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A model of international currency with private information ☆

Chenxi Wang

The School of Finance, Renmin University of China, 59 Zhongguancun St, Haidian District, Beijing 100872, China

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ABSTRACT

This paper studies a model in which nominal exchange rate is determined by asymmetric information. Within a two-country two-currency search-theoretic model, buyers have complete information, while sellers have incomplete information regarding the real value of foreign currencies in decentralized meetings. With no restrictions on which currency to use to settle transactions, sellers' domestic currency turns out to be the preferred means of payment in equilibrium and various currency regimes emerge endogenously. The degree of information asymmetry and economic openness have ambiguous impacts on the nominal exchange rate due to the variety of currency regimes. When two countries interact in a policy game of setting inflation targets, the equilibrium inflation targets also depend on the degree of information asymmetry and economic openness, since these factors affect the seigniorage revenue and accordingly central banks' temptation to inflate.

1. Introduction

As pointed out by Kareken and Wallace (1981), when agents have unrestricted access to currency markets and are free to use any currency as medium of exchange, all currencies are perfect substitutes and the nominal exchange rate between these currencies is indeterminate. To break this indeterminacy, models in mainstream international monetary economics usually impose exogenous restrictions on the type of currencies that may be used in certain kinds of trades/meetings. The indeterminacy can also be resolved by the threat of currency counterfeiting (Gomis-Porqueras et al. (2017)) or the costly verification of foreign currency (Zhang (2014)) following the new monetarist approach. In this paper, I overcome the indeterminacy problem by information asymmetry regarding the real value of foreign currency.

When a seller meets with a buyer who holds multiple currencies, the seller is less willing to accept foreign currencies compared to its domestic one, since he is not certain about the real value of foreign currency when he spends it, due to future fluctuations in the nominal exchange rate. To capture this friction, I consider a two-country two-currency version of the monetary search model of Lagos and Wright (2005). Agents first interact in decentralized international trade markets, and then participate in a Walrasian foreign exchange market, in addition to the centralized markets in their home countries. There are no restrictions on which currency to use to settle transactions, but agents have asymmetric information regarding the real value of currencies in decentralized meetings. Specifically, suppose sellers know the real value of their domestic currencies only, while buyers are fully informed. The terms of trade in decentralized meetings are determined by a bargaining game with private information (Nosal and Rocheteau (2011)). In equilibrium, buyers first use the sellers' domestic currency as the medium of exchange, and turn to the sellers' foreign currency only if their domestic balance is not enough. That is, there is a *pecking-order theory of payments*. When buyers do not have sufficient domestic currency to purchase the

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E-mail address: chenxi.wang@ruc.edu.cn

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surplus-maximizing level of output, they choose to signal the high value of foreign currency by retaining part of their foreign currency holding. Thus information asymmetry reduces the liquidity of foreign currency in decentralized meetings. Indeed, the liquidity of foreign currency decreases with its real value uncertainty, which eventually stems from the uncertainty of foreign inflation.

The model generates various currency regimes in equilibrium, which give rise to different behaviors of the nominal exchange rate. To be concrete, let's consider two countries, A and B. When both currencies are held by some buyers, they would substitute currency B for currency A if the uncertainty carried by currency A increases, leading to a depreciation of currency A. However, the substitution would not occur if buyers only hold their domestic currencies. In this circumstance, country-A buyers need to signal the high value of currency A in meetings with country-B sellers by retaining a larger fraction of their currency holding. This would increase the demand for currency A and lead to an appreciation. Therefore, the impact of uncertainty on nominal exchange rate relies on the type of currency regime. Alternatively, suppose country-A buyers go abroad more frequently. If they hold both currencies, they would substitute currency B for currency A, leading to a depreciation of currency A. But if they only hold their domestic currency, they would demand more currency A to signal its high value in foreign meetings, leading to an appreciation. That is, the impact of economic openness on nominal exchange rate also depends on the type of currency regimes.

When the two monetary authorities interact strategically to set their inflation targets, the degree of uncertainty and economic openness also matter for the equilibrium outcome. The tradeoff lies in the increasing seigniorage revenue and the shrinking purchasing power of domestic currency when a monetary authority raises its inflation target. The monetary authority of country B, for example, would set a higher inflation target when the uncertainty of currency A increases, since country-A buyers would demand more currency B to overcome the information asymmetry in decentralized meetings. This raises the seigniorage income of country B and leads to a higher equilibrium inflation target. The monetary authority of country A, on the other hand, would reduce its inflation target, as country-B buyers reduce their holdings of currency A. That is, the temptation to inflate decreases with the uncertainty of monetary policy (relative to the other country).

In contrast, the degree of economic openness has uniform impacts on the optimal inflation targets of both countries. With more frequent occurrence of foreign meetings, buyers increase their holdings of foreign currencies, leading to higher seigniorage revenue and higher inflation targets in equilibrium. This result holds when buyers from one particular country go abroad more frequently, as well as when both countries open up at the same time (as in (Liu and Shi, 2010)).

Related Literature. This paper broadly fits in the growing literature on monetary search theory applied in international macroeconomics, such as the study of home bias puzzle (Geromichalos and Simonovska (2014)), uncovered interest parity puzzle (Jung and Lee (2020)), trade finance (Liu et al. (2019)), over-the-counter foreign exchange market (Geromichalos and Jung (2018)), etc.

In particular, this paper is related to approaches to breaking "the curse of Kareken and Wallace". Rather than imposing restrictions on the number of currencies that can be held by buyers (Head and Shi (2003)) or accepted by sellers (Liu and Shi (2010)), or introducing fixed costs on counterfeiting (Gomis-Porqueras et al. (2017)) or the verification of foreign currency (Zhang (2014)), this paper explicitly models the information problem between buyers and sellers. More importantly, the information asymmetry does not comes from the possibility of counterfeiting, which is not standard in international monetary economics[1]. It instead stems from the fluctuations of nominal exchange rate, which is a salient feature of the real world economy and studied extensively in the literature.

This paper builds on studies on imperfect information in search-theoretic models. Lester et al. (2011, 2012) study how recognizability affects assets' liquidity, which are then extended to an open-economy setting by Zhang (2014). Nosal and Wallace (2007) and Li et al. (2012) consider the possibility of asset fraud, which are later applied in an international setting by Gomis-Porqueras et al. (2017). This paper closely follows Rocheteau (2011) in terms of the bargaining game under incomplete information, which is formulated as an adverse-selection problem with uncertainty on the future value of assets. Other papers concerned with the adverse selection problem in search-theoretic literature include Guerrieri et al. (2010), Bajaj (2018), Choi (2018), Madison (2019), Wang (2020), etc.

Lastly, this paper is related to the literature on exchange rate determination, such as the traditional monetary theory (Frenkel and Mussa (1985)), the portfolio balance theory (Branson and Henderson (1985)), and recent studies on the role of financial factors (Valchev (2020), Itskhoki and Mukhin (2021), among others). Notably, Gabaix and Maggiori (2015) provide a theory of exchange rate determination based on capital flows in imperfect financial markets. They show that, in the presence of intermediation frictions, shocks to financiers' risk-bearing capacity affect the level and volatility of exchange rates. Mueller et al. (2017), on the other hand, construct a model in which financiers are uncertain about the future path of monetary policy and show that such uncertainty leads to depreciation of the currency on central bank announcement days using high-frequency data. This paper complements the literature by offering an alternative theory in support of the documented relationship between monetary policy uncertainty and exchange rate determination.

The paper proceeds as follows. Section 2 describes the environment. Section 3 defines and solves for the equilibrium. Section 4 investigates different currency regimes and the determinants of nominal exchange rate. Section 5 studies a policy game between the two monetary authorities. Finally, Section 6 concludes.

[1] In real life, payments for international trade and investment are mostly transferred through financial institutions electronically, hence counterfeiting should not be a major concern in the determination of nominal exchange rate.

2. Environment

Time is discrete and continues forever. There are two countries, A and B, populated with a continuum of 2 and 2N agents, respectively, where $N \in (0,1]$ denotes relative country size. Agents live forever with discount factor $\beta \in (0,1)$. In each country, agents are permanently and equally divided into buyers and sellers. Sellers have immobile factors of production and stay in their home countries. Buyers are free to move across the border.

Each period is divided into two subperiods: the first is for decentralized trade between sellers and buyers; the second is for centralized settlement and currency exchange. At the beginning of the decentralized market (DM), each buyer is matched with a seller. Country-j ($j = A, B$) buyers meet with foreign sellers with probability σ_F^j and meet with domestic sellers with probability $1 - \sigma_F^j$. Without loss of generality, buyers are assumed to stay in their home countries more frequently, i.e. $0 \leqslant \sigma_F^j \leq 1/2$. The magnitude of σ_F^j captures the degree of economic openness. When $\sigma_F^A = \sigma_F^B = \frac{1}{2}$, there is perfect economic integration. When $\sigma_F^A = \sigma_F^B = 0$, each country is isolated from the rest of the world. Sellers can produce a specialized good of its own country, but do not want to consume, while buyers want to consume special goods but cannot produce. In the centralized market (CM), all agents can produce and consume a common general good, which is produced according to a linear production function in labor.

Agents meet pairwise and at random in DM. The number of trade matches in DM of country-j is given by the matching function $\mathscr{M}^j \equiv \mathscr{M}(\mathscr{B}^j, \mathscr{S}^j) = \frac{\mathscr{B}^j \mathscr{S}^j}{\mathscr{B}^j + \mathscr{S}^j}$, where $\mathscr{B}^j$ and $\mathscr{S}^j$ denote the measures of buyers and sellers in the DM of country-j. In country-A, $\mathscr{S}^A = 1, \mathscr{B}^A = 1 - \sigma_F^A + N\sigma_F^B$. In country-B, $\mathscr{S}^B = N, \mathscr{B}^B = N(1 - \sigma_F^B) + \sigma_F^A$. Conditional on being in country-j, a buyer meets a seller with probability $\frac{\mathscr{M}^j}{\mathscr{B}^j}$. Let μ^{ij} be the probability that a country-i buyer meets with a country-j seller. The value of μ^{ij} are given in Table 1.

For tractability, the instantaneous utilities of buyers and sellers are assumed to be

$$\mathscr{U}^B = u(q) + x - n \tag{1}$$

$$\mathscr{U}^S = -c(q) + x - n \tag{2}$$

where x is the CM consumption of general good, n is the CM disutility from labor, and q is the quantity of special good produced in DM. To ease presentation, functional forms of utilities and cost functions are assumed to be the same across countries. $u(\cdot)$ and $c(\cdot)$ are C^2 and satisfy conventional assumptions: $u' > 0, u'' < 0, c' > 0, c'' > 0, u(0) = c(0) = 0$ and $u'(0) > c'(0)$. The first-best quantity is $q^* = \text{argmax}_q[u(q) - c(q)]$. All goods are perishable. Unsecured credit cannot be used since agents lack commitment and individual histories are private information in DM, thus a medium of exchange is essential for trade.

Currencies and Information. Each country issues its own currency, which is perfectly divisible and storable. There is no exogenous restriction on the types of currencies accepted in decentralized meetings. Let ϕ_t^j be the price of currency j in terms of the CM general good. The nominal exchange rate is defined as the price of currency B in terms of currency A: $e_t = \frac{\phi_t^B}{\phi_t^A}$. Agents adjust their currency holdings in CM through trading of the general good at the market clearing exchange rate, hence CM also functions as a foreign exchange market.

The stock of money at the beginning of period t in country-j is denoted as M_t^j, which grows at rate $\gamma_t^j = \frac{M_{t+1}^j}{M_t^j}$. We assume that γ_t^j is i.i.d over time and takes two possible values [2]:

$$\gamma_t^j = \begin{cases} \gamma_H^j & \text{with prob.} \pi_H^j \\ \gamma_L^j & \text{with prob.} \pi_L^j \end{cases}, \ j = \text{A, B}.$$

With probability $\pi_H^j \in (0,1)$ the growth rate is high at γ_H^j and with complement probability π_L^j, the growth rate is low at γ_L^j, where $0 < \gamma_L^j < \gamma_H^j$. Define the average growth rate of money supply $\overline{\gamma}^j$ as

$$\overline{\gamma}^j = \left(\frac{\pi_H^j}{\gamma_H^j} + \frac{\pi_L^j}{\gamma_L^j} \right)^{-1}$$

Changes in the money supply of country j are implemented through lump-sum monetary transfers or taxes to country-j buyers in CM.

The level of γ_t^j is realized at the beginning of DM period in period t. The distributions are common knowledge. However, the realized value is fully revealed only to buyers, who are free to move across the border and could potentially hold both currencies. Sellers learn the growth rate of their domestic money supply at the beginning of DM, since they stay at their home country due to immobile factors of production and they generally do not carry currencies into the next period (shown

[2] As long as γ_t^A and γ_t^B are not perfectly correlated, i.e. there is some degree of information asymmetry, all implications follow similarly.

Table 1
meeting probabilities.

	country-A seller	country-B seller
country-A buyer	$\mu^{AA} = (1-\sigma_F^A)\frac{\mathscr{M}^A}{\mathscr{B}^A}$	$\mu^{AB} = \sigma_A^F\frac{\mathscr{M}^B}{\mathscr{B}^B}$
country-B buyer	$\mu^{BA} = \sigma_F^B\frac{\mathscr{M}^A}{\mathscr{B}^A}$	$\mu^{BB} = (1-\sigma_B^F)\frac{\mathscr{M}^B}{\mathscr{B}^B}$

later).[3] Therefore, there is asymmetric information in DM trade. At the beginning of the following CM period, monetary transfers take place and the growth rates γ_t^j are revealed to everyone. Fig. 1 summarizes the market structure and timing of events.

I focus on stationary equilibria where the real value of money supply (in terms of the common general good) is constant over time, i.e. $\phi_t^j M_{t+1}^j = Z^j$, where ϕ_t^j and M_{t+1}^j are the real value and total stock of currency j after lump-sum transfers are carried out in period-t CM. Accordingly the growth rate γ_t^j can also be interpreted as the inflation rate in country-j, since $\gamma_t^j = \frac{M_{t+1}^j}{M_t^j} = \frac{\phi_{t-1}^j}{\phi_t^j}$. Thus the nominal exchange rate $e_t = \frac{\phi_t^B}{\phi_t^A} = \frac{Z^B}{Z^A}\frac{M_{t+1}^A}{M_{t+1}^B}$ is determined by money supply in both countries, as well as their real values in terms of the CM general good. Uncertainty in foreign inflation rate leads to uncertainty in the nominal exchange rate, which corresponds to the exchange rate risk faced by firms and investors in real life.

3. Equilibrium

3.1. Centralized market

Let $W^j\left(m_t^a\phi_t^A, m_t^b\phi_t^B\right)$ denote the value function of a country-j buyer with money holdings (m_t^a, m_t^b) after lump-sum taxes/-transfers. The country-j buyer solves the following maximization problem:

$$W^j\left(m_t^a\phi_t^A, m_t^b\phi_t^B\right) = \max_{x,n,m_{t+1}^a\phi_t^A,m_{t+1}^b\phi_t^B} x - n + \beta\mathbb{E}\left[V^j\left(m_{t+1}^a\phi_t^A, m_{t+1}^b\phi_t^B, \gamma_{t+1}^A, \gamma_{t+1}^B\right)\right]$$
$$\text{s.t.} \qquad x + m_{t+1}^a\phi_t^A + m_{t+1}^b\phi_t^B = n + m_t^a\phi_t^A + m_t^b\phi_t^B$$

where $V^j\left(m_{t+1}^a\phi_t^A, m_{t+1}^b\phi_t^B, \gamma_{t+1}^A, \gamma_{t+1}^B\right)$ denotes the value function of a country-j buyer with money holdings (m_{t+1}^a, m_{t+1}^b) at the beginning of DM in period $t+1$, after information regarding the money growth rates is revealed to respective agents, but before any trades or meetings take place. The expectation $\mathbb{E}$ is taken with respect to money growth rates in the next period, $(\gamma_{t+1}^A, \gamma_{t+1}^B)$. Eliminating $x - n$ using the budget constraint yields

$$W^j\left(m_t^a\phi_t^A, m_t^b\phi_t^B\right) = m_t^a\phi_t^A + m_t^b\phi_t^B + \max_{m_{t+1}^a\phi_t^A,m_{t+1}^b\phi_t^B} -m_{t+1}^a\phi_t^A - m_{t+1}^b\phi_t^B + \beta\mathbb{E}\left[V^j\left(m_{t+1}^a\phi_t^A, m_{t+1}^b\phi_t^B, \gamma_{t+1}^A, \gamma_{t+1}^B\right)\right]$$
$$\bar{m}_t^a\phi_t^A + m_t^b\phi_t^B + W^j(0,0) \tag{3}$$

The CM value function has several useful properties. First, $W^j\left(m_t^a\phi_t^A, m_t^b\phi_t^B\right)$ is linear in the total real balance $m_t^a\phi_t^A + m_t^b\phi_t^B$. Second, the optimal choice of (m_{t+1}^a, m_{t+1}^b) does not depend on (m_t^a, m_t^b), which follows from the linearity of the CM utility function. So all buyers from the same country will carry the same real balances into the next period DM. Moreover, as long as holding money is costly, that is, $\overline{\gamma}^j \geqslant \beta$, sellers will leave the CM with no money holdings.

3.2. Payments under private information

In DM meetings, buyers and sellers have asymmetric information regarding the real value of currencies. The bargaining game between buyers and sellers takes the structure of a signaling game[4].

Consider the bargaining game between a seller and a buyer with currency holdings (m_t^a, m_t^b). Without loss of generality, assume the seller is from country A, so that γ_t^A is in his information set and the information asymmetry is only about γ_t^B. I take the bargaining protocol in which the buyer makes a take-it-or-leave-it offer, so that all gains from trade goes to the buyer. A strategy for the buyer is an offer $(q, p_a, p_b) \in \mathscr{F} \equiv [0, \infty) \times [0, m_t^a] \times [0, m_t^b]$, where q is the amount of special good produced by the seller, and p_a and p_b are the amount of currencies delivered to the seller. A strategy for the seller is an acceptance rule specifying a set $\mathscr{A} \subseteq \mathscr{F}$ of acceptable offers.

[3] In reality, people who hold dollars (euros) know how many goods and services that are denominated in dollars (euros) they can purchase. Correspondingly, in the model we assume that sellers know how many general goods they can purchase in CM using their domestic currency. This implies that sellers know the growth rate of domestic money supply.

[4] See Rocheteau (2011) for details.

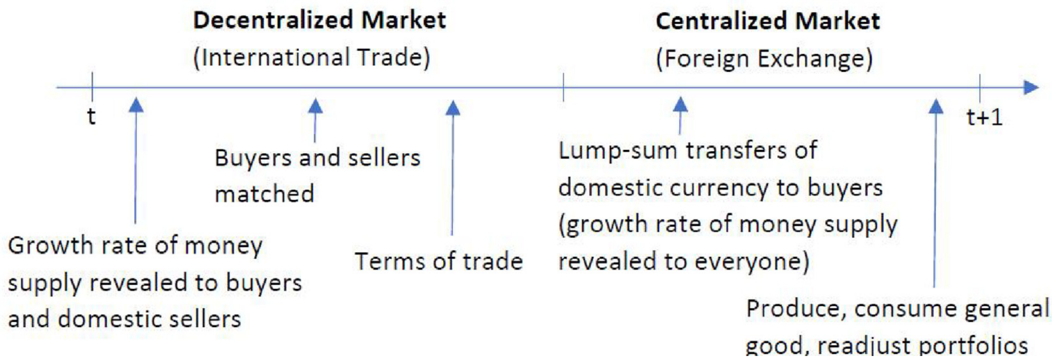

Fig. 1. Timing of a representative period.

The payoff of a country-j buyer who offers (q, p_a, p_b) is

$$\left[u(q) + W^j\left((m_t^a - p_a)\phi_t^A, (m_t^b - p_b)\phi_t^B\right)\right]\mathbb{1}_{\mathscr{A}}(q, p_a, p_b) + W^j\left(m_t^a\phi_t^A, m_t^b\phi_t^B\right)[1 - \mathbb{1}_{\mathscr{A}}(q, p_a, p_b)]$$

where $\mathbb{1}_{\mathscr{A}}(q, p_a, p_b)$ is an indicator function that equals to one when the offer is accepted by the seller. From the linearity of the value function W^j, this can be written as

$$\left[u(q) - p_a\phi_t^A - p_b\phi_t^B\right]\mathbb{1}_{\mathscr{A}}(q, p_a, p_b) + W_j\left(m_t^a\phi_t^A, m_t^b\phi_t^B\right)$$

Therefore, the gain from trade for the buyer is $\left[u(q) - p_a\phi_t^A - p_b\phi_t^B\right]\mathbb{1}_{\mathscr{A}}(q, p_a, p_b)$.

The expected payoff for the seller facing an offer (q, p_a, p_b) is

$$\left[-c(q) + p_a\phi_t^A + p_b\mathbb{E}[\phi_t^B]\right]\mathbb{1}_{\mathscr{A}}(q, p_a, p_b)$$

where the expectation $\mathbb{E}$ is formed according to the offer proposed. Let $\lambda(q, p_a, p_b) \in [0, 1]$ represent the updated belief of a seller that $\gamma_t^B = \gamma_L^B$, conditional on the offer (q, p_a, p_b) being made. Then

$$\mathbb{E}[\phi_t^B] \equiv \mathbb{E}_\lambda[\phi_t^B] = \lambda(q, p_a, p_b)\frac{\phi_{t-1}^B}{\gamma_L^B} + [1 - \lambda(q, p_a, p_b)]\frac{\phi_{t-1}^B}{\gamma_H^B}$$

The seller accepts the offer (q, p_a, p_b) as long as the expected gain from trade is nonnegative. Thus for a given belief system λ, the set of acceptable offers for a seller is

$$\mathscr{A}(\lambda) = \left\{(q, p_a, p_b) \in \mathscr{F} : -c(q) + p_a\phi_t^A + p_b\mathbb{E}_\lambda[\phi_t^B] \geqslant 0\right\}. \tag{4}$$

That is, the seller's cost of production in DM, $-c(q)$, must be compensated for by the expected value of currency payments in the next CM, $p_a\phi_t^A + p_b\mathbb{E}_\lambda[\phi_t^B]$, for an offer to be acceptable. Assuming a tie-breaking rule according to which a seller accepts an offer when he is indifferent between accepting or rejecting, the problem of a buyer is then

$$\max_{q, p_a, p_b}\left[u(q) - p_a\phi_t^A - p_b\phi_t^B\right]\mathbb{1}_{\mathscr{A}}(q, p_a, p_b) \text{s.t.} (q, p_a, p_b) \in [0, \infty) \times [0, m_t^a] \times [0, m_t^b]. \tag{5}$$

The equilibrium concept is perfect Bayesian equilibrium (PBE) refined by the Intuitive Criterion of Cho and Kreps (1987). An equilibrium of the bargaining game is a profile of strategies for the sellers and buyers, and a belief system, λ, which is derived according to the Bayes' rule. The Intuitive Criterion[5] is used to discipline out-of-equilibrium beliefs.

Definition 1. An equilibrium of the bargaining game is a pair of strategies and a belief system, $\langle[q(\gamma_t^B), p_a(\gamma_t^B), p_b(\gamma_t^B)], \mathscr{A}, \lambda\rangle$, such that $[q(\gamma_t^B), p_a(\gamma_t^B), p_b(\gamma_t^B)]$ is a solution to (5), with $\gamma_t^B \in \{\gamma_L^B, \gamma_H^B\}$; $\mathscr{A}$ is given by (4); $\lambda : \mathscr{F} \to [0, 1]$ satisfies Bayes's rule whenever possible and the Intuitive Criterion.

Equilibria of the bargaining game can be characterized in two steps. First, we show that all but the Pareto-efficient separating PBE can be dismissed by the Intuitive Criterion (Lemma 1). Then, we construct a system of belief and derive the acceptance rule of sellers to support the Pareto-efficient strategy.

[5] See Appendix A for description.

Lemma 1. An optimal offer of the buyer when $\gamma_t^B = \gamma_H^B$ is

$$(q^H, p_a^H, p_b^H) \in arg\max_{q,p_a,p_b}\left[u(q) - p_a\phi_t^A - p_b\frac{\phi_{t-1}^B}{\gamma_H^B}\right] \tag{6}$$

$$s.t. - c(q) + p_a\phi_t^A + p_b\frac{\phi_{t-1}^B}{\gamma_H^B} \geqslant 0 \tag{7}$$

$$0 \leqslant p_a \leqslant m_t^a, 0 \leqslant p_b \leqslant m_t^b \tag{8}$$

An optimal offer of the buyer when $\gamma_t^B = \gamma_L^B$ is

$$(q^L, p_a^L, p_b^L) \in arg\max_{q,p_a,p_b}\left[u(q) - p_a\phi_t^A - p_b\frac{\phi_{t-1}^B}{\gamma_L^B}\right] \tag{9}$$

$$s.t. - c(q) + p_a\phi_t^A + p_b\frac{\phi_{t-1}^B}{\gamma_L^B} \geqslant 0 \tag{10}$$

$$u(q) - p_a\phi_t^A - p_b\frac{\phi_{t-1}^B}{\gamma_H^B} \leqslant u(q^H) - c(q^H) \tag{11}$$

$$0 \leqslant p_a \leqslant m_t^a, 0 \leqslant p_b \leqslant m_t^b \tag{12}$$

When $\gamma_t^B = \gamma_H^B$, the buyer makes the complete-information offer (which is always acceptable irrespective of sellers' beliefs). The solution to (6)–(8) has the following properties:

(i) if $m_t^a\phi_t^A + m_t^b\frac{\phi_{t-1}^B}{\gamma_H^B} \geqslant c(q^*)$, then $q^H = q^*$ and $p_a\phi_t^A + p_b\frac{\phi_{t-1}^B}{\gamma_H^B} = c(q^*)$.

(ii) if $m_t^a\phi_t^A + m_t^b\frac{\phi_{t-1}^B}{\gamma_H^B} < c(q^*)$, then $p_a = m_t^a, p_b = m_t^b$ and $q^H = c^{-1}\left(m_t^a\phi_t^A + m_t^b\frac{\phi_{t-1}^B}{\gamma_H^B}\right)$

That is, the surplus-maximizing level of output q^* is achieved if the buyer holds enough real balances. Otherwise, the buyer spends all its currency holdings to achieve the maximum possible level of output.

When $\gamma_t^B = \gamma_L^B$, the only possible offer that a buyer can make in equilibrium is the one that maximizes its gain from trade in the class of all offers that satisfy the participation constraint (10) and the incentive-compatibility (11) of the seller, where the seller has the correct belief that it is in the $\gamma_t^B = \gamma_L^B$ state.

Next, we construct a belief system that generates an acceptance rule for sellers that is consistent with the buyer' offers in Lemma 1 and that satisfies the Intuitive Criterion. Formally,

$$\lambda(q, p_a, p_b) = 0, \forall(q, p_a, p_b) \notin \mathcal{O} \text{s.t.} u(q) - p_a\phi_t^A - p_b\tfrac{\phi_{t-1}}{\gamma_H^B} > u(q^H) - c(q^H)$$
$$\lambda(q, p_a, p_b) = 1, \forall(q, p_a, p_b) \notin \mathcal{O} \text{s.t.} u(q) - p_a\phi_t^A - p_b\tfrac{\phi_{t-1}}{\gamma_H^B} \leqslant u(q^H) - c(q^H)$$

where $\mathcal{O}$ is the set of equilibrium offers. That is, all out-of-equilibrium offers that would raise the payoff of buyers in the high-inflation state relative to their complete-information payoff are attributed to high-inflation state, and all other out-of-equilibrium offers are attributed to the low-inflation state. This system of beliefs is derived from the Bayes's rule and satisfies the Intuitive Criterion by construction.

Under this belief system, the set of acceptable offers is

$$\mathcal{A} = \left\{(q, p_a, p_b) \in \mathcal{F} : u(q) - p_a\phi_t^A - p_b\frac{\phi_{t-1}^B}{\gamma_H^B} \leqslant u(q^H) - c(q^H) \text{and} - c(q) + p_a\phi_t^A + p_b\frac{\phi_{t-1}^B}{\gamma_L^B} \geqslant 0\right\}.$$

Feasible offers that violate the incentive-compatibility constraint (11) are attributed to the high-inflation state. They are rejected since they violate the seller's participation constraint (7). All offers that satisfy (11) are attributed to the low-inflation state, except for (q^H, p_a^H, p_b^H). They must also satisfy (10) in order to be accepted.

Proposition 1. Consider a match between a country-A seller and a buyer with currency portfolio (m_t^a, m_t^b) in period t. There is a solution (q^L, p_a^L, p_b^L) to (9)–(12), and it has the following properties:

(i) If $m_t^a\phi_t^A \geqslant c(q^*)$, then $q^L = q^*, p_a^L\phi_t^A + p_b^L\frac{\phi_{t-1}^B}{\gamma_L^B} = c(q^*)$ and $p_b^L = 0$.

(ii) If $m_t^a\phi_t^A < c(q^*)$, then $p_a^L = m_t^a$ and $(q^L, p_b^L) \in [0, q^H] \times [0, m_t^b]$ is the unique solution to:

$$p_b^L\frac{\phi_{t-1}^B}{\gamma_L^B} = c(q^L) - m_t^a\phi_t^A \tag{13}$$

$$u(q^L) - c(q^L) + \left(1 - \frac{\gamma_L^B}{\gamma_H^B}\right)\left[c(q^L) - m_t^a\phi_t^A\right] = u(q^H) - c(q^H) \tag{14}$$

where $q^H = \min\left[q^*, c^{-1}\left(m_t^a \phi_t^A + m_t^b \frac{\phi_{t-1}^B}{\gamma_H^B}\right)\right]$. Moreover, if $m_t^b > 0$, then $q^L < q^H$ and $p_b^L \in (0, m_t^b)$.

This proposition offers a *pecking-order theory of payment choices*. When the real value of foreign currency is low (i.e. foreign inflation is high), there is no information asymmetry, and the two currencies play the same role in DM transactions. But when the real value of foreign currency is high (i.e. foreign inflation is low), the buyer offers the seller's domestic currency first, and then use the seller's foreign currency as a last resort. Hence, the seller's domestic currency is the preferred means of payment. Even when the buyer does not have enough domestic currency to purchase the optimal level of output q^*, it chooses not to spend all its foreign currency. By retaining part of its foreign currency holding, the buyer signals the high value of foreign currency, and hence secures better terms of trade.[6]

The intuition of the pecking-order theory of payment choices is similar to that of the Gresham's Law. When there is asymmetric information regarding the real value of currency, people will hold on to the "good money" (with high value) and spend the "bad money" (with low value) first. The fact that the buyer is less willing to spend currency B signals its high value to the seller.

Proposition 2. (Currency liquidity and monetary policy uncertainty). Assume $m_t^a \phi_t^A < c(q^*)$ and $m_t^b > 0$. Then:

$$\left.\frac{d p_b^L}{d \gamma_H^B}\right|_{\frac{\pi_H^B}{\gamma_H^B} + \frac{\pi_L^B}{\gamma_L^B} = \frac{1}{\bar{\gamma}^B}} < 0.$$

Suppose that the buyer holds both currencies, and that using currency A only is not enough to reach q^* in the low-inflation state, then the usage of currency B decreases as the spread of inflation in country B increases. That is, the liquidity of currency B decreases with the monetary policy uncertainty in country B. In order to separate the high-inflation state and low-inflation state, buyers incur a signaling cost, which is equal to the difference between the buyer's surplus in the two states $p_b^L\left(\frac{\phi_{t-1}^B}{\gamma_L^B} - \frac{\phi_{t-1}^B}{\gamma_H^B}\right) > 0$. As this difference increases, the information asymmetry becomes more severe, which tightens the incentive-compatibility constraint, raises the signaling cost and reduces the liquidity of currency B.

Proposition 3. (Hyperinflation). Assume $m_t^a \phi_t^A < c(q^*)$ and $m_t^b > 0$. As $\gamma_H^B \to \infty, p_b^L \to 0$.

In the extreme case where $\gamma_H^B \to \infty$, currency B becomes valueless in the high-inflation state. The information asymmetry is so severe that currency B ceases to serve as a medium of exchange. In this scenario, currency A becomes the only means of payment in both high-inflation and low-inflation states, even when the average level of inflation $\bar{\gamma}^B$ is at a reasonable level.

Proposition 4. (Currency composition). If $m_t^a \phi_t^A < c(q^*)$ and $m_t^b > 0$, then

$$\frac{\partial\left(p_b^L \phi_{t-1}^B / \gamma_L^B\right)}{\partial\left(m_t^a \phi_t^A\right)} = \frac{\frac{u'(q^H)}{c'(q^H)} - \frac{u'(q^L)}{c'(q^L)}}{\frac{u'(q^L)}{c'(q^L)} - \frac{\gamma_B^L}{\gamma_B^H}} < 0 \tag{15}$$

If $m_t^a \phi_t^A + m_t^b \frac{\phi_{t-1}^B}{\gamma_H^B} < c(q^*)$, then

$$\frac{\partial p_b^L}{\partial m_t^b} = \frac{\frac{u'(q^H)}{c'(q^H)} - 1}{\frac{\gamma_H^B}{\gamma_L^B} \frac{u'(q^L)}{c'(q^L)} - 1} \in (0,1). \tag{16}$$

As the buyer's holding of currency A increases, its use of currency B as means of payment decreases. In the low-inflation state, the buyer can reduce its signaling cost by substituting information-insensitive currency A for information-sensitive currency B, thereby relaxing the incentive-compatibility constraint. In addition, when the current currency holding is not enough to reach the surplus-maximizing q^* in the high-inflation state, the marginal propensity of a buyer to spend currency B in the low-inflation state is less than one. An additional unit of currency B would raise the surplus that the buyer can obtain in the high-inflation state. As a result, the buyer in the low-inflation state that receives an additional unit of currency B can spend a fraction of it without giving others incentives to imitate. If, on the other hand, $m_t^a \phi_t^A + m_t^b \frac{\phi_{t-1}^B}{\gamma_H^B} \geqslant c(q^*)$, then $q^H = q^*$ and $\partial p_b^L / \partial m_t^b = 0$. That is, when the liquidity needs in the high-inflation state are satiated, an additional unit of currency B does not affect the incentive-compatibility constraint, and hence, the terms of trade, in the low-inflation state.

[6] The model can be extended to include a vehicle currency C, such as USD. The real value of currency C is unknown to both the buyer and the seller in DM, hence information asymmetry exists only for currency B. With both agents being risk neutral in CM, the buyer is indifferent between currency A and C. They are used in payments first, while currency B serves only as a last resort.

3.3. Decentralized market

Consider a buyer entering DM with currency holding (m_t^a, m_t^b) before inflation rates are revealed. There are 4 possible states of the world: $(\gamma_H^A, \gamma_H^B), (\gamma_H^A, \gamma_L^B), (\gamma_L^A, \gamma_H^B), (\gamma_L^A, \gamma_L^B)$. Define $\pi_{ij} = \pi_i^A \pi_j^B$ where $i = H, L$ and $j = H, L$. Then

$$\mathbb{E}\left[V^j\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B, \gamma_t^A, \gamma_t^B\right)\right] = \pi_{HH}V^j\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B, \gamma_H^A, \gamma_H^B\right) + \pi_{LH}V^j\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B, \gamma_L^A, \gamma_H^B\right) + \pi_{HL}V^j\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B, \gamma_H^A, \gamma_L^B\right) + \pi_{LL}V^j\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B, \gamma_L^A, \gamma_L^B\right)$$

where $V^j\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B, \gamma_t^A, \gamma_t^B\right)$ is the DM value function for a country-j buyer when the state of the world is (γ_t^A, γ_t^B).

The buyer could meet with sellers in both countries. For example, the value function of country-A buyer when the state of the world is (γ_H^A, γ_L^B) is

$$V^A\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B, \gamma_H^A, \gamma_L^B\right) = \mu^{AA}S^{HLA}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right) + \mu^{AB}S^{HLB}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right) + m_t^a\frac{\phi_{t-1}^A}{\gamma_H^A} + \left(1 - \frac{1}{\gamma_H^A}\right)Z^A + m_t^b\frac{\phi_{t-1}^B}{\gamma_L^B} + W^A(0,0)$$

where $S^{HLj} = u(q^{HLj}) - c(q^{HLj})$ is the gain from trade in a DM meeting between the buyer and a country-j ($j = A, B$) seller when the state of the world is (γ_H^A, γ_L^B), and $\left(1 - \frac{1}{\gamma_H^A}\right)Z^A$ denotes the monetary transfer to domestic buyers from the government.

There are 8 possible scenarios in total and they come in pairs in terms of the information structure: (HHA,HLA), (LHA,LLA), (HHB,LHB), (HLB,LLB). In the first two pairs, information asymmetry is about country-B inflation. In the last two pairs, information asymmetry is about country-A inflation. Appendix C lists the bargaining solutions for all possible DM meetings. Accordingly, the expected DM value function for the country-A buyer can be written as follows:

$$\begin{aligned}\mathbb{E}\left[V^A\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B, \gamma_t^A, \gamma_t^B\right)\right] &= m_t^a\frac{\phi_{t-1}^A}{\bar{\gamma}^A} + \left(1 - \frac{1}{\bar{\gamma}^A}\right)Z^A + m_t^b\frac{\phi_{t-1}^B}{\bar{\gamma}^B} + W^A(0,0)\\
&+ \mu^{AA}\Big[S^{HHA}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\pi_{HH} + S^{HLA}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\pi_{HL}\\
&+ S^{LHA}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\pi_{LH} + S^{LLA}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\pi_{LL}\Big]\\
&+ \mu^{AB}\Big[S^{HHB}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\pi_{HH} + S^{HLB}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\pi_{HL}\\
&+ S^{LHB}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\pi_{LH} + S^{LLB}\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\pi_{LL}\Big]\end{aligned} \tag{17}$$

From (3), $\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)$ solves

$$\max_{m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B} -m_t^a\phi_{t-1}^A - m_t^b\phi_{t-1}^B + \beta\left\{\mathbb{E}^A\left[S\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\right] + m_t^a\frac{\phi_{t-1}^A}{\bar{\gamma}^A} + \left(1 - \frac{1}{\bar{\gamma}^A}\right)Z^A + m_t^b\frac{\phi_{t-1}^B}{\bar{\gamma}^B} + W^A(0,0)\right\}$$

where the superscript of $\mathbb{E}^A$ reflects the meeting probabilities of country-A buyers, μ^{AA} and μ^{AB}. Rearranging the above optimization problem yields

$$\max_{m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B} \mathbb{E}^A\left[S\left(m_t^a\phi_{t-1}^A, m_t^b\phi_{t-1}^B\right)\right] - \left(\frac{\bar{\gamma}^A}{\beta} - 1\right)m_t^a\frac{\phi_{t-1}^A}{\bar{\gamma}^A} - \left(\frac{\bar{\gamma}^B}{\beta} - 1\right)m_t^b\frac{\phi_{t-1}^B}{\bar{\gamma}^B} + \left(1 - \frac{1}{\bar{\gamma}^A}\right)Z^A + W^A(0,0) \tag{18}$$

The term $(\bar{\gamma}^j/\beta - 1)$ reflects the cost of holding currency j ($j = A, B$).

Define

$$z_a^A = m_t^a\phi_{t-1}^A \text{ and } z_b^A = m_t^b\phi_{t-1}^B$$

which are the real value of currency holdings for a country-A buyer at the end of period $t-1$. Similarly, let (z_a^B, z_b^B) be the real value of currency holdings for a country-B buyer, which can be obtained by solving a country-B buyer's optimization problem. Then, in equilibrium the value of total money supply is determined endogenously by

$$Z^A = z_a^A + z_a^B N \tag{19}$$
$$Z^B = z_b^A + z_b^B N \tag{20}$$

Definition 2. Given the distributions of inflation in both countries, a stationary equilibrium is a list of quantities traded in all possible meetings, buyers' strategies, seller's strategies and beliefs, and currency holdings $\{z_a^A, z_b^A, z_a^B, z_b^B\}$ such that

1. The strategies of the buyers and sellers, and the belief system constitute a Perfect Bayesian Equilibrium that satisfies the Intuitive Criterion.
2. $\{z_a^A, z_b^A, z_a^B, z_b^B\}$ solves the buyers' portfolio choice problem.

Moreover, a stationary equilibrium is monetary if at least one money is valued, i.e. $Z^A > 0$ or $Z^B > 0$.

The following proposition provides necessary and sufficient conditions for an optimal portfolio.

Proposition 5. (Buyers' portfolio choices). When $\overline{\gamma}^A \geqslant \beta$ and $\overline{\gamma}^B \geqslant \beta$, $\left(z_a^j, z_b^j\right)$ is a solution to the country-j buyer's portfolio problem, (18), if and only if

$$-\left(\frac{\overline{\gamma}^A}{\beta} - 1\right)\frac{1}{\overline{\gamma}^A} + \mathbb{E}^j\left[S_a\left(z_a^j, z_b^j\right)\right] \leqslant 0, = \text{ if } z_a^j > 0 \tag{21}$$

$$-\left(\frac{\overline{\gamma}^B}{\beta} - 1\right)\frac{1}{\overline{\gamma}^B} + \mathbb{E}^j\left[S_b\left(z_a^j, z_b^j\right)\right] \leqslant 0, = \text{ if } z_b^j > 0 \tag{22}$$

where S_a and S_b are the first-order partial derivatives with respect to z_a^j and z_b^j, respectively. If $\overline{\gamma}^A > \beta$ and $\overline{\gamma}^B > \beta$, then $\left(z_a^j, z_b^j\right)$ is unique. If $\overline{\gamma}^A = \beta$ or $\overline{\gamma}^B = \beta$, then $z_a^j/\gamma_H^A + z_b^j/\gamma_H^B \geqslant c(q^*)$.

For a currency to be held, its expected holding cost must be equal to the expected marginal benefit that the currency confers in DM. If the cost of holding a currency is zero, e.g. $\overline{\gamma}^j = \beta$, buyers will carry enough balance of currency j so that they can obtain the optimal level of special goods q^* in all possible meetings. Otherwise, q^* is not attainable in certain kinds of meetings.

When $\overline{\gamma}^A > \beta$ and $\overline{\gamma}^B > \beta$, buyers' holdings of currencies are uniquely determined in equilibrium, so are the real value of money supply Z^j and the nominal exchange rate e_t. Hence the indeterminacy of nominal exchange rate (Kareken and Wallace (1981)) is resolved when the cost of holding currency is positive in both countries. Otherwise, buyers' holding of currency j is indeterminate when $\overline{\gamma}^j = \beta$, so that the indeterminacy of nominal exchange rate still holds.

4. Currency regimes and the nominal exchange rate

In this section, we explore the variety of currency regimes in stationary equilibria. For simplicity, we start from a symmetric case. The following lemma summarizes some basic properties of the optimal portfolio holdings when the two countries are symmetric.

Lemma 2. Consider a stationary monetary equilibrium where country A and country B are symmetric, with $N = 1, \overline{\gamma}^A = \overline{\gamma}^B > \beta, \gamma_H^A/\overline{\gamma}^A = \gamma_H^B/\overline{\gamma}^B > 1$ and $\sigma_F^A = \sigma_F^B$.

1. The domestic/foreign composition of buyers' portfolios are the same for both countries. That is, $z_a^A = z_b^B, z_b^A = z_a^B$.
2. When $\sigma_F^A = \sigma_F^B = \frac{1}{2}$, all buyers hold the same portfolio. That is, $z_a^A = z_b^B = z_b^A = z_a^B$.
3. In a neighbourhood of $\sigma_F^A = \sigma_F^B = 1/2$, buyers hold both currencies; and the exchange rate e_t increases with N and σ_F^A.
4. In a neighbourhood of $\sigma_F^A = \sigma_F^B = 0$, buyers only hold their domestic currency; and the exchange rate e_t increases with N but decreases with σ_F^A.

When the two countries are perfectly integrated, all buyers share the same probability of meeting with domestic/foreign sellers, hence they hold the same portfolio. By continuity, buyers hold both currencies in a neighborhood of this special circumstance. As the probability of foreign meetings increases, buyers hold less domestic currency and more foreign currency, since the sellers' domestic currency is the preferred means of payment. Thus as σ_F^A increases, currency B appreciates and currency A depreciates, leading to a higher nominal exchange rate.

When the two countries are isolated from each other, buyers only meet with domestic sellers, so they only hold their domestic currency. By continuity, this also holds in a neighborhood of the perfect isolation case. As the probability of foreign meeting increases, buyers demand more of their domestic currency, since the liquidity of domestic currency is low in foreign meetings. Buyers need to retain part of their domestic currency holding to signal its high value (low inflation state) in foreign meetings. Thus as σ_F^A increases, country-A buyers demand more currency A, leading to an appreciation of currency A and in turn a lower nominal exchange rate.

Therefore, the impact of σ_F^A on nominal exchange rate depends on the currency regime. In addition, the relative size of country B also affects the equilibrium nominal exchange rate through meeting probabilities. As N increases, it is more likely for all buyers to meet with country-B sellers and less likely to meet with country-A sellers. When buyers hold both currencies, they demand more currency B and less currency A, pushing up the nominal exchange rate. When buyers only hold their domestic currency, country-A buyers would demand more currency A to signal its high value in meetings with country-B sellers; and country-B buyers would demand less currency B, as the probability of foreign meetings decreases, reducing the incidence of information asymmetry. Hence larger size of country-B raises the nominal exchange rate in both circumstances.

4.1. Currency regimes in (σ_F^A, σ_F^B)-space

Fig. 2 shows the types of currency regimes in the (σ_F^A, σ_F^B)-space, keeping other parameters fixed and symmetric. The red (blue) lines depict the currency regimes for country A (B). For instance, when (σ_F^A, σ_F^B) is between the two blue lines, country-B buyers hold both currencies; when the (σ_F^A, σ_F^B) is above (below) the two blue lines, country-B buyers only hold currency A (B). We focus on the lower left corner of the figure since $\sigma_F^j \leqslant 1/2$. Buyers always hold their domestic currency because they meet with domestic sellers more frequently, hence relevant currency regimes include (A,B), (A,AB), (AB,B) and (AB,AB).[7] In addition to the symmetric cases analyzed in Lemma 2, Fig. 2 also demonstrates the possibility of asymmetric currency regimes. For instance, when σ_F^B is large but σ_F^A is small, country-B buyers hold both currencies since they go abroad more frequently, while country-A buyers only hold their domestic currency since they stay in their home countries more often.

4.2. Currency regimes in $(\overline{\gamma}^A, \overline{\gamma}^B)$-space

Next we consider currency regimes in the $(\overline{\gamma}^A, \overline{\gamma}^B)$-space, shown in the left panel of Fig. 3. When the mean inflation of country-j is very high, currency j is no longer valued since the cost of holding currency j exceeds the marginal benefits it confers in DM meetings. Specifically, $\gamma_{max}^{A,j}$ ($\gamma_{max}^{B,j}$) denotes the cutoff country-A (country-B) mean inflation such that currency-A (currency-B) is no longer held by country-j buyers. Thus the existence of a monetary equilibrium requires $\overline{\gamma}^A < \gamma_{max}^{A,A}$ or $\overline{\gamma}^B < \gamma_{max}^{B,B}$.[8]

Moreover, the marginal benefit conferred by currency j in DM meetings differs for buyers from the two countries. For example, the benefit brought by an additional unit of currency B is higher for country-B buyers, since currency B is more liquid in meetings with country-B sellers, which happens more often for country-B buyers, compared to country-A buyers. Hence, as $\overline{\gamma}^B$ increases, country-A buyers cease to hold currency B first. Country-B buyers abandon currency B only when $\overline{\gamma}^B$ is very high.

How does the boundaries of currency regimes in the $(\overline{\gamma}^A, \overline{\gamma}^B)$-space vary with the spread of inflation? In the right panel of Fig. 3, I compare the currency regimes when γ_H^A is smaller (the dashed lines) with the baseline result (the solid lines). As the spread of country-A inflation decreases, all boundaries shift to the right/downward. Intuitively, the mean and spread of inflation outcomes are two aspects of the currency quality. As the spread decreases, buyers are willing to accept higher mean inflation of currency A, for a given quality of currency B. Alternatively, for a given $\overline{\gamma}^A$, lower spread of currency A improves its quality, raising the quality of currency B required by buyers, which leads to lower cutoff mean inflation of currency B.

Fig. 4 further demonstrates the comparative statics of currency regimes in $(\overline{\gamma}^A, \overline{\gamma}^B)$-space. The left panel shows the impact of higher degree of economic openness. When country-A buyers meet with foreign sellers more frequently, the marginal benefit conferred by currency B increases, pushing up the level of $\overline{\gamma}^B$ accepted by country-A buyers, so the boundaries in red shift up. For country-B buyers, the probability of meeting with foreign sellers increases as more country-A buyers go abroad. This raises the marginal benefit conferred by currency A in DM meetings. Thus for a given $\overline{\gamma}^B$, country-B buyers are willing to accept a higher $\overline{\gamma}^A$, and the boundaries in blue shift to the right. Note that as the degree of economic openness of one country increases, buyers from both countries increase their holdings of foreign currency, so that all boundaries shift to the middle. The right panel shows the impact of larger country size. As N gets larger, the mass of country-B sellers increases, raising the probability of meetings with country-B sellers for all buyers. Consequently, the marginal benefit of currency B in DM meetings increases and all boundaries shift upward.

To summarize, when the spread of inflation is smaller, or the country size is relatively larger, all buyers tend to hold more currency issued by that country. However, higher degree of economic openness of a country increases all buyers' holding of foreign currencies.

[7] The first argument in each parenthesis denotes the types of currency held by country-A buyers, and the second argument denotes the types of currency held by country-B buyers.

[8] Specific forms of $\gamma_{max}^{A,j}$ and $\gamma_{max}^{B,j}$ are derived in Appendix B.1.

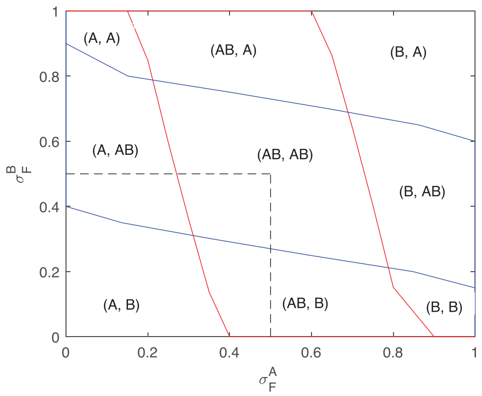

Fig. 2. Types of equilibria in (σ_F^A, σ_F^B)-space.

How does mean inflation affect the equilibrium nominal exchange rate? The impact of mean inflation resembles that of the country size. Proposition 6 proves that the impact of mean inflation is independent of the type of currency regimes, which is in stark contrast with the impact of economic openness.

Proposition 6. In a stationary monetary equilibrium with $\overline{\gamma}^A > \beta, \overline{\gamma}^B > \beta$ and both currencies being valued, the nominal exchange rate e_t increases with $\overline{\gamma}^A$ and decreases with $\overline{\gamma}^B$ regardless of the currency regime.

As the mean inflation of currency A increases, agents substitute into currency B. The increasing demand leads to an appreciation of currency B and a higher nominal exchange rate. The substitution does not happen in the (A,B) regime. In this scenario, country-A buyers simply reduce their holding of the currency, since the cost is too high. This leads to the depreciation of currency A and again a rise in the nominal exchange rate.

4.3. Currency regimes in $(\gamma_H^A/\overline{\gamma}^A, \gamma_H^B/\overline{\gamma}^B)$-space

Lastly, we look into currency regimes in the $(\gamma_H^A/\overline{\gamma}^A, \gamma_H^B/\overline{\gamma}^B)$-space, shown in Fig. 5. The pattern is similar to that in the $(\overline{\gamma}^A, \overline{\gamma}^B)$-space. Buyers only hold domestic currency when the spread of foreign inflation is very high; and only hold foreign currency when the spread of domestic inflation is very high. When spread of inflation is about the same for the two countries, buyers hold their domestic currency when uncertainty is relatively low; and hold both currencies when uncertainty is relatively high.

In contrast, in Fig. 3, when mean inflation is about the same for the two countries, buyers hold their domestic currency when the mean inflation is relatively high; and hold both currencies when mean inflation is relatively low. Hence, when both

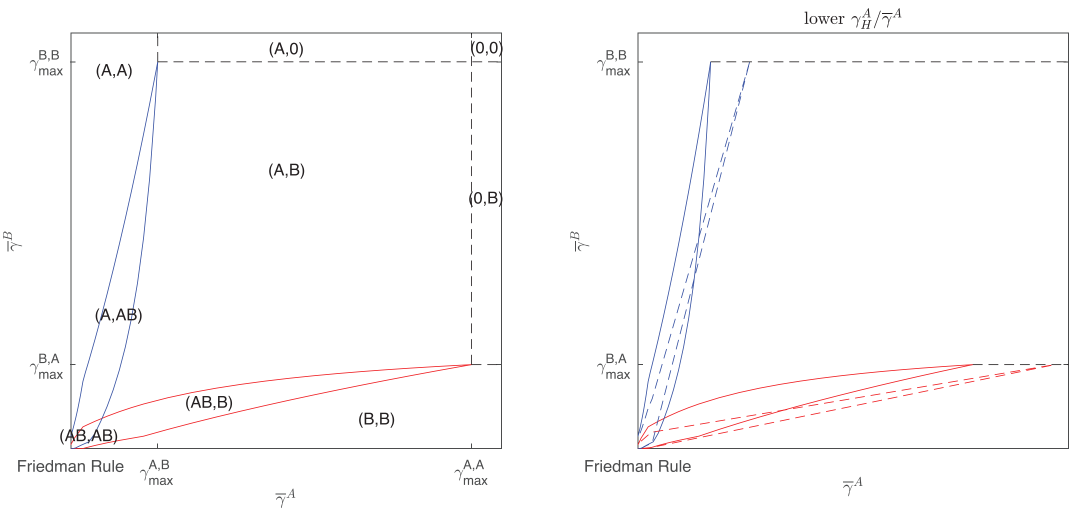

Fig. 3. Types of equilibria in $(\overline{\gamma}^A, \overline{\gamma}^B)$-space.

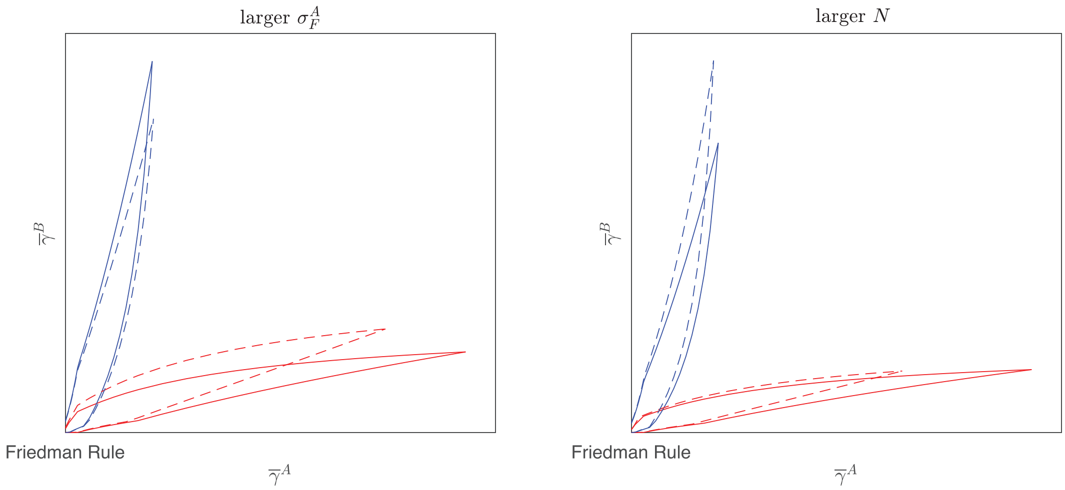

Fig. 4. Types of equilibria in $(\overline{\gamma}^A, \overline{\gamma}^B)$-space.

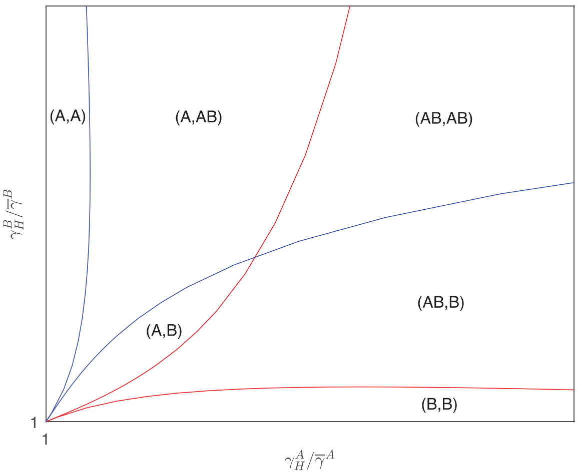

Fig. 5. Types of equilibria in $(\gamma_H^A/\overline{\gamma}^A, \gamma_H^B/\overline{\gamma}^B)$-space.

currencies are of poor quality, people tend to hold both of them when the poor quality comes from the inflation spread, and tend to hold only their domestic currency when the poor quality comes from high mean inflation. Therefore, although the mean and spread of inflation are two alternative characteristics describing the quality of a currency, there exist some key differences in the way they work.

Holding both currencies help alleviates the information asymmetry in DM meetings. When the costs of holding currencies are not too high, buyers would hold both of them. Since $\sigma_F^j < 1/2$, buyers' holding of domestic currency is higher than their holding of foreign currency. As the mean inflation increases, buyers' currency holdings drop accordingly, with the foreign currency holding reaches zero first. So buyers only hold their domestic currencies when mean inflation is relatively high.

The spread of inflation represents the degree of information asymmetry in DM meetings. When the spread is very high, buyers would like to hold both currencies to overcome the information asymmetry in foreign meetings. If the spread of inflation is relatively low, buyers have less incentive to hold the foreign currency, because the probability of foreign meeting is relatively low.

4.4. Nominal exchange rate

While country size and mean inflation have uniform impacts on the nominal exchange rate, inflation uncertainty and economic openness do not. Table 2 illustrates how the impact on nominal exchange rate depends on currency regimes.[9]

[9] Results based on numerical exercises.

Table 2
determinants of nominal exchange rate.

Currency regime	$N\uparrow$	$\overline{\gamma}^A\uparrow$	$\sigma_F^A\uparrow$	$\gamma_H^A/\overline{\gamma}^A\uparrow$
(AB, B)	$e\uparrow$	$e\uparrow$	$e\uparrow$	$e\uparrow$
(A, B)	$e\uparrow$	$e\uparrow$	$e\downarrow$	$e\downarrow$
(A, AB)	$e\uparrow$	$e\uparrow$	$e\downarrow$	$e\uparrow$
(AB, AB)	$e\uparrow$	$e\uparrow$	$e\uparrow$	$e\uparrow$

If country-A buyers hold both currencies, as the openness of country A increases, they would like to substitute currency A for currency B, since the probability of meeting foreign sellers is higher. As a result, currency A depreciates and the exchange rate rises. If country-A buyers only hold domestic currency, they would demand more real balance to signal its high value in foreign meetings, thus currency A appreciates and the exchange rate falls.

The increase of σ_F^A corresponds to the increase of import demand in mainstream international economics literature. So the pattern for (AB,B) and (AB,AB) regimes is in agreement with the standard view that higher import leads to depreciation of the currency, assuming exporters' domestic currency is required to settle transactions (producer currency pricing, PCP). This assumption is also in line with the *pecking-order theory of payments* derived in Section 3, where sellers' domestic currency is the preferred means of payment in foreign transactions. However, certain trades are instead settled using the destination country's currency (local currency pricing, LCP), such as the (A,B) and (A,AB) regimes.[10] Our model implies that higher import may also leads to appreciation of the currency when the other currency is of poor quality (high $\overline{\gamma}^B$ or high $\gamma_H^B/\overline{\gamma}^B$) due to the information asymmetry.

In addition, as the distribution of inflation becomes more spread out, its impact on exchange rate also depends on the currency regimes. When the distribution is more spread out (higher $\gamma_H^A/\overline{\gamma}^A$), the degree of information asymmetry is more severe. If at least one currency is circulating internationally, buyers could switch from currency A to currency B as the quality of currency A deteriorates, so the exchange rate increases. If buyers only hold their domestic currencies, in other words, if both countries employ LCP in foreign transactions, country-A buyers would demand more real balance to signal its high value in foreign meetings, since the liquidity of currency A falls as γ_H^A increases, which is shown in Proposition 2.

5. A monetary policy game

In this section, we consider a policy game where each monetary authority strategically sets its mean inflation, taking as given the other monetary authority's mean inflation, as well as the spread of distributions. In reality, a monetary authority can only set a specific inflation target. There is no guarantee that the target will be achieved precisely. This constitutes the economic background for this monetary policy game.

We first derive the objective functions of monetary authorities, which is the steady state welfare of its citizens. The welfare of country j ($\mathscr{W}^j$) is defined as the sum of buyers' and sellers' utilities in country j, weighted by their respective measures. Since all gains from DM trade goes to the buyer, the utilities of sellers' are always zero. Hence

$$\mathscr{W}^A = \mathbb{E}\left[V^A\right]\text{and}\mathscr{W}^B = N\mathbb{E}\left[V^B\right]$$

It is shown in Appendix B.2 that

$$\mathscr{W}^{\mathscr{A}} = \frac{1}{1-\beta}\left\{\mathbb{E}^A\left[S(z_a^A, z_b^A)\right] - \underbrace{z_b^A\left(1-\frac{1}{\overline{\gamma}^B}\right)}_{\text{A's seigniorage transfer to B}} + \underbrace{Nz_a^B\left(1-\frac{1}{\overline{\gamma}^A}\right)}_{\text{A's seigniorage revenue from B}}\right\}$$

$$\mathscr{W}^{\mathscr{B}} = \frac{N}{1-\beta}\left\{\mathbb{E}^B\left[S(z_a^B, z_b^B)\right] - \underbrace{z_a^B\left(1-\frac{1}{\overline{\gamma}^A}\right)}_{\text{B's seigniorage transfer to A}} + \underbrace{\frac{z_b^A}{N}\left(1-\frac{1}{\overline{\gamma}^B}\right)}_{\text{B's seigniorage revenue from A}}\right\}$$

Welfare can be decomposed into two components: (i) DM surplus and (ii) net seigniorage revenue or transfer. Total welfare of the world is the sum of DM surplus from both countries, as seiginorage revenues and transfers cancel out.

$$\mathscr{W} = \mathscr{W}^{\mathscr{A}} + \mathscr{W}^{\mathscr{B}} = \frac{1}{1-\beta}\left\{\mathbb{E}^A\left[S(z_a^A, z_b^A)\right] + N\mathbb{E}^B\left[S(z_a^B, z_b^B)\right]\right\}$$

[10] This could happen when country A in an advanced economy like the U.S. and country B in an emerging market economy.

5.1. Social planer's problem and cooperative policy

As a benchmark, we first consider the social planner' problem of choosing inflation targets for the two countries $(\overline{\gamma}^A, \overline{\gamma}^B)$, by maximizing the total welfare of the world.

Proposition 7. The social planner's choice of mean inflation satisfies the joint Friedman Rule, $\left(\overline{\gamma}^A, \overline{\gamma}^B\right) = (\beta, \beta)$, which achieves the first-best level of output q^* in all meetings. In that case, both currencies are valued, even when they differ in terms of the spread of inflation distribution.

At the Friedman Rule, there is no expected cost of holding money into the next period. So buyers hold enough real balances $\left(z_a^j, z_b^j\right)$ to overcome various constraints and achieve the optimum q^*. The real balances are not uniquely determined, but all allocations are payoff and welfare equivalent.

When the two countries coordinate in setting their inflation targets, they will also choose the joint Friedman Rule since their objective is the same as the social planner. There is no longer redistributive gains from inflation and hence no more temptation to inflate.

5.2. Non-cooperative Game

Suppose the two monetary authorities do not cooperate. Setting a higher inflation target will raise the country's seigniorage revenue and the currency's holding cost at the same time. So monetary authorities face a tradeoff when choosing the inflation targets. The next proposition shows that the Friedman Rule cannot be an equilibrium in this non-cooperative policy game. At the Friedman Rule, DM surplus is maximized so the temptation to inflate overweighs the fear of declining purchasing power of currency.

Proposition 8. Suppose the two countries set $\overline{\gamma}^j = \beta, j = A, B$. Then country A can benefit by setting $\overline{\gamma}^A > \beta$, that is $\frac{\partial \mathscr{W}^{\mathscr{A}}}{\partial \overline{\gamma}^A}|_{\overline{\gamma}^A = \overline{\gamma}^B = \beta} > 0$, as long as the two countries are not completely isolated. Moreover, the incentive to deviate decreases with the spread of inflation in country A.

By inflating currency A, country A can raise more seigniorage revenue. But holding currency-A is costly when $\overline{\gamma}^A > \beta$, so buyers may not achieve the optimum q^* when they meet with country-A sellers. When the inflation spread of currency-A is small, the signaling cost in DM meetings is low. This leads to a lower cost of inflating currency A and a higher temptation to deviate from the Friedman Rule.

Next, we argue that the in a Nash equilibrium of this policy game, at least one currency must be circulating internationally. First, any point in region $(0,0)$ cannot be an equilibrium. In this region, the two countries have zero welfare. Country A can attain a positive welfare by reducing mean inflation and entering region $(A, 0)$. Similarly, any point in region $(0, B)/(A, 0)$ cannot be an equilibrium, since country A/B can benefit by deviating to region (A, B). Second, any point in the interior of region (A, B) cannot be an equilibrium. In this region, each country holds its domestic currency, so seigniorage revenue is zero. Country A/B can benefit by reducing its mean inflation and increasing its holding of domestic currency. That is, for a small $\epsilon > 0$

$$\mathscr{W}^{\mathscr{A}'} = \tfrac{1}{1-\beta}\mathbb{E}^A\left[S\left(z_a^A + \epsilon, 0\right)\right] > \tfrac{1}{1-\beta}\mathbb{E}^A\left[S\left(z_a^A, 0\right)\right] = \mathscr{W}^{\mathscr{A}}$$
$$\mathscr{W}^{\mathscr{B}'} = \tfrac{1}{1-\beta}\mathbb{E}^B\left[S\left(0, z_b^B + \epsilon\right)\right] > \tfrac{1}{1-\beta}\mathbb{E}^B\left[S\left(0, z_b^B\right)\right] = \mathscr{W}^{\mathscr{B}}$$

Finally, note that at any point in the interior of region (B, B), country A has no incentive to deviate since welfare in this region only depends on $\overline{\gamma}^B$:

$$\mathscr{W}^{\mathscr{A}} = \frac{1}{1-\beta}\left\{\mathbb{E}^A\left[S\left(0, z_b^A\right)\right] - z_b^A\left(1 - \frac{1}{\overline{\gamma}^B}\right)\right\}$$

The same argument applies to region (A, A). Hence without loss of generality, we assume that whenever a money authority is indifferent, it sets its inflation target to the lowest possible level. Therefore, we can restrict attention to regions (AB, AB), (A, AB) and (AB, B).

Fig. 6 plots the best response functions in a symmetric non-cooperative policy game. By symmetry, the Nash equilibrium lies in the region where both currencies circulate internationally and the two inflation targets are identical. What's the role of inflation uncertainty and economic openness in this policy game? I explore this question in Fig. 7, assuming the two countries are symmetric and using dashed lines to depict the shifts of currency regime boundaries under alternative parametrizations.

In the left panel, as the uncertainty of inflation decreases, the areas of (A, AB) and (AB, B) become thinner, the region where both currencies are circulating internationally becomes smaller, and the inflation targets in Nash equilibrium become lower. Intuitively, buyers hold less foreign currencies as the uncertainty of inflation decreases, because it mitigates the information asymmetry problem. This reduces the seigniorage revenue from inflation, as well as the temptation to inflate in the policy game. Therefore, monetary uncertainty could potentially lead to higher inflation.

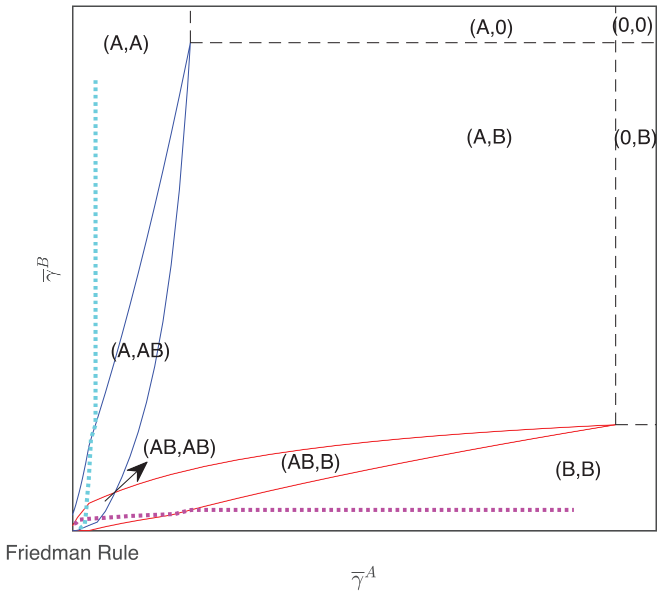

Fig. 6. Best response functions in a symmetric policy game.

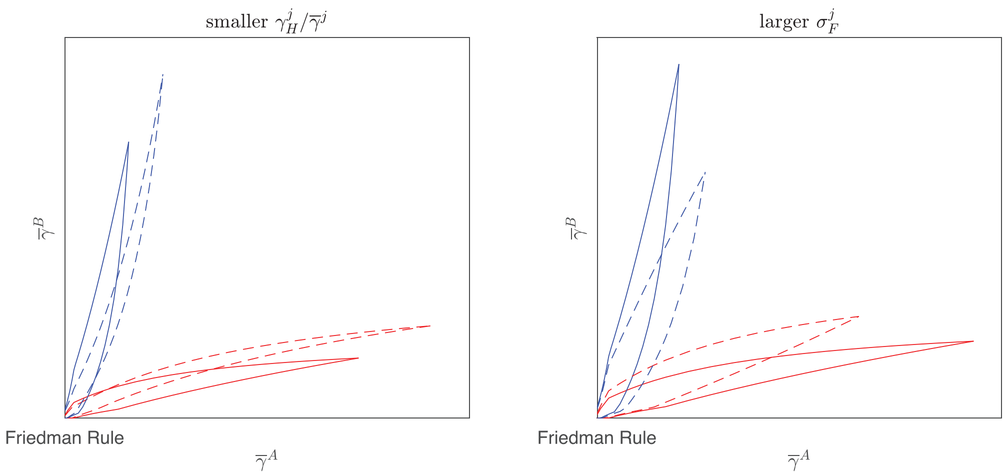

Fig. 7. Shifts of currency regimes in symmetric equilibria.

Alternatively, as σ_F^j decreases, the areas of (A, AB) and (AB, B) become further apart, the region where both currencies are circulating internationally becomes smaller, and the inflation targets in Nash equilibrium become lower. In the extreme case where the two countries are isolated, the equilibrium is both countries following the Friedman Rule. The reason is that seigniorage concern becomes less important when σ_F^j decreases, hence the incentive to inflate becomes smaller, leading to low inflation targets in equilibrium. This result is consistent with Liu and Shi (2010), who conclude that the gain from coordination increases as the two countries become more integrated.

I further explore asymmetric policy games in Fig. 8, in which the red (blue) line denotes the inflation target set by country A (B). As the uncertainty of inflation in country A decreases, buyers in both countries substitute currency A for currency B. This raises the seigniorage revenue of country A and reduces that of country B. Consequently, country A sets a higher inflation target due to higher benefit from inflation, while country B chooses a lower inflation target. In other words, a country' temptation to inflate decreases with the uncertainty of its monetary policy (relative to the other country), conforming with the result of Proposition 8. This can also be seen in the right panel of Fig. 3. As $\gamma_H^A/\overline{\gamma}^A$ decreases, the areas with at least one currency circulating internationally shifts to the right/ downward, leading to higher $\overline{\gamma}^A$ and lower $\overline{\gamma}^B$.

When country-A buyers go abroad more frequently, they increase their holding of currency B, which raises the seigniorage revenue of country B. With less country-A buyers staying at home, country-B buyers that enter the country-A DM has a

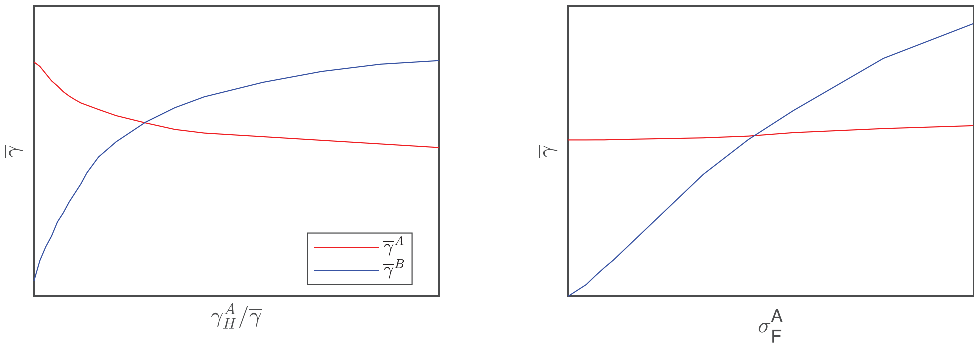

Fig. 8. Nash equilibrium in asymmetric equilibria.

higher probability of meeting with sellers. Thus they demand more currency-A, increasing the seigniorage revenue of country A. As a result, both $\bar{\gamma}^A$ and $\bar{\gamma}^B$ increases with σ_F^A since higher seigniorage revenue drives up the temptation to inflate for both countries. This result echoes with the left panel of Fig. 4, where the areas with at least one currency circulating internationally shifts to the middle as σ_F^A increases. This is complementary to the result of Liu and Shi (2010)[11]. The opening up of single country could push up the equilibrium inflation targets worldwide.

5.3. Discussion

Analysis of the monetary policy game provides some new insights on the international impact of (permanent) monetary expansion. In the New Open Economy Macroeconomics literature, the impact depends on the currency of export pricing and the cross-country substitutability.[12] By paying attention to the microfoundation of international trade and currency exchanges, this model identifies another set of factors that may affect the international impacts of monetary expansions, such as the uncertainty of monetary policy, the degree of economic openness, and the current monetary stances in the two countries. For instance, if the current inflation target is below the optimal $\bar{\gamma}^A$, given the monetary policy of country B, monetary expansion would be welfare-improving. On the other hand, if the current inflation target is higher than the optimal $\bar{\gamma}^A$, then monetary expansion is a beggar-thyself policy.

6. Conclusion

In this paper, I study an international search-theoretic model in which nominal exchange rate is determined by asymmetric information. Agents are free to choose any currency as medium of exchange and various currency regimes emerge endogenously. With the presence of private information, currency holdings not only provide medium of exchange and thus enable buyers to benefit from the gain in DM trades, but also signal the high value of domestic currency in foreign meetings and thus secure better terms of trade. The degree of information asymmetry and economic openness have ambiguous effect on the nominal exchange rate due to the variety of currency regimes. They also affect the trade-off faced by monetary authorities when they decide on the inflation targets. Low uncertainty in monetary policies would result in low inflation targets for both countries, while economic integration leads to inflationary pressures worldwide.

This paper provides a new theoretic framework on the determination of nominal exchange rate and offers a new perspective to understand the impacts of monetary policy uncertainty and economic openness on exchange rate and inflation. For future research, it would be interesting to study the impact of immigration policy on nominal exchange rate and currency regimes by allowing buyers to migrate to the other country. Capital and heterogeneous productivity may also be introduced in the CM to facilitate the comparison with standard macroeconomic models of migration.

Data availability

No data was used for the research described in the article.

[11] Liu and Shi (2010) suppose both countries increase their probability of going abroad simultaneously.

[12] Obstfeld and Rogoff (1995) show that monetary expansion increases domestic and foreign welfare equally, in cases where the cross-country substitutability of goods is high, and export prices are set in the producer's currency (PCP). On the other hand, Corsetti and Pesenti (2001) and Tille (2001) find that if the cross-country substitutability is low, then monetary expansion is a beggar-thyself policy that reduces domestic welfare and increases foreign welfare. Betts and Devereux (2000) show that if export prices are set in the local currency of the consumer (LCP), a monetary expansion is a beggar-thy-neighbour policy.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proofs

Intuitive Criterion. Specifically, let U_H be the surplus of buyer when $\gamma_t^B = \gamma_H^B$ and U_L the surplus of buyer when $\gamma_t^B = \gamma_L^B$ in a proposed equilibrium of the bargaining game. The proposed equilibrium fails the Intuitive Criterion if there is an out-of-equilibrium offer, $(\tilde{q}, \tilde{p_a}, \tilde{p_b})$ and an inflation level $\chi \in H, L$, such that the following is true:

$$u(\tilde{q}) - \tilde{p_a}\phi_t^A - \tilde{p_b}\frac{\phi_{t-1}^B}{\gamma_\chi^B} > U_\chi \tag{A.1}$$

$$u(\tilde{q}) - \tilde{p_a}\phi_t^A - \tilde{p_b}\frac{\phi_{t-1}^B}{\gamma_{-\chi}^B} < U_{-\chi} \tag{A.2}$$

$$-c(\tilde{q}) + \tilde{p_a}\phi_t^A + \tilde{p_b}\frac{\phi_{t-1}^B}{\gamma_\chi^B} \geqslant 0 \tag{A.3}$$

where $\{-\chi\} = \{L, H\}/\chi$. That is, the out-of-equilibrium offer $(\tilde{q}, \tilde{p_a}, \tilde{p_b})$ would make the buyer strictly better off when $\gamma_t^B = \gamma_\chi^B$ and strictly worse off when $\gamma_t^B = \gamma_{-\chi}^B$, if it were accepted. And the offer is acceptable provided that the seller believes $\gamma_t^B = \gamma_\chi^B$.

Proof of Lemma 1. We first show that any equilibrium with a pooling offer $(\overline{q}, \overline{p_a}, \overline{p_b})$ such that $p_b > 0$ can be dismissed by the Intuitive Criterion. It consists in constructing a non-empty set of feasible offers distinct from $(\overline{q}, \overline{p_a}, \overline{p_b})$ that satisfy the requirements in (A.1)–(A.3) for $\chi = L$.

The buyers' payoffs at the proposed equilibrium are $U_H = u(\overline{q}) - \overline{p_a}\phi_t^A - \overline{p_b}\frac{\phi_{t-1}^B}{\gamma_H^B} \geqslant 0$ and $U_L = u(\overline{q}) - \overline{p_a}\phi_t^A - \overline{p_b}\frac{\phi_{t-1}^B}{\gamma_L^B} \geqslant 0$. A necessary condition for $(\overline{q}, \overline{p_a}, \overline{p_b})$ to be acceptable when $\lambda(\overline{q}, \overline{p_a}, \overline{p_b}) < 1$ is $-c(\overline{q}) + \overline{p_a}\phi_t^A + \overline{p_b}\frac{\phi_{t-1}^B}{\gamma_L^B} > 0$. Denote $\mathcal{O}_1$ the set of feasible offers that generate a strictly positive surplus to sellers when country-B experiences a low inflation,

$$\mathcal{O}_1 \equiv \left\{(q, p_a, p_b) \in \mathcal{F} : -c(q) + p_a\phi_t^A + p_b\frac{\phi_{t-1}^B}{\gamma_L^B} > 0\right\}.$$

Then $\mathcal{O}_1$ is open in $\mathcal{F}$, and it contains $(\overline{q}, \overline{p_a}, \overline{p_b})$. Denote $\mathcal{O}_2$ the set of feasible offers that, if they were accepted, would make buyers with $\gamma_t^B = \gamma_L^B$ strictly better-off relative to their equilibrium payoff but would make buyers with $\gamma_t^B = \gamma_H^B$ strictly worse-off,

$$\mathcal{O}_2 \equiv \left\{(q, p_a, p_b) \in \mathcal{F} : \frac{\phi_{t-1}^B}{\gamma_H^B}(\overline{p_b} - p_b) < \left[u(\overline{q}) - \overline{p_a}\phi_t^A\right] - \left[u(q) - p_a\phi_t^A\right] < \frac{\phi_{t-1}^B}{\gamma_L^B}(\overline{p_b} - p_b)\right\}.$$

Then $\mathcal{O}_2$ is open in $\mathcal{F}$, it is not empty, and its closure is $\overline{O}_s \ni (\overline{q}, \overline{p_a}, \overline{p_b})$[13]. Consequently, any open ball, $\mathcal{B}((\overline{q}, \overline{p_a}, \overline{p_b}), \epsilon)$, centered at $(\overline{q}, \overline{p_a}, \overline{p_b})$ has a non-empty intersection with $\mathcal{O}_2$. So one can find feasible offers distinct from $(\overline{q}, \overline{p_a}, \overline{p_b})$ that satisfy (A.1) and (A.2). Moreoever, by definition of an open set, there exists a radius $\epsilon > 0$ such that the open ball $\mathcal{B}((\overline{q}, \overline{p_a}, \overline{p_b}), \epsilon) \subset \mathcal{O}_1$. Consequently, there is $(\tilde{q}, \tilde{p_a}, \tilde{p_b}) \in \mathcal{O}_1 \bigcap \mathcal{O}_2$ that satisfies (A.1)–(A.3) with $\chi = L$, and the proposed pooling equilibrium fails the Intuitive Criterion.

Next, we show that among all separating PBE, all but the Pareto-efficient one can be dismissed by the Intuitive Criterion. First, it establishes that the offer by buyers when $\gamma_t^B = \gamma_H^B$ corresponds to their complete-information offer. Second, the offer made by buyers when $\gamma_t^B = \gamma_L^B$ is the least-costly separating offer, i.e., it is the offer preferred by buyers with $\gamma_t^B = \gamma_L^B$ among all separating offers. Third, the proof specifies a belief system consistent with the equilibrium offers.

First part of the proof: Offer by buyers with $\gamma_t^B = \gamma_H^B$. Any offer that satisfies (7) is acceptable since

$$-c(q) + p_a\phi_t^A + p_b\left\{\lambda(q, p_a, p_b)\frac{\phi_{t-1}^B}{\gamma_L^B} + [1 - \lambda(q, p_a, p_b)]\frac{\phi_{t-1}^B}{\gamma_H^B}\right\} \geqslant -c(q) + p_a\phi_t^A + p_b\frac{\phi_{t-1}^B}{\gamma_H^B} \geqslant 0$$

[13] To show that $\mathcal{O}_2$ is not empty, one can construct an element $(y, p_a, p_b) \in \mathcal{F}$ such that $p_a = \overline{p_a}$ and $\frac{\phi_{t-1}^B}{\gamma_H^B} < \frac{u(\overline{q}) - u(q)}{\overline{p_b} - p_b} < \frac{\phi_{t-1}^L}{\gamma_L^B}$where $u(\overline{q}) - u(q) > 0$ and $\overline{p_b} - p_b > 0$ can be made arbitrarily small. To show that $(\overline{q}, \overline{p_a}, \overline{p_b})$ is in the closure of $\mathcal{O}_2$, consider a sequence $\{(q_n, p_{a,n}, p_{b,n})\}_{n=1}^\infty$ such that $p_{a,n} = \overline{p_a}, u(\overline{q}) - u(q_n) > 0, \overline{p_b} - d_{p,n} > 0$ and $\frac{\phi_{t-1}^B}{\gamma_H^B} < \frac{u(\overline{q}) - u(q_n)}{\overline{p_b} - p_{b,n}} < \frac{\phi_{t-1}^B}{\gamma_L^B}$for all $n \in \mathbb{N}$ and $(q_n, p_{b,n}) \to (\overline{q}, \overline{p_b})$. All the terms of the sequence are in $\mathcal{O}_2$ and it converges to $(\overline{q}, \overline{p_a}, \overline{p_b})$.

for all $\lambda(q,p_a,p_b) \in [0,1]$. Consequently, the complete-information payoff for buyers with $\gamma_t^B = \gamma_H^B$, given by the solution to (6)–(8), can always be achieved. A payoff strictly greater than the complete-information payoff is obtained only if (7) is violated, i.e.

$$-c(q) + p_a\phi_t^A + p_b\frac{\phi_{t-1}^B}{\gamma_H^B} < 0. \tag{A.4}$$

But an offer is acceptable if

$$-c(q) + p_a\phi_t^A + p_b\left\{\lambda(q,p_a,p_b)\frac{\phi_{t-1}^B}{\gamma_L^B} + [1-\lambda(q,p_a,p_b)]\frac{\phi_{t-1}^B}{\gamma_H^B}\right\} \geqslant 0. \tag{A.5}$$

From (A.4) and (A.5), $p_b\frac{\phi_{t-1}^B}{\gamma_H^B} < p_b\left\{\lambda(q,p_a,p_b)\frac{\phi_{t-1}^B}{\gamma_L^B} + [1-\lambda(q,p_a,p_b)]\frac{\phi_{t-1}^B}{\gamma_H^B}\right\}$and hence $p_b > 0$ and $\lambda(q,p_a,p_b) > 0$. This pooling offer has been ruled out. *Second part of the proof: Offer by buyers with* $\gamma_t^B = \gamma_L^B$. Suppose there is an equilibrium where buyers with $\gamma_t^B = \gamma_H^B$ achieve its complete information payoff, $U_H = u(q^H) - c(q^H)$, and the expected payoff of buyers with $\gamma_t^B = \gamma_L^B$ is $U_L \in [0,\overline{U})$, where $\overline{U}$ is the payoff associated with the solution to (9)–(12), i.e. it is the highest payoff a buyer with $\gamma_t^B = \gamma_L^B$ can obtain in a separating equilibrium. It will be shown that there is an out-of-equilibrium offer that would make buyers with $\gamma_t^B = \gamma_L^B$ better-off and buyers with $\gamma_t^B = \gamma_H^B$ worse-off. For $\epsilon > 0$, define U^ϵ as

$$\begin{aligned}
U^\epsilon &= \max_{q,p_a,p_b}\left[u(q) - p_a\phi_t^A - p_b\tfrac{\phi_{t-1}^B}{\gamma_L^B}\right] \\
&s.t. -c(q) + p_a\phi_t^A + p_b\tfrac{\phi_{t-1}^B}{\gamma_L^B} \geqslant 0 \\
&u(q) - p_a\phi_t^A - p_b\tfrac{\phi_{t-1}^B}{\gamma_H^B} \leqslant U_H - \epsilon \\
&0 \leqslant p_a \leqslant m_t^a, 0 \leqslant p_b \leqslant m_t^b.
\end{aligned}$$

The set of acceptable and feasible offers is compact, and it is nonempty provided that $\epsilon < U_H$. From the Theorem of the Maximum, U^ϵ is continuous in ϵ, and $lim_{\epsilon\to 0}U^\epsilon = \overline{U}$. Hence, there is an $\epsilon > 0$ such that $U^\epsilon > U_L$. The associated offer satisfies (A.1)–(A.3), i.e. the proposed equilibrium violates the Intuitive Criterion. Finally, the payoff of a buyer with $\gamma_t^B = \gamma_L^B$ in a separating equilibrium cannot be greater than $\overline{U}$ since otherwise either the firm's participation constraint, (10), or the incentive-compatibility constraint, (11), would be violated. Consequently, in any equilibrium $U_L = \overline{U}$, and the offer of buyers with $\gamma_t^B = \gamma_L^B$ is the solution to (9)–(12).

Third part of the proof: Beliefs. A belief system consistent with the offers in Lemma 1 is such that $\lambda(q^L,p_a^L,p_b^L) = 1$ and $\lambda(q^H,p_a^H,p_b^H) = 0$ if $(q^L,p_a^L,p_b^L) \neq (q^H,p_a^H,p_b^H)$, and $\lambda(q^L,p_a^L,p_b^L) = \pi_L^B$ and $\lambda(q^H,p_a^H,p_b^H) = \pi_H^B$ if $(q^L,p_a^L,p_b^L) = (q^H,p_a^H,p_b^H)$ (from Bayes' rule). For out-of-equilibrium offers,

$$\begin{aligned}
\lambda(q,p_a,p_b) &= 1 \text{ if } (3.9) \text{ holds} \\
&= 0 \text{ otherwise.}
\end{aligned}$$

Offers that violate (11) also violates (7), and since they are attributed to buyers with $\gamma_t^B = \gamma_H^B$, they are rejected. □

Proof of Proposition 1. The household's objective function in (9) is continuous, and it is maximized over a non-empty, compact set. Hence, by the Theorem of the Maximum, there is a solution to (9)–(12). If $m_t^b = 0$, then the maximization of (9) subject to (10) gives $q^L = min\left[q^*, c^{-1}\left(p_a\phi_t^A\right)\right] = q^H, p_a^L = c(q^L)/\phi_t^A$, and $p_b^L = 0$. The incentive-compatibility condition (11) implies

$$u(q^L) - c(q^L) \leqslant u(q^H) - c(q^H),$$

which is satisfied. In the following, I focus on the case where $m_t^b > 0$.

In this part, we investigate the conditions under which the constraints (10) and (11) are slack. First, suppose that the incentive-compatibility condition (11) is slack. Then $q^L = min[q^*,\ c^{-1}\left(\phi_t^A m_t^a + \frac{\phi_{t-1}^B}{\gamma_L^B}m_t^b\right)] \geqslant q^H$. Since, from (10), $c(q^L) = \phi_t^A p_a^L + \frac{\phi_{t-1}^B}{\gamma_L^B}p_b^L$, then (11) becomes

$$u(q^L) - c(q^L) + p_b^L\left(\frac{\phi_{t-1}^B}{\gamma_L^B} - \frac{\phi_{t-1}^B}{\gamma_H^B}\right) \leqslant u(q^H) - c(q^H).$$

If $\phi_t^A m_t^a < c(q^*)$, then $p_b^L > 0$ and (11) is violated, which is a contradiction. If $\phi_t^A m_t^a \geqslant c(q^*)$, then $q^H = q^L = q^*$ and the inequality above implies $p_b^L = 0$.

Second, suppose that the seller's participation constraint (10) is slack. Substitute $u(q^L)$ by its expression given by (11) at equality into the objective function (9) to get

$$U_L = \max_{p_b \in [0, m_t^b]} \left[\left(\frac{\phi_{t-1}^B}{\gamma_H^B} - \frac{\phi_{t-1}^B}{\gamma_L^B} \right) p_b + U_H \right] = U_H,$$

and $p_b^L = 0$. But if $p_b^L = 0, U_L \leqslant \max \left[u(q) - \phi_t^A p_a \right]$ subject to $-c(q) + \phi_t^A p_a \geqslant 0$. Hence, $U_L = U_H$ if and only if $\phi_t^A m_t^a \geqslant c(q^*)$. In that case, $q^L = q^*$ and $\phi_t^A p_a^L = c(q^*)$.

Next we show first that the constraint $p_a^L \leqslant m_t^a$ is binding when $\phi_t^A m_t^a < c(q^*)$. If $p_a^L \leqslant m_t^a$is slack, then $q^L = q^*, p_b^L = 0$ and $\phi_t^A p_a^L = c(q^*)$. This solution maximizes (9) subject to (10), and $p_b^L = 0$ guarantees that (11) holds. However, $\phi_t^A p_a^L = c(q^*)$ is in contradiction with $\phi_t^A m_t^a < c(q^*)$.

Since (10) is binding and $p_a^L = m_t^a, p_b^L$ is given by (13). Substitute p_b^L by its expression into (11) at equality to get (14). For all $q^L \in [0, q^H]$ the left side of (14) is strictly increasing. It is nonpositive at $q^L = 0$, and greater than $u(q^H) - c(q^H)$ at $q^L = q^H$ if $c(q^H) > \phi_t^A m_t^a$. This last inequality holds from (6)–(8). Indeed, if $\phi_t^A m_t^a < c(q^*)$, then $c(q^H) = min\left[c(q^*), \phi_t^A m_t^a + \frac{\phi_{t-1}^B}{\gamma_H^B} m_t^b\right] > \phi_t^A m_t^a$since the focus is on the case $m_t^b > 0$. Hence, there is a unique $q^L \in (0, q^H)$ solution to (14). The objective in (9) $u(q^L) - c(q^L) = u(q^H) - c(q^H) - \left(1 - \frac{\gamma_L^B}{\gamma_H^B}\right)\left[c(q^L) - m_t^a \phi_t^A\right]$ is decreasing in q^L for any solution to (14). Hence, the unique solution in $(0, q^H)$ delivers a maximum to the problem (9)–(12). Given a unique q^L, d_b^L is determined by (13). Finally, $c(q^L) = \phi_t^A m_t^a + \frac{\phi_{t-1}^B}{\gamma_L^B} p_b^L < c(q^H) = \phi_t^A p_a^H + \frac{\phi_{t-1}^B}{\gamma_H^B} p_b^H \leqslant \phi_t^A m_t^a + \frac{\phi_{t-1}^B}{\gamma_H^B} m_t^b$ implies $p_b^L < m_t^b$. From (14), $q^L < q^H$ implies $c(q^L) - \phi_t^A m_t^a > 0$ and, from (13), $p_b^L > 0$. □

Proof of Proposition 2. From Proposition 1, if $m_t^a \phi_t^A < c(q^*)$, then q^L is the unique solution in $[0, q^H]$ to (14). Differentiating (14) to obtain

$$\frac{\partial q^L}{\partial \gamma_H^B} = -\frac{\left[\frac{u'(q^H)}{c'(q^H)} - 1\right] m_t^b \frac{\phi_{t-1}^B}{(\gamma_H^B)^2} + p_b^L \frac{\phi_{t-1}^B}{(\gamma_H^B)^2}}{u'(q^L) - \frac{\gamma_L^B}{\gamma_B^H} c'(q^L)} < 0$$

$$\frac{\partial q^L}{\partial \gamma_L^B} = \frac{p_b^L \frac{\phi_{t-1}^B}{\gamma_H^B \gamma_L^B}}{u'(q^L) - \frac{\gamma_L^B}{\gamma_H^B} c'(q^L)} > 0$$

where $q^H = min\left[q^*, c^{-1}\left(m_t^a \phi_t^A + m_t^b \frac{\phi_{t-1}^B}{\gamma_H^B}\right)\right]$ and $p_b^L > 0$ (from Proposition 1 and the assumption $m_t^b > 0$). From (13), we have

$$\frac{\partial p_b^L}{\gamma_H^B} = \frac{c'(q^L)\gamma_L^B}{\phi_{t-1}^B} \frac{\partial q^L}{\partial \gamma_H^B} < 0$$

and

$$\frac{\partial p_b^L}{\partial \gamma_L^B} = \frac{u'(q^L)}{u'(q^L) - \frac{\gamma_L^B}{\gamma_H^B} c'(q^L)} \frac{p_b^L}{\gamma_L^B} > 0$$

Since $\frac{\pi_H^B}{\gamma_H^B} + \frac{\pi_L^B}{\gamma_L^B} = \frac{1}{\gamma^B}$, then

$$\frac{dp_b^L}{d\gamma_H^B} = \frac{\partial p_b^L}{\gamma_H^B} - \frac{\pi_H^B}{\pi_L^B}\left(\frac{\gamma_L^B}{\gamma_H^B}\right)^2 \frac{\partial p_b^L}{\partial \gamma_L^B} < 0.$$

Proof of Proposition 3. From (14), as $\gamma_H^B \to \infty, q^L$ tends to the solution to

$$u(q^L) - m_t^a \phi_t^A = u(q^H) - c(q^H) = u(q^H) - m_t^a \phi_t^A$$

where $q^H = c^{-1}\left(m_t^a \phi_t^A\right)$ when $m_t^a \phi_t^A < c(q^*)$. Consequently, $q^L \to q^H$ and from (13),

$$p_b^L = \left[c(q^L) - m_t^a \phi_t^A\right] \frac{\gamma_L^B}{\phi_{t-1}^B} \to 0$$

Proof of Proposition 4. From Proposition 1, if $m_t^a \phi_t^A < c(q^*)$, then q^L is the unique solution in $[0, q^H]$ to (14). Differentiating (14), we have

$$\frac{\partial q^L}{\partial\left(m_t^a\phi_t^A\right)} = \frac{\frac{u'\left(q^H\right)}{c'\left(q^H\right)} - \frac{\gamma_L^B}{\gamma_H^B}}{u'(q^L) - \frac{\gamma_L^B}{\gamma_H^B}c'(q^L)} > 0.$$

From (13), $\frac{\partial\left(p_b^L\frac{\phi_{t-1}^B}{\gamma_L^B}\right)}{\partial\left(m_t^a\phi_t^A\right)} = c'\left(q^L\right)\frac{\partial q^L}{\partial\left(m_t^a\phi_t^A\right)} - 1$, and hence (15). The assumption $m_t^b > 0$ implies $q^H > q^L$ and $\frac{\partial\left(p_b^L\frac{\phi_{t-1}^B}{\gamma_L^B}\right)}{\partial\left(m_t^a\phi_t^A\right)} < 0$. The expression for $\frac{\partial p_b^L}{\partial m_t^b}$ in (16) can be obtained in a similar way.

Proof of Proposition 5. We first show that the objective function (18) is jointly concave in (z_a^j, z_b^j) so that the first-order conditions of the buyer's problem are necessary and sufficient. Define

$$\begin{aligned}\mathbb{E}^j\left[S\left(z_a^j,z_b^j\right)\right] &= \pi_{HH}\left[\mu^{jA}S^{HHA}\left(z_a^j,z_b^j\right)+\mu^{jB}S^{HHB}\left(z_a^j,z_b^j\right)\right]+\pi_{HL}\left[\mu^{jA}S^{HLA}\left(z_a^j,z_b^j\right)+\mu^{jB}S^{HLB}\left(z_a^j,z_b^j\right)\right]\\ &+\pi_{LH}\left[\mu^{jA}S^{LHA}\left(z_a^j,z_b^j\right)+\mu^{jB}S^{LHB}\left(z_a^j,z_b^j\right)\right]+\pi_{LL}\left[\mu^{jA}S^{LLA}\left(z_a^j,z_b^j\right)+\mu^{jB}S^{LLB}\left(z_a^j,z_b^j\right)\right]\end{aligned}$$

Then the Hessian Matrix associated with (18) is

$$\mathbb{H} = \begin{pmatrix}\mathbb{E}^j\left[S_{aa}\left(z_a^j,z_b^j\right)\right] & \mathbb{E}^j\left[S_{ab}\left(z_a^j,z_b^j\right)\right]\\ \mathbb{E}^j\left[S_{ba}\left(z_a^j,z_b^j\right)\right] & \mathbb{E}^j\left[S_{bb}\left(z_a^j,z_b^j\right)\right]\end{pmatrix}$$

Based on the calculation in Appendix C, $S_{aa}^{HHA} < 0$ for all (z_a, z_b) such that $\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B} < c(q^*)$. Consequently, $\mathbb{E}^j\left[S\left(z_a^j,z_b^j\right)\right] \leqslant 0$, and it is strictly negative when $\frac{z_a^j}{\gamma_H^A} + \frac{z_b^j}{\gamma_H^B} < c(q^*)$. The determinant of the Hessian matrix is

$$|\mathbb{H}| = \mathbb{E}^j\left[S_{aa}\left(z_a^j,z_b^j\right)\right]\mathbb{E}^j\left[S_{bb}^j\left(z_a^j,z_b^j\right)\right] - \mathbb{E}^j\left[S_{ab}^j(z_a,z_b)\right]^2$$

It can be decomposed as

$$|\mathbb{H}| = \Gamma_1+\Gamma_2+\Gamma_3+\Gamma_4+\Gamma_5+\Gamma_6$$

where

$$\begin{aligned}\Gamma_1 &= \left(\pi_{HH}\mu^{jA}\right)^2\left[S_{aa}^{HHA}S_{bb}^{HHA}-\left(S_{ab}^{HHA}\right)^2\right]+\left(\pi_{HH}\mu^{jB}\right)^2\left[S_{aa}^{HHB}S_{bb}^{HHB}-\left(S_{ab}^{HHB}\right)^2\right]+\left(\pi_{LH}\mu^{jA}\right)^2\left[S_{aa}^{LHA}S_{bb}^{LHA}-\left(S_{ab}^{LHA}\right)^2\right]\\ &+\left(\pi_{HL}\mu^{jB}\right)^2\left[S_{aa}^{HLB}S_{bb}^{HLB}-\left(S_{ab}^{HLB}\right)^2\right]=0\end{aligned}$$

$$\begin{aligned}\Gamma_2 &= \left(\pi_{HL}\mu^{jA}\right)^2\left[S_{aa}^{HLA}S_{bb}^{HLA}-\left(S_{ab}^{HLA}\right)^2\right]+\left(\pi_{LH}\mu^{jB}\right)^2\left[S_{aa}^{LHB}S_{bb}^{LHB}-\left(S_{ab}^{LHB}\right)^2\right]+\left(\pi_{LL}\mu^{jA}\right)^2\left[S_{aa}^{LLA}S_{bb}^{LLA}-\left(S_{ab}^{LLA}\right)^2\right]\\ &+\left(\pi_{LL}\mu^{jB}\right)^2\left[S_{aa}^{LLB}S_{bb}^{LLB}-\left(S_{ab}^{LLB}\right)^2\right]\geqslant 0\end{aligned}$$

since each term is non-negative. For instance,

$$\left[S_{aa}^{LLA}S_{bb}^{LLA}-\left(S_{ab}^{LLA}\right)^2\right] = \left(\pi_{LL}\mu^{jA}\right)^2\left(1-\frac{\gamma_L^B}{\gamma_H^B}\right)\Delta^B(q^{LLA})\Delta^{B'}(q^{LLA})S''\left(\frac{z_a^j}{\gamma_L^A}+\frac{z_b^j}{\gamma_H^B}\right)\frac{1}{\gamma_L^A\gamma_H^B}\left(\frac{q_a^{LLA}}{\gamma_H^B}-\frac{q_b^{LLA}}{\gamma_L^A}\right)\geq 0$$

$$\begin{aligned}\Gamma_3 &= (\pi_{HH})^2\mu^{jA}\mu^{jB}\left[S_{aa}^{HHA}S_{bb}^{HHB}+S_{aa}^{HHB}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{HHB}\right]+\pi_{HH}\pi_{LH}\left(\mu^{jA}\right)^2\left[S_{aa}^{HHA}S_{bb}^{LHA}+S_{aa}^{LHA}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{LHA}\right]\\ &+\pi_{HH}\pi_{HL}\mu^{jA}\mu^{jB}\left[S_{aa}^{HHA}S_{bb}^{HLB}+S_{aa}^{HLB}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{HLB}\right]+\pi_{HH}\pi_{LH}\mu^{jA}\mu^{jB}\left[S_{aa}^{HHB}S_{bb}^{LHA}+S_{aa}^{LHA}S_{bb}^{HHB}-2S_{ab}^{HHB}S_{ab}^{LHA}\right]\\ &+\pi_{HH}\pi_{HL}\left(\mu^{jB}\right)^2\left[S_{aa}^{HHB}S_{bb}^{HLB}+S_{aa}^{HLB}S_{bb}^{HHB}-2S_{ab}^{HHB}S_{ab}^{HLB}\right]+\pi_{HL}\pi_{LH}\mu^{jA}\mu^{jB}\left[S_{aa}^{HLB}S_{bb}^{LHA}+S_{aa}^{LHA}S_{bb}^{HLB}-2S_{ab}^{HLB}S_{ab}^{LHA}\right]\geqslant 0\end{aligned}$$

since each term is non-negative. For instance,

$$\left[S_{aa}^{HLB}S_{bb}^{LHA}+S_{aa}^{LHA}S_{bb}^{HLB}-2S_{ab}^{HLB}S_{ab}^{LHA}\right] = \left(\frac{1}{\gamma_L^A\gamma_L^B}-\frac{1}{\gamma_H^A\gamma_H^B}\right)^2 S''\left(\frac{z_a^j}{\gamma_H^A}+\frac{z_b^j}{\gamma_L^B}\right)S''\left(\frac{z_a^j}{\gamma_L^A}+\frac{z_b^j}{\gamma_H^B}\right)\geqslant 0$$

$$\begin{aligned}\Gamma_4 = {} & \pi_{HH}\pi_{HL}\left(\mu^{jA}\right)^2\left[S_{aa}^{HHA}S_{bb}^{HLA}+S_{aa}^{HLA}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{HLA}\right]+\pi_{HH}\pi_{LH}\mu^{jA}\mu^{jB}\left[S_{aa}^{HHA}S_{bb}^{LHB}+S_{aa}^{LHB}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{LHB}\right]\\ & +\pi_{HH}\pi_{LL}\left(\mu^{jA}\right)^2\left[S_{aa}^{HHA}S_{bb}^{LLA}+S_{aa}^{LLA}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{LLA}\right]+\pi_{HH}\pi_{LL}\mu^{jA}\mu^{jB}\left[S_{aa}^{HHA}S_{bb}^{LLB}+S_{aa}^{LLB}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{LLB}\right]\\ & +\pi_{HH}\pi_{HL}\mu^{jA}\mu^{jB}\left[S_{aa}^{HHB}S_{bb}^{HLA}+S_{aa}^{HLA}S_{bb}^{HHB}-2S_{ab}^{HHB}S_{ab}^{HLA}\right]+\pi_{HH}\pi_{LH}\left(\mu^{jB}\right)^2\left[S_{aa}^{HHB}S_{bb}^{LHB}+S_{aa}^{LHB}S_{bb}^{HHB}-2S_{ab}^{HHB}S_{ab}^{LHB}\right]\\ & +\pi_{HH}\pi_{LL}\mu^{jA}\mu^{jB}\left[S_{aa}^{HHB}S_{bb}^{LLA}+S_{aa}^{LLA}S_{bb}^{HHB}-2S_{ab}^{HHB}S_{ab}^{LLA}\right]+\pi_{HH}\pi_{LL}\left(\mu^{jB}\right)^2\left[S_{aa}^{HHB}S_{bb}^{LLB}+S_{aa}^{LLB}S_{bb}^{HHB}-2S_{ab}^{HHB}S_{ab}^{LLB}\right]\geqslant 0\end{aligned}$$

since each term is non-negative. For instance,

$$\begin{aligned}&\left[S_{aa}^{HHA}S_{bb}^{LLA}+S_{aa}^{LLA}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{LLA}\right]\\ &=\Delta^{B'}(q^{LLA})S''\left(\tfrac{z_a^j}{\gamma_H^A}+\tfrac{z_b^j}{\gamma_H^B}\right)\tfrac{1}{\gamma_L^A\gamma_H^B}\left(\tfrac{q_a^{LLA}}{\gamma_H^B}-\tfrac{q_b^{LLA}}{\gamma_H^A}\right)\left[\left(1-\tfrac{\gamma_L^B}{\gamma_H^B}\right)+S'\left(\tfrac{z_a^j}{\gamma_L^A}+\tfrac{z_b^j}{\gamma_H^B}\right)\left(1-\tfrac{\gamma_L^A}{\gamma_H^A}\right)\right]\geqslant 0\end{aligned}$$

$$\begin{aligned}\Gamma_5 = {} & (\pi_{HL})^2\mu^{jA}\mu^{jB}\left[S_{aa}^{HLB}S_{bb}^{HLA}+S_{aa}^{HLA}S_{bb}^{HLB}-2S_{ab}^{HLB}S_{ab}^{HLA}\right]+\pi_{HL}\pi_{LH}\left(\mu^{jB}\right)^2\left[S_{aa}^{HLB}S_{bb}^{LHB}+S_{aa}^{LHB}S_{bb}^{HLB}-2S_{ab}^{HLB}S_{ab}^{LHB}\right]\\ & +\pi_{HL}\pi_{LL}\mu^{jA}\mu^{jB}\left[S_{aa}^{HLB}S_{bb}^{LLA}+S_{aa}^{LLA}S_{bb}^{HLB}-2S_{ab}^{HLB}S_{ab}^{LLA}\right]+\pi_{HL}\pi_{LL}\left(\mu^{jB}\right)^2\left[S_{aa}^{HLB}S_{bb}^{LLB}+S_{aa}^{LLB}S_{bb}^{HLB}-2S_{ab}^{HLB}S_{ab}^{LLB}\right]\\ & +\pi_{HL}\pi_{LH}\left(\mu^{jA}\right)^2\left[S_{aa}^{LHA}S_{bb}^{HLA}+S_{aa}^{HLA}S_{bb}^{LHA}-2S_{ab}^{LHA}S_{ab}^{HLA}\right]+(\pi_{LH})^2\mu^{jA}\mu^{jB}\left[S_{aa}^{LHA}S_{bb}^{LHB}+S_{aa}^{LHB}S_{bb}^{LHA}-2S_{ab}^{LHA}S_{ab}^{LHB}\right]\\ & +\pi_{LH}\pi_{LL}\left(\mu^{jA}\right)^2\left[S_{aa}^{LHA}S_{bb}^{LLA}+S_{aa}^{LLA}S_{bb}^{LHA}-2S_{ab}^{LHA}S_{ab}^{LLA}\right]+\pi_{LH}\pi_{LL}\mu^{jA}\mu^{jB}\left[S_{aa}^{LHA}S_{bb}^{LLB}+S_{aa}^{LLB}S_{bb}^{LHA}-2S_{ab}^{LHA}S_{ab}^{LLB}\right]\geqslant 0\end{aligned}$$

since each term is non-negative. For instance,

$$\begin{aligned}&\left[S_{aa}^{HLB}S_{bb}^{LLA}+S_{aa}^{LLA}S_{bb}^{HLB}-2S_{ab}^{HLB}S_{ab}^{LLA}\right]=S''\left(\tfrac{z_a^j}{\gamma_H^A}+\tfrac{z_b^j}{\gamma_L^B}\right)S''\left(\tfrac{z_a^j}{\gamma_L^A}+\tfrac{z_b^j}{\gamma_H^B}\right)\Delta^B(q^{LLA})\left(\tfrac{1}{\gamma_L^A\gamma_L^B}-\tfrac{1}{\gamma_H^A\gamma_H^B}\right)^2\\ &+\Delta^{B'}(q^{LLA})S''\left(\tfrac{z_a^j}{\gamma_H^A}+\tfrac{z_b^j}{\gamma_L^B}\right)\tfrac{1}{\gamma_L^A\gamma_L^B}\left(\tfrac{q_a^{LLA}}{\gamma_L^B}-\tfrac{q_b^{LLA}}{\gamma_H^A}\right)\left[\left(1-\tfrac{\gamma_L^B}{\gamma_H^B}\right)+S'\left(\tfrac{z_a^j}{\gamma_L^A}+\tfrac{z_b^j}{\gamma_H^B}\right)\left(1-\tfrac{\gamma_L^A\gamma_L^B}{\gamma_H^A\gamma_H^B}\right)\right]\geqslant 0\end{aligned}$$

$$\begin{aligned}\Gamma_6 = {} & \pi_{HL}\pi_{LH}\mu^{jA}\mu^{jB}\left[S_{aa}^{HLA}S_{bb}^{LHB}+S_{aa}^{LHB}S_{bb}^{HLA}-2S_{ab}^{HLA}S_{ab}^{LHB}\right]+\pi_{HL}\pi_{LL}\left(\mu^{jA}\right)^2\left[S_{aa}^{HLA}S_{bb}^{LLA}+S_{aa}^{LLA}S_{bb}^{HLA}-2S_{ab}^{HLA}S_{ab}^{LLA}\right]\\ & +\pi_{HL}\pi_{LL}\mu^{jA}\mu^{jB}\left[S_{aa}^{HLA}S_{bb}^{LLB}+S_{aa}^{LLB}S_{bb}^{HLA}-2S_{ab}^{HLA}S_{ab}^{LLB}\right]+\pi_{LH}\pi_{LL}\mu^{jA}\mu^{jB}\left[S_{aa}^{LHB}S_{bb}^{LLA}+S_{aa}^{LLA}S_{bb}^{LHB}-2S_{ab}^{LHB}S_{ab}^{LLA}\right]\\ & +\pi_{LH}\pi_{LL}\left(\mu^{jB}\right)^2\left[S_{aa}^{LHB}S_{bb}^{LLB}+S_{aa}^{LLB}S_{bb}^{LHB}-2S_{ab}^{LHB}S_{ab}^{LLB}\right]+(\pi_{LL})^2\mu^{jA}\mu^{jB}\left[S_{aa}^{LLA}S_{bb}^{LLB}+S_{aa}^{LLB}S_{bb}^{LLA}-2S_{ab}^{LLA}S_{ab}^{LLB}\right]\geqslant 0\end{aligned}$$

since each term on the RHS is non-negative. For instance,

$$\begin{aligned}&\left[S_{aa}^{LLA}S_{bb}^{LLB}+S_{aa}^{LLB}S_{bb}^{LLA}-2S_{ab}^{LLA}S_{ab}^{LLB}\right]=\\ &S''\left(\frac{z_a^j}{\gamma_L^A}+\frac{z_b^j}{\gamma_H^B}\right)\left[1-\frac{\gamma_L^A}{\gamma_H^A}+S'\left(\frac{z_a^j}{\gamma_H^A}+\frac{z_b^j}{\gamma_L^B}\right)\left(1-\frac{\gamma_L^A\gamma_L^B}{\gamma_H^A\gamma_H^B}\right)\right]^2\left(\frac{1}{\gamma_L^A\gamma_L^B}\right)^2\Delta^B(q^{LLA})\Delta^{A'}(q^{LLB})\frac{1}{u'(q^{LLB})-\frac{\gamma_L^A}{\gamma_H^A}c'(q^{LLB})}\\ &+S''\left(\frac{z_a^j}{\gamma_H^A}+\frac{z_b^j}{\gamma_L^B}\right)\left[1-\frac{\gamma_L^B}{\gamma_H^B}+S'\left(\frac{z_a^j}{\gamma_H^A}+\frac{z_b^j}{\gamma_H^B}\right)\left(1-\frac{\gamma_L^A\gamma_L^B}{\gamma_H^A\gamma_H^B}\right)\right]^2\left(\frac{1}{\gamma_L^A\gamma_L^B}\right)^2\Delta^A(q^{LLB})\Delta^{B'}(q^{LLA})\frac{1}{u'(q^{LLA})-\frac{\gamma_L^B}{\gamma_H^B}c'(q^{LLA})}\\ &+S'\left(\frac{z_a^j}{\gamma_L^A}+\frac{z_b^j}{\gamma_H^B}\right)S'\left(\frac{z_a^j}{\gamma_H^A}+\frac{z_b^j}{\gamma_L^B}\right)\Delta^A\left(q^{LLB}\right)\Delta^B(q^{LLA})\left(\frac{1}{\gamma_L^A\gamma_L^B}-\frac{1}{\gamma_H^A\gamma_H^B}\right)^2\\ &+\left\{\left[S'\left(\frac{z_a^j}{\gamma_L^A}+\frac{z_b^j}{\gamma_H^B}\right)+1-\frac{\gamma_L^B}{\gamma_H^B}\right]\left[S'\left(\frac{z_a^j}{\gamma_H^A}+\frac{z_b^j}{\gamma_L^B}\right)+1-\frac{\gamma_L^A}{\gamma_H^A}\right]-S'\left(\frac{z_a^j}{\gamma_L^A}+\frac{z_b^j}{\gamma_H^B}\right)S'\left(\frac{z_a^j}{\gamma_H^A}+\frac{z_b^j}{\gamma_L^B}\right)\frac{\gamma_L^A\gamma_L^B}{\gamma_H^A\gamma_H^B}\right\}^2\\ &\cdot\left(\frac{1}{\gamma_L^A\gamma_L^B}\right)^2\Delta^{A'}(q^{LLB})\Delta^{B'}(q^{LLA})\frac{1}{u'(q^{LLB})-\frac{\gamma_L^A}{\gamma_H^A}c'(q^{LLB})}\frac{1}{u'(q^{LLA})-\frac{\gamma_L^B}{\gamma_H^B}c'(q^{LLA})}\\ &\geqslant 0\end{aligned}$$

Hence $|\mathbb{H}|\geqslant 0$. Note that when $\frac{z_a}{\gamma_H^A}+\frac{z_b}{\gamma_H^B}<c(q^*), S''\left(\frac{z_a}{\gamma_H^A}+\frac{z_b}{\gamma_H^B}\right)<0$, thus the first term in Γ_4 is positive:

$$\left[S_{aa}^{HHA}S_{bb}^{HLA}+S_{aa}^{HLA}S_{bb}^{HHA}-2S_{ab}^{HHA}S_{ab}^{HLA}\right]=\Delta^{B'}(q^{HLA})S''\left(\frac{z_a^j}{\gamma_H^A}+\frac{z_b^j}{\gamma_H^B}\right)\frac{1}{\gamma_H^A\gamma_H^B}\left(\frac{q_a^{HLA}}{\gamma_H^B}-\frac{q_b^{HLA}}{\gamma_H^A}\right)\left(1-\frac{\gamma_L^B}{\gamma_H^B}\right)>0$$

Consequently $|\mathbb{H}|>0$ when $\frac{z_a}{\gamma_H^A}+\frac{z_b}{\gamma_H^B}<c(q^*)$. The first-order conditions in (21) and (22) are necessary and sufficient for an optimum to the buyer's problem.

Next, we show that when $\overline{\gamma}^A > \beta$ and $\overline{\gamma}^B > \beta$, there is a unique solution to (18). First note that the solution to (18) is such that $\frac{z_a^j}{\gamma_H^A} + \frac{z_b^j}{\gamma_H^B} \leqslant c(q^*)$. Suppose $\frac{z_a^j}{\gamma_H^A} + \frac{z_b^j}{\gamma_H^B} > c(q^*)$, then $\mathbb{E}^j\left[S_a\left(z_a^j, z_b^j\right)\right] = \mathbb{E}^j\left[S_b\left(z_a^j, z_b^j\right)\right] = 0$, which implies $z_a^j = z_b^j = 0$. But $\mathbb{E}^j[S_a(0,0)] > 0$ and $\mathbb{E}^j[S_b(0,0)] > 0$. A contradiction.

So one can restrict $\left(z_a^j, z_b^j\right)$ to the compact set $\left\{\left(z_a^j, z_b^j\right) \in \mathbb{R}^{2+} : \frac{z_a^j}{\gamma_H^A} + \frac{z_b^j}{\gamma_H^B} \leqslant c(q^*)\right\}$ and, from the Theorem of the Maximum, a solution to (18) exists, and it satisfies the first-order conditions (21)–(22). Since $\mathbb{H}$ is negative definite for all $\left(z_a^j, z_b^j\right)$ such that $\frac{z_a^j}{\gamma_H^A} + \frac{z_b^j}{\gamma_H^B} < c(q^*)$, i.e., the leading principal minors of $\mathbb{H}$ alternate in sign with the first one being negative, the solution to (18) is unique.

When $\overline{\gamma}^A = \beta$, from (21), $\mathbb{E}^j\left[S_a\left(z_a^j, z_b^j\right)\right] = 0$ implying that $\frac{z_a^j}{\gamma_H^A} + \frac{z_b^j}{\gamma_H^B} \geqslant c(q^*)$. When $\overline{\gamma}^B = \beta$, the same conclusion follows. □

Proof of Lemma 2.

When the two countries are symmetric, $\sigma_F^A = \sigma_F^B = \sigma_F$,

$$\mu^{AA} = \mu^{BB} = \frac{1}{4}(1 - \sigma_F) \text{and} \mu^{AB} = \mu^{BA} = \frac{1}{4}\sigma_F$$

According to Appendix C, we have

$$\begin{aligned}
&S_a^{HHA} = S_b^{HHA}, S_a^{LLA} > S_b^{LLA}, S_a^{HLA} > S_b^{HLA}, S_a^{LHA} > S_b^{LHA}\\
&S_a^{HHB} = S_b^{HHB}, S_a^{LLB} < S_b^{LLB}, S_a^{HLB} < S_b^{HLB}, S_a^{LHB} < S_b^{LHB},\\
&S_{aa}^{HHA} = S_{ab}^{HHA} = S_{bb}^{HHA}, S_{aa}^{HLA} > S_{ab}^{HLA} > S_{bb}^{HLA}, S_{aa}^{LHA} > S_{ab}^{LHA} > S_{bb}^{LHA}, S_{aa}^{LLA} > S_{ab}^{LLA} > S_{bb}^{LLA}\\
&S_{aa}^{HHB} = S_{ab}^{HHB} = S_{bb}^{HHB}, S_{aa}^{HLB} < S_{ab}^{HLB} < S_{bb}^{HLB}, S_{aa}^{LHB} < S_{ab}^{LHB} < S_{bb}^{LHB}, S_{aa}^{LLB} < S_{ab}^{LLB} < S_{bb}^{LLB}
\end{aligned}$$

Hence, $S_a^{XXA} = S_b^{XXB} > S_a^{XXB} = S_b^{XXA}$ and $S_{aa}^{XXA} = S_{bb}^{XXB} > S_{ab}^{XXA} = S_{ab}^{XXB} > S_{bb}^{XXA} = S_{aa}^{XXB}$ in a symmetric equilibrium, where the superscript XXj represents expectation over the four possible states of the world, when the meeting happens in country j. For instance,

$$S_a^{XXj} = S_a^{HHj}\pi_{HH} + S_a^{HLj}\pi_{HL} + S_a^{LHj}\pi_{LH} + S_a^{LLj}\pi_{LL}.$$

The exchange rate in a stationary equilibrium is given by

$$e_t = \frac{\phi_t^B}{\phi_t^A} = \frac{Z^B}{Z^A}\frac{M_{t+1}^A}{M_{t+1}^B} = \frac{z_b^A + z_b^B N}{z_a^A + z_a^B N}\frac{M_{t+1}^A}{M_{t+1}^B}$$

Since M_{t+1}^A/M_{t+1}^B is determined exogenously, we only focus on the value of the first term in the proof. In addition, we list properties of meeting probabilities μ^{ij} here, which will be useful later.

$$\begin{aligned}
\mu^{AA} &= \left(1 - \sigma_F^A\right)\tfrac{\mathscr{M}^A}{\mathscr{B}^A} = \tfrac{1-\sigma_F^A}{1-\sigma_F^A+N\sigma_F^B+1}\\
\mu^{AB} &= \sigma_A^F\tfrac{\mathscr{M}^B}{\mathscr{B}^B} = \tfrac{\sigma_F^A}{1-\sigma_F^B+\frac{1}{N}\sigma_F^A+1}\\
\mu^{BA} &= \sigma_F^B\tfrac{\mathscr{M}^A}{\mathscr{B}^A} = \tfrac{\sigma_F^B}{1-\sigma_F^A+N\sigma_F^B+1}\\
\mu^{BB} &= \left(1 - \sigma_B^F\right)\tfrac{\mathscr{M}^B}{\mathscr{B}^B} = \tfrac{1-\sigma_F^B}{1-\sigma_F^B+\frac{1}{N}\sigma_F^A+1}
\end{aligned}$$

$$\partial\begin{pmatrix}\mu^{AA}\\ \mu^{AB}\end{pmatrix}/\partial N = \begin{pmatrix}-\frac{\left(1-\sigma_F^A\right)\sigma_F^B}{\left(1-\sigma_F^A+N\sigma_F^B+1\right)^2}\\ \frac{\left(\sigma_F^A\right)^2}{\left(1-\sigma_F^B+\frac{1}{N}\sigma_F^A+1\right)^2}\frac{1}{N^2}\end{pmatrix} \underset{=}{symmetric} \tfrac{\sigma_F}{4}\begin{pmatrix}\sigma_F - 1\\ \sigma_F\end{pmatrix}$$

$$\partial\begin{pmatrix}\mu^{BA}\\ \mu^{BB}\end{pmatrix}/\partial N = \begin{pmatrix}-\frac{\left(\sigma_F^B\right)^2}{\left(1-\sigma_F^A+N\sigma_F^B+1\right)^2}\\ \frac{\left(1-\sigma_F^B\right)\sigma_F^A}{\left(1-\sigma_F^B+\frac{1}{N}\sigma_F^A+1\right)^2}\frac{1}{N^2}\end{pmatrix} \underset{=}{symmetric} \tfrac{\sigma_F}{4}\begin{pmatrix}-\sigma_F\\ 1 - \sigma_F\end{pmatrix}$$

$$\partial\begin{pmatrix}\mu^{AA}\\ \mu^{AB}\end{pmatrix}/\partial\sigma_F^A = \begin{pmatrix}-\frac{1}{\left(1+\frac{N\sigma_F^B+1}{1-\sigma_F^A}\right)^2}\frac{N\sigma_F^B+1}{\left(1-\sigma_F^A\right)^2}\\ \frac{1}{\left(\frac{2-\sigma_F^B}{\sigma_F^A}+\frac{1}{N}\right)^2}\frac{2-\sigma_F^B}{\left(\sigma_F^A\right)^2}\end{pmatrix} \underset{=}{symmetric} \tfrac{1}{4}\begin{pmatrix}-1 - \sigma_F\\ 2 - \sigma_F\end{pmatrix}$$

$$\partial\begin{pmatrix}\mu^{BA}\\ \mu^{BB}\end{pmatrix}/\partial\sigma_F^A = \begin{pmatrix}\frac{\sigma_F^B}{\left(1-\sigma_F^A+N\sigma_F^B+1\right)^2}\\ -\frac{1-\sigma_F^B}{\left(1-\sigma_F^B+\frac{1}{N}\sigma_F^A+1\right)^2}\frac{1}{N}\end{pmatrix} \underset{=}{symmetric} \tfrac{1}{4}\begin{pmatrix}\sigma_F\\ -1 + \sigma_F\end{pmatrix}$$

1) and 2) are obvious by asymmetry.

3) Consider a buyer from country A, (z_a^A, z_b^A) satisfies

$$\left(\frac{1}{\beta}-\frac{1}{\bar{\gamma}^A}\right) \geqslant \mathbb{E}^A\left[S_a\left(z_a^A, z_b^A\right)\right] = \frac{1}{2}\left(1-\sigma_F^A\right)S_a^{XXA}\left(z_a^A, z_b^A\right) + \frac{1}{2}\sigma_F^A S_a^{XXB}\left(z_a^A, z_b^A\right)$$
$$\left(\frac{1}{\beta}-\frac{1}{\bar{\gamma}^B}\right) \geqslant \mathbb{E}^A\left[S_b\left(z_a^A, z_b^A\right)\right] = \frac{1}{2}\left(1-\sigma_F^A\right)S_b^{XXA}\left(z_a^A, z_b^A\right) + \frac{1}{2}\sigma_F^A S_b^{XXB}\left(z_a^A, z_b^A\right)$$

When $\sigma_F = \frac{1}{2}, z_a^A = z_b^A > 0$ in any monetary equilibrium. By continuity, the above first-order conditions hold with equalities in a neighborhood of $\sigma_F^A = \sigma_F^B = \frac{1}{2}$.

Full differentiation of the FOCs yields

$$\Delta\mu^{jA}S_a^{XXA} + \Delta\mu^{jB}S_a^{XXB} + \mathbb{E}^j[S_{aa}]\Delta z_a^j + \mathbb{E}^j[S_{ab}]\Delta z_b^j = 0$$
$$\Delta\mu^{jA}S_b^{XXA} + \Delta\mu^{jB}S_b^{XXB} + \mathbb{E}^j[S_{ba}]\Delta z_a^j + \mathbb{E}^j[S_{bb}]\Delta z_b^j = 0$$

Solving for $\left(\Delta z_a^j, \Delta z_b^j\right)$ leads to

$$\begin{pmatrix}\Delta z_a^j \\ \Delta z_b^j\end{pmatrix} = -\frac{1}{|\mathbb{H}|}\begin{pmatrix}\mathbb{E}^j[S_{bb}] & -\mathbb{E}^j[S_{ab}] \\ -\mathbb{E}^j[S_{ba}] & \mathbb{E}^j[S_{aa}]\end{pmatrix}\begin{pmatrix}S_a^{XXA} & S_a^{XXB} \\ S_b^{XXA} & S_b^{XXB}\end{pmatrix}\begin{pmatrix}\Delta\mu^{jA} \\ \Delta\mu^{jB}\end{pmatrix}$$

where $|\mathbb{H}|$ is the determinant of the Hessian matrix.

When $\sigma_F = \frac{1}{2}$,

$$\mathbb{E}^A[S_{aa}] = \mathbb{E}^B[S_{aa}] = \frac{1}{4}S_{aa}^{XXA} + \frac{1}{4}S_{aa}^{XXB}$$
$$\mathbb{E}^A[S_{ab}] = \mathbb{E}^B[S_{ab}] = \frac{1}{4}S_{ab}^{XXA} + \frac{1}{4}S_{ab}^{XXB} = \frac{1}{2}S_{ab}^{XXA}$$
$$\mathbb{E}^A[S_{bb}] = \mathbb{E}^B[S_{bb}] = \frac{1}{4}S_{bb}^{XXA} + \frac{1}{4}S_{bb}^{XXB}$$

thus we have

$$\mathbb{E}^j[S_{aa}] = \mathbb{E}^j[S_{bb}] > \mathbb{E}^j[S_{ab}]$$

$$\begin{pmatrix}\mathbb{E}^j[S_{bb}] & -\mathbb{E}^j[S_{ab}] \\ -\mathbb{E}^j[S_{ba}] & \mathbb{E}^j[S_{aa}]\end{pmatrix}\begin{pmatrix}S_a^{XXA} & S_a^{XXB} \\ S_b^{XXA} & S_b^{XXB}\end{pmatrix} = \begin{pmatrix}\mathbb{E}^j[S_{bb}]S_a^{XXA} - \mathbb{E}^j[S_{ab}]S_b^{XXA} & \mathbb{E}^j[S_{bb}]S_a^{XXB} - \mathbb{E}^j[S_{ab}]S_b^{XXB} \\ \mathbb{E}^j[S_{aa}]S_b^{XXA} - \mathbb{E}^j[S_{ab}]S_a^{XXA} & \mathbb{E}^j[S_{aa}]S_b^{XXB} - \mathbb{E}^j[S_{ab}]S_a^{XXB}\end{pmatrix}$$
$$= \begin{pmatrix}\mathbb{E}^j[S_{aa}]S_a^{XXA} - \mathbb{E}^j[S_{ab}]S_b^{XXA} & \mathbb{E}^j[S_{aa}]S_b^{XXA} - \mathbb{E}^j[S_{ab}]S_a^{XXA} \\ \mathbb{E}^j[S_{aa}]S_b^{XXA} - \mathbb{E}^j[S_{ab}]S_a^{XXA} & \mathbb{E}^j[S_{aa}]S_a^{XXA} - \mathbb{E}^j[S_{ab}]S_b^{XXA}\end{pmatrix}$$

Note that

$$\partial\begin{pmatrix}\mu^{AA} \\ \mu^{AB}\end{pmatrix}/\partial N = \partial\begin{pmatrix}\mu^{BA} \\ \mu^{BB}\end{pmatrix}/\partial N = \frac{1}{16}\begin{pmatrix}-1 \\ 1\end{pmatrix}$$
$$\partial\begin{pmatrix}\mu^{AA} \\ \mu^{AB}\end{pmatrix}/\partial\sigma_F^A = \frac{3}{8}\begin{pmatrix}-1 \\ 1\end{pmatrix}, \partial\begin{pmatrix}\mu^{BA} \\ \mu^{BB}\end{pmatrix}/\partial\sigma_F^A = \frac{1}{8}\begin{pmatrix}1 \\ -1\end{pmatrix}$$

Therefore,

$$\partial\begin{pmatrix}z_a^j \\ z_b^j\end{pmatrix}/\partial N = -\frac{1}{16|\mathbb{H}|}\left(\mathbb{E}^j[S_{aa}] + \mathbb{E}^j[S_{ab}]\right)\left(S_a^{XXA} - S_b^{XXA}\right)\begin{pmatrix}-1 \\ 1\end{pmatrix}$$
$$\partial\begin{pmatrix}z_a^A \\ z_b^A\end{pmatrix}/\partial\sigma_F^A = -\frac{3}{8|\mathbb{H}|}\left(\mathbb{E}^j[S_{aa}] + \mathbb{E}^j[S_{ab}]\right)\left(S_a^{XXA} - S_b^{XXA}\right)\begin{pmatrix}-1 \\ 1\end{pmatrix}$$
$$\partial\begin{pmatrix}z_a^B \\ z_b^B\end{pmatrix}/\partial\sigma_F^A = -\frac{1}{8|\mathbb{H}|}\left(\mathbb{E}^j[S_{aa}] + \mathbb{E}^j[S_{ab}]\right)\left(S_a^{XXA} - S_b^{XXA}\right)\begin{pmatrix}1 \\ -1\end{pmatrix}$$

Accordingly,

$$\frac{\partial e_t}{\partial N} = \frac{\left(\frac{\partial z_b^A}{\partial N} + \frac{\partial z_b^B}{\partial N}\right) - \left(\frac{\partial z_a^A}{\partial N} + \frac{\partial z_a^B}{\partial N}\right)}{z_a^A + z_a^B}\frac{M_{t+1}^A}{M_{t+1}^B} > 0$$
$$\frac{\partial e_t}{\partial\sigma_F^A} = \frac{\left(\frac{\partial z_b^A}{\partial\sigma_F^A} + \frac{\partial z_b^B}{\partial\sigma_F^A}\right) - \left(\frac{\partial z_a^A}{\partial\sigma_F^A} + \frac{\partial z_a^B}{\partial\sigma_F^A}\right)}{z_a^A + z_a^B}\frac{M_{t+1}^A}{M_{t+1}^B} > 0$$

By continuity, these results hold in a neighborhood of $\sigma_F^A = \sigma_F^B = \frac{1}{2}$.

4) When $\sigma_F = 0$, we have

$$\left(\frac{1}{\beta} - \frac{1}{\overline{\gamma}^A}\right) = \frac{1}{2} S_a^{XXA}\left(z_a^A, z_b^A\right)$$
$$\left(\frac{1}{\beta} - \frac{1}{\overline{\gamma}^B}\right) > \frac{1}{2} S_b^{XXA}\left(z_a^A, z_b^A\right)$$

for a country-A buyer, since $S_a^{XXA} > S_b^{XXA}$. This means $z_a^A > 0$ and $z_b^A = 0$. By continuity, $z_a^A = z_b^B > 0$ and $z_b^A = z_a^B = 0$ in a neighbourhood of $\sigma_F^A = \sigma_F^B = 0$.

Full differentiation of the FOCs yields

$$\Delta \mu^{AA} S_a^{XXA}\left(z_a^A, 0\right) + \Delta \mu^{AB} S_a^{XXB}\left(z_a^A, 0\right) + \mathbb{E}^A\left[S_{aa}\left(z_a^A, 0\right)\right] \Delta z_a^A = 0$$
$$\Delta \mu^{BA} S_b^{XXA}\left(0, z_b^B\right) + \Delta \mu^{BB} S_b^{XXB}\left(0, z_b^B\right) + \mathbb{E}^B\left[S_{bb}\left(0, z_b^B\right)\right] \Delta z_b^B = 0$$

Hence,

$$\frac{\partial e_t}{\partial N} = \left(1 + \frac{\frac{\partial z_b^B}{\partial N} - \frac{\partial z_a^A}{\partial N}}{z_a^A}\right) \frac{M_{t+1}^A}{M_{t+1}^B} > 0 \text{when } \sigma^F \text{ is sufficiently small}$$

since

$$\frac{\partial z_a^A}{\partial N} = \frac{1}{\mathbb{E}^A\left[S_{aa}\left(z_a^A, 0\right)\right]} \left[(1 - \sigma_F) S_a^{XXA}\left(z_a^A, 0\right) - \sigma_F S_a^{XXB}\left(z_a^A, 0\right)\right] \frac{\sigma_F}{4}$$
$$\frac{\partial z_b^B}{\partial N} = \frac{1}{\mathbb{E}^B\left[S_{bb}\left(0, z_b^B\right)\right]} \left[\sigma_F S_b^{XXA}\left(0, z_b^B\right) - (1 - \sigma_F) S_b^{XXB}\left(0, z_b^B\right)\right] \frac{\sigma_F}{4}$$

In addition, we have

$$\frac{\partial e_t}{\partial \sigma_F^A} = \frac{-\sigma_F S_a^{XXA}\left(z_a^A, 0\right) + (1 - \sigma_F) S_a^{XXB}\left(z_a^A, 0\right)}{2\mathbb{E}^B\left[S_{bb}\left(0, z_b^B\right)\right] z_a^A} \frac{M_{t+1}^A}{M_{t+1}^B} < 0 \quad \text{when } \sigma^F \text{ is sufficiently small}$$

since

$$\frac{\partial z_a^A}{\partial \sigma_F^A} = \frac{1}{4\mathbb{E}^A\left[S_{aa}\left(z_a^A, 0\right)\right]} \left[(1 + \sigma_F) S_a^{XXA}\left(z_a^A, 0\right) - (2 - \sigma_F) S_a^{XXB}\left(z_a^A, 0\right)\right]$$
$$\frac{\partial z_b^B}{\partial \sigma_F^A} = \frac{1}{4\mathbb{E}^B\left[S_{bb}\left(0, z_b^B\right)\right]} \left[-\sigma_F S_b^{XXA}\left(0, z_b^B\right) + (1 - \sigma_F) S_b^{XXB}\left(0, z_b^B\right)\right]$$

□

Proof of Proposition 6.

$$e_t = \frac{\phi_t^B}{\phi_t^A} = \frac{Z^B}{Z^A} \frac{M_{t+1}^A}{M_{t+1}^B} = \frac{Z^B}{Z^A} \frac{M_t^A}{M_t^B} \frac{\gamma_t^A}{\gamma_t^B} = \left(\frac{Z^B}{Z^A} \frac{\overline{\gamma}^A}{\overline{\gamma}^B}\right) \frac{M_t^A}{M_t^B} \left(\frac{\gamma_t^A}{\overline{\gamma}^A} \frac{\overline{\gamma}^B}{\gamma_t^B}\right)$$

M_t^A/M_t^B is predetermined at the end of period $t-1$. $\gamma_t^A/\overline{\gamma}^A$ and $\gamma_t^B/\overline{\gamma}^B$ are stochastic and depend on the spread of inflation distributions. Therefore, we focus on the first term, which is determined endogenously.

Let $d_a = z_a/\overline{\gamma}^A$ and $d_b = z_b/\overline{\gamma}^B$. Then the FOCs become

$$-\left(\frac{\overline{\gamma}^A}{\beta} - 1\right) + \mathbb{E}^j\left[S_a\left(d_a^j, d_b^j\right)\right] \leqslant 0, = \text{ if } \ d_a^j > 0$$
$$-\left(\frac{\overline{\gamma}^B}{\beta} - 1\right) + \mathbb{E}^j\left[S_b\left(d_a^j, d_b^j\right)\right] \leqslant 0, = \text{ if } \ d_b^j > 0$$

and

$$\frac{Z^B \overline{\gamma}^A}{Z^A \overline{\gamma}^B} = \frac{d_b^A + d_b^B N}{d_a^A + d_a^B N}$$

First, suppose $d_a^j > 0$ and $d_b^j > 0$. Both FOCs hold with equality. Full differentiation yields

$$\begin{pmatrix} \mathbb{E}^j\left[S_{aa}\left(d_a^j, d_b^j\right)\right] & \mathbb{E}^j\left[S_{ab}\left(d_a^j, d_b^j\right)\right] \\ \mathbb{E}^j\left[S_{ba}\left(d_a^j, d_b^j\right)\right] & \mathbb{E}^j\left[S_{bb}\left(d_a^j, d_b^j\right)\right] \end{pmatrix} \begin{pmatrix} \Delta d_a^j \\ \Delta d_b^j \end{pmatrix} = \begin{pmatrix} \Delta \overline{\gamma}^A \\ \Delta \overline{\gamma}^B \end{pmatrix} \frac{1}{\beta}$$

Hence

$$\begin{pmatrix} \Delta d_a^j \\ \Delta d_b^j \end{pmatrix} = \frac{1}{\beta|\mathbb{H}|} \begin{pmatrix} \mathbb{E}^A\left[S_{bb}\left(d_a^A, d_b^A\right)\right] & -\mathbb{E}^A\left[S_{ab}\left(d_a^A, d_b^A\right)\right] \\ -\mathbb{E}^A\left[S_{ba}\left(d_a^A, d_b^A\right)\right] & \mathbb{E}^A\left[S_{aa}\left(d_a^A, d_b^A\right)\right] \end{pmatrix} \begin{pmatrix} \Delta \overline{\gamma}^A \\ \Delta \overline{\gamma}^B \end{pmatrix}$$

We have

$$\frac{\partial d_a^j}{\partial \overline{\gamma}^A} \leqslant 0, \frac{\partial d_b^j}{\partial \overline{\gamma}^A} \geqslant 0, \frac{\partial d_a^j}{\partial \overline{\gamma}^B} \geqslant 0, \frac{\partial d_b^j}{\partial \overline{\gamma}^B} \leqslant 0$$

where the inequalities hold strictly when $d_a^j \frac{\overline{\gamma}^A}{\gamma_H^A} + d_b^j \frac{\overline{\gamma}^B}{\gamma_H^B} < c(q^*)$.

Next, suppose $d_a^j = 0$ and $d_b^j > 0$, then

$$-\left(\frac{\overline{\gamma}^A}{\beta} - 1\right) + \mathbb{E}^j\left[S_a\left(0, d_b^j\right)\right] < 0$$
$$-\left(\frac{\overline{\gamma}^B}{\beta} - 1\right) + \mathbb{E}^j\left[S_b\left(0, d_b^j\right)\right] = 0$$

We have

$$\frac{\partial d_a^j}{\partial \overline{\gamma}^A} = 0, \frac{\partial d_b^j}{\partial \overline{\gamma}^A} = 0, \frac{\partial d_a^j}{\partial \overline{\gamma}^B} = 0, \frac{\partial d_b^j}{\partial \overline{\gamma}^B} \leqslant 0$$

where the inequality holds strictly when $d_a^j \frac{\overline{\gamma}^A}{\gamma_H^A} + d_b^j \frac{\overline{\gamma}^B}{\gamma_H^B} < c(q^*)$.

Finally, suppose $d_a^j > 0$ and $d_b^j = 0$, then

$$-\left(\frac{\overline{\gamma}^A}{\beta} - 1\right) + \mathbb{E}^j\left[S_a\left(d_a^j, 0\right)\right] = 0$$
$$-\left(\frac{\overline{\gamma}^B}{\beta} - 1\right) + \mathbb{E}^j\left[S_b\left(d_a^j, 0\right)\right] < 0$$

We have

$$\frac{\partial d_a^j}{\partial \overline{\gamma}^A} \leqslant 0, \frac{\partial d_b^j}{\partial \overline{\gamma}^A} = 0, \frac{\partial d_a^j}{\partial \overline{\gamma}^B} = 0, \frac{\partial d_b^j}{\partial \overline{\gamma}^B} = 0$$

where the inequality holds strictly when $d_a^j \frac{\overline{\gamma}^A}{\gamma_H^A} + d_b^j \frac{\overline{\gamma}^B}{\gamma_H^B} < c(q^*)$.

Note that $d_a^j \frac{\overline{\gamma}^A}{\gamma_H^A} + d_b^j \frac{\overline{\gamma}^B}{\gamma_H^B} < c(q^*)$ holds as long as $\overline{\gamma}^A > \beta$ and $\overline{\gamma}^B > \beta$.

Suppose $\overline{\gamma}^A$ increases, then

- region (AB,AB), (A,AB) and (AB,A): $d_b^A + d_b^B N$increases, $d_a^A + d_a^B N$decreases
- region (A,B): $d_b^A + d_b^B N$no change, $d_a^A + d_a^B N$decreases

Therefore $\frac{Z^B \overline{\gamma}^A}{Z^A \overline{\gamma}^B}$ increases with $\overline{\gamma}^A$, regardless of the currency regime. □

Proof of Proposition 7.

$$\mathscr{W} = \mathscr{W}^{\mathscr{A}} + \mathscr{W}^{\mathscr{B}} = \frac{1}{1-\beta}\left\{\mathbb{E}^A\left[S\left(z_a^A, z_b^A\right)\right] + N\mathbb{E}^B\left[S\left(z_a^B, z_b^B\right)\right]\right\}$$

Total welfare $\mathscr{W}$ is maximized at the joint Friedman Rule, since q^* is achieved in all DM meetings. By the Envelope Theorem, the welfare effect of a change in $\overline{\gamma}^j$ in the neighborhood of $\overline{\gamma}^j = \beta$ can be evaluated by totally differentiating the planner's welfare function. Differentiating $\mathscr{W}$ with respect to $\overline{\gamma}^A$ and $\overline{\gamma}^B$ respectively produces $\frac{d\mathscr{W}}{d\overline{\gamma}^A}|_{\overline{\gamma}^A=\beta} = 0$ and $\frac{d\mathscr{W}}{d\overline{\gamma}^A}|_{\overline{\gamma}^A=\beta} = 0$, since $S'(q^*) = 0$. Hence, the social planner that maximizes total welfare for the world has no temptation to inflate. □

Proof of Proposition 8. Country B follows the Friedman Rule, so there is no cost of holding currency B. Buyers from both countries will have a real balance of currency-B such that the relevant constraints are no longer binding and q^* is produced in bilateral meetings. Relevant cases are the ones in which he meets with a country-A seller when country B is in the low inflation state, i.e. HLA and LLA meetings. All other cases result in the optimal output q^*, since there is no cost of holding currency-B.

$$\frac{\partial \mathscr{W}^{\mathscr{A}}}{\partial \overline{\gamma}^A}|_{\overline{\gamma}^B=\beta} = \frac{1}{1-\beta}\left\{\mathbb{E}^A\left[S_a\left(z_a^A, z_b^A\right)\right]\frac{\partial z_a^A}{\partial \overline{\gamma}^A} + N\frac{\partial z_a^B}{\partial \overline{\gamma}^A}\left(1 - \frac{1}{\overline{\gamma}^A}\right) + N z_a^B \frac{1}{\left(\overline{\gamma}^A\right)^2}\right\}$$
$$\frac{\partial \mathscr{W}^{\mathscr{A}}}{\partial \overline{\gamma}^A}|_{\overline{\gamma}^A=\overline{\gamma}^B=\beta} = \frac{N}{1-\beta}\left\{\frac{\partial z_a^B}{\partial \overline{\gamma}^A}\left(1 - \frac{1}{\beta}\right) + z_a^B \frac{1}{\beta^2}\right\}$$

The FOC for a country-B buyer when $\overline{\gamma}^B = \beta$ is

$$\mu^{BA}\left[\pi_{HL}S_a^{HLA}(z_a^B,z_b^B)+\pi_{LL}S_a^{LLA}(z_a^B,z_b^B)\right]=\frac{1}{\beta}-\frac{1}{\overline{\gamma}^A}$$

To derive $\partial z_a^B/\partial\overline{\gamma}^A$, we differentiate the FOC and evaluate at $\overline{\gamma}^A=\overline{\gamma}^B=\beta$:

$$\mu^{BA}\left[\pi_{HL}S_{aa}^{HLA}(z_a^B,z_b^B)+\pi_{LL}S_{aa}^{LLA}(z_a^B,z_b^B)\right]\frac{\partial z_a^B}{\partial\overline{\gamma}^A}=\frac{1}{\beta^2} \tag{A.6}$$

where

$$S_{aa}^{HLA}=\left(1-\frac{\gamma_L^B}{\gamma_H^B}\right)\frac{1}{\gamma_A^H}\Delta^{B'}(q^{HLA})q_a^{HLA}\leqslant 0$$
$$S_{aa}^{LLA}=\left(1-\frac{\gamma_L^B}{\gamma_H^B}\right)\frac{1}{\gamma_A^L}\Delta^{B'}(q^{LLA})q_a^{LLA}\leqslant 0$$

Therefore $\partial z_a^B/\partial\overline{\gamma}^A\leqslant 0$ and $\frac{\partial\mathscr{W}^{\mathscr{A}}}{\partial\overline{\gamma}^A}|_{\overline{\gamma}^A=\overline{\gamma}^B=\beta}>0$.

Note that

$$S_{aa}^{HLA}|_{\overline{\gamma}^A=\overline{\gamma}^B=\beta}=\left(\frac{1}{\gamma_A^H}\right)^2\left(1-\frac{\gamma_L^B}{\gamma_H^B}\right)^2\frac{u''(q^*)-c''(q^*)}{\left[u'(q^*)-\frac{\gamma_L^B}{\gamma_H^B}c'(q^*)\right]^2}$$
$$S_{aa}^{LLA}|_{\overline{\gamma}^A=\overline{\gamma}^B=\beta}=\left(\frac{1}{\gamma_A^L}\right)^2\left(1-\frac{\gamma_L^B}{\gamma_H^B}\right)^2\frac{u''(q^*)-c''(q^*)}{\left[u'(q^*)-\frac{\gamma_L^B}{\gamma_H^B}c'(q^*)\right]^2}$$

then Eq. (A.6) becomes

$$\mu^{BA}\pi_L^B\left(1-\frac{\gamma_L^B}{\gamma_H^B}\right)^2\frac{u''(q^*)-c''(q^*)}{\left[u'(q^*)-\frac{\gamma_L^B}{\gamma_H^B}c'(q^*)\right]^2}\left[\pi_H^A\left(\frac{\overline{\gamma}^A}{\gamma_A^H}\right)^2+\pi_L^A\left(\frac{\overline{\gamma}^A}{\gamma_A^L}\right)^2\right]\frac{\partial z_a^B}{\partial\overline{\gamma}^A}=1$$

where

$$\pi_H^A\left(\frac{\overline{\gamma}^A}{\gamma_A^H}\right)^2+\pi_L^A\left(\frac{\overline{\gamma}^A}{\gamma_A^L}\right)^2=Var\left(\frac{\overline{\gamma}^A}{\gamma_t^A}\right)+1$$

Therefore, when the spread of country-A inflation is larger, the incentive to raise mean inflation $\frac{\partial\mathscr{W}^{\mathscr{A}}}{\partial\overline{\gamma}^A}|_{\overline{\gamma}^A=\overline{\gamma}^B=\beta}$ is smaller. □

Appendix B. Derivations

B.1. Derivation of $\gamma_{max}^{A,j}$ and $\gamma_{max}^{B,j}$

$\gamma_{max}^{A,j}$ and $\gamma_{max}^{B,j}$ satisfy $\frac{1}{\beta}-\frac{1}{\overline{\gamma}^A}=\mathbb{E}^j[S_a(0,0)]$ and $\frac{1}{\beta}-\frac{1}{\overline{\gamma}^B}=\mathbb{E}^j[S_b(0,0)]$.

1. $\gamma_t^A=\gamma_H^A,\gamma_t^B=\gamma_H^B$, the buyer meets with a seller from a country A.

$$S_a^{HHA}=\frac{1}{\gamma_H^A}S'(0),S_b^{HHA}=\frac{1}{\gamma_H^B}S'(0)$$

2. $\gamma_t^A=\gamma_H^A,\gamma_t^B=\gamma_L^B$, the buyer meets with a seller from country A.

$$S_a^{HLA}=\left[S'(0)+1-\frac{\gamma_L^B}{\gamma_H^B}\right]\frac{1}{\gamma_A^H}\Delta^B(0),S_b^{HLA}=S'(0)\frac{1}{\gamma_B^H}\Delta^B(0)$$

3. $\gamma_t^A=\gamma_A^H,\gamma_t^B=\gamma_B^H$, the buyer meets with a seller from country B.
   Same as case 1.
4. $\gamma_t^A=\gamma_A^L,\gamma_t^B=\gamma_B^H$, the buyer meets with a seller from country B.

$$S_a^{LHB}=S'(0)\frac{1}{\gamma_H^A}\Delta^A(0),S_b^{LHB}=\left[S'(0)+1-\frac{\gamma_L^A}{\gamma_H^A}\right]\frac{1}{\gamma_H^B}\Delta^A(0)$$

5. $\gamma_t^A=\gamma_A^L,\gamma_t^B=\gamma_B^H$, the buyer meets with a seller from country A.

$$S_a^{LHA} = \frac{1}{\gamma_A^L} S'(0), S_b^{LHA} = \frac{1}{\gamma_B^H} S'(0)$$

6. $\gamma_t^A = \gamma_A^L, \gamma_t^B = \gamma_B^L$, the buyer meets with a seller from country A.

$$S_a^{LLA} = \left[S'(0) + 1 - \frac{\gamma_L^B}{\gamma_H^B}\right] \frac{1}{\gamma_A^L} \Delta^B((0)), S_b^{LLA} = S'(0) \frac{1}{\gamma_B^H} \Delta^B((0))$$

7. $\gamma_t^a = \gamma_a^H, \gamma_t^b = \gamma_b^L$, the buyer meets with a seller from country B.

$$S_a^{HLB} = \frac{1}{\gamma_A^H} S'(0), S_b^{HLB} = \frac{1}{\gamma_B^L} S'(0)$$

8. $\gamma_t^A = \gamma_A^L, \gamma_t^B = \gamma_B^L$, the buyer meets with a seller from country B.

$$S_a^{LLB} = S'(0) \frac{1}{\gamma_H^A} \Delta^A(0), S_b^{LLB} = \left[S'(0) + 1 - \frac{\gamma_L^A}{\gamma_H^A}\right] \frac{1}{\gamma_L^B} \Delta^A(0)$$

$$\begin{aligned}
&\mathbb{E}^j[S_a(0,0)] = \mu^{jA}\left[S_a^{HHA}(0,0)\pi_{HH} + S_a^{HLA}(0,0)\pi_{HL} + S_a^{LHA}(0,0)\pi_{LH} + S_a^{LLA}(0,0)\pi_{LL}\right] \\
&+\mu^{jB}\left[S_a^{HHB}(0,0)\pi_{HH} + S_a^{HLB}(0,0)\pi_{HL} + S_a^{LHB}(0,0)\pi_{LH} + S_a^{LLB}(0,0)\pi_{LL}\right] \\
&= \left(\frac{u'(0)}{c'(0)} - 1\right) \frac{1}{\bar{\gamma}^A} \left\{ \mu^{jA} + \mu^{jB} \frac{\bar{\gamma}^A}{\gamma_H^A} \left\{ \pi_H^A + \pi_L^A \frac{\frac{u'(0)}{c'(0)} - 1}{\frac{u'(0)}{c'(0)} - \frac{\gamma_L^A}{\gamma_H^A}} \right\} \right\}
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}^j[S_b(0,0)] = \mu^{jA}\left[S_b^{HHA}(0,0)\pi_{HH} + S_b^{HLA}(0,0)\pi_{HL} + S_b^{LHA}(0,0)\pi_{LH} + S_b^{LLA}(0,0)\pi_{LL}\right] \\
&+\mu^{jB}\left[S_b^{HHB}(0,0)\pi_{HH} + S_b^{HLB}(0,0)\pi_{HL} + S_b^{LHB}(0,0)\pi_{LH} + S_b^{LLB}(0,0)\pi_{LL}\right] \\
&= \left(\frac{u'(0)}{c'(0)} - 1\right) \frac{1}{\bar{\gamma}^B} \left[\mu^{jB} + \mu^{jA} \frac{\bar{\gamma}^B}{\gamma_H^B} \left(\pi_H^B + \frac{u'(0) - c'(0)}{u'(0) - \frac{\gamma_L^B}{\gamma_H^B} c'(0)} \pi_L^B \right) \right]
\end{aligned}$$

Rearranging yields

$$\gamma_{max}^{A,j} = \beta \left\{ 1 + \left[\frac{u'(0)}{c'(0)} - 1\right] \left[\mu^{jA} + \mu^{jB} \frac{\bar{\gamma}^A}{\gamma_H^A} \left(\pi_H^A + \pi_L^A \frac{u'(0) - c'(0)}{u'(0) - \frac{\gamma_L^A}{\gamma_H^A} c'(0)} \right) \right] \right\}$$

$$\gamma_{max}^{B,j} = \beta \left\{ 1 + \left[\frac{u'(0)}{c'(0)} - 1\right] \left[\mu^{jB} + \mu^{jA} \frac{\bar{\gamma}^B}{\gamma_H^B} \left(\pi_H^B + \pi_L^B \frac{u'(0) - c'(0)}{u'(0) - \frac{\gamma_L^B}{\gamma_H^B} c'(0)} \right) \right] \right\}$$

Especially, when $u'(0)/c'(0) \to \infty, \gamma_{max}^{A,j} \to \infty$ and $\gamma_{max}^{B,j} \to \infty$. In this case, a monetary stationary equilibrium always exists.

B.2. Derivation of welfare function

Consider country A buyers. The steady-state version of Eq. (17) is

$$\mathbb{E}\left[V^A\right] = \frac{z_a^A}{\bar{\gamma}^A} + \frac{z_b^A}{\bar{\gamma}^B} + \left(1 - \frac{1}{\bar{\gamma}^A}\right) Z^A + W^A(0,0) + \mathbb{E}^A\left[S(z_a^A, z_b^A)\right]$$

where, from (3),

$$W^A(0,0) = -z_a^A - z_b^A + \beta \mathbb{E}\left[V^A\right]$$

Thus

$$\begin{aligned}
\mathbb{E}\left[V^A\right] &= \frac{1}{1-\beta}\left\{ \left(1 - \frac{1}{\bar{\gamma}^A}\right)\left(Z^A - z_a^A\right) - \left(1 - \frac{1}{\bar{\gamma}^B}\right) z_b^A + \mathbb{E}^A\left[S(z_a^A, z_b^A)\right] \right\} \\
&= \frac{1}{1-\beta}\left\{ \left(1 - \frac{1}{\bar{\gamma}^A}\right) z_a^B - \left(1 - \frac{1}{\bar{\gamma}^B}\right) z_b^A + \mathbb{E}^A\left[S(z_a^A, z_b^A)\right] \right\}
\end{aligned}$$

where the last line follows from (19). The welfare function for country B be can be derived in a similar way.

Appendix C. Bargaining solutions in all possible meetings

For $x \geqslant 0$, define

$$S(x) \equiv \begin{cases} u \circ c^{-1}(x) - x & x < c(q^*) \\ u(q^*) - c(q^*) & \text{otherwise} \end{cases}$$

then

$$S'(x) = \begin{cases} \frac{u' \circ c^{-1}(x)}{c' \circ c^{-1}(x)} - 1 & x < c(q^*) \\ 0 & \text{otherwise} \end{cases}$$

$$S''(x) = \begin{cases} \frac{u''c' - c''u'}{(c')^3} \circ c^{-1}(x) & x < c(q^*) \\ 0 & \text{otherwise} \end{cases}$$

Note that $S'' \leqslant 0$ and it is strictly negative for all $x < c(q^*)$.

For $j = A, B$, define

$$\Delta^j(q) \equiv \frac{u'(q) - c'(q)}{u'(q) - \frac{\gamma_L^j}{\gamma_H^j} c'(q)} = 1 - \frac{1 - \frac{\gamma_L^j}{\gamma_H^j}}{\frac{u'(q)}{c'(q)} - \frac{\gamma_L^j}{\gamma_H^j}} \in [0,1] \text{forall} q \in [0, q^*]$$

then $\Delta^j(q) \in [0,1]$ for all $q \in [0, q^*]$. Since $u'(q)/c'(q)$ is decreasing for all $q \in [0, q^*]$ and $u'(q^*)/c'(q^*) = 1$,

$$\Delta^{j'}(q) = \frac{[u''(q) - c''(q)]\left[u'(q) - \frac{\gamma_L^j}{\gamma_H^j} c'(q)\right] - [u'(q) - c'(q)]\left[u''(q) - \frac{\gamma_L^j}{\gamma_H^j} c''(q)\right]}{\left[u'(q) - \frac{\gamma_L^j}{\gamma_H^j} c'(q)\right]^2} < 0$$

1. $\gamma_t^A = \gamma_H^A, \gamma_t^B = \gamma_H^B$, the buyer meets with a seller from country A.
   When
   $$\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B} < c(q^*)$$
   we have
   $$q^{HHA} = c^{-1}\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right), q_a^{HHA} = \frac{1}{\gamma_A^H} \frac{1}{c'(q^{HHA})}, q_b^{HHA} = \frac{1}{\gamma_B^H} \frac{1}{c'(q^{HHA})}$$
   Otherwise, $q^{HHA} = q^*, q_a^{HHA} = q_b^{HHA} = 0$.
   $$S^{HHA} = u(q^{HHA}) - c(q^{HHA}) = S\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right)$$
   $$S_a^{HHA} = \frac{1}{\gamma_A^H} S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right), S_b^{HHA} = \frac{1}{\gamma_B^H} S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right)$$
   $$S_{aa}^{HHA} = \frac{1}{(\gamma_H^A)^2} S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right), S_{bb}^{HHA} = \frac{1}{(\gamma_H^B)^2} S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right), S_{ab}^{HHA} = \frac{1}{\gamma_H^A \gamma_H^B} S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right).$$

2. $\gamma_t^A = \gamma_H^A, \gamma_t^B = \gamma_L^B$, the buyer meets with a seller from country A.
   When
   $$c(q^*) > \frac{z_a}{\gamma_H^A}$$
   q^{HLA} is determined by

$$u(q^{HLA}) - c(q^{HLA}) + \left(1 - \frac{\gamma_L^B}{\gamma_H^B}\right)\left[c(q^{HLA}) - \frac{z_a}{\gamma_H^A}\right] = u(q^{HHA}) - c(q^{HHA}) = S\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right)$$

and

$$\begin{aligned} q_a^{HLA} &= \left[S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^B}{\gamma_H^B}\right]\tfrac{1}{\gamma_H^A}\frac{1}{u'(q^{HLA}) - \frac{\gamma_L^B}{\gamma_H^B}c'(q^{HLA})} \\ q_b^{HLA} &= S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^B}\frac{1}{u'(q^{HLA}) - \frac{\gamma_L^B}{\gamma_H^B}c'(q^{HLA})} \end{aligned}$$

Otherwise, $q^{HLA} = q^*, q_a^{HLA} = q_b^{HLA} = 0$.

$$\begin{aligned} S^{HLA} &= u(q^{HLA}) - c(q^{HLA}) \\ S_a^{HLA} &= [u'(q^{HLA}) - c'(q^{HLA})]q_a^{HLA} = \left[S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^B}{\gamma_H^B}\right]\tfrac{1}{\gamma_A^H}\Delta^B(q^{HLA}) \\ S_b^{HLA} &= [u'(q^{HLA}) - c'(q^{HLA})]q_b^{HLA} = S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_B^H}\Delta^B(q^{HLA}) \\ S_{aa}^{HLA} &= S''\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\left(\tfrac{1}{\gamma_H^A}\right)^2\Delta^B(q^{HLA}) + \left[S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^B}{\gamma_H^B}\right]\tfrac{1}{\gamma_A^H}\Delta^{B'}(q^{HLA})q_a^{HLA} \\ S_{bb}^{HLA} &= S''\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\left(\tfrac{1}{\gamma_H^B}\right)^2\Delta^B(q^{HLA}) + S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^B}\Delta^{B'}(q^{HLA})q_b^{HLA} \\ S_{ab}^{HLA} &= S''\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^A\gamma_H^B}\Delta^B(q^{HLA}) + S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^B}\Delta^{B'}(q^{HLA})q_a^{HLA} \\ &= S''\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^A\gamma_H^B}\Delta^B(q^{HLA}) + \left[S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^B}{\gamma_H^B}\right]\tfrac{1}{\gamma_H^A}\Delta^{B'}(q^{HLA})q_b^{HLA} \end{aligned}$$

3. $\gamma_t^A = \gamma_A^H, \gamma_t^B = \gamma_B^H$, the buyer meets with a seller from country B.
   Same as case 1.
4. $\gamma_t^A = \gamma_A^L, \gamma_t^B = \gamma_B^H$, the buyer meets with a seller from country B.
   When

$$c(q^*) > \frac{z_b}{\gamma_H^B}$$

q^{LHB} is determined by

$$u(q^{LHB}) - c(q^{LHB}) + \left(1 - \frac{\gamma_L^A}{\gamma_H^A}\right)\left[c(q^{LHB}) - \frac{z_b}{\gamma_H^B}\right] = u(q^{HHB}) - c(q^{HHB}) = S\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_H^B}\right)$$

and

$$\begin{aligned} q_a^{LHB} &= S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^A}\frac{1}{u'(q^{LHB}) - \frac{\gamma_L^A}{\gamma_H^A}c'(q^{LHB})} \\ q_b^{LHB} &= \left[S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^A}{\gamma_H^A}\right]\tfrac{1}{\gamma_H^B}\frac{1}{u'(q^{LHB}) - \frac{\gamma_L^A}{\gamma_H^A}c'(q^{LHB})} \end{aligned}$$

Otherwise, $q^{LHB} = q^*, q_a^{LHB} = q_b^{LHB} = 0$.

$$\begin{aligned} S^{LHB} &= u(q^{LHB}) - c(q^{LHB}) \\ S_a^{LHB} &= [u'(q^{LHB}) - c'(q^{LHB})]q_a^{LHB} = S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^A}\Delta^A(q^{LHB}) \\ S_b^{LHB} &= [u'(q^{LHB}) - c'(q^{LHB})]q_b^{LHB} = \left[S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^A}{\gamma_H^A}\right]\tfrac{1}{\gamma_H^B}\Delta^A(q^{LHB}) \\ S_{aa}^{LHB} &= S''\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\left(\tfrac{1}{\gamma_H^A}\right)^2\Delta^A(q^{LHB}) + S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^A}\Delta^{A'}(q^{LHB})q_a^{LHB} \\ S_{bb}^{LHB} &= S''\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\left(\tfrac{1}{\gamma_H^B}\right)^2\Delta^A(q^{LHB}) + \left[S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^A}{\gamma_H^A}\right]\tfrac{1}{\gamma_B^H}\Delta^{A'}(q^{LHB})q_b^{LHB} \\ S_{ab}^{LHB} &= S''\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^A\gamma_H^B}\Delta^A(q^{LHB}) + S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^A}\Delta^{A'}(q^{LHB})q_b^{LHB} \\ &= S''\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^A\gamma_H^B}\Delta^A(q^{LHB}) + \left[S'\left(\tfrac{z_a}{\gamma_H^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^A}{\gamma_H^A}\right]\tfrac{1}{\gamma_H^B}\Delta^{A'}(q^{LHB})q_a^{LHB} \end{aligned}$$

5. $\gamma_t^A = \gamma_A^L, \gamma_t^B = \gamma_B^H$, the buyer meets with a seller from country A.
   When

$$\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B} < c(q^*)$$

we have

$$q^{LHA} = c^{-1}\left(\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B}\right), q_a^{LHA} = \frac{1}{\gamma_A^L}\frac{1}{c'(q^{LHA})}, q_b^{LHA} = \frac{1}{\gamma_B^H}\frac{1}{c'(q^{LHA})}$$

Otherwise, $q^{LHA} = q^*, q_a^{LHA} = q_b^{LHA} = 0$.

$$S^{LHA} = u(q^{LHA}) - c(q^{LHA}) = S\left(\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B}\right)$$

$$S_a^{LHA} = \frac{1}{\gamma_A^L}S'\left(\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B}\right), S_b^{LHA} = \frac{1}{\gamma_B^H}S'\left(\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B}\right)$$

$$S_{aa}^{LHA} = \frac{1}{\left(\gamma_L^A\right)^2}S''\left(\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B}\right), S_{bb}^{LHA} = \frac{1}{\left(\gamma_H^B\right)^2}S''\left(\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B}\right), S_{ab}^{LHA} = \frac{1}{\gamma_L^A\gamma_H^B}S''\left(\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B}\right).$$

6. $\gamma_t^A = \gamma_A^L, \gamma_t^B = \gamma_B^L$, the buyer meets with a seller from country A.
   When

$$c(q^*) > \frac{z_a}{\gamma_L^A}$$

q^{LLA} is determined by

$$u(q^{LLA}) - c(q^{LLA}) + \left(1 - \frac{\gamma_L^B}{\gamma_H^B}\right)\left[c(q^{LLA}) - \frac{z_a}{\gamma_L^A}\right] = u(q^{LHA}) - c(q^{LHA}) = S\left(\frac{z_a}{\gamma_L^A} + \frac{z_b}{\gamma_H^B}\right)$$

and

$$q_a^{LLA} = \left[S'\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^B}{\gamma_H^B}\right]\tfrac{1}{\gamma_L^A}\frac{1}{u'\left(q^{LLA}\right) - \frac{\gamma_L^B}{\gamma_H^B}c'\left(q^{LLA}\right)}$$

$$q_b^{LLA} = S'\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^B}\frac{1}{u'\left(q^{LLA}\right) - \frac{\gamma_L^B}{\gamma_H^B}c'\left(q^{LLA}\right)}$$

Otherwise, $q^{LLA} = q^*, q_a^{LLA} = q_b^{LLA} = 0$.

$$S^{LLA} = u(q^{LLA}) - c(q^{LLA})$$

$$S_a^{LLA} = \left[u'\left(q^{LLA}\right) - c'\left(q^{LLA}\right)\right]q_a^{LLA} = \left[S'\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^B}{\gamma_H^B}\right]\tfrac{1}{\gamma_A^L}\Delta^B\left(q^{LLA}\right)$$

$$S_b^{LLA} = \left[u'\left(q^{LLA}\right) - c'\left(q^{LLA}\right)\right]q_b^{LLA} = S'\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_B^H}\Delta^B\left(q^{LLA}\right)$$

$$S_{aa}^{LLA} = S''\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right)\left(\tfrac{1}{\gamma_L^A}\right)^2\Delta^B\left(q^{LLA}\right) + \left[S'\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^B}{\gamma_H^B}\right]\tfrac{1}{\gamma_A^L}\Delta^{B'}\left(q^{LLA}\right)q_a^{LLA}$$

$$S_{bb}^{LLA} = S''\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right)\left(\tfrac{1}{\gamma_H^B}\right)^2\Delta^B\left(q^{LLA}\right) + S'\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^B}\Delta^{B'}\left(q^{LLA}\right)q_b^{LLA}$$

$$S_{ab}^{LLA} = S''\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_L^A\gamma_H^B}\Delta^B\left(q^{LLA}\right) + S'\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_H^B}\Delta^{B'}\left(q^{LLA}\right)q_a^{LLA}$$

$$= S''\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right)\tfrac{1}{\gamma_L^A\gamma_H^B}\Delta^B\left(q^{LLA}\right) + \left[S'\left(\tfrac{z_a}{\gamma_L^A} + \tfrac{z_b}{\gamma_H^B}\right) + 1 - \tfrac{\gamma_L^B}{\gamma_H^B}\right]\tfrac{1}{\gamma_L^A}\Delta^{B'}\left(q^{LLA}\right)q_b^{LLA}$$

7. $\gamma_t^a = \gamma_a^H, \gamma_t^b = \gamma_b^L$, the buyer meets with a seller from country B.
   When

$$\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B} < c(q^*)$$

we have

$$q^{HLB} = c^{-1}\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right), q_a^{HLB} = \frac{1}{\gamma_A^H}\frac{1}{c'(q^{HLB})}, q_b^{HHA} = \frac{1}{\gamma_B^L}\frac{1}{c'(q^{HLB})}$$

Otherwise, $q^{HLB} = q^*, q_a^{HLB} = q_b^{HLB} = 0$.

$$S^{HLB} = u(q^{HLB}) - c(q^{HLB}) = S\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)$$

$$S_a^{HLB} = \frac{1}{\gamma_A^H}S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right), S_b^{HLB} = \frac{1}{\gamma_B^L}S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)$$

$$S_{aa}^{HLB} = \frac{1}{(\gamma_H^A)^2}S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right), S_{bb}^{HLB} = \frac{1}{(\gamma_L^B)^2}S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right), S_{ab}^{HLB} = \frac{1}{\gamma_H^A\gamma_L^B}S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right).$$

8. $\gamma_t^A = \gamma_A^L, \gamma_t^B = \gamma_B^L$, the buyer meets with a seller from country B.
When

$$c(q^*) > \frac{z_b}{\gamma_L^B}$$

q^{LLB} is determined by

$$u(q^{LLB}) - c(q^{LLB}) + \left(1 - \frac{\gamma_L^A}{\gamma_H^A}\right)\left[c(q^{LLB}) - \frac{z_b}{\gamma_L^B}\right] = u(q^{HLB}) - c(q^{HLB}) = S\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)$$

and

$$q_a^{LLB} = S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)\frac{1}{\gamma_H^A}\frac{1}{u'(q^{LLB}) - \frac{\gamma_L^A}{\gamma_H^A}c'(q^{LLB})}$$

$$q_b^{LLB} = \left[S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right) + 1 - \frac{\gamma_L^A}{\gamma_H^A}\right]\frac{1}{\gamma_L^B}\frac{1}{u'(q^{LLB}) - \frac{\gamma_L^A}{\gamma_H^A}c'(q^{LLB})}$$

Otherwise, $q^{LLB} = q^*, q_a^{LLB} = q_b^{LLB} = 0$.

$$S^{LLB} = u(q^{LLB}) - c(q^{LLB})$$

$$S_a^{LLB} = [u'(q^{LLB}) - c'(q^{LLB})]q_a^{LLB} = S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)\frac{1}{\gamma_H^A}\Delta^A(q^{LLB})$$

$$S_b^{LLB} = [u'(q^{LLB}) - c'(q^{LLB})]q_b^{LLB} = \left[S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right) + 1 - \frac{\gamma_L^A}{\gamma_H^A}\right]\frac{1}{\gamma_L^B}\Delta^A(q^{LLB})$$

$$S_{aa}^{LLB} = S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)\left(\frac{1}{\gamma_H^A}\right)^2\Delta^A(q^{LLB}) + S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)\frac{1}{\gamma_H^A}\Delta^{A'}(q^{LLB})q_a^{LLB}$$

$$S_{bb}^{LLB} = S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)\left(\frac{1}{\gamma_L^B}\right)^2\Delta^A(q^{LLB}) + \left[S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right) + 1 - \frac{\gamma_L^A}{\gamma_H^A}\right]\frac{1}{\gamma_B^L}\Delta^{A'}(q^{LLB})q_b^{LLB}$$

$$S_{ab}^{LLB} = S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)\frac{1}{\gamma_H^A\gamma_L^B}\Delta^A(q^{LLB}) + S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)\frac{1}{\gamma_H^A}\Delta^{A'}(q^{LLB})q_b^{LLB}$$

$$= S''\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right)\frac{1}{\gamma_H^A\gamma_L^B}\Delta^A(q^{LLB}) + \left[S'\left(\frac{z_a}{\gamma_H^A} + \frac{z_b}{\gamma_L^B}\right) + 1 - \frac{\gamma_L^A}{\gamma_H^A}\right]\frac{1}{\gamma_L^B}\Delta^{A'}(q^{LLB})q_a^{LLB}$$

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