



Liquidity-constrained consumers and optimal monetary policy in a currency union [☆]

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ABSTRACT

This paper examines optimal monetary policy in a currency union model with liquidity-constrained (LC) consumers. The result demonstrates that given LC consumers in the home country, the response of macroeconomic variables to a foreign structural shock is generally dampened as a share of foreign LC consumers increases. This paper finds that the costs of discretionary policy are generally not greater in a currency union in which LC consumers exist in both countries. We address that the presence of nominal wage rigidity significantly lowers welfare losses in our model.

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1. Introduction

This study investigates the impact of liquidity-constrained (LC) consumers on optimal monetary policy in a currency union model. Although the standard new Keynesian (NK) model has been analyzed with the assumption of representative households (Woodford, 2003), the importance of LC consumers has been pointed out in several studies (Campbell and Mankiw, 1989; Bilbiie, 2008; Galí et al., 2004; Galí et al., 2007). LC consumers refer to households with limited access to financial assets who cannot intertemporally behave according to an optimal decision based on a consumption Euler equation.¹ This indicates that their disposable income constrains their consumption in the current period. Conversely, households with free access to financial markets are referred to as Ricardian households. Therefore, the transmission mechanism of monetary policy into the real economy naturally differs in the presence of LC consumers. The impact of LC consumers on monetary policy has been discussed in a closed economy model, but not fully in an open economy model.

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¹ Unless otherwise noted, we refer to households that do not have free access to financial assets as LC consumers. Several naming schemes for LC households have been proposed. For instance, Galí et al. (2007) refer to LC households as rule of thumb households.

The motivation of this paper is based on the empirical fact that the share of LC consumers differs across the member countries in the Euro area. For instance, Kaplan, Violante and Weidner (2014) pointed out that for the United States, Canada, Australia, the United Kingdom, Germany, France, Italy, and Spain, the share of households that face liquidity constraints is between 20% and 35%. Moreover, Almgren et al. (2019) reported that the fraction of households that face liquidity constraints ranges from 10% in Malta to almost 65% in Latvia. In addition, several studies argued that the presence of LC consumers generates a non-negligible effect on the fiscal policy (Coenen et al., 2012; Forni et al., 2009). We relate the motivation of this paper to these empirical findings in examining the optimal monetary policy for a currency union.

We also focus on the role of nominal wage rigidity in a currency union. Several studies have examined the role of nominal wage rigidity in a dynamic stochastic general equilibrium (DSGE) model (Christiano et al., 2005). Recently, Iwasaki et al. (2021) argued the significant role of wage inflation in Europe using a non-linear DSGE model. In addition, Palomino et al. (2020) focused on the effect of the lockdown and social distancing measures precipitated by the coronavirus disease 2019 (COVID-19) pandemic on wage inequality in Europe. These studies enable us to address the role of nominal wage rigidity in a currency union. We demonstrate that the presence of nominal wage rigidity can reduce welfare loss in our model.

The present study examines optimal monetary policy in a two-country NK model under a currency union, considering both LC consumers and nominal wage rigidity. Specifically, the model focuses on a currency union in the standard two-country model developed by Clarida et al., 2002) and Benigno (2004) and introduces LC consumers (Bilbiie, 2008). We also introduce nominal wage rigidity to the two countries based on Ascari et al., 2017). To our knowledge, no study has attempted such an extension.

The main findings of this study are as follows. We demonstrate that consumption risk-sharing in the two countries is significantly affected by shares of LC consumers. We show that although Ricardian households have complete risk-sharing both domestically and internationally, the level of aggregate consumption in one country is affected by the share of LC consumers in both countries. Thus, we underline that the presence of LC consumers in each country makes the international financial market incomplete in terms of aggregate consumption.

We show that LC consumers in both countries affect both price and wage NK Phillips curves through changes in both home and foreign real wages. We derive a well-defined loss function by calculating a quadratic approximation of the household utility function. Our loss function contains inflation and the output gap for both countries (Benigno, 2004; Clarida et al., 2002). It also includes the stabilization terms of wage inflation and real wages. The former is due to the presence of nominal wage rigidities (Erceg, Henderson and Levin, 2000), whereas the latter is due to the presence of LC consumers (Ascari et al., 2017). Notably, we address the stabilization term on the co-movement in home and foreign real wages, which is the source of the welfare gains in our model.

In our model, we emphasize the importance of exploring the welfare comparison between commitment and discretionary policies. In general, the performance of a commitment policy is considered to be superior to that of a discretionary policy (McCallum and Nelson, 2004; Woodford, 2003). Contrary to a discretionary policy, a commitment policy can alleviate fluctuations in macrovariables because it introduces policy inertia by manipulating the expectations of the private sector, as argued in Woodford (2003). This argument is incorporated into the open economy model. However, in our model, the ability of the central bank to manage the private sector's expectations is hampered by the increasing proportion of LC consumers in both countries. Moreover, nominal wage rigidity undermines the ability of a commitment policy to impart policy inertia to the economy. Therefore, it is not evident that a discretionary policy results in substantial welfare costs relative to a commitment policy in our model.

Our model shows that an increased share in LC consumers can contribute to a reduction in welfare losses under both discretion and commitment policies. Indeed, given the share in LC consumers in the home country, optimal monetary policy implies that the response of macroeconomic variables to foreign structural shocks is generally attenuated as the share in foreign LC consumers increases. We find that nominal wage stickiness in both countries significantly reduces the costs arising from discretion. In particular, unlike previous studies specifying the substantial costs of discretion, the discretionary policy costs are not greater than commitment policy costs, unless nominal wages are highly sticky only in the foreign country.

The remainder of this paper is structured as follows. Section 2 briefly reviews the related literature. Section 3 describes the two-country NK model with LC consumers. This paper focuses on deriving the two-country model under a currency union. Section 4 discusses optimal monetary policy in the model. Section 5 reports the main results, and Section 6 provides additional results. Finally, Section 7 concludes the paper.

2. Related literature

This section briefly reviews previous studies on the effect of LC consumers on monetary policy in an NK model. More concretely, Section 2.1 reviews the impact of LC consumers on monetary policy. Section 2.2 discusses the role of LC consumers in open economy models. Section 2.3 addresses the contribution of this paper to previous studies.

2.1. Impact of LC consumers on monetary policy

First, several previous studies have examined the impact of LC consumers on monetary policy (Airaud, 2013; Amato and Laubach, 2003; Bilbiie, 2008; Galí et al., 2004). Unlike the standard representative agent NK (RANK) model, the presence of LC

consumers significantly affects the uniqueness of the rational expectations equilibrium (Airaudo, 2013; Bilbiie, 2008; Galí et al., 2004) and optimal monetary policy (Amato and Laubach, 2003; Bilbiie, 2008). In addition, Galí et al. (2007) demonstrated a mechanism in which the fiscal policy shock causing a decline in consumption is eliminated by assuming the presence of LC consumers in the standard NK model. Colciago (2011) focused on the role of nominal wage rigidity in an NK model with LC consumers. Moreover, Areosa and Areosa (2016) examined optimal monetary policy when the presence of LC consumers generated income inequality as measured by the Gini coefficient. Ascari et al. (2017) explored the effect of LC consumers on optimal monetary policy in an economy with nominal price and wage rigidity.

2.2. The role of LC consumers in an open economy

How important are LC consumers considered to be in an open economy? To the best of our knowledge, very few previous studies have addressed this issue. Several studies have analyzed optimal monetary policy in a two-country NK model (Benigno, 2004; Benigno and López-Salido, 2006; Clarida et al., 2002; Pappa, 2004). In an open economy, the key to the international monetary transmission can be captured by the presence of international consumption risk-sharing. Benigno (2004) and Benigno and López-Salido (2006) examined optimal monetary policy in a currency union. Meanwhile, Groll and Monacelli (2020) explored the cost of implementing a currency union compared to implementing an exchange rate float in a two-country NK model.² Clarida et al. (2002) showed that, given the existence of a complete market in which the home and foreign households can trade state-contingent securities domestically and internationally, the home and foreign countries will have identical consumption, under the assumption of producer currency pricing and no home bias. They demonstrated the gains from international policy coordination when home and foreign central banks jointly implemented an optimal discretionary policy. Pappa (2004) examined the gain from policy coordination in a two-country NK model in which the consumption basket was constructed by a constant elasticity of substitution (CES) aggregate. Moreover, Campolmi (2014) and Rhee and Turdaliev (2013) focused on the role of nominal wage rigidity in a small-open NK model. They showed that stabilizing consumer price index (CPI) inflation led to outcomes preferable to those under producer price inflation (PPI) inflation targeting.³ This study is closely related to Iyer (2016), who examined the role of LC consumers in a small-open NK model. Iyer (2016) analytically derived the central bank's loss function and investigated the effect of LC consumer shares on optimal monetary policy. We address that except for Iyer (2016), the above studies focus on the RANK model.

2.3. Contribution of this paper to previous studies

The main contribution of this paper is thoroughly examining the impact of LC consumers on optimal monetary policy in an NK model under a currency union. Our motivation is the fact that a share of LC consumers differs between the two countries. Further, we examine different degrees of wage stickiness in Europe and introduce wage stickiness to the two countries. More precisely, the contributions of this paper is threefold.

First, as noted earlier, this study is closely related to Iyer (2016). Iyer (2016) focused on the case for emerging countries, whereas we consider how shares of home and foreign LC consumers change the properties of optimal monetary policy in a currency union. Iyer (2016) derived the central bank's loss function by calculating the second-order approximation of the utility function. Similarly, we also derive the loss function in a currency union. In our model, both price and wage NK Phillips curves are affected by a share in LC consumers due to incomplete international financial markets. In contrast to the model of Iyer (2016), we consider the interaction between LC consumers and nominal wage rigidities. Therefore, our model is suitable for considering optimal monetary policy in a currency union, such as the Euro area.

Second, this study is related to Ascari et al. (2017) and Bilbiie (2008), who analyzed LC consumers and optimal monetary policy, but in a closed economy model. In terms of the currency union model, this paper is also related to Bhattarai et al. (2015). Palek and Schwanebeck (2019) introduced heterogeneity by incorporating financial friction, but they did not consider the role of LC consumers in a currency union. In their model, heterogeneity exists among households and financial institutions. In contrast to these studies, this paper shows that the international financial market becomes incomplete when LC consumers are present in both countries. We address that a rise in LC consumers in both countries helps reduce welfare losses in our model.

Third, this paper is also related to Campolmi (2014), Galí and Monacelli (2016), Groll and Monacelli (2020), Rhee and Turdaliev (2013). In particular, Campolmi (2014) examined optimal monetary policy in a small-open economy with nominal price and wage rigidity. She showed that stabilizing CPI inflation led to outcomes preferable to those under PPI inflation targeting. As pointed out by Rhee and Turdaliev (2013), the terms of trade depend on the real wage gap in an open economy. Meanwhile, Groll and Monacelli (2020) argued the importance of terms of trade inertia in a two-country NK model with complete wage flexibility. However, we underscore the role of LC consumers and therefore emphasize the substantial impact of the nominal wage rigidity on the welfare costs arising from discretion in our model. This is because in the two-country

² Groll and Monacelli (2020) compared the costs of a currency union with the costs of an exchange rate float in both the discretionary and commitment cases. When the terms of trade are characterized by inertial behavior, the home and foreign central banks that conduct monetary policy with discretion can benefit from currency union. This comparison is an intriguing topic, but it is outside the scope of this paper.

³ Note that since we focus on the case for optimal monetary policy, we do not consider specifically which inflation central banks should target using simple rules.

model, the terms of trade change in response to fluctuations in home and foreign output. Moreover, inertial changes in wage rates now translate into changes in the terms of trade. This mechanism was not addressed in the aforementioned studies. As a result, both LC consumers and nominal wage rigidity in both countries significantly impact the structural equations and the central bank's loss function derived by the second-order approximation of the household's utility function. Although [Galí and Monacelli \(2016\)](#) focused on the role of wage flexibility in a currency union, they did not examine the impact of LC consumers on optimal monetary policy.

Finally, we would like to comment on the heterogeneous agent NK (HANK) model, which could be used to analyze optimal monetary policy by considering the existence of more heterogeneous households ([Kaplan et al., 2014](#)). First, intuitively understanding the economy's structure in this model is difficult, since the central bank's policy objectives cannot be explicitly derived. In the TANK model, the log-linearized structural equations are derived more easily and intuitively than in the HANK model. Second, since the model is a currency union and its size is larger than the closed NK model, taking HANK into account would be computationally burdensome, and optimal monetary policy may not be derived, even with a numerical solution. Third, the optimal monetary policy for a closed economy cannot be easily and appropriately evaluated. Fourth, according to [Debortoli and Galí \(2017\)](#), although no clear difference exists between HANK and TANK in terms of aggregate shocks, we can adopt the TANK model as an approximation in this study.

3. The model

As mentioned earlier, we incorporate nominal wage rigidity and LC consumers into the two-country NK model developed by [Clarida et al. \(2002\)](#). As in [Benigno \(2004\)](#), we focus on the case for a currency union.⁴ The model developed here differs from the existing literature in that we investigate optimal monetary policy in a currency union with LC consumers.

Consider an economy with two large symmetrical countries: home and foreign. The economy sizes for home and foreign are $1 - \gamma$ and γ , respectively. Each country has two production sectors: a final goods sector characterized by perfect competition and an intermediate goods sector in which firms face monopolistic competition and set prices according to [Calvo \(1983\)](#)-type nominal price rigidity. This paper assumes that our model holds the law of one price (LOOP). This setting is based on [Clarida et al. \(2002\)](#).

In each country, households obtain utility from consumption and disutility from supplying labor. A fraction of $1 - \lambda$ is comprised of Ricardian households that can freely access financial markets in each country. Ricardian households in each country have access to a complete set of state-contingent securities traded domestically and internationally. Meanwhile, in each country, a fraction of λ represents LC consumers that cannot trade financial assets in the financial market. Following [Ascari et al. \(2017\)](#), we assume that there exists a labor union that determines the nominal wage. Each union faces Calvo-type nominal wage rigidity.⁵

Finally, unless otherwise noted, analogous equations hold for the foreign country. Asterisks represent foreign variables.

3.1. Households

3.1.1. Consumption preferences

Preferences for consumption in the home country are given by:

$$C_t \equiv C_{H,t}^{1-\gamma} C_{F,t}^\gamma, \quad (1)$$

where $C_{H,t}$ and $C_{F,t}$ are the consumption of home and foreign goods, respectively. The price index in the home country is given by:

$$P_t = \kappa^{-1} P_{H,t}^{1-\gamma} P_{F,t}^\gamma = \kappa^{-1} P_{H,t} S_t^\gamma, \quad (2)$$

where $\kappa \equiv (1 - \gamma)^{(1-\gamma)} \gamma^\gamma$, $P_{H,t}$ is the price of home goods, and $P_{F,t}$ is the price of foreign goods. In addition, S_t represents the terms of trade, which is given by:

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}. \quad (3)$$

3.1.2. Labor market

Following [Ascari et al. \(2017\)](#), we assume that each household's nominal wage is set by the labor type-specific union, indexed by $j \in [0, 1]$. The nominal wage is fixed by union j . With this setup, the labor supply is given by:

⁴ As will be mentioned in the conclusion, if we introduce an exchange rate float into the two-country model with LC consumers, we cannot explicitly derive the central bank's loss function by implementing a second-order approximation of the household utility function. In this case, we have to solve optimal monetary policy under the non-linear Ramsey policy. Although this is beyond the scope of this paper, we would like to consider this issue as future work.

⁵ See also [Erceg et al. \(2000\)](#) for a detailed discussion of nominal wage rigidity and nominal price rigidity.

$$L_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} L_t^d, \quad (4)$$

where W_t^j is the nominal wage set by union j , W_t denotes the aggregate nominal wage, L_t^j denotes the labor supply in union j , and L_t^d denotes the demand for labor. The parameter θ_w is the elasticity of substitution for individual labor. Following [Ascari et al. \(2017\)](#), we can eliminate the discrepancy in the wage difference between households since we assume that L_t^j is identical for both Ricardian and LC consumers:

$$L_t = \int_0^1 L_t^j dj. \quad (5)$$

Therefore, we obtain the following common labor income:

$$\int_0^1 W_t^j L_t^j dj = L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj. \quad (6)$$

3.1.3. Ricardian households

A fraction of $1 - \lambda$ is comprised of Ricardian households that can freely access financial markets. The intertemporal utility of Ricardian households is as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t = E_0 \sum_{t=0}^{\infty} \beta^t \{u(C_{o,t}) - V(L_{o,t})\}, \quad (7)$$

where $C_{o,t}$ and $L_{o,t}$ denote Ricardian household's consumption and labor supply, respectively. We assume that the utility function, $u(\cdot)$, is strictly concave and continuously differentiable, whereas the disutility of supplying labor, $V(\cdot)$, is strictly convex and continuously differentiable.

The representative household maximizes the above utility function, subject to the following budget constraint:

$$P_t C_{o,t} + E_t [Q_{t,t+1} B_{o,t+1}] = B_{o,t} + L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj + \Gamma_{o,t} - T_{o,t}, \quad (8)$$

where $B_{o,t}$ represents nominal bonds held for one period, and $\Gamma_{o,t}$ denotes the dividends earned from home firms. $T_{o,t}$ denotes a lump-sum tax. We assume that Ricardian households can access state-contingent bonds traded domestically and internationally in a complete market and introduce the following stochastic discount factor:

$$E_t(Q_{t,t+1}) = \frac{1}{1 + r_t}, \quad (9)$$

where $E_t(Q_{t,t+1})$ denotes a stochastic discount factor, and r_t is the union-wide, risk-free, short-term nominal interest rate.

From the optimal condition, we obtain the familiar Euler consumption equation.

$$E_t(Q_{t,t+1}) = \frac{1}{1 + r_t} = \beta E_t \left[\frac{u_c(C_{o,t+1})}{u_c(C_{o,t})} \frac{P_t}{P_{t+1}} \right]. \quad (10)$$

Ricardian households in the foreign country can trade state-contingent bonds both domestically and internationally; thus, the consumption Euler equation holds for foreign Ricardian households as follows:

$$E_t(Q_{t,t+1}) = \frac{1}{1 + r_t} = \beta E_t \left[\frac{u_c(C_{o,t+1}^*)}{u_c(C_{o,t}^*)} \frac{P_t^*}{P_{t+1}^*} \right], \quad (11)$$

where an asterisk denotes a foreign variable. At the level of Ricardian households, complete markets for state-contingent nominal bonds exist both domestically and internationally, so complete consumption risk-sharing can be established between Ricardian households in the two countries. Thus, we obtain the following:

$$C_{o,t} = C_{o,t}^*. \quad (12)$$

If LC consumers are not present in both countries, the risk-sharing condition corresponds to the case presented by [Benigno \(2004\)](#) and [Clarida et al. \(2002\)](#). In addition, $P_t = P_t^*$ holds in a currency union. However, as we will demonstrate, international consumption risk-sharing is not feasible at the aggregate consumption level because LC households exist in both countries in this paper.

3.1.4. LC consumers

Following Galí et al. (2007), Bilbiie (2008), and Ascari et al. (2017), this paper assumes that a fraction of λ cannot access financial markets in each country. Therefore, LC consumers face the same periodic utility function as Ricardian households,⁶ but their budget constraints are given by:

$$P_t C_{r,t} = L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{\bar{W}_t} \right)^{-\theta_w} dj. \tag{13}$$

Thus, in contrast to Ricardian households, since LC consumers cannot implement intertemporal consumption smoothing by trading state-contingent bonds in bond markets, they consume their entire disposable income every period.

3.1.5. Wage determination

Both Ricardian and LC consumers delegate the role of wage determination to a labor union.⁷ Following Ascari et al. (2017), the wage setting is subject to Calvo-type staggered wage contracts. Thus, a fraction of $1 - \zeta$ can change nominal wages in its union, whereas a remaining fraction of ζ cannot do so. With this setting, the labor union for each country solves the following maximization problem:

$$\max_{\bar{W}_t} E_t \sum_{k=0}^{\infty} (\zeta \beta)^k [(1 - \lambda)u(C_{0,t+k}) + \lambda u(C_{r,t+k}) - V(L_{t+k})]. \tag{14}$$

where \bar{W}_t denotes the optimal nominal wage. The first-order condition of this problem is given by:

$$E_t \sum_{k=0}^{\infty} (\zeta \beta)^k V_L(L_{t+k}) L_{t+k}^d W_{t+k}^{\theta_w} \left[\left(\frac{\lambda}{MRS_{0,t+k}} + \frac{1 - \lambda}{MRS_{r,t+k}} \right) \frac{\bar{W}_t}{P_{t+k}} - \mu^w \right] = 0, \tag{15}$$

where $\mu^w = \theta_w / (\theta_w - 1)$.

The aggregate wage index is defined as follows:

$$W_t = \left[\frac{1}{1 - \gamma} \int_0^1 W_t(j)^{1 - \theta_w} dj \right]^{\frac{1}{1 - \theta_w}}. \tag{16}$$

Under a Calvo-type wage setting, this equation can be rewritten as

$$W_t = \left[(1 - \zeta) \bar{W}_t^{1 - \theta_w} + \zeta W_{t-1}^{1 - \theta_w} \right]^{\frac{1}{1 - \theta_w}}. \tag{17}$$

3.2. Firms

Each country has two production sectors: the final and intermediate goods sectors. The final goods sector produces final goods using intermediate goods and is characterized by perfect competition. Meanwhile, in the intermediate goods sector, firms face monopolistic competition and Calvo (1983)'s nominal price rigidity.

3.2.1. Final goods firms

Final goods are produced according to the following CES aggregate:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta_{p,t} - 1}{\theta_{p,t}}} di \right]^{\frac{\theta_{p,t}}{\theta_{p,t} - 1}}, \tag{18}$$

where Y_t is aggregate output, $Y_t(i)$ is demand for intermediate goods produced by firm i , and $\theta_{p,t}$ is the elasticity of substitution, which is time-varying, as assumed by Steinsson (2003).⁸ Note that Y_t and $Y_t(i)$ are normalized by $1 - \gamma$.

Under the CES aggregate, the demand function is given by:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta_{p,t}} Y_t, \tag{19}$$

and the domestic price level is defined as follows:

⁶ Thus, the properties of the Ricardian household's utility function are carried over to the LC consumers.

⁷ See Ascari et al. (2017) and Galí et al. (2007) for a detailed discussion of the labor union specification in the model.

⁸ Clarida et al. (2002) presumed a time-varying wage markup to introduce an exogenous cost-push shock. In this study, instead of introducing a wage markup, a price markup derived from a time-varying elasticity of substitution $\theta_{p,t}$ is introduced to examine the effect of a cost-push shock on optimal monetary policy. See Clarida et al. (2002) for a detailed discussion of the presence of a wage markup shock.

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\theta_{p,t}} di \right]^{\frac{1}{1-\theta_{p,t}}}, \quad (20)$$

where $P_{H,t}(i)$ is the price for intermediate goods produced by firm i .

3.2.2. Intermediate goods firms

The intermediate goods sector is characterized by monopolistic competition, and each firm produces a differentiated intermediate good. Firm i 's production function is given by:

$$Y_t(i) = A_t L_t(i), \quad (21)$$

where $L_t(i)$ denotes labor supply employed in firm i and A_t denotes an aggregate productivity disturbance, which follows an autoregressive (AR)(1) process given by $\log A_t = \rho_a \log A_{t-1} + \epsilon_t^a$ with $0 \leq \rho_a < 1$, where ϵ_t^a is an independent and identically distributed (i.i.d.) shock with constant variance σ_a^2 .

The intermediate firm's real marginal cost φ_t is given as follows:

$$\varphi_t = (1 - \tau) \frac{W_t}{P_{H,t}} \frac{1}{A_t}, \quad (22)$$

where τ denotes the subsidies, which is used to eliminate the price and wage markup distortions arising from the monopolistic competition in both intermediate goods sector and a labor union.

Following Calvo (1983), we assume that nominal price rigidity is present in the intermediate goods sector. The following explanation focuses on the home country, whereas we can also consider a similar case for the foreign country. Thus, a fraction $1 - \omega$ of all firms adjusts their price, whereas the remaining fraction of firms ω does not.

We now consider the intermediate firms that can adjust their prices. When revising their prices, these firms account for the uncertainty regarding when they will next be able to adjust prices. In this case, the intermediate firm's optimization problem for the home country is given by:

$$E_t \sum_{k=0}^{\infty} \omega^k Q_{t,t+k} Y_{t+k}(i) \left(P_{H,t}^{opt} - P_{H,t+k} \varphi_{t+k} \right), \quad (23)$$

where $P_{H,t}^{opt}$ is the firm's optimal price. The first-order condition of this maximization problem is as follows:

$$E_t \sum_{k=0}^{\infty} \omega^k Q_{t,t+k} Y_{t+k}(i) \left(P_{H,t}^{opt} - \mu_{t+k}^p P_{H,t+k} \varphi_{t+k} \right) = 0, \quad (24)$$

where the variable $\mu_t^p = \theta_{p,t} / (\theta_{p,t} - 1)$ is the time-varying price markup.

3.3. Equilibrium

We now describe the equilibrium conditions in an open economy. The equilibrium conditions for the goods market are given as:

$$(1 - \gamma) Y_t = (1 - \gamma) C_{H,t} + \gamma C_{H,t}^*, \quad (25)$$

$$\gamma Y_t^* = (1 - \gamma) C_{F,t} + \gamma C_{F,t}^*. \quad (26)$$

Under the Cobb–Douglas consumption index, we obtain the following demand function for $C_{H,t}$ and $C_{F,t}$:

$$C_{H,t} = (1 - \gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t, \quad (27)$$

$$C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t. \quad (28)$$

Substituting these equations and the corresponding foreign ones into (25) yields:

$$Y_t = \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t^u, \quad (29)$$

where $C_t^u = (1 - \gamma) C_t + \gamma C_t^*$. Similarly,

$$Y_t^* = \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t^u. \quad (30)$$

At this point, the home terms of trade are represented by the ratio of home output to foreign output as follows:⁹

$$S_t = \frac{Y_t}{Y_t^*}. \quad (31)$$

Eq. (31) indicates that holding home output constant, an increase in foreign output leads to an appreciation of the home terms of trade.

The Ricardian households in each country can trade state-contingent bonds both domestically and internationally; hence, the market-clearing conditions for the bond markets are as follows:

$$\begin{aligned} B_{o,t} &= (1 - \lambda)B_t, \\ B_{o,t}^* &= (1 - \lambda^*)B_t^*, \\ (1 - \gamma)B_t + \gamma B_t^* &= 0. \end{aligned}$$

Aggregate consumption and dividends are given as

$$C_t = (1 - \lambda)C_{o,t} + \lambda C_{r,t}, \quad (32)$$

$$C_t^* = (1 - \lambda^*)C_{o,t}^* + \lambda^* C_{r,t}^*, \quad (33)$$

$$\Gamma_{o,t} = (1 - \lambda)\Gamma_t,$$

$$\Gamma_{o,t}^* = (1 - \lambda^*)\Gamma_t^*.$$

3.4. Log-linearization

3.4.1. The effect of LC consumers in a currency union

This section provides the log-linearized equations derived in the previous section. We define $\hat{H}_t = \log(H_t/\bar{H})$ as the deviation of H_t from the steady state. \bar{H} represents the value of the steady state.

The log-linearization of Eqs. (32) and (33) leads to the following:

$$\hat{C}_t = (1 - \lambda)\hat{C}_{o,t} + \lambda\hat{C}_{r,t}, \quad (34)$$

$$\hat{C}_t^* = (1 - \lambda^*)\hat{C}_{o,t}^* + \lambda^*\hat{C}_{r,t}^*. \quad (35)$$

Ricardian households in both countries can trade state-contingent bonds both domestically and internationally. Thus, international consumption risk-sharing holds for each country's Ricardian households as follows:

$$\hat{C}_{o,t} = \hat{C}_{o,t}^*. \quad (36)$$

Importantly, this paper addresses that even if consumption risk-sharing holds for Ricardian households in each country, it cannot hold at the aggregate consumption level. More precisely, log-linearizing Eq. (29) and the corresponding equation for the foreign country and using Eqs. (34)–(36) yield the consumption level for Ricardian households:

$$\hat{C}_{o,t} = \frac{1}{\Omega} \left[(1 - \gamma)(\hat{Y}_t - \lambda\hat{C}_{r,t}) + \gamma(\hat{Y}_t^* - \lambda^*\hat{C}_{r,t}^*) \right], \quad (37)$$

$$\hat{C}_{o,t}^* = \frac{1}{\Omega} \left[\gamma(\hat{Y}_t^* - \lambda^*\hat{C}_{r,t}^*) + (1 - \gamma)(\hat{Y}_t - \lambda\hat{C}_{r,t}) \right], \quad (38)$$

where $\Omega = (1 - \gamma)(1 - \lambda) + \gamma(1 - \lambda^*)$. It follows from these equations that if home or foreign LC consumers' consumption increases, home Ricardian households' consumption decreases.

In addition, according to budget constraints for LC households, LC consumers' consumption increases in response to an increase in both home and foreign real wages. However, an increase in real wages leads to a decline in Ricardian consumption. To confirm this mechanism, we substitute log-linearization of budget constraints for LC consumers and production functions for both countries into Eqs. (37) and (38), which leads to

$$\hat{C}_{o,t} = \frac{1}{\Omega} \left[(1 - \gamma)(1 - \lambda)\hat{Y}_t - \lambda(1 - \gamma)(\hat{w}_t - A_t) + \gamma(1 - \lambda^*)\hat{Y}_t^* - \lambda^*\gamma(\hat{w}_t^* - A_t^*) \right], \quad (39)$$

$$\hat{C}_{o,t}^* = \frac{1}{\Omega} \left[\gamma(1 - \lambda^*)\hat{Y}_t^* - \lambda^*\gamma(\hat{w}_t^* - A_t^*) + (1 - \gamma)(1 - \lambda)\hat{Y}_t - \lambda(1 - \gamma)(\hat{w}_t - A_t) \right]. \quad (40)$$

These equations indicate that the home country's Ricardian consumption is reduced not only by the home country's real wage but also by the increase in the foreign country's real wage. This is because international consumption risk-sharing becomes incomplete when LC consumers are present in both countries. Therefore, the fact that $C_{o,t} = C_{o,t}^*$ holds does not mean that $C_t = C_t^*$ also holds.

3.4.2. Equilibrium under flexible prices and wages

When prices and wages are flexible in both countries, the natural rate of output for each country is

$$(\eta + \sigma + D_0 - D_1)\widehat{Y}_t^n + (D_1 - D_0)\widehat{Y}_t^{n,*} = (1 + \eta + D_0)A_t - D_0\widehat{W}_t^n + \frac{\lambda^*(1-\lambda)\gamma\sigma}{\Omega}(\widehat{W}_t^{n,*} - A_t^*), \tag{41}$$

$$(\eta + \sigma + D_0^* - D_1^*)\widehat{Y}_t^n + (D_1^* - D_0^*)\widehat{Y}_t^{n,*} = (1 + \eta + D_0^*)A_t^* - D_0^*\widehat{W}_t^{n,*} + \frac{\lambda(1-\lambda^*)(1-\gamma)\sigma}{\Omega}(\widehat{W}_t^n - A_t), \tag{42}$$

where

$$1 - \psi = \frac{(1-\gamma)(1-\lambda)}{\Omega}, \quad \psi = \frac{\gamma(1-\lambda^*)}{\Omega}, \quad D_1 = \sigma\psi - \gamma, \quad D_0 = \sigma\psi\lambda, \quad D_1^* = \sigma(1 - \psi) - (1 - \gamma), \quad D_0^* = \sigma(1 - \psi)\lambda^*.$$

Also, $\sigma = -u_{cc}C/u_c$ and $\eta = v_{ll}L/v_l$. The open economy effect is captured by parameter D_1 , which consists of both the terms of trade and the international risk-sharing effects of Ricardian households in both countries. Importantly, unlike the standard two-country NK model (Clarida et al., 2002), D_1 is affected by a change in LC consumers for both countries through a change in ψ . The share of LC consumers in each country also significantly impacts the value of D_0 . Importantly, even when LC consumers completely disappear in the home country, $1 - \psi$ does not correspond to $1 - \gamma$, as long as the share of LC consumers in the foreign country is not zero. Consequently, the presence of home and foreign LC consumers causes fluctuations in macrovariables in both countries even under flexible price and wage equilibrium. Note that the case in which $\lambda = \lambda^* = 0$ corresponds to the standard two-country NK model (Clarida et al., 2002). Similar discussions hold for the foreign country.

To understand the role of LC consumers in our model, we solve the above equations for the natural rate of output, which is given as follows:

$$\widehat{Y}_t^n = \frac{Y_{ya}^h A_t - Y_{ya,*}^h A_t^*}{Y}, \tag{43}$$

$$\widehat{Y}_t^{n,*} = \frac{Y_{ya,*}^f A_t^* - Y_{ya}^f A_t}{Y}, \tag{44}$$

where

$$Y_{ya}^h = [(1 + \eta) - D_0](\sigma + \eta + D_0^* - D_1^*), \quad Y_{ya,*}^h = [(1 + \eta) - D_0^*](D_1 - D_0), \\ Y_{ya,*}^f = [(1 + \eta) - D_0^*](\sigma + \eta + D_0 - D_1), \quad Y_{ya}^f = [(1 + \eta) - D_0](D_1^* - D_0^*), \\ Y = (\sigma + \eta + D_0 - D_1)(\sigma + \eta + D_0^* - D_1^*) - (D_1 - D_0)(D_1^* - D_0^*).$$

Since we easily confirm that the denominator becomes positive, we focus on the numerator in the following discussion. As in the standard two-country model (Clarida et al., 2002; Pappa, 2004), our model demonstrates that both home and foreign natural output depend only on productivity shocks in each country. On the one hand, if the CRRA coefficient (σ) does not take an extremely larger value, we can say $Y_{ya}^h > 0$. Therefore, home positive productivity shock generally increases the home natural rate of output. In contrast to the standard two-country NK model, an increase in λ weakens the impact of the home productivity shock on home natural rate of output through a change in D_0 , whereas an increase in λ^* magnifies its impact through a change in D_0^* . On the other hand, if $Y_{ya,*}^h > 0$, the foreign positive productivity shock leads to a decline in the home natural rate of output. Since we can easily confirm that $1 + \eta > D_0^*$, this response corresponds to the case where $D_1 > D_0$. If $D_1 < D_0$, the foreign positive productivity shock induces an increase in the home natural rate of output.

These results are interpreted as follows. In our model, real wages positively move with a change in the marginal product of labor proportionally affected by productivity shocks. Such a change in real wages is translated into a fluctuation in LC consumers' consumption via their disposable income. Accordingly, Eqs. (41) and (42) imply that when incomplete aggregate consumption risk-sharing is caused by the presence of LC consumers in both countries, home and foreign real wages affect home and foreign natural output through a change in consumption of each country's LC consumers. These channels disappear when LC consumers are absent in both countries.

We also demonstrate consumption under flexible price and wage equilibrium. In this explanation, we focus on home aggregate consumption:

$$\widehat{C}_t^n = \frac{[(1 - \gamma)(1 - \lambda) + \gamma\lambda(1 - \lambda^*)]\widehat{Y}_t^n + \gamma(1 - \lambda)(1 - \lambda^*)\widehat{Y}_t^{n,*}}{\Omega}. \tag{45}$$

It follows from Eq. (45) that a change in the positive foreign productivity shock causes a fluctuation in home consumption through a change in $\widehat{Y}_t^{n,*}$. More concretely, by substituting Eq. (43) into Eq. (45), we can show that home consumption is affected by both home and foreign productivity shocks as follows:

$$\widehat{C}_t^n = \frac{Y_{ca}^h A_t - Y_{ca,*}^h A_t^*}{\Omega Y}, \tag{46}$$

where

$$Y_{ca}^h = (1 - \gamma)(1 - \lambda)Y_{ya}^h - \gamma(1 - \lambda^*) \left[1 - \lambda(Y_{ya}^h + Y_{ya}^f) \right],$$

$$Y_{ca,*}^h = (1 - \gamma)(1 - \lambda)Y_{ya,*}^h - \gamma(1 - \lambda^*) \left[1 - \lambda(Y_{ya,*}^h + Y_{ya,*}^f) \right].$$

We cannot proceed with further manipulations because of the complexity of these coefficients. However, we may provide an intuitive explanation of the effect of productivity shocks on aggregate consumption based on (46). Eq. (46) states that an increase in home productivity positively affects home aggregate consumption and that increased foreign productivity generally seems to induce a decline in home consumption. However, this equation indicates that whether an increase in foreign productivity causes a decline in home consumption depends on a share in LC consumers in both countries due to incomplete international risk-sharing in aggregate consumption. A similar discussion holds for foreign aggregate consumption.

Finally, if LC consumers are absent in both countries, Eqs. (41) and (42) reduce to the following:

$$[\eta + \sigma + \gamma(\sigma - 1)]\hat{Y}_t^n + \gamma(\sigma - 1)\hat{Y}_t^{n,*} = (1 + \eta)A_t,$$

$$(1 - \gamma)(\sigma - 1)\hat{Y}_t^n + [\eta + \sigma + (1 - \gamma)(\sigma - 1)]\hat{Y}_t^{n,*} = (1 + \eta)A_t^*,$$

which corresponds to the results derived by Clarida et al. (2002).

3.4.3. Equilibrium under sticky prices

Aggregate employment in the home country can be rewritten in terms of price and wage dispersion:

$$L_t = \int_0^1 L_t^j dj = \frac{\Delta_{p,t} \Delta_{w,t} Y_t}{A_t}. \tag{47}$$

Price and wage dispersion is defined as follows:

$$\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta_{p,t}} di, \tag{48}$$

$$\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj. \tag{49}$$

At a sticky price equilibrium, the real marginal cost is expressed by the “gap” term as follows:

$$\hat{\phi}_t = (\eta + \sigma + D_0 - D_1)x_t + (D_1 - D_0)x_t^* + D_0\tilde{w}_t - \frac{\sigma\gamma(1 - \lambda)\lambda^*}{\Omega}\tilde{w}_t^*, \tag{50}$$

where x_t is the home output gap, x_t^* is the foreign output gap, \tilde{w}_t is the home real wage gap, and \tilde{w}_t^* is the foreign real wage gap. Following this equation, we characterized the home real marginal cost by the inclusion of the home country’s real wage and the foreign country’s real wage gap. The home country’s real wage gap positively affects the home country’s marginal costs. The first channel is that the home country’s real wages push up the real marginal cost by increasing the home country’s cost of production. The second channel implies that a rise in foreign real wages decreases the home real marginal cost and is associated with incomplete international risk-sharing caused by foreign LC consumers. As the share of LC consumers rises in the foreign country, the volatility of their consumption increases in that country. However, it also lowers the real marginal cost in the home country through Ricardian households’ risk-sharing. If a sizable share in LC consumers is present in the foreign country, a rise in the foreign real wages reduces the home country’s real marginal cost.

Log-linearizing the firm’s optimal price-setting rule (24) leads to the following price NK Phillips curve (PNKPC), expressed by the real marginal cost as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \delta_p \hat{\phi}_t + u_t, \tag{51}$$

where $\pi_t = \log(P_{H,t}/P_{H,t-1})$ and u_t is the price markup shock associated with a time-varying elasticity of substitution. In addition, $\delta_p = (1 - \omega)(1 - \omega\beta)/\omega$. Substituting Eq. (22) into Eq. (51) yields the following PNKPC in terms of the “gap” term:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_1 x_t + \kappa_2 x_t^* + \kappa_3 \tilde{w}_t - \kappa_4 \tilde{w}_t^* + u_t, \tag{52}$$

where

$$\kappa_1 = \delta_p(\eta + \sigma + D_0 - D_1), \quad \kappa_2 = \delta_p(D_1 - D_0), \quad \kappa_3 = \delta_p D_0, \quad \kappa_4 = \frac{\delta_p \sigma \gamma (1 - \lambda) \lambda^*}{\Omega}.$$

In the two-country model, home inflation is affected by the foreign output gap due to the existence of the terms of trade (Benigno, 2004; Clarida et al., 2002; Pappa, 2004).¹⁰ Whether the foreign output gap positively or negatively impacts the home country depends on the magnitude of D_1 and D_0 . Unlike Benigno (2004) and Clarida et al. (2002), our model characterizes the effect of LC consumers on the structural equations in a currency union; thus, these coefficients are affected by home and foreign LC consumers (i.e., shares of LC consumers affect D_0 and D_1). In addition, LC consumers in both countries causes home and real

¹⁰ See Clarida et al. (2002) for an intuitive discussion of international spillover through the terms of trade and international consumption risk-sharing.

wages to affect the home inflation rate. Specifically, the home and foreign country's real wage gaps positively and negatively affect inflation in the home country, respectively. The latter effect is again caused by incomplete international consumption risk-sharing. Note that the transmission channel from the real wage gap to inflation disappears without LC consumers in both countries.

We also derive the wage NK Phillips curve (WNKPC) because this paper's model has the effect of sticky wages:

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_{w,1} x_t + \kappa_{w,2} x_t^* + \kappa_{w,3} \tilde{w}_t - \kappa_{w,4} \tilde{w}_t^*, \quad (53)$$

where $\delta_w = (1 - \zeta)(1 - \zeta\beta)/\zeta$. Also,

$$\kappa_{w,1} = \delta_w(\eta + \sigma + D_0 - D_1), \quad \kappa_{w,2} = \delta_w(D_1 - D_0), \quad \kappa_{w,3} = \delta_w D_0, \quad \kappa_{w,4} = \frac{\delta_w \sigma \gamma (1 - \lambda) \lambda^*}{\Omega}.$$

The WNKPC in this paper differs from [Erceg et al. \(2000\)](#), [Campolmi \(2014\)](#), and [Rhee and Turdaliev \(2013\)](#) in that home wage inflation depends positively on the foreign output gap and negatively on the foreign real wage gap. The former is associated with the impact of the terms of trade and the latter with the presence of foreign LC consumers. As in the PNKPC, the impact of the foreign output gap and real wage gap on home wage inflation depends on the magnitude of D_0 and D_1 . In this economy, we show that movements in the real wage rate substantially impact optimal monetary policy. The law of motion for the definition of the home real wage evolves as follows:

$$w_t = \frac{W_t}{P_t} = \frac{W_t S_t^\gamma}{P_{H,t}}. \quad (54)$$

As pointed out in [Campolmi \(2014\)](#) and [Rhee and Turdaliev \(2013\)](#), the terms of trade affect the real wage rates in an open economy model. Unlike the aforementioned studies, our model is based on a currency union model, and thus, the home and foreign real wages affect their interdependence through a change in the terms of trade. Log-linearizing Eq. (54) leads to the following law of motion for the home real wage:

$$\hat{w}_t = \hat{w}_{t-1} + \gamma(\hat{S}_t - \hat{S}_{t-1}) + \pi_t^w - \pi_t. \quad (55)$$

Thus, the terms of trade being characterized by the ratio of home output to foreign output implies that the foreign output gap indirectly affects the home real wage gap through the law of motion of home real wages due to changes in the terms of trade.

Finally, using home and foreign consumption Euler equations and the definition of the terms of trade, we derive the union-wide investment-saving equation as follows:

$$x_t^u = E_t x_{t+1}^u + \vartheta_1 E_t \Delta x_{t+1}^* + \vartheta_2 E_t \Delta x_{t+1} - \sigma_u (\hat{r}_t - E_t \pi_{t+1}^u - r_{t+1}^u) - \vartheta_3 E_t \Delta \tilde{w}_{t+1} - \vartheta_4 E_t \Delta \tilde{w}_{t+1}^*, \quad (56)$$

where variables $x_t^u = (1 - \gamma)x_t + \gamma x_t^*$ and $\pi_t^u = (1 - \gamma)\pi_t + \gamma\pi_t^*$ denote the union-wide output gap and inflation, respectively. In addition, the parameters are defined as follows:

$$\vartheta_1 = \frac{(1-\gamma)(\sigma\psi-\gamma)}{\sigma_0}, \quad \vartheta_2 = \frac{\gamma(\sigma(1-\psi)-(1-\gamma))}{\sigma_0}, \quad \sigma_0 = \sigma(1-\psi) - \gamma, \quad \sigma_0^* = \sigma(1-\psi) - (1-\gamma),$$

$$\sigma_u = \frac{\sigma_0 + \sigma_0^*}{\sigma_0 \sigma_0^*}, \quad \vartheta_3 = \frac{\sigma_u \sigma (1 - \gamma) \lambda}{\Omega}, \quad \vartheta_4 = \frac{\sigma_u \sigma \gamma \lambda^*}{\Omega}$$

Since the model in this paper is built under a currency union, the two countries have a common policy rate, which is given by \hat{r}_t . The central bank under the currency union determines the path of the worldwide policy rate by implementing monetary policy.

4. Optimal monetary policy

This section examines the optimal monetary policy in a currency union with LC consumers. In Section 4.1, we derive the central bank's loss function by calculating a second-order approximation of the household utility function around the efficient steady state. Section 4.2 explains the optimal monetary policy under a commitment policy.

4.1. The central bank's loss function

This section derives a well-defined loss function with a micro-foundation to examine the optimal monetary policy in our model. [Woodford \(2003\)](#) showed that the second-order approximation of the household utility function corresponds to the central bank's loss function. In a two-country framework, [Clarida et al. \(2002\)](#) and [Engel \(2011\)](#) derived the central bank's loss function using a second-order approximation of the utility function.

To derive a well-defined loss function for the central bank from a quadratic approximation of the household utility function without calculating the second-order approximation of structural equations, we perform a Taylor expansion around an efficient steady state. In this paper, an efficient steady state that removes the following distortions is necessary to derive the central bank's loss function. In each country, a distortion of markups arose from the monopolistic competition by interme-

diate goods firms and a distortion of wage markups emerged by labor unions. As shown in the online appendix, the optimal subsidies that attain an efficient steady state are given as follows:¹¹

$$\tau = \tau^* = 1 - \frac{1}{\kappa\mu^p\mu^w}. \tag{57}$$

In deriving these subsidies, we assumed that the government in each country will calculate the optimal subsidy. If these subsidies do not offset a distorted steady state, we have to use the second-order approximation of both the PNKPC and WNKPC to obtain a well-defined central bank loss function in this economy (Benigno and Woodford, 2005).¹² In fact, except for the presence of nominal wage rigidity and LC consumers, Benigno and Benigno (2006) derived the central bank's loss function by calculating the second-order conditions of the PNKPC in a two-country economy with a distorted steady state.¹³ However, this task is beyond the scope of this paper. Therefore, we assume these subsidies offset the distortions caused by the respective markups on prices and wages due to monopolistic competition between intermediate firms and labor unions.

With optimal subsidies, we derive the central bank's policy objective in a currency union by implementing the second-order approximation of the following household utility function.

$$\begin{aligned} \mathcal{W}_t \simeq & \mathcal{W} - \frac{u_c \zeta \Omega}{2} \sum_{t=0}^{\infty} \left\{ (1 - \psi) \left[\alpha_\pi^p \pi_t^2 + \alpha_\pi^w (\pi_t^w)^2 + \alpha_x x_t^2 + \alpha_w \tilde{w}_t^2 \right] + \psi \left[\alpha_\pi^{p,*} (\pi_t^*)^2 + \alpha_\pi^{w,*} (\pi_t^{w,*})^2 + \alpha_x^* (x_t^*)^2 + \alpha_w^* (\tilde{w}_t^*)^2 \right] \right. \\ & \left. - 2\Lambda_1 x_t x_t^* - 2\Lambda_2 \tilde{w}_t \tilde{w}_t^* - 2\Lambda_3 (\chi \tilde{w}_t - \chi^* \tilde{w}_t^*) (x_t - x_t^*) \right\} + t.i.p. + O(\|\xi\|^3), \end{aligned} \tag{58}$$

where¹⁴

$$\begin{aligned} \alpha_\pi^p &= \frac{\omega\theta_p}{(1-\omega)(1-\omega\beta)}, \quad \alpha_\pi^w = \frac{\zeta\theta_w}{(1-\zeta)(1-\zeta\beta)}, \quad \alpha_x = \sigma + \eta - \frac{(1-\sigma)(1-\psi(1-\lambda))}{1-\lambda}, \quad \alpha_w = (\sigma - 1)\lambda\Phi, \\ \alpha_\pi^{p,*} &= \frac{\omega^*\theta_p}{(1-\omega^*)(1-\omega^*\beta)}, \quad \alpha_\pi^{w,*} = \frac{\zeta^*\theta_w}{(1-\zeta^*)(1-\zeta^*\beta)}, \quad \alpha_x^* = \sigma + \eta - \frac{(1-\sigma)(1-(1-\psi)(1-\lambda^*))}{1-\lambda^*}, \quad \alpha_w^* = (\sigma - 1)\lambda^*\Phi^*, \\ \Lambda_1 &= (1 - \sigma)\psi(1 - \psi), \quad \Lambda_2 = \frac{(1-\sigma)\psi(1-\psi)\lambda\lambda^*}{(1-\lambda)(1-\lambda^*)}, \quad \Lambda_3 = \frac{1-\sigma}{\Omega}, \\ \Phi &= \frac{(1-\gamma)\lambda+(1-\lambda)\Omega}{(1-\lambda)\Omega}, \quad \Phi^* = \frac{\gamma^*\lambda^*+(1-\lambda^*)\Omega}{(1-\lambda^*)\Omega}, \quad \chi = (1 - \gamma)\psi\lambda, \quad \chi^* = \gamma(1 - \psi)\lambda^*. \end{aligned}$$

In addition, *t.i.p.* includes terms independent of monetary policy, and $O(\|\xi\|^3)$ indicates the terms of third or higher orders. The online appendix provides a derivation of the loss function by calculating the second-order approximation of the household utility function.

As in the standard two-country NK model, our loss function contains several standard policy objectives. As in Benigno (2004) and Clarida et al. (2002), inflation and output gap objectives are assigned to both the home and foreign countries. The first objective is related to price dispersions caused by nominal price rigidity. In addition, as in Erceg et al. (2000), the loss functions for each country contain the wage inflation stabilization associated with wage stickiness. This objective is associated with wage dispersion caused by staggered wage contracts. In addition, as in Ascari et al. (2017), our loss function includes stabilizing the real wage gap due to LC consumers. As in Clarida et al. (2002), a stabilizing term for the co-variation of the home and foreign output gaps is also derived in this loss function. Summing up, in the presence of both LC consumers and nominal wage rigidities, our loss function corresponds to the following standard policy objectives. The terms α_π^w and $\alpha_\pi^{w,*}$ disappear in the absence of nominal wage rigidities. In addition of the absence of nominal wage rigidities in both countries, we obtain the standard loss function in a two-country model derived by Benigno (2004) and Clarida et al. (2002) when LC consumers are not present in both countries (i.e., $\lambda = \lambda^* = 0$).¹⁵

In contrast to the aforementioned standard policy objectives derived in previous studies, we address several characteristics of our loss function. First, we address that except for the stabilizing coefficients of price and wage inflation, all coefficients of its loss function are affected by home and foreign LC consumers. We emphasize the fact that the share of LC consumers' impact in both countries on the loss function's stabilization weights is more complex than that considered in Ascari et al. (2017) and Clarida et al. (2002). For example, as long as $\sigma > 1$, the coefficient of output gap stabilization in the home country rises as a share in LC consumers increases. This is because an increase in the share of LC consumers in the home country will cause fluctuations in output through changes in their consumption. An increase in λ also increases the stabilizing weight of the real wage gap, as long as $\sigma > 1$.¹⁶ The same is true for foreign country. Moreover, in the loss function, the coefficient ψ indicating the weights foreign policy objectives are affected by a change in both λ and λ^* . This implies that the union-wide central bank adjusts the weights on home and foreign policy objectives by considering a share in LC consumers in each country.

¹¹ See also Ascari et al. (2017) for a detailed discussion about optimal subsidies in an economy with LC consumers and nominal price and wage rigidity.

¹² Fujiwara and Wang (2017) obtained a well-defined central bank loss function in a two-country NK model with local currency pricing.

¹³ To assess the gain from international monetary policy coordination in a two-country model under producer currency pricing, Benigno and Benigno (2006) derived the central bank's loss function by calculating a second-order approximation of the PNKPC under both policy coordination and no policy coordination.

¹⁴ If $\gamma = 0.5$ and $\lambda = \lambda^*$, we can rewrite the final term in the loss function as $\chi(\tilde{w}_t - \tilde{w}_t^*)(x_t - x_t^*)$.

¹⁵ See Clarida et al. (2002) for a detailed discussion of the central bank's loss function in an open economy.

¹⁶ The opposite logic is applicable when $\sigma < 1$.

Second, as shown in the coefficient Λ_2 , LC consumers in the home and foreign countries have a co-movement effect through the risk-sharing conditions of Ricardian households. In other words, the presence of this stabilization term is associated with the incompleteness of international financial markets. More concretely, this term is reflected by the observation that LC consumers' consumption harms Ricardian consumption through a change in real wages. This term implies that the co-movement of home and foreign LC consumers' consumption translates into a stabilizing weight for the cross-term of the home and foreign real wage gaps. The magnitude of this weight is greater if shares in LC consumers are higher in both countries.

Third, the final term in the loss function stabilizes the co-movement of the relative real wages between home and foreign countries and the terms of trade. The incompleteness of international financial markets also derives this term. More precisely, as long as LC consumers are present in each country, a fluctuation in consumption of LC consumers causes a change in the output gap. This is because their consumption proportionally moves with a change in their disposable income. As mentioned earlier, a change in LC consumers' consumption is also converted into one in real wages. Furthermore, a fluctuation in both home and foreign output gaps changes the terms of trade expressed by the ratio of home and foreign output. Accordingly, the union-wide central bank cares about the co-movement of the relative real wages between two countries and the terms of trade. The stabilization weight on this term will be larger if both countries realize a greater share of LC consumers.

For the second and third points, the stabilization weights Λ_2 and Λ_3 are associated with the incompleteness of international financial markets that also affect price and wage NKPCs through a change in real wages. In the presence of LC consumers in both countries, the central bank considers the effect of those on price and wage dynamics through international financial market incompleteness. In our model, in the presence of nominal wage rigidity and incomplete international financial markets, strict inflation targeting is no longer optimal because divine coincidence does not hold. As a result, compared to the standard two-country model, the central bank's optimal monetary policy is more completed in our model than in the standard two-country model without nominal wage rigidity and incomplete international financial markets.

4.2. The central bank's optimization problem

This paper first considers the commitment solution under a currency union. Under a currency union, the benevolent central bank chooses the union-wide interest rate path by intertemporally solving the minimization problem of the central bank's loss function. We focus on the commitment solution as a benchmark case because no studies have investigated the performance of a commitment policy in a currency union under LC consumers. However, since the commitment solution is a powerful tool for determining an optimal stabilization policy (McCallum and Nelson, 2004; Woodford, 2003), we also explore a comparison of the commitment and discretionary solutions in Sections 5.4 and 5.5.

Under a commitment policy, central banks can commit to future monetary policy stances in the current period. This paper regards the commitment policy as optimal from a timeless perspective as suggested by Woodford (2003).¹⁷ If central banks commit to a policy, they can introduce policy inertia into the economy. This policy inertia then enables central banks to influence the private sector's expectations.

Combining the first-order conditions of the central bank's optimization problem with structural equations, we obtain the following:

$$A_0 X_t = A_1 X_{t-1} + B_1 R_t + \Gamma_t, \quad (59)$$

with

$$\begin{aligned} X_t &= [X_{1t+1} \ E_t X_{2t+1}]', \\ X_{1t} &= [A_t \ A_t^* \ u_t \ u_t^* \ \phi_{1t} \ \phi_{2t} \ \phi_{3t} \ \phi_{4t}]', \\ X_{2t} &= [\tilde{w}_t \ \tilde{w}_t^* \ \pi_t^w \ \pi_t^{w,*} \ \pi_t \ \pi_t^* \ x_t \ x_t^* \ \phi_{5t} \ \phi_{6t}]'. \end{aligned}$$

A_0, A_1 , and B_1 are coefficient matrices constructed by deep parameters. X_{1t} denotes the vector for the predetermined and state variables, X_{2t} represents that for the jump variables, R_t is the vector of the central bank's instrumental variables, and Γ_t denotes the vector of structural shocks. ϕ_{1t} , and ϕ_{2t} denote the Lagrange multipliers for the PNKPCs, ϕ_{3t} , and ϕ_{4t} represent those for the WNKPCs. Finally, ϕ_{5t} and ϕ_{6t} are the Lagrange multipliers for the real wage identities. The properties of optimal monetary policy are simulated using the Dynare software package.¹⁸

5. Main results

This section provides the main results. In particular, Section 5.1 presents the calibrated deep parameters used in this paper. Section 5.2 examines the role of LC consumers in a currency union under a simple Taylor rule. Section 5.3 presents

¹⁷ See Chapter 7 in Woodford (2003) for a detailed discussion of optimal monetary policy from a timeless perspective.

¹⁸ Dynare is available at <http://www.dynare.org/>.

the results of impulse response analysis under a commitment policy for the cases of productivity and price markup shocks occurring in a foreign country. Sections 5.4 and 5.5 report welfare losses under optimal monetary policy caused by the presence of LC consumers in each country. Finally, Section 5.6 provides several intuitive discussions.

5.1. Calibration

This section describes the deep parameters used in this paper. The discount factor is set to 0.99. The relative risk aversion coefficient for consumption, σ , is 2.0, based on Pappa (2004). In addition, this paper assumes that parameter η equals 1.0. Following Pappa (2004), we set the elasticity of substitution between goods, θ_p , to 7.88. The elasticity of substitution among labor varieties is set to 5.0. Meanwhile, parameter γ , which denotes the degree of openness, is set to 0.4 as a baseline parameter value. Following Pappa (2004), we set the degree of nominal price rigidity for both countries to 0.75. Both countries' degree of nominal wage rigidity is set to 0.5 as a benchmark parameter value.

We provide the calibrated value of λ for both countries and select four candidates for λ : 0, 0.25, 0.5, and 0.75. The first value implies the standard two-country NK economy without LC consumers. The second value is based on the estimated value reported by Kaplan et al. (2014) and Almgren et al. (2019). The third and fourth values correspond to economies such as Latvia, where the share of liquidity constraints is considerably high (Almgren et al., 2019). We examine the impulse response and welfare costs under several combinations of λ and λ^* . In the impulse response analysis under optimal monetary policy, the calibrated value for home LC consumers is fixed at 0.25 as a benchmark value.

Finally, the standard deviations of a productivity shock, σ_a , and price markup shock, σ_u , are both set to 0.02. Analogous values are assumed for the foreign country. Regarding the autoregressive coefficients for structural shocks, we set ρ_a and ρ_u to 0.8 and 0, respectively. These calibrated values for structural shocks are presumed to be the same in the foreign country.

In the following, we report only on foreign structural shocks when conducting impulse response and welfare analysis because we focus on the international spillover effects of foreign structural shocks.

5.2. Monetary policy under a simple Taylor rule

Before examining optimal monetary policy, we focus on the role of LC consumers by exploring the endogenous response of monetary policy to the shock when considering a simple monetary policy rule. More precisely, we assume that the central bank implements the following Taylor rule:

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1 - \phi_r)(\phi_\pi \pi_t^u + \phi_x x_t^u) + v_t, \quad (60)$$

where ϕ_π and ϕ_x denote the stabilization coefficients for worldwide inflation and the worldwide output gap, respectively. ϕ_r represents the degree of interest rate smoothing. v_t denotes an exogenous monetary policy shock, which follows an AR(1) process given by $v_t = \rho_v v_{t-1} + \epsilon_t^v$ with $0 \leq \rho_v < 1$, where ϵ_t^v is an independent and identically distributed (i.i.d.) shock with constant variance σ_v^2 . Following the standard DSGE literature (Christiano et al., 2005), we set ϕ_π , ϕ_x , and ϕ_r to 1.5, 0.125, 0.8, respectively. The auto-correlation of v_t is set to 0.5, and σ_v is set to 0.02.

We examine the impact of a change in foreign LC consumers on the international transmission of foreign structural shocks using a simple Taylor rule. To do this, we assume $\lambda = 0$ in this exercise. Fig. 1 shows the impulse response under a simple Taylor rule to an increase in foreign productivity.¹⁹ Consider the case of $\lambda^* = 0$. A rise in foreign productivity causes a decline in the foreign output gap, lowering the response of foreign inflation through the PNKPC. In addition, with the presence of the WNKPC, a decline in inflation decreases wage inflation through a reduction in real wages in the foreign country. Meanwhile, a decline in the output gap leads to an appreciation in the home terms of trade. Therefore, the home output gap decreases, thereby reducing home inflation through the PNKPC. Similar to the foreign country, a decline in inflation decreases both wage inflation and real wages in the home country.

After a rise in foreign productivity, although we do not report in this figure, the central bank reduces its policy rate in response to a decline in foreign inflation by following the Taylor rule. In the Ricardian household's Euler equation, the negative income effect via a decline in labor income dominates the substitution effect that a reduction in the nominal interest rate pushes current consumption. As a result, as shown in Appendix B2, foreign Ricardian consumption declines following a positive shock to foreign productivity.²⁰ Accordingly, when a share in LC households does not exist, foreign aggregate consumption falls in response to an increase in foreign productivity. Home aggregate consumption also decreases through international consumption risk-sharing between home and foreign Ricardian households. This is due to the fact that a decrease in home Ricardian consumption simply induces a decrease in home aggregate consumption when Ricardian households are only present in the home country. When a share in LC consumers does not exist, the risk-sharing incompleteness, defined as a difference between home and foreign aggregate consumption, disappears.

Fig. 1 also illustrates that the impulse response of other endogenous variables for both countries are generally muted as λ^* increases. This is because in a forward-looking model, an increased share in LC consumers attenuates the expectations channel of monetary policy. In contrast, the degree of risk-sharing incompleteness widens as a share in LC households increases in

¹⁹ See an online appendix for additional reports on the response of macrovariables to other structural shocks under a simple Taylor rule.

²⁰ See also the online appendix for a detailed discussion of the consumption response of both Ricardian and LC consumers under the Taylor rule.

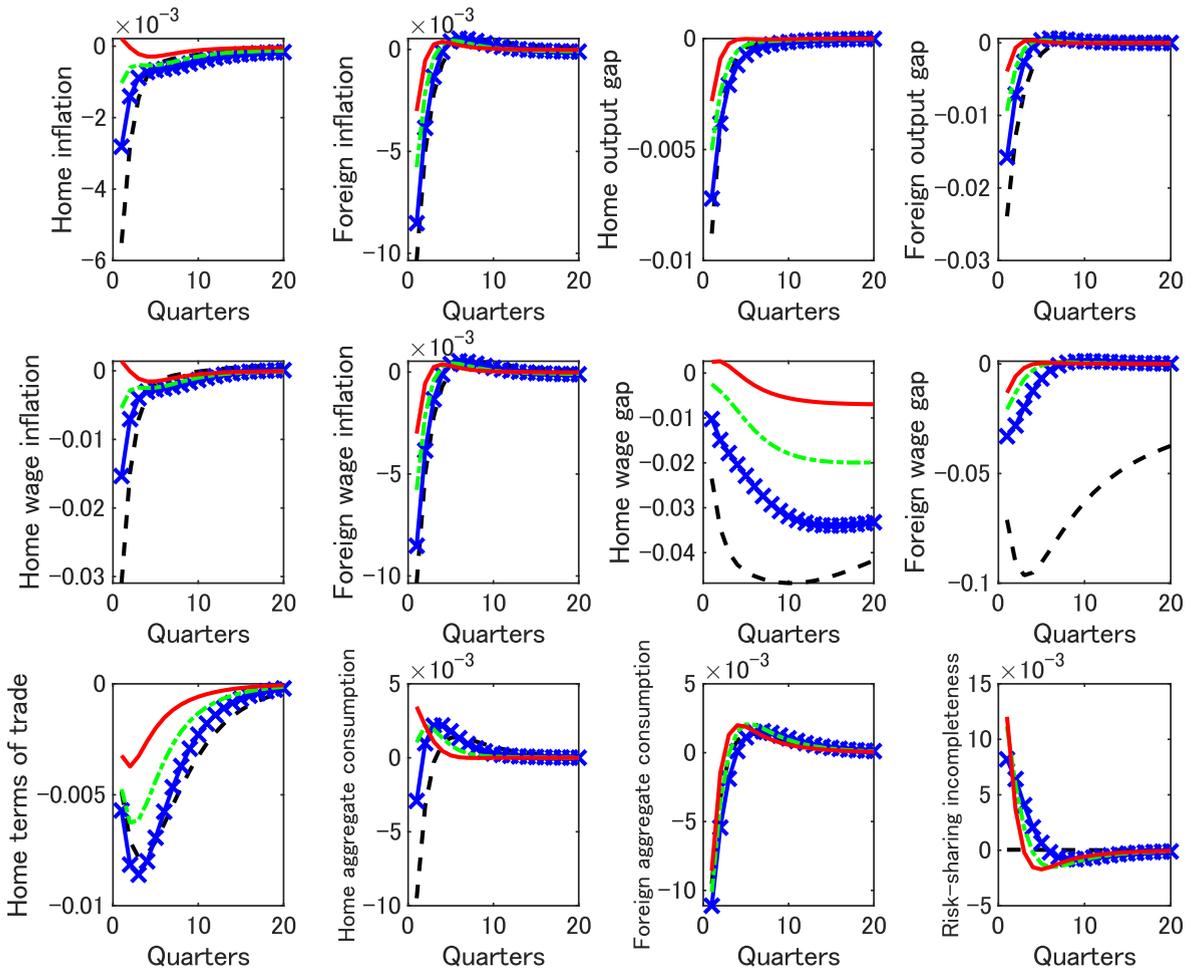


Fig. 1. Impulse response to a foreign positive productivity shock: A simple Taylor rule. (Note) Dashed line denotes $\lambda^* = 0$, line with (x) denotes $\lambda^* = 0.25$, dash-dotted line denotes $\lambda^* = 0.5$, and solid line denotes $\lambda^* = 0.75$, respectively.

the foreign country. Notably, in the case of $\lambda^* = 0.5$ and $\lambda^* = 0.75$, a rise in foreign productivity results in a negative response of foreign aggregate consumption, boosting home aggregate consumption. Fig. 1 demonstrates that a larger value of λ^* mitigates a decline in foreign real wages. In the foreign Ricardian household's Euler equation, the substitution effect dominates the income effect, so foreign Ricardian consumption rises. However, when λ^* takes a higher value, foreign aggregate consumption decreases because a decline in LC households' consumption outweighs an increase in Ricardian households' consumption. Conversely, a rise in foreign Ricardian consumption increases home Ricardian consumption through international Ricardian's consumption risk-sharing. Home aggregate consumption increases due to the absence of LC consumers in the home country.

5.3. Optimal monetary policy under commitment

This section explores the impulse response under a commitment policy when a share in LC consumers changes. More concretely, we examine how the central bank that follows a commitment policy attempts to stabilize the union-wide economy by managing the expectations of firms and households in our model.

Fig. 2 shows the impulse response function under the commitment solution when an exogenous increase in foreign productivity occurs. First, consider the case for $\lambda^* = 0$. A rise in foreign productivity causes a decline in both inflation and the output gap in the foreign country. That shock produces the negative response of wage inflation and real wages through the WNKPC. A rise in foreign productivity further increases foreign aggregate consumption under a commitment policy. Under a commitment policy, home and foreign central banks adhere to an optimal targeting rule that has policy inertia through several endogenous variables. Although we cannot explicitly derive a targeting rule from our model, we can consider that, unlike a simple Taylor rule, a foreign commitment policy can alleviate a reduction in foreign real wages. In fact, Fig. 2 demonstrates that, unlike a simple Taylor rule, a commitment policy mitigates fluctuations in home and foreign

macrovariables. Hence, foreign Ricardian consumption increases because the substitution effect dominates the income effect in the Euler equation for the Ricardian household in the foreign country.

Moreover, that shock increases home aggregate consumption through an international consumption risk-sharing between Ricardian households. Thus, in the home country, boosting Ricardian consumption increases aggregate consumption when the proportion of Ricardian households exceeds that of LC consumers. In addition, as long as a share in LC consumers is smaller in both countries, the effect of the foreign productivity shock on risk-sharing incompleteness seems negligible.²¹ In the home country, the positive foreign productivity shock boosts the home output gap, leading to home inflation through the home PNKPC. A rise in home inflation increases both wage inflation and real wages in home country.

Next, we consider the case for $\lambda^* = 0.25$. On the one hand, foreign aggregate consumption falls after a foreign productivity shock because a decline in consumption by LC consumers outweighs a rise in Ricardian consumption. On the other hand, a rise in foreign productivity boosts home aggregate consumption via the international risk-sharing condition between home and foreign Ricardian households. This is because foreign Ricardian consumption continues to rise after the shock. Figure B3 in the online appendix illustrates these responses. Moreover, the degree of risk-sharing incompleteness becomes a non-negligible response to a positive foreign productivity shock. Thus, when $\lambda = \lambda^* = 0.25$, a foreign positive productivity shock generates a wedge in the co-movement between home and foreign aggregate consumption. Even in this case, the home output gap is still positive in response to that shock. Compared to the case where LC consumers is absent in the foreign country, except for the degree of risk-sharing incompleteness, the response of macrovariables are basically muted when $\lambda = \lambda^* = 0.25$. This implies that an increased value in λ^* weakens the effect of a commitment policy on manipulating the private sector's expectations.

However, when λ^* is above 0.25, a rise in foreign productivity lowers both home and foreign aggregate consumption. Naturally, the degree of imperfections in international financial markets also widens. In this case, the output gap in both countries declines in response to a positive foreign productivity shock. Finally, a larger value of λ^* renders the response of macrovariables to the shock more attenuated.

Fig. 3 illustrates an exogenous increase in a cost-push shock in a foreign country. First, we consider the case for $\lambda^* = 0$. As argued in the standard NK literature, a cost-push shock generates a trade-off between price inflation and the output gap in the foreign country. We observe that as long as the economic structure is forward-looking, the central bank can reduce the welfare loss associated with a cost-push shock by appropriately introducing policy inertia. In addition, when nominal wage rigidity is present, this shock generates a trade-off between wage inflation and real wages through the WNKPC. Although a rise in a foreign cost-push disturbance induces a depreciation in the home terms of trade, decreasing home aggregate consumption due to international risk-sharing between Ricardian households. In this figure, the output gap and inflation increase in the home country because depreciation of the home terms of trade dominates a reduction in home consumption. An increase in home inflation also increases in both wage inflation and real wages in the home country.

Next, we examine the case for $\lambda^* = 0.25$. Compared to the case of $\lambda^* = 0$, as noted earlier, an increase in λ^* weakens the central bank's ability to manipulate the expectations of private sectors. Except for the degree of risk-sharing incompleteness, the response of macrovariables to an increase in foreign cost-push disturbance are also muted in this case. In particular, a rise in λ^* considerably attenuates home and foreign real wage gap fluctuations. In addition, home price and wage inflation decrease with a rise in a share in foreign LC consumers.

Finally, consider the case where λ^* is above 0.5. Similar to the foreign productivity shock, the degree of risk-sharing incompleteness also widens. Fig. 3 shows that since a decline in aggregate consumption is larger in the foreign country than in the home country, the degree of risk-sharing incompleteness increases. In this case, the output gap in both countries further declines in response to a foreign cost-push shock. Finally, as shown in Figs. 1 and 2, a larger value of λ^* also counteracts the response of macrovariables to the shock.

5.4. LC consumers and costs of discretion

We concentrated on the case of a commitment policy that is optimal from a timeless perspective. A commitment policy involves the central bank minimizing its loss function at some initial point and committing to a time-contingent strategy to influence private sector expectations. Several studies have compared the performance of a commitment policy to that of a purely discretionary policy in a purely forward-looking model. In contrast to a commitment policy, a discretionary policy allows the central bank to re-optimize its loss function in each period based on the private sector's future expectations. The welfare loss under a commitment policy appears to be less than that under a discretionary policy (McCallum and Nelson, 2004; Woodford, 2003; Walsh, 2010).²² Moreover, Pappa (2004) pointed out that a commitment policy can eliminate the deflationary bias induced by a discretionary policy, which is a source of additional gain from commitment associated with an open economy.

²¹ See also online appendix for a detailed discussion about the response of consumption in response to a foreign positive productivity shock.

²² If we introduce endogenous persistence into the forward-looking model, then the gain from commitment decreases (Steinsson, 2003).

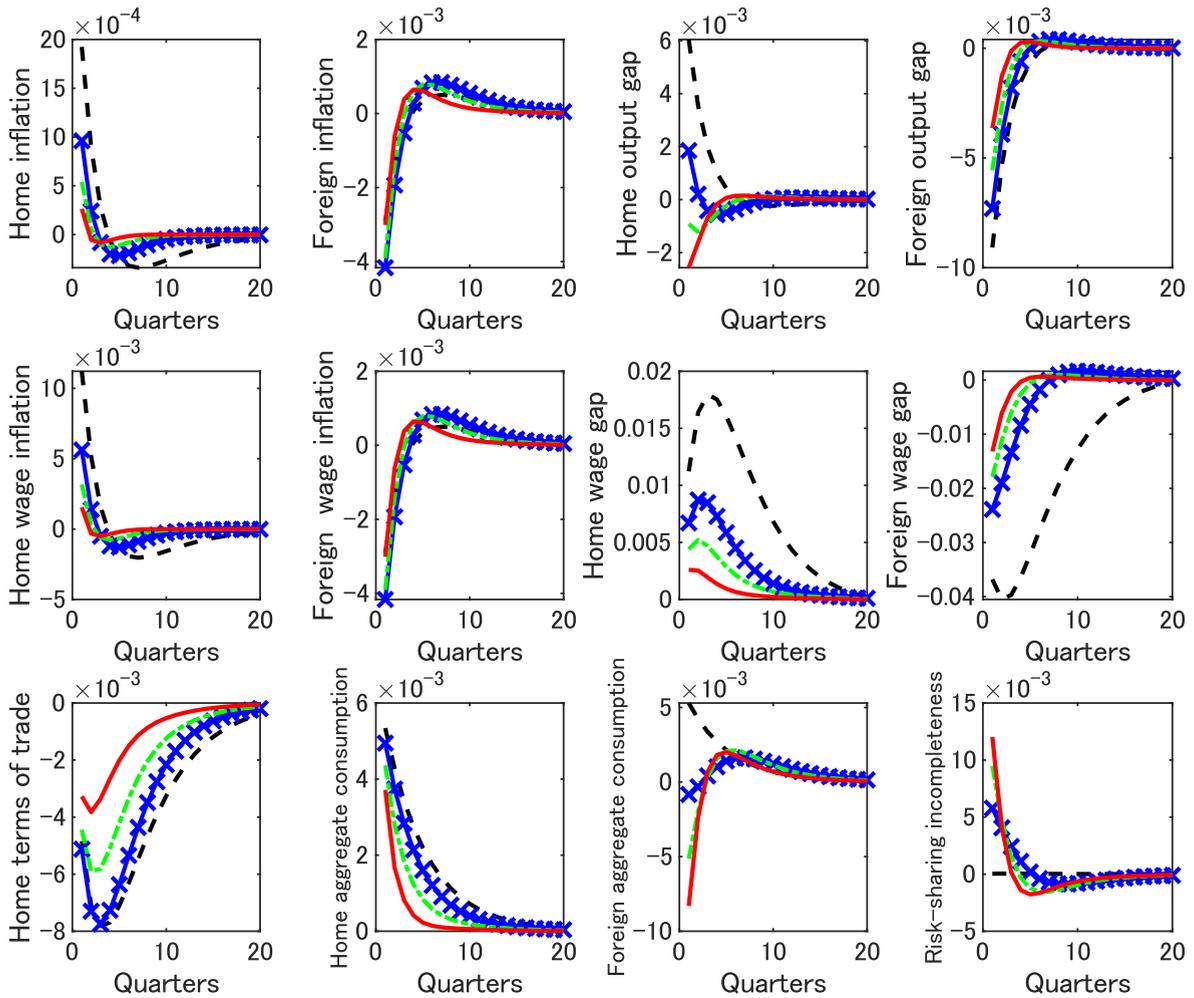


Fig. 2. Impulse response to a foreign positive productivity shock: Commitment policy. (Note) Dashed line denotes $\lambda^* = 0$, line with (x) denotes $\lambda^* = 0.25$, dash-dotted line denotes $\lambda^* = 0.5$, and solid line denotes $\lambda^* = 0.75$, respectively.

Generally, the central banks cannot easily implement time-consistent commitment solutions. There seems to be a practical aspect to implementing discretionary solutions, because a central bank has difficulty making a strong commitment to the private sector.²³ Moreover, it is challenging for the central bank to commit strongly to the private sector in a framework such as the two-country model. As noted earlier, one may conjecture that in a TANK model, the central bank's ability to manipulate the expectations of the private sector is dampened as the share in LC consumers increases.

This argument naturally casts the following questions. Are the costs of implementing a discretionary policy in an economy with a currency union high? How does the share of LC consumers in both countries affect this cost? Although [Ascari et al. \(2017\)](#) examined welfare losses under commitment, they did not calculate the cost of discretionary policy in an NK model with LC consumers. [Galí and Monacelli \(2016\)](#) also calculated the gain from commitment in a currency union model with nominal wage rigidity. [Groll and Monacelli \(2020\)](#) compared the cost of the discretionary policy in both cases of an exchange rate float and a currency union. However, these studies did not consider the effect of LC consumers on the costs from discretion in a two-country NK model. The present study focuses on whether the costs of discretionary policy are negligible when LC consumers play a significant role in a currency union.

[Table 1](#) calculates the effect of a change in LC consumers on the costs from discretionary policy. In calculating the costs of the discretionary policy, we expressed the welfare losses of both commitment and discretionary policy for each case according to the relative losses normalized by the commitment solution in the absence of home and foreign LC consumers (i.e., the

²³ When the central bank cannot implement a commitment policy, the government may delegate the loss function with policy inertia to it. See [Bilbie \(2014\)](#) and [Walsh \(2003\)](#) for a detailed discussion of this delegation problem. See also [Ida and Okano \(2021\)](#) for a detailed discussion of the delegation problem in a small-open economy.

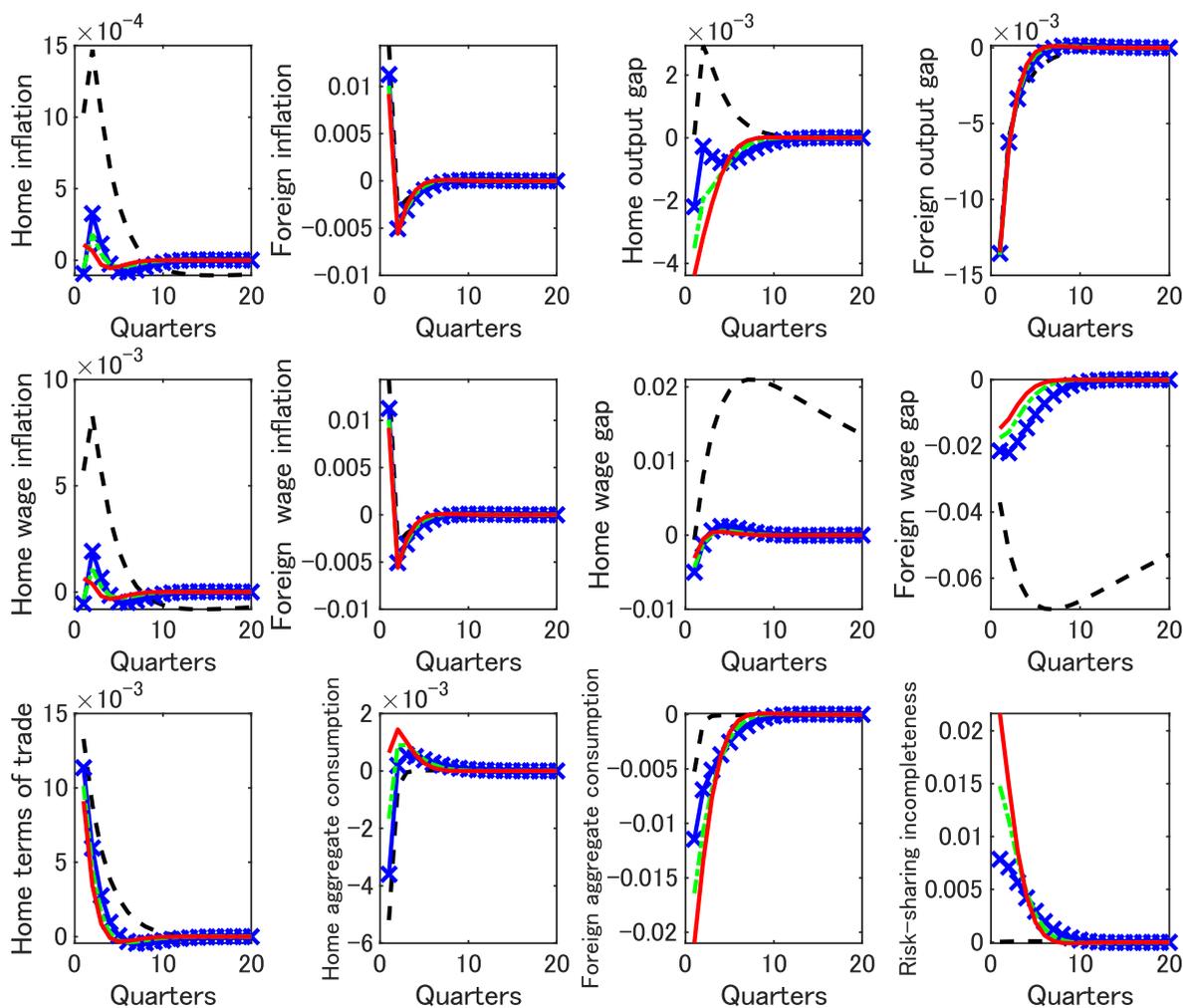


Fig. 3. Impulse response to a foreign positive price markup shock: Commitment policy. (Note) Dashed line denotes $\lambda^* = 0$, line with (x) denotes $\lambda^* = 0.25$, dash-dotted line denotes $\lambda^* = 0.5$, and solid line denotes $\lambda^* = 0.75$, respectively.

case for $\lambda = \lambda^* = 0$. To do so, we can assess how the costs from discretion increase following a change in the share of LC consumers in both countries. The differences (%) in Table 1 also report the costs from a discretionary policy.

We obtain the following results from Table 1. First, importantly, given the share of LC consumers in the home country, the welfare losses of both the commitment and the discretionary solutions are improved relative to the benchmark case in which a share in foreign LC consumers increase. Second, compared to the result of Table 1(a), the welfare loss is greater when the share of LC consumers in the home country increases, given the share of LC consumers in the foreign country. Third, the cost of discretionary policy is not great in our currency union model. For example, Table 1(a) shows that in the absence of liquidity-constrained consumers in both countries, a cost of a discretionary policy becomes negligible in the presence of nominal wage rigidity. This is because the nominal wage rigidity reduces the central bank’s ability to introduce policy inertia into the economy. It also shows that the cost from discretionary policy is approximately 3.7 % in the case where $\lambda = 0$ and $\lambda^* = 0.75$. Table 1(d) reports the case in which $\lambda = \lambda^* = 0.75$, but, even in that case, the cost from the discretionary policy is approximately 6 %. Therefore, the discretionary policy costs are not substantially large compared to the standard two-country RANK model in which LC consumers are not present.

5.5. Role of nominal wage rigidity

We also examine how LC consumers and wage stickiness in both countries affect the cost of discretionary solutions. It has been established that wage stickiness significantly affects optimal monetary policy (Erceg et al., 2000). In particular, it is useful to consider the relationship between LC consumers and wage stickiness in our model. Branten et al. (2018) and Fabiani et al. (2010) examined the severity of wage rigidity in the European region. Fabiani et al. (2010) confirmed the existence of

Table 1
Welfare costs from discretionary policy.

(a) $\lambda = 0$		Commitment	Discretion	Difference (%)
λ^*				
0		1.0000	1.0060	0.60
0.25		0.5235	0.5351	2.18
0.5		0.3281	0.3369	2.64
0.75		0.1786	0.1853	3.68
(b) $\lambda = 0.25$				
λ^*				
0		1.0862	1.0957	0.86
0.25		0.6188	0.6351	2.58
0.5		0.4035	0.4164	3.15
0.75		0.2271	0.2361	3.91
(c) $\lambda = 0.5$				
λ^*				
0		1.2509	1.2655	1.16
0.25		0.7753	0.8000	3.13
0.5		0.5346	0.5558	3.89
0.75		0.3194	0.3345	4.61
(d) $\lambda = 0.75$				
λ^*				
0		1.4927	1.5154	1.51
0.25		1.0606	1.1007	3.71
0.5		0.8054	0.8473	5.06
0.75		0.5465	0.5807	6.08

(Note) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of discretionary policy, we expressed the welfare losses of both commitment and discretionary policies for each case according to the relative losses normalized by the commitment solution in the absence of liquidity-constrained consumers (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from discretionary policy.

downward rigidity in nominal and real wages in the European economy. Galí (2013) examined the impact of wage stickiness on social welfare. Galí and Monacelli (2016) analyzed how the degree of wage flexibility affected optimal monetary policy in a currency union. Assessing the effect of nominal wage rigidity on optimal monetary policy remains an open question in a currency area in which LC consumers exist.

Table 2 reports how changes in home and foreign nominal wage rigidities affect the costs from a discretionary policy in a model with and without LC consumers. In particular, Table 2(a) reports the welfare costs under discretion relative to commitment in a model without LC consumers. When LC consumers are absent in both countries, given ζ , an increase in ζ^* generally raises the costs from a discretionary policy. The observation that a rise in nominal wage stickiness generates welfare loss is consistent with that argued in Galí (2013). However, our result underlines that the costs from a discretionary policy decrease as both ζ and ζ^* take a larger value.

Tables 2(b)–(d) demonstrate the welfare costs from a discretionary policy in a model with LC consumers. In this simulation, we do not allow household heterogeneities between two countries. Compared to the model without LC consumers, the costs from a discretionary policy increase as home and foreign countries face a larger share in LC consumers. However, even in this case, an increase in both ζ and ζ^* can generally reduce the welfare loss from a discretionary policy. In the case where $\lambda = \lambda^* = 0.25$, Table 2(b) demonstrates that the welfare cost of discretionary policies is lowest when $\zeta = 0.75$ and $\zeta^* = 0.5$. Tables 2(c)–(d) indicate that if the proportions of home and foreign LC consumers are greater than 0.25, the central bank incurs the lowest welfare costs from discretionary policies when $\zeta = 0.75$ and $\zeta^* = 0.25$. While an increase in λ and λ^* generates a wedge on international consumption risk-sharing, an increase in nominal wage rigidity in both countries helps reduce the welfare losses in both cases of commitment and discretion. Moreover, the presence of nominal wage rigidities can reduce the costs from a discretionary policy since an increased share in LC consumers in both countries counteracts the expectations channel of a commitment policy.

5.6. Discussion

This paper demonstrates that considering the share of LC consumers provides important implications for optimal monetary policy in a currency union. An increase in LC consumers in both countries enhances social welfare in a currency union model. More precisely, the presence of international consumption risk-sharing incompleteness allows the central bank to reduce welfare loss under both discretion and commitment. Why does a higher share of LC consumers in both countries enhance social welfare? This is an interesting result that has not been considered in previous studies. Indeed, in impulse response function under the commitment solution, we find that, given a share of LC consumers in the home country, macroeconomic fluctuations in the home and foreign countries are generally attenuated with the increase in a share of foreign liquidity constraints. Our results also show that this is especially true in cases where the share of LC consumers is

Table 2
Welfare costs from discretionary policy: The role of nominal wage rigidity.

(a) $\lambda = \lambda^* = 0$				
ζ/ζ^*				
0	0	0.25	0.5	0.75
0.25	0.46	12.90	11.47	8.06
0.5	3.44	0.26	1.52	1.81
0.75	2.10	0.12	0.22	0.58
	0.49	0.21	0.53	1.74
(b) $\lambda = \lambda^* = 0.25$				
ζ/ζ^*				
0	0	0.25	0.5	0.75
0.25	3.04	11.02	17.38	20.60
0.5	2.90	3.68	7.05	9.75
0.75	2.62	1.89	2.76	4.57
	2.07	1.31	1.28	2.52
(c) $\lambda = \lambda^* = 0.5$				
ζ/ζ^*				
0	0	0.25	0.5	0.75
0.25	4.41	10.32	16.30	21.40
0.5	3.60	5.39	8.66	12.21
0.75	2.63	2.94	4.16	6.41
	1.60	1.48	1.80	3.30
(d) $\lambda = \lambda^* = 0.75$				
ζ/ζ^*				
0	0	0.25	0.5	0.75
0.25	6.72	9.93	15.81	27.71
0.5	5.85	7.29	10.67	17.77
0.75	4.80	5.22	6.44	9.25
	4.81	4.21	3.78	4.44

(Note) We calculate the welfare costs under discretion relative to commitment in a model without liquidity-constrained consumers. The welfare costs in this table are expressed in the percentage difference.

predominately large. This naturally implies the possibility that the central bank can substantially improve social welfare when the share of LC consumers is high in both countries.

One might think that the result of welfare loss decreases as the international financial market becomes incomplete seems inconsistent with the standard two-country NK model. In fact, Benigno (2004) argued that inflation targeting is optimal when heterogeneity of nominal price rigidity is absent in a two-country model where the economy size is the same. In our model, apart from the presence of incomplete international risk-sharing, heterogeneities of both nominal price and wage rigidities also exist in both countries. These distortions make achieving optimal monetary policy more difficult in a currency union.

When a share in LC consumers is larger in both countries, according to price and wage NKPCs, the sensitivity of real wages to price and wage inflation is amplified, given the degree of nominal wage rigidities in both countries. In our model, this implies that a change in real wages can alleviate price and wage inflation fluctuations through price and wage NKPCs. These effects are significantly reflected by the presence of a stabilization weight on the co-movement between home and foreign real wages (Λ_2). Furthermore, since the loss function also contains the stabilization term for the co-movement between the difference in home and foreign real wages and that in home and foreign output gaps (Λ_3), this term can help reduce welfare losses through price and wage NKPCs. Hence, an increased share in home and foreign LC consumers can enlarge social welfare by changes in these weights in the loss function. This implies that the central bank can exploit this channel to reduce volatiles of union macrovariables. Accordingly, when LC consumers are present in both countries, a larger share of home and foreign LC consumers contributes to a reduction in welfare loss. In such a case, sticker nominal wages in both countries can reduce the costs from a discretionary policy even in the presence of an attenuated expectations channel of a commitment policy.

Several previous studies have reported that discretionary policy costs are significant (McCallum and Nelson, 2004). Meanwhile, in some NK models, such as those with strong inflation inertia and consumption habit formation, there may be no difference in the performance of discretionary and commitment policies (Amato and Laubach, 2003; Steinsson, 2003). As noted earlier, Ascari et al. (2017) showed that a higher share of LC consumers leads to more significant welfare losses under a commitment policy. However, they did not consider the impact of the increased share of LC consumers on a discretionary policy. Although Groll and Monacelli (2020) argued for the importance of endogenous inertia in the terms of trade in a two-country NK model with full wage flexibility, they did not consider the role of LC consumers. The contribution of this paper is to demonstrate that in a currency union, generally small differences exist in the performance of discretionary and commitment policies. More concretely, this paper shows that the source of reducing welfare losses is associated with the presence of both LC consumers and nominal wage rigidities in both countries. To the best of our knowledge, these are our most important contributions to previous studies.

6. Additional results

This section provides several robustness checks. First, we examine the impact of economic openness and the share of home and foreign LC consumers on the cost of discretionary policy. Second, we explore the role of nominal wage rigidity in each country to assess the cost of discretionary policy when LC consumers are present only in the foreign country.

6.1. LC consumers and the degree of openness

In addition to a change in the share of LC consumers in each country, we focus on the openness parameter, γ , which plays a significant role in the two-country NK model. Fig. 4 shows the discretionary policy costs under combinations of home and foreign LC consumers in the case of $\gamma = 0$. This case can be regarded as a closed economy model. This exercise is useful for assessing the role of LC consumers in a two-country framework. Again, in calculating the cost from this discretionary policy, the welfare loss in each case is normalized by the loss of the commitment solution in the absence of LC consumers in both countries. Fig. 4 illustrates that regardless of a share in home LC consumers, an increase in the share of foreign LC consumers

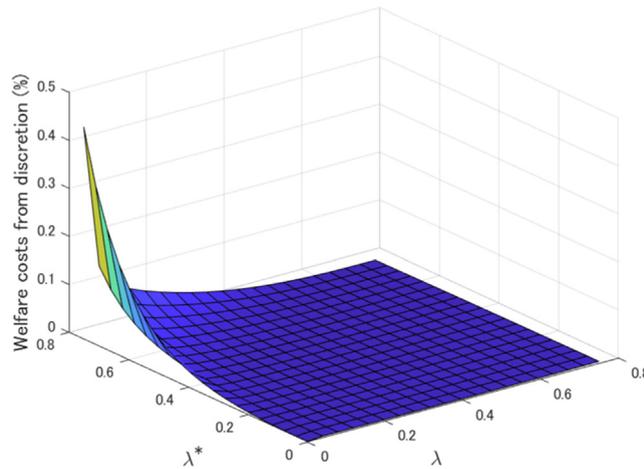


Fig. 4. Welfare costs from discretion ($\gamma = 0$) (Note) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of the discretionary policy, we expressed the welfare losses of both commitment and discretionary policies for each case according to the relative losses normalized by the commitment solution in the absence of liquidity-constrained consumers (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from discretionary policy..

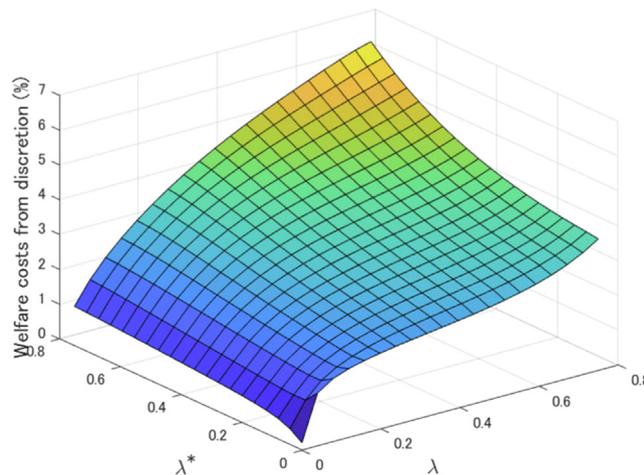


Fig. 5. Welfare costs from discretion ($\gamma = 0.4$) (Note) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of the discretionary policy, we expressed the welfare losses of both commitment and discretionary policies for each case according to the relative losses normalized by the commitment solution in the absence of liquidity-constrained consumers (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from discretionary policy..

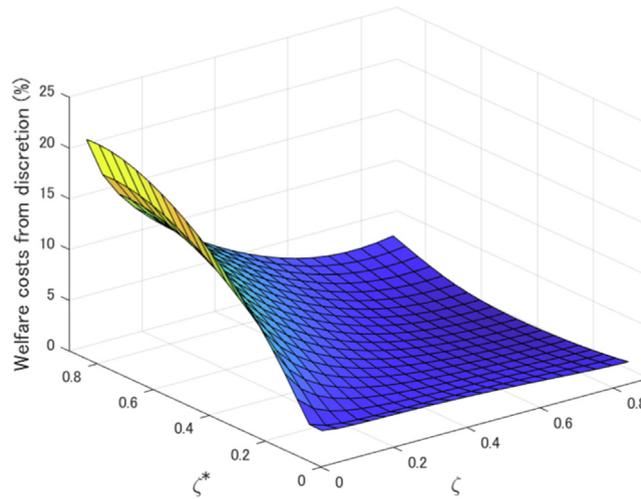


Fig. 6. Welfare costs from discretion ($\lambda = 0$ and $\lambda^* = 0.25$) (Note) The values for both the commitment and discretion policies represent welfare losses under each regime. In calculating the costs of the discretionary policy, the welfare losses of both commitment and discretionary policies for each case are expressed according to the relative losses normalized by the commitment solution in the absence of liquidity-constrained consumers (i.e., the case for $\lambda = \lambda^* = 0$). The percentage difference implies the costs from the discretionary policy..

generates an increased welfare loss under a discretionary policy, but the magnitude is negligible. The maximum value is approximately 0.4 % at $\lambda^* = 0.8$.

Fig. 5 illustrates the welfare costs from a discretionary policy in the case of $\gamma = 0.4$.²⁴ Naturally, since the scale of the home country increases, the international spillover effect is larger for $\gamma = 0.4$ than for $\gamma = 0$. In Fig. 5, compared to the case of Fig. 4, the costs from discretion become non-negligible as λ and λ^* increase. The costs are around 6% at the maximum level when $\lambda = \lambda^* = 0.8$. This result is consistent with the results of Table 1.

6.2. LC consumers and nominal wage rigidity

In Table 2, we reported the welfare costs of a discretionary policy when λ and λ^* take same values. In the following analyses, we explore how the presence of nominal wage rigidity affects the welfare cost of a discretionary policy when a share in LC consumers is different between two countries. Previous studies reported a wide range of estimates of the share of LC consumers in the European region. Kaplan et al. (2014) reported that the proportion of LC consumers ranges from 20%-35%, mainly in developed countries. Since we focus on the case for developed countries, we select two values of LC consumers: 0 and 0.25.

Fig. 6 shows the welfare costs from a discretionary policy, given values of $\lambda = 0$ and $\lambda^* = 0.25$. In this figure, the welfare costs from discretion are normalized by the loss of the commitment solution in the cases of $\lambda = \lambda^* = 0$ and $\zeta = \zeta^* = 0$. To do so, given values for λ and λ^* , we can evaluate the costs arising from discretion under several parameterizations of ζ and ζ^* . This simulation reveals how the degree of wage stickiness in both countries affects the welfare costs of discretion when the share of LC consumers is higher in the foreign country than in the home country.

Fig. 6 shows that regardless of the home country's wage stickiness, the higher the nominal wage stickiness is in the foreign country, the greater the welfare costs that arise from discretionary policy. At the maximum level, approximately 20% of welfare costs is generated when $\zeta^* = 0.8$ when flexible wages hold for the home country. However, larger values of ζ and ζ^* can help reduce welfare costs from a discretionary policy. This result is consistent with that in Table 2.

This exercise implies that it is robust for the costs of discretionary policies to be small when nominal wage stickiness is high in both countries even when an asymmetry share in LC consumers is introduced. Needless to say, it follows from Table 2 that, given the greater value of wage stickiness for the foreign country, we can easily conjecture that the welfare costs from discretion are greater for any value of λ and λ^* above 0.25.

7. Concluding remarks

This paper examined optimal monetary policy in a two-country NK model with LC consumers. We explored the properties of optimal monetary policy under a currency union because we focused on the empirical fact that the share of LC consumers differs across the member countries of the European economy.

²⁴ While we also examine the case for $\gamma = 0.5$, we confirm that the main result under $\gamma = 0.4$ is unaffected by this alternative calibration for γ .

The main findings of this study are as follows. We demonstrated that given LC consumers in the home country, the response of macroeconomic variables to a foreign structural shock is generally attenuated as the share of foreign LC consumers increases. We found that the costs of discretionary policy are generally not greater in a currency union in which LC consumers exist in both countries. Moreover, we emphasize that the presence of nominal wage rigidity significantly lowers welfare losses in our model.

Finally, we would like to mention a few extensions to this paper. The model in this paper focuses on a currency union. This restriction was required to successfully derive a log-linearized structural model and a loss function for the central bank. However, it would be interesting to analyze how the presence of LC consumers interacts between two countries under an exchange rate float. Are the gains from the cooperative solution greater in the presence of LC consumers in both countries? This topic is interesting, but beyond the scope of this paper, so it will be left for future work.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.jimonfin.2022.102787>.

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