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Monetary Policy & Anchored Expectations An Endogenous Gain Learning Model*

Laura Gáti

[a] *European Central Bank, Directorate General Research, Sonnemannstraße 20, Frankfurt am Main, 60314, HE, Germany*

Abstract

Monetary policy is analyzed in a model with a potential unanchoring of inflation expectations. The degree of unanchoring is given by how sensitively the public's long-run inflation expectations respond to inflation surprises. I find that optimal policy moves the interest rate aggressively when expectations unanchor, allowing the central bank to accommodate inflation fluctuations when expectations are well-anchored. Furthermore, I estimate the model-implied unanchoring process. The data suggest that unanchoring is nonlinear and asymmetric: expectations respond more sensitively to large or downside surprises than to smaller or upside ones.

Keywords: anchored expectations, behavioral macro, monetary policy
E52, E71, D84

1. Introduction

There is broad consensus among policymakers that anchoring inflation expectations is central to the modern conduct of monetary policy. Policymakers think of expectations as anchored when long-run expectations do not fluctuate systematically with short-run inflation surprises. If expectations were to unanchor, policymakers fear this would result in an adverse cycle of self-enforcing movements in expectations.

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Email address: `laura_veronika.gati@ecb.europa.eu` (Laura Gáti)

This paper studies how a concern to anchor inflation expectations affects the conduct of monetary policy. I augment a New Keynesian model with inflation expectations that can unanchor to varying degrees. I use survey data on inflation expectations to discipline how unanchoring happens in the model, and solve the Ramsey problem of optimal monetary policy. The main contribution of the paper is to provide analytical and numerical prescriptions for monetary policy when the degree of expectations anchoring may vary.

To capture unanchoring, I build on new work by Carvalho et al. (2023), who model the anchoring of expectations as a discrete sensitivity to surprises. In their model, agents choose whether to revise their estimate of long-run inflation weakly or strongly in response to past expectations errors, yielding a well-anchored and an unanchored expectations regime. In order to be able to study optimal monetary policy analytically, I extend this work by considering a smooth, continuous sensitivity. This allows me to take derivatives of all equations, while maintaining the property that agents choose what weight to put on past expectations errors based on recent inflation surprises.

As an empirical contribution, I employ a simulated method of moments strategy (Duffie and Singleton, 1990, Lee and Ingram, 1991, Smith, 1993) on Survey of Professional Forecasters (SPF) data to estimate how expectations unanchor. I find that unanchoring in the data is nonlinear and asymmetric. On the one hand, larger inflation surprises upset the public's view on long-run inflation much more than smaller ones. On the other hand, I find that inflation surprises on the downside unanchor expectations more than same-sized surprises on the upside.

I then turn to the main contribution of the paper, the question of how to conduct policy when expectations can unanchor. First, I present an analytical characterization of the Ramsey problem of the monetary authority. This prescribes that the central bank should smooth out the effects of shocks over time.

Second, I solve the optimality conditions of the Ramsey problem numerically. Because the anchoring function renders the model nonlinear, I rely on global methods to obtain the optimal interest-rate policy function. The key takeaway is that the optimal interest-rate setting is time-varying. When expectations unanchor, the monetary authority responds with large movements in the interest rate. When expectations are well-anchored, by contrast, the central bank accommodates fluctuations in inflation. This state-dependent behavior allows the central bank to anchor expectations in volatile times, but avoid inflicting volatility in stable times.

The paper is structured as follows. Section 2 introduces the model. Section 3 describes how anchoring works in the model. Section 4 estimates the anchoring process. Section 5 presents the optimal Ramsey policy. Section 6 concludes.

1.1. Related literature

The paper is related to two strands of literature. The model is a behavioral version of the standard New Keynesian (NK) model. Monetary policy in the rational expectations (RE) version of this model has been studied extensively, for example in Woodford (2003).

The behavioral part of the model is allowing the public's expectation of long-run inflation to fluctuate based on the history of observed inflation in the adaptive learning tradition advocated by Evans and Honkapohja (2001). This literature assumes that agents use a forecasting rule to form expectations and that they update it in every period using observed data.

There are three main reasons for why adaptive learning is a suitable framework to study anchoring. First, many studies document the ability of adaptive learning models to match empirical properties of both expectations and macro aggregates. These models imply that forecast errors are correlated with forecast revisions (Coibion and Gorodnichenko, 2015), and that, in response to shocks, expectations initially underreact, and then overshoot (Angeletos et al., 2021). They match the persistence of inflation without recourse to backward-looking elements in the Phillips curve (Milani, 2007), and lead to persistent and hump-shaped responses to iid shocks even in the simple RBC model (Eusepi and Preston, 2011).

Secondly, there is strong empirical backing for state-dependent expectations. Milani (2014) documents that state-dependence in expectations can generate endogenous time-varying volatility in models without any time-variation in the exogenous processes. Carvalho et al. (2023) show that state-dependent sensitivity of inflation expectations provides great out-of-sample fit to both inflation and inflation expectations.

Thirdly, an extensive experimental literature in the spirit of Anufriev and Hommes (2012) demonstrates that simple, state-dependent forecasting rules provide the best fit among competing models to how individuals form expectations in controlled lab settings.

Within the adaptive learning literature, the paper touches base with two sets of papers. First, Molnár and Santoro (2014) and Mele et al. (2019) show that adaptive learning in general introduces an intertemporal tradeoff to monetary policy. Erceg and Levin (2003) and King and Lu (2021) also investigate monetary policy when the public learns from observed data and the policymaker seeks to influence the learning process. Orphanides and Williams (2004) and Eusepi et al. (2020) look at the optimal responsiveness of Taylor rules to inflation under different specifications for the public's learning. My paper is distinct from these in that I zoom in on the interaction of monetary policy and state-dependent anchoring.

Second, Marcet and Nicolini (2003), Cho and Kasa (2015), Kostyshyna (2012), Milani (2014) and Carvalho et al. (2023) study state-dependent sensitivities of expectations to surprises. My paper is intimately connected to Carvalho et al. (2023), who propose endogenous sensitivity as a model of anchoring. While their focus is validating the empirical performance of their model, I modify their framework to provide what I think is a novel contribution: the study of monetary policy for varying degrees of unanchoring.

2. The model

Apart from expectation formation, the model is a standard New Keynesian (NK) model with nominal frictions à la Calvo (1983). Since the model follows Preston (2005), I proceed directly to the aggregate laws of motion and refer the reader to Preston (2005) for details.[1] The IS and Phillips curves take the following form:

$$x_t = \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_T - \pi_{T+1}) + \sigma r_T^n\big), \tag{1}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big). \tag{2}$$

Here x_t, π_t and i_t are the log-deviations of the output gap, inflation and the nominal interest rate from their steady state values, and σ is the intertemporal elasticity of substitution. The variables r_t^n and u_t represent a natural-rate shock and a cost-push shock.

To simplify notation, I gather the exogenous state variables in the vector s_t and observables in the vector z_t as

$$s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \qquad \text{and} \qquad z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix}, \tag{3}$$

where $\bar{i}_t$ is a shock to the interest rate that only shows up in the model for particular specifications of monetary policy. This allows me to denote long-horizon expectations by

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} \qquad \text{and} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} z_{T+1}. \tag{4}$$

As detailed in Appendix A, one can use this notation to reformulate the laws of motion of jump variables and a given monetary policy rule compactly as

$$z_t = A_a f_{a,t} + A_b f_{b,t} + A_s s_t, \tag{5}$$

where the matrices $A_i,\ i = \{a, b, s\}$ gather coefficients and are given in Appendix A. Assuming that exogenous variables evolve according to independent AR(1) processes, I write the state transition matrix equation as

$$s_t = h s_{t-1} + \epsilon_t \qquad \text{with} \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma), \tag{6}$$

[1] There are alternatives to Preston (2005)'s long-horizon learning approach in the adaptive learning literature. One example is the Euler-equation approach (Bullard and Mitra, 2002). Appendix S shows that the conclusions of the model remain unchanged under Euler-equation learning.

where h gathers the autoregressive coefficients ρ_j, ϵ_t the Gaussian innovations ε_t^j, and η the standard deviations σ_t^j, for $j = \{r, i, u\}$. $\Sigma = \eta\eta'$ is the variance-covariance matrix of disturbances.[2]

3. The unanchoring of inflation expectations

The informational assumption of the model is that agents do not know the equilibrium mapping between states and jumps, as is standard in the adaptive learning literature (Evans and Honkapohja, 2001). Therefore, instead of forming rational expectations forecasts, agents postulate a forecasting relationship and refine it in light of incoming data.

3.1. Perceived law of motion

I assume agents consider a forecasting model of the form

$$\hat{\mathbb{E}}_t z_{t+1} = a_{t-1} + b_{t-1} s_t, \tag{7}$$

where a and b are estimated coefficients of dimensions 3×1 and 3×3 respectively. This perceived law of motion (PLM) reflects the assumption that agents forecast jumps using a linear function of current states and a constant, with last period's estimated coefficients. Note that a can be interpreted as long-run expectations of the observables z. Since inflation is the first element of z, the first element of a corresponds to long-run inflation expectations. This object, which I will denote by $\bar{\pi}$, will be the main focus of the paper.

Summarizing the estimated coefficients as $\phi_{t-1} \equiv \begin{bmatrix} a_{t-1} & b_{t-1} \end{bmatrix}$, here 3×4, I can rewrite Equation (7) as

$$\hat{\mathbb{E}}_t z_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix}. \tag{8}$$

I also assume that

$$\hat{\mathbb{E}}_t \phi_{t+k} = \phi_t \quad \forall\, k \geq 0. \tag{9}$$

This assumption, known as anticipated utility (Kreps, 1998), is standard in the learning literature. Since the states s_t are exogenous, I assume that agents know Equation (6), the equation governing the evolution of s_t.[3] Then, the PLM together with anticipated utility implies that k-period-ahead forecasts in the beginning of period t are constructed as

$$\hat{\mathbb{E}}_t z_{t+k} = a_{t-1} + b_{t-1} h^{k-1} s_t \quad \forall k \geq 1. \tag{10}$$

[2] Appendix A writes out these expressions in full.

[3] This is another common simplifying assumption in adaptive learning. Relaxing it only affects model dynamics by muting the model's responses to shocks until agents have learned the evolution of state variables, which happens after the first few observations.

The timing assumptions of the model are as follows. In the beginning of period t, the current state s_t is realized. Agents then form expectations according to (7) using last period's estimate ϕ_{t-1} and the current state s_t. Given exogenous states and expectations, today's jump vector z_t is realized. This leads to the most recent forecast error

$$f_{t|t-1} \equiv z_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix}, \tag{11}$$

which agents use to update their forecasting rule.[4] The estimate is updated according to the following recursive least-squares algorithm:

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} f'_{t|t-1} \right)', \tag{12}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right). \tag{13}$$

R_t is the 4×4 variance-covariance matrix of the regressors and k_t is the gain.

3.2. Endogenous gain as a metric of unanchoring

It is common practice to specify the gain either as a constant, $\bar{g}$, or decreasing with time, so that $k_t = t^{-1}$, where t indexes time. As Carvalho et al. (2023) point out, one can think of the gain as a formal notion of the degree of unanchoring, because it captures the sensitivity of expectations to surprises. To have time-varying degrees of unanchoring, then, I follow Carvalho et al. (2023) in allowing the gain to fluctuate in response to short-run forecast errors. I assume the gain evolves as

$$k_t = \mathbf{g}(f_{t|t-1}), \tag{14}$$

where $\mathbf{g}(\cdot)$ is a smooth, continuous function that I refer to as the anchoring function. Apart from smoothness and continuity, I also assume that

$$\mathbf{g}_{ff} \geq 0. \tag{15}$$

Equation (15) states that $\mathbf{g}(\cdot)$ is not concave, meaning that the gain is weakly increasing in the absolute value of forecast errors. While this assumption helps identification in my baseline estimation, an alternative estimation strategy in Appendix F shows that it is a robust feature of the data.

The anchoring function is an extension to the decreasing or constant gain specifications, as it nests both as special cases. The usefulness of specifying the evolution of the gain as endogenous is that it offers a state-dependent sensitivity of the expectations process to surprises. In both of the exogenous gain schemes, the gain is divorced from the current environment. It either decreases deter-

[4] Alternatively, one could assume that agents use the forecast they made in the morning of the current period. This does not change the dynamics of the model.

ministically ($k_t = t^{-1}$), or is a constant ($k_t = \bar{g}$). This means that the level of unanchoring is either deterministic or constant.

An endogenous gain, instead, generates periods of well-anchored expectations (low gain) as well as unanchored episodes (high gain), and everything in-between. The endogenous gain can thus be interpreted as a metric of the varying degrees of unanchoring. Furthermore, since monetary policy influences the volatility of the economic environment, an endogenous gain framework allows monetary policy to affect the anchoring and unanchoring of expectations directly.

To be clear, I am not the first to use an endogenous gain learning model in the macro context. Marcet and Nicolini (2003) and Milani (2014) propose models in which agents switch between a constant and a decreasing gain based on a switching criterion. In Kostyshyna (2012), agents use a Kushner and Yin (2003) algorithm to choose the size of the gain.[5] Carvalho et al. (2023) propose an endogenous gain model as a metric of unanchoring.[6] Their focus is to show that an anchoring expectation formation can match untargeted moments of long-run inflation expectations from surveys.

I instead analyze the interaction between expectations unanchoring and monetary policy. The modification of Carvalho et al. (2023)'s anchoring theory using a smooth gain function makes sure that the derivatives of $\mathbf{g}(\cdot)$ exist, allowing me to formally consider the Ramsey problem. Thus, the paper sheds light on what varying degrees of expectations unanchoring means for monetary policy.

Conceptualizing unanchoring as the sensitivity of long-run expectations to short-run surprises through an endogenous gain is very attractive for studying monetary policy with unanchoring. Unlike alternative definitions of unanchoring, such as the distance of expectations from or the distribution of expectations around the target, the endogenous gain framework provides a simple structure that captures the feedback between policy and anchoring. While it is simple enough to be studied analytically in a Ramsey problem, it introduces the possibility that the monetary authority influences not just the input into the expectations process, but the very expectations process itself through the gain function. Thus, it captures the feature of anchoring policymakers worry the most about, namely how their actions will influence the way the public forms its expectations.

3.3. Actual law of motion

I now characterize the evolution of the jump variables under learning. Using the PLM from (7), I write the long-run expectations in (4) as

$$f_{a,t} \equiv \frac{1}{1-\alpha\beta} a_{t-1} + b_{t-1}(I_3 - \alpha\beta h)^{-1} s_t \qquad \text{and} \qquad f_{b,t} \equiv \frac{1}{1-\beta} a_{t-1} + b_{t-1}(I_3 - \beta h)^{-1} s_t. \tag{16}$$

[5] I consider Kostyshyna (2012)'s gain framework in Appendix I.

[6] See Appendix C for a description of Carvalho et al. (2023)'s anchoring function.

Substituting these into the law of motion of observables, Equation (5), yields the actual law of motion (ALM):

$$z_t = g^l_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix}, \tag{17}$$

where g^l is a 3×4 matrix given in Appendix B. Thus, instead of the state-space solution of the RE version of the model, Equations (6) and (B.1), the state-space solution for the learning model is characterized by the pair of equations (6) and (17), together with the PLM (10), the learning equations (12) and (13), as well as the anchoring function (14).

3.4. Simplifying assumption

To simplify the analytical work in Section 5.1, I make one assumption:

Assumption 1. $a_t = \begin{pmatrix} \bar{\pi}_t \\ 0 \\ 0 \end{pmatrix}$, *and* $b_t = g\ h, \quad \forall t.$

Assumption 1 amounts to restricting the intercepts in the forecasts of the output gap and the interest rate, as well as the slope coefficients of all forecasts to what they would be under rational expectations. This means that instead of learning the intercept and slope parameters for all three endogenous variables, the private sector only learns the intercept of the inflation process. Thus the single learning parameter is the long-run inflation expectation, $\bar{\pi}$.[7]

The rationale is that this is the smallest possible deviation from rational expectations that has enough flexibility to study the unanchoring of inflation expectations. It renders the Ramsey problem tractable and makes the comparison with the rational expectations benchmark more transparent. In particular, since the inflation intercept is learned using my anchoring model, the formulation is able to capture the time-varying sensitivity of long-run inflation expectations to short-run forecast errors in inflation.

Relaxing the assumption that slope coefficients are not learned does not change the implications of the model because it only makes impulse responses to shocks more bumpy after impact. The assumption on whether all variables are learned has stronger implications because interest-rate expectations are intricately linked with the optimal aggressiveness of monetary policy responses. Thus, when the interest-rate intercept is learned (as in Appendix L), or when the output gap intercept is learned, optimal policy becomes much less aggressive. I maintain this assumption for the sake of tractability of the Ramsey problem, and for ease of comparison with rational expectations.

With Assumption 1, the updating equations (12) and (13) simplify to a single equation:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1}, \tag{18}$$

[7] Appendix L considers a case where also the interest-rate intercept is learned.

where, using the notation that b_1 is the first element of the vector b, the k-period-ahead inflation forecast in the beginning of period t is given by

$$\hat{\mathbb{E}}_t \pi_{t+k} = \bar{\pi}_{t-1} + b_1^k s_t, \quad \forall k \geq 1. \tag{19}$$

Lastly, the one-period-ahead forecast error in inflation from the end of last period simplifies to

$$f_{t|t-1} = \pi_t - \big(\bar{\pi}_{t-1} + b_1 s_{t-1}\big). \tag{20}$$

The simplifying assumption also offers insights into how the anchoring process works. Recursively substituting Equation (18) into (20) yields

$$f_{t|t-1} = f_{t|t-1}^{RE} - \sum_{\tau=0}^{t-1} k_\tau f_{\tau|\tau-1}. \tag{21}$$

Here $f_{t|t-1}^{RE}$ stands for the rational expectations forecast error and captures unforecastable variation in inflation. Furthermore, I have initialized the long-run inflation expectation and the forecast error at the beginning of time with the rational expectations values of zero.

Equation (21) says that a forecast error today can arise from two sources. First, it can reflect a realization of shocks that was not foreseeable last period. This has the interpretation that tail events can unanchor expectations. The other source is the sum of the full history of gain-weighted forecast errors. This means that unanchoring is more likely to happen if the public has seen a long streak of similar surprises in the past, or if its expectations were highly or persistently unanchored in the past. In this way, even though the gain function only depends on the most recent forecast error, it still captures the dependence of anchoring on the full history of surprises.

4. Quantification of the anchoring channel

The numerical analysis of monetary policy requires a functional specification for the anchoring function. For this reason, I estimate the form of $\mathbf{g}(\cdot)$ in two steps. I first calibrate the parameters of the underlying New Keynesian model. Conditional on these values, I estimate the anchoring function by simulated method of moments (SMM) à la Lee and Ingram (1991), Duffie and Singleton (1990) and Smith (1993). I target the autocovariance structure of the Baxter-King filtered observables of the model and expectations. The observables are CPI inflation from the Bureau of Labor Statistics (BEA), the output gap and the federal funds rate from the Board of Governors of the Federal Reserve System.[8] For expectations, I rely on 12-month-ahead CPI inflation forecasts from the Survey of Professional Forecasters (SPF). The dataset is quarterly and ranges

[8] The output gap measure is constructed as the difference between real GDP from the Bureau of Economic Analysis (BEA) and the Congressional Budget Office's (CBO) estimate of real potential output.

from 1981-Q3 to 2020-Q1. Appendix E contains a detailed description of the estimation methodology.

4.1. Calibration

For the calibration of the New Keynesian core, I split the parameters into two subsets. The first is calibrated using values from the literature (Chari et al., 2000), while the second is calibrated to match the moments outlined above, for an initial set of expectations parameters. Tables 1 and 2 show the two subsets, and Appendix D expands on the details of the calibration. Appendix Q investigates alternative parameter values.[9]

Table 1: Parameters calibrated from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.085	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule*
$\bar{g}$	0.145	initial value of the gain

*Parameters with an asterisk refer to sections of the paper where a Taylor rule is in effect.

Table 2: Parameters set to match data moments

ψ_x	0.3	coefficient of the output gap in Taylor rule*
σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock*
σ_u	0.5	standard deviation, cost-push shock

*Parameters with an asterisk refer to sections of the paper where a Taylor rule is in effect.

4.2. Estimation

I employ a piecewise linear approximation of the form:

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}). \tag{22}$$

Here $b_i(\cdot)$ is a piecewise linear basis and γ is a vector of approximating coefficients. The index i refers to the breakpoints of the piecewise linear approximation. As explained in Appendix E, I estimate γ by simulated method of

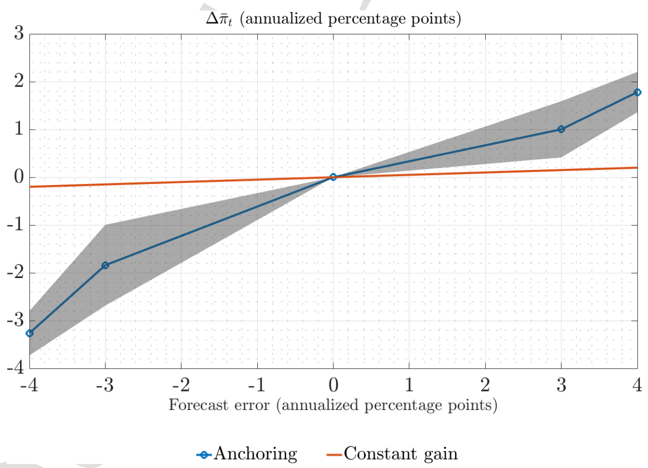
[9] In particular, $\beta = 0.99$ raises the pass-through between expectations and observables and renders optimal policy much more aggressive.

moments, targeting the autocovariance structure of the observables of the model and expectations.

The moments provided by the autocovariances constrain the number of parameters I can estimate. Therefore, to add power, I impose as an additional moment that the gain function should not be concave, corresponding to Equation (15). In Appendix F, I investigate whether this is an assumption that is supported by the data. There, I use the cross-section of the individual responses in the SPF to estimate the updating equation directly, using individual 10-year-ahead inflation expectations data. This provides many more moments. Thus, I can not only to do away with the no-concavity assumption, but also estimate a more general functional form for the anchoring function. In particular, I can also add persistence to the gain function by including one lag of the gain as an additional input.

I obtain that convexity is highly supported by the data. However, the alternative estimation strategy in Appendix F has an important drawback since it requires individual-level long-run inflation expectations that are only available starting in the early 1990s. As the appendix shows, this is prohibitive because long-run expectations are very stable over the entire sample period. Thus, the sample excludes episodes of unanchoring, which one would need in order to reliably estimate the anchoring process. Therefore I use the SMM approach as my benchmark, with no-concavity as an identifying restriction that I can rest assured is in line with the data.

Figure 1: Changes in long-run inflation expectations for various forecast errors given the estimated gain function



Estimates for 5 knots (indicated by circles), cross-section of size $N = 1000$. The shaded gray areas are 95% confidence intervals constructed using Wald's method from 100 bootstrap samples. The constant gain is calibrated to the consensus value of 0.05.

The estimated coefficients are $\hat{\gamma} = (0.82; 0.61; 0; 0.33; 0.45)$. The elements of $\hat{\gamma}$ represent the value of the gain the private sector chooses when it observes a forecast error of a particular magnitude. For example, a forecast error of -4 pp in inflation is associated with a gain of 0.82.

To make these numbers easier to interpret, Figure 1 depicts what the elements of $\hat{\gamma}$ imply for the updating of long-run inflation expectations (blue dotted line). For comparison, the red line plots a constant gain model with the gain calibrated to the consensus value from the empirical literature (see below). The circles on the blue line indicate the nodes of the piecewise linear approximation. For instance, a gain of 0.82 that arises when the private sector observes a -4 pp forecast error implies a downward revision in long-run inflation expectations by about 3.2 pp. For the constant gain model, the -4 pp forecast error implies a 0.1 pp downward revision in long-run expectations.

Let us put these numbers in perspective. The consensus in the literature on estimating constant gains is a gain between 0.01-0.05. Branch and Evans (2006) obtain a constant gain on inflation of 0.062. Milani (2007) finds 0.0183. The estimates of the maximal value for endogenous gains are 0.082 in Milani (2014) and at 0.145 in Carvalho et al. (2023).

Given the wide array of approaches to modeling and estimating gains, it is no surprise that researchers find very different numbers. Since the literature has settled on 0.05 in particular as the benchmark, I calibrate the constant gain model I use for comparison throughout the paper using this value.

The crucial message of Figure 1 is that there is considerable nonlinearity and asymmetry in the gain function. On the one hand, this means that large and downward surprises unanchor expectations more than small or upward ones. One can interpret the nonlinearity through the lens of rational inattention: people pay more attention to larger mistakes. By contrast, the constant gain model is by construction linear, failing to account for this feature of the data. The improved fit of the anchoring model is also reinforced in the autocovariogram, shown in Appendix G.

5. Monetary policy and anchoring

This section sets up and solves the optimal monetary policy problem in the model with anchoring. I begin by analyzing the Ramsey problem of the central bank. I then solve for the interest-rate sequence that implements optimal policy. In Appendix M, I also consider optimal Taylor rules.

5.1. The Ramsey policy under anchoring

I assume the monetary authority seeks to maximize welfare of the representative household under commitment. As shown in Woodford (2003), a second-order Taylor approximation of household utility delivers a central bank loss function of the form

$$L^{CB} = \mathbb{E}_t \sum_{T=t}^{\infty} \{(\pi_T - \pi^*)^2 + \lambda_x (x_T - x^*)^2\}, \tag{23}$$

where λ_x is the weight the central bank assigns to stabilizing the output gap. Just as under rational expectations, one can show that it is optimal to set the central bank's targets, π^* and x^*, to zero. The central bank's problem, then, is to determine paths for inflation, the output gap and the interest rate that minimize the loss in Equation (23), subject to the model equations (1) and (2), as well as the PLM (10), the learning equations (12) and (13), and the anchoring function (14). The full statement of the Ramsey problem is

$$\min_{\{\pi_t, x_t, i_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \lambda_x x_t^2) \qquad \text{s.t.}$$
$$x_t = \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}\big((1-\beta)x_{T+1} - \sigma(\beta i_T - \pi_{T+1}) + \sigma r_T^n\big),$$
$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T\big),$$
$$\hat{\mathbb{E}}_t \pi_{t+k} = \bar{\pi}_{t-1} + b_1^k s_t \quad \forall k \geq 1,$$
$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1},$$
$$f_{t|t-1} = \pi_t - \hat{\mathbb{E}}_{t-1}\pi_t,$$
$$k_t = \mathbf{g}(f_{t|t-1}), \tag{24}$$

where $\mathbb{E}$ is the central bank's expectation. Following Gaspar et al. (2010), I assume that the central bank has model-consistent expectations and observes the private sector's expectations.

5.1.1. Optimal Ramsey policy as a target criterion

The solution of the Ramsey problem is stated in the following proposition.

Proposition 1. *Target criterion in the anchoring model*
The targeting rule in the anchoring model is given by

$$\pi_t + \frac{\lambda_x}{\kappa} x_t = c\Big(k_t + f_{t|t-1}\mathbf{g}'_t\Big)\mathbb{E}_t\Big((1-\beta)x_{t+1} + \beta \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} 1 - k_{t+j} - f_{t+j|t-1+j}\mathbf{g}'_{t-1+j}\Big), \tag{25}$$

$$c \equiv \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta^2}{1-\alpha\beta}. \tag{26}$$

Proved in Appendix H. For versions of the target criterion for more general specifications of the gain function, see Appendix I.

Here $\mathbf{g}'_t$ denotes the derivative of $\mathbf{g}(\cdot)$ with respect to the forecast error $f_{t|t-1}$, evaluated at time t. Note that $\mathbf{g}' = \mathbf{g}_\pi = -\mathbf{g}_{\bar{\pi}}$.

The interpretation of (25) is that the *intratemporal* tradeoff between inflation and the output gap due to cost-push shocks is complemented by two *intertem-*

poral tradeoffs. One is due to learning in general (Molnár and Santoro, 2014), and one is due to anchoring in particular, and is thus new here.

Let me investigate these channels in isolation. Consider first the special case of constant gain adaptive learning. In this case (25) boils down to

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t + \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta^2}{1-\alpha\beta} k \mathbb{E}_t \left((1-\beta) x_{t+1} + \sum_{i=0}^{\infty} x_{t+i+1} (1-k)^i \right), \quad (27)$$

which replicates Molnár and Santoro (2014)'s expression, with the exceptions that the parameters $\frac{(1-\alpha)\beta^2}{1-\alpha\beta}$ and an extra $(1-\beta)x_{t+1}$ appear.[10]

This result suggests that learning by itself is responsible for the first intertemporal tradeoff between inflation and output gap stabilization. The central bank now has future output gaps as a margin of adjustment and therefore does not have to face the full tradeoff in the current period. In other words, adaptive learning allows the central bank to postpone the current tradeoff to the future.

Contrasting Equations (27) and (25) highlights the novel role of the anchoring channel. With anchoring, the extent to which policy can transfer the intratemporal tradeoff to future periods depends not only on the stance of the learning process, as in (27), but also on whether expectations are anchored, and in which direction they are moving.

Anchoring, however, can both alleviate or worsen the tradeoff. One can see this on the fact that forecast errors and the derivatives of the anchoring function are able to flip the sign of the second term in (25). To see the intuition, consider the equation governing the dynamics of observables from the first-order conditions of the Ramsey problem (Appendix H).

$$2\pi_t = -2\frac{\lambda_x}{\kappa} x_t + \varphi_{5,t} k_t + \varphi_{6,t} \mathbf{g}'_t. \quad (28)$$

The Lagrange multipliers $\varphi_5 \geq 0$ and $\varphi_6 \geq 0$ are the multipliers of the updating equation (12) and the anchoring function respectively.[11]

Whether the anchoring equation alleviates or exacerbates the inflation-output gap tradeoff depends on the sign of $\mathbf{g}'_t$. If the derivative is positive, the effect is the same as above, and the central bank has more leeway to postpone the tradeoff to the future. By contrast, if the derivative is negative, that is expectations are becoming anchored, the intratemporal tradeoff is worsened.

[10] Since Molnár and Santoro (2014) have no equation number at the relevant expression, I refer the reader to Equation (24) of Gaspar et al. (2010), who provide a parsimonious summary of Molnár and Santoro (2014). These differences come from the fact that in my model the entire term structure of expectations enters the evolution equations of inflation and output, while in Molnár and Santoro (2014), only one-period-ahead expectations do.

[11] φ_5 and φ_6 are zero if the learning process has converged. Convergence to a rational expectations equilibrium happens in this model if shocks are small enough for forecast errors to be decreasing over time. But because shocks can raise forecast errors at any point, the gain can rise again, restarting the convergence process. Strong expectational stability is thus only a limiting case.

Why do unanchored expectations give the central bank the possibility to postpone its current inflation-output gap tradeoff? The reason is that when expectations become unanchored, the learning process is restarted. A not-yet converged learning process implies that postponing the tradeoff is possible. This seems to suggest that from a smoothing standpoint, there is some benefit to having unanchored expectations. However, unanchoring is very costly from a volatility standpoint, as I show next.

5.2. Implementing the Ramsey policy: the optimal interest rate sequence

How should the central bank set its interest rate tool in order to implement the target criterion? The nonlinearity of the model does not admit an analytical answer to this question. I therefore solve for the optimal interest rate policy numerically using global methods. I rely on the calibration presented in Tables 1 and 2 and the estimated parameters of the expectations process in Section 4.[12] Furthermore, I set the central bank's weight on output gap fluctuations, λ_x, to 0.05 (Rotemberg and Woodford, 1997).

Appendix J outlines my preferred solution procedure, the parameterized expectations approach, while Appendix K gives the details of the parametric value function iteration approach I implement as a robustness check. The main output of this procedure is an approximation of the optimal interest rate policy as a function of the vector of state variables. The relevant state variables are expected mean inflation and the exogenous states at time t and $t-1$, rendering the state vector five-dimensional:

$$X_t = (\bar{\pi}_{t-1}, r_t^n, u_t, r_{t-1}^n, u_{t-1}). \tag{29}$$

As a first step, I plot how the approximated policy function depends on $\bar{\pi}_{t-1}$, while keeping all the other states at their mean. The result, depicted on Panel (a) of Figure 2, suggests that optimal interest-rate setting responds very sensitively to the stance of expectations, $\bar{\pi}_{t-1}$. If expected mean inflation increases by 5 basis points, the interest rate rises by about 250 basis points.[13]

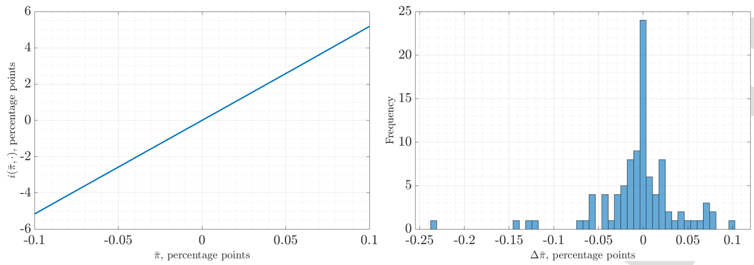
This is a large response. Clearly, optimal policy involves subduing unanchored expectations by injecting massive negative feedback to the system. Why is optimal policy so aggressive when anchoring can alleviate the stabilization tradeoff between output and inflation? The reason is that unanchoring introduces another intertemporal tradeoff to monetary policy: a volatility tradeoff. One can see this on Figure 3, portraying the dynamics of the system following a two-standard-deviation inflationary cost-push shock, conditional on a Taylor rule with baseline parameters. The figure contrasts the rational expectations version of the model with a well-anchored, weakly anchored and strongly unanchored scenario in the anchoring model, as well as with a constant gain model.

The same shock that under rational expectations completely vanishes by the second period, triggers a large, persistent and oscillatory response in the

[12] See Appendix Q for robustness to parameter values.

[13] This quantitative prediction is qualified if one relaxes Assumption 1. See Appendix L.

Figure 2: Policy function and implied volatility in long-run expectations



(a) $i(\bar{\pi}, \cdot)$, holding the other states fixed at their means (b) Histogram of changes in $\bar{\pi}$ under the optimal policy rule

anchoring model.[14] This is because the more expectations are unanchored, the more they become volatile, raising the pass-through from shocks to observables, thus amplifying shocks. Having unanchored expectations, then, comes at a volatility cost.

This volatility cost is also present in the constant gain model, and its size depends on the size of the gain. But as the literature on estimating constant gain models shows (Branch and Evans, 2006, Milani, 2007), a too large constant gain is inconsistent with data, thus putting an upper bound on how much excess volatility can come from a constant gain model. Also, the constant gain model implies the same volatility cost to a particular shock in any state of the world. A state-dependent gain, instead, allows the same shock to have different effects. As seen on the difference between the blue and yellow lines, the same shock leads to much more volatility if expectations are unanchored when the shock hits than if they are strongly anchored.

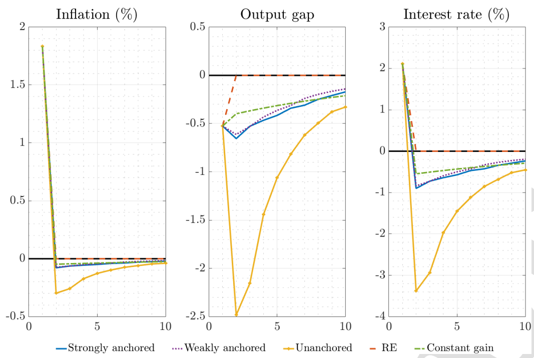
Therefore, in the long run, the central bank wishes to have expectations anchored. At the same time, anchoring expectations itself comes at a cost. Anchoring expectations requires an aggressive interest response because by these means the central bank can introduce negative feedback to the system. But changes in the interest rate surprise the private sector, raising forecast errors. The more unanchored expectations are, the more volatility the interest-rate movement inflicts on the economy.

We can see from the policy function how optimal policy resolves this tradeoff: it reacts extremely aggressively to movements in long-run expectations. This way, the central bank avoids even larger interventions that would become necessary were expectations to unanchor further. In other words, the central bank trades off short-run volatility for long-run stability.

Figure 2 shows that this is a successful strategy. As Panel (b) depicts, with the optimal policy in place, the model mostly displays tiny fluctuations in $\bar{\pi}$. In other words, the aggressive nature of optimal policy allows the central bank to

[14]To understand oscillatory dynamics in adaptive learning, see Appendix N.

Figure 3: Impulse responses after a cost-push shock



10 periods of impulse responses, shock imposed at $t = 25$ of a sample length of $T = 600$ (with 200 additional burn-in periods), cross-sectional average with a cross-section size of $N = 100$. The remark on whether expectations are anchored refers to whether the gain is below the 10^{th}, below the 50^{th}, or above the 90^{th} percentile of simulated gains at the time the shock hits. For the constant gain model, the gain is set to the consensus value of 0.05.

keep expectations anchored or quickly reanchor them.

6. Conclusion

Central bankers frequently voice a concern to anchor expectations. The contribution of this paper is to investigate how this affects the conduct of monetary policy. I use a simple behavioral model with a notion of anchoring as the time-varying sensitivity of long-run expectations to short-run surprises. I quantify my anchoring channel and establish that unanchoring is nonlinear and asymmetric in the data.

The main result is that optimal policy is state-dependent. The central bank responds aggressively when expectations unanchor in order to suppress excess volatility and to reanchor expectations. When expectations are well-anchored, by contrast, the central bank does not need to intervene and can accommodate inflation fluctuations.

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- Optimal monetary policy is considered when inflation expectations can unanchor.
- Unanchoring is given by the sensitivity of long-run inflation expectations to short-run surprises.
- Optimal policy responds aggressively to shocks when expectations are unanchored.
- State-dependent policy anchors expectations, which stabilizes inflation and output.

Laura Gáti
Directorate General Research, European Central Bank, Sonnemannstraße 20, 60314 Frankfurt am Main, Germany, +49 160 669 1757, laura_veronika.gati@ecb.europa.eu

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