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The low-magnitude and high-magnitude asymmetries in tail dependence structures in international equity markets and the role of bilateral exchange rate

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ABSTRACT

This paper introduces a new Markov-switching mixture copula model with a GAS mechanism in the weighting process to investigate the structures of the upper-tail dependence and lower-tail dependence in a low-magnitude asymmetry state and a high-magnitude asymmetry state for international equity markets. Three important findings arise. First, there are obvious low-magnitude and high-magnitude asymmetries in the tail dependence between equity markets of the U.S. and four nations (Canada, France, Germany, and the U.K.). Second, the difference between the upper-tail dependence and lower-tail dependence increases substantially around after 2000. In general, the importance of the lower-tail dependence is stronger than that of the upper-tail dependence in the high-magnitude asymmetry state. However, the Brexit panic does not change the importance of the tail dependence for the U.S. and European equity markets. Third, the evidence for the impact of the bilateral exchange rate on the tail dependence appears only for some international equity markets during the period of a high-magnitude asymmetry state. The bilateral exchange rate has a significant asymmetric impact on the upper-tail dependence and lower-tail dependence for the U.S. and Canadian equity markets. There is weak evidence of an asymmetric effect of the bilateral exchange rate for the U.S. and U.K. equity markets.

1. Introduction

Compared to the effect of the exchange rate on the equity market for a single country, the impact of the bilateral exchange rate for domestic and foreign countries on the dependence between domestic and foreign equity markets is a more considerable issue for cross-border investment decisions. Before discussing this important topic, depicting the dynamic dependence between domestic and foreign equity markets needs to be handled appropriately; otherwise, the influence of the bilateral exchange rate will be misestimated, resulting in the implementation of unsuitable portfolios and government policies. There are two main purposes of this paper: (1) to examine the time-varying and asymmetric tail dependence between international equity markets in high-degree and low-degree asymmetry states; and (2) to explore whether and how the bilateral exchange rate affects the tail structures between international equity markets in two asymmetric states.

The study of tail dependence between international equity markets has a long history, and its importance has continued to increase in recent decades due to the high degree of co-movements between international equity markets via a sharp

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surge in equity prices and a severe decline in equity prices.[1] The relationships between international equity markets become closer in bad situations, such as bear markets and periods of high volatility, than in good situations (Ramchand and Susmel, 1998; Longin and Solnik, 2001; Ang and Bekaert, 2002). In other words, a closer dependence appears in the lower tail of the joint distribution of international equity returns than in the upper tail of the joint distribution of international equity returns.

The copula method with the Markov-switching mechanism has become a popular approach for investigating tail dependence in recent decades due to its superior ability in capturing both non-linearity and asymmetry. There is much evidence in support of regime-dependent tail dependence between international equity markets (Jondeau and Rockinger, 2006; Rodriguez, 2007; Okimoto, 2008; Chollete et al., 2009; Garcia and Tsafack, 2011; Silva Filho et al., 2012; BenSaida et al., 2018; Ji et al., 2020). The advantage of the copula method with the Markov-switching mechanism is that the dependence intensity in each state can be different.

The generalized autoregressive score (GAS) updating mechanism, introduced by Creal et al. (2013), is a popular way to capture the time-varying dynamics of parameters. The GAS mechanism belongs to the so-called observation-driven approach suggested by Cox (1981) and has been employed by many empirical studies.[2] The superiority of the copula framework with GAS dynamics has been documented. For example, Creal et al. (2013) compared the relative performance between the copula method using the GAS process and the copula method using the dynamic process of Patton (2006a).[3] They observed that the GAS approach can more accurately capture the upper-tail dependence and lower-tail dependence compared to Patton's (2006a) approach, and found that the persistence of the copula parameter is slightly lower in Patton's (2006a) framework than in the GAS framework. In examining the hedging performance for crude oil futures and natural gas futures, Xu and Lien (2020) found that the copula method using the GAS process provides a better hedging performance than the copula method using the traditional GARCH process.

The Markov-switching property has been incorporated into the GAS process in univariate and bivariate frameworks. In the univariate framework, Blazsek and Ho (2017) allowed score-driven parameters to change between high-volatility and low-volatility states. In a study of the S&P 500 stock market, they found that the Markov-switching GAS model yields a better density forecast performance than the single-state GAS model. Blazsek et al. (2018) extended the specification of Blazsek and Ho (2017) by allowing a variety of distributions of random error terms. The evaluation of the value-at-risk and expected shortfall for S&P 500 stock returns showed that skewed distributions have better performance than the student-t distribution.[4] In the bivariate framework, Bernardi and Catania (2019) developed a Markov-switching copula model in which the copula parameter follows a Markov-switching GAS process; that is, the time-varying dependence for international stock markets can switch between low-dependence and high-dependence states. Their empirical results showed that, in general, the student-t copula and Clayton copula provide better fitting performance than the Gaussian, Frank, and Plackett copulas.

Due to the appealing advantages of the copula function, the Markov-switching mechanism, and the GAS updating process, this paper develops a new Markov-switching mixture copula mechanism with a regime-switching GAS dynamic evolution process to capture the pattern of asymmetric dependence between international equity markets in two distinct dependence states (a low-magnitude asymmetry state and a high-magnitude asymmetry state). In order to simultaneously emphasize time-varying and asymmetric changes in the upper-tail and lower-tail dependences, a mixture of the Gumbel copula (which has a larger mass in the upper tail than in the lower tail) and its survival copula (which has a larger mass in the lower tail than in the upper tail) is employed, and the weights of the mixture copula are assumed to obey the GAS updating processes. To take the Markov-switching mechanism into account, the parameters for each copula function and the GAS dynamic behavior are assumed to follow a first-order two-state Markov chain process. In doing so, the relative importance between the time-varying lower-tail dependence and time-varying upper-tail dependence can vary between the low-magnitude asymmetry state and the high-magnitude asymmetry state.

There are two differences between the new specification and the specification of Bernardi and Catania (2019). One is the use of a copula function. As opposed to the single copula of Bernardi and Catania (2019), the new specification employs a mixture copula, providing an easy way to capture the asymmetric dependence in upper and lower tails. The second is the Markov-switching process in the parameters. Bernardi and Catania (2019) assumed that the copula parameter has a Markov-switching GAS process; this paper assumes that the mixing parameter follows a Markov-switching GAS process. Compared to the high dependence state and low dependence state of Bernardi and Catania (2019), the magnitude of asymmetry in the upper-tail dependence and lower-tail dependence can be measured in the new specification. Accordingly, moderate and intense asymmetries in the tail dependence between international equity markets are analyzed in this paper.

[1] Another strand of literature investigates the possible driving forces that cause an increase in the relationships between international equity markets. For example, Bekaert et al. (2009) considered capital and trade integrations to be important channels. Walti (2011) observed that trade linkage, financial integration, and monetary integration play important roles. Asgharian et al. (2013) found that the exchange rate volatility, inflation expectation divergence, interest rate differential, bilateral trade, foreign direct investment, and geographical distance can affect the co-movements. Huang (2020) demonstrated the importance of foreign direct investment and foreign portfolio investment. Qiu et al. (2022) found that the driving force is related to the capital outflow from the U.S. to other countries.

[2] For instance, Creal et al. (2011), Avdulaj and Barunik (2015), Salvatierra and Patton (2015), Blazsek and Ho (2017), Blazsek et al. (2018), Tafakori et al. (2018), Bernardi and Catania (2019), Gong et al. (2019), Manguzvane and Mwamba (2020), and Xu and Lien (2020).

[3] The main difference between the dynamic process of Patton (2006a) and the GAS process is that the former uses the past history of variables as explanatory variables and the latter uses the information on the joint probability distribution of variables to depict the time-varying behavior.

[4] The skewed generalized t, exponential generalized beta, and normal-inverse Gaussian are employed to represent the skewed distributions.

The role of the bilateral exchange rate on the relationship between domestic and foreign equity markets has been investigated by Malliaroupulos (1998), Moore and Wang (2014), and Hau and Rey (2006).[5] For example, Malliaroupulos (1998) proposed a negative relationship between the bilateral exchange rate and the difference between domestic equity market returns and foreign equity market returns, based on the relative purchasing power parity and the negative relationship between inflation rate and stock price. Based on the framework of Malliaroupulos (1998), a similar conclusion was documented by Moore and Wang (2014); on the contrary, an opposing claim was developed by Hau and Rey (2006). Based on the incomplete foreign exchange risk, there is a positive relationship between the bilateral exchange rate and the difference between domestic equity market returns and foreign equity market returns.

From the development of the relationship between the bilateral exchange rate and the difference between international stock market returns, this paper further empirically investigates whether the bilateral exchange rate can affect the upper-tail dependence and lower-tail dependence, and examines whether the direction and magnitude of the influence power of the bilateral exchange rate change between the low-magnitude and high-magnitude asymmetry states.

The empirical analysis has three important findings. First, the dependence between the U.S. and four developed countries (Canada, France, Germany, and the U.K.) exhibits significantly asymmetric and non-linear patterns. The dependence between international equity markets varies between the low-intensity asymmetry state and the high-intensity asymmetry state. The absolute value of the difference between the upper-tail dependence and lower-tail dependence is larger in the high-intensity asymmetry state than in the low-intensity asymmetry state.

Second, the time point for the transition from a low-intensity asymmetry state to a high-intensity asymmetry state occurs around 2000. In the high-intensity asymmetry state, the lower-tail dependence is larger than the upper-tail dependence, irrespective of the pair of international stock markets. On the contrary, in general, the lower-tail dependence is bigger than the upper-tail dependence in the low-intensity asymmetry state, except for the US-UK pair. Specifically, the dominance of the upper tail dependence exists in the low-intensity asymmetry state for the US-UK pair; however, the dominance of the upper tail dependence disappears and the importance of lower tail dependence increases for the US-UK pair during the period of high-intensity asymmetry state. Hence, in the low-intensity asymmetric state, the tail dependence characteristic for the U.S. and U.K. is different from that for the U.S. and other European nations.

Third, evidence for the asymmeric impact of the bilateral exchange rate on the tail dependence is observed for the U.S. and Canada, for the U.S. and Germany, and for the U.S. and U.K. The impacts of the bilateral exchange rate on the upper-tail dependence and lower-tail dependence are different. Appreciation of the US dollar increases the lower-tail dependence between the U.S. and international markets but decreases the upper-tail dependence.

The remainder of this research is structured as follows. The Markov-switching mixture copula model with the GAS process is introduced in Section 2. The empirical results for the marginal specification and mixture copula framework are discussed in Section 3. The conclusions and discussions are provided in Section 4.

2. The model

This subsection introduces the Markov-switching mixture copula model using the GAS process. Let y_{1t} and y_{2t} be the equity returns for the U.S. and other countries, respectively. Assume that s_t is the state variable with the two values of 1 and 2. As Chollete et al. (2009) and Bernardi and Catania (2019) pointed out, Sklar's (1959) theorem can be applied to the Markov-switching copula specification. Following the similar spirit of the above two studies, the relationship between the joint probability density function and the mixture copula density, which is conditional on the state variable (s_t) and information set (Ω_{t-1}), can be expressed in the following form:

$$\mathrm{f}(r_{1t}, r_{2t}|s_t = j, \Omega_{t-1}) = c_m^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \boldsymbol{\theta}^{(\mathbf{j})}\right)$$
$$\times \prod_{i=1}^{2} f_i(y_{it}|\Omega_{t-1}; \psi_{\mathbf{i}}) \tag{1}$$

where, $F_i(r_{it}|\Omega_{t-1})$ is the conditional cumulative distribution function of r_{it}; $f_i(y_{it}|\Omega_{t-1}; \varnothing_i)$ is the conditional probability density function of r_{it}; η_1 and η_2 are parameter vectors for the marginal functions of r_{1t} and r_{2t}, respectively; $\boldsymbol{\theta}^{(\mathbf{j})}$ is the parameter vector for the mixture copula density, given the j$^{\text{th}}$ outcome of the state variable; and $c_m^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \boldsymbol{\theta}^{(\mathbf{j})}\right)$ is the conditional mixture copula density, given $s_t = j$ and Ω_{t-1}.

The evolution process of s_t is governed by a time-homogenous first-order Markov chain process with two states. The four transition probabilities can be expressed as follows:

$$P(s_t = 1|s_{t-1} = 1) = \frac{\exp(\mathrm{g}_{11})}{1 + \exp(\mathrm{g}_{11})}, -\infty < \mathrm{g}_{11} < \infty \tag{2}$$

$$P(s_t = 2|s_{t-1} = 1) = \frac{1}{1 + \exp(\mathrm{g}_{11})}, -\infty < \mathrm{g}_{11} < \infty \tag{3}$$

[5] For comparison purpose, the bilateral exchange rate is defined as the value of one US dollar unit to the value of the domestic currency. When the bilateral exchange rate increases, the US dollar appreciates and the domestic currency depreciates.

$$P(s_t = 1|s_{t-1} = 2) = \frac{1}{1 + \exp(g_{22})}, \ -\infty < g_{22} < \infty \tag{4}$$

$$P(s_t = 2|s_{t-1} = 2) = \frac{\exp(g_{22})}{1 + \exp(g_{22})}, -\infty < g_{22} < \infty \tag{5}$$

2.1. The Markov-switching mixture copula with the GAS process

Many empirical studies (Okimoto, 2008; Chollete et al., 2009; Chollete et al., 2011; Silva Filho et al., 2012; Ji et al., 2020) have observed that international equity markets have strong positive dependence patterns. There are two kinds of positive relationships: the stock prices of two international equity markets increasing simultaneously, and the equity prices of two international equity markets decreasing simultaneously. The Gumbel copula is suitable for capturing the former situation, while the latter situation can be successfully captured by the survival Gumbel copula. Consequently, a combination of the Gumbel and survival Gumbel copulas can simultaneously measure both the magnitude of the movement of two stock markets toward large and positive returns and the magnitude of the movement of two stock markets toward large and negative returns.

The Markov-switching property is a popular way to depict non-linear dependence. Combining the mixture copula with the Markov-switching mechanism can allow asymmetric dependence patterns to switch between two market states. Due to the advantages of the mixture copula and the Markov-switching mechanism, the regime-dependent mixture copula density shown in Equation (1) is proposed as follows:

$$\begin{aligned} & c_m^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \boldsymbol{\theta^{(j)}}\right) \\ & = \omega_t^{(j)} c_G^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \theta_G^{(j)}\right) \\ & + \left(1 - \omega_t^{(j)}\right) c_{SG}^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \theta_{SG}^{(j)}\right) \end{aligned} \tag{6}$$

where $\omega_t^{(j)}$ $(1 - \omega_t^{(j)})$ is the weight for the Gumbel (survival Gumbel) copula density with time-varying and regime-switching properties. The components of the regime-dependent parameter vector $\boldsymbol{\theta^{(j)}}$ include $\theta_G^{(j)}$, $\theta_{SG}^{(j)}$, and parameters for the weight function. The two regime-dependent copula functions are respectively defined as follows:

$$\begin{aligned} & C_G^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \theta_G^{(j)}\right) \\ & = exp\left(-\left[[-ln(F_1(r_{1t}|\Omega_{t-1}))]^{\theta_G^{(j)}} + [-ln(F_2(r_{2t}|\Omega_{t-1}))]^{\theta_{G,t}^{(j)}}\right]^{1/\theta_G^{(j)}}\right) \end{aligned} \tag{7}$$

$$\begin{aligned} & C_{SG}^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \theta_{SG}^{(j)}\right) \\ & = F_1(r_{1t}|\Omega_{t-1}) + F_2(r_{2t}|\Omega_{t-1}) - 1 \\ & + C_G^{(j)}\left(1 - F_1(r_{1t}|\Omega_{t-1}), 1 - F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \theta_{SG}^{(j)}\right) \end{aligned} \tag{8}$$

where $\theta_G^{(j)}$ and $\theta_{SG}^{(j)}$ are larger than or equal to 1.

Without imposing restrictions on parameters $\theta_G^{(j)}$ and $\theta_{SG}^{(j)}$ in the estimation procedure, this paper provides the following transformations:

$$\theta_G^{(j)} = 1 + \exp\left(\widetilde{\theta}_{G,t}^{(j)}\right), -\infty < \widetilde{\theta}_{G,t}^{(j)} < \infty \tag{9}$$

$$\theta_{SG}^{(j)} = 1 + \exp\left(\widetilde{\theta}_{SG,t}^{(j)}\right), -\infty < \widetilde{\theta}_{SG,t}^{(j)} < \infty \tag{10}$$

This paper allows the tail dependence structure to be time-varying in each state through the GAS updating mechanism of $\omega_t^{(j)}$.[6] The dynamics of $\omega_t^{(j)}$ extend the framework of Catanlia (2021) by allowing a Markov-switching process. In order to satisfy the condition that the weight lies between 0 and 1, the following transform is employed:

[6] As shown by Bernardi and Catania (2019), allowing copula parameters to have GAS dynamics can generate time-varying asymmetries in the tail dependence. However, when weights and copula parameters are allowed to follow GAS processes, there will be too many parameters in the econometric model, making it difficult to implement a quasi-maximum likelihood estimation procedure. As opposed to the method of Bernardi and Catania (2019), the GAS updating mechanism of $\omega_t^{(j)}$ is easier to handle and can largely reduce the estimation complexity while maintaining the ability to describe the characteristics of asymmetric tail dependence in a given state.

$$\omega_t^{(j)} = \Lambda\left(\widetilde{\omega}_t^{(j)}\right) = \frac{1}{1+\exp\left(\widetilde{\omega}_t^{(j)}\right)} \tag{11}$$

in which $\widetilde{\omega}_t^{(j)}$ has the following GAS evolution dynamics:

$$\widetilde{\omega}_t^{(j)} = \pi_{0,\omega}^{(j)} + \pi_{1,\omega}^{(j)}\widetilde{s}_{\omega,t-1}^{(j)} + \pi_{2,\omega}^{(j)}\widetilde{\omega}_{t-1}^{(j)},\ \pi_{1,\omega}^{(j)} > 0 \textit{ and } 0 < \pi_{2,\omega}^{(j)} < 1 \tag{12}$$

with

$$\widetilde{s}_{\omega,t-1}^{(j)} = \widetilde{S}_{\omega,t-1}^{(j)}\widetilde{\nabla}_{\omega,t-1}^{(j)}$$

where $\widetilde{\nabla}_{\omega,t-1}^{(j)}$ and $\widetilde{S}_{\omega,t-1}^{(j)}$ are the regime-dependent score function and the regime-dependent scaling function, respectively, which are defined as follows::

$$\widetilde{\nabla}_{\omega,t-1}^{(j)} = \frac{\partial \ln\left(c_m^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1};\boldsymbol{\theta}^{(j)}\right)\right)}{\partial\widetilde{\omega}_{t-1}^{(j)}}$$

$$= \frac{\partial \ln\left(c_m^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1};\boldsymbol{\theta}^{(j)}\right)\right)}{\partial\omega_{t-1}^{(j)}} \times \frac{\partial\Lambda\left(\widetilde{\omega}_{t-1}^{(j)}\right)}{\partial\widetilde{\omega}_{t-1}^{(j)}} \tag{13}$$

$$= \nabla_{\omega,t-1}^{(j)} \times \frac{\partial\Lambda\left(\widetilde{\omega}_{t-1}^{(j)}\right)}{\partial\widetilde{\omega}_{t-1}^{(j)}}$$

$$\widetilde{S}_{\omega,t-1}^{(j)} = \left(\widetilde{I}_{t-1|t-2}^{(j)}\right)^{-k} \tag{14}$$

where $\frac{\partial\Lambda\left(\widetilde{\omega}_{t-1}^{(j)}\right)}{\partial\widetilde{\omega}_{t-1}^{(j)}}$ is the regime-dependent value of Jacobian and $\widetilde{I}_{t-1|t-2}^{(j)}$ is the regime-dependent information function.

Because the regime-dependent GAS process cannot be implemented, the collapsing method of Gray (1996) is employed here to make Equation (12) feasible. The regime-dependent score function $\left(\widetilde{s}_{\omega,t-1}^{(j)}\right)$ and the regime-dependent weight function $\left(\widetilde{\omega}_{t-1}^{(j)}\right)$ are respectively replaced by the regime-independent score function ($\widetilde{s}_{\omega,t-1}$) and the regime-independent weight function ($\widetilde{\omega}_{t-1}$), as shown below:

$$\widetilde{s}_{\omega,t-1} \cong E[\widetilde{s}_{\omega,t-1}|\Omega_{t-2}] = \sum\nolimits_j P(s_{t-1}=j|\Omega_{t-2}) \times \widetilde{s}_{\omega,t-1}^{(j)} \tag{15}$$

$$\widetilde{\omega}_{t-1} \cong E\left[\widetilde{\omega}_{t-1}|\Omega_{t-2}\right] = \sum\nolimits_j P(s_{t-1}=j|\Omega_{t-2}) \times \widetilde{\omega}_{t-1}^{(j)} \tag{16}$$

where $P(s_{t-1}=j|\Omega_{t-2})$ is the probability of $s_{t-1}=j$, given the information set Ω_{t-2}.[7]

In the specification of Creal et al. (2013), the parameter k has three distinct values: 0, 0.5, and 1, reflecting the step size in the updating process. Existing empirical analyses find no consistent evidence for the optimal value of k. For example, k=0 has been used in empirical studies, such as Blazsek et al. (2018), Manguzvane and Mwamba (2020), and Catanlia (2021); the use of k = 0.5 has been employed by Salvatierra and Patton (2015), Avdulaj and Barunik (2015), and Catanlia (2021); and $k=1$ has been chosen by Bernardi and Catania (2019). Under general conditions, the information matrix is equal to the negative expectation value of the Hessian matrix. However, in this paper, the Hessian matrix is a zero matrix. Therefore, k is set to 0 in this paper.

2.2. The AR-Garch model with skewed student's t innovation

The conditional probability density functions for r_{1t} and r_{2t} are also components of the Markov-switching mixture copula model. The time-series process of stock returns is assumed to have an AR-GARCH process. The dynamic process for r_{it} is given as follows:

$$r_{i,t} = b_{0i} + b_{1i}r_{i,t-1} + \varepsilon_{i,t} \tag{17}$$

$$\varepsilon_{i,t} = \sqrt{\sigma_{i,t}^2} \times v_{i,t} \tag{18}$$

[7] The predicting probabilities for $P(s_{t-1}=1|\Omega_{t-2})$ and $P(s_{t-1}=2|\Omega_{t-2})$ can be obtained through the formulae of Gray (1996).

$$\sigma_{i,t}^2 = \beta_{0i} + \beta_{1i}\left(v_{i,t}\right)^2 + \beta_{2i}\sigma_{i,t-1}^2 \tag{19}$$

where $\sigma_{i,t}^2$ is the conditional variance with the GARCH process, and $v_{i,t}$ is a random innovation.

In order to consider the skewed and asymmetric characteristics of equity returns, the skewed student-t distribution of Fernandez and Steel (1998) is employed here. Therefore, the probability density function of $v_{i,t}$ has the following expression:

$$f\left(v_{i,t}\right) = \begin{cases} \frac{2}{\eta_i + \frac{1}{\eta_i}} t_{d_i}\left(\eta_i v_{i,t}\right) & if \quad v_{i,t} < 0 \\ \frac{2}{\eta_i + \frac{1}{\eta_i}} t_{d_i}\left(\frac{v_{i,t}}{\eta_i}\right) & if \quad v_{i,t} > 0 \end{cases} \tag{20}$$

where η_i is the asymmetric parameter,[8] and t_{d_i} is a student's t distribution with degree of freedom d_i.

According to Castillo et al. (2011), the cumulative distribution function of the skewed student-t distribution is defined as follows:

$$F\left(v_{i,t}\right) = \begin{cases} \frac{2}{1+\eta_i^2} T_{d_i}\left(\eta_i v_{i,t}\right) & if \quad v_{i,t} < 0 \\ \frac{1-\eta_i^2}{1+\eta_i^2} + \frac{2\eta_i^2}{1+\eta_i^2} T_{d_i}\left(\frac{v_{i,t}}{\eta_i}\right) & if \quad v_{i,t} > 0 \end{cases} \tag{21}$$

where T_{d_i} is the cumulative distribution function of the student-t distribution.

2.3. Estimation procedures

Given the regime-dependent probability density function of r_{1t} and r_{2t} as shown in Equation (1), the joint conditional density function of r_{1t} and r_{2t}, given information set Ω_{t-1}, can be expressed as follows:

$$\mathrm{f}(r_{1t}, r_{2t}|\Omega_{t-1}) = \sum_{j=1}^{2} c_m^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \boldsymbol{\theta}^{(j)}\right) \tag{22}$$

$$\times \prod_{i=1}^{2} f_i(y_{it}|\Omega_{t-1}; \psi_{\mathbf{i}}) \times P(s_t = j|\Omega_{t-1})$$

where $\psi_{\mathbf{i}}$ includes b_{0i}, b_{1i}, β_{0i}, β_{1i}, η_i, and d_i, and $P(s_t = j|\Omega_{t-1})$ is the probability of jth state occurring at time t, conditional on the information set available at time t − 1. The calculation of $\mathrm{P}(s_t = j|\Omega_{t-1})$ is related to the transition probabilities shown in Equations (2)–(5).

The log-likelihood function of the empirical specification can be expressed as follows:

$$\mathrm{lnL} = \sum_{t=1}^{T} ln\mathrm{f}(r_{1t}, r_{2t}|\Omega_{t-1})$$

$$= \sum_{t=1}^{T} ln\left\{\sum_{j=1}^{2} c_m^{(j)}\left(F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}; \boldsymbol{\theta}^{(j)}\right) \times P(s_t = j|\Omega_{t-1})\right\}$$

$$+ \sum_{t=1}^{T} \{lnf_1(y_{1t}|\Omega_{t-1}; \psi_1) + lnf_2(y_{2t}|\Omega_{t-1}; \psi_2)\} \tag{23}$$

As Chollete et al. (2009) noted, the parameters can be estimated through the Inference Functions for Margins (IFM) method of Patton (2006a; 2006b) even when the joint distribution is composed of a regime-dependent copula function and regime-independent margin specifications. Accordingly, this paper employs the IFM procedure. In IFM, the first step is to estimate the parameters for marginal distributions, as shown below:

$$\widehat{\psi}_i \equiv argmax \sum_{t=1}^{T} lnf_i(y_{it}|\Omega_{t-1}; \psi_{\mathbf{i}})$$

$$\psi_{\mathbf{i}} = \{b_{0i}, b_{1i}, \beta_{0i}, \beta_{1i}, \beta_{2i}, d_i, \eta_i\}$$

The second step, given the parameter estimates obtained in step 1, is to estimate the parameters for the Markov-switching mixture copula and transition probabilities, as follows:

[8] The density function is well-defined when the value of η_i is greater than 0. The density is symmetric as $\eta_i = 1$. The density is left-skewed (right-skewed) when $\eta_i < 1$ ($\eta_i > 1$).

$$\widehat{\Theta} \equiv argmax \sum_{t=1}^{T} ln \left\{ \sum_{j=1}^{2} c_m^{(j)} (F_1(r_{1t}|\Omega_{t-1}), F_2(r_{2t}|\Omega_{t-1})|\Omega_{t-1}) \times P(s_t = j|\Omega_{t-1}) \right\}$$

Here, $\Theta = \left\{ \pi_{0,\omega}^{(1)}, \pi_{1,\omega}^{(1)}, \pi_{2,\omega}^{(1)}, \theta_G^{(1)}, \theta_{SG}^{(1)}, \pi_{0,\omega}^{(2)}, \pi_{1,\omega}^{(2)}, \pi_{2,\omega}^{(2)}, \theta_G^{(2)}, \theta_{SG}^{(2)}, g_{11}, g_{22} \right\}$.[9]

3. Empirical analysis

3.1. Data

This research investigates the patterns for the asymmetric tail dependence between the U.S. stock market and four international stock markets (Canada, France, Germany, and the U.K.). France, Germany, and the U.K. are the three largest economies in Europe, while Canada has a high connection with the U.S. in many areas, such as economics, culture, diplomacy, trade, and the military. This paper thus investigates whether the dependence between the U.S. and European countries is different from that between the U.S. and Canada.

The monthly equity price indices are collected from the Monthly Monetary and Financial Statistics (MEI) dataset of the Organisation for Economic Cooperation and Development (OECD).[10] The equity index is a capitalization-weighted index, and is defined as 100 in 2015 (The base year is 2015). The equity prices of related companies are collected in local currency and then, for ease of cross-country comparisons, are expressed in the market price index in terms of OECD form.[11]

The sample period is from January 1971 to December 2021, with a total of 612 observations. Fig. 1 displays the equity price indices. The international equity price indices have an increasing trend starting from the beginning of the 1970s, followed by a sudden decline around 2000. After prices drop, the share price indices still show an increasing pattern but with larger variations, compared to the period before 2000. Fig. 2 shows the equity market returns.[12]

Table 1 reports the summary statistics for equity returns. The sample standard deviation is the largest for the French equity market, whereas the standard deviation is the smallest for the U.S. equity market. The five stock markets have negative skewness. The magnitude of negative skewness is most severe for the U.S. market and is less severe for the U.K. market. The kurtosis coefficient is the largest for the U.K. market, followed by the U.S. market, the Canadian market, the German market, and the French market, which has the smallest kurtosis. The Jarque-Bera statistics are statistically significant at the 1 % level, indicating the non-normality of equity returns for all international equity markets investigated herein. Furthermore, the augmented Dickey–Fuller (ADF) test and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test show that the equity return series is stationary for each country. On the other hand, the Ljung-Box tests show an autocorrelation in the squared returns for each equity market, showing the presence of time-varying variances. These findings show the inappropriateness of symmetric distribution and homoscedasticity in capturing the equity return dynamics. Hence, in order to consider the volatility clustering as well as the asymmetric and fat-tailed property, the AR-GARCH model with skewed student-t error is employed to model the time-series dynamics of equity returns.

3.2. Results for the AR-GARCH model with skewed student-t error

The estimation results for the AR-GARCH model are shown in Table 2. Volatility persistence is obvious in each stock market. It is highest for the French and U.K. equity market, while the German equity market shows the smallest persistence in the volatility process. Moreover, the Canadian and U.S. stock markets have similar volatility persistence.

The estimated parameters for asymmetric parameter η and degree of freedom t_d show evidence in support of the departure from the symmetric distribution. The parameter estimate of η is smaller than 1 for each equity market, showing the left skewness shape of the equity returns. The left skewness for the probability distribution function of the equity returns refers to a large mass in negative returns than in positive returns. The U.S. stock market has the most serious left skewness, followed by the German equity market, the French equity market, and the Canadian equity market, and the magnitude of left skewness is less serious for the U.K. equity market. The estimate for t_d ranges from 4.861 to 6.387. The smaller the degrees of freedom is, the greater the fat-tailed pattern is. The U.S. stock market has the smallest degrees of freedom, while the German stock market has the largest degrees of freedom. In sum, the marginal distribution of stock returns shows a significantly asymmetric and fat-tailed shape for each international equity market.

The goodness-of-fit framework of Berkowitz (2001), an extension of the concept of Diebold et al. (1998), is employed to test the fitting ability of the AR-GARCH model with the skewed student-t error. Proposition 1 of Berkowitz (2001) points out that the inverse standard normal distribution of the conditional integral transformation error term is an uncorrelated ran-

[9] At time 1, the predicting probability $P(s_1 = j|\Omega_0)$ is replaced by the ergodic probability.

[10] The OECD publishes equity price indices in monthly, quarterly, and yearly frequencies. High frequency data (weekly and daily indices) are not available.

[11] The target companies of the OECD equity index are related to companies included in the S&P/TSX composite index for Canada, companies included in the SBF250 index for France, companies included in the CDAX-GESAMTINDEX (KURS) index for Germany, companies included in the FTSE-100 index for the UK, and companies included in the NYSE Composite index for the US. Please refer to the MEI database for more information about the construction of the equity index of each country.

[12] The return on equity price is defined as 100 times the change of the logarithm of the equity price index between two consecutive months.

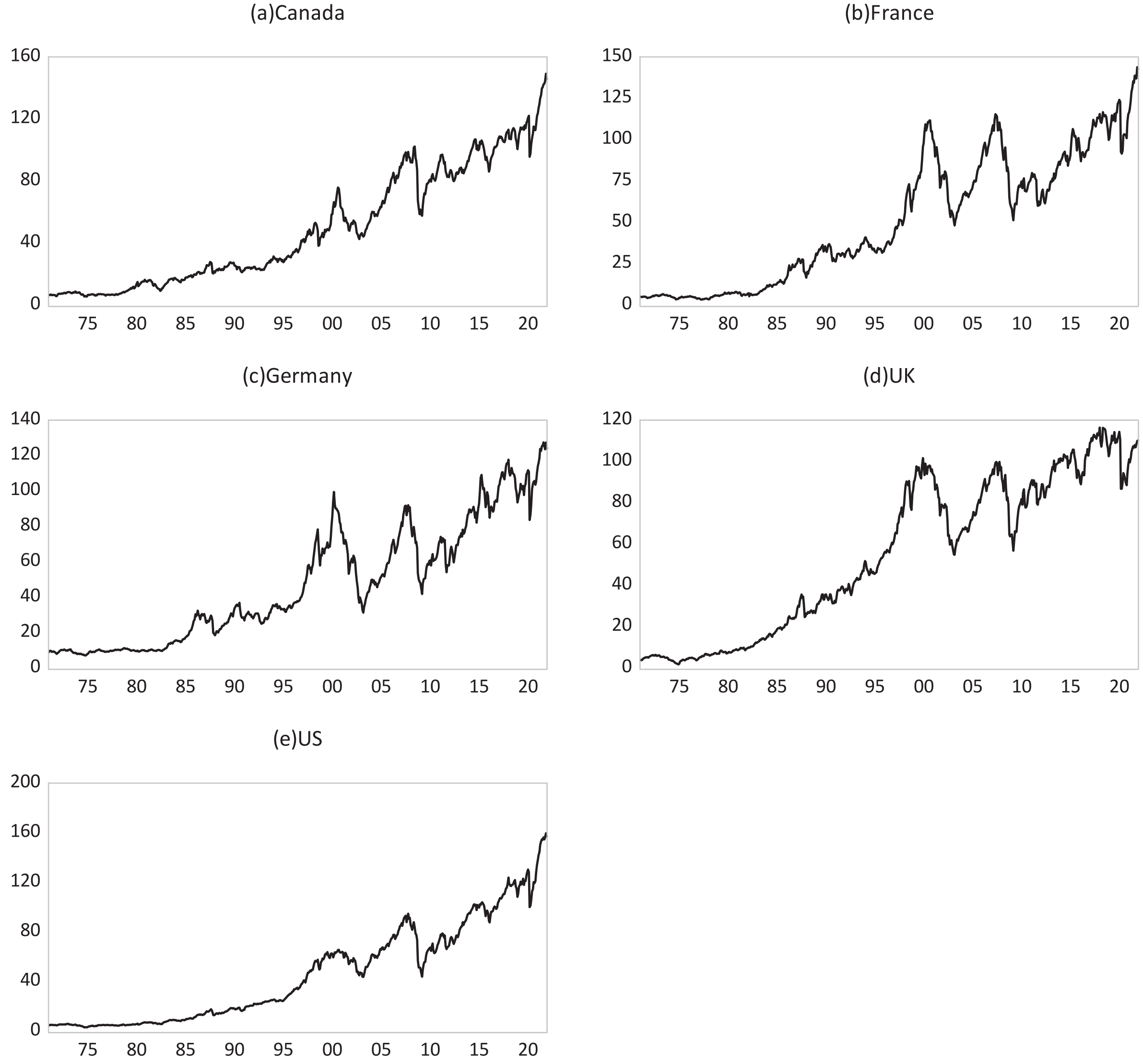

Fig. 1. Equity price indices.

dom variable when the conditional integral transformation error term is an independent and identical distributed random variable with uniform distribution over [0,1]. The Ljung-Box statistics for the transformed variable and the squared transformed variable are listed in Table 3. The test results show that the Ljung-Box statistics are not statistically significant at the 1 % level, signifying that, for each international equity market, the transformed variable and squared transformed variable are serially uncorrelated. Accordingly, the AR-GARCH model with the skewed student-t error is a suitable marginal specification for equity returns.

3.3. Results for transition probabilities

The estimation results for the Markov-switching mixture copula, estimated through the quasi-maximum likelihood estimation approach, are reported in Table 4.[13] Panel C of Table 4 displays the estimates for g_{11} and g_{22}. State 1 and state 2 are

[13] Because the OECD does not provide weekly equity indices and no existing equity indices can represent the OECD equity indices, empirical results obtained by existing equity indices cannot compare with those obtained by OECD equity indices. In order to examine whether the Markov-switching mixture copula can be applied to the weekly data, this paper arbitrarily chooses the weekly S&P500 index and weekly S&P/TSX Composite index for the period from January 1, 1993 to December 31, 2021. The data are collected from the Taiwan Economic Journal (TEJ) database. Due to data availability, the sample period starts from January 1, 1993. The estimation result indicates that the U.S. and Canadian equity markets exist as asymmetric dependence structures, but their asymmetric structures differ from that shown in the OECD monthly data. Due to space limitations, empirical results are not reported here but are available upon request.

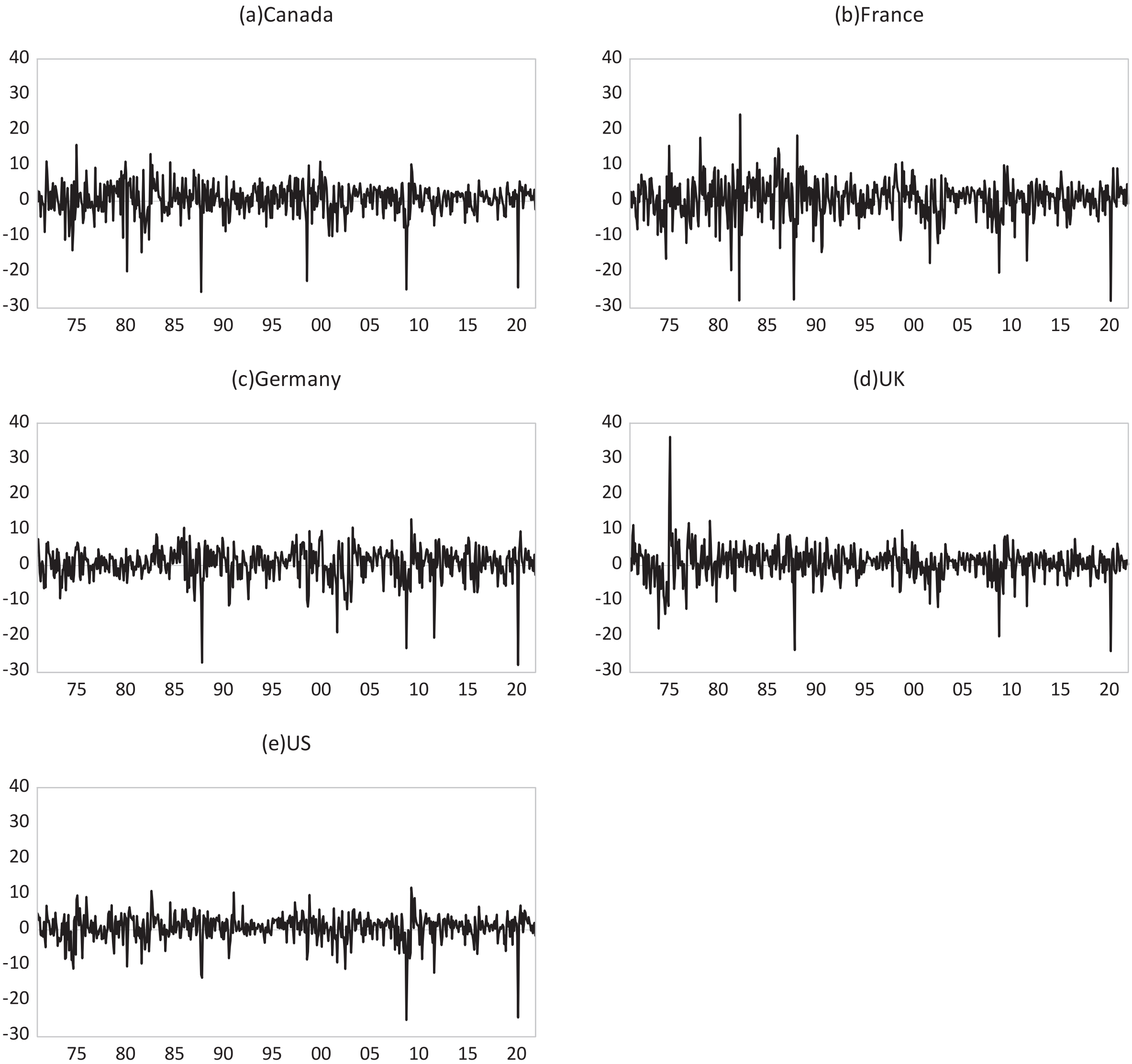

Fig. 2. Equity market returns.

Table 1
Summary Statistics.

	Canada	France	Germany	UK	US
Mean	0.056	0.540	0.436	0.538	0.568
Median	0.995	1.077	0.969	0.816	0.895
Maximum	15.921	24.466	13.042	36.291	11.927
Minimum	−25.657	−28.185	−28.157	−24.193	−25.472
SD	4.458	5.457	4.542	4.513	3.751
Skewness	−1.338	−0.864	−1.463	−0.091	−1.499
Kurtosis	9.574	7.470	9.300	12.812	10.629
JB	1284.744***	585.694***	1230.379***	2456.063***	1713.343***
ADF	−21.732***	−21.776***	−18.571***	−16.842***	−19.026***
KPSS	0.031	0.078	0.036	0.244	0.058
LB2(1)	0.321	14.483***	3.889**	19.896***	7.788***
LB2(2)	5.273*	15.226***	4.396	23.176***	7.790**

Notes: SD represents the standard deviation. JB refers to the Jarque-Bera test for normality. ADF and KPSS refer to the augmented Dickey–Fuller test and Kwiatkowski–Phillips–Schmidt–Shin test, respectively. LB2(k) is the Ljung-Box statistics for squared returns at lag k. "***" indicates significance at the l % level.

Table 2
Estimation Results for Marginal Specifications.

Parameters	Canada	France	Germany	UK	US
b_0	1.636***	1.943***	1.657***	1.328***	1.680***
	(0.266)	(0.310)	(0.288)	(0.240)	(0.230)
b_1	0.080*	0.130***	0.235***	0.144***	0.173***
	(0.044)	(0.040)	(0.040)	(0.042)	(0.040)
β_0	2.340**	1.935**	3.044**	1.584**	1.366**
	(0.969)	(0.926)	(1.192)	(0.687)	(0.612)
β_1	0.178***	0.150***	0.199***	0.193***	0.127***
	(0.057)	(0.045)	(0.066)	(0.053)	(0.046)
β_2	0.680***	0.766***	0.598***	0.723***	0.731***
	(0.087)	(0.060)	(0.109)	(0.069)	(0.081)
d	4.906***	5.136***	6.387***	4.894***	4.861***
	(0.880)	(0.946)	(1.246)	(1.049)	(0.937)
η	0.814***	0.803***	0.795***	0.848***	0.768***
	(0.045)	(0.041)	(0.046)	(0.042)	(0.044)
Log_L	−1700.597	−1826.777	−1696.665	−1683.653	−1576.479

Notes: *, **, and *** refer to significance at 10 %, 5 %, and 1 % levels, respectively. The value in parentheses below the coefficient estimate is the standard error. Log_L refers to the log-likelihood function.

Table 3
Ljung-Box statistics.

	Canada	France	Germany	UK	US
Panel A: Transformed variable					
LB(5)	4.695	11.263	3.460	8.689	9.441
LB(10)	5.153	21.329	15.911	16.179	14.066
Panel B: Squared transformed score					
LB(5)	7.709	1.993	2.061	4.148	3.014
LB(10)	17.068	9.378	7.823	7.467	9.297

Notes: LB(k) refers to the Ljung-Box statistics at lag k. The critical value at 1 % significance level is 15.086 for k = 5 and 23.209 for k = 10.

identified as the high-magnitude asymmetry state and the low-magnitude asymmetry state, respectively.[14] For each pair of international equity markets, the coefficients for g_{11} and g_{22} are statistically different from 0, and the estimated coefficients for g_{11} and g_{22} are close to each other. These findings reveal that there are two distinct states in the tail dependence structures of the joint dynamic system, and the persistence of each state is extremely high. According to coefficients for g_{11} and g_{22}, the transition probability for occurring in the same state is at least higher than 0.997, irrespective of the state and the pair of international equity markets. Such astonishing persistence means the situation in which the joint dynamics switch from one state to the other state rarely occurs. The transition probabilities for staying in the same state are highest for the U.S. and Canada, while the transition probabilities are the smallest for the U.S. and Germany.

The smoothing algorithm of Kim (1993) is employed to calculate the occurrence probability of a specific state at time t by using all information available in the dynamic system. The smoothed probabilities for state 1 (the high-magnitude asymmetry state) are plotted in Fig. 3. The time series plot shows that the probability roughly stays at a low level before 2000 and then increases almost to nearly 1 and maintains its high level until the end of the sample period. The changing point occurs around the dot-com bubble of 2000. Specifically, the changing dates are February 1998 for the US-UK pair, June 2000 for the US-France pair and the US-Germany pair, and December 2000 for the US-Canada pair.

3.4. Results for asymmetries in tail dependence

The calculation for the upper-tail and lower-tail dependence measures is related to the copula parameters, mixture weights, and smoothed probabilities. The ex-post upper-tail dependence (UTD) and lower-tail dependence (LTD) are calculated according to the following formulae:[15]

$$\mathrm{UTD} = \sum_{j=1}^{2} \omega_t^{(j)} \times \left(2 - 2^{1/\theta_G^{(j)}}\right) \times P(s_t = j|\Omega_T) \tag{24}$$

$$\mathrm{LTD} = \sum_{j=1}^{2} \left(1 - \omega_t^{(j)}\right) \times \left(2 - 2^{1/\theta_{SG}^{(j)}}\right) \times P(s_t = j|\Omega_T) \tag{25}$$

[14] The characteristics of these two different states are discussed in details in subsection 3.4.

[15] The term ex-post is used in this paper, as smoothed probabilities are employed to calculate the weighting tail dependence measure.

Table 4
Estimation Results for mixture copulas.

	US-Canada	US-France	US-Germany	US-UK
Panel A: Sate 1				
$\widetilde{\theta}_{G,t}^{(1)}$	0.581	1.389***	2.608***	1.238***
	(0.645)	(0.344)	(0.547)	(0.321)
$\widetilde{\theta}_{SG,t}^{(1)}$	0.606***	0.509***	0.348***	0.417***
	(0.169)	(0.132)	(0.121)	(0.161)
$\pi_{0,\omega}^{(1)}$	0.026	2.431**	2.320	0.842
	(0.243)	(1.016)	(2.275)	(0.550)
$\pi_{1,\omega}^{(1)}$	0.000	19.795	3.319	0.659
	(6.537)	(23.872)	(2.829)	(3.397)
$\pi_{2,\omega}^{(1)}$	0.959***	–	0.000	0.078
	(0.174)		(0.833)	(0.237)
Panel B: State 2				
$\widetilde{\theta}_{G,t}^{(2)}$	1.273**	−0.705	1.084	−0.587**
	(0.534)	(2.294)	(3.554)	(0.131)
$\widetilde{\theta}_{SG,t}^{(2)}$	−0.704***	−1.365***	−1.182***	−1.879***
	(0.138)	(0.393)	(0.217)	(0.540)
$\pi_{0,\omega}^{(2)}$	2.906	1.786	0.274	−0.719*
	(2.393)	(1.831)	(2.400)	(0.385)
$\pi_{1,\omega}^{(2)}$	42.295	0.000	0.000	41.218
	(107.989)	(21.384)	(49.095)	(43.715)
$\pi_{2,\omega}^{(2)}$	0.330	–	0.964**	0.808***
	(0.393)		(0.467)	(0.052)
Panel C: Transition probabilities				
g_{11}	6.439**	6.366**	5.775***	6.422***
	(2.928)	(2.764)	(1.993)	(1.935)
g_{22}	6.558***	6.382***	5.851***	5.878***
	(1.550)	(1.963)	(1.235)	(0.858)
Log_L	219.078	188.957	187.223	239.185

Notes: *, **, and *** refer to significance at 10 %, 5 %, and 1 % levels, respectively. The value in parentheses below the coefficient estimate is the standard error. Log_L refers to the log-likelihood function.

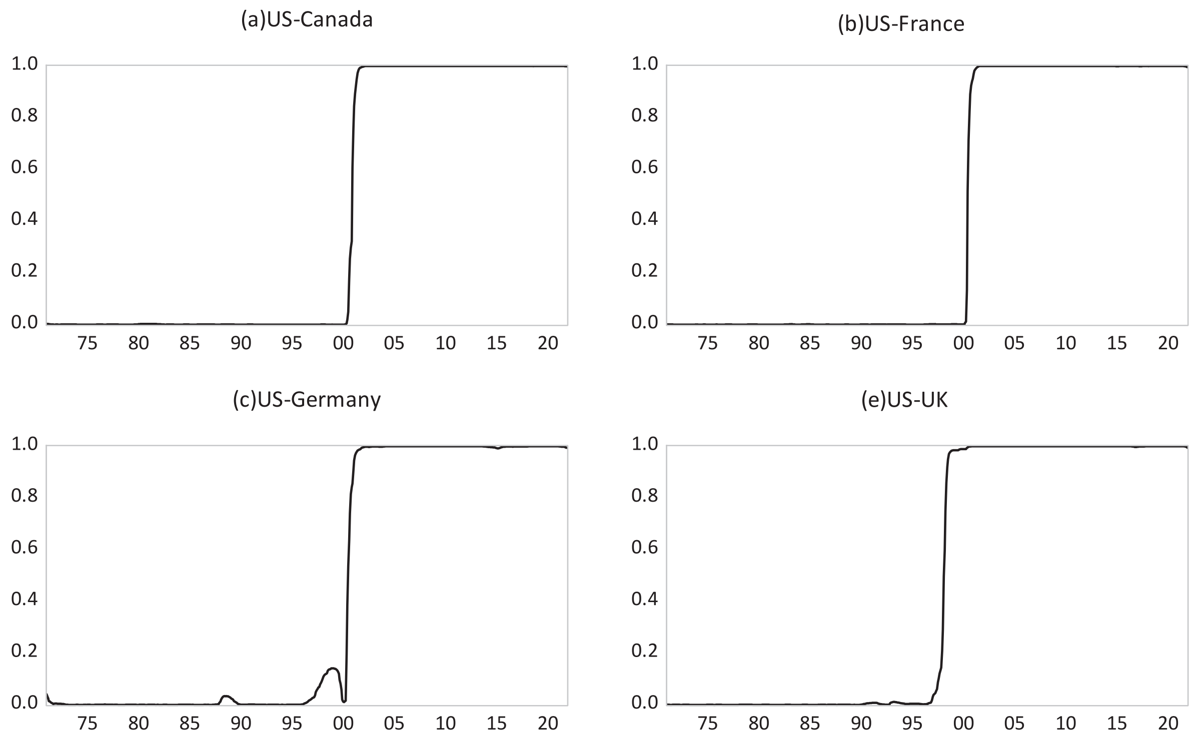

Fig. 3. Smoothed probabilities for state 1 (high-magnitude asymmetry state).

where $P(s_t = j|\Omega_T)$ is the smoothed probability for $s_t = j$ conditional on Ω_T.

Before discussing the magnitude of the upper-tail dependence and lower-tail dependence, the evolution processes for the regime-dependent weights are analyzed first. The time-varying plots of the weights are displayed in Fig. 4.

The time-series dynamics for the weight of the Gumbel copula show greatly different patterns in different states and different pairs of international equity markets. For the US-Canada pair, in state 1 the weight is volatile before 2000, and there is an increasing pattern after 2000. In state 2, the dynamics of the weight change frequently. The weight of the Gumbel copula is slightly larger in state 1 than in state 2 for the US-Germany pair. For the US-UK pair, the sample mean and variation for the weight of the Gumbel copula is larger in state 2 than in state 1. For the US-France pair, the variation in the weight of the Gumbel copula is larger in state 1. However, the US-France weight for state 2 is around 0.144 for the whole period, compared to other pairs which exhibit variations in weight. Specifically, for the US-France pair, the dominant role of the survival Gumbel copula remains unchanged during the period of state 2. The almost time-invariant weight could be related to Huang et al.'s (2022) viewpoint of a low trade linkage between France and the US.[16] The low trade linkage between France and the U.S. leads to a more stable relationship between the U.S. and French equity markets. In sum, the GAS process of the weight reveals that the time-varying relative importance between the Gumbel copula and survival Gumbel copula varies across different pairs.

Fig. 5 shows the time-series plots for the ex-post upper-tail dependence and lower-tail dependence. Two surprising findings, which are not observed in previous empirical studies, are indicated. First, the relative magnitude between the upper-tail dependence and lower-tail dependence changes between the two states. In general, the lower-tail dependence is larger than the upper-tail dependence during the period of state 2,[17] and the upper-tail dependence is close to zero, except for the US-UK pair. On the other hand, the lower-tail dependence dominates the upper-tail dependence during the period of state 1. The lower-tail dependence increases noticeably in state 1. Second, in general, the absolute value of the difference between the lower-tail dependence and upper-tail dependence is smaller in state 2 than in state 1. The asymmetry in tail dependence structures is moderate during the period of state 2, and the asymmetry is more intense during the period of state 1; that is, the difference in the lower-tail dependence and upper-tail dependence becomes larger after 2000. Accordingly, state 1 can be identified as the state with high-magnitude asymmetry in tail dependence structures, while state 2 is the state with low-magnitude asymmetry in tail dependence structures. In comparison to previous studies that classify the high dependence state and low dependence state, this paper identifies a moderate asymmetry state and intense asymmetry state.

The finding that the structural change in the dependence structure of international equity markets occurs around 2000 is approximately related to that of Cappielo et al. (2006), Aslanidis et al. (2010), and Marfatia (2020).[18] For example, Cappielo et al. (2006) observed that the correlation between the US and EMU (Economic and Monetary Union) stock markets increases strongly in January 1999, and they considered that globalization, market asymmetry, and volatility rise are possible causes of increasing relationship between international stock markets. Aslanidis et al. (2010) found a significant increase in conditional correlation between the U.K. and U.S. stock markets around 2000, and that factors such as globalization and market integration are import for increasing correlation. Marfatia (2020) demonstrated that the risk perception of the U.S. stock market affects the correlations between the U.S. and international stock markets, with the magnitude for the impact of risk perception changing around 1999.

The above findings indicate that, before 2000, the co-movement between the U.S. equity market and other international equity markets during a bear market is more important than the co-movement in a bull market, except for the US-UK pair. On the other hand, the magnitude of co-movement between U.S. and international equity markets changes remarkably after 2000. The magnitude of the tail dependence between U.S. and international equity markets increases more in bad market conditions, but the magnitude increases slightly in good market conditions, except for the U.K. equity market . Specifically, for the US-Canada pair, the lower-tail dependence is far stronger than the upper-tail dependence in state 1. A similar finding can be observed for the U.S. and European equity market pairs and this finding does not change during 2016-2018, which is related to the uncertainty and panic related to Brexit.[19]

It is worth emphasizing that the findings of existing empirical studies on the dependence between international stock markets becoming higher during bad market conditions than in good market conditions can be observed during the period of state 2, except for the US-UK pair. On the other hand, during the period of state 1, the possibility of stock prices in the U.S. and international equity markets rising together is lower than the possibility of stock prices falling together.

[16] Huang et al. (2022) investigated the impacts of the disagreement on the U.S. economy and the disagreement on the domestic economy on the domestic stock markets for non-U.S. G7 countries. The impact of the disagreement on the U.S. economy play a dominant role for non-U.S. G7 countries, except for France. Specifically, the impact of French economy disagreement is more important than the impact of the U.S. economy disagreement, revealing the predominant role of the French economy in affecting the French equity market. Huang et al. (2022) considered that the smaller impact of the U.S. economy disagreement can be attributed to the lower dependence on the U.S. economy. Among Canada, France, Germany, and the U.K., the proportion of domestic exports to the U.S. to domestic GDP ranked from largest to smallest is Canada, Germany, the U.K., and France.

[17] The period for large lower-tail dependence and small upper-tail dependence is similar to the period of occurring in state 2 for the US-France and US-Germany pairs. The only exception is US-Canada. For US-Canada, the period for a large lower-tail dependence and small upper-tail dependence is from January 1971 to June 2001 and this period is slightly longer than the period of occurring in state 2.

[18] On the other hand, Lane and Milesi-Ferretti (2007) examined the magnitude of international financial integration in industrial countries and in emerging and developing countries over the period 1970–2002. The index of international financial integration is defined as the ratio of the sum of external assets and external liabilities to GDP. They observed an increasing magnitude, with a structural change in magnitude taking place around 1999.

[19] The United Kingdom European Union membership referendum was held in June 2016 and subsequently caused equity market uncertainty and panic for the U.K. and European nations.

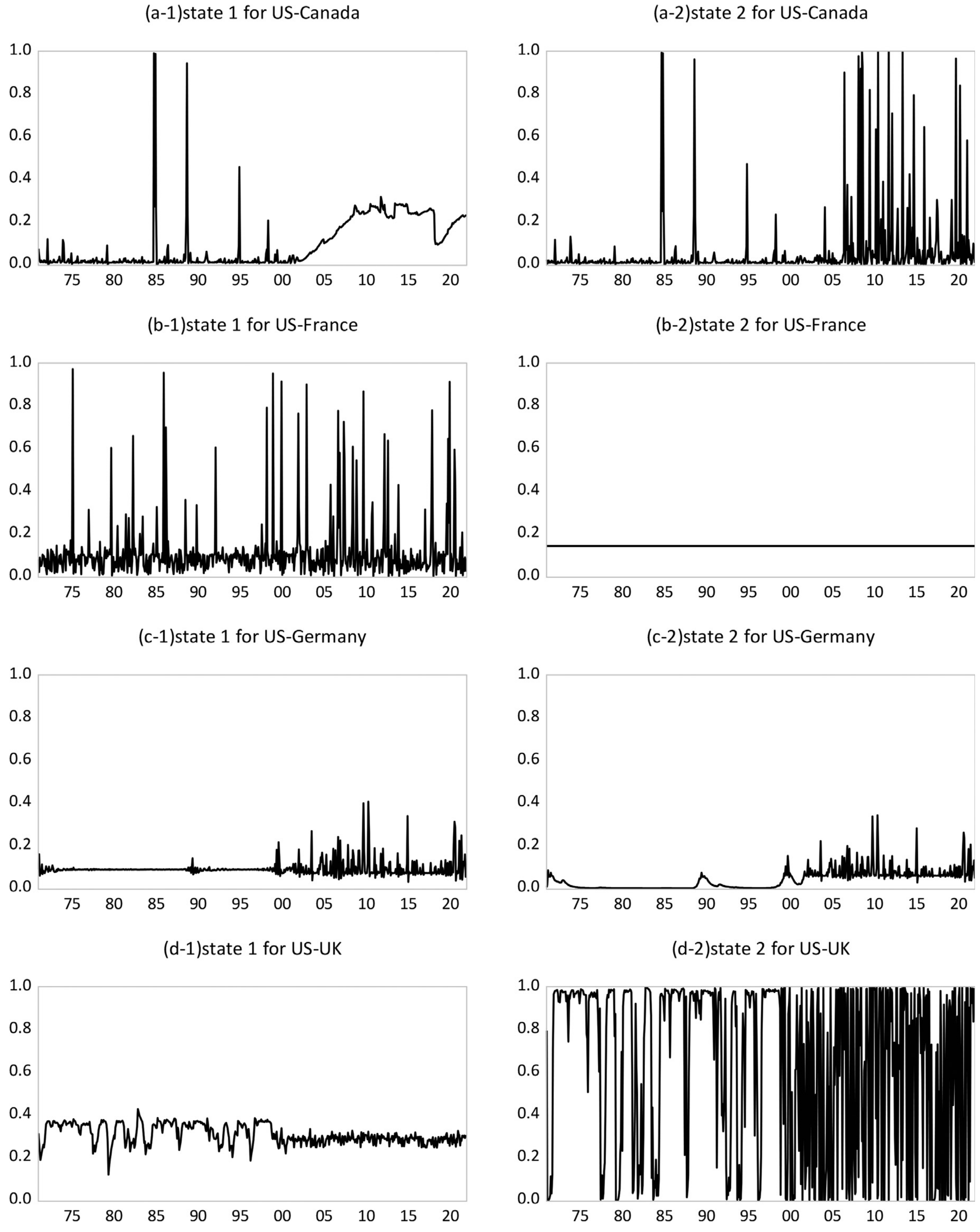

Fig. 4. Time-series plots for regime-dependent weights.

In sum, there is an astonishing change in the upper-tail dependence and lower-tail dependence around 2000. Before 2000, the co-movement between U.S. and international stock markets in extremely bad conditions is stronger than that in extremely good conditions, and the difference between the lower-tail dependence and upper-tail dependence is moderate. Con-

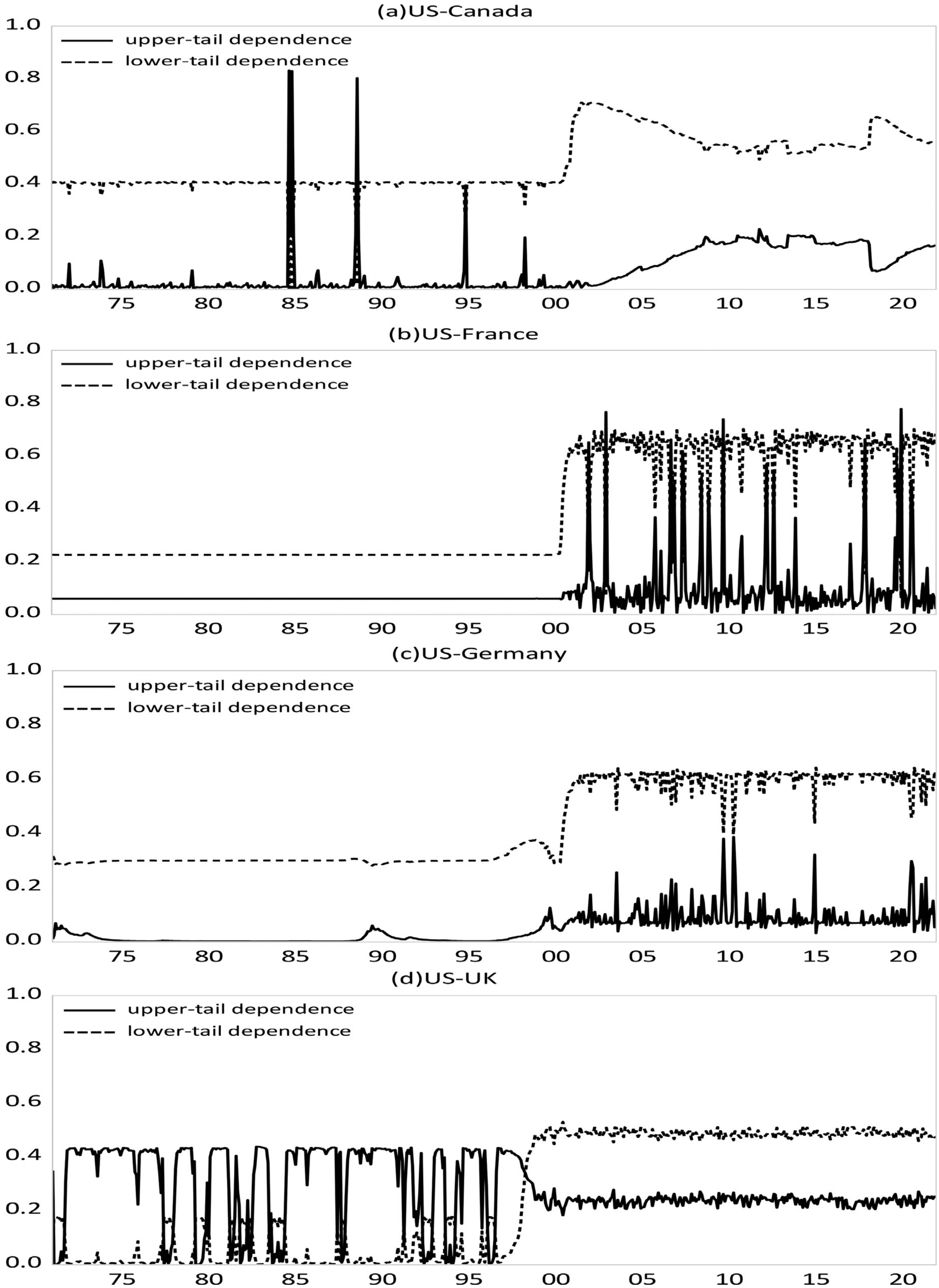

Fig. 5. Time-series plots for upper-tail dependence and lower-tail dependence.

trarily, after 2000, the difference between the upper-tail dependence and lower-tail dependence becomes larger, and the co-movement is stronger in extremely bad conditions than in extremely good conditions.

3.5. Results for the impacts of bilateral exchange rate

Each international equity index employed in this paper is essentially denominated in local currency. Whether the foreign exchange rate can affect the relationship between international stock markets is important for international investors. According to the viewpoint of Malliaroupulos (1998) and Moore and Wang (2014), when the domestic currency devaluates, the domestic stock price declines in terms of the mean-reverting hypothesis of the exchange rate. Subsequently, the difference between domestic stock returns and foreign stock returns declines, revealing that the co-movement between domestic and foreign stock markets in a bad direction increases. On the other hand, Hau and Rey (2006) corroborated that when the domestic currency devaluates, the domestic stock price increases. In the assumption of uncovering equity parity, the difference between the domestic stock return and foreign stock return increases, implying that the co-movement between domestic and foreign stock markets in a good direction decreases.

This subsection further investigates whether the bilateral exchange rate can affect the dependence between U.S. and international stock returns.[20] In order to consider the change in asymmetric dependence structures, this paper employs a threshold model to separate the impacts of bilateral exchange rates in a low-magnitude asymmetry state and a high-magnitude asymmetry state.

The threshold model is given as follows:

$$TD_t = \begin{cases} a_{20} + a_{21} \times ER_{t-1} + \varepsilon_t & if \quad P(s_t = 1|\Omega_T) < 0.5 \\ a_{10} + a_{11} \times ER_{t-1} + \varepsilon_t & if \quad P(s_t = 1|\Omega_T) \geq 0.5 \end{cases} \tag{24}$$

where TD_t is the weighted upper-tail dependence measure (UTD) or weighted lower-tail dependence measure (LTD) and ER_{t-1} is the bilateral exchange rate between the U.S. and another country at time t-1. The exchange rate is defined as the number of units of a specified currency per unit of U.S. dollar.

The threshold model is estimated based on the consideration of Newey-West heteroskedasticity autocorrelation consistent (Newey-West HAC) standard errors and heterogeneous error distributions. The estimation results are reported in Table 5. For the US-Canada pair, the impact of exchange rate on the tail relationship is insignificant during the period of state 2. The importance of the bilateral exchange rate between the US dollar and the Canadian dollar on the tail dependence measures can be observed after 2000. The bilateral exchange rate has a statistically significant impact on the upper-tail dependence and lower-tail dependence in the high-magnitude asymmetry state. The impact of the exchange rate on the lower-tail dependence is positive, and the impact of the exchange rate on the upper-tail dependence is negative. The appreciation of the US dollar against the Canadian dollar increases the lower-tail dependence between the U.S. and Canadian equity markets, supporting the argument of Malliaroupulos (1998) and Moore and Wang (2014). On the other hand, the appreciation of the US dollar decreases the upper-tail dependence between the U.S. and Canadian equity markets, showing evidence in support of the viewpoint of Hau and Rey (2006).

The exchange rate of the British pound has a positive impact on the tail dependence between the U.S. and U.K. equity markets only during the period of a high-magnitude asymmetry state, but not during the period of a low-magnitude asymmetry state. In other words, a depreciation in the British pound causes an increase in the lower-tail dependence, conforming to the standpoint of Malliaroupulos (1998) and Moore and Wang (2014). On the other hand, the tail dependence is not affected by the exchange rate in state 2. For the remaining two pairs (US-France and US-Germany), there is no evidence that the bilateral exchange rate can affect the upper-tail dependence and lower-tail tail dependence, except for the upper-tail dependence in state 1 for the US-Germany pair.

The above empirical findings provide important implications for international investors. The role of bilateral exchange rates in the tail dependence between cross-country equity markets is not always important for international stock markets. For example, the foreign exchange risk is almost negligible for international portfolios based on the U.S. and France stock markets as well as those based on the U.S. and German stock markets. Specifically, the diversified benefit will not change no matter whether the stock returns are denominated in U.S. dollars or local currency.

On the other hand, in the last 20 years, the diversification benefit has been affected by the foreign exchange rate for portfolios investing in the U.S. and Canadian stock markets and portfolios based on the U.S. and U.K. stock markets. When the local currency depreciates, the lower tail dependence increases, resulting in a worse diversification benefit in bad stock market environments. On the other hand, a depreciation in local currency reduces the upper tail dependence, causing an increase in diversification benefits in good stock market environments.

[20] Another way to investigate the impact of the foreign exchange risk on the linkages between international equity markets is to employ the stock indices denominated in local currency and in U.S. dollars. When the empirical finding obtained through the dollar-denominated equity indices differs from those obtained through the equity indices composed of the local currency, the importance of exchange rate risk cannot be ignored. As mentioned in subsection 3.1, the OECD equity index is local currency-denominated index. Hence, the dependence between dollar-denominated international stock returns cannot be addressed due to data limitations. Furthermore, the OECD equity index is a transformed index in terms of the OECD form. Hence, it is not reasonable to transform the OECD equity index into a dollar-denominated index based on the bilateral exchange rate.

Table 5
Estimation Results for the effect of the bilateral exchange rate.

	Lower tail dependence		Upper tail dependence	
	State 1	State 2	State 1	State 2
(1)US-Canada	0.240***	−0.005	−0.254***	0.027
	(0.042)	(0.022)	(0.054)	(0.026)
(2)US-France	−0.043	0.005	0.019	-3.104×10^{-5}(5.431×10^{-5})
	(0.067)	(0.067)	(0.088)	
(3)US-Germany	−0.017	−0.018	−0.041*	0.006
	(0.029)	(0.049)	(0.022)	(0.012)
(4)US-UK	0.033*	0.018	−0.033*	−0.017
	(0.017)	(0.120)	(0.020)	(0.290)

Notes: * and *** refer to significance at 10 % and 1 % levels, respectively. The value in parentheses below the coefficient estimate is the standard error.

4. Conclusions and discussions

This paper designs a Markov-switching mixture copula model, in which the Gumbel and survival Gumbel copulas are components of the regime-switching mixture copula. The time-varying mixture weight is assumed to follow a regime-switching GAS process in order to explore the characteristics of non-linear and asymmetric tail dependences between the U.S. and four international stock markets (Canada, France, Germany, and the U.K.). In doing so, the difference in structures between the upper-tail dependence and lower-tail dependence between two distinct states can be investigated. Furthermore, this paper also examines whether the role of the bilateral exchange rate has different impacts on the upper-tail dependence and lower-tail dependence for different pairs of equity markets.

Three interesting findings are observed in this paper. First, the Markov-switching mechanism reveals two different dependence structures: a low-magnitude asymmetry state and a high-magnitude asymmetry state. The distance between the lower-tail dependence and the upper-tail dependence is smaller in the low-magnitude asymmetry state; however, this difference is larger in the high-magnitude asymmetry state.

Second, the asymmetric characteristic is different between the two states. In the high-magnitude asymmetry state, the lower-tail dependence is larger than the upper-tail dependence. This finding supports the viewpoint of existing empirical studies, which state that international stock markets experience stronger co-movements during extremely pessimistic markets than in extremely optimistic markets. In contrast, there is a distinct finding that shows more dependence in the upper tails than in the lower tails during the period of a low-magnitude asymmetry state for the US-UK pair.

Third, the importance of the bilateral exchange rate on the upper-tail dependence and the lower-tail dependence can be clearly observed for the US-Canada pair and the US-UK pair, but not for the US-France and US-Germany pairs. The asymmetric effect of the bilateral exchange rate is especially significant during the period of a high-magnitude asymmetry state, but the asymmetric effect cannot be observed during the period of a low-magnitude asymmetry state. The appreciation in the U.S. dollar compared to the Canadian dollar results in an increase in the lower-tail dependence between the U.S. and Canadian equity markets and a decrease in the upper-tail dependence. For the US-UK pair, the appreciation in the U.S. dollar can increase the lower-tail dependence, and the impact of appreciation on the upper-tail dependence can also be found.

The new empirical findings provide valuable information to investors and policy makers. The U.S. equity market has a higher degree of linkage with four international equity markets after 2000, and the magnitude for co-movements is smaller in a bull market than in a bear market, showing a greater possibility that the U.S. and international equity returns decrease simultaneously, compared to the situation in which the U.S. and international equity returns increase simultaneously. Accordingly, the diversification benefit drastically decreases when the U.S. and international equity markets are in a bear market, while a lesser degree of reduction in diversification appears in a bull market. In sum, the bilateral exchange rate can partly affect the diversification benefit.

The equity price is a leading index of economic activity. The rise (or crash) in equity prices will accelerate (or slow) economic growth, resulting in an overheated economy (or economic recession). When the government authority of a developed country adopts a contractionary policy to cool down the equity market without considering the U.S. equity market condition, the spillover effect from rising U.S. equity prices will partly offset the policy strength of the initial contraction policy. On the other hand, a drop in equity prices will lead to an economic recession. A similar offset effect can be made for the expansion policy due to its high linkage with the U.S. equity market. Therefore, the authorities cannot design policies without considering the possible spillover effect from the U.S. equity market.

These empirical findings are observed from developed equity markets. The financial markets of BRICS economies have received more attention in the last decade; however, the equity markets and exchange rate markets of the BRICS countries are not fully liberalized. Whether similar results can be observed among the BRICS economies is worthy of future research. On the other hand, whether and how dependence structures between international equity markets vary across frequencies and sources of equity indices is worthy of further investigation in the future..

CRediT authorship contribution statement

Kuang-Liang Chang: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Funding acquisition, Data curation.

Data availability

The authors do not have permission to share data.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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