrowers—transact through banks that are subject to lending frictions. Monopolistic firms produce and set prices subject to nominal rigidities. The central bank sets its policy tool—the interest rate on reserves—optimally.

Households. Two types of households—savers and borrowers—are distinguished by their relative patience with discount factors β and β_b , respectively, where $0 < \beta_b < \beta < 1$.

A representative saver household is composed of a fraction f workers and $1 - f$ bankers with perfect consumption insurance. Workers and bankers switch with probability $1 - \theta$. When they do, bankers transfer retained profits to the household. Households consume, $C_{s,t}$, supply labor, $L_{s,t}$, and save in bank deposits, D_t , with gross nominal return $R_{d,t}$. Financial markets are incomplete. The saver household's problem is given by

$$V_{s,t} = \max_{\{C_{s,t}, L_{s,t}, D_t\}} \left(\frac{C_{s,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{s,t}^{1+\varphi}}{1+\varphi} \right) + \beta \exp(s_t) \mathbb{E}_t V_{s,t+1}, \quad (1)$$

subject to

$$P_t C_{s,t} + D_t = P_t W_{s,t} L_{s,t} + R_{d,t-1} D_{t-1} + \Omega_{1,t} - \Omega_{2,t}, \quad (2)$$

where s_t is an AR(1) time-preference shock that generates exogenous movements in the natural real rate, P_t is the aggregate price level, $W_{s,t}$ is the real wage, $\Omega_{1,t}$ are firm and bank profits, and $\Omega_{2,t} = R_{b,t-1} P_{t-1} B_{t-1} - P_t W_{b,t} L_{b,t}$ is a lump-sum transfer from savers to borrowers both households take as given. While contrived, this transfer facilitates a clean set of equilibrium conditions that maintain focus on the key features of the model related to negative interest rates. The first-order conditions are given by $1 = \mathbb{E}_t \Lambda_{t,t+1} R_{d,t} / \Pi_{t+1}$, and $\chi L_{s,t}^\varphi = C_{s,t}^{-\sigma} W_{s,t}$, where $\Lambda_{t-1,t} \equiv \beta \exp(s_t) (C_{s,t} / C_{s,t-1})^{-\sigma}$ is the saver household's real stochastic discount factor and $\Pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate. A ZLB on the deposit rate, $R_{d,t} \geq 1$, arises as cash offers a zero nominal net return.

The representative borrower household only consists of workers. Its problem is given by

$$V_{b,t} = \max_{\{C_{b,t}, L_{b,t}, B_t\}} \left(\frac{C_{b,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{b,t}^{1+\varphi}}{1+\varphi} \right) + \beta_b \exp(s_t) \mathbb{E}_t V_{b,t+1}, \quad (3)$$

subject to

$$P_t C_{b,t} + R_{b,t-1} P_{t-1} B_{t-1} = P_t W_{b,t} L_{b,t} + P_t B_t + \Omega_{2,t}, \quad (4)$$

⁶ Bonciani and Oh (2023) argue smoothing and QE resolve several new-Keynesian ZLB puzzles.

where variables have subscript b . B_t are bank loans with gross nominal interest rate $R_{b,t}$. The first-order conditions are given by $C_{b,t}^{-\sigma} = \beta_b e^{s_t} \mathbb{E}_t C_{b,t+1}^{-\sigma} \frac{R_{b,t}}{\Pi_{t+1}}$ and $\chi L_{b,t}^\varphi = C_{b,t}^{-\sigma} W_{b,t}$.

Banks. The balance sheet of banker j is given by

$$B_t(j) + A_t(j) = D_t(j) + N_t(j), \quad (5)$$

where $N_t(j)$ is net worth and $A_t(j)$ are central bank reserves that earn the gross nominal return R_t . We assume a banker's demand for central bank reserves is given by

$$A_t(j) = \alpha(x_t) D_t(j), \quad (6)$$

where $x_t \equiv R_t/R_{d,t}$, $\alpha(1) = \alpha$, $\alpha(x_t) > 0$, $\alpha'(x_t) > 0$, and $\alpha''(x_t) > 0$. This demand schedule captures the trade-off between banks' preference for holding reserves to self-insure against idiosyncratic liquidity risk and the cost of holding reserves.⁷

Within a period, the timing is as follows: i) Bankers receive loan payments and repay depositors. ii) Bankers exit with probability $1 - \theta$. An exiting banker is replaced by a worker with an endowment of net worth, \bar{N} . iii) Bankers accept new deposits and demand reserves. iv) A banker can divert a fraction λ of its assets (net of reserves) to its household, in which case, the depositors force bankruptcy and recover the remaining assets.

This agency problem creates a financial friction and makes bankers' net worth a relevant determinant of equilibrium outcomes. The banker problem is given by

$$V_{n,t}(j) = \max_{\{B_t(j), A_t(j), D_t(j), N_t(j)\}} \mathbb{E}_t \Lambda_{t,t+1} ((1 - \theta)N_{t+1}(j) + \theta V_{n,t+1}(j)), \quad (7)$$

subject to the banker's balance sheet, (5), reserve demand, (6), incentive compatibility constraint, (8), and net worth accumulation equation, (9), with the latter two given by

$$V_{n,t}(j) \geq \lambda B_t(j), \quad (8)$$

$$N_t(j) = \frac{R_{b,t-1}}{\Pi_t} (1 - \tau) B_{t-1}(j) + \frac{R_{t-1}}{\Pi_t} A_{t-1}(j) - \frac{R_{d,t-1}}{\Pi_t} D_{t-1}(j), \quad (9)$$

where the tax $\tau \equiv 1 - \frac{\beta_b}{\beta}$ ensures the steady state is undistorted by the financial friction. The central bank sets the reserve rate and supplies reserves elastically. Since banks are competitive, arbitrage ensures $R_t = R_{d,t}$ when $R_{d,t} > 1$. In a symmetric equilibrium, bankers have a common leverage ratio, denoted $\Phi_t \equiv B_t/N_t = B_t(j)/N_t(j)$. The banking sector can thus be summarized in two equations.⁸ Aggregate net worth is given by

$$N_{t+1} = \theta \left(\frac{R_{b,t}}{\Pi_{t+1}} (1 - \tau) \Phi_t - \frac{R_{d,t} - \alpha(x_t) R_t}{(1 - \alpha(x_t)) \Pi_{t+1}} (\Phi_t - 1) \right) N_t + (1 - \theta) \bar{N}, \quad (10)$$

and, if the incentive constraint binds, (11) holds, otherwise arbitrage ensures (12) holds,

$$\lambda \Phi_t = \mathbb{E}_t \Lambda_{t,t+1} \frac{1 - \theta + \theta \lambda \Phi_{t+1}}{\Pi_{t+1}} \left(R_{b,t} (1 - \tau) \Phi_t - \frac{R_{d,t} - \alpha(x_t) R_t}{1 - \alpha(x_t)} (\Phi_t - 1) \right), \quad (11)$$

$$R_{b,t} (1 - \tau) = \frac{R_{d,t} - \alpha(x_t) R_t}{1 - \alpha(x_t)}. \quad (12)$$

Production. Intermediate firm i produces output $X_t(i) = L_{s,t}(i)^\omega L_{b,t}(i)^{1-\omega}$, hiring workers in a competitive labor market. Retail firms repackage intermediate output one-for-one, $Y_t(i) = X_t(i)$. Final output, $Y_t = (\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di)^{\epsilon/(\epsilon-1)}$, is a CES aggregate of retail firm output, where $\epsilon > 0$. Cost minimization results in demand for good i given by $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$, where $P_t = (\int_0^1 P_t(i)^{1-\epsilon} di)^{1/(1-\epsilon)}$. Subject to a Calvo nominal price rigidity, each period, retail firms adjust their prices with probability $1 - \iota$. In doing so, they solve $\max_{P_t(i)} \mathbb{E}_t \sum_{\tau=0}^{\infty} \iota^\tau \Lambda_{t,t+\tau} \left(\frac{P_t(i)}{P_{t+\tau}} - \mathcal{M}_{t+\tau} \right) Y_{t+\tau}(i)$ subject to the demand for good i , where $\mathcal{M}_t = W_{s,t}^\omega W_{b,t}^{1-\omega} / (\omega^\omega (1-\omega)^{1-\omega})$ denotes marginal cost.

The aggregate resource constraint is given by $C_{s,t} + C_{b,t} = Y_t$. To close the model, the central bank sets the reserve rate, R_t , optimally as described in Section 2.3.

2.2. Log-linear equilibrium

The beauty of this stylized model is that its log-linear form is very similar to the canonical 3-equation new-Keynesian model.⁹ In the following, we focus on the case when $\theta = 0$ where bankers survive a single period. In this case, when the financial sector incentive compatibility constraint binds, the private-sector equilibrium conditions are given by

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t, \quad (13)$$

⁷ A simple model to microfound this functional form is presented in Appendix A.1. For other models of idiosyncratic liquidity risk and reserve demand see Güntner (2015) and Bianchi and Bigio (2022).

⁸ A full derivation of the banker's problem for the quantitative model can be found in Appendix B.1.

⁹ Appendix A.2 provides the full derivation of the log-linear model.

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-c}{\sigma} (r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t) - c(\mathbb{E}_t \phi_{t+1} - \phi_t), \quad (14)$$

$$\phi_t = \tau_1 \mathbb{E}_t \phi_{t+1} - \tau_2 (r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t) - \tau_3 (r_{d,t} - r_t), \quad (15)$$

where lower-case letters are the log-levels of their upper-case counterparts and c is the steady state consumption share of borrowers. The other parameters are given by

$$\kappa = \frac{(1-l\beta)(1-l)(\varphi+\sigma)}{l}, \quad \tau_1 = \frac{\Phi\sigma}{1+\Phi\sigma}, \quad \tau_2 = \frac{\Phi}{1+\Phi\sigma}, \quad \tau_3 = \frac{\Phi-1}{1+\Phi\sigma} \frac{\alpha}{1-\alpha}.$$

Eq. (13) is the standard Phillips curve. Since we only have time-preference shocks, output and output gap coincide. Eq. (14) is the IS curve. When $c=0$, it reduces to the standard IS curve. Two more points are worth making: First, the deposit rate, $r_{d,t}$, rather than the policy rate, r_t , enters the IS curve. Second, the IS curve features an endogenous demand shifter, $c(\mathbb{E}_t \phi_{t+1} - \phi_t)$, which result from leverage fluctuations. Eq. (15) is the banks' incentive constraint and determines leverage. Its final term captures the costly interest margin channel: When the reserve and deposit rates deviate, inefficient fluctuations in bank leverage feed through into aggregate demand fluctuations.

The costly interest margin channel even operates in the absence of financial frictions because banks, active in a competitive environment, still need to break-even.¹⁰ When the incentive constraint does not bind, (15) disappears and (14) can be rewritten as

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma} (r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t) - \phi (r_{d,t} - r_t), \quad (16)$$

where $\phi \equiv (c/\sigma)\alpha/(1-\alpha)$. In this case, endogenous demand shifts in the IS curve occur when the reserve rate deviates from the deposit rate. All else equal, when the reserve rate, r_t , turns negative and the deposit rate, $r_{d,t}$, is bounded by zero, this pushes down output, y_t . To see why, we can write the credit spread, $r_{b,t} - r_{d,t}$, as

$$r_{b,t} - r_{d,t} = \frac{\alpha}{1-\alpha} (r_{d,t} - r_t). \quad (17)$$

When $r_t < r_{d,t}$, banks pass on the cost of the negative rate policy into higher borrowing rates, $r_{b,t}$, resulting in a reduction in consumption demand by borrowers. The pass-through from negative rates to the credit spread—the *costly interest margin channel*—is increasing in the quantity of reserves in the banking system, α .

Proposition 1. *The costly interest margin channel of negative interest rates is exacerbated by an increase in the quantity of reserves, α , in the banking system.*

This result is analogous to tax theory, with the reserve rate as a tax and the quantity of reserves as the tax base. It illustrates why a negative rate policy may be contractionary.

2.3. Analytical results

This section studies the conditions for negative interest rates to be a tool in an *optimal* policymaker's toolkit and introduces our theory of the signalling channel.¹¹

Optimal policy. To study optimal policy, we assume social welfare is approximated by a quadratic function in inflation and the output gap,

$$V_t^{SW} = -\frac{1}{2} (\pi_t^2 + \lambda y_t^2) + \beta \mathbb{E}_t V_{t+1}^{SW}, \quad (18)$$

where $\lambda = \frac{\xi}{\xi}$. A model-consistent welfare function depends on the welfare weights of savers and borrowers. For tractability, we use a policy-relevant function consistent with i) the microfounded welfare function of the canonical new-Keynesian model, and ii) many central banks' dual mandate. The policymaker maximizes (18)—setting the reserve rate, r_t —subject to the private-sector equilibrium conditions, and three constraints given by

$$r_{d,t} \geq 0, \quad r_{d,t} - r_t \geq 0, \quad r_{d,t} (r_{d,t} - r_t) = 0. \quad (19)$$

The first constraint in Eq. (19) is the ZLB on the deposit rate. The second states the deposit rate cannot be below the reserve rate. The third ensures the reserve and deposit rate can only diverge when the deposit rate is at zero. In sum, while the reserve rate can turn negative, away from the ZLB on the deposit rate, arbitrage equates the two.

In the following, we consider an optimal policymaker that maximizes Eq. (18), first, under full commitment, and, second, in a time-consistent (discretionary) manner.

¹⁰ In the quantitative model in Section 3, we ensure the incentive constraint binds and the financial accelerator operates. In this section we pursue the frictionless case as it provides clean analytical insights.

¹¹ Appendix A.3 reports the behaviour of the model under a simple Taylor-type rule.

Proposition 2. *Under commitment—when the policymaker solves for a state-contingent plan $\{\pi_t, y_t, r_t, r_{d,t}\}_{t=0}^{\infty}$ by maximizing (18) subject to the sequence of constraints (13), (16), (19)—it follows that $r_t \geq 0 \forall s_t$.*

Proof. See Appendix A.4. \square

Proposition 2 states that with full commitment, a policymaker will never use negative interest rates. The intuition is relatively simple. Under commitment, the central bank can credibly promise to hold the deposit rate lower-for-longer in the future in order to compensate, in part, for the presence of the ZLB. Setting a negative reserve rate results in a cost via the interest margin channel without any further benefit.

Proposition 3. *Under discretion—when the policymaker solves for $\{\pi_t, y_t, r_t, r_{d,t}\}$ re-optimizing (18) every period subject to (13), (16), (19), and the actions of future policymakers—it follows that $r_t \geq 0 \forall s_t$.*

Proof. See Appendix A.4. \square

Proposition 3 states that negative rates are also not part of the optimal time-consistent policymaker's toolkit. Under discretion, the policymaker cannot commit to future actions and so a negative rate does not signal lower rates in the future. Setting a negative reserve rate results in a cost via the interest margin channel without any further benefit.

Propositions 2 and 3 suggest that negative rates are never optimal in an environment with deposit rates bounded by zero and no alternative transmission to lending rates—for example, through monopolistic competition among banks as in Ulate (2021b). However, welfare can sometimes be raised by appointing a central banker whose preferences do not coincide with the social welfare function (Rogoff, 1985). In the following, we show that delegating policy to a central banker that places a weight on smoothing policy will—under certain conditions—allow negative rates to increase welfare. With a preference for smoothing interest rates, lowering policy rates today signals lower policy rates tomorrow. This is the essence of the expansionary signalling channel of negative interest rates.

Technically, smoothing gives the policymaker an endogenous state variable that allows it to signal. Whether another state variable in the model structure will do the job is an open question. However, not any endogenous state variable will do. Propositions 2 and 3 also hold when a hybrid Phillips curve makes lagged inflation a state.

Optimal policy with delegation. Woodford (2003) and Nakata and Schmidt (2019), amongst others, show that under discretion delegating policy to a policymaker with a preference for smoothing can be desirable. We therefore introduce a delegated central bank loss function (that deviates from the social welfare function) given by

$$V_t = -\frac{1}{2} \left((1 - \psi)(\pi_t^2 + \lambda y_t^2) + \psi (r_t - r_{t-1})^2 \right) + \beta \mathbb{E}_t V_{t+1}, \quad (20)$$

with a preference for interest rate smoothing weighted by $\psi \in (0, 1)$. Proposition 4 states a set of necessary conditions for negative rates to be in the optimal policymaker's toolkit.

Proposition 4. *Two necessary conditions for the optimality of negative interest rates in this framework are i) a discretionary policy setting, and ii) the delegation of policy to a policymaker with a preference for smoothing interest rates ($\psi > 0$).*

The first necessary condition prevents the policymaker from exploiting “open-mouth” forward guidance to ease policy at the ZLB. The second enables the policymaker to use a change in the current level of the policy rate, r_t , to signal a change in future deposit rates.

The intuition for Proposition 4 is as follows. The discretionary policymaker reoptimizes every period, taking the policy functions of future policymakers as given. When $\psi > 0$, r_{t-1} becomes an endogenous state variable making negative rates a tangible signal of future rates in a time-consistent equilibrium. To be more precise—conditional on the “regime” the reserve rate r_t is in—when maximizing (20) subject to (13), (14), and (19) the first-order conditions of the optimal policy problem can be written as follows:

Regime I: ($r_t > 0$)

$$0 = \psi(1 + \beta)r_t - \psi r_{t-1} - \psi \beta \mathbb{E}_t r_{t+1} + (1 - \psi) \beta \mathbb{E}_t \frac{\partial \pi(r_t, s_{t+1})}{\partial r_t} \\ + (1 - \psi) \left(\mathbb{E}_t \frac{\partial y(r_t, s_{t+1})}{\partial r_t} + \sigma^{-1} \mathbb{E}_t \frac{\partial \pi(r_t, s_{t+1})}{\partial r_t} - \sigma^{-1} \right) (\lambda y_t + \kappa \pi_t), \quad r_{d,t} = r_t.$$

Regime II: ($r_t < 0$)

$$0 = \psi(1 + \beta)r_t - \psi r_{t-1} - \psi \beta \mathbb{E}_t r_{t+1} + (1 - \psi) \beta \mathbb{E}_t \frac{\partial \pi(r_t, s_{t+1})}{\partial r_t} \\ + (1 - \psi) \left(\mathbb{E}_t \frac{\partial y(r_t, s_{t+1})}{\partial r_t} + \sigma^{-1} \mathbb{E}_t \frac{\partial \pi(r_t, s_{t+1})}{\partial r_t} + \phi \right) (\lambda y_t + \kappa \pi_t), \quad r_{d,t} = 0.$$

Regime III: ($r_t = r_{d,t} = 0$),

where $\pi_t = \pi(r_{t-1}, s_t)$, for example, denotes the solution for inflation as a function of the state vector. In any period, the economy can be in three possible regimes. I: The ZLB does not bind, II: The deposit rate ZLB binds and the reserve rate is

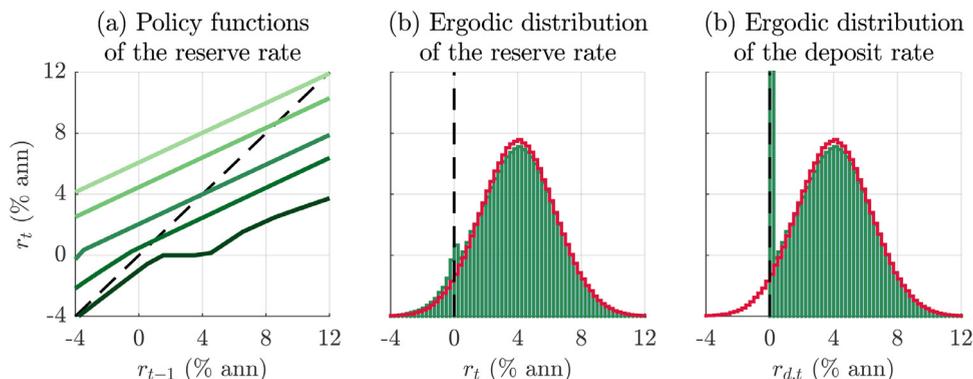


Fig. 2. Optimal policy solution. *Note:* (a) plots policy functions for five different s_t values. The black-dash is the 45-degree line. (b) and (c) plot ergodic distributions generated from simulations of length 10^6 with a burn-in of 10^3 . The filled-green plots the distribution with negative rates, the red line the distribution without a ZLB. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

set negative, or III: The deposit rate ZLB binds and the reserve rate is set to zero. Regime III allows for the possibility that a negative rate policy is feasible but not optimal. For example, we will see that if ψ is sufficiently small or ϕ is sufficiently large, Regime II is never visited and at the ZLB the reserve rate is kept at zero. The first-order condition also illustrates the role of policy smoothing in generating the signalling channel. When $\psi = 0$, it reduces to a static condition: $y_t = -\frac{\kappa}{\lambda} \pi_t$. When $\psi > 0$, the policymaker takes account of the actions of future policymakers and past actions influence current decisions.

2.4. Numerical exercise and comparative statics

The previous section showed negative rates can be optimal when policy is discretionary and delegated to a policymaker with a preference for smoothing policy. This section illustrates their optimal use with a numerical exercise and comparative statics.

Numerical exercise. We solve the model with the Endogenous Grid Method and take parameter values from Nakata and Schmidt (2019).¹² Risk aversion is $\sigma = 0.5$, the discount factor of savers is $\beta = 0.99$, the Phillips curve slope is $\kappa = 0.008$, and the output gap weight in welfare is 7.85×10^{-4} . In addition, we set the consumption share of borrowers to $c = 0.4$ and the natural rate, s_t , follows an AR(1) process with persistence 0.85 and variance 0.0016, approximated using the Tauchen quadrature algorithm with 21 nodes.

We set the reserve-to-deposit ratio to $\alpha = 0.2$, implying $\phi = 0.2$. All else equal, a 25 basis point (bp) gap between the deposit and reserve rate widens the output gap by 5 bps. The smoothing preference, $\psi = 0.029$, is set to maximize welfare in the absence of negative rates.¹³ Since ϕ and ψ are crucial for the strength of the signalling and costly interest margin channel, respectively, we illustrate the comparative statics of changing both below.

Figure 2 (a) plots policy functions $r_t = r(r_{t-1}, s_t)$, and reveals three new insights into optimal discretionary policy with smoothing. First, the policy functions turn negative, illustrating that an optimal policymaker uses negative rates in these conditions. Second, there are “inaction” regions where the policy function is flat. For a large fall in s_t , the policymaker initially drops the reserve rate to zero and only subsequently sets it negative. Third, the policy function slope is steeper to the left of the inaction region, suggesting that once the policymaker passes into negative territory, it will cut the reserve rate more aggressively than if unconstrained by the ZLB. Proposition 6 (below) rationalizes this.

Panels (b) and (c) display the ergodic distributions (in green) for r_t and $r_{d,t}$, respectively. The deposit rate distribution is truncated by the ZLB whereas the reserve rate has some mass below zero. Due to the observed inaction, the r_t distribution is non-symmetric, with more mass at both $r_t = 0$ and $r_t < 0$ relative to the unconstrained distribution (red line). Comparing the distributions with and without negative rate policies, the deposit rate is expected to bind 4.4% and 3.7%, respectively. This increased frequency at the ZLB with negative rates is welfare improving. With negative rates, households would forgo 2.33% of consumption per period to avoid uncertainty, compared to 2.57% without negative rates.

Comparative statics. We make two further assumptions: i) s_t becomes iid; ii) the policymaker disregards the output gap ($\lambda = 0$) and only cares about smoothing policy between periods 2 and 1. This effectively reduces the model to a 2-period problem since $\{\pi_t, y_t\} = \{0, 0\}$ for $t \geq 3$, allowing for closed-form solutions. First, we derive two additional analytical results to complement Proposition 4 in this environment.

¹² Appendix A.6 describes the algorithm to solve the time-consistent optimal policymaker’s problem.

¹³ Appendix A.7 derives the consumption equivalent welfare measure and shows it is hump-shaped and concave in the smoothing parameter, ψ . Thus, policy is optimally delegated to a policymaker with a positive but finite smoothing preference. Appendix A.8 shows the optimal response to a natural rate shock.

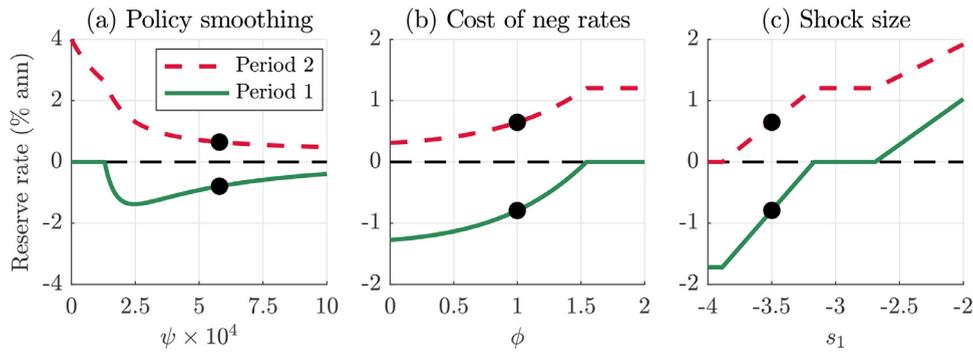


Fig. 3. Optimal policy sensitivity analysis. *Note:* The black-dot refers to the baseline parameterization across the three panels with the weight on policy smoothing scaled down by 50 ($\psi = \psi/50$) and the cost of negative rates scaled up by 5 ($\phi = \phi \times 5$). The period 1 natural rate, s_1 , is set to -3.5 . We rescale these parameters to visually highlight the trade-offs at play. This adjustment is not necessary for the qualitative nature of the results but allows to zoom in on a range of the parameter space that captures the full spectrum of comparative statics results.

Proposition 5. *There exists a threshold, ϕ^* , for the cost of negative rates, ϕ , such that a negative interest rate policy raises both inflation and output if and only if $\phi < \phi^* \equiv \frac{\psi}{\psi + (1-\psi)(\kappa\sigma^{-1})^2} \times \sigma^{-1}(1 + \kappa\sigma^{-1})$.*

Proof. See Appendix A.9. \square

Proposition 5 shows that the threshold on the cost of negative rates is monotonically increasing in the degree of policy smoothing, ψ . For any $\phi < \phi^*$, the marginal cost of lowering the reserve rate in negative territory, ϕ , is lower than the marginal benefit in terms of inflation and output, given by the product of two terms: i) the optimal marginal effect of lowering today’s policy rate on tomorrow’s rate (when $\psi = 0$, signalling is inactive and $\phi^* = 0$); and ii) today’s effect of tomorrow’s marginally lower rate via expected inflation and output. Thus, intuitively, Proposition 5 can also be interpreted as a sufficient condition for negative rates to be welfare improving.

Proposition 6. *For $\phi < \phi^*$, the response of policy in negative territory to offset a marginal change in the natural rate ($dy_1/ds_1 = 0$) is given by $\frac{\partial r_1}{\partial s_1} |_{r_1 < 0} > \frac{\partial r_1}{\partial s_1} |_{r_1 > 0}$.*

Proof. See Appendix A.9. \square

Proposition 6 states that, conditional on rate cuts being effective ($\phi < \phi^*$), the cut in the policy rate needed to generate the same effect on output is larger in negative than in positive territory.¹⁴ Intuitively, away from the ZLB, a marginal cut in the policy rate has an additional direct benefit on output of σ^{-1} via the IS equation. In contrast, in negative territory, the overall effect of a cut is smaller because the only direct effect is the cost $-\phi$.

We conclude with three comparative statics exercises. Fig. 3(a) illustrates the non-monotonic effect of varying the smoothing parameter, ψ , on the optimal period-1 reserve rate, r_1 , when the natural rate is deeply negative. Consistent with Proposition 5, when ψ is low, the benefit of signalling is outweighed by the cost of negative rates and the reserve rate is set to zero. As the smoothing parameter increases, however, negative rates become optimal. With $r_{d,1}$ at zero, lowering r_1 lowers r_2 , raises expected inflation and lowers the real interest rate. With a moderate ψ , the policymaker needs a very negative rate to lower r_2 , but, as ψ increases, signalling becomes more powerful and a smaller decline is sufficient to achieve the same fall in r_2 , resulting in the observed non-monotonicity.

Panel (b) shows r_1 is increasing and convex in the cost parameter, ϕ . From the Phillips curve, inflation is linear in ϕ , which means welfare is quadratic in ϕ . Thus, as ϕ increases, the policymaker rapidly reduces the degree of negative rates it willingly deploys. Finally, Panel (c) varies the natural rate, s_1 . As s_1 falls, the policy rate falls to accommodate it. However, there exists an inaction region where the policy rate is zero. Only for sufficiently large shocks does the policymaker use negative rates. Consistent with Proposition 6, the slope ($\partial r_1/\partial s_1$) is steeper to the left of the inaction region.

3. Quantitative model

The previous section set up a stylized model to qualitatively study the optimality of negative rates. This section develops a richly specified and carefully estimated medium-scale model to quantitatively assess their effectiveness and the relative strength of the costly interest margin and signalling channels. Using a novel decomposition of bank profits and various model modifications, we further elucidate the transmission of negative rates.

¹⁴ The inequality in the proposition holds equally for inflation with $d\pi_1/ds_1 = 0$.

3.1. Set up

The basis of the model is a financial-friction new-Keynesian model as in [Gertler and Karadi \(2011\)](#). In contrast to our stylized model in [Section 2](#), we dispense with borrower households and instead have firms borrowing from banks to finance the rent of capital. We introduce endogenous capital formation, investment adjustment costs, consumption habits, and, instead of studying optimal policy, we endow the central bank with an inertial Taylor-type rule to set the reserve rate. For compactness, rather than specifying the entire model below, we only focus on features that differ markedly from the stylized model.¹⁵

Households. Three changes are made to households. One, only a representative (saver) household exists. Two, preferences exhibit consumption habits, $\tilde{C}_t \equiv C_t - \tilde{h}C_{t-1}$. Three, we introduce a [Smets and Wouters \(2007\)](#) AR(1) risk premium shock, ζ_t , to generate the ZLB scenario.¹⁶ Thus, the household problem is given by

$$V_t = \max_{\{C_t, L_t, D_t\}} \left(\log \tilde{C}_t - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right) + \beta \mathbb{E}_t V_{t+1}, \quad (21)$$

subject to

$$P_t C_t + D_t = P_t W_t L_t + \exp(\zeta_{t-1}) R_{d,t-1} D_{t-1} + \Omega_t. \quad (22)$$

Bankers. We make three changes to the banking sector. One, we assume banker j buys $S_t(j)$ units of firm equity at price Q_t (rather than lending to borrower households). Firm equity pays a stochastic real return, $R_{k,t+1}$. Thus, the banker solves

$$V_{n,t}(j) = \max_{\{S_t(j), A_t(j), D_t(j), N_t(j)\}} \mathbb{E}_t \Lambda_{t,t+1} ((1-\theta)N_{t+1}(j) + \theta V_{n,t+1}(j)), \quad (23)$$

subject to

$$Q_t S_t(j) + A_t(j) = D_t(j) + N_t(j), \quad (24)$$

$$V_{n,t}(j) \geq \lambda Q_t S_t(j), \quad (25)$$

$$A_t(j) = \alpha(x_t) D_t(j), \quad (26)$$

$$N_t(j) = R_{k,t} Q_{t-1} S_{t-1}(j) + (R_{t-1}/\Pi_t) A_{t-1}(j) - (R_{d,t-1}/\Pi_t) D_{t-1}(j). \quad (27)$$

The incentive constraint always binds in our baseline parameterization. Two, in equilibrium, $S_t = K_t$, where $S_t = \int_j S_t(j) dj$ and K_t is aggregate capital. Three, exiting bankers are replaced by workers with initial net worth equal to a fraction ω of total firm equity in the previous period. Hence, the evolution of aggregate net worth is given by

$$N_t = \theta \left(R_{k,t} \Phi_{t-1} - \frac{R_{d,t-1} - \alpha(x_t) R_{t-1}}{(1 - \alpha(x_t)) \Pi_t} (\Phi_{t-1} - 1) \right) N_{t-1} + \omega Q_t K_{t-1}. \quad (28)$$

Capital goods firms. Capital goods firms are new to the model, repair depreciated capital, and produce new capital. Existing capital depreciates at rate δ and is refurbished at unit cost. New capital, $K_{n,t}$, is produced using technology $K_{n,t} = f(I_{n,t}, I_{n,t-1})$, where $I_{n,t}$ is investment in new capital formation. Accordingly, capital goods firms solve $V_{k,t} = \max_{I_{n,t}} (Q_t K_{n,t} - I_{n,t}) + \mathbb{E}_t \Lambda_{t,t+1} V_{k,t+1}$. Production technology comes with quadratic adjustment costs, $f(\cdot) \equiv (1 - (\eta/2))((I_{n,t} + I)/(I_{n,t-1} + I) - 1)^2 I_{n,t}$, where $I = \delta K$ is defined as steady state gross investment given by $I_t = f(I_{n,t}, I_{n,t-1}) + \delta K_{t-1}$. Thus, capital accumulation follows $K_t = K_{t-1} + f(I_{n,t}, I_{n,t-1})$.

Intermediate goods firms. Intermediate goods firms produce and sell intermediate output, $Y_t = K_{t-1}^\gamma L_t^{1-\gamma}$, at price $P_{m,t}$. Their labor demand is $W_t = P_{m,t} (1 - \gamma) Y_t / L_t$ and profits per unit of capital are $P_{m,t} \gamma Y_t / K_{t-1}$. Capital is purchased using external finance. At the start of the period, firms issue S_t units of equity to bankers at price Q_t . In return, bankers receive next period's realized return per unit of capital, $R_{k,t} = \frac{P_{m,t} \gamma Y_t / K_{t-1} + Q_t - \delta}{Q_{t-1}}$.

Other. Retail firms are unchanged except we introduce a cost-push shock by making the elasticity of substitution between goods time-varying. The aggregate resource constraint is $Y_t = C_t + I_t + G$, where G is exogenous government spending, set at $G/Y = 0.2$.

Monetary policy. The central bank sets the reserve rate, which when unconstrained follows a Taylor-type inertial policy rule given by

¹⁵ Appendix B.1 derives the financial sector equations and Appendix B.2 lists the full set of equilibrium conditions.

¹⁶ While both risk premium and discount factor shocks are common in the literature to induce a demand-driven ZLB scenario, the risk premium shock is preferable in a model with endogenous capital formation as it induces a positive co-movement of consumption and investment.

Table 1
Structural parameter values.

Block A. Standard parameters					
β	Discount factor	0.990	\hbar	Habit parameter	0.815
φ	Inverse Frisch elasticity	0.276	γ	Capital share	0.330
δ	Depreciation rate	0.025	ϵ	Elasticity of substitution	4.167
ι	Probability of fixed prices	0.900	θ	Survival probability of bankers	0.975
ϕ_π	Policy rule inflation response	1.500	ϕ_x	Policy rule output response	0.125
ρ_ζ	Persistence of risk premium shocks	0.800	ρ_ϵ	Persistence of cost-push shocks	0.800
Block B. Steady state calibrated parameters					
χ	Utility weight on labor	3.411	α	Reserve-to-deposit ratio	0.200
λ	Fraction of divertible assets	0.411	ω	Transfer to new bankers	0.001
Block C. Estimated parameters					
η	Inverse investment elasticity	1.617	ρ	Policy rule inertia	0.856
σ_ζ	S.d. of risk premium innovations	0.002	σ_ϵ	S.d. of cost-push innovations	0.033

$$R_{T,t} = \left(R \Pi_t^{\phi_\pi} (X_t/X)^{\phi_x} \right)^{1-\rho} R_{T,t-1}^\rho \exp(\varepsilon_{m,t}), \quad (29)$$

where $R_{T,t}$ is the rate implied by the policy rule, $X_t = 1/\mathcal{M}_t$ is the mark-up and—in the absence of sticky wages—a good proxy of the output gap, and $\varepsilon_{m,t}$ is an iid shock. The degree of inertia is given by ρ and the inertial term is the lagged reserve rate. The policy rule is not inertial when the policy rate is bounded at zero.¹⁷ In what follows, we compare three policy scenarios equivalent to the regimes in Section 2 and Ulate (2021b):

- I. The unconstrained (“No ZLB”) scenario, in which both the reserve and deposit rate are unconstrained and can turn negative, is given by $R_t = R_{d,t} = R_{T,t}$.
- II. The deposit rate-only ZLB (“ZLB: R_d only”) scenario—our baseline to study the effects of negative interest rates—in which the deposit rate is bounded by zero, but the reserve rate can turn negative, is given by $R_t = R_{T,t}$ and $R_{d,t} = \max\{1, R_{T,t}\}$.
- III. The standard ZLB scenario (“ZLB: R_d & R ”), in which both the reserve and deposit rate are constrained by zero, is given by $R_t = R_{d,t} = \max\{1, R_{T,t}\}$.

3.2. Parameterization

Table 1 summarizes our baseline parameter values, distinguishing between those that are standard (Block A), calibrated (B), and estimated (C). The model is set up at quarterly frequency. We solve the model using the Guerrieri and Iacoviello (2015) toolkit—a piecewise first-order perturbation approach—to account for the occasionally binding ZLB. The sensitivity and robustness of our main results is extensively documented in Section 3.4.

Block A parameters are assigned standard values from the literature. Consumption habits are $\hbar = 0.815$ and the inverse Frisch elasticity is set to $\varphi = 0.276$, a relatively low value we pick as a stand-in for nominal wage rigidities in the model. The elasticity of substitution is $\epsilon = 4.167$, in line with a steady state mark up of around 30%. The Calvo parameter is $\iota = 0.9$, implying prices adjust on average every 10 quarters. This relatively high degree of price stickiness compromises between micro evidence on the frequency of price changes and macro evidence for a flat Phillips curve. Harding et al. (2022) show this trade-off results from using a CES rather than a Kimball aggregator as in Smets and Wouters (2007). Finally, bankers’ survival probability is $\theta = 0.975$, implying an average tenure of 10 years.

Block B parameters are calibrated to match steady state values with long-run averages in the data. The utility weight on labor is $\chi = 3.411$ to normalize steady state labor supply to 1/3. Based on US financial balance sheet and interest rate data (see Appendix B.3), the two financial sector parameters, $\lambda = 0.411$ and $\omega = 0.001$, are calibrated to match a steady state leverage ratio of 4 and a credit spread, $400(\frac{R_k}{R_d} - 1)$, of 1%. The reserve ratio is $\alpha = 0.2$, in line with the post-financial crisis average for both the euro area and US.

Block C parameters are estimated using the simulated method of moments.¹⁸ We target ten US time-series moments and five yield curve moments to estimate four parameters $\Theta = \{\eta, \rho, \sigma_\zeta, \sigma_\epsilon\}$, the inverse investment elasticity parameter, the policy rule inertia coefficient, and the standard deviations of risk premium and cost-push innovations. We estimate the inverse investment elasticity ($\eta = 1.617$) as its value is not well-informed by the literature and it determines the financial accelerator and the interest margin channel. We also estimate policy inertia ($\rho = 0.856$) because it is key for the signalling

¹⁷ Other studies have considered policy rules in which the inertial term is on the Taylor-rule implied rate, $R_{T,t}$, rather than the actual policy rate, R_t . To the extent that such a rule is credible ($R_{T,t}$ is a latent variable), it also increases the effectiveness of monetary policy in a standard ZLB scenario. Thus, this latter formulation is more akin to explicit forward guidance, whereas in our specification inertia is a structural feature of monetary policy that is orthogonal to whether the economy is at the ZLB or not.

¹⁸ Appendix B.3 documents the data sources and transformations, estimation methodology, and results.

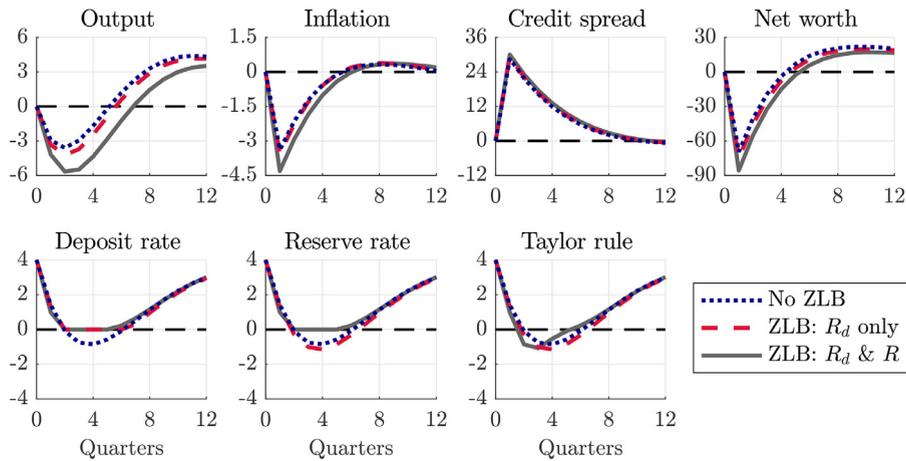


Fig. 4. Risk premium shock with inertia in the policy rule. *Note:* $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a risk premium shock that brings the economy to the ZLB for 4 quarters. All interest rates displayed are in annualized percent. All other variables are in $100 \times \log$ -deviation from steady state. Inflation is annualized. Log-deviations are a good approximation of percent deviations when the deviation is small. For net worth, the -80 log-deviation, however, translates to a more modest 55 percent drop.

channel. The estimation suggests a significant amount of policy smoothing. Appendix B.4 shows this aligns with the existing literature and suggestive negative rates evidence from Sweden.

3.3. Main results

This section presents our main results on the effectiveness of negative interest rates and illustrates the transmission mechanism using a novel decomposition of bank profits.

Effectiveness of negative rates. Our baseline crisis experiment is a risk premium shock that drives the economy to the ZLB for 4 quarters.¹⁹ Figure 4 shows the impulse responses for our policy scenarios. In all three, households cut consumption, bank net worth declines, and investment demand falls. Like the natural rate shock in Section 2, the risk premium shock shifts aggregate demand, depressing both output and inflation.²⁰

The smaller fall in output and inflation in the deposit rate-only ZLB scenario (II, red-dash) as compared to the standard ZLB scenario with both rates constrained (III, black-solid) indicates that negative rates—in our baseline parameterization—are expansionary. Unsurprisingly, the unconstrained scenario (I, blue-dot), where deposit rates can turn negative to offset the fall in aggregate demand, results in the smallest fall in output. When policy is constrained by the ZLB, the fall in output is largest. However, when the central bank can decrease the policy rate into negative territory—despite the deposit rate being bounded by zero—it is able to extend the ZLB duration by 1 quarter and lower the post-ZLB deposit rate path providing additional stimulus.²¹ Thus, a negative rate policy is expansionary even when the current deposit rate, which is relevant for households' intertemporal substitution decision, is constrained. In terms of welfare, the steady state consumption loss for the representative household to be indifferent between the unconstrained and the ZLB scenario is 0.185%, but only 0.051% with negative rates. Negative rates are expansionary (in terms of output and inflation) and welfare improving.

To isolate the quantitative response to negative interest rates in crisis times, we introduce an extra -25 bp iid monetary policy shock in period 2 when the economy is at the ZLB. Figure 5 reports the impulse responses to the policy shock stripping out the effect of the underlying risk premium shock. When both the deposit and reserve rates are constrained by zero, a shock to the Taylor-rule implied rate has no effect on equilibrium outcomes (III, black-solid). Figure 5(a) shows this is not the case when the reserve rate can turn negative (II, red-dash). The policy shock becomes expansionary with peak output and inflation responses of 51% and 67% of an unconstrained shock (I, blue-dot), respectively. The deposit rate path is key to understanding this. While being constrained from period 2 (when the shock hits) until period 7, it then drops and stays persistently lower thereafter.

To explicitly identify the role of the signalling and costly interest margin channels, Fig. 5(b) removes policy inertia and re-runs the previous experiment. Crucially, under the deposit rate-only ZLB scenario (II, red-dash), negative rates are now contractionary rather than expansionary, resulting in a fall in output and inflation. There are two reasons for this. First, by

¹⁹ The size of the shock is calibrated to deliver 4 periods at the ZLB in the standard ZLB scenario (III). In using a single large shock, we are trading off realism for expositional clarity.

²⁰ For comparison, Appendix A.3 replicates Fig. 4 with the stylized model closed with the Taylor rule.

²¹ The effectiveness of negative rates largely appears in the output response rather than inflation, reflecting the flatness of the Phillips curve. Credit spreads are mostly driven by the exogenous risk premium shock, which is why the responses are very similar across scenarios. The lower-for-longer effect in interest rates mirrors the empirical evidence for Sweden presented in Appendix B.4.

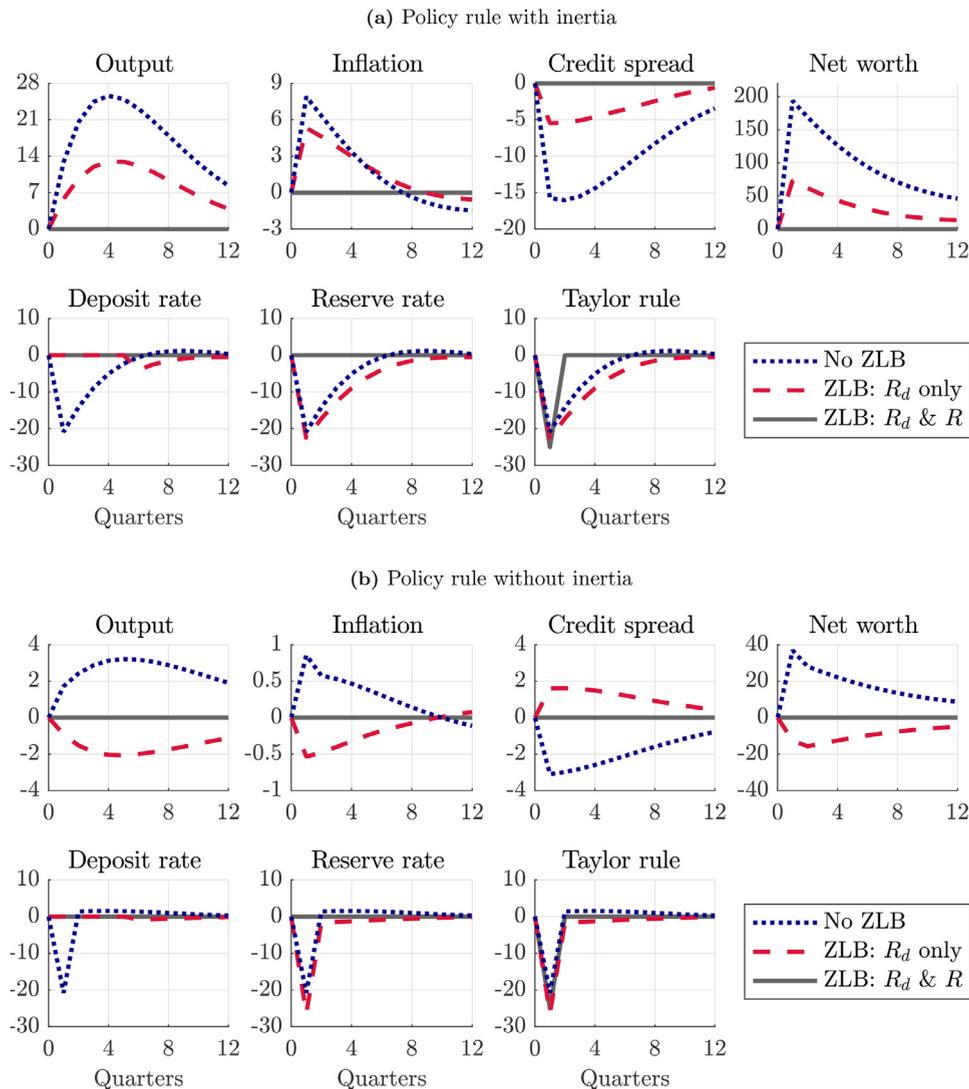


Fig. 5. Monetary policy shock in negative territory. *Note:* (a) $\alpha = 0.2$ and $\rho = 0.85$, (b) $\alpha = 0.2$ and $\rho = 0$. Impulse responses to a -25 bp iid monetary policy shock at the ZLB. All interest rates displayed are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

setting $\rho = 0$ we have switched off the signalling channel and so the fall in the reserve rate has no effect on the path of the deposit rate. Second, the costly interest margin channel results in bank net worth falling, tightening banks' incentive constraints, and causing credit spreads to rise. With the deposit rate constrained, a rise in credit spreads implies higher lending rates for firms which depresses investment demand.

Figure 6 combines these results, decomposing the baseline deposit rate-only ZLB scenario into the signalling (blue-dot) and costly interest margin channel (red-dash). The 13bp peak output response is decomposed into a 16bp expansionary signalling channel and a -3 bp contractionary interest margin channel contribution. The 5bp peak inflation response is almost completely explained by the signalling channel.²²

Decomposition of bank profits. Bank net worth is key for the transmission of negative rates. We explore this with a novel decomposition of bank profits, prof_t , defined as the gross growth rate of a nominal net worth, conditional on not exiting.

²² Appendix B.5 replicates this decomposition for $\rho = 0$, showing that the signalling channel is indeed inactive without policy inertia. This is in a model with many endogenous state variables, which provides further evidence that "not any endogenous state variable will do" in order to generate a signalling channel.

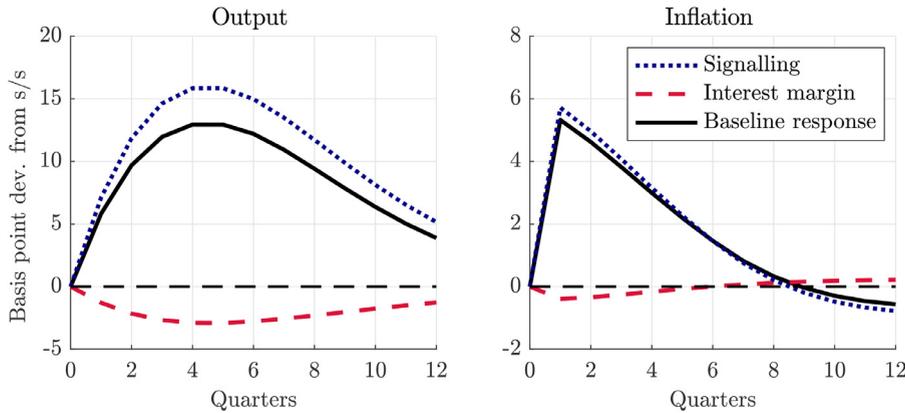


Fig. 6. Contribution of signalling and interest margin channels. *Note:* Impulse responses to a -25 bp iid monetary policy shock at the ZLB. Inflation is annualized. We linearly decompose the baseline response into “Signalling”— $\alpha = 0$ and $\rho = 0.85$, i.e. no costly interest margin channel—and “Interest margin”—difference between the baseline and “Signalling”.

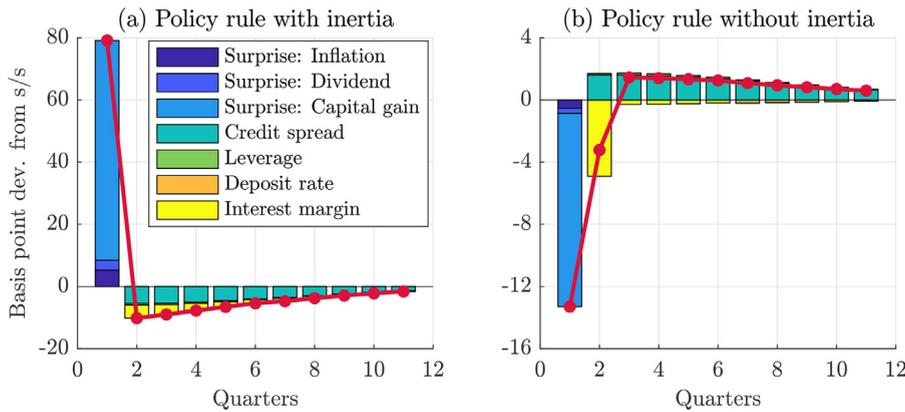


Fig. 7. Decomposition of bank profits. *Note:* (a) $\alpha = 0.2$ and $\rho = 0.85$, (b) $\alpha = 0.2$ and $\rho = 0$. The red-dot line plots the impulse response of bank profits to a -25 bp iid monetary policy shock at the ZLB. Stacked bars decompose the impulse response for every period. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Its log-linear form can be decomposed into 3 windfall (or “surprise”) and 4 predetermined terms given by

$$\begin{aligned} \widehat{\text{prof}}_t = & \underbrace{\frac{R_k \Phi}{\text{prof}} (\hat{\pi}_t - \mathbb{E}_{t-1} \hat{\pi}_t)}_{\text{Surprise: Inflation}} + \underbrace{\frac{\text{mpk} \Phi}{\text{prof}} (\widehat{\text{mpk}}_t - \mathbb{E}_{t-1} \widehat{\text{mpk}}_t)}_{\text{Surprise: Dividend}} + \underbrace{\frac{\Phi}{\text{prof}} (\hat{q}_t - \mathbb{E}_{t-1} \hat{q}_t)}_{\text{Surprise: Capital gain}} \\ & + \underbrace{\frac{cs \Phi}{\text{prof}} \hat{c}s_{t-1}}_{\text{Credit spread}} + \underbrace{\frac{cs \Phi}{\text{prof}} \hat{\phi}_{t-1}}_{\text{Leverage}} + \underbrace{\frac{R_d}{\text{prof}} \hat{r}_{d,t-1}}_{\text{Deposit rate}} - \underbrace{\frac{\alpha}{1-\alpha} \frac{R_d (\Phi - 1)}{\text{prof}} (\hat{r}_{d,t-1} - \hat{r}_{t-1})}_{\text{Interest margin channel}}, \end{aligned} \quad (30)$$

where hats denote log-deviations from steady state, variables without subscripts are steady states, $cs_t \equiv \mathbb{E}_t \Pi_{t+1} R_{k,t+1} - R_{d,t}$ is the nominal credit spread, and $\text{mpk}_t \equiv P_{m,t} \gamma Y_t / K_{t-1}$ is the marginal product of capital.²³ In general, the return on any asset can be split into a dividend payout and a capital gain. Accordingly, for banks’ assets, we term the surprise change in the marginal product of capital the “dividend” and the leveraged surprise change in the asset price the “capital gain”. The third windfall term is surprise inflation since we decompose nominal profits. The four predetermined terms are the evolution of i) the credit spread, ii) leverage, iii) the risk-free rate, and iv) the partial equilibrium effect of negative rates on interest margins (i.e. the costly interest margin channel).

Figure 7 plots the decomposition of bank profits in response to a -25 bp iid monetary policy shock at the ZLB. In Panel (a), with the signalling channel switched on, on impact, profits sharply increase due, almost entirely, to the surprise capital gain—i.e., a revaluation of the banks’ assets in response to the monetary easing. With the signalling channel op-

²³ Appendix B.6 derives the decomposition, both for the baseline model and an extended version with firm equity and loan finance introduced in Section 3.4 below.

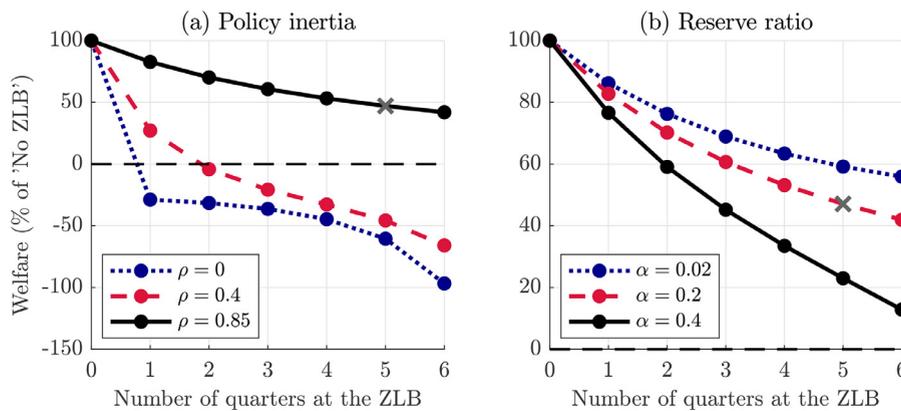


Fig. 8. Policy inertia, reserve ratio, and ZLB duration. *Note:* The x-axis scales with the size of the initial risk premium shock. The y-axis reports welfare in consumption equivalent units in response to a -25 bp iid monetary policy shock for the corresponding ZLB duration relative to the welfare effect of an unconstrained monetary policy shock. The \times denotes the baseline experiment.

erative, a lowering of the reserve rate into negative territory depresses the future expected deposit rate path. Households bring forward consumption causing aggregate production and the price of capital to increase instantaneously, driving up bank profits. From period 2 on, tighter credit spreads (the revaluation of bank assets raises net worth, slackens the banks' incentive compatibility constraint, contracting credit spreads) and the costly interest margin channel reduce profits to bring net worth back to steady state. This decomposition of how negative rates affect different parts of banks' balance sheets is consistent with empirical evidence in, for example, [Altavilla et al. \(2018\)](#).

In Panel (b), with signalling switched off, bank profits fall in response to the negative rate shock. Without policy inertia, negative rates come without an expansionary aggregate demand effect but solely reduce net worth via the costly interest margin channel. Lower net worth implies rising credit spreads that affect the profit decomposition in two ways. One, on impact, higher expected credit spreads depress firms' investment demand and induce capital losses. Two, from period 2 on, higher realized credit spreads generate additional profits that bring net worth back to steady state. Compared to Panel (a), all but one partial equilibrium term switch sign—the only consistently contractionary term is the interest margin channel which reduces bank profits irrespective of ρ .

3.4. Sensitivity, forward guidance puzzle, and equity vs loans

The results above show negative rates can be both expansionary and contractionary. This section investigates more thoroughly the factors that determine their effectiveness.

Sensitivity analysis. [Figure 8](#) plots the welfare gain from a -25 bp iid monetary policy shock at the ZLB for different values of policy inertia, ρ , banks' reserve-to-deposit ratio, α , and the ZLB duration. The x-axis plots the number of quarters the ZLB is expected to bind when the monetary policy shock is introduced. It scales with the size of the risk premium shock and proxies the severity of the crisis. The y-axis is the welfare gain from the monetary policy shocks as a percentage of the welfare gain from an unconstrained monetary policy shock. When the ZLB binds for zero quarters (the model is unconstrained) the value reported is 100%. This normalization ensures we strip out the effect of parameter changes on the effectiveness of "conventional" monetary policy in the model.²⁴

The figure shows negative rates are less effective if the ZLB is expected to bind for longer. If the deposit rate is constrained for a long period, the effect of reducing the reserve rate today only marginally lowers the future expected deposit rate path. By increasing the crisis severity (from 5 to 6 quarters at the ZLB), the welfare gain of negative rates drops from 47% to 42% of an unconstrained policy easing when $\rho = 0.85$. Panel (a) also shows negative rates are less effective for a central bank with a lower degree of policy inertia. For example, with an inertia of $\rho = 0.4$, negative rates are only welfare improving in a 1-period ZLB scenario. Panel (b) shows that when banks hold a larger reserve ratio the effectiveness of negative rates diminishes as the costly interest margin channel is stronger. For example, doubling reserve holdings to an extreme value of $\alpha = 0.4$ results in negative rates being only marginally welfare improving in a 6-period ZLB scenario. In this case, the signalling channel only just dominates the interest margin channel.

Overall though, our main result that negative rates are an effective policy tool is fairly robust. Even with $\rho = 0.8$ (the lowest degree of policy inertia documented in [Appendix B.4](#)), $\alpha = 0.27$ (the largest reserve ratio documented in [Appendix B.3](#)), and a severe crisis with 6 periods at the ZLB, a negative rate policy is still welfare improving in our model.

²⁴ [Figure 5](#) illustrates the need for this. The peak output response to an unconstrained monetary policy shock (1, blue-dot) is 26 bps when $\rho = 0.85$ but only 3 bps when $\rho = 0$. To remove this effect in our sensitivity analysis, we report results relative to unconstrained policy with the same parameter values.

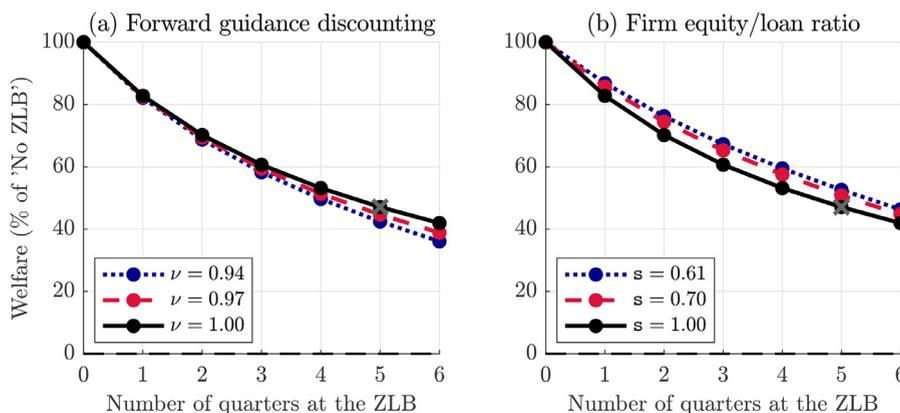


Fig. 9. Forward guidance discounting, firm equity/loan ratio, and ZLB duration. *Note:* The x-axis scales with the size of the initial risk premium shock. The y-axis reports welfare in consumption equivalent units in response to a -25 bp iid monetary policy shock for the corresponding ZLB duration relative to the welfare effect of an unconstrained monetary policy shock. The \times denotes the baseline experiment.

Forward guidance puzzle. One criticism of the new-Keynesian paradigm is that equilibrium outcomes are too sensitive to future interest rate changes (Del Negro et al., 2012). Following McKay et al. (2017), we resolve the forward guidance puzzle with additional discounting, $\nu \leq 1$, in the consumption Euler equation, dampening households' sensitivity to expected future interest rate changes. The augmented Euler equation can be written as $1 = \mathbb{E}_t \beta \frac{\mu_{t+1}^\nu}{\mu_t^{\nu-1} \mu_t} \frac{\exp(\zeta_t) R_{d,t}}{\Pi_{t+1}}$, where μ_t is the marginal utility of consumption and the additional discounting is introduced in such a way so as not to distort the steady state. A first-order approximation yields the same equation as in McKay et al. (2017).

Figure 9 (a) shows the signalling channel is both qualitatively and quantitatively robust to the introduction of discounting. The differences between our baseline model ($\nu = 1$), the value in McKay et al. (2017) ($\nu = .97$), and a more extreme version ($\nu = .94$) are small. For example, in our crisis scenario, dampening forward guidance only reduces the relative welfare gain from negative rates from 47% to 45% and 42%, respectively.

Equity vs. loan finance. In common with Gertler and Karadi (2011), banks provide external finance to firms by purchasing their equity in our model. This gives rise to a stochastic return on bank assets and windfall dividends and capital gains in the profit decomposition (Fig. 7). In reality, a large share of firms' external finance is loans. We therefore augment our model by making a fraction of banks' assets loans that earn a predetermined return. This reduces the role of capital gains in the transmission of negative rates but it does not imply negative rates become unattractive. Figure 9(b) plots results for two equity-to-loan ratios, $s = .61$ (euro area data) and $s = .70$ (US), respectively, and shows our results regarding the effectiveness of negative rates remain robust. If anything, negative rates are relatively more effective than in our baseline parameterization ($s = 1$).²⁵

In summary, since we report the effectiveness of negative rates normalized relative to the effectiveness of unconstrained policy, modifications of the model, such as ν or s , only affect our main results in so far as they have differential effects on monetary policy in negative territory and in normal times. In contrast, changes to ρ , α , and the ZLB duration do affect our conclusions regarding the effectiveness of negative rates as they directly determine the strength of the signalling and costly interest margin channels.

Further robustness. We document four more robustness exercises in Appendix B.7. First is the Frisch labor supply elasticity. In the absence of wage rigidities, our baseline parameterization contains a relatively high labor supply elasticity. Lowering it to unity—a common compromise between micro and macro estimates as in Hazell et al. (2022)—decreases output and inflation responses to monetary policy but does little to alter the relative effectiveness of a negative rates. Second is the Phillips curve slope. Targeting an empirically realistic unemployment-inflation trade-off of 0.0062 as in Hazell et al. (2022), we simultaneously change the Frisch elasticity and Calvo parameter to generate a steeper Phillips curve slope of 0.023—the upper bound in Harding et al. (2022). This impacts the co-movement of output and inflation but not the relative effectiveness of negative rates.

Third is the investment elasticity since it largely determines the financial accelerator. In our baseline parameterization, net worth falls 194 bp on impact in response to an unconstrained 25 bp monetary policy shock, a little below the 210 bp response estimated by Jaroćinski and Karadi (2020), suggesting our estimated elasticity is marginally too high.²⁶ Decreasing the elasticity weakens the impact but increases the persistence of responses to monetary policy. In terms of bank profitability, it increases windfall capital gains (as asset prices are more responsive) but lowers windfall dividends (as investment is

²⁵ Appendix B.6 derives the augmented model and reproduces Fig. 7-type profit decompositions for $s = 0.61$ and 0.70. In the limit, with only loans, windfall capital gains (and dividends) are zero.

²⁶ As we do not target this moment in our estimation though, we take the close fit of model-implied and empirical impulse responses as supportive external validation of our baseline parameterization.

less responsive) to negative rates. Fourth, we augment the model with nominal wage rigidity. This allows a lower Frisch elasticity while preserving the baseline effects of monetary policy.

4. Conclusion

While QE has become a relatively standard policy tool, negative rate policies remain controversial and less well understood, adopted only by a few central banks. We highlight the signalling channel of negative interest rates and show it can dominate the costly interest margin channel—exemplifying the importance of general equilibrium effects and cautioning against partial equilibrium policy evaluation. Many commercial banks have criticized the contractionary effects of negative rates on their interest margins and profits. However, we show signalling has positive general equilibrium effects on banks' asset values and balance sheet health that are not easily attributable to negative rates.

Since our quantitative results rely on a non-optimized inertial policy rule, we also study optimal policy. Abstracting from monopolistic competition among banks, we first prove negative rates are redundant under the extreme assumption of full commitment. However, under more realistic conditions in which policymakers cannot fully commit but have a preference for policy smoothing, we show negative rates can be welfare improving.

Data availability

Data will be made available on request.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:[10.1016/j.jmoneco.2023.05.011](https://doi.org/10.1016/j.jmoneco.2023.05.011).

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