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Why are BHCs organized as parent-subsidiaries? How do they grow in value? $\ensuremath{^{\diamond}}$

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ABSTRACT

We rationalize the organization of US banking groups into a holding company with subsidiaries – instead of branches or stand-alone units – subject to regulatory provisions of the "source-of strength" type. We show that their value increases with debt diversity among affiliates and with complexity, as measured by the number of subsidiaries.

Regulatory interventions that are aimed at ring fencing reduce (increase) the shareholder value, whenever the Governmental leniency to bailout is low (high).

Branches become more valuable when there is no full commitment to internal rescue and Government bailout occurs with certainty.

1. Introduction

Large banking groups in the United States are generally organized as BHCs. A BHC is a corporation that controls one or more banks. It owns a number of domestic bank affiliates that are engaged in commercial banking, i.e. lending and deposit-taking, but also nonbanking, and foreign affiliates that cover a number of businesses, from insurance to securities trading and underwriting, mutual funds, realestate funds, private equity and venture capital, as well as asset and wealth management, including trusts.

Certain BHCs are made of relatively independent stand-alone affiliates, which are able to default individually, without threatening the other entities under the umbrella of the BHC's or the "home" control, but do not receive assets or cash transfers from the home when they are close to defaulting. As an alternative, BHCs are sometimes organized as branches, i.e. merged entities, which can only default jointly. However, stand-alone or merged BHCs are considered more the exception than the rule. The standard situation is that of the parent-subsidiary organization. The parent-subsidiary structure is characterized by the fact that limited liability protects the holding company from the collapse of its subsidiaries, but asset transfers from the parent to the subsidiaries are provided whenever the latter are close to defaulting. In the US, this happens because of the "source-of-strength" doctrine and the Financial Institutions Reform, Recovery, and Enforcement (FIRREA) Act. Already in 2012, at least half a dozen of the top US banks were BHCs, according to Avraham et al. (2012). Over the period 1996-2018, the US BHCs with real assets over \$25 billion were on average 35, according to Correa and Goldberg (2022).

In order to explain the prevalence of parent-subsidiaries, in this paper we have built a theoretical model whereby banks can choose from three different types of organizational forms: ring-fenced – or individual, stand-alone – banks, branches, or parent-subsidiaries. The rescue arrangements these organizations adopt for their affiliates, whenever one of them is in default, differ. No rescue operation is performed for ring-fenced affiliates, while unconditional rescue takes place for the overall survival of the group in merged banks, and conditional rescue is performed for joint survival in holding-subsidiary structures. By calibrating the model to US BHCs, we show that the parent-subsidiary structure creates the highest shareholder value of the three, at least when the commitment to rescue is high, as imposed by the FIRREA Act. This offers an explanation for its pervasiveness.

The comparative advantage for shareholders of BHCs organized as parent-subsidiaries increases with the number of (equally sized and

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¹ Clas passed away in March 2021.

equally levered) subsidiaries, and their complexity. We consider this a rationale for the increase in BHC complexity over time and for seeing them become "too large to fail", or, when complexity is measured by organizational complexity, "too complex to fail".

Moreover, the comparative advantage is also increasing in the wedge between the leverage on the parent and the subsidiaries, which we call "debt diversity". This advantage is maximal when either the parent is responsible for the whole debt and the subsidiaries are unlevered, or vice-versa. Therefore, it is spurred by the presence of internal capital markets. We have considered this as a rationale for the heterogeneity of the leverage on BHCs, as reported by Correa and Goldberg (2022). Moreover, we have considered this as evidence that shows how the regulatory interventions that are aimed at constraining the leverage of BHCs – if differentiated across group members – come at a cost, that is, a reduction in shareholder value.

These results are robust, on condition the commitment to rescue within the group is full, while a central entity – a Government – is unwilling to rescue failed entities in any circumstance. If these two conditions do not hold, branches "free ride" on Governmental bailout processes and become more valuable.

Regulatory interventions of the ring-fencing type reduce shareholder value, if the willingness of the Government to bailout is low, because they lower the value of free riding, while they do the opposite when that willingness is high.

The paper is structured as follows: Section 2 briefly reviews the background literature on the organizational forms of banks and on the complexity measure of BHCs. Section 3 describes the actual rescue functioning mechanisms in banking groups under different jurisdictions, including the US, as a background for our model. Section 4 models such rescues and their effects on the shareholders' value, in the different organizational forms for banks: individual, ring-fenced, parent-subsidiaries and branches. Section 5 rationalizes the evidence using the model presented in Section 4. Section 6 provides an empirical assessment of the value for the average US BHC. Section 7 discusses the effect of policy interventions of the ring-fencing or leverage-constraint types and concludes the paper.

2. Background literature

The theoretical literature on the comparison of organizational forms for risky projects and banks is rich.

Banal-Estanol et al. (2013) compared the joint and separate financing of risky projects. The former can be more or less valuable than the latter, because the merged projects cannot take advantage of limited liability. Thus, any losses that arise from one of these projects reduce the joint capital base. This "contamination" - or negative contamination - effect might be greater than the positive contamination or "coinsurance" effect - that exists in joint financing, but not in separate financing - of helping a project out of a financial distress situation by resorting to the joint capital base. The problem reminds the one of branches and separate activities in banking, but not the parent-subsidiaries one. Also, given the generality of the approach, no Governmental bailout is taken into account, and the focus is more on the probabilistic properties of project returns and on how they interact with the prevalence of contamination vs. diversification.

Luciano and Wihlborg (2018) built on a structural default model in the spirit of Merton (1974), as studied by Leland (2007), and developed in Luciano and Nicodano (2014) for non-banking groups. They compared two banks that were organized as parent-subsidiaries, branches or ring-fenced activities, in the presence of Governmental bailout and with full commitment by one affiliate to rescue another one. They showed that – if the leverage in each affiliate is the same across organizational forms and there are no taxes – branches are dominated by parent-subsidiaries, in the sense that they have lower shareholder value. The reason why parent-subsidiaries dominate branches is that limited liability annihilates negative "contamination" and makes "coinsurance" prevail. When the leverage is different and taxes, which produce a tax shield for debt, exist, the lower value of branches is no longer the rule. The theoretical analysis is limited by the effect of tax shields, which have been shown to hinder analytical conclusions but not to be empirically relevant. Other limitations, with respect to the current analysis, are full commitment to rescue and the fact that the two affiliates should have the same size. In the current paper, to make the situation more realistic, we assume that commitment can also be very limited, as in the "comfort letters" case and, to represent a BHC situation, we allow the subsidiary to grow through the acquisition of several affiliates. As a consequence, the parent can also be much smaller than the rest of the group.

Some of the papers that have addressed the comparison between a parent-subsidiary and merged branches - excluding ring-fenced ones are those by Loranth and Morrison (2007) and Calzolari and Loranth (2011), following an unpublished earlier paper by Harr and Ronde (2003). The main incentive in this strand of literature is the presence of deposit insurance, together with capital requirements. In our paper, deposit insurance is represented by Governmental rescue. In Loranth and Morrison (2007), banks had accepted deposits and could choose a lending or asset policy. Deposit insurance spurs overinvestment, capital requirements induce underinvestment. The effect of deposit insurance is lower in parent-subsidiaries, because their limited liability already prevents them from failing as a whole when either the parent or the subsidiary fails. In branches, where there is less natural protection from losses derived from unlimited liability within the group, deposit insurance is more valuable, especially with different capital requirements across organizations. As a consequence, Loranth and Morrison (2007) explained the prevalence of parent subsidiaries observed in reality as due to the fact that equal capital requirements across organizations lower the value of deposit insurance in branches and induce banks to opt for a subsidiary structure.² A similar mechanism applies to our Governmental bailout, which, however, we allow to interplay with the level of internal willingness to rescue. Governmental rescue is in fact important to understand the arrangements of BHCs. Some BHCs are classified as Systemically Important Financial Institutions (SIFIs) or Globally Systemically Important Banks (GSIBs). As such, they generally receive substantial implicit Government support. In our model, the Government is assumed to bail out banks if internal rescue has failed to protect them from default, with a probability that may depend on the organizational form and on the bank being a home or affiliate. When the size of the two differs, this includes the potential bias towards SIFIs and GSIBs.

The Financial Stability Board (FSB) (2012) and the Basel Committee on Banking Supervision (2014) have defined a number of characteristics, including size, cross-jurisdictional activity, interconnectedness and complexity, which they use to classify financial institutions as either SIFIs or GSIBs. The academic literature to which we adhere, which was initiated by Cetorelli and Goldberg (2014), and continued by Carmassi and Herring (2016) and Laeven et al. (2014), instead points out the importance of the so-called "complexity", including organizational complexity, which is measured by the number of 50+%owned affiliates under a parent organization, as a stylized incentive for Governmental support. Since it is difficult to incorporate the FSB characteristics in a single theoretical model, organizational complexity – or the number of affiliates – is generally considered as a rough but efficient proxy. This is especially true when – as in our model – each single affiliate has equally sized assets.

² Calzolari and Loranth (2011) explained the presence of the opposite outcome, namely banks opting for branches, in an international context, assuming that home banks could rescue their subsidiaries, while the latter could not fund the rescue of the parent. However, we have not considered this situation, because of the evidence presented in Correa and Goldberg (2022).

We consider as a basis of our model the long-standing evidence of asset transfers between BHCs on the one side and their affiliates on the other. We review this evidence, together with the guarantees adopted in other banking groups that do not have a holding on top, in Section 3 below.

3. Rescues in banking

The provision of guarantees from one affiliate to another in a banking group may be the result of a regulatory or voluntary intervention, and has a long-standing tradition.

In the US, the Federal Reserve's source-of-strength doctrine, contained in Regulation Y of 1984 and its amendments, states that a BHC with subsidiaries "should stand ready to use any available resources to provide adequate capital funds to its subsidiary banks during periods of financial stress", while preserving limited liability, and therefore the safety of the parent itself in case a weak subsidiary defaults. This regulation has been complemented by the FIRREA Act of 1989, through the introduction of a cross-guarantee authority granted to the FDIC. The latter states that any expected loss from a failed banking subsidiary should be charged off with the capital of any non-failing affiliate bank. Ashcraft (2004) showed that the FIRREA Act has in fact strengthened the Federal Reserve's source-of-strength doctrine, so that a bank affiliated with a holding company is much safer than a standalone bank. This happens as a result of distressed, affiliated banks receiving capital injections and recovering more quickly than other banks, as we assume in our model.

Ashcraft (2003) recalled that the FDIC used cross-guarantees to close thirty-eight subsidiaries of the First Republic Bank Corporation in 1988 and eighteen subsidiaries of the First City Bancorporation in 1992, when the leading banks from each of these bank holding companies were declared insolvent. This testifies that, in the source-of-strength case, limited liability is strictly preserved.

The Dodd–Frank Act made it clear that the source-of-strength is an ongoing obligation, and that the parent may be asked to file evidence of support.

In regions other than the US, rescues have been observed over time, even in the absence of a compulsory regulation, or when regulators were doubtful about their appropriateness. This is the case of the parallel-owned banking structures examined by the BIS in 2003. These structures are not part of the same group for regulatory purposes, but have the same owners. The BIS was worried about their opaqueness, because it could provide an incentive to the owners to use some banks to provide undisclosed support mechanisms to the others within the group. This is also the case of the SIVs that were rescued during the Great Recession. Such SIVs had issued asset-backed commercial paper or medium term notes that they found difficult to refinance. In spite of explicit guarantees from the sponsor that covered no more than 30% of those assets but none of the notes, the SIVs were eventually rescued by the sponsor itself.

The practice of rescuing by the parent is so common that rating agencies provide both an "individual" and an "all-in" rating to affiliates. The latter contains an uplift, resulting from the parental rescue as well as from Government bailout, and is the rating that is actually provided, sometimes the only one, for rated debtors. Schich and Kim (2012), using Moody's data for OECD countries, distinguish the uplift from the fact that the parent rescues its subsidiary and the uplift due to Government bailout. The former amounts to 17% of the uplift, with the Government bailout counting for 63% of the total. They do the same split for branches, and obtain respectively 4% for the cooperative uplift from the other branches and 96% for Government bailout.

Rescue or close-to-rescue operations often arise in multinational banking. Jeon et al. (2013) used bank-level data on 368 foreign

subsidiaries of 68 multinational banks in 47 emerging economies over the 1994–2008 period to provide evidence of internal capital markets contributing to the transmission of financial shocks from parent banks to foreign subsidiaries. These shocks include both negative and positive ones, which amplify the distance from default of the subsidiaries. The Authors showed that such a transmission is stronger in multinational banking when the subsidiaries rely on funds from their parent bank. Therefore, rescue is part of the functioning practices of the internal capital market.

Cetorelli and Goldberg (2011) showed that these transfers may take place from the parent to the subsidiaries, or viceversa, as we assume hereafter, and that they are strategic. When reallocating capital – which, in their case, is liquidity, while we instead consider a single type of asset – because of a shock at the parent level, affiliates that produce revenues for the group are kept relatively protected, while the affiliates used for funding under normal circumstances – because they have a relative advantage in doing so – are the most involved ones. Therefore, "liquidity management is driven by each bank's assessment of the marginal conditions of each foreign location along both funding and investment dimensions". This is why we provide a rationally-based decision model for rescue and bailout in this paper.

4. Modeling rescue: individual, ring-fenced, branch and subsidiary structures

First, we formalize the value of an individual, or stand alone (*SA*) bank, then the value of a ring-fenced group (RF), a branch structure (BR) and, finally, a parent-subsidiary arrangement (PS).

4.1. Stand-alone bank

In order to make the model as lean as possible, we have worked in a static economy. There are two points in time: 0 and T, where default may occur. For simplicity reasons, debt is created through deposits, with a face value of F. Whether deposits are given back or not depends on whether the value of the bank assets or loans at time T, L(T), is greater than F, or not. Any exogenous delinquency on loans makes default possible, because it deprives the bank of the assets needed to pay back its own deposits.

The bank liabilities at time 0 are made up of debt and equity, D_0 and E_0 . Debt D_0 is the expected present value of the payoff to depositors under the risk-neutral measure, and equity is the expected present value of payoffs to equity holders, under the same measure.

At time *T*, the bank collects the random value of loans L(T), net of taxes at the $0 \le k \le 1$ rate. Because there is a tax shield on passive interest rates, $F - D_0$, the bank's assets at *T*, net of taxes, are

$$L(T) = (1 - k)L(T) + k(F - D_0),$$
(1)

The $\overline{L}(T)$ assets are distributed to depositors and equity holders as follows. Depositors receive F, either when this is greater or equal to the asset value $\overline{L}(T)$, or when it is smaller and the Government bails the bank out. If $\overline{L}(T) < F$ and there is no bailout, default occurs, with costs that are proportional to the total assets at T, $\alpha \overline{L}(T)$, where $0 \leq \alpha \leq 1$. Equity holders receive the difference between the asset value at T and the face value of the debt, when default does not occur, and zero otherwise.

r

We denote the probability of bailout with $0 \le \pi \le 1$,³ and set the riskless interest rate to zero. Thus, the expected present value of assets to depositors is

$$D_0 = \begin{cases} F - (1 - \pi) \mathbb{E} \max(0, F - \bar{L}(T)) \\ -\alpha(1 - \pi) \mathbb{E} \left[\bar{L}(T) \mathbf{1}_{\{\bar{L}(T) < F\}} \right] \end{cases}$$
(2)

where $\mathbf{1}_{\{E\}}$ is the indicator of event E, which takes the value 1 when the event is true and 0 otherwise. Debt holders are long the face value of debt F – the first term – and short a put on the asset value with strike F, which enters the second term as $-\mathbb{E} \max(0, F - \overline{L}(T))$. The second term also incorporates the effect of bailout, which is an option-like payoff, $\pi \mathbb{E} \max(0, F - \overline{L}(T))$. The third term represents default costs in the case default occurs ($\overline{L}(T) < F$) but no bailout takes place, since the latter occurs with probability $1 - \pi$.⁴

The payoffs to equity holders of the bank at *T* are represented by the standard payoff of a call on the asset value. The equity value at time 0, E_0 , is therefore

$$E_0 = \mathbb{E} \max\left[\bar{L}(T) - F, 0\right]. \tag{3}$$

The value of the amount the stand-alone bank pays to its shareholders, who have the equity and raise the debt, and therefore receive its present value, is simply $V_{SA} = D_0 + E_0$. This value can also be written as the net initial loan value plus the expected value of the Government bailout minus the expected value of the default costs:

$$V_{SA} = \bar{L}_{0}$$

$$+ \underbrace{\pi \mathbb{E} \max(0, F - \bar{L}(T))}_{\text{Government bailout}}$$

$$- \underbrace{\alpha(1 - \pi) \mathbb{E} \left[\bar{L}(T) \mathbf{1}_{\{\bar{L}(T) < F\}} \right]}_{\text{default costs}}$$

$$(4)$$

where, consistently with (1), $\bar{L}_0 = (1 - k)L(0) + k(F - D_0)$ is the initial asset value, net of taxes.

4.2. Multiple banks

Let us now consider a banking group made up of two affiliates, one of which will be made up of N-1 subaffiliates. This serves the purpose of describing a "little" home bank and the potentially "large" portfolio of its subaffiliates, and of being able to vary the size of the portfolio – N as a total – by changing the number of subaffiliates. The size of each member is measured by the initial value of loans L(0).

As far as the "little" vs. "large" affiliates are concerned, we have named the former affiliate "home". The latter, made up of several subaffiliates, can work as stand-alone bank, branch or subsidiary.

We assume that the bailout probability can differ between the home and the portfolio: we have π_h versus π_i , with the latter constant for i = 1, ...N - 1. The parameters α and k can also vary between the home and the subaffiliates.⁵

⁵ The formulas in this Section can also be adapted to the case in which π_i, α_i, k_i differ among the subaffiliates. We present the simplest case in the

4.3. Ring-fenced structure

When the "small" and the potentially "large" affiliates are standalone banks, they do not rescue each other, because they are ringfenced, i.e. they provide no guarantee to rescue each other. As a consequence of this lack of guarantees, their total value – home plus subaffiliates – is equal to N times the value of each stand-alone bank

$$GV_{RF} = N \times V_{SL}$$

4.4. Branch structure

We know from previous studies and suggestions – see, for instance, Banal-Estanol et al. (2013) – that any unconditional support from branches runs the risk of creating negative contamination. One affiliate in default can in fact cause the default of the whole group. There is also an opportunity of positive contamination or coinsurance.

Rescue is offered from the home bank to a branch whenever the former's assets are greater than the face value of its debt, $\bar{L}_h(T) > F_h$, so that the bank can pay its depositors back, while the opposite inequality holds true for the portfolio branches, $\sum_{b=1}^{N-1} \bar{L}_b(T) < \sum_{b=1}^{N-1} F_b$:

$$R_{BR} \triangleq \begin{cases} \bar{L}_{h}(T) > F_{h} \\ \sum_{b=1}^{N-1} \bar{L}_{b}(T) < \sum_{b=1}^{N-1} F_{b} \end{cases}$$
(5)

Support from the subaffiliates to the home occurs whenever the opposite inequalities hold:

$$R'_{BR} \triangleq \begin{cases} \bar{L}_h(T) < F_h\\ \sum_{b=1}^{N-1} \bar{L}_b(T) > \sum_{b=1}^{N-1} F_b \end{cases}$$
(6)

Any transfer in the two events does not necessarily cover the difference between the face value of the debt of the guaranteed company and its own assets, since rescue occurs even when the assets of the guarantor are not sufficient to avoid default, but it does cover the minimum between that difference and the extra-cashflows of the guarantor. Rescue by the home can produce negative contamination, if the home has not enough free assets to save the other affiliates, and this is analogous for branches. When *R* or *R'* hold true, and funds cannot avoid default, namely, when

$$Q \triangleq \begin{cases} \bar{L}_{h}(T) > F_{h} \\ \sum_{b=1}^{N-1} \bar{L}_{b}(T) < \sum_{b=1}^{N-1} F_{b} \\ \bar{L}_{h}(T) - F_{h} < \sum_{b=1}^{N-1} \left(F_{b} - \bar{L}_{b}(T) \right) \end{cases}$$

in *R* and analogously for R'

$$Q' \triangleq \begin{cases} \bar{L}_{h}(T) < F_{h} \\ \sum_{b=1}^{N-1} \bar{L}_{b}(T) > \sum_{b=1}^{N-1} F_{b} \\ F_{h} - \bar{L}_{h}(T) > \sum_{b=1}^{N-1} \left[\bar{L}_{b}(T) - F_{b} \right] \end{cases}$$

negative contamination occurs. The whole bank defaults when either the home or the branch become insolvent, and their affiliate does not have enough assets to rescue the other (negative contamination), or has enough but does not fulfill its commitment. Let us call β the probability of fulfilling the commitment. Bailout of the home (portfolio) occurs, with probability $\pi_h(\pi_b)$, if either Q'(Q) is true or $R'_{BR}(R_{BR})$ is, but the commitment has not been respected.

$$\begin{aligned} \beta_{hBR} &\triangleq \mathbf{1}_{\{Q'\}} + (1-\beta)\mathbf{1}_{\{R'_{BR}\}} \\ \beta_{bBR} &\triangleq \mathbf{1}_{\{Q\}} + (1-\beta)\mathbf{1}_{\{R_{BR}\}} \end{aligned}$$

The overall group value, GV_{BR} , which is the sum of the equity and debt values of all the affiliates, can be written as the sum of their

³ In principle, we can distinguish two kinds of debt; insured deposits and non-insured loan funding. We assume here that all the deposits are guaranteed with a certain probability because, in the current economic environment, the implicit insurance of creditors of all types seems to be the rule rather than the exception. However, implicit guarantees from bailout cannot be certain. This is why we add a π parameter. We do not explicitly model a price for deposit insurance. However, if deposit insurance is paid as a percentage *d* of the face value of the deposits, *dF*, charged to debt holders, and if they receive the whole face value, or a percentage of their recovery, *dL*, all the results below still hold, provided that the reader reinterprets *F* as (1 - d)F.

⁴ The fact that debt is evaluated as the expected present value of future payoffs to debt holders makes the percentage interest rate on deposits, $(F - D_0)/D_0$, respond to the level of deposits themselves. Increasing deposits *F* causes a higher interest rate $(F - D_0)/D_0$, or increasing funding costs.

theoretical part, while we allow parameters to differ across banks in some of the calibrated examples below.

asset values plus the Government bailout minus the default costs (see Appendix A):

$$GV_{BR}$$

$$= \bar{L}_{h0} + \sum_{b=1}^{N-1} \bar{L}_{i0}$$

$$+ \pi_{h} \beta_{hBR} \mathbb{E} \max(0, F_{h} - \bar{L}_{h}(T))$$
Government bailout home
$$- \alpha_{h}(1 - \pi_{h})\beta_{hBR} \mathbb{E} \left[\bar{L}_{h}(T) + \max\left(0, \sum_{b=1}^{N-1} \left(\bar{L}_{b}(T) - F_{b}\right)\right) \right]$$
default cost home
$$+ \mathbb{E} \max \sum_{b=1}^{N-1} \pi_{b} \beta_{bBR} (F_{b} - \bar{L}_{b}(T), 0)$$
Government bailout branch
$$-\mathbb{E} \left[\sum_{b=1}^{N-1} \alpha_{b}(1 - \pi_{b})\beta_{bBR} \left[\bar{L}_{b}(T) + \max(0, \bar{L}_{h}(T) - F_{h}) \right] \right]$$
default cost branch
$$(7)$$

where the asset value at 0, comprehensive of all the tax effects, is

$$\begin{split} \bar{L}_{i0} &= (1-k_i)L_i(0) + k_i(F_i - D_{0i}), \\ i &= h, b. \end{split}$$

4.5. Subsidiary structure

Rescue exists in the *PS* and acts together with limited liability constraints. Rescue or coinsurance of the subsidiary occurs if, and only if, the home bank is not in default or distress, the subsidiary portfolio is in default, because its asset value is below the default level, and rescuing the subsidiary does not drive the home bank into default, so that it is not endangered by rescue. Using its surplus, the home bank pays that part of the subsidiary's deposits that is not covered by its own activities. These conditions can be reduced to the event

$$R_{PS} \triangleq \begin{cases} \sum_{s=1}^{N-1} \bar{L}_{s}(T) < \sum_{s=1}^{N-1} F_{s} \\ \bar{L}_{h}(T) - F_{h} > \sum_{s=1}^{N-1} \left(F_{s} - \bar{L}_{s}(T) \right) \end{cases}$$
(8)

Rescue of the home bank by the subsidiary takes place when the latter is not in default and is not endangered by rescue:

$$R'_{PS} \triangleq \begin{cases} \bar{L}_{h}(T) < F_{h} \\ \sum_{s=1}^{N-1} \left(\bar{L}_{s}(T) - F_{s} \right) > F_{h} - \bar{L}_{h}(T) \end{cases}$$
(9)

Rescue of the subsidiary-portfolio in the parent-subsidiary excludes the cases of negative contamination in which the home survives, the portfolio does not survive on its own, and the home has not enough extra assets to save the latter:

$$Q \triangleq \begin{cases} \bar{L}_{h}(T) > F_{h} \\ \sum_{s=1}^{N-1} \bar{L}_{s}(T) < \sum_{s=1}^{N-1} F_{s} \\ \bar{L}_{h}(T) - F_{h} < \sum_{s=1}^{N-1} (F_{s} - \bar{L}_{s}(T)) \end{cases}$$

An analogous definition of Q' holds for the home bank.

A defaulting home (subsidiary) is not rescued by the subsidiary (home) if the latter does not have enough extra cash-flows to survive on its own and successfully rescues its affiliate, namely in the same event Q'(Q) as in the branch case. If either Q'(Q) occurs or $R'_{PS}(R_{PS})$ does, but rescue is not fulfilled – with probability β – the state bails out the home (subsidiary) with probability $\pi_h(\pi_s)$.

$$\beta_{hPS} \triangleq \mathbf{1}_{\{Q'\}} + (1-\beta)\mathbf{1}_{\{R'_{PS}\}}$$
$$\beta_{sPS} \triangleq \mathbf{1}_{\{Q\}} + (1-\beta)\mathbf{1}_{\{R_{PS}\}}$$

The overall group value of the parent-subsidiary structure, GV_{PS} , can be written as the asset value plus the appropriate Government

bailout extra-value minus the default costs:

$$GV_{PS} = \overline{L}_{h0} + \sum_{s=1}^{N-1} \overline{L}_{i0} + \frac{\pi_h \beta_{hPS} \mathbb{E} \max(0, F_h - \overline{L}_h(T))}{\text{Government bailout home}} + \frac{-\alpha_h (1 - \pi_h) \beta_{hPS} \mathbb{E} \overline{L}_h(T)}{\text{default cost home}} + \mathbb{E} \max \sum_{s=1}^{N-1} \pi_s \beta_{sPS} (F_s - \overline{L}_s(T), 0) - \frac{1}{\text{Government bailout subsidiary}} - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^{N-1} (1 - \pi_s) \alpha_s \beta_{sPS} \overline{L}_s(T) - \mathbb{E} \sum_{s=1}^$$

default cost subsidiary

A limited liability prevents the occurrence of negative contamination: an affiliate cannot lead the other into insolvency. However, positive contamination still exists because one affiliate can support another. In the absence of tax distortions, for a given face value of the deposits in the affiliates, the absence of negative contamination and the persistence of positive contamination could render a subsidiary structure more valuable than a branch one. However, the mechanism is also influenced by bailout, which restores the branch value. Therefore, a more careful examination of negative and positive contamination consistent with the empirical evidence is necessary. We perform such an examination in the next Section.

5. Rationalizing the empirical evidence

This Section is dedicated to rationalizing the prevalence of holdingsubsidiaries in the US (and beyond, see Buch and Goldberg (2022)) using the model presented in Section 4.

In order to make such a comparison possible, we assume that the home and the subaffiliates have the same pre-tax assets at time 0, regardless of whether they are home or members of the portfolio of subaffiliates, and of which organizational form they belong to: $L_h(0) = L_i(0) = L(0), i = 1, 2, ... N - 1, i = s, b, a$, where *a* is the stand-alone subaffiliate. We consider the case in which the tax rate is small, or $k \rightarrow 0$, so that the after-tax initial asset value of all the affiliates is the same, regardless of whether they are home, or members of the portfolio of subaffiliates, and of which organizational form they belong to. Consistently, we also assume that the home and the subaffiliates have equally distributed returns, regardless of whether they are home, or members of the portfolio of subaffiliates and of subaffiliates, and of the organizational form they belong to: $L_h(T) = L_i(T) = L(T), i = 1, 2, ... N - 1$ in distribution. For simplicity reasons, we assume that all the states of the world at time *T* have the same probability of occurring.

In order to ensure that the current comparison is unaffected by any possible Government attitude toward specific organizational forms, we also assume that the home and affiliates' probabilities of being bailed out are independent of whether they are branches or parentsubsidiaries, namely π_h and π_i are the same for the different organizations, even though they can be different for the home and the affiliates, $\pi_h \neq \pi_i$. We introduce a symmetric assumption on α_h , α_i to exclude the distortionary effects of different default costs. Unless stated otherwise, all these parameters and the commitment β are positive, in order to include all the effects at play: bailout, default costs and rescue.

First, we assume that all the affiliates, with the same asset value at 0 and T, are equally levered: $F_h = F_i = F, i = 1, 2, ..N - 1$ in all the organizations, then it will be possible to see what the organization-dependent choice of leverage delivers.

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5.1. Leverage-free advantage of PS vs. BR and RF

This Section rationalizes the prevalence of *PS*. The following properties are proved in Appendix B.

Proposition 1. Let us consider a home bank and N - 1 subaffiliates, which can be organized as parent-subsidiaries or branches. The home and the subaffiliates have the same initial and final loan values (in distribution), commitment level, default costs and probability of bailout, regardless of their organization. When the tax rate is small and commitment is high, the PS structure is at least as valuable as the BR one.

The intuition that arises for this result is the following. The value of each organization is the sum of the initial value and of the positive value of bailout of the home and subsidiaries or branches, minus their default costs. Remember that, for comparison purposes, we set the initial value to be the same, by eliminating the tax effect. If there is a tendency to full commitment, the Government intervenes in the same circumstances in both organizations, namely when the internal funds are not able to rescue the failed affiliate (Q' or Q occur). This, together with the initial value equality, makes the bailout value the same in both organizations. However, the default costs are greater in BR, because they occur when there is negative contamination. As we know, negative contamination never occurs in the PS case.

This explains the role of the "source of strength" and the FIRREA Act: with the latter in place, $\beta = 1$, and all the other external values being equal (default costs per dollar, bailout probability, taxes), the Proposition shows the enhanced and greater value of *PS*.

Under the FIRREA-Act-alike provisions, the superiority of *PS* is greater, if there is no sure Government intervention, and is annihilated if the latter occurs. This is the content of Corollary 2:

Corollary 2. Under the assumptions of Proposition 1, the PS structure is worth as much as the BR one if Government bailout is certain ($\pi_h = \pi_i = 1$).

The intuition of this result is that, with no certain public intervention, the *PS* relies on the absence of negative contamination; a certain bailout permits the *BR* to acquire value by shifting the burden of negative contamination onto the external funds.

In our opinion, Proposition 1 and its Corollary 2 are able to explain the prevalence of *PS* in the US environment.

The next Proposition also provides conditions under which PS can be less valuable than BR :

Proposition 3. If there is not full commitment, BR are more valuable than PS, if one of the following situations holds true:

(a) there are no default costs;

(b) the default costs are positive, but the bailout probability is one, for all the entities, $\pi_h = \pi_i = 1$.

When $\beta < 1$, since the rescue events of the home and portfolio in the *BR* structure include the rescue of the *PS* – the two differ in negative contamination – both default costs and bailout value are greater than in the *PS*, because β_{hBR} and β_{iBR} are greater in the *BR*. With no default costs, bailout makes the *BR* more valuable. If the bailout probability is 1 for each member of the group, the default costs never occur, and bailout makes the *BR* more valuable. If $\beta < 1$ and $\pi_h = \pi_i = 0$, the default costs still make the *BR* less valuable.

Another effect that has been observed over time is the increase in the complexity of BHCs, as measured by the numbers of affiliates. The following Corollary explains this effect:

Corollary 4. Under the assumptions of *Proposition* 1, the difference between merged and parent-subsidiary values weakly increases with the number of subaffiliates. If they are unlevered, such a difference increases.

Therefore, one way of increasing value is to increase the organizational complexity of BHCs organized through a parent, thus leaving to it the burden of raising capital for the group. It should be noted that the value of branches does not necessarily increase with the number of affiliates, because they can generate more negative default costs than bailout advantages. The role of bailout is also important in the comparison with ring-fenced banks:

Corollary 5. When the assumptions of Proposition 1 also hold for ringfenced banks, branches and parent-subsidiaries are less (more) valuable than ring-fenced banks if $\pi_h = \pi_i = 1$ ($\pi_h = \pi_i = 0$).

This Corollary may explain the cases in which banking groups in the US are organized as ring-fenced entities. The latter are penalized by the absence of internal rescue when they do not rely on bailout, but are definitely more valuable when they can rely on bailout.

5.2. Leverage-dependent advantage of PS vs. BR

How does the possibility of choosing leverage and of it possibly being different between the home and the subaffiliates affect these results?

The last property considered in Appendix B is the following:

Proposition 6. Under the assumptions of *Proposition* 1, including the low tax rate and full commitment, but excluding an equal choice of leverage, the value of the PS is greater than BR:

(a) if the home is unlevered and the subsidiaries are equally levered,

(b) if the home is levered, but the subsidiaries are not.

Therefore, leverage – and debt diversity in particular – can be used to maximize the value of the guarantees. The Proposition in fact states that PS can also outperform the branches when they are equally valued with equal debt, provided that the leverage is on either only the affiliates (case (a)) or only on the home (case (b)). The option value of the guarantees, which we have illustrated for the stand-alone case, is boosted by this choice. It should be noted that leverage increases the value of PS, thanks to coinsurance and bailout, but the value of debt does not grow linearly with its face value (see Appendix A). The incentive to raise it decreases, because of default costs.

The next Section applies these properties and extends them.

6. US BHCs

We now calibrate the above model to US holding companies, using the statistics presented in Correa and Goldberg (2022), who analyzed BHCs with at least \$25 billion in total real assets over the 1996Q1 to 2018Q2 period. We show that, in order to explain their prevalence, even when some of the assumptions in the Propositions above are not respected, for instance because the tax rate is not zero, and the other parameters are set to "realistic" values, *PS* structures are still largely superior in value to *BR* and *RF* banks.

First we conduct the calibration assuming that banks do not optimize their leverage – i.e., all BHCs are equally levered – and then that they do. Last, we study the effect of increasing the organizational complexity of BHCs.

6.1. Basic calibration

We normalize the initial value of the loans from each affiliate to $L_i(0) = 100, i = h, s, b, a$ for the home, subsidiary, branch and affiliates in a ring-fenced group, respectively.

We assume that the log returns on loans, X_i in $L_i(T) = L_i(0) \exp(X_i)$, are Gaussian with a risk-neutral mean $\mu = (r - \sigma^2/2)T$ and variance $\sigma^2 T$. Correa and Goldberg (2022) reported that the standard deviation of returns on assets for US BHCs was 1% over the 1996-2018Q2 period, so we set $\sigma = 1\%$.⁶

Consistently with this choice, we use an extension of the above formulas with a positive riskless rate and set the interest rate r = 2.59%, which is the GDP growth rate over the same period provided by Correa and Golberg. By so doing, we imagine an equilibrium situation in which the GDP growth rate is equal to the riskless rate.

The time horizon is set to five years, T = 5, which is also the average duration of debt for the non-financial sector in the USA.

Knowing that the correlation between the assets involved has little effect on the results, according to the structural model of Leland (2007), we set it to a mild $\rho = .2$, as in Leland (2007).

The tax rate k is equal to 5%, on condition the effective tax rate is lower than the nominal.

The default costs rate is set to $\alpha = 15\%$, so that the recovery rate on the *SA* bank, computed according to Merton's rule, in the case of Government bailout, is as high as 85%. Therefore, we do not consider the chosen level of α too low.

Finally, the probability of the bailout of each single affiliate is $\pi_i = \pi_h = 5\%$, i = 1, 2, ..N - 1. This figure reflects the decreased value of guarantees from Government bailout in OECD countries after the Great Recession, as documented by Schich and Kim (2012).

We also assume that the commitment to rescue is full ($\beta = 1$) in order to reflect the FIRREA Act pertaining to *PS*, and to put the branches at the same level of commitment.

Table 1 illustrates the results obtained when leverage of the parent is the double of the affiliate. We compute – for each rescue type and each affiliate – their current value of debt D_{0i} for the given face value of deposits F_i , the corresponding equity value, E_{0i} , the single member and group values, GV_{PS} , GV_{BR} , GV_{RF} . The obtained results are presented in two columns, one for the home and the other for the affiliate, and in three blocks, for the *PS*, *BR* and *RF* activities.

In Table 1, we set the level of deposits F_i in the affiliate, for i = s, b, a, so that, in its present value, it produces the ratio to assets reported by Correa and Goldberg for the US BHCs, namely 61%. The face level of deposits is set so that its present value at the rate 2.59% for five years is 61%. Indeed, $F_i \exp(-2.59\% \times 5)/100 = 61\%$ if $F_i = 79$. Therefore, the subsidiary/branch/ring-fenced bank is indebted in the same way as the average bank presented in Correa and Goldberg.

The home leverage is arbitrarily set to the double, that is, $F_h = 158$, so that we can represent the capital provision by the home and observe the debt diversity and internal capital markets at play.⁷

The overall GV value of RF is the highest, and it is closely followed by PS and, albeit further away, by BR. This shows that the superiority of PS with respect to BR also holds beyond Proposition 1, when we go in the direction of Proposition 6 using debt diversity. Ringfencing is more valuable than the other organizations because we are not considering the same case as that of Proposition 6 (the bailout probability is not zero). Even a small, positive bailout probability, with debt diversity, makes ring-fencing valuable.

It is possible to perceive, from the debt and equity row of each block, how nuanced the value impact that we were able to point out is in terms of split between the two affiliates, and between the payoffs to debt and equity holders. The first, striking result is that, in spite of an equal probability of bailout, the same face value of deposits corresponds to different expected values for the depositors D_0 in different organizational forms. However, this does not happen for both the more and the less indebted banks. The value for the more indebted affiliate (the home) is smaller in the *RF* case (no rescue),

Table 1

Parent subsidiary			
Variables	Symbols	Values	
		Home bank	Subsidiary
Face value of deposits	F	158	79
Current value of deposits	D_0	86.1998	69.4042
Discounted expected loss	DeL_0	53.6851	0.5382
Equity	E_0	0.0000	26.0170
Single bank value	$V_0 = E_0 + D_0$	86.1998	95.4212
Group value	$GV_{PS} = V_{0h} + V_{0s}$	181.6210	181.6210
Branch			
Variables	Symbols	Values	
		Home bank	Branch
Face value of deposits	F	158	79
Current value of deposits	D_0	106.4890	69.4042
Discounted expected loss	DeL_0	33.3959	0.5382
Equity	E_0	0.0000	0.0000
Single bank value	$V_0 = E_0 + D_0$	106.4890	69.4042
Group value	$GV_{BR} = V_{0h} + V_{0b}$	175.8933	175.8933
Ring Fenced			
Variables	Symbols	Values	
		Home bank	Affiliate
Face value of deposits	F	158	79
Current value of deposits	D_0	86.1997	69.4042
Discounted expected loss	DeL_0	53.6852	0.5382
Equity	E_0	0.0000	26.0173
Single bank value	$V_0 = E_0 + D_0$	86.1997	95.4215
Group value	$GV_{RF} = V_{0h} + V_{0a}$	181.6212	181.6212

Comparison of the GV value, the $DeL_0 = F - D_0$ default loss and other features with non-optimized leverage; k = 5%, $\pi = 5\%$, $\alpha = 15\%$, $\sigma = 1\%$ /year, equal across affiliates, $\rho = 0.2$, T = 5, initial, pre-tax asset value (100) equal across affiliates.

slightly higher in the *PS* (coinsurance) case and even higher in the *BR* organization, for which contamination here does not prevail. The value of the deposits for the least indebted subsidiaries is the same across the organization, because, in this case, no rescue is likely to be needed.

If we look at equity across the same three organizations, we discover that the equity of the most indebted affiliate is always zero: therefore high deposits, which, as we mentioned above, are partially paid back through bailout and rescue (for *PS* and *BR*), do not leave any expected payoffs to the shareholders. This points to the subtlety of the value split between equity and debt, something which is not visible in the closed-form formulas of the previous section.

6.2. Exploiting optimal debt diversity

We now deal with the calibration of the US scenario in more depth by considering that, within the sample of Correa and Goldberg, there was a great deal of heterogeneity in the actual deposit-to-asset ratio, presumably because banks optimize their deposit policy and leverage. This might explain why, in the BHCs studied by Correa and Goldberg, different entities emerged as heterogeneous, in that they ended up choosing different levels of the deposit-to-asset ratio.

Even in the model presented in Section 4, the amount of the deposits counts for the overall shareholders' value. It affects value because higher deposits increase the tax savings of debt (through \bar{L}_{i0} , if k > 0) and the Government bailout value, but also the default costs. The trade off between the former two – or the former, if taxes are mute as in our Propositions and Corollaries – and the latter determines whether value increases or decreases with deposits, and this trade off works differently, because of the guarantees, in the *PS*, *BR* or *RF* banks. We now consider the case in which banks maximize value by choosing the level of deposits in the home and subaffiliates, and present the results in Table 2.

Debt diversity was considered to already exist, by assumption, in Table 1. Here we show that an optimized choice of leverage exacerbates

⁶ We therefore extend also the assumption of states of the world of equal probability of the theoretical part.

⁷ Since all the parameters (default costs, bailout probability, tax rate) and their sizes are the same across the affiliates, it is possible to switch the debt between the parent and the subsidiary without changing the comments.

Table 2

Parent subsidiary			
Variables	Symbols	Values	
		Home bank	Subsidiary
Face value of deposits	F^*	205	0
Current value of deposits	D_0^*	180.0703	0.0000
Discounted expected loss	DeL_0^*	1.4259	0.0000
Equity	E_0^*	0.0000	14.8092
Single bank value	$V_0^* = E_0^* + D_0^*$	180.0703	14.8092
Group value	$GV_{PS}^* = V_{0h}^* + V_{0s}^*$	194.8795	194.8795
Branch			
Variables	Symbols	Values	
		Home bank	Branch
Face value of deposits	F^*	107	98
Current value of deposits	D_0^*	93.9990	86.0964
Discounted expected loss	DeL_0^*	0.7332	0.6677
Equity	E_0^*	1.8470	9.1477
Single bank value	$V_0^* = E_0^* + D_0^*$	95.8460	95.2440
Group value	$GV_{BR}^* = V_{0h}^* + V_{0b}^*$	191.0900	191.0900
Ring Fenced			
Variables	Symbols	Values	
		Home bank	Affiliate
Face value of deposits	F^*	101	101
Current value of deposits	D_0^*	88.7260	88.7260
Discounted expected loss	DeL_0^*	0.6941	0.6941
Equity	E_0^*	6.8074	6.8074
Single bank value	$V_0^* = E_0^* + D_0^*$	95.5334	95.5334
Group value	$GV_{RF}^{*} = V_{0h}^{*} + V_{a}^{*}$	191.07	191.07

Comparison of the *GV* value, the *DeL* default loss and other features with optimized leverage; k = 5%, $\pi = 5\%$, $\alpha = 15\%$, $\sigma = 1\%$ /year, equal across affiliates, $\rho = 0.2$, T = 5, initial asset value (100) equal across affiliates.

debt diversity. We expect debt diversity to make the total deposits in a parent-subsidiary even greater than in ring-fenced banks or branches, even for equal tax rates and default costs, if the parent or the portfolio is unlevered, as in Table 2. This is the reasoning behind Proposition 6.

In the *PS* case, there is always an incentive to concentrate debt in just one entity, in order to increase coinsurance. Indeed, if the other affiliate is unlevered or only slightly levered, it does not have to pay back or to pay back too many deposits and it can therefore provide a great deal of coinsurance. Contamination does not occur, so that one highly levered affiliate does not endanger the survivorship and the value of another.

In *BR*, because of contamination, there is no incentive to differentiate deposits in the home and branches, i.e. to create debt diversity. This happens in spite of the many non-linearities inherent to the functioning of the default costs and tax savings,⁸ if none of the affiliates has particularly favorable conditions, such as a lower proportional default cost and a higher tax rate.

In the ring-fenced case, since there are no guarantees, the optimal face value is the same as the one that would maximize the value of one affiliate.

When the *PS* fully exploits its coinsurance-without-contamination profile, by pushing debt diversity to the limit in an affiliate without deposits, the *PS* group value rises with respect to Table 1 and with respect to the other two arrangements: *BR* and *SA*. Indeed, the *BR*, because of contamination, exploits very little debt diversity and ends up with a lower value than the *PS*, but which is higher than the *RF*, which does not exploit debt diversity to any extent whatsoever, because it has no rescue.

Table 3	
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Symbols	Values	
	Home bank	Subsidiary
F^*	550	193
D_0^*	116.1807	169.5572
DeL_0^*	369.8751	1.3149
E_0^*	0.0000	21.4726
$V_0^* = E_0^* + D_0^*$	116.1807	191.0298
$GV_{PS}^* = V_{0h}^* + V_{0s}^*$	307.2105	307.2105
Symbols	Values	
	Home bank	Branch
F^*	111	197
D_0^*	97.5079	173.0712
DeL_0^*	0.0094	0.0001
E_0^*	0.2167	15.8393
$V_0^* = E_0^* + D_0^*$	97.7246	188.9105
$GV_{BR}^* = V_{0h}^* + V_{0b}^*$	286.6351	286.6351
Symbols	Values	
	Affiliate	Home ban
F^*	201	101
D_0^*	176.5802	88.7260
DeL_0^*	0.0053	0.6041
E_0^*	14.4874	6.8074
$V_0^* = E_0^* + D_0^*$	191.0676	95.5334
$GV_{RF}^* = V_{0h}^* + V_{0a}^*$	286.6	286.6
	$F^{*} D_{0}^{*} DeL_{0}^{*} E_{0}^{*} + D_{0}^{*} DeL_{0}^{*} E_{0}^{*} + D_{0}^{*} GV_{PS}^{*} = E_{0}^{*} + D_{0}^{*} GV_{PS}^{*} = V_{0h}^{*} + V_{0s}^{*}$ Symbols $F^{*} DeL_{0}^{*} E_{0}^{*} + D_{0}^{*} GV_{BR}^{*} = V_{0h}^{*} + V_{0b}^{*}$ Symbols $F^{*} D_{0}^{*} E_{0}^{*} + V_{0b}^{*} E_{0}^{*} + V_{0b}^{*}$ Symbols	$\begin{tabular}{ c c c c c } \hline Home bank \\ \hline F^* 550 \\ D_0^* 116.1807 \\ DeL_0^* 369.8751 \\ E_0^* 0.0000 \\ $V_s^* = E_0^* + D_0^*$ 116.1807 \\ $GV_{PS}^* = V_{0h}^* + V_{0s}^*$ 307.2105 \\ \hline \\ $

Comparison of the *GV* value, the *DeL* default loss and other features with optimized leverage; k = 5%, $\pi = 5\%$, $\alpha = 15\%$, $\sigma = 1\%$ /year, equal across affiliates, $\rho = 0.2$, T = 5, the initial asset value of the subsidiary/branch/affiliate (200) is the double of the home value (100) and their correlation is negative.

6.3. Increasing the number of subaffiliates

The top BHCs in Avraham et al. (2012) had more than 1000 subsidiaries. The 29 groups classified as GSIBs by the FSB, according to the same Authors, had on average 1002 subsidiaries, and they had 2.6x more subsidiaries than the non-financial institutions that were largest in market capitalization. The average number of affiliates in Correa and Goldberg (2022) was 404, with a standard deviation of 715, a minimum of 4, and a maximum of 4494. These affiliates, unlike the ones in our model, did not have the same "size", as measured by the initial assets, since the minimum asset size was \$25 billion and the maximum was 2542, that is, 100 times greater.

We now investigate the effect of increasing the number of affiliates, while keeping their individual initial asset value, which represents their size, equal, according to Corollary 4. In order to facilitate the comparison with the simulations presented so far, we here raise the number of subaffiliates from 1 to 2. The home affiliate remains the same as above but the other affiliate is twice its size, because it is made up of two subaffiliates. This permits the total value of assets of the group, which is now 300, to be kept similar to the average asset size of Correa and Goldberg, 278 billion. In all the Tables, we put an exogenous constraint of 550 (slightly below twice the initial asset value) on the deposits of the *PS*. Otherwise, the parent would choose very large values for deposits.

Tables 3–5 correspond to different correlations between the two subaffiliates and therefore to different volatilities of the subsidiary, branch affiliate or affiliate tout court in the *RF* case. The largest affiliate includes two activities with return correlations -1/2, 0 or 1, and the volatility of the large affiliate obviously increases as the correlation increases. In Table 3, the volatility for the large affiliate is half of the smaller one in percentage of the initial asset values. In

⁸ Both rescues and bailout take the form of options, where the face value of deposits in each bank determines the strike *F* and the underlying \bar{L} , in a non-linear way.

Table 4

Parent subsidiary			
Variables	Symbols	Values	
		Home bank	Subsidiary
Face value of deposits	F^*	550	195
Current value of deposits	D_0^*	116.2574	171.3047
Discounted expected loss	DeL_0^*	370.6837	1.3380
Equity	E_0^*	0.0000	19.7270
Single bank value	$V_0^* = E_0^* + D_0^*$	116.2574	191.0317
Group value	$GV_{PS}^* = V_{0h}^* + V_{0s}^*$	307.2891	307.2891
Branch			
Variables	Symbols	Values	
		Home bank	Branch
Face value of deposits	F^*	114	189
Current value of deposits	D_0^*	100.1396	166.0429
Discounted expected loss	DeL_0^*	0.0133	0.0001
Equity	E_0^*	0.0145	20.4083
Single bank value	$V_0^* = E_0^* + D_0^*$	100.1541	186.4512
Group value	$GV_{BR}^* = V_{0h}^* + V_{0b}^*$	286.6053	286.6053
Ring Fenced			
Variables	Symbols	Values	
		Affiliate	Home bar
Face value of deposits	F^*	196	101
Current value of deposits	D_0^*	172.1799	88.7260
Discounted expected loss	DeL_0^*	0.0129	0.6941
Equity	E_0^*	18.8543	6.8074
Single bank value	$V_0^* = E_0^* + D_0^*$	191.0341	95.5334
Group value	$GV_{BR}^* = V_{0h}^* + V_{0a}^*$	286.57	286.57

Comparison of the *GV* value, the *DeL* default loss and other features with optimized leverage; k = 5%, $\pi = 5\%$, $\alpha = 15\%$, $\sigma = 1\%$ /year, equal across affiliates, $\rho = 0.2$, T = 5, the initial asset value of the subsidiary/branch/affiliate (200) is the double of the home value (100) and they are decorrelated.

Table 4, the volatility of the large affiliate is still lower than the initial one in percentage assets. In Table 5, the two volatilities are the same, because there is no diversification.

Table 3 shows, for the *PS* structure, that doubling the size of the subsidiary with a reduced volatility in comparison to the home entity leads to an exceptionally large increase in the leverage of the smaller home affiliate as well as a huge increase in the leverage of the large subsidiary (from 205 and 0 to 550 and 193, with 550 imposed as a constraint). The powerfulness of debt diversity in *PS* is very much at play, with the large subsidiary capable of rescuing the smaller home one and of levering up itself: the high leverage in the smaller home leads to an increase in the group value, which is now higher than the total initial asset value (307 and 300, while it was 194, all the others being equal, when the assets were 200). The difference in value between the three blocks becomes more remarkable than in Tables 1 and 2, which points to the magnitude of the effect in realistic situations of hundreds of subsidiaries.

Increasing the correlation between the activities in the large affiliate in the *PS* case, as shown in Tables 4 and 5, changes value very little. Thus, in *PS* the large increase in value is explained primarily by the change in the number of the banks in the portfolio of affiliates, and therefore by the group size, and not by the correlation of its affiliates.

Turning to the *BR* structure, Tables 3 to 5 show that this structure cannot benefit as much as the *PS* structure from internal rescue arrangements between different-sized affiliates. The *BR* structure still obtains a lower group value than the *PS* structure. As in the *PS* case, correlation does not affect these remarks to any great extent.

Turning to the RF structure, it should be pointed out that the home bank is identical, in all respects, to that shown in Table 2. The other stand-alone bank is twice as large. In Table 3, there is diversification

Table 5	
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Parent subsidiary			
Variables	Symbols	Values	
		Home bank	Subsidiary
Face value of deposits	F^*	550	187
Current value of deposits	D_0^*	116.2574	164.2733
Discounted expected loss	DeL_0^*	370.6837	1.2867
Equity	E_0^*	0.0000	26.7131
Single bank value	$V_0^* = E_0^* + D_0^*$	116.2574	190.9864
Group value	$GV_{PS}^* = V_{0h}^* + V_{0s}^*$	307.2438	307.2438
Branch			
Variables	Symbols	Values	
		Home bank	Branch
Face value of deposits	F^*	176	119
Current value of deposits	D_0^*	154.6064	104.5456
Discounted expected loss	DeL_0^*	0.0157	0.0000
Equity	E_0^*	0.0000	27.4087
Single bank value	$V_0^* = E_0^* + D_0^*$	154.6064	131.9543
Group value	$GV_{BR}^* = V_{0h}^* + V_{0b}^*$	286.5607	286.5607
Ring Fenced			
Variables	Symbols	Values	
		Affiliate	Home banl
Face value of deposits	F^*	188	101
Current value of deposits	D_0^*	165.1498	88.7260
Discounted expected loss	DeL_0^*	0.0147	0.6941
Equity	E_0^*	25.8404	6.8074
Single bank value	$V_0^0 = E_0^* + D_0^*$	190.9902	95.5334
Group value	$GV_{RF}^* = V_{0h}^* + V_{0a}^*$	286.52	286.52

Comparison of the *GV* value, the *DeL* default loss and other features with optimized leverage; k = 5%, $\pi = 5\%$, $\alpha = 15\%$, $\sigma = 1\%$ /year, equal across affiliates, $\rho = 0.2$, T = 5, the initial asset value of the subsidiary/branch/affiliate (200) is the double of the home value (100) and their correlation is one.

and lower volatility. In Tables 4 and 5, the volatility of the portfolio increases, albeit with mild effects.

In short, the difference in size provides strong incentives to create high leverage in the small affiliates, somewhat independently of the correlation, even beyond the hypotheses of Corollary 4. Size does not affect the value, in percentage, to any great extent. This should be kept in mind, especially when the number of subsidiaries becomes large, as in reality, and when the highest possible value for deposits is not bound.

The Tables show that, even though the tax rates are positive and the bailout probability is not extreme, and therefore the simple conditions of the above Propositions are not satisfied, the incentive to organize groups as *PS* remains, and even more so when the portfolio of subaffiliates has greater assets than the parent.

7. Policy interventions and conclusions

We now address the efficacy of different policy instruments. Bearing in mind Corollary 5 and Proposition 6, we focus on ring-fencing and leverage constraints as available instruments.

An obvious policy response to the objective of reducing the spillover effects of a bank's default is to ring-fence legal entities and, therefore, to prevent asset transfers with the objective of rescuing a defaulting affiliate. This is the purpose of the Volcker rule in the US, the Vickers proposal in the UK, and the Liikanen report in the EU. The Volcker rule (Section 619 of the Dodd–Frank Act), among other things, prohibits banks from entering into transactions with the institutions they advise, but also from rescuing them. Similarly, the Vickers Commission in the UK prohibits transactions – other than market-price-based ones – between regulated commercial banks and other affiliates in the same BHC. Since the prohibited transactions may include rescue actions, the

Vickers Commission de facto rules out rescues. The Liikanen report is also directed at banning rescue.

In terms of our model, if rescue is eliminated, the legal entities within a banking group become RF affiliates. Tables 2 to 5, in which only Government bailout is carried out in 5 cases out of 100 ($\pi = 5\%$ both for the home and the subaffiliates), show that - when leverage is optimized - there is a reduction in the group value. This happens with home and subsidiary of equal size – both if the group is a PS one and if it is a BR one. This is evident, for instance, from Table 2, where it is possible to observe that, going from a PS or BR to a RF group, the group value decreases by 2 and .5%, respectively. However, the PS phenomenon is even more important when the size of the portfolio of affiliates increases. In Table 5, for instance, with a portfolio that is twice the size of the home, the value decreases by 6.5% from PS to RF. Therefore, ring fencing has strong effects, especially when starting from PS, and especially when the size is large, both under and beyond the conditions of Proposition 5. Indeed, from the latter we discover that *RF* is more valuable that *BR* or *PS*, with high π , while the opposite emerges here for low π , for full commitment and no taxes.

Our model has obviously been simplified, in that, in reality, there may be a mismatch between the legal structure and the business function, and not all services are critical for default. However, it is likely that, as in our model, strong ring-fencing is highly beneficial in reducing expected losses but also highly detrimental to value, at least when the consequences are not nullified by the intervention of the Government. If, instead, bailout occurs in all cases, and nullifies the default costs, ring-fencing is optimal for shareholders and depositors, because internal guarantees are substituted by centralized bailout, at no extra cost. According to our model, when in practice we see equity values going up after ring-fencing, an implicit reliance on bailout is at play.

Another policy response to default loss may be to impose a leverage constraint on each legal entity. Broadly speaking, this is the case of the different capital ratios that have been imposed in the EU over the last few decades. The previous arguments show that it is not sufficient to impose a constraint on the banking group as a whole, since rescue arrangements between different legal entities can be used to allocate deposits of the individual legal entities differently and to keep default losses high, as in *PS*.

Tables 3 to 5 show that a capital constraint of 550, with respect to an initial asset value of 100 in the home, leads to a much higher BHC value, if it is organized as *PS* with respect to the other two organizations, than when the debt is artificially set to 158, as in Table 1, or when, because the portfolio of subsidiaries is smaller, as in Table 2, it only grows up to 205, thus leaving the subsidiary unlevered. However, these increases in value go hand in hand with an increase in a magnitude that we have not observed so far, the discounted expected loss due to default, DeL_0 , namely the difference between the face value of debt, *F*, and the present value of what depositors obtain, D_0 . This is the loss on creditors generated by default. In *PS*, higher value goes together with greater DeL. Tables 3 to 5 suggest that limits on leverage in individual subsidiaries may, in principle, be an efficient way of dealing with default losses generated by *PS*, which affect their creditors.

In conclusion, this paper has examined the likely effect of rescue arrangements, and justified their importance because of both the rescue practices observed over time and their effect on a theoretical model calibrated to US BHCs. The calibration showed that such a value is higher in *PS* than in *BR* and *RF* groups, especially when leverage is optimized and the portfolio of affiliates is larger than the home bank (twice in size or more). This may explain the dominance of *PS* arrangements in BHCs, and the risk associated with complex organizations, such as BHCs, especially those with a large number of affiliates. Herein, we

have provided sufficient conditions for the phenomenon to occur.

We have shown, by means of the calibrated examples, and in theory, that a constraint on the home bank level of deposits may be effective in reducing default losses, and may be much more effective than other organizational forms. Therefore, for BHCs organized as *PS*, constraints on leverage, when appropriately localized, are likely to be effective in reducing default losses, even though they reduce value, as ring-fencing does, whenever the bailout probability is low.

Appendix A

Let i = s, b, a depending on the case we study. In the branch case, the capital transfers are as follows, since the transfer is conditional to the R_{BR}, R'_{BR} events. In the case of rescue from the home bank, the capital transfer is

$$\min\left(\bar{L}_{h}(T) - F_{h}, \sum_{i=1}^{N-1} \left[F_{i} - \bar{L}_{i}(T)\right]\right),$$
(11)

In the case of rescue from the portfolio to the home bank, the transfer is

$$\min\left(\sum_{i=1}^{N-1} \left[\bar{L}_i(T) - F_i\right], F_h - \bar{L}_h(T)\right).$$
(12)

The *Q* and *Q'* events are as follows. In *Q*, bailout of the portfolio occurs if the latter is not able to pay depositors back, because $\sum_{i=1}^{N-1} [F_i - \bar{L}_i(T)] > 0$, and the home has not enough assets – once it has paid its own depositors – to cover the lack of funds of the portfolio, $\bar{L}_h(T) - F_h < \sum_{i=1}^{N-1} [F_i - \bar{L}_i(T)]$. In *Q'*, the home is not able to pay depositors back, because $F_h - \bar{L}_h(T) > 0$, and the branch, which is obliged to try to rescue the home, does not have enough capital – once it has paid its own depositors – to cover the lack of funds of the home, $\bar{L}_h(T) - F_h > \sum_{i=1}^{N-1} [F_i - \bar{L}_i(T)]$.

In the parent-subsidiary case, rescue of the subsidiary occurs if, and only if, the home bank is not in default or distress $(\bar{L}_h(T) > F_h)$, the subsidiary portfolio is in default, because their asset value is below the default level $(\sum_{i=1}^{N-1} (\bar{L}_i(T) - F_i) < 0)$, and rescuing the subsidiary does not drive the home bank into default. In this case, rescue means that, using its surplus $\bar{L}_h(T) - F_h$, the home bank pays the part of the subsidiary's deposits that is not covered by its own activities, $\sum_{i=1}^{N-1} (F_i - \bar{L}_i(T))$. The home-bank can do this without facing default if its surplus is greater than the amount needed for the rescue, $\bar{L}_h(T) - F_h > \sum_{i=1}^{N-1} (F_i - \bar{L}_i(T))$. The conditions can be reduced to the R_{PS} event in the text, and in the same way for R'_{PS} .

The group value of all the organizations is the sum of the values of its stakeholders, namely

$$GV_j = D_{0h} + E_{0h} + \sum_{i=1}^{N-1} \left(D_{0i} + E_{0i} \right)$$
(13)

where j = PS in the parent-subsidiary structure and *BR* in the branch structure. Let us now use the events and payoffs to stakeholders described in the main text to derive the overall expressions for the branch group value (7) and parent-subsidiary value (10) from the above expression.

In the branch case, rescue of the home bank by the portfolio of subaffiliates affects the payoffs to the debt holders of the home bank positively – compared to a stand alone bank – since it entails a transfer to them if the event R' occurs. The home debt value is therefore the value it would have without bailout and rescue, plus the Government bailout – which the home would also enjoy as an individual bank, although in different circumstances, but not in Q' – plus the value of rescue received from the portfolio banks, minus default costs. Default costs do not occur in the same circumstances as for an individual bank, because default is avoided if the Q' event – unsuccessful rescue from the portfolio – occurs:

$$D_{0h} = + \left[F_h - \mathbb{E} \max(0, F_h - \bar{L}_h(T)) \right]$$
(14)

value without bailout and rescue

$$+ \underbrace{\pi_{h}\mathbb{E}\left\{\max(F_{h} - \bar{L}_{h}(T), 0)\mathbf{1}_{\{Q'\}}\right\}}_{\text{Government bailout}} \\ + \underbrace{\mathbb{E}\left\{\left[F_{h} - \bar{L}_{h}(T)\right]\mathbf{1}_{\{R'\}}\right\}}_{\text{rescue received}} \\ -(1 - \pi_{h})\alpha_{h}\mathbb{E}\left[\bar{L}_{h}(T)\mathbf{1}_{\{Q'\}}\right]. \\ \underbrace{\text{default costs}}_{\text{default costs}}$$

Since the rescue of the branch is made by the home bank, the rescue diminishes the equity value of the home-bank, in comparison with the situation of an individual bank. The home equity value is the individual value minus the expected injection of assets to the subaffiliates to rescue them

$$\begin{split} E_{0h} &= \underbrace{\mathbb{E} \max\left[F_h - \bar{L}_h(T), 0\right]}_{\text{value without rescue of the subaffiliates}} \\ &= \underbrace{\mathbb{E} \left\{\sum_{i=1}^{N-1} \left[F_i - \bar{L}_i(T)\right] \mathbf{1}_{\{R\}}\right\}}_{i=1} \quad , \end{split}$$

capital given to the subaffiliates to rescue them

The debt value of the portfolio of subaffiliates is the value they would enjoy without bailout and rescue, namely upon no default, plus the assets they receive in the case of default and Government bailout, plus what they receive in the case of default and home rescue, minus the default costs that occur in the case neither rescue from the home or Government rescue hold true:

$$\sum_{i=1}^{N-1} D_{0s} = \underbrace{\left[\sum_{i=1}^{N-1} F_{i} - \mathbb{E} \max(0, \sum_{i=1}^{N-1} \left[F_{i} - \bar{L}_{i}(T)\right])\right]}_{\text{value without bailout and rescue}} + \underbrace{\mathbb{E} \left\{ \max(\sum_{i=1}^{N-1} \pi_{i} \left[F_{i} - \bar{L}_{i}(T)\right], 0)\mathbf{1}_{\{Q\}}\right\}}_{\text{Government bailout}} + \underbrace{\mathbb{E} \left\{\sum_{i=1}^{N-1} \left[F_{i} - \bar{L}_{i}(T)\right]\mathbf{1}_{\{R\}}\right\}}_{\text{rescue from the home}} - \underbrace{\mathbb{E} \left[\sum_{i=1}^{N-1} (1 - \pi_{i})\alpha_{i}\bar{L}_{i}(T)\mathbf{1}_{\{Q\}}\right]}_{\text{default costs}}$$
(15)

The equity value of the portfolio banks is the one they would have without the provision of rescue to the home, minus what they inject into the home in the case the portfolio rescues its home bank, namely when the event R' occurs:

$$\sum_{i=1}^{N-1} E_{0i} = \mathbb{E} \max \left[\sum_{i=1}^{N-1} \left[\bar{L}_i(T) - F_i \right], 0 \right] \\ - \mathbb{E} \left\{ \left[F_h - \bar{L}_h(T) \right] \mathbf{1}_{\{R'\}} \right\},$$

By substituting R, R', Q, Q' for the events, we obtain (7), and in the same way for (10).

Appendix B

This Appendix proves the Propositions and Corollaries of Section 5. We begin with Proposition 1. **Proof.** Under the stated assumptions, the values of PS and BR (10) and (7) in the text, have equal parameters across organizations:

$$GV_{PS} =
\bar{L}_{h0} + \sum_{i=1}^{N-1} \bar{L}_{i0}
+ \underbrace{\pi_h \beta_{hPS} \mathbb{E} \max(0, F_h - \bar{L}_h(T))}_{\text{Government bailout home}}
- \underbrace{\alpha_h (1 - \pi_h) \beta_{hPS} \mathbb{E} \bar{L}_h(T)}_{\text{default cost home}}
+ \mathbb{E} \max \sum_{i=1}^{N-1} \pi_i \beta_{iPS} (F_i - \bar{L}_i(T), 0)
\underbrace{-\mathbb{E} \sum_{i=1}^{N-1} (1 - \pi_i) \alpha_i \beta_{iPS} \bar{L}_i(T)}_{\text{Government bailout subsidiary}}$$
(17)

and

default cost subsidiary

CV

$$= \bar{L}_{h0} + \sum_{i=1}^{N-1} \bar{L}_{i0}$$

$$= \bar{L}_{h0} + \sum_{i=1}^{N-1} \bar{L}_{i0}$$

$$+ \pi_h \beta_{hBR} \mathbb{E} \max(0, F_h - \bar{L}_h(T))$$
Government bailout home
$$- \alpha_h (1 - \pi_h) \beta_{hBR} \mathbb{E} \left[\bar{L}_h(T) + \max\left(0, \sum_{i=1}^{N-1} \left(\bar{L}_i(T) - F_i\right)\right) \right]$$
default cost home
$$+ \mathbb{E} \max \sum_{i=1}^{N-1} \pi_i \beta_{iBR} (F_i - \bar{L}_i(T), 0)$$
Government bailout branch
$$- \mathbb{E} \left[\sum_{i=1}^{N-1} \alpha_i (1 - \pi_i) \beta_{iBR} \left[\bar{L}_i(T) + \max(0, \bar{L}_h(T) - F_h) \right] \right]$$
default cost branch
$$(18)$$

When the tax rate drops to 0, $k \to 0$, the initial asset value for the home and subaffiliates is the same across the organizations, and the first lines of the two values are the same. Moreover, the bailout values of the two organizations, lines 2 and 4 of each expression, are also the same for $\beta \to 1$, since when the home and the portfolio coincide, the bailout events β_{hBR} and β_{iBR} for the two organizations coincide too. The default costs for the two organizations, lines 3 and 5 of each expression, are smaller or equal in *PS* than in *BR* because they do not contain the non-negative terms max $\left(0, \sum_{i=1}^{N-1} (\bar{L}_i(T) - F_i)\right)$, max $(0, \bar{L}_h(T) - F_h)$, derived from negative contamination. The statement follows.

We now prove Corollary 2.

Proof. Under the assumptions of the Proposition and $\pi_h = \pi_i = 1$, default never occurs, default costs are never paid, and the two organizations have the same value (lines 3 and 5 are the same).

We now prove Corollary 4

Proof. The term by which the *BR* and *PS* differ, under the assumption of the Proposition, weakly increases with the number of subaffiliates, since it depends on $\bar{L}_i(T) - F_i$, which is equal across *i*. If they are unlevered, the $\bar{L}_i(T) - F_i$ terms are strictly positive.

The Proof of Proposition 5 instead requires that we write the value of N for ring-fenced entities

Proof. The group value of N for ring-fenced entities is

$$V_{Na} = N\bar{L}_0$$

$$+\underbrace{N\pi_{a}\mathbb{E}\max(0, F - \bar{L}_{a}(T))}_{\text{Government bailout}}$$

$$-\underbrace{N\alpha_{a}(1 - \pi_{a})\mathbb{E}\left[\bar{L}_{a}(T)\mathbf{1}_{\{\bar{L}_{a}(T) < F_{a}\}}\right]}_{\text{default costs}}$$
(19)

If its α_a, k_a and π_a parameters are the same as those of the single members of *PS* and *BR*, and taxes are annihilated, the initial value of this configuration is equal to that of the others. For zero bailout probability, we need to compare

$$-\underbrace{N\alpha_{a}\mathbb{E}\left[\bar{L}_{a}(T)\mathbf{1}_{\left\{\bar{L}_{a}(T)< F_{a}\right\}}\right]}_{\text{default costs}}$$

with the default costs of PS and BR, respectively

$$-\underbrace{\alpha_{h}\beta_{hPS}\mathbb{E}\bar{L}_{h}(T)}_{\text{default cost home}} -\mathbb{E}\sum_{s=1}^{N-1}\alpha_{s}\beta_{sPS}\bar{L}_{s}(T)$$
(20)

default cost subsidiary

and

$$- \underbrace{\alpha_{h}\beta_{hBR}\mathbb{E}\left[\bar{L}_{h}(T) + \max\left(0, \sum_{i=1}^{N-1} \left(\bar{L}_{i}(T) - F_{i}\right)\right)\right]}_{\text{default cost home}}$$

$$-\mathbb{E}\left[\sum_{b=1}^{N-1} \alpha_{b}\beta_{bBR}\left[\bar{L}_{b}(T) + \max(0, \bar{L}_{h}(T) - F_{h})\right]\right]$$
(21)

default cost branch

For bailout, we need to compare

$$+N\mathbb{E}\max(0, F - \tilde{L}_a(T))$$

Government bailout

with

$$\underbrace{ \begin{array}{c} \underbrace{\beta_{hPS} \mathbb{E} \max(0, F_h - \bar{L}_h(T))}_{\text{Government bailout home}} \\ + \mathbb{E} \max \sum_{i=1}^{N-1} \beta_{sPS}(F_i - \bar{L}_s(T), 0) \end{array}}_{i=1}$$

Government bailout subsidiary

and

 $\underbrace{\frac{\beta_{hBR}\mathbb{E}\max(0, F_h - \bar{L}_h(T))}{\text{Government bailout home}}}_{+\mathbb{E}\max\sum_{k=1}^{N-1} \beta_{bBR}(F_i - \bar{L}_b(T), 0)}$ (23)

Government bailout branch

The statement follows.

Last, we prove Proposition 6

Proof. Let us once again consider the (17) and (18) values, together with the value of the *N* stand-alone banks, (19). If we insert the hypotheses of leverage (a), bailout of the home never occurs and has no value (2nd line vanishes), *R'*, rescue of the home, never takes place, so that, with full commitment, $\beta_{hPS} = \beta_{hBR} = 0$ and there are no home default costs. The comparison between the remaining terms of the *BR* and *PS* values follows. If we assume (b), there are neither portfolio rescue nor default costs, since, with full commitment, $\beta_{iPS} = \beta_{iBR} = 0$. The comparison between the remaining terms of the *BR* and *PS* values follows. This proves the Proposition.

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