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# GDP-linked bonds and economic growth

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ABSTRACT

We analyze the implications of introducing GDP-linked bonds for economic growth. First, we model a stochastically growing small open economy. The government borrows from the international financial market, collects tax revenue, and provides a public infrastructure good. Sovereign debt may be both conventional and indexed to GDP. Second, we calibrate the model for a developing country. The introduction of GDP-linked bonds increases the optimal debt-to-GDP ratio, public-to-private capital ratio, and tax rate. It also exerts a small negative effect on the mean GDP growth rate as well as a small positive welfare impact.

## 1. Introduction

Sovereign debt levels of low- and middle-income countries have been steadily increasing during the last decade (Weiss, 2021). The Covid-19 pandemic of 2020–2023 caused a further surge in sovereign borrowing. As a result, several countries have declared bankruptcy during the pandemic, including Argentina, Ecuador, Ethiopia, and Lebanon (Arellano et al., 2021). As a means to improve debt sustainability and prevent sovereign defaults, GDP-linked bonds have been proposed in academia and are under discussion in politics.[2]

The advantages and disadvantages of GDP-linked bonds in the short and medium run have been studied extensively in the literature, beginning with the notion of state-contingent debt instruments covering risk that is out of control to public agents (Krugman, 1988; Shiller, 1993). Specifically, while growth indexation may lead to consumption smoothing for households for various reasons, a reduced default risk might lead to higher indebtedness, see Durdu (2009); Hatchondo and Martinez (2012). The latter could result in a larger public sector, potentially affecting growth. So, welfare might not only be altered by consumption smoothing but also by changes in the level of income. Nevertheless, to the best of our knowledge, the long-term perspective concentrating on growth has not been

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[2] GDP-linked bonds fall into the wider category of state-contingent debt instruments. Indexation to GDP growth, as opposed to commodity prices or inflation, is of particular interest to policymakers since macroeconomic shocks are more frequent and among the most severe events affecting sovereign debt sustainability. In the toolkit of sovereign debt management, they would serve as insurance against a loss of the ability to service debt obligations, sharing the risk with investors. Therefore, they have caught considerable attention in academics and entered policy discussions, though political will for implementation has been scarce so far (IMF, 2017; G20_2017), exceptions being Portugal, where GDP-linked debt instruments were issued in 2013 and 2017 (Pina, 2020), or debt restructurings in Argentina, Bosnia and Herzegovina, Bulgaria and Costa Rica (Durdu, 2009).

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adopted yet, see Borensztein and Mauro (2004) and Acalin (2018).

Therefore, we employ a small-open-economy, stochastic growth model with a public and a private sector adapted from Chung and Turnovsky (2010), who focus on imperfect capital markets. It suits our research question since it incorporates the main elements and assumptions to assess the long-term effects of debt indexation on growth: endogenous fiscal policy, the modeling of a sovereign debt market, a focus on consumption smoothing and lenders that fear default. The latter two features have been considered in the literature on GDP-linked bonds.[3] This way, we are able to consider the feedback loop between the primary balance, debt stock, interest rate, and growth. Moreover, we can address whether indexation disincentivizes growth policy or would lead to (excessive) use of additional fiscal space (Kim and Ostry, 2020). In our model economy, a benevolent government chooses the optimal levels of sovereign debt, public infrastructure expenditure, and an income tax.[4] Funds for sovereign debt are supplied by an imperfect capital market, which is modeled as by Chung and Turnovsky (2010). The interest investors ask for sovereign debt depends on the perceived probability of default, which results in an upward-sloping *fund supply curve*.[5] Output is produced by a representative local firm that uses public infrastructure costlessly and local private capital to produce a domestically consumed good. An infinitely-lived private household receives the firm's returns, demands its output, pays taxes to the local government, and makes a savings decision. We introduce GDP-linked bonds as an exogenous proportion of total debt. By varying this share, we evaluate the effects of indexation on optimal fiscal policies and household savings behavior and the resulting implications for growth and welfare.

We then calibrate the model to a developing economy. Our main findings indicate that output growth and its variance both fall when the share of GDP-linked bonds in total public debt rises, though both results are quantitatively small. Raising the share of GDP-linked bonds in the debt portfolio from 0% to 10%, the mean GDP growth rate declines negligibly by less than 0.01 percentage points. Thus, we confirm the finding that consumption, which moves along with growth on the balanced growth path, is smoother. Albeit, in our model with endogenous growth, this comes at the price of lower growth rates. Moreover, we observe increases in the optimal debt-to-output ratio, the tax rate, and the public-to-private capital stock ratio. Thus, the policymaker in our model makes use of additional fiscal space. Finally, welfare is slightly higher, by 0.002%, than in the reference scenario without GDP-linked bonds.

The rationale behind these results is the following. At a given debt-to-GDP ratio, introducing GDP-linked bonds lowers the perceived default risk, which depends on the uncertainty of the future debt-to-GDP ratio. This effect raises the optimal debt level. Moreover, it lowers the variance of the expected growth rate, which stimulates infrastructure investment. Consequently, the optimal tax also increases. While a higher public capital stock raises GDP growth, a higher tax lowers it. The tax effect dominates, which reduces the expected growth rate. The lower growth volatility and the higher public-to-private capital ratio are responsible for an increase in consumption (despite the higher tax rate), which increases welfare.

Finally, we present two extensions to the model. First, we consider the case where the government provides a public consumption good. The main results stay qualitatively and quantitatively unchanged, with the only exception that the welfare effect turns slightly negative. In the second extension, which can be found in the online Appendix, we incorporate diminishing returns to investment as proposed by Barlevy (2004). In this second extension, findings indicate that there is no perceptible (negative) impact of introducing GDP-linked bonds on the growth rate, while welfare is again found to increase only slightly, albeit by more than the baseline model.

As opposed to our approach, the benefits of stabilizing public debt are often assessed by computing projections of the debt-to-GDP ratio, see Borensztein and Mauro (2004); Benford et al. (2016); Blanchard et al. (2016); Cabrillac and Sumner (2016); Carnot et al. (2017); Bonfim and Pereira (2018). Results indicate that the variance of the debt-to-GDP ratio is reduced considerably since indexation to growth smooths the debt path during times of notably elevated or subdued growth rates. During downturns, indexation could be especially helpful by containing excessive debt pile-ups. In the case of Portugal, Pina (2020) finds that the debt-to-GDP ratio would have been some 15 percentage points lower in 2011 than it was if the country had issued only GDP-linked bonds, starting in 1999 and at a growth risk premium of 100 basis points. And Bonfim and Pereira (2018) show in an ex-post analysis that Eurozone members would have saved interest payments, albeit small amounts (0.18% of GDP on average), if they had used GDP-linked bonds exclusively in the period of 2000–2015.

While most projections assume an exogenous growth rate and sovereign borrowing, Barr et al. (2014) allow for the government to raise debt levels by adjusting the primary balance. Yet, when considering the feedback of this fiscal reaction on GDP, Barr et al. (2014) index debt to (exogenous) total factor productivity. In the present model, we close the feedback loop by endogenizing tax and debt policy (as well as expenditure policy) while indexing debt on output. Though we confirm their main results of increased consumption smoothing and increased debt levels, we present a richer set of effects at work.

Also, Hatchondo and Martinez (2012) consider the endogeneity of borrowing accounting for borrowing costs rising with the degree of indebtedness, just as the present article. While their contribution concentrates on default stressing that governments cannot commit, they assume that the government has no impact on growth. In contrast, we concentrate on the steady state with and without growth-indexed bonds while having to abstract from default for sake of tractability. Hatchondo and Martinez (2012) find that debt indexation does not only eliminate defaults by overcoming the commitment problem, but it also leads to consumption smoothing. We share the latter result yet show that it materializes alongside lower growth.

[3] See Hatchondo and Martinez (2012); Durdu (2009); Barr et al. (2014).

[4] The earmarking of funds raised through GDP-linked bonds can be motivated by the historical example of the industrial and agricultural bonds issued by France in 1956, see Pina (2020). These bonds, whose coupon was tied to an index of industrial production, served to finance the outfitting of public enterprises in the primary and secondary sectors.

[5] We adopt this term from Chung and Turnovsky (2010). It refers to the notion of investors demanding premia that increase with the ratio of debt to output reflecting the increasing perceived risk.

Moreover, pricing is often seen as a critical issue since high premia vis-à-vis conventional bonds may prevent sovereigns from tapping into this type of instrument. Investors might ask for a premium for growth risk, compensating them for sharing the risk via the equity-like instrument. On the other side, the insurance effect leading to lower default probabilities lowers the default risk premium, see Chamon and Mauro (2005); Barr et al. (2014); Blanchard et al. (2016).[6]

When modeling the bond market, we follow Turnovsky and Chattopadhyay (2003) as well as Hatchondo and Martinez (2012), assuming that (foreign) lenders' supply of funds depends on their zero-expected-profit condition and on indebtedness, where debt beyond a certain threshold is perceived as potentially leading to default. The latter circumstance causes lenders to require a premium.[7] In the baseline calibration, we find that the premium on top of the risk-free rate increases from 48.7 to 54.1 basepoints looking at the average interest rate paid by the sovereign owed to higher indebtedness. This is comparable to what is found by Barr et al. (2014).[8]

Finally, in our model, shocks occur via stochastic depreciation of private and public capital stocks. Guimaraes (2011) argues that interest rate shocks cause a much greater impact on the economy than productivity shocks. Yet, in our approach, a sudden depreciation of either capital stock translates into increased indebtedness through reduced output, which again reflects in an upward adjustment of interest rates. So, without introducing explicit interest rate shocks, our model does provide the interest rate channel.

The rest of the paper is structured as follows. Section 2 presents the model and Section 3 derives the optimal government policy. Section 4 derives the equilibrium interest rates for conventional and GDP-linked bonds. Section 5 presents the numerical results. Section 6 presents the extension to the model, while Section 7 concludes.

## 2. The model

Consider a representative-agent, dynamic small open economy. Time $t$ evolves continuously over the interval $[0,\infty[$. The representative firm produces output at time $t$, $Y_t$, according to the CES production function

$$Y_t = F(K_t, G_t) = \alpha\left[\eta G_t^{-\mu} + (1-\eta)K_t^{-\mu}\right]^{-\frac{1}{\mu}}, \alpha > 0, \eta \in ]0,1[, \mu > -1, \tag{1}$$

where $\alpha > 0$ is a constant, $K_t$ denotes the private capital stock, $G_t$ is a public capital stock (infrastructure), and $\in \equiv 1/(1+\mu) > 0$ is the elasticity of substitution between private and public capital in production. The representative household owns capital and earns a return on capital, $r_K$, over the period $(t, t+dt)$, determined by its marginal product

$$r_K K_t \mathrm{dt} = F_K K_t \mathrm{dt} \ , \tag{2}$$

where a subscript to the production function denotes a partial derivative. Following Koethenbuerger and Lockwood (2010), the return to public capital, $r_G$, is earned by a third fixed factor of production (labor). Over the period $(t, t+dt)$, labor income equals

$$r_G G_t \mathrm{dt} = F_G G_t \mathrm{dt}. \tag{3}$$

The sum of capital and labor income over $(t, t+dt)$ is spent on consumption, $C_t$, investment in the private capital stock, $I_t$, and tax payments $dT_t$:

$$(r_K K_t + r_G G_t)\mathrm{dt} = C_t \mathrm{dt} + I_t \mathrm{dt} + \mathrm{d}T_t \ . \tag{4}$$

Similar to Turnovsky and Chattopadhyay (2003), tax payments have a deterministic and a stochastic component:

$$\mathrm{d}T_t = \tau Y_t \mathrm{dt} + \sigma_V Y_t \mathrm{d}V_t, \tag{5}$$

where $\tau > 0$ is an endogenous income tax rate, $\sigma_V > 0$, and $dV_t$ is a standard Brownian motion. The term $\sigma_V dV_t$ is endogenously determined on the balanced growth path. Using (1), we can rewrite (5) as

$$\mathrm{d}T_t = \xi K_t(\tau \mathrm{dt} + \sigma_V dV_t), \tag{5.1}$$

where $\xi \equiv \alpha(\eta(G_t/K_t)^{-\mu} + (1-\eta))^{(-1/\mu)}$. Private capital accumulation over the time interval $(t, t+dt)$ is determined by the difference between investment and depreciation

$$\mathrm{d}K_t = I_t \mathrm{dt} - K_t(\delta_K dt + \sigma_W dW_t), \tag{6}$$

where $\delta_K$ denotes the deterministic rate of depreciation, $\sigma_W > 0$, and $dW_t \sim N(0, dt)$ is a stochastic rate of depreciation. The assumption of a stochastic depreciation rate follows Wälde (2011). We insert (4) and (5.1) in (6) and define $n_G \equiv G_t/K_t$. The resulting

[6] An overview can be found in Consiglio and Zenios (2018). Growth risk premia are found to be almost always positive (sometimes prohibitively high), yet estimates vary substantially, see Borensztein and Mauro (2004); Bowman and Naylor (2016). Liquidity and novelty premia might also be noticeable at the beginning and hinder the get-going of the issuing of GDP-linked bonds, yet to the best of our knowledge, the literature does not provide any estimates.

[7] Considering foreign lenders we abstract from the effects of indexation on lenders' return and capital income stream or feedback of these on sovereign borrowing. Durdu (2009) in contrast assumes, that the lender is domestic and shows that for low degrees of indexation, consumption smoothing is attained, yet for high degrees, the dynamic adjustment of the coupon renders consumption more volatile and households need to step up precautionary saving leading to a welfare loss.

[8] Given a relative risk aversion of 4, they find a markup of 35 basepoints. We assume a value of 3 for relative risk aversion.

expression determines the growth rate of capital as

$$\frac{dK_t}{K_t} = \left( r_K - \delta_K + r_G n_G - \frac{C_t}{K_t} - \xi\tau \right) \mathrm{dt} - \xi\sigma_V \mathrm{d}V_t - \sigma_W \mathrm{d}W_t. \tag{7}$$

The consumer's expected discounted utility $\Omega$ is given by

$$\Omega = E_0 \int_0^\infty \frac{1}{1-\gamma} C^{1-\gamma} e^{-\beta t} \mathrm{dt} \ , \tag{8}$$

where $\gamma > 0$ denotes the degree of relative risk aversion and $\beta > 0$ is the time discount rate. The consumer maximizes (8) over consumption $C_t$ subject to the constraint (7). Denote the optimal consumption as $\widehat{C}$. It is derived in Appendix A and is determined by

$$\widehat{C} = \mathrm{aK} \ , \tag{9}$$

where $a$ is a constant given by

$$a = \frac{\beta}{\gamma} - \frac{1-\gamma}{\gamma}(r_K - \delta_K + r_G n_G - \xi\tau) + \frac{1}{2}(1-\gamma)\sigma_K^2 \ , \tag{10}$$

with $\sigma_K^2$ defined in Appendix A as the variance of the growth rate of the capital stock.

## 3. The government

The government in this model has an ample role. It provides the public (capital) good $G_t$ and finances its expenditures by raising taxes and issuing debt to the end of maximizing welfare. Its budget constraint writes

$$dZ_t + d\widehat{Z}_t = Z_t dR_Z + \widehat{Z}_t d\widehat{R}_Z + dH_t - dT_t \ , \tag{11}$$

where $Z_t$ and $\widehat{Z}_t$ denote the stocks of conventional and GDP-linked bonds respectively, $dR_Z$ and $d\widehat{R}_Z$ denote the respective interest payments on each debt type, and $dH_t$ denotes primary spending.

Let us first turn to the two types of debt instruments the government issues on the international capital market. The first is conventional debt, with a principal equal to $Z_t$. Its return in the period $(t, t+dt)$ is deterministic and given by $dR_Z = r_Z dt$, where $r_Z$ is the coupon rate. The coupon rate is endogenously determined on the credit market but taken as given by the local government. The second debt instrument is GDP-linked.

Its principal, $\widehat{Z}_t$, is not constant and instead grows at the rate of GDP growth.[9] On the balanced growth path, the capital stock and GDP grow at the same deterministic and stochastic rates. We define in (A.1) in Appendix A $\psi$ as the expected growth rate of the private capital stock. Thus, it follows from (7) and (A.1) that the principal $\widehat{Z}_t$ evolves according to

$$\frac{d\widehat{Z}_t}{\widehat{Z}_t} = \psi dt - \xi\sigma_V dV_t - \sigma_W dW_t \ , \tag{12}$$

Moreover, the GDP-linked bond pays a coupon at a rate equal to $\widehat{r}_Z$. Similar to the interest rate $r_Z$, $\widehat{r}_Z$ is also endogenously determined on the international credit market but viewed as exogenous by the government. Thus, the overall return to the GDP-linked bond, $d\widehat{R}_Z$, is given by

$$d\widehat{R}_Z = \frac{d\widehat{Z}_t}{\widehat{Z}_t} + \frac{\widehat{r}_Z \widehat{Z}_t dt}{\widehat{Z}_t} = (\widehat{r}_Z + \psi)dt - \xi\sigma_V dV_t - \sigma_W dW_t \ . \tag{13}$$

Furthermore, the government issues each debt instrument at a constant proportion of total debt. We denote the total public debt as $Z_t^{\mathrm{total}} = Z_t + \widehat{Z}_t$ and the share of GDP-linkers as $\nu$, such that

$$\widehat{Z}_t = \nu Z_t^{\mathrm{total}}. \tag{14}$$

Thus, the case $\nu = 0$ corresponds to the baseline scenario without GDP-linked bonds. An increase in $\nu$ corresponds to a higher share of GDP-linked and a lower share of conventional bonds, respectively.

The expenditures $dH_t$ are spent on investment in infrastructure, $G_t$ (in Section 6, we extend the model to allow for public spending on a consumption good). Infrastructure evolves according to

$$dG_t = dH_t - G_t(\delta_G dt + \sigma_U dU_t) \ , \tag{15}$$

[9] If a country issues GDP-linkers, they would likely be linked to a proportion of the GDP-growth rate. This is the case in Portugal, which issued contingent bonds in 2013 and 2017 that were linked to 80% and 40% of the GDP growth rate, respectively (Pina, 2020). We assume a link of 100% to GDP growth to keep the analytical solution simple. Taking partial linking into account would not affect our results qualitatively. It would only mitigate our numerical results. Because it also makes the exposition of the model much more cumbersome, we neglect it in the model.

where $\delta_G > 0$ denotes the deterministic rate of depreciation, $\sigma_U$ is a positive constant, and $dU_t$ is a standard Brownian motion that is independent of $dW_t$. Suppose the government's expenditure consists of a deterministic and a stochastic component such that

$$dH_t = G_t\left(\chi dt + \sigma_\chi dX_t\right) \ , \tag{16}$$

where $\chi$ and $\sigma_\chi dX_t$ are to be determined.

In Appendix B, we derive the economy's balanced growth path (BGP). Denote.

$n_Z^{total} \equiv \frac{Z_t^{total}}{K_t}$ as the debt-to-private capital ratio. Using (14), we define as

$n_Z \equiv \frac{Z_t}{K_t} = (1-\nu)n_Z^{total}$ the conventional debt-to-GDP ratio. On the BGP, the mean growth rate of the economy, $\psi$, and its variance, $\sigma_K^2$, are given by (see Appendix B)

$$\psi = \frac{1}{\gamma}[r_K - \delta_K + n_G r_G - \xi\tau - \beta] - \frac{1-\gamma}{2}\sigma_K^2 \ , \tag{17}$$

$$\sigma_K^2 = \frac{n_G^2\sigma_U^2 + \sigma_W^2}{(1 - n_Z + n_G)^2} \ . \tag{18}$$

Moreover, on the balanced growth path, the public-to-private capital ratio $n_G$ can be defined as the following implicit function of the debt-to-capital ratio $n_Z^{total}$ and the tax rate $\tau$:

$$\xi\tau + n_Z^{total}\psi = n_Z^{total}[(1-\nu)r_Z + \nu(\hat{r}_Z + \psi)] + n_G(\delta_G + \psi). \tag{19}$$

Equation (19) stems from the condition that on the BGP, public capital, private capital, and debt must grow at the same deterministic rate. It is a balanced growth path representation of the government's budget constraint with the following interpretation. Over a time period $(t, t+dt)$, the government collects revenues equal to a share $\xi\tau$ of the capital stock. Moreover, public debt grows at the economy's growth rate to keep $n_z^{total}$ constant and provides additional financing to the government equal to $n_Z^{total}\psi$. Hence, the left-hand side of (19) gives the government revenues. The right-hand side of (19) shows the public expenditures on the BGP. The first term represents the interest costs on conventional and GDP-linked bonds. The second term gives the necessary public investment to keep the public capital stock at a constant proportion relative to private capital. Because public capital depreciates at a deterministic rate $\delta_G$ and private capital grows at the rate $\psi$, public investment is given by $n_G(\delta_G + \psi)$.

Equation (19) depends on the three policy parameters $n_G, n_Z^{total}$, and $\tau$. When the government sets two of these parameters, the third would be determined by (19). We choose, without loss of generality, $\tau$ and $n_Z^{total}$ as the government's control variables and use (19) to implicitly define $n_G\left(n_Z^{\text{total}}, \tau\right)$.

### 3.1. Optimal policy

The government chooses the total debt-to-capital ratio $n_Z^{total}$ and tax rate $\tau$ to maximize the expected lifetime utility of the household, taking into account $n_G\left(n_Z^{\text{total}}, \tau\right)$ as defined by (19). The expected lifetime utility is given by

$$\Omega = \frac{a^{-\gamma}K_0^{1-\gamma}}{(1-\gamma)}, \tag{20}$$

see Appendix C for a derivation of this term and all further expressions of this section.

Departing from the first-order conditions of the government's maximization problem, we obtain the following conditions for an optimal fiscal policy

$$\psi - [(1-\nu)r_Z + \nu(\hat{r}_Z + \psi)] = \frac{1}{2}\gamma\frac{\partial\sigma_K^2}{\partial n_Z^{total}}(1 - n_Z + n_G), \tag{21}$$

$$r_G - (\delta_G + \psi) = \frac{1}{2}\gamma\frac{\partial\sigma_K^2}{\partial n_G}(1 - n_Z + n_G). \tag{22}$$

Equation (21) determines the optimal debt-to-capital ratio $n_Z^{total}$. Total debt grows at the rate $\psi$ to keep $n_Z^{\text{total}}$ constant. Thus, it provides government financing at the rate $\psi$ per period, which is expressed as a marginal benefit of debt on the left-hand side of (21). However, debt financing imposes costs, given by (i) the weighted average coupon rate (second term on the left-hand side of (21)) and (ii) higher uncertainty of the GDP growth, $\sigma_K^2$. The relative importance of the effect on uncertainty depends on the risk aversion parameter, $\gamma$, as shown by the right-hand side of (21).

Equation (22) determines the optimal public capital stock $n_G$. The marginal benefit of a higher public capital is its return, $r_G$. The necessary expenditures to keep $n_G$ constant are $\delta_G + \psi$. Moreover, a marginal change in $n_G$ affects the variance of the GDP growth rate, $\sigma_K^2$ (this effect is again weighted by the risk aversion parameter $\gamma$).

Together, Equations (17), (18), (19), (21), and (22) determine the three policy parameters, as well as the growth rate of GDP and its variance, as functions of the interest rates $r_Z, \hat{r}_Z$. We determine the interest rates in the next section.

## 4. The interest rates

Our modeling of the financial market follows Chung and Turnovsky (2010). That is, we model imperfect capital markets that are characterized by an upward-sloping supply of funds for sovereign debt. The literature on imperfect capital markets usually takes a reduced form approach and assumes an exogenous upward-sloping fund supply curve. Chung and Turnovsky (2010) derive the equilibrium borrowing premium that international lenders ask for because of a perceived positive default probability. As in our model, they model imperfect capital markets, defined by the characteristic that creditors perceive a positive risk of default despite the fact that the developing country does not default. However, Chung and Turnovsky (2010) find empirical support that their model's fund supply curve can explain the risk premia that developing debtor countries face. Therefore, we also employ it in our analysis.

To derive the interest rates $r_Z$ and $\widehat{r}_Z$, we first define a hypothetical risk-free debt with a principal $Z^*$. It pays a coupon equal to $dR^* = r^* Z_t^* dt$ over the period $(t, t+dt)$, where $r^*$ is the risk-free interest rate.

In period $t$, the government issues conventional bonds with a constant principal $Z$ and GDP-linked bonds with a principal $\widehat{Z}$ linked to the GDP growth rate. Without loss of generality, they have the same maturity $M$. When a debt contract expires at maturity, it is rolled over with an identical contract. Foreign lenders are represented by foreign-owned, risk-neutral financial intermediaries that operate on a competitive capital market.

For a conventional bond issued in period t with maturity M, the government owes a coupon $r_Z Z_{t+m}$ in period $t+m, m \in [0, M]$ and the principal $Z_{t+M}$ at expiration (the amount owed for GDP-linkers is defined analogously). In line with Chung and Turnovsky (2010), we assume that the lender has incomplete information about the borrower and, therefore, will charge an interest rate that compensates for potential losses from unforeseen adverse events. Concretely, the lender assumes that the borrower may default if the total debt-to-GDP ratio $Z_t^{\text{total}}/Y_t$ exceeds a certain threshold $\theta > 0$. In that case, the sovereign would, given its reduced capabilities, only pay a share of $\lambda \in [0, 1[$ of the coupon (and principal in period $M$). If the debt-to-GDP ratio drops below $\theta$, then the financial intermediary expects the payments to be as stipulated in the contract.

Denote the basis of the payment assessment in period $t+m$ as $\widetilde{Z}_{t+m}^{total}$. It is given by,

$$\widetilde{Z}_{t+m}^{\text{total}} = \begin{cases} Z_{t+m}^{\text{total}} & \text{if} \quad \dfrac{Z_{t+m}^{\text{total}}}{Y_{t+m}} < \theta, \\ \lambda Z_{t+m}^{\text{total}} & \text{if} \quad \dfrac{Z_{t+m}^{\text{total}}}{Y_{t+m}} > \theta. \end{cases} \tag{23}$$

Denote an expected value based on a risk-neutral probability measure as $E^*(\cdot)$. Then, the expected payoff for the financial intermediary in time period $t$ from lending a conventional risky debt with maturity $M$ is

$$E_t^*(\text{Payoff}) = E_t^*\left[\int_0^M r_Z \widetilde{Z}_{t+m} e^{-r^* m} dm + \widetilde{Z}_{t+m} e^{-r^* M}\right], \tag{24}$$

where $\widetilde{Z}_{t+m}$ is the basis of the payment assessment in period $t+m$ for a conventional bond and is defined analogously to $\widetilde{Z}^{\text{total}}$ in (23). The expected payoff of a GDP-linked bond is defined analogously to (24), where $r_Z$ and $\widetilde{Z}$ are replaced by $\widehat{r}_Z$ and $\widehat{\widetilde{Z}}$, respectively.

The financial intermediaries operate under perfect competition and thus earn zero profits in equilibrium. Hence, the equilibrium interest rates $r_Z$ and $\widehat{r_Z}$ must satisfy $E_t^*(\text{Payoff}) = Z_t$ and $E_t^*(\widehat{\text{Payoff}}) = \widehat{Z}_t$, respectively; that is, the present value of expected profits, measured using a risk-neutral probability measure, must be zero.

In Appendix D, we derive the equilibrium interest rates. Moreover, we focus in the remaining analysis on the realistic case $\nu < 1$. The financial intermediaries know that GDP-linked bonds grow at the growth rate of GDP. Hence, they know in period $t$ that $Z_{t+m}^{\text{total}}/Y_{t+m}$ is constant for the corner case $\nu = 1$ and any $m > 0$. Thus, as long as the initial debt-to-GDP ratio $Z_t^{\text{total}}/Y_t$ is below $\theta$, the perceived probability of default is zero for.

$\nu = 1$. In the plausible and nontrivial case $\nu < 1$, Appendix D shows that the interest rates are given by

$$r_Z = \frac{1 - e^{-r^* M}[1 - \Phi(h_M)(1-\lambda)]}{\int_0^M e^{-r^* m}[1 - \Phi(h_m)(1-\lambda)]dm}, \tag{25}$$

$$\widehat{r}_Z = \frac{1 - e^{-(r^* - \psi)M}[1 - \Phi(g_M)(1-\lambda)]}{\int_0^M e^{-(r^* - \psi)m}[1 - \Phi(g_m)(1-\lambda)]dm}. \tag{26}$$

Where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution and $\Phi(h_m)(\Phi(g_m))$ determines the perceived probability of losing a share $1-\lambda$ of the initial amount lent in period $t+m$ of conventional (GDP-linked) bonds. In the case of a zero perceived loss probability, $\Phi(h_m) = \Phi(g_m) = 0$ for all $m$. Then, (25) and (26) give $r_Z = r^*$ and $\widehat{r}_Z + \psi = r^*$, i.e., when debt is risk-free, it yields the risk-free interest rate. Moreover, the interest rates are increasing in the loss probabilities.

Together, Equations (17)-(19), (21)-(22), and (25)-(26) determine the optimal policy parameters $\tau, n_G, n_Z^{\text{total}}$, expected growth and

**Table 1**
Benchmark parameter values.

| | |
|---|---|
| Production parameters | $\alpha = 0.25, \eta = 0.22, \mu = -1/2,$ |
| Preference parameters | $\beta = 0.06, \gamma = 3,$ |
| Risk-free return | $r^* = 0.02,$ |
| Depreciation rates | $\delta_K = 0.04, \delta_G = 0.04,$ |
| Volatility parameters | $\sigma_U = 0.05, \sigma_W = 0.05$ |
| Financial market | $\theta = 0.6, \lambda = 0.9$ |
| Debt maturity | $M = 5$ |

**Table 2**
Balanced Growth Path.

| $\nu$ | $\psi$ | $\sigma_K$ | $Z^{total}/Y$ | $\tau$ | $r_Z$ | $\hat{r}_Z + \psi$ | $\bar{r}_Z$ | $Y/K$ | $C/Y$ | $n_G$ | $\Delta\%\Omega$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 2.986 | 4.08 | 0.612 | 0.1597 | 2.487 | – | 2.487 | 0.22 | 0.522 | 0.512 | |
| 0.1 | 2.982 | 4.04 | 0.622 | 0.1605 | 2.558 | 2.386 | 2.541 | 0.22 | 0.522 | 0.514 | 0.002 |

Legend: $\nu$ – Share of GDP-linked bonds in the portfolio of sovereign debt, $\psi$ – Steady-state growth rate, $\sigma_K$ – Standard deviation in steady state, debt-to-output ratio, $r_Z$ – Interest rate of conventional bonds, $\tau$ – Income tax rate, $\hat{r}_Z + \psi$ – Interest rate of GDP-linked bonds, $\bar{r}_Z$ – Average interest rate, $Y/K$ – Output-to-private-capital ratio, $C/Y$ – Consumption-to-output ratio, $\Delta\%\Omega$ – Change in welfare vis-à-vis business-as-usual case (0% GDP-linked bonds).

its variance, $\psi, \sigma_K^2$, and the interest rates $r_Z, \hat{r}_Z$. While in general the introduction of GDP-linked bonds has ambiguous effects on these seven endogenous variables, we can derive one unambiguous result:

**Proposition 1.** *For any constant policy parameters $\tau, n_G, n_Z^{\text{total}}$, and $\nu < 1$, the perceived probability of loss for conventional bonds, $\Phi(h_m)$, is larger than the perceived probability of incurring losses for GDP-linked bonds, $\Phi(g_m)$.*

**Proof:** The condition $\Phi(h_m) > \Phi(g_m)$ holds iff $h_m > g_m$. Holding $\tau, n_G, n_Z^{total}$ and $\nu < 1$ constant, and using the definitions of $h_m$ and $g_m$ from Appendix D, it is straightforward to show that $h_m > g_m$ is satisfied when $(1-\nu)m\sigma_K^2 > 0$, which is true. ■

The intuition behind Proposition 1 is the following. Note first that each bond instrument is perceived to be risky because the future total debt-to-GDP ratio is perceived as stochastic from financial markets. That is, if $Z_{t+m}^{\text{total}}/Y_{t+m}$ rises above $\theta$, creditors expect the government to lower the interest payment on both types of bonds by a factor of $1-\lambda$. However, while conventional bonds have a constant principal and investors expect to lose a share $1-\lambda$ of the constant value $r_Z Z_t$, GDP-linked bonds have a principal that grows at the rate of GDP. This characteristic counteracts the perceived risk of GDP-linked bonds compared to conventional bonds because in the state of the world where GDP is low enough to trigger a default, the principal of GDP-linked bonds would also have declined. Hence, whenever a default occurs, investors would lose a share $1-\lambda$ of a smaller payment $\hat{r}_Z \hat{Z}_{t+m}$. On the other hand, in the states of nature where GDP and $\hat{Z}_{t+m}$ grow, no default is expected (because in these states $Z_{t+m}^{\text{total}}/Y_{t+m}$ declines).

To understand how the share of GDP-linked bonds affects all policy variables, the growth rate, and welfare, we calibrate the model in the next section.

## 5. Calibration

To compare the two debt instruments, we calibrate the model. In the benchmark simulation, the degree of relative risk aversion is $\gamma = 3$ and the time preference parameter is $\beta = 0.06$, which are standard values. An et al. (2019) empirically estimate the elasticity of substitution between private and public capital, $\epsilon$, to be equal to one for advanced countries, three for emerging countries, and two for low-income developing countries. Since we consider a developing country, we take the value $\epsilon = 2$ in the benchmark simulation. This value implies $\mu = -1/2$. Furthermore, $\alpha = 0.25$ and $\eta = 0.22$, which produces a value $n_G = 0.52$ in the benchmark case that is within the range of estimates for emerging and low-income developing countries (An et al., 2019). The rates of depreciation are standard with $\delta_K = \delta_G = 0.04$. The risk-free interest rate is $r^* = 0.02$, representing the low level of interest rates prevailing during the 2010 s. We also set the standard deviations to common values of $\sigma_U = \sigma_W = 0.05$.[10] Furthermore, we set the threshold debt-GDP-ratio, above which credit markets view debt as risky, at 60%; that is, $\theta = 0.6$.[11] The repayment share of the restraint sovereign $\lambda$ is derived in the following way. Edwards (2015) studies 180 sovereign defaults and finds the average "haircut" to be 0.37. Moreover, Standard & Poors estimate

[10] These values have also been chosen by Chung and Turnovsky (2010).

[11] The 60%-threshold for debt is motivated by the European Union Maastricht criteria on indebtedness. Some of its members have been considered emerging markets, such as the eastern member states. On the other side, no such standard with respect to an indebtedness threshold is known to us for other (developing) countries. In any case, the sensitivity analysis shows that also thresholds of 40% or 80% qualitatively yield the results.

the 15-year cumulative default rate of countries with speculative ratings equal to 25.1% (Standard and Poors, 2018). Hence, the financial intermediaries expect to receive $0.251(1-0.37)+(1-0.251)1=0.907$. We have thus set $\lambda=0.9$. Lastly, the debt maturity is medium-term and equals 5 years, $M=5$.

In the benchmark calibration, we initially compare two cases: one without GDP-linked bonds ($\nu=0$) and one where 10% of the total government debt is GDP-linked ($\nu=0.1$), meant to represent a cautious introduction of this type of bond.[12] The results from these two cases are reported in Table 2. One new variable we introduce in Table 2 is $\bar{r}_Z$, which gives the average deterministic interest on public debt: $\bar{r}_Z=(1-\nu)r_Z+\nu(\widehat{r}_Z+\psi)$.

In the first case, the growth rate is 2.986%, which is similar to the result of Chung and Turnovsky (2010) who also study a developing country. The standard deviation of the growth rate is 4.08%. The optimal total debt-to-GDP ratio and tax rate are 0.612 and 0.16, respectively. The interest rate on conventional debt is 2.49%.

Allowing for GDP-linked bonds that equal 10% of total indebtedness leads to an increase in the debt ratio by one percentage point ($Z^{\text{total}}/Y=0.622$) and a slightly higher tax rate. The reason for the higher borrowing is the lower total deterministic return on GDP-linkers ($\widehat{r}_Z+\psi=2.39\%$). However, the higher borrowing pushes the average interest rate on public debt to a higher value than in the first case: $\bar{r}_Z=2.54>2.49$. Moreover, the higher tax rate allows for a slightly higher public-to-private capital ratio $n_G$. The growth rate decreases slightly to 2.982%, as does its standard deviation. However, the change in welfare is slightly positive. Lifetime utility increases insignificantly by 0.002%.[13]

All in all, the effects of introducing GDP-linked bonds are relatively small. The largest change is in the debt-to-GDP ratio, which increases from 61.2% to 62.2%. More importantly, the incentives to invest in productive capital do not decline, as the public-to-private capital ratio increases. While there is a negative impact on growth, though, of insignificant magnitude, the introduction of GDP-linked bonds lowers the variance of the expected growth rate and increases welfare slightly.

The intuition of these results is the following. First, as Proposition 1 states, the perceived probability of incurring losses from GDP-linkers is smaller than that from conventional bonds. Hence, financial markets require a smaller premium over the risk-free rate for GDP-linkers. Therefore, an increase in the share of GDP-linked bonds in the debt portfolio $\nu$ lowers the average interest costs and thus increases the optimal debt-to-GDP ratio. This result is consistent with previous findings that GDP-linked debt boosts fiscal space (Durdu, 2009; Kim and Ostry, 2020).

Next, consider how $\nu$ affects the conventional debt-to-capital ratio $n_Z=(1-\nu)n_Z^{\text{total}}$. On the one hand, a positive share of GDP-linkers directly lowers $n_Z$, while, on the other hand, it increases $n_Z^{\text{total}}$, which may raise the ratio of conventional bonds. In the calibration, the direct effect dominates. Thus, $n_Z$ declines. Because the variance of the growth rate is increasing in the conventional debt-to-capital ratio, the lower $n_Z$ leads to a lower variance $\sigma_K^2$. Furthermore, lower $\sigma_K^2$ lowers the expected GDP growth rate $\psi$, which incentivizes the government to invest more in infrastructure according to (22) ($n_Z$ also affects (22) though its right-hand side in an ambiguous way, however, in the calibration, the growth effect dominates). Hence, $n_G$ goes up. Lastly, the tax rate must increase because of both the higher interest payments and infrastructure spending. Together, the reduction in $\sigma_K^2$ and the higher tax rate lower the expected GDP growth $\psi$, even though the higher infrastructure ratio $n_G$ counteracts these effects.

To summarize, GDP-linked debt disincentivizes the accumulation of private capital, $\psi$, by lowering its volatility, $\sigma_K^2$. While this effect stimulates more public investment and a higher public capital ratio, the crowding-in through more public investment is insufficient to reverse the growth rate decrease.[14]

Fig. 1 shows that these effects are close to linear in $\nu$. In panels (a) to (g), we vary $\nu$ in the interval $[0,0.2]$. The most significant absolute change is in the debt-to-GDP ratio, which increases by almost 2 percentage points as the share of GDP-linkers goes up from 0% to 20% (panel (c)). At the same time, the average interest rate $\bar{r}_Z$ increases by only ten basis points (panel (f)), while the mean GDP growth rate declines from 2.986% to 2.976% (panel (a)). The welfare effect (panel (g)) is small but positive and increasing in $\nu$.

### 5.1. Sensitivity analysis

To assess the robustness of our findings, we analyze changes in the results from varying a series of parameters: (i) debt maturity (considering short-term debt with $M=1$ and long-term debt with $M=10$), (ii) risk aversion ($\gamma$), (iii) the threshold debt-to-GDP level above which default is expected ($\theta$), (iv) the expected payment fraction when debt is above the critical value ($\lambda$), (v) the share of public capital in production ($\eta$), and finally (vi) the complementarity of public and private capital in production ($\mu=(1-\epsilon)/\epsilon$). The results indicate that our findings from above are robust (see Appendix E, Table E1).

In all cases, with one exception, the results remain both qualitatively and quantitatively the same. The only exception is when we

[12] A wider range of values for $\nu$ is considered in Fig. 1.

[13] The welfare change is measured as an equivalent variation. It is given by the change in the initial capital stock, $K_0$, in the situation without GDP-linkers that leads to the same welfare effect as the introduction of GDP-linked bonds.

[14] If we allow the private household to also borrow on the international capital market, assuming that private debt is not GDP-linked, the introduction of GDP-linked public bonds would raise the interest costs that private borrowers face on the capital market (because of the increase in the public debt-toGDP ratio). This is likely to further disincentivize private investment. An extension of the model in this direction is left for future research.

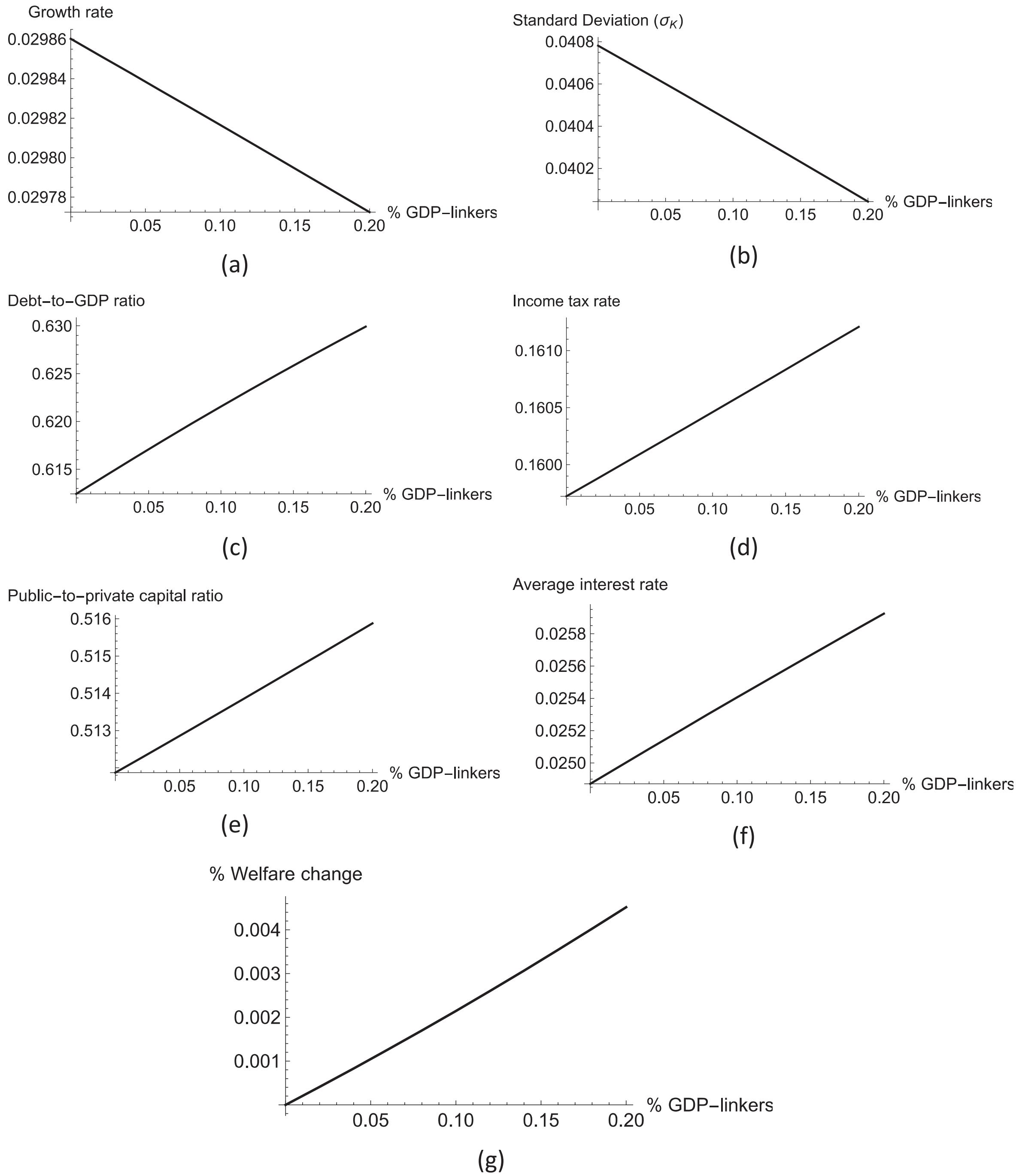


**Fig. 1.** Variation of Share of GDP-linked Bonds in Debt Portfolio ($\nu$). Legend: % GDP-linkers – $\nu$ (share of GDP-linked bonds in the portfolio of sovereign debt), Growth rate – $\psi$, Standard deviation of growth – $\sigma_K$, Debt-to-GDP ratio - $Z^{\text{total}}/Y$, Income tax rate – $\tau$, Public-to-private capital ratio – $n_G$, Average interest rate – $\bar{r}_Z$, % Welfare change – $\Delta\%\Omega$

vary the elasticity of substitution between public and private capital. At $\in = 3$ (the estimate of An et al. (2019) for emerging markets), welfare increases by 0.008% when $\nu$ goes up from zero to 0.1, while at $\in = 1$, welfare decreases insignificantly by 0.002%.[15] The only effect that may become quantitatively larger, is the change in public borrowing. In the case of high risk aversion ($\gamma = 3.5$ in Table E1) and in the case of long-term debt ($M = 10$ in Table E1), an increase in $\nu$ from zero to 0.1 raises the optimal debt-to-GDP ratio by about 2 percentage points. Thus, all results reported in Table 2 are robust. In the next section, we analyze whether the type of government expenditure matters for the results.

## 6. Public consumption good

In the main model, the government can only spend the tax revenues on infrastructure. However, many types of public spending are directed toward consumption public goods. In this section, we consider whether the type of government spending affects our results. Therefore, we consider the other extreme case, where $G$ is a pure consumption public good.

In the absence of public capital, we define the production function as in Wälde (2011):

$$Y_t = \alpha K_t, \quad \alpha > 0. \tag{27}$$

Thus, the return on capital is $r_K = \alpha$ and the budget constraint over the period $(t, t + dt)$ becomes

$$r_K K_t dt = C_t dt + I_t dt + dT_t, \tag{28}$$

where taxes $dT_t$ are defined analogously to (5). Capital evolves according to (6). Together, (5), (6), (27) and (28) give

$$\frac{dK_t}{K_t} = \left[\alpha(1-\tau) - \frac{C_t}{K_t} - \delta_K\right]dt - \alpha\sigma_V dV_t - \sigma_W dW_t. \tag{29}$$

Moreover, expected utility is now defined as

$$\Omega = E_0 \int_0^\infty \frac{1}{1-\gamma}(CG^\zeta)^{1-\gamma} e^{-\beta t} dt, \quad \gamma, \zeta > 0, \quad \zeta(1-\gamma) < 1, \quad (1+\zeta)(1-\gamma) < 1, \tag{30}$$

where the constraints on the parameters $\gamma, \zeta$ ensure that utility is increasing and strictly concave in $C$ and $G$. Similar to tax revenues, government spending is proportional to capital. Over the period $t, t + dt$, the government spends

$$dG_t = G_t dt = gK_t dt, \tag{31}$$

where $g$ is the degree of proportionality and is endogenously determined. Thus, public consumption evolves deterministically, analogously to private consumption.

To solve the model, we first derive the utility-maximizing private consumption in Appendix F. It equals $\widehat{C} = cK$, where

$$c = \frac{1-\gamma}{\gamma(1+\zeta)}\left[\frac{\beta}{1-\gamma} - (1+\zeta)(\alpha(1-\tau) - \delta_K) + \frac{1}{2}(1+\zeta)(1-(1+\zeta)(1-\gamma))\sigma_K^2\right]. \tag{32}$$

The government's budget constraint is defined by (11) when we replace $dH_t$ by $dG_t$ from (31). Following the same steps as in the main model, we derive the BGP (see Appendix F). Analogously to equation (19), we can derive an implicit function that determines the relationship between $\tau, g$ and $n_Z^{\text{total}}$ along the balanced growth path and which write

$$\alpha\tau + n_Z^{total}\psi = n_Z^{total}[(1-\nu)r + \nu(\widehat{r}_Z + \psi)] + g. \tag{33}$$

Equation (33) is the analog to (19) from the main model. Here, the output-to-capital ratio $\xi$ is substituted by $\alpha$ because of the change in the production function. Thus, the lefthand side of (33) has the same interpretation as the left-hand side of (19). Moreover, its right-hand side contains the expenditures on interest (first term) and public consumption, $g$. Thus, (33) determines the public expenditure proportion $g$ as a function of the tax rate $\tau$ and debt ratio $n_Z^{\text{total}}$, i.e., $g(n_Z^{\text{total}}, \tau)$. The government maximizes expected utility (30) over $\tau$ and $n_Z^{\text{total}}$, taking (33) into account. The optimal policy is described by (see Appendix F)

$$\frac{g}{c} = \zeta\left[1 - \frac{n_Z}{\gamma}\right]\psi, \tag{34}$$

$$\psi - [(1-\nu)r_Z + \nu(\widehat{r}_Z + \psi)] = (1-n_Z)\frac{1-(1+\zeta)(1-\gamma)}{2}\frac{\partial\sigma_K^2}{\partial n_Z^{total}}. \tag{35}$$

Equation (34) is the Samuelson rule for optimal public good provision, while (35) has the same interpretation as (21).

Moreover, the equilibrium interest rates are determined analogously to the main model, following the same steps as in Appendix D.

[15] With a low substitutability (i.e., high complementarity) between $G$ and $K$, the crowding in of public capital is weakened (in comparison to the baseline parameter setting). Recall that private capital is decreased due to GDP-linked bonds. Therefore, the crowding in of public capital can compensate less for the negative effect of private capital and the change of welfare turns negative.

**Table 3**
Balanced Growth Path for the Extension with Public Consumption Good.

| $\nu$ | $\psi$ | $\sigma_K$ | $Z^{total}/Y$ | $\tau$ | $r_Z$ | $\hat{r}_Z + \psi$ | $\bar{r}_Z$ | $C/Y$ | $g$ | $\Delta\%\Omega$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 3.583 | 5.72 | 0.569 | 0.207 | 2.275 | – | 2.275 | 0.4481 | 0.0472 | – |
| 0.1 | 3.568 | 5.67 | 0.595 | 0.208 | 2.424 | 2.3 | 2.41 | 0.4478 | 0.0473 | −0.007 |

Legend: $\nu$ − Share of GDP-linked bonds in the portfolio of sovereign debt, $\psi$ − Steady-state growth rate, $\sigma_K$− Standard deviation of growth in steady state, debt-to-output ratio, $r_Z$ − Interest rate of conventional bonds, $\tau$ − Income tax rate, $\hat{r}_Z + \psi$ − Interest rate of GDP-linked bonds, $\bar{r}_Z$ − Average interest rate, $g$ − Provision of public (consumption) good as ratio to output ($g = G/Y$), $C/Y$ − Consumption-to-output ratio, $\Delta\%\Omega$ − Change in welfare vis-à-vis business-as-usual case (0% GDP-linked bonds).

In the next section, we calibrate the model to evaluate the implications of varying the proportion of GDP-linked bonds, $\nu$.

### *6.1. Calibration*

We calibrate analogously to the main model. The parameters $\beta, \gamma, r^*, \delta_K, \sigma_W, \theta, \lambda,$ and $M$ take the same values as in Table 1. The parameter $\alpha$ now determines $Y/K$. Therefore, instead of setting it at 0.25 as in Table 1, we choose $\alpha = 0.22$, which is the BGP value of $Y/K$ in the main model (see Table 2). The only new parameter $\zeta$ is chosen to equal 0.5. Thus, public consumption has half the value of private consumption in utility. The resulting balanced growth paths for $\nu = 0$ and $\nu = 0.1$ are presented in Table 3.

Almost all results are identical qualitatively and quantitatively to those from the main model: the introduction of GDP-linked bonds has a negligible negative effect on the expected growth rate and its variance. Also, it raises the debt-to-GDP ratio by three percentage points and the average interest rate by 14 basis points. The only qualitative difference is that now welfare declines slightly. The reason is that private consumption goes down, while the share spent on public consumption increases only marginally.

Furthermore, sensitivity analysis in Appendix F shows that the effects found in Table 3 are monotone when varying the share of GDP-linked bonds from zero to 0.2 (see Fig. F1). Moreover, in Tables F1 and F2, we show that the results remain qualitatively unchanged when varying the preference for public consumption to $\zeta = 0.4$ or $\zeta = 0.6$.

## 7. Diminishing returns to investment

As a second extension, we consider diminishing returns to investment, as in Barlevy (2004), who shows that diminishing returns have important implications for the effects of uncertainty on welfare and growth.

Following Barlevy (2004), we introduce a production function for new capital $\Phi(I_t, K_t)$ that determines how much capital is produced for a given level of investment $I_t$. It is increasing in its first argument and homogeneous of degree one. Analogously, there is a capital production function $\Phi_G(\bullet)$ in the public sector.

This extension complicates the solution of the model, as it introduces three new endogenous variables: Tobin's $q$ and the investment-to-capital ratios in the private and the public sector, respectively (see the Online Appendix for a full description and solution of the extension). The presence of these variables complicates the government's first-order conditions enough to make them analytically intractable and makes an analysis similar to the interpretation of equations (21) and (22) in Section 3.1 impossible. However, a numerical calibration shows that the results remain mostly unchanged. Moreover, the negative growth effects and the impact on uncertainty ($\sigma_K^2$) become even smaller, while the positive welfare effects increase to about 0.2%. The stronger the diminishing returns to investment are, the more the effects on the growth rate and on uncertainty diminish and the stronger the positive welfare impact is. However, the quantitative effects of GDP-linked bonds remain small.

The small quantitative effects do not necessarily mean that uncertainty plays a small role in the model. It is the government following an optimal policy and adjusting its tax rate and borrowing as a response to a change in the ratio of GDP-linked bonds that mitigate the overall impacts. To see this, consider the corner case of no uncertainty. Setting uncertainty equal to zero (and assuming there are no GDP-linked bonds), the government will choose public debt such that the growth rate equals the interest rate on government bonds $\psi = r_Z$ (according to the government's first-order condition (21)). As long as this debt is perceived as risk-free, we will have $r_Z = r^*$, and the growth rate $\psi$ will equal the risk-free interest rate $r^*$. In effect, we would ignore the meaningful notion of a fund-supply function as put forward by Chung and Turnovsky (2010), which poses interest rates rising with the level of indebtedness. Moreover, introducing GDP-linked debt in this setting would not have any repercussions on growth or welfare because the effective interest rate on GDP-linked bonds would again be the risk-free interest rate.

Hence, the endogeneity of government policy makes it difficult to isolate the insurance effects of GDP-linked bonds. To analyze such effects, future work should combine our long-term analysis with a short-term analysis where the government may follow a time-varying policy. In such a model, one could assume that at the time of the introduction of GDP-linked bonds, the government does not change its policy and only smoothly approaches its new long-term tax, debt, and spending levels.

## 8. Conclusion

This paper analyzes how the introduction of GDP-linked bonds affects the economic growth and welfare of a small open, developing country. We find that the government's incentives to invest in growth-enhancing infrastructure are increasing in the share of GDP-linked bonds. However, public debt also increases, which leads to a higher tax rate with a net negative impact on growth. On the other hand, private consumption increases, which leads to higher welfare. However, all results are quantitatively small. If the government spends on a consumption public good, the welfare results reverse but remain negligible.

First and foremost, our results show that short-term benefits of indexing to growth, such as debt stabilization, do not come at the cost of markedly lower output growth or welfare, independently of whether public money is spent on infrastructure or a public consumption good. While the policymaker uses fiscal space gained through GDP-linkers to increase indebtedness, this leads to higher expenses, promoting growth. On the other side, we find that GDP-linked debt disincentivizes the accumulation of private capital, $\psi$, by lowering its volatility, $\sigma_K$. However, the crowding in of public capital cannot fully compensate for the subdued growth of private capital, and the net effect on growth is slightly negative in the main model variant.

Yet, our analysis also shows that the effects of indexation on growth are rather small. This shows that such a policy is rather an instrument aimed at short to medium-term policy goals. Moreover, this explains why considering diminishing returns to investment, by which Barlevy (2004) motivates high costs if business cycle fluctuations, do not show big improvements concerning growth, its variance, or welfare.

Lastly, we find stronger indications for positive welfare effects when government outlays are dedicated to public infrastructure, as is the case in some developing countries. It would be worthwhile to test whether debt-financed public infrastructure investments do show these effects on the rate and variance of economic growth.

Our model thus also makes several predictions. Specifically, in all scenarios, the model predicts that the introduction of GDP-linked debt raises the overall debt-to-GDP ratio, government expenditures, and tax revenues. Future work should test these predictions empirically.

To keep some focus of the paper, we concentrate on the effect of policy (the introduction of indexed bonds) in an uncertain environment. Therefore, we do not address the role of uncertainty itself, as does, for example, Barlevy (2004). To isolate the insurance effects of GDP-linked bonds, our model should be extended to consider time-varying government policy by, for example, assuming that the government does not change its policy at the instance of the introduction of GDP-linked bonds. In such an analysis, our predictions would be the long-term effects when the government has fully changed its policy.

As concerns caveats, an important implicit assumption in our model is the earmarking of public revenues for infrastructure expenditures and the risk neutrality of lenders. Pina (2021) shares the first assumption in a broad sense yet shows, albeit not in a fully-fledged growth model, that risk-averse lenders could, under certain parameter constellations, prefer conventional to contingent debt. A promising direction for future research is to introduce risk-averse lenders in a growth model and analyze how both our and Pina's (2021) results are affected.

Finally, while this paper analyzes GDP-linked bonds by comparing balanced growth paths, an important direction for future research is to analyze the transitional path between steady states. Such an analysis will produce estimates regarding the short-term effects of GDP-linked bonds, which are beyond the scope of our paper. It will also link our long-term results to the short-term results from the existing literature.

## CRediT authorship contribution statement

**Zarko Y. Kalamov:** Conceptualization, Methodology, Software, Formal analysis, Writing – original draft, Writing – review & editing. **Karl J. Zimmermann:** Conceptualization, Validation, Writing – original draft, Writing – review & editing.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. Moreover, the authors take full responsibility for the content of this publication.

## Appendix A. Solution of the utility maximization problem

To solve the utility maximization problem, we first define

$$\psi \equiv r_K - \delta_K + r_G n_G - \frac{C_t}{K_t} - \xi, \tag{A.1}$$

$$\sigma_K^2 \equiv Var(-\xi\sigma_V dV_t - \sigma_W dW_t), \tag{A.2}$$

where $\psi$ and $\sigma_K^2$ are the expected growth rate and variance of the growth rate of private capital. The solution procedure follows Turnovsky (1999). Denote the maximized lifetime utility by the value function $V(K,t)$ and define its differential generator $L[V(K,t)]$ as

$$L[V(K,t)] \equiv \frac{\partial V}{\partial t} + \psi K \frac{\partial V}{\partial K} + \frac{1}{2}\sigma_K^2 K^2 \frac{\partial^2 V}{\partial K^2}. \tag{A.3}$$

Suppose that $V(K,t)$ is separable, such that

$$V(K,t) = e^{-\beta t} X(K). \tag{A.4}$$

The optimization problem consists of choosing $C_t$ to maximize

$$\frac{1}{1-\gamma}C^{1-\gamma}e^{-\beta t}+L\left[e^{-\beta t}X(K)\right]=\frac{1}{1-\gamma}C^{1-\gamma}e^{-\beta t}-\beta V+\psi Ke^{-\beta t}\frac{\partial X}{\partial K}+\frac{1}{2}\sigma_K^2K^2e^{-\beta t}\frac{\partial^2 X}{\partial K^2}, \tag{A.5}$$

where we used (A.4) to derive $\partial V/\partial t=-\beta V$. The first-order condition with respect to $C$ gives

$$C^{-\gamma}e^{-\beta t}-\frac{1}{K}Ke^{-\beta t}\frac{\partial X}{\partial K}=0. \tag{A.6}$$

Simplifying and using a subscript to denote partial derivatives, we get

$$C=X_K^{-\frac{1}{\gamma}}. \tag{A.7}$$

Additionally, the value function must satisfy the Bellman Equation

$$\max_C\left\{\frac{1}{1-\gamma}C^{1-\gamma}e^{-\beta t}+L\left[e^{-\beta t}X(K)\right]\right\}=0. \tag{A.8}$$

Equation (A.8) states that the maximized value of the left-hand side equals zero. Denote the optimal consumption from (A.7) as $\widehat{C}$. Then, we can reformulate (A.8)[16]:

$$\frac{1}{1-\gamma}\widehat{C}^{1-\gamma}-\beta X(K)+\psi(\widehat{C})KX_K+\frac{1}{2}\sigma_K^2K^2X_{KK}=0. \tag{A.9}$$

Equation (A.5) is a differential equation for $X(K)$. To solve it, we make an educated guess regarding $X(K)$. Suppose that

$$X(K)=\frac{a^{-\gamma}K^{1-\gamma}}{1-\gamma}, \tag{A.10}$$

where $a$ is a constant to be determined. Then, we can derive the following partial derivatives:

$$X_K=a^{-\gamma}K^{-\gamma},\quad X_{KK}=-a^{-\gamma}\gamma K^{-\gamma-1}\ . \tag{A.11}$$

Now, we can plug (A.7), (A.10) and (A.11) in (A.9) to derive the following expression:

$$\begin{aligned}&\frac{1}{1-\gamma}\left[\left(a^{-\gamma}K^{-\gamma}\right)^{-\frac{1}{\gamma}}\right]^{1-\gamma}-\frac{\beta\ a^{-\gamma}K^{1-\gamma}}{1-\gamma}\ldots\\ &\ldots+\left[r_K-\delta_K+r_Gn_G-\frac{\left(a^{-\gamma}K^{-\gamma}\right)^{-\frac{1}{\gamma}}}{K}-\xi\tau\right]Ka^{-\gamma}K^{-\gamma}-\frac{1}{2}\sigma_K^2K^2a^{-\gamma}\gamma K^{-\gamma-1}=0.\end{aligned} \tag{A.12}$$

Simplifying, we get

$$\frac{1}{1-\gamma}a^{1-\gamma}K^{1-\gamma}-\beta\frac{a^{-\gamma}K^{1-\gamma}}{1-\gamma}+[r_K-\delta_K+r_Gn_G-a-\xi\tau]a^{-\gamma}K^{1-\gamma}-\frac{1}{2}\sigma_K^2a^{-\gamma}\gamma K^{1-\gamma}=0. \tag{A.13}$$

All the terms in (A.13) contain $a^{-\gamma}K^{1-\gamma}$, which can be canceled out. Then, we can solve for $a$:

$$a=\frac{\beta}{\gamma}-\frac{1-\gamma}{\gamma}(r_K-\delta_K+r_Gn_G-\xi\tau)+\frac{1}{2}(1-\gamma)\sigma_K^2. \tag{A.14}$$

Using (A.7), (A.11) and (A.14), we can also solve for $\widehat{C}$:

$$\widehat{C}=\left(a^{-\gamma}K^{-\gamma}\right)^{-\frac{1}{\gamma}}=aK. \tag{A.15}$$

## Appendix B. Derivation of the balanced growth path

We combine Equations (5.1), (13), (14), (11), (16) and $dR_Z=r_Zdt$ to derive

$$\begin{aligned}\frac{dZ_t}{Z_t}=&\left[(1-\nu)\left(r_Z+\frac{n_G}{n_Z}\chi-\xi\frac{\tau}{n_Z}\right)+\nu(\widehat{r_Z}+\psi)\right]dt\ \ldots\\ &\ldots\ +(1-\nu)\frac{n_G}{n_Z}\sigma_\chi dX_t-\left(\nu+\frac{1-\nu}{n_Z}\right)\xi\sigma_VdV_t-\nu\sigma_WdW_t\ .\end{aligned} \tag{B.1}$$

[16] In deriving (A.9), we canceled out the exponential terms $e^{-\beta t}$.

Moreover, (15) and (16) give

$$\frac{dG_t}{G_t} = (\chi - \delta_G)dt + \sigma_\chi dX_t - \sigma_U dU_t. \tag{B.2}$$

On a balanced growth path, (7), (B.1) and (B.2) must grow at the same rate. This condition ensures the constancy of $n_G$ and $n_Z$. Equating the stochastic parts of these three equations determines the endogenous $\sigma_V dVt$ and $\sigma_\chi dXt$:

$$\sigma_V dV_t = \frac{n_G\sigma_U dU_t + (n_Z - n_G)\sigma_W dW_t}{\xi(1 - n_Z + n_G)}, \tag{B.3}$$

$$\sigma_\chi dX_t = \frac{(1 - n_Z)\sigma_U dU_t - \sigma_W dW_t}{1 - n_Z + n_G}. \tag{B.4}$$

Equating the deterministic components of (7), (B.1) and (B.2), we get

$$\chi = \frac{n_Z\left[(1-\nu)\left(r_Z + \frac{\delta_G}{1-\nu} - \frac{\xi\tau}{n_Z}\right) + \nu(\widehat{r}_Z + \psi)\right]}{n_Z - (1-\nu)n_G}, \tag{B.5}$$

$$0 = n_Z\left(r_Z + \frac{\nu}{1-\nu}\widehat{r}_Z\right) - \xi\tau + n_G\delta_G - (n_Z - n_G)\psi. \tag{B.6}$$

Equation (19) is (B.6), when we substitute $n_Z$ by $(1-\nu)n_Z^{total}$ and rearrange the terms. The mean growth rate $\psi$ is determined by (A.1) and (A.14) and is

$$\psi = \frac{1}{\gamma}(r_K - \delta_K + r_G n_G - \xi\tau - \beta) - \frac{1-\gamma}{2}\sigma_K^2. \tag{B.7}$$

Moreover, the variance of the growth rate $\sigma_K^2$ can be calculated using (7) and (B.3):

$$\sigma_K^2 = \frac{n_G^2\sigma_U^2 + \sigma_W^2}{(1 - n_Z + n_G)^2}. \tag{B.8}$$

Finally, we differentiate (19) with respect to $n_G, n_Z^{\text{total}}$ and $\tau$, taking into account (B.7) and (B.8), to derive

$$\frac{dn_G}{dn_Z^{total}} = -\frac{(1-\nu)r_Z + \nu\widehat{r}_Z - (1-\nu)\psi + \frac{(n_Z-n_G)(1-\gamma)}{2}\frac{\partial\sigma_K^2}{\partial n_Z^{total}}}{\delta_G + \psi - r_G\tau - \frac{n_Z-n_G}{\gamma}r_G(1-\tau) + \frac{(n_Z-n_G)(1-\gamma)}{2}\frac{\partial\sigma_K^2}{\partial n_G}}, \tag{B.9}$$

$$\frac{dn_G}{d\tau} = \frac{\xi\left(1 - \frac{n_Z-n_G}{\gamma}\right)}{\delta_G + \psi - r_G\tau - \frac{n_Z-n_G}{\gamma}r_G(1-\tau) + \frac{(n_Z-n_G)(1-\gamma)}{2}\frac{\partial\sigma_K^2}{\partial n_G}}. \tag{B.10}$$

Equation (B.9) determines the effect of public debt on the stock of public capital consistent with a balanced growth path, while (B.10) determines the effect of a higher tax rate on the stock of public capital. Moreover, in deriving (B.9) and (B.10), we used $\partial\xi/\partial n_G = \alpha\eta n_G^{-\mu-1}\left[\eta n_G^{-\mu} + (1-\eta)\right]^{-(1+\mu)/\mu} = r_G$.

## Appendix C. Derivation of the optimal policies

First, we determine the expected lifetime utility

$$\Omega = E\int_0^\infty \frac{1}{1-\gamma}C^{1-\gamma}e^{-\beta t}dt = E\int_0^\infty \frac{1}{1-\gamma}(aK_t)^{1-\gamma}e^{-\beta t}dt$$

$$= \frac{1}{1-\gamma}(aK_0)^{1-\gamma}E\int_0^\infty e^{(1-\gamma)\left(\psi - \frac{\sigma_K^2}{2}\right)t - (1-\gamma)\frac{n_G\sigma_U(U_t-U_0)+\sigma_W(W_t-W_0)}{1-n_Z+n_G}}e^{-\beta t}dt$$

$$= \frac{1}{1-\gamma}(aK_0)^{1-\gamma}\int_0^\infty e^{(1-\gamma)\left(\psi - \frac{\sigma_K^2}{2}\right)t + (1-\gamma)^2\frac{\sigma_K^2}{2}t - \beta t}dt = \frac{1}{1-\gamma}(aK_0)^{1-\gamma}\int_0^\infty e^{(1-\gamma)\left(\psi - \gamma\frac{\sigma_K^2}{2}\right)t - \beta t}dt$$

$$\Omega = \frac{(aK_0)^{1-\gamma}}{(1-\gamma)\left[\beta - (1-\gamma)\left(\psi - \gamma\frac{\sigma_K^2}{2}\right)\right]}. \tag{C.1}$$

Using Equations (A.1) and (A.14), we can see that

$$\beta-(1-\gamma)\left(\psi-\gamma\frac{\sigma_K^2}{2}\right)=a\ .$$

Thus, expected lifetime utility can be simplified to

$$\Omega=\frac{a^{-\gamma}K_0^{1-\gamma}}{(1-\gamma)}. \tag{C.2}$$

The government maximizes $\Omega$ over $n_Z^{\text{total}}$ and $\tau$, taking into account (B.9) and (B.10). First, we note that a depends additionally on $n_G$ through $r_K$ and $r_G$ which are given by (2) and (3) and can be written as

$$r_K=F_K=\alpha(1-\eta)[\eta n_G^{-\mu}+(1-\eta)]^{-\frac{1+\mu}{\mu}}, \tag{C.3}$$

$$r_G=F_G=\alpha\eta[\eta+(1-\eta)n_G^{\mu}]^{-\frac{1+\mu}{\mu}}. \tag{C.4}$$

Thus, we have the following first-order conditions for the government:

$$\frac{\partial\Omega}{\partial n_Z^{total}}=\frac{-\gamma a^{-\gamma-1}K_0^{1-\gamma}}{(1-\gamma)}\frac{\partial a}{\partial n_Z^{total}} \tag{C.5}$$

$$=\frac{-\gamma a^{-\gamma-1}K_0^{1-\gamma}}{(1-\gamma)}\left[\frac{1-\gamma}{2}\frac{\partial\sigma_K^2}{\partial n_Z^{total}}+\frac{dn_G}{dn_Z^{total}}\cdot\left(\left(-\frac{1-\gamma}{\gamma}\left(\frac{\partial r_K}{\partial n_G}+\frac{\partial r_G}{\partial n_G}n_G+r_G-\frac{\partial\xi}{\partial n_G}\tau\right)\right)+\frac{1-\gamma}{2}\frac{\partial\sigma_K^2}{\partial n_G}\right)\right]=0,$$

$$\frac{\partial\Omega}{\partial\tau}=\frac{-\gamma a^{-\gamma-1}K_0^{1-\gamma}}{(1-\gamma)}\frac{\partial a}{\partial\tau} \tag{C.6}$$

$$=\frac{-\gamma a^{-\gamma-1}K_0^{1-\gamma}}{(1-\gamma)}\left[\frac{1-\gamma}{\gamma}\xi+\frac{dn_G}{d\tau}\cdot\left(\left(-\frac{1-\gamma}{\gamma}\left(\frac{\partial r_K}{\partial n_G}+\frac{\partial r_G}{\partial n_G}n_G+r_G-\frac{\partial\xi}{\partial n_G}\tau\right)\right)+\frac{1-\gamma}{2}\frac{\partial\sigma_K^2}{\partial n_G}\right)\right]=0.$$

Furthermore, the last rows of (C.5) and (C.6) simplify owing to $\partial r_K/\partial n_G + n_G\partial r_G/\partial n_G = 0$ and $\partial\xi/\partial n_G = r_G$. Putting (C.5) and (C.6) together and using (B.9) and (B.10) in the resulting expression gives (21) from the main text. Moreover, inserting (B.10) in (C.6) gives (22).

## Appendix D. Derivation of the interest rates $r_Z$ and $\widehat{r}_Z$

The derivation in this Appendix follows Chung and Turnovsky (2010). Denote the first case in (23) as $A_{t+m}$ and the second case as $A_{t+m}^C$. Define furthermore the indicator function $I_{A_{t+m}}$ to equal one in the case of $A_{t+m}$ and zero otherwise. Then, (24) can be rewritten as

$$E_t^*(\text{Payoff})=\int_0^M r_Z E_t^*\left[\widetilde{Z}_{t+m}\right]e^{-r^*m}dm+E_t^*\left[\widetilde{Z}_{t+M}\right]e^{-r^*M}$$

$$=\int_0^M r_Z e^{-r^*m}E_t^*[Z_{t+m}I_{A_{t+m}}+\lambda Z_{t+m}(1-I_{A_{t+m}})]dm$$

$$+e^{-r^*M}E_t^*[Z_{t+M}I_{A_{t+M}}+\lambda Z_{t+M}(1-I_{A_{t+M}})]. \tag{D.1}$$

We now derive the expectation terms in (D.1). We first solve for the value of the total principal in period $t+m$, $Z_{t+m}^{total}$, and the value of GDP, $Y_{t+m}$. To do so, we rewrite the equations of motion, which are given by

$$\frac{dZ^{total}}{Z^{total}}=\nu\frac{d\widehat{Z}}{\widehat{Z}}=\nu\left(\psi dt-\frac{n_G\sigma_U dU_t+\sigma_W dW_t}{1-n_Z+n_G}\right), \tag{D.2}$$

$$\frac{dY}{Y}=(1-\eta)\frac{dK}{K}+\eta\frac{dG}{G}=\psi dt-\frac{n_G\sigma_U dU_t+\sigma_W dW_t}{1-n_Z+n_G}. \tag{D.3}$$

The solutions to these equations are given by:

$$Z_{t+m}^{\text{total}}=Z_t^{\text{total}}e^{\left(\nu\psi-\nu^2\frac{\sigma_K^2}{2}\right)m-\nu\frac{n_G\sigma_U(U_{t+m}-U_t)+\sigma_W(W_{t+m}-W_t)}{1-n_Z+n_G}}, \tag{D.4}$$

$$Y_{t+m}=Y_t e^{\left(\psi-\frac{\sigma_K^2}{2}\right)m-\frac{n_G\sigma_U(U_{t+m}-U_t)+\sigma_W(W_{t+m}-W_t)}{1-n_Z+n_G}}. \tag{D.5}$$

Hence, the condition of full repayment $A_{t+m}$ can be rewritten as

$$\theta Y_t \geq Z_t^{\text{total}} e^{\left((\nu-1)\psi-\left(\nu^2-1\right)\frac{\sigma_K^2}{2}\right)m-(\nu-1)\frac{n_G\sigma_U(U_{t+m}-U_t)+\sigma_W(W_{t+m}-W_t)}{1-n_Z+n_G}}. \tag{D.6}$$

Note that the exponential term in (D.6) equals unity in the special case $\nu = 1$. In this case, total debt is GDP-linked and financial markets know that it grows at the rate of GDP growth. Hence, they know that $Z_t^{\text{total}}/Y_t$ remains constant in all future periods $t+m$, where $m > 0$. Hence, either the government is perceived to never default if $Z_t^{\text{total}}/Y_t < \theta$ or to always be in default if $Z_t^{\text{total}}/Y_t \geq \theta$. Hence, the corner case of $\nu = 1$ is trivial and we neglect it in the remaining analysis.

Consider the case $\nu < 1$. We now undertake a change of variables. Suppose that the financial intermediary takes $\psi$ as given. Define

$$d\widetilde{W}_t \equiv \frac{n_G\sigma_U dU_t + \sigma_W dW_t}{1-n_Z+n_G}. \tag{D.7}$$

Then, $\widetilde{W}_t$ is normally distributed with mean zero and variance $\left(n_G^2\sigma_U^2 + \sigma_W^2\right)/(1-n_Z+n_G)^2$.
Define

$$B_t = \frac{(1-n_Z+n_G)\widetilde{W}_t}{\sqrt{n_G^2\sigma_U^2+\sigma_W^2}} - \frac{(1-n_Z+n_G)((1-\nu)\psi - r^*)}{(1-\nu)\sqrt{n_G^2\sigma_U^2+\sigma_W^2}}t\ . \tag{D.8}$$

There exists a risk-neutral probability measure that is equivalent to the natural probability measure, under which $B_t$ is a standard Brownian motion. Then, (D.6) becomes

$$\theta Y_t \geq Z_t^{total} e^{\left(-r^*-\left(\nu^2-1\right)\frac{\sigma_K^2}{2}\right)m+(1-\nu)\frac{\sqrt{n_G^2\sigma_U^2+\sigma_W^2}}{1-n_Z+n_G}(B_{t+m}-B_t)}. \tag{D.9}$$

Define now the following variable:

$$z_2 = \frac{B_{t+m}-B_t}{\sqrt{m}}. \tag{D.10}$$

Since under the risk-neutral probability measure, we have $B_{t+m} - B_t \sim N(0,m)$, the variable $z_2$ is standard normally distributed, $z_2 \sim N(0,1)$. Taking logarithms of the both sides of (D.9), we get

$$-\frac{(1-\nu)\sqrt{n_G^2\sigma_U^2+\sigma_W^2}}{1-n_Z+n_G}\sqrt{m}z_2 \geq \left[-r^* - \left(\nu^2-1\right)\frac{\sigma_K^2}{2}\right]m + \ln\left(\frac{Z_t^{\text{total}}}{\theta Y_t}\right). \tag{D.11}$$

Analogously, we can derive $A_{t+m}^C$ as

$$-\frac{(1-\nu)\sqrt{n_G^2\sigma_U^2+\sigma_W^2}}{1-n_Z+n_G}\sqrt{m}z_2 \leq \left[-r^* - \left(\nu^2-1\right)\frac{\sigma_K^2}{2}\right]m + \ln\left(\frac{Z_t^{\text{total}}}{\theta Y_t}\right). \tag{D.12}$$

Now, we turn to the expected payoff of a conventional bond $E_t^*\left[\widetilde{Z}_{t+m}\right]$. We express the expected payoff as

$$\begin{aligned}
\mathrm{E}_t^*\left[\widetilde{Z}_{t+m}\right] &= E_t^*\left[Z_{t+m}I_{A_{t+m}} + \lambda Z_{t+m}(1-I_{A_{t+m}})\right] = E_t^*\left[Z_t I_{A_{t+m}} + \lambda Z_t(1-I_{A_{t+m}})\right] \\
&= \int_{A_{t+m}} Z_t \frac{1}{\sqrt{2\pi}}e^{-\frac{z_2^2}{2}}dz_2 + \int_{A_{t+m}^C} \lambda Z_t \frac{1}{\sqrt{2\pi}}e^{-\frac{z_2^2}{2}}dz_2,
\end{aligned} \tag{D.13}$$

where we used $Z_{t+m} = Z_t$ for all $m \geq 0$ because the principal of conventional debt is constant. Expressing $Z^{total}/(\theta Y) = n_Z/(\theta\xi(1-\nu))$ and using (B.8), equations (D.11) and (D.12) become

$$z_2 \leq -\frac{1}{(1-\nu)\sigma_K\sqrt{m}}\left[\left(-r^* - \left(\nu^2-1\right)\frac{\sigma_K^2}{2}\right)m + ln\left(\frac{n_Z}{\theta\xi(1-\nu)}\right)\right], \tag{D.14}$$

$$z_2 \geq -\frac{1}{(1-\nu)\sigma_K\sqrt{m}}\left[\left(-r^* - \left(\nu^2-1\right)\frac{\sigma_K^2}{2}\right)m + \ln\left(\frac{n_Z}{\theta\xi(1-\nu)}\right)\right]. \tag{D.15}$$

Define the negative of the right-hand side of (D.14) (and (D.15)) as $h_m$. Denote furthermore the cdf of the standrard normal distribution as $\Phi(\cdot)$. Then, (D.13) becomes

$$E_t^*\left[\widetilde{Z}_{t+m}\right] = Z_t\Phi(-h_m) + \lambda Z_t\Phi(h_m) = Z_t[1-\Phi(h_m)(1-\lambda)]. \tag{D.16}$$

Now, we can plug (D.16) in (24) to get

$$E_t^*(\text{Payoff}) = r_Z\int_0^M e^{-r^*m}Z_t[1-\Phi(h_m)(1-\lambda)]dm + e^{-r^*M}Z_t[1-\Phi(h_M)(1-\lambda)]. \tag{D.17}$$

Perfect competition in the financial intermediaries' market leads to zero profits, i.e., $E_t^*(\text{Payoff}) = Z_t$. Solving the zero-profit condition for $r_Z$, we get

$$r_Z = \frac{1 - e^{-r^* M}[1 - \Phi(h_M)(1-\lambda)]}{\int_0^M e^{-r^* m}[1 - \Phi(h_m)(1-\lambda)]dm} \ . \tag{D.18}$$

We now consider GDP-linked bonds. Here, the expected value $E_t^*\left[\widetilde{\widehat{Z}}_t\right]$ is given by

$$E_t^*\left[\widetilde{\widehat{Z}}_{t+m}\right] = E_t^*[\widehat{Z}_{t+m} I_{A_{t+m}} + \lambda \widehat{Z}_{t+m}(1 - I_{A_{t+m}})]$$

$$= E_t^*\left[\widehat{Z}_t e^{\left(\psi - \frac{\sigma_K^2}{2}\right)m - \frac{n_G\sigma_U(U_{t+m}-U_t)+\sigma_W(W_{t+m}-W_t)}{1-n_Z+n_G}} I_{A_{t+m}} \ \dots\right.$$

$$\left. .. \ + \lambda \widehat{Z}_t e^{\left(\psi - \frac{\sigma_K^2}{2}\right)m - \frac{n_G\sigma_U(U_{t+m}-U_t)+\sigma_W(W_{t+m}-W_t)}{1-n_Z+n_G}} (1 - I_{A_{t+m}})\right] . \tag{D.19}$$

We define now

$$\widetilde{W}_t = \frac{n_G\sigma_U U_t + \sigma_W W_t}{1 - n_Z + n_G}, \tag{D.20}$$

$$\widetilde{B}_t = \frac{\widetilde{W}_t(1 - n_Z + n_G)}{\sqrt{n_G^2\sigma_U^2 + \sigma_W^2}}, \tag{D.21}$$

$$z_3 = \frac{\widetilde{B}_{t+m} - \widetilde{B}_t}{\sqrt{m}} \sim N(0,1). \tag{D.22}$$

Thus, (D.19) becomes

$$E_t^*\left[\widetilde{\widehat{Z}}_{t+m}\right] = E_t^*\left[\widehat{Z}_t e^{\left(\psi - \frac{\sigma_K^2}{2}\right)m - \frac{\sqrt{n_G^2\sigma_U^2+\sigma_W^2}}{1-n_Z+n_G}\sqrt{m}z_3} I_{A_{t+m}} + \lambda \widehat{Z}_t e^{\left(\psi - \frac{\sigma_K^2}{2}\right)m - \frac{\sqrt{n_G^2\sigma_U^2+\sigma_W^2}}{1-n_Z+n_G}\sqrt{m}z_3} (1 - I_{A_{t+m}})\right]$$

$$= \int_{A_{t+m}} \widehat{Z}_t e^{\left(\psi - \frac{\sigma_K^2}{2}\right)m - \frac{\sqrt{n_G^2\sigma_U^2+\sigma_W^2}}{1-n_Z+n_G}\sqrt{m}z_3} \frac{1}{\sqrt{2\pi}} e^{-\frac{z_2^2}{2}} dz_2 + \int_{A_{t+m}^C} \lambda \widehat{Z}_t e^{\left(\psi - \frac{\sigma_K^2}{2}\right)m - \frac{\sqrt{n_G^2\sigma_U^2+\sigma_W^2}}{1-n_Z+n_G}\sqrt{m}z_3} \frac{1}{\sqrt{2\pi}} e^{-\frac{z_2^2}{2}} dz_2. \tag{D.23}$$

Furthermore, from (D.7), (D.8), and (D.9), together with (D.20)-(D.22), we know that the correlation between $z_2$ and $z_3$ equals one. Under the risk-neutral probability measure, $B_t$ is a standard Brownian motion, and thus $z_2$ is standard normally distributed. The term $z_3$ is also standard normal. Thus, we can conclude that, under the risk-neutral probability measure for $B_t$, $z_2 = z_3$. Hence, (D.23) simplifies to

$$E_t^*\left[\widetilde{\widehat{Z}}_{t+m}\right] = \int_{A_{t+m}} \widehat{Z}_t e^{\psi m - \frac{(z_2 + \sigma_K\sqrt{m})^2}{2}} \frac{1}{\sqrt{2\pi}} dz_2 + \int_{A_{t+m}^C} \lambda \widehat{Z}_t e^{\psi m - \frac{(z_2 + \sigma_K\sqrt{m})^2}{2}} \frac{1}{\sqrt{2\pi}} dz_2, \tag{D.24}$$

where $\sigma_K$ is defined in (B.8). We now define $z_2^{'} \equiv z_2 + \sigma_K\sqrt{m}$, which simplifies (D.24) to

$$E_t^*\left[\widetilde{\widehat{Z}}_{t+m}\right] = \int_{A_{t+m}} \widehat{Z}_t e^{\psi m - \frac{\left(z_2^{'}\right)^2}{2}} \frac{1}{\sqrt{2\pi}} dz_2^{'} + \int_{A_{t+m}^C} \lambda \widehat{Z}_t e^{\psi m - \frac{\left(z_2^{'}\right)^2}{2}} \frac{1}{\sqrt{2\pi}} dz_2^{'}, \tag{D.25}$$

while (D.11) and (D.12) become

$$z_2^{'} \le -\frac{\left[-r^* - (1-\nu)^2\frac{\sigma_K^2}{2}\right]m + ln\left(\frac{n_Z}{\theta\xi(1-\nu)}\right)}{(1-\nu)\sigma_K\sqrt{m}} \ , \tag{D.26}$$

$$z_2^{'} \ge -\frac{\left[-r^* - (1-\nu)^2\frac{\sigma_K^2}{2}\right]m + ln\left(\frac{n_Z}{\theta\xi(1-\nu)}\right)}{(1-\nu)\sigma_K\sqrt{m}}. \tag{D.27}$$

Define the negative of the right-hand sides of (D.26) and (D.27) as $g_m$. Then, we have

$$\mathrm{E}_t^*\left[\widetilde{\widehat{Z}}_{t+m}\right] = \widehat{Z}_t e^{\psi m}\Phi(-g_m) + \lambda \widehat{Z}_t e^{\psi m}\Phi(g_m) = \widehat{Z}_t e^{\psi m}[1 - \Phi(g_m)(1-\lambda)] \;. \tag{D.28}$$

The expected payoff of the GDP-linked bond is thus

$$E_t^*(\widehat{\text{Payoff}}) = \widehat{r}_Z \int_0^M e^{-(r^*-\psi)m} \widehat{Z}_t[1 - \Phi(g_m)(1-\lambda)]dm + e^{-(r^*-\psi)M}\widehat{Z}_t[1 - \Phi(g_M)(1-\lambda)] \;. \tag{D.29}$$

Perfect competition in the financial intermediaries market leads to zero profits, i.e., $E_t^*(\widehat{\text{Payoff}}) = \widehat{Z}_t$. Solving the zero-profit condition for $\widehat{r}_Z$, we get

$$\widehat{r}_Z = \frac{1 - e^{-(r^*-\psi)M}[1 - \Phi(g_M)(1-\lambda)]}{\int_0^M e^{-(r^*-\psi)m}[1 - \Phi(g_m)(1-\lambda)]dm} \;. \tag{D.30}$$

## Appendix E. Sensitivity analysis

Table E1 presents the results of the sensitivity analysis to the simulation of the main model.

**Table E1**
Sensitivity Analysis.

| $\nu$ | $\psi$ | $\sigma_K$ | $Z^{\text{total}}/Y$ | $r_Z$ | $\bar{r}_Z$ | $n_G$ | $\Delta\%\Omega$ |
|---|---|---|---|---|---|---|---|
| Short-term debt: $M = 1$ | | | | | | | |
| 0 | 2.983 | 4.05 | 0.572 | 0.1603 | 2.491 | 0.514 | – |
| 0.1 | 2.978 | 4.01 | 0.577 | 0.161 | 2.543 | 0.515 | 0.002 |
| Long-term debt: $M = 10$ | | | | | | | |
| 0 | 2.993 | 4.132 | 0.688 | 0.1586 | 2.48 | 0.509 | – |
| 0.1 | 2.988 | 4.094 | 0.706 | 0.1594 | 2.535 | 0.511 | 0.002 |
| Low risk aversion: $\gamma = 2.5$ | | | | | | | |
| 0 | 3.423 | 4.19 | 0.646 | 0.1506 | 2.984 | 0.444 | – |
| 0.1 | 3.419 | 4.15 | 0.651 | 0.1513 | 3.032 | 0.445 | 0.001 |
| High risk aversion: $\gamma = 3.5$ | | | | | | | |
| 0 | 2.67 | 3.99 | 0.572 | 0.167 | 2.112 | 0.573 | – |
| 0.1 | 2.666 | 3.96 | 0.59 | 0.168 | 2.172 | 0.574 | 0.0025 |
| $\nu$ | $\psi$ | $\sigma_K$ | $Z^{\text{total}}/Y$ | $r_Z$ | $\bar{r}_Z$ | $n_G$ | $\Delta\%\Omega$ |
| Low threshold of repayment restraint: $\theta = 0.4$ | | | | | | | |
| 0 | 2.97 | 3.94 | 0.41 | 0.162 | 2.5 | 0.519 | – |
| 0.1 | 2.967 | 3.912 | 0.416 | 0.163 | 2.55 | 0.52 | 0.002 |
| High threshold of repayment restraint: $\theta = 0.8$ | | | | | | | |
| 0 | 3 | 4.22 | 0.813 | 0.157 | 2.469 | 0.504 | – |
| 0.1 | 2.997 | 4.17 | 0.826 | 0.158 | 2.527 | 0.507 | 0.002 |
| Low repayment share $\lambda = 0.8$ | | | | | | | |
| 0 | 2.984 | 4.06 | 0.59 | 0.16 | 2.49 | 0.513 | – |
| 0.1 | 2.98 | 4.03 | 0.6 | 0.161 | 2.54 | 0.515 | 0.002 |
| High repayment share $\lambda = 0.95$ | | | | | | | |
| 0 | 2.99 | 4.1 | 0.648 | 0.159 | 2.48 | 0.51 | – |
| 0.1 | 2.98 | 4.06 | 0.657 | 0.16 | 2.54 | 0.513 | 0.002 |
| $\nu$ | $\psi$ | $\sigma_K$ | $Z^{\text{total}}/Y$ | $r_Z$ | $\bar{r}_Z$ | $n_G$ | $\Delta\%\Omega$ |
| Low share of public capital: $\eta = 0.2$ | | | | | | | |
| 0 | 3.08 | 4.26 | 0.614 | 0.126 | 2.54 | 0.392 | – |
| 0.1 | 3.075 | 4.22 | 0.624 | 0.127 | 2.59 | 0.393 | 0.003 |
| High share of public capital: $\eta = 0.25$ | | | | | | | |
| 0 | 2.895 | 3.89 | 0.611 | 0.217 | 2.44 | 0.742 | – |
| 0.1 | 2.895 | 3.86 | 0.62 | 0.2175 | 2.49 | 0.744 | 0.00005 |
| Lower substitutability: $\mu = 0$ | | | | | | | |
| 0 | 2.93 | 3.902 | 0.613 | 0.214 | 2.47 | 0.728 | – |
| 0.1 | 2.926 | 3.87 | 0.622 | 0.214 | 2.52 | 0.729 | −0.002 |
| Higher substitutability: $\mu = -2/3$ | | | | | | | |
| 0 | 3.045 | 4.35 | 0.609 | 0.112 | 2.48 | 0.344 | – |
| 0.1 | 3.04 | 4.3 | 0.619 | 0.113 | 2.54 | 0.346 | 0.008 |

Legend: $\nu$ – Share of GDP-linked bonds in the portfolio of sovereign debt, $\psi$ – Steady state growth rate, $\sigma_K$ – Standard deviation of growth in steady state, $Z^{\text{total}}/Y$ - Debt-to-output ratio, $\tau$ – Income tax rate, $r_Z$ – Average interest rate for conventional and GDP-linked bonds; $n_G$– Ratio of public capital to private capital.
$\Delta\%\Omega$ – Change in welfare vis-à-vis businesses-usual case (0% GDP-linked bonds).

## Appendix F. Consumption public good

First, we derive the optimal consumption $\widehat{C}$ similarly to Appendix A. Define the expected growth rate and its variance as

$$\psi \equiv \alpha(1-\tau) - \delta_K - \frac{C_t}{K_t}\ , \tag{F.1}$$

$$\sigma_K^2 \equiv Var(-\alpha\sigma_V dV_t - \sigma_W dW_t). \tag{F.2}$$

Then, we define the value function as $V(K,t) = e^{-\beta t}X(K)$ as in (A.4) and its differential generator $L[V(K,t)]$ as in (A.3). The first-order condition with respect to $C$ is analogous to (A.7) and gives an optimal $\widehat{C}$ defined by

$$\widehat{C} = \left(\frac{X_K}{(gK)^{\zeta(1-\gamma)}}\right)^{-\frac{1}{\gamma}}, \tag{F.3}$$

where we used $G_t = gK_t$. Then, we assume

$$X(K) = \frac{a^{-\gamma}K^{(1+\zeta)(1-\gamma)}}{1-\gamma}, \tag{F.4}$$

where $a$ is a constant to be determined. Together (F.4) and the maximized Bellman.
Equation (defined analogously to (A.8)) give a solution to $a$, given by

$$a = \frac{(1-\gamma)(1+\zeta)^{\frac{1-\gamma}{\gamma}}}{\gamma g^{\frac{\zeta(1-\gamma)}{\gamma}}}\left[\frac{\beta}{1-\gamma} - (1+\zeta)(\alpha(1-\tau)-\delta_K) + \frac{1}{2}(1+\zeta)(1-(1+\zeta)(1-\gamma))\sigma_K^2\right]. \tag{F.5}$$

Together the derivative of (F.4) with respect to $K$, (F.3), and (F.5) give a solution for $C$, which is of the form $\widehat{C} = cK$, where $c$ is defined in (32).

Turning to the government, its budget constraint over period $(t, t+dt)$ is given by

$$dZ_t^{total} = Z_t dR_Z + \widehat{Z}_t d\widehat{R}_t + dG_t - dT_t \tag{F.6}$$

where the return on conventional bonds, $dR_Z$, is the same as in the main model, while $d\widehat{R}_Z$ is defined analogously to (13). Thus, we can derive, analogously to (B.1) in Appendix B, the following growth rate of conventional bonds:

$$\frac{dZ_t}{Z_t} = \left[(1-\nu)\left(r_Z - \frac{\alpha\tau - g}{n_Z}\right) + \nu(\widehat{r}_Z + \psi)\right]dt - \left(\nu + \frac{1-\nu}{n_Z}\right)\alpha\sigma_V dV_t - \nu\sigma_W dW_t. \tag{F.7}$$

Because capital and debt must grow at the same rates on a balanced growth path, the deterministic and stochastic components of (29) and (F.7) must be equal. Equating these components and using $n_Z = (1-\nu)n_Z^{\text{total}}$, we get

$$\alpha\tau + n_Z^{total}\psi = n_Z^{total}[(1-\nu)r_Z + \nu(\widehat{r}_Z + \psi)] + g, \tag{F.8}$$

$$\sigma_V dV_t = \frac{n_Z\sigma_W dW_t}{\alpha(1-n_Z)}. \tag{F.9}$$

Moreover, together (F.2) and (F.9) give

$$\sigma_K^2 = \frac{\sigma_W^2}{(1-n_Z)^2}. \tag{F.10}$$

Equation (F.8) determines the optimal relation between $g, \tau$, and $n_Z^{\text{total}}$ on the balanced growth path. Assuming without loss of generality that the government chooses $\tau$ and $n_Z^{\text{total}}$ as its control variables, (F.8) determines the share of expenditures on the public good as a function of the optimal tax and debt ratio, i.e., $g(\tau, n_Z^{\text{total}})$. A total differential of (F.8) with respect to $g, \tau$ and $n_Z^{\text{total}}$ gives

$$\frac{\partial g}{\partial \tau} = \alpha\left[1 - \frac{(1-\nu)n_Z^{total}}{\gamma}\right], \tag{F.11}$$

$$\frac{\partial g}{\partial n_Z^{total}} = (1-\nu)\left[\psi - r_Z - \frac{\nu}{1-\nu}\widehat{r}_Z - \frac{n_Z^{total}(1-\gamma)(1-(1+\zeta)(1-\gamma))}{2\gamma}\frac{\partial\sigma_K^2}{\partial n_Z^{total}}\right]. \tag{F.12}$$

The government chooses $\tau$ and $n_Z^{\text{total}}$ to maximize welfare, taking into account (F.11) and (F.12). Using (30) and following the same steps as in Appendix C, we show that expected utility is given by

$$\Omega = \frac{c^{-\gamma} g^{\zeta(1-\gamma)} K_0^{(1+\zeta)(1-\gamma)}}{(1+\zeta)(1-\gamma)} \tag{F.13}$$

The maximization of (F.13) over $\tau$ and $n_Z^{\text{total}}$ (taking (32), (F.11), and (F.12) into account) gives after simplification of the first-order conditions:

$$\frac{\mathrm{g}}{\mathrm{c}} = \zeta\left[1 - \frac{(1-\nu)\mathrm{n}_{\mathrm{Z}}^{\text{total}}}{\gamma}\right], \tag{F.14}$$

$$(1-\nu)\left[\psi - \mathrm{r}_{\mathrm{Z}} - \frac{\nu}{1-\nu}\hat{\mathrm{r}}_{\mathrm{Z}}\right] = \left(1 - (1-\nu)\mathrm{n}_{\mathrm{Z}}^{\text{total}}\right)\frac{1-(1+\zeta)(1-\gamma)}{2}\frac{\partial\sigma_{\mathrm{K}}^2}{\partial \mathrm{n}_{\mathrm{Z}}^{\text{total}}}. \tag{F.15}$$

Taking further into account that $n_Z = (1-\nu)n_Z^{\text{total}}$, we can simplify (F.14) and (F.15). The resulting expressions are (34) and (35).

**Calibration: Sensitivity Analysis** Here, we report the sensitivity analysis to the simulation of the model with a consumption public good.

Fig. F1 displays the effects of a variation in the share of GDP-linked bonds $\nu$ from zero to 0.2 on the mean growth rate $\psi$ (panel (a)), the standard deviation of growth $\sigma_K$ (panel (b)), the optimal public debt-to-GDP ratio $n_Z^{\text{total}}$ (panel (c)), the optimal tax rate $\tau$ (panel (d)), the ratio of public consumption to private capital $g$ (panel (e)), the average interest rate $r_Z$ (panel (f)), and welfare $\Omega$ (panel (g)).

**Table F1**
Balanced Growth Path ($\zeta = 0.4$).

| $\nu$ | $\psi$ | $\sigma_K$ | $Z^{total}/Y$ | $\tau$ | $r_Z$ | $\hat{r}_Z + \psi$ | $\bar{r}_Z$ | $C/Y$ | $g$ | $\Delta\%\Omega$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | *3.711* | 5.75 | 0.592 | 0.174 | *2.45* | – | 2.45 | 0.475 | 0.04 | – |
| 0.1 | 3.695 | 5.69 | 0.612 | 0.175 | 2.607 | 2.415 | 2.588 | 0.475 | 0.04 | −0.008 |

Legend: $\nu$ – Share of GDP-linked bonds in the portfolio of sovereign debt, $\psi$ – Steady-state growth rate, $\sigma_K$ – Standard deviation of growth in steady state, debt-to-output ratio, $r_Z$ – Interest rate of conventional bonds, $\tau$ – Income tax rate, $\hat{r}_Z + \psi$ – Interest rate of GDP-linked bonds, $\bar{r}_Z$ – Average interest rate, $g$ – Provision of public (consumption) good as ratio to output ($g = G/Y$), $C/Y$ – Consumption-to-output ratio, $\Delta\%\Omega$ – Change in welfare vis-à-vis business-as-usual case (0% GDP-linked bonds).

**Table F2**
Balanced Growth Path ($\zeta = 0.6$).

| $\nu$ | $\psi$ | $\sigma_K$ | $Z^{total}/Y$ | $\tau$ | $r_Z$ | $\hat{r}_Z + \psi$ | $\bar{r}_Z$ | $C/Y$ | $g$ | $\Delta\%\Omega$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | *3.462* | *5.67* | *0.537* | 0.237 | *2.11* | – | 2.11 | 0.4238 | 0.0537 | – |
| 0.1 | *3.454* | 5.64 | 0.576 | 0.238 | 2.26 | 2.19 | 2.25 | 0.4236 | 0.0538 | −0.003 |

Legend: $\nu$ – Share of GDP-linked bonds in the portfolio of sovereign debt, $\psi$ – Steady-state growth rate, $\sigma_K$ – Standard deviation of growth in steady state, debt-to-output ratio, $r_Z$ – Interest rate of conventional bonds, $\tau$ – income tax rate, $\hat{r}_Z + \psi$ – Interest rate of GDP-linked bonds, $\bar{r}_Z$ – Average interest rate, $g$ – Provision of public (consumption) good as ratio to output ($g = G/Y$), $C/Y$ – Consumption-to-output ratio, $\Delta\%\Omega$ – Change in welfare vis-à-vis business-as-usual case (0% GDP-linked bonds).

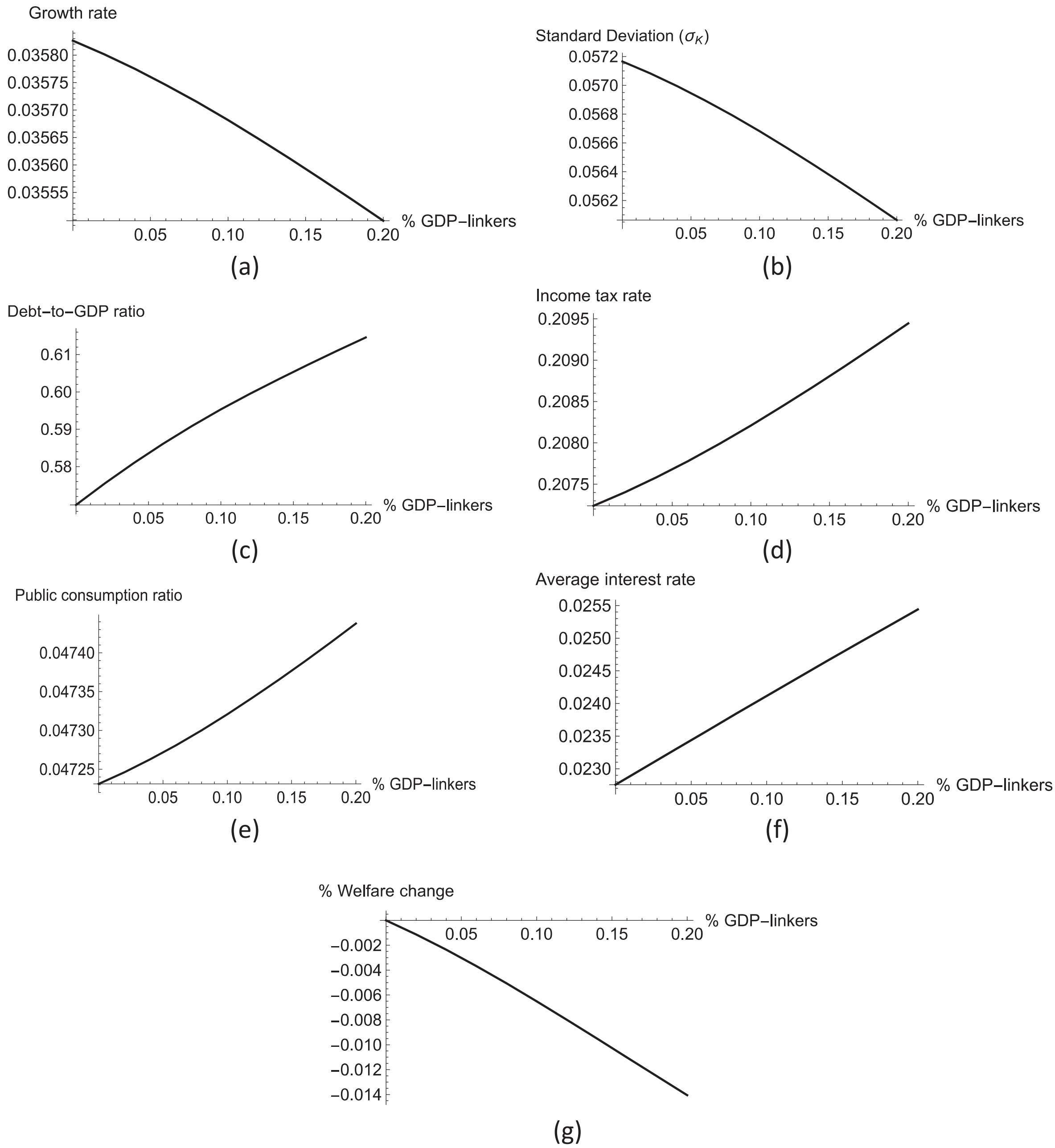


**Fig. F1.** Variation in the share of GDP-linked bonds for the model variant with a public consumption good. Legend: % GDP-linkers – $\nu$ (share of GDP-linked bonds in the portfolio of sovereign debt), Growth rate – $\psi$, Standard deviation of growth – $\sigma_K$, Debt-to-GDP ratio - $Z^{total}/Y$, Income tax rate – $\tau$, Public consumption ratio – $g$, Average interest rate – $\bar{r}_Z$, % Welfare change – $\Delta\%\Omega$

## Appendix G. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jimonfin.2023.102918.

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