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Bank solvency stress tests with fire sales[☆]

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ABSTRACT

We present a new framework combining current methods of bank solvency stress tests with a model of fire sales. We apply the framework to the stress tests conducted by the European Banking Authority. Fire sales are described by an equilibrium model balancing leverage improvements and drops in security prices. Additional bank losses caused by fire sales are significant and go beyond the trivial fact that with fire sales we will get bigger losses. Ignoring potential fire sales effects may lead to a false sense of resilience by assuming that institutions, which are in fact fragile, are resilient.

1. Introduction

Fire sales of assets and the resulting amplification of losses is a well-known and broadly studied element of banking and financial crises. The global financial crisis of 2007–2008 led to renewed interest and a large wave of modern research literature analyzing and modeling of fire sales as part of crises in the financial system.

Bank solvency stress tests have been established to analyze the consequences of potential losses on the solvency and viability of banks during a financial crisis. Since fire sales have played such an important role for losses incurred during a crisis, they should be a part of loss assessment in stress testing.

When we look at current stress testing practice, there seems to be, however, a gap between the amount of academic literature about fire sales and the actual application of loss analysis taking into account fire sales in actual stress testing.[1] The reason for stress testing practitioners' general reluctance to engage in a broader take up of the literature on fire sales, is perhaps a certain fear to overburden an already complex framework.

We offer a framework which can be applied in a straightforward way to the current stress test procedures by building on the large literature on fire selling. We hope to overcome stress testing practitioners' skepticism about the applicability of fire sale analysis and thus to support a wider adoption of stress tests which include fire sale losses as an integral part of risk assessment. We do so by showing how modeling ideas from the literature on fire sales can be applied to the stress test conducted by the European Banking Authority (EBA). The framework can be easily integrated into current stress testing methodology.

In the first part we present a model, building on previous contributions to the literature. We strive to build a simple and applicable bridge to current stress testing practice. One of the perhaps most general theoretical contributions focused on fire sales in a stress-test-like framework is a recent paper by Banerjee and Feinstein (2021). It serves as a general reference for us which contains our model as a special case.

We apply our framework to an actual stress testing context by analyzing the published EBA stress test data to compare a stress test that consider and do not consider fire sales. We demonstrate that the

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[1] The Bank of England seems to be an exception. It has taken a pioneering role building on the excellent work developed by Cont and Schaanning (2017).

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outcome differs materially. Not only does it differ in the trivial way of leading to larger losses but also with respect to the institutions that are classified as fragile by the stress test. Institutions can incur losses indirectly because of price impact effects on their assets. Thus, banks looking resilient in the traditional EBA approach look fragile when losses are evaluated more comprehensively. The results suggest that the traditional approach might create an illusion of resilience, which is in fact not as strong as it appears to be. The code as well as all the compiled and raw data we use for this paper are available to the interested reader in a public GitHub repository.[2]

We do not discuss how systemic risk exposure can be structurally assessed ex-ante. Nor do we discuss potential policy measures to mitigate such exposures, because this has already been exhaustively covered in the literature (for instance see Greenwood et al., 2015 or Duarte and Eisenbach, 2021, Cont and Schaanning, 2017).

Related literature. Fire sales have been a very prominent topic in the literature of the last ten years. Our paper builds on Cont and Schaanning (2017) as well as on the theory paper by Banerjee and Feinstein (2021). It differs from Cont and Schaanning (2017) in two main respects. In their paper the results of fire sales on prices are based on simulation, while our paper embeds this approach into a theoretical framework, which both reveals more clearly what is going on and at the same time allows for a more efficient computation of price impact. In terms of theory our framework is a special case of the more general theory of Banerjee and Feinstein (2021). The contribution here is mainly our exposition of the model, which is formulated to build a bridge to applications as clearly visible as possible. We also believe that simplicity in application and parsimony in computation are important to make the model attractive for stress testing practitioners. In contrast to Banerjee and Feinstein (2021), we therefore use a very simple specific rather than a general model of security supply in a fire sale. Given the very coarse precision of the data and a lack of systematic empirical evidence on how banks actually behave in an actual fire sale situation, it seems reasonable to use a simple rule, and make the model thus easier to use in actual stress tests. In the data part we take a perspective that differs from Cont and Schaanning (2017). While they focus on triggers of fire sale waves and the role of liquidity-weighted overlapping portfolios, we directly compare a loss evaluation with and without fire sales using EBA data.

Two seminal and pioneering papers in the applied literature on fire sales of banking assets were Greenwood et al. (2015) for Europe and Duarte and Eisenbach (2021) for the US. Both papers use – as a behavioral assumption for banks – a form of leverage targeting, in which banks try to maintain a constant leverage ratio in response to a shock. Adrian and Shin (2010) have documented this kind of behavior for large US investment banks over the medium term. It leads to a linear fire sales rule. We believe, however that such a rule is not the relevant behavioral assumption in times of distress. In contrast to this assumption, and in line with Cont and Schaanning (2017), we assume that banks react to losses in asset values or to sudden outflows of funding. The one-sided nature of the constraints we apply in our analysis leads to an asymmetry in gains and losses. Cont and Schaanning (2017) have pointed out the importance of this fact for loss analysis: Leverage targeting models tend to overestimate the magnitude of fires sales for small shocks and underestimated them for larger shocks. We thus do not work with the leverage targeting model in this paper. In contrast to our approach for the quantification of fire sale impact, both Duarte and Eisenbach (2021) as well as Greenwood et al. (2015) rely in a similar way as Cont and Schaanning (2017) on a simulation model. In line with these papers, we focus on fire sales and do not model indirect effects from direct interbank debt relations. There is a literature combining both of these aspects. See for instance Aymanns et al. (2018), Feinstein and El-Masri (2017) as well as Cifuentes et al. (2010) and Veraart (2020). Here we focus on fire sales only.

We analyze the effect of fire sales using a fixed point argument. There are a number of papers that use similar ideas in slight model specific variations. Examples are Detering et al. (2021, 2022), Feinstein (2017), Weber and Weske (2017), Feinstein and El-Masri (2017), Banerjee and Feinstein (2021), Cifuentes et al. (2010) and Cont and Wagalath (2016), Braouezec and Wagalath (2018). We also use a fixed point or equilibrium idea, because we believe, that, with this approach it is easier to understand what is going on in the computation of fire sale impacts compared to a pure simulation approach.

The papers in the market microstructure literature, which our price impact model is based on are mainly Kyle and Obizhaeva (2016) and Bouchaud (2010).

Table A.1
Balance sheet of bank b at $t = 0$. The data provide information on S_b^0, L_b^0, and e_b^0 for all banks b. Debt D_b^0 is the aggregate residual figure $S_b^0\mathbf{1}_I + L_b^0\mathbf{1}_J - e_b^0$.

Assets	Liabilities
$S_b^0\mathbf{1}_I$	D_b^0
$L_b^0\mathbf{1}_J$	e_b^0
$a_b^0 = S_b^0\mathbf{1}_I + L_b^0\mathbf{1}_J$	
$\lambda_b^0 = (S_b^0\mathbf{1}_I + L_b^1\mathbf{1}_J)/e_b^0$	

2. The modeling framework

The banking system

We apply a bank solvency stress test to a banking system described by balance sheet positions. The assets can be broken down by loan and security exposures. On the liability side only the equity position is used. These are the standard data that are commonly available across jurisdictions conducting bank solvency stress tests.

Formally we describe these balance sheet exposures across the set of banks included in the stress test by a $B \times I$ matrix of securities, where each row is one of B banks and each column is one of I securities which can be sold anytime in corresponding markets. We also have a $B \times J$ matrix of loans, with an analogous interpretation. Each row is a bank and each column is a loan in one of J particular exposure categories. In contrast to securities it is assumed that loans cannot be sold over the time horizon considered in the stress test. In the same way as assets of different categories, equity across the banking system is represented by a $B \times 1$ vector $\mathbf{e}^0$. We define *leverage* as the ratio of total assets over equity.

This description of the data results in a vector of total assets $\mathbf{a}^0$ and a vector of leverage λ^0 at the observation period $t = 0$ given by

$$\mathbf{a}^0 = \left(S^0\mathbf{1}_I + L^0\mathbf{1}_J\right) \tag{1}$$

$$\lambda^0 = \left(E^0\right)^{-1}\left(S^0\mathbf{1}_I + L^0\mathbf{1}_J\right) \tag{2}$$

where E^0 is a $B \times B$ diagonal matrix with $\mathbf{e}^0$ in the diagonal. The corresponding balance sheet can be visualized in Table A.1.

Stress tests usually assume that there is a future time horizon, which we call $t = 1$, at which the banking system incurs losses defined by a loss scenario. The construction of loss scenarios is an integral – and perhaps the most involved and time consuming – part of stress tests in practice. Here we take a loss scenario at the future time horizon $t = 1$ as given.

[2] https://github.com/Martin-Summer-1090/syslosseval. From the GitHub repository one can download the source code as a zip archive. The raw data and the R-scripts which compile the datasets used in the paper are in a tar.gz archive in the folder "data-raw" (syslossevel_raw_data.tar.gz). For instructions on how to untar and unzip this archive on Windows and Mac, see for example: https://www.uubyte.com/extract-tar-gz-bz2-on-windows-mac.html; or on Linux: https://smarttechnicalworld.com/how-to-extract-unzip-tar-gz-file/.

Table A.2
Balance sheet of bank b at $t = 1$. The value of its total assets is a_b^1 and its leverage is λ_b^1. These values depend on the stress scenario.

Assets	Liabilities
$S_b^1 \mathbf{1}_I$	D_b^1
$L_b^1 \mathbf{1}_J$	$e_b^1 = e_b^0 + (S_b^1 \mathbf{1}_I + L_b^1 \mathbf{1}_J) - (S_b^0 \mathbf{1}_I + L_b^0 \mathbf{1}_J)$
$a_b^1 = S_b^1 \mathbf{1}_I + L_b^1 \mathbf{1}_J$	
$\lambda_b^1 = (S_b^1 \mathbf{1}_I + L_b^1 \mathbf{1}_J)/e_b^1$	

When asset losses are incurred banks end up with a new state with diminished asset values and as a consequence also with a changed leverage. We describe this new state by

$$\mathbf{a}^1 = \left(S^1 \mathbf{1}_I + L^1 \mathbf{1}_J\right) \tag{3}$$

$$\lambda^1 = \left(E^1\right)^{-1} \left(S^1 \mathbf{1}_I + L^1 \mathbf{1}_J\right) \tag{4}$$

At this stage the standard stress test ends. When fire sales are considered, we need an additional step. Table A.2 visualizes the corresponding balance sheet.

In line with the literature on fire sales, we assume that there is a critical threshold of leverage, which we call λ^*, such that banks liquidate fractions θ_b of marketable assets once their leverage falls below this critical threshold. In such a situation of crisis it is assumed that it is their prior aim to bring down their leverage. Since raising new equity is usually not possible in a situation of financial distress, the options are reduced to liquidating assets.

We think of fire selling taking place after losses have occurred at $t = 1$. The time horizon over which liquidations are taking place is much shorter than the time horizon over which exogenous losses are considered in a usual bank solvency stress test. The period $t = 0$ to $t = 1$ spans the time horizon of credit risk, which is of the order of one year. In contrast, the period from $t = 1$ to $t = 1 + \tau$ is of the order of days or weeks over which security sales take place.

To highlight this we use the notation that in addition to the period $t = 0$ to $t = 1$ there is a second shorter period of liquidations from $t = 1$ to $t = 1 + \tau$, where τ described the new and shorter (liquidation) time horizon.

Note that this liquidation rule is compatible with different stories of fire sales. One that focuses on the aim of banks is the motive to stay above some thresholds which banks themselves see as critical for their own operations as for instance in Cont and Schaanning (2017). Our model would also be consistent with a story where upon the breach of some threshold other investors withdraw funding from the bank, because they become concerned with the viability of that institution, which must then sell assets to raise the cash to fill the funding gap. This is referred to as the interaction of market and funding liquidity by Brunnermeier and Pedersen (2009).[3]

Price impact of fire sales

At the time of fire sales the value of an exposure in security i equals only $1 - \delta_i$ times its value before fire sales, where $0 < \delta_i < 1$ is the price drop due to fire sales. We assume that we can interpret the change in the value of the position as coming from a liquidation impact on prices. So we interpret δ_i as the price impact of fire sales on security i. The price impact on all securities is denoted by the vector $\boldsymbol{\delta} = (\delta_1, \dots, \delta_I)$ which is the vector of fire sales discounts as percentage of pre-sales prices.

Fire sales discounts imply that security exposures do not have a value of $S_b^1 \mathbf{1}_I$ but $S_b^1(\mathbf{1}_I - \boldsymbol{\delta})$. This has consequences on the value of equity, which changes to:

$$e_b^{1+\tau} = S_b^1(\mathbf{1}_I - \boldsymbol{\delta})(1 - \theta_b) + L_b^1 \mathbf{1}_J - (D_b^0 - \theta_b S_b^1(\mathbf{1}_I - \boldsymbol{\delta}))$$

Table A.3
Balance sheet of bank b at $t = 1 + \tau$ with price impact. The value of its total assets is $a^{1+\tau}$ and its leverage is $\lambda^{1+\tau}$. The asset sale has an ambiguous effect on the new leverage. On the one hand, debt can be reduced with the proceeds from the sale. On the other hand, by the price impact the bank is facing a valuation loss on its assets at $t = 1 + \tau$ relative to $t = 1$. The loss affects the share of the assets sold as well as the value of the shares kept on the balance sheet. This loss has to be absorbed by equity and increases leverage. The total effect depends on the size of the price impact.

Assets	Liabilities
$S_b^1(\mathbf{1}_I - \boldsymbol{\delta})(1 - \theta_b)$	$D_b^0 - \theta_b S_b^1 (1 - \boldsymbol{\delta})$
$L_b^1 \mathbf{1}_J$	$e_b^1 - S_b^1 \boldsymbol{\delta}$
$a_b^{1+\tau} = S_b^1(\mathbf{1}_I - \boldsymbol{\delta})(1 - \theta_b) + L_b^1 \mathbf{1}_J$	
$\lambda_b^{1+\tau} = \frac{S_b^1(\mathbf{1}_I - \boldsymbol{\delta})(1-\theta_b) + L_b^1 \mathbf{1}_J}{e_b^1 - S_b^1 \boldsymbol{\delta}}$	

$$= S_b^1(\mathbf{1}_I - \boldsymbol{\delta}) + L_b^1 \mathbf{1}_J - D_b^0 \tag{5}$$

The corresponding balance sheet is shown in Table A.3

Following the market microstructure literature we assume that the price impact on a security asset class i is some function of the total volume of this security sold in the market. We denote this function by φ and make the following:

Assumption 1. The price impact of selling a certain volume of security i is described by a function φ_i from $\mathbb{R}_+$ to $[0, 1]$ which we assume to have the following properties: $\varphi_i(0) = 0$, φ_i is strictly increasing and continuous and for all $i \in \mathcal{I}$, $\varphi_i < 1$, more specifically

$$\varphi_i\left(\sum_b S_{b,i}^1\right) =: \delta_{i,max} < 1. \tag{6}$$

This assumption corresponds to Assumption 2.1 in Banerjee and Feinstein (2021). It implies that no direct cross price effects are taken into account.[4]

Fire sales behavior

Assume that at $t = 1$ there is a bank b for which $\lambda_b^1 > \lambda^*$. This bank will then begin to sell securities to achieve the target leverage λ^*.

The bank understands that selling assets at time $t = 1$ will have a price impact on the sold asset classes at $t = 1 + \tau$ expressed as a vector $\boldsymbol{\delta} \in [0, 1]^I$. Individual banks are price takers. In their liquidation decisions $\boldsymbol{\delta}$ cannot be influenced by their individual actions. It is not the choice of the bank but is determined in the market.[5] A leverage threshold λ^* specifies the maximum leverage a bank is allowed to have. If the leverage of the bank is below λ^* the bank is fine and there is no further need for action. If the leverage is above λ^* the bank has to sell part of its security portfolio to bring its leverage back to λ^*.

We denote by $\lambda_{b,\min}$ the leverage after fire sales if bank b sold its whole security portfolio, and by $\lambda_{b,\max}$ the leverage if bank b sold no securities. Both $\lambda_{b,\min}$ and $\lambda_{b,\max}$ depend on the fire sale price discount $\boldsymbol{\delta}$.

$$\lambda_{b,\min}(\boldsymbol{\delta}) = \frac{L_b^1 \mathbf{1}_J}{e_b^1 - S_b^1 \boldsymbol{\delta}}, \qquad \lambda_{b,\max}(\boldsymbol{\delta}) = \frac{S_b^1(\mathbf{1}_I - \boldsymbol{\delta}) + L_b^1 \mathbf{1}_J}{e_b^1 - S_b^1 \boldsymbol{\delta}}. \tag{7}$$

$\lambda_{b,\max}(\mathbf{0}) = \lambda_b^1$, which is the leverage of bank b at $t = 1$.

[3] One example of the detailed modeling of such a mechanism is given, for example, in Cont and Wagalath (2013).

[4] The no-cross impacts assumption is common in the literature. Some works do allow for cross impacts (e.g., Feinstein and El-Masri, 2017) and the recent work of Bichuch and Feinstein (2022), finds theoretical structures so that cross price impacts would be allowed for.

[5] This assumption is similar to assuming price parametric behavior in models of perfect competition. In our case it is also a conceptual approach allowing simplicity in application. This is an important aspect for us, since we would like to offer practitioners a framework that is easy to apply. In the fire sales literature a strategic equilibrium concept has been analyzed by Braouezec and Wagalath (2019). Also Banerjee and Feinstein (2021) analyze the case of strategic behavior in Proposition 3.7.

Assumption 2. Banks sell the same proportion θ_b of all securities. If $\lambda^* < \lambda_{b,\min}(\delta)$ or if $e_b^1 \leq 0$ then the bank has no way to meet the leverage constraint and sells all its securities and $\theta_b(\delta) = 1$. If $\lambda^* \in [\lambda_{b,\min}(\delta), \lambda_{b,\max}(\delta)]$ then the bank sells the share θ_b of all its securities that brings leverage exactly to λ^*. In this case $\theta_b = 1 - \frac{\lambda^*(e_b^1 - S_b^1\delta) - L_b^1 \mathbf{1}_J}{S_b^1(\mathbf{1}_I - \delta)}$. If $\lambda^* > \lambda_{b,\max}(\delta)$ the bank sells nothing. We assume that bank b must not go short in securities ($\theta_b \leq 1$) and that it does not buy securities ($0 \leq \theta_b$) in a fire sale when it already violates the leverage constraint. We assume the loan portfolio cannot be sold on the time scale τ of fire sales. The proceeds from selling the proportions θ_b are used to reduce debt.

The liquidation strategy described in Assumption 2 fulfills the minimal liquidation condition (Assumption 2.6) in Banerjee and Feinstein (2021).

Balance sheets after fire sales

If banks follow the liquidation strategies θ_b and cause price impact δ, the state of bank balance sheets at $t = 1 + \tau$ is given by

$$S_b^{1+\tau} = S_b^1(\mathbf{1}_J - \delta)(1 - \theta_b) \tag{8}$$

$$L_b^{1+\tau} = L_b^1 \tag{9}$$

$$e_b^{1+\tau} = e_b^1 - S_b^1\delta \tag{10}$$

$$\lambda_b^{1+\tau} = \frac{S_b^1(\mathbf{1} - \delta)(1 - \theta_b) + L_b^1\mathbf{1}_J}{e_b^1 - S_b^1\delta}. \tag{11}$$

Fire sales equilibrium

In the literature on fire sales the values of δ and θ_b have been determined by two approaches. The first one, dominating the literature in economics and finance relies on behavioral assumptions and computer simulations (see for example Greenwood et al., 2015; Duarte and Eisenbach, 2021; Cont and Schaanning, 2017). The second one, coming from mathematical finance and operations research, is based on fixed point arguments. This literature is well developed and mature (see for example Feinstein and El-Masri, 2017, Banerjee and Feinstein, 2021, Detering et al., 2021, Braouezec and Wagalath, 2019). We do not contribute new results there. Our aim is rather to build on these insights to show stress testing practitioners that these insights can be easily applied in practice.

To make a specific loss assessment for a given stress scenario we determine the price discount by applying an equilibrium idea. Given a discount vector δ, we can think of liquidation behavior as a security supply decision by banks. The impact function φ can be thought of as an inverse demand function which describes the price reactions that can be expected in the security markets for a given volume sold. In a fire-sale equilibrium supply and demand balance. The ultimate price impact is the discount vector which achieves this balance. Note that a bank only knows its own fire sales behavior but not the behavior of other banks, which depends on their balance sheets. Our concept therefore applies a non-strategic, competitive-equilibrium idea. We have the following

Definition 1. Given a stress scenario at $t = 1$ a fire-sale equilibrium is given by a pair $(\mathbf{q}(\delta^*), \delta^*)$ such that:

1. For every bank $b \in \mathcal{B}$:

$$\theta_b(\delta^*) = \begin{cases} 1 & \text{if } \lambda^* < \lambda_{b,\min}(\delta^*) \text{ or if } e_b^1 \leq 0 \\ 1 - \frac{\lambda^*(e_b^1 - S_b^1\delta^*) - L_b^1\mathbf{1}_J}{S_b^1(\mathbf{1}_I - \delta^*)} & \text{if } \lambda^* \in [\lambda_{b,\min}(\delta^*), \lambda_{b,\max}(\delta^*)] \\ 0 & \text{if } \lambda^* > \lambda_{b,\max}(\delta^*) \end{cases} \tag{12}$$

with $\mathbf{q}(\delta^*) = (S^1)^T\theta(\delta^*)$, where $(S^1)^T$ denotes the transposed security holdings matrix S^1.

2. Security supply equals security demand: $\left(\varphi_1(q_1^*(\delta^*)), \dots, \varphi_I(q_I^*(\delta^*))\right) = \delta^*$

The notation $\mathbf{q}(\delta^*)$ is used for the vector of total volume of securities. For example, the component $q_i(\delta^*)$ is the volume of security i which is sold on the market in equilibrium:

$$q_i(\delta^*) = \sum_b S_{bi}^1\theta_b(\delta^*) \tag{13}$$

This definition can be compared to Eq. (2) in Banerjee and Feinstein (2021) who call their condition a "clearing solution". Our formulation emphasizes the analogy with the usual economic equilibrium concept and is equivalent. In contrast to our approach Banerjee and Feinstein (2021) distinguish between the mark to market value and the value recovered from selling. Such a distinction would be possible in principle also in our framework, if we interpreted the iterative computation of the fixed point as a model of a price adjustment dynamics during the fire sale.

The characterization of a fire sale equilibrium by fixed point or equilibrium arguments can be found frequently in the literature. We have listed the references to this literature in the introduction and refer for details to this literature. The paper containing a model that can directly be applied to our setup is Banerjee and Feinstein (2021). The idea of the proof uses the fact that the map $\varphi \circ q$ turns out to be an order preserving self map on the complete lattice $D := \{\delta = (\delta_1, \dots, \delta_I) : 0 \leq \delta_i \leq \delta_{i,max}\}$. By the Knaster-Tarski fixed point theorem, we can then deduce that a fixed point exists and that the set of fixed points of $\varphi \circ q$ contains a minimal and a maximal element.[6] The order structure of the complete lattice and the order preserving property of $\varphi \circ q$ allows then a computation of the fixed points by an iterative algorithm.[7]

We could not find conditions on φ which would guarantee that the fixed point is unique. With the assumptions we have imposed on φ uniqueness will not hold in general. The appendix of the working paper version of this paper (see Breuer et al., 2021) we give an example where multiple equilibria occur. The only paper giving conditions for a unique fixed point we are aware of, is Banerjee and Feinstein (2021), Theorem 3.2. We did, however, not try to analyze how these conditions would precisely translate to our framework, because we believe that for practical purposes we do not have to assure uniqueness of the fixed point in general.

In applications, for the cases where we do not get a unique fixed point, since the least and the greatest fixed point can be computed by iteration easily, and since the fixed points can be ordered by the lattice structure we can always compute a lower and an upper bound for the losses. Thus for the cases with a non-unique fixed point we can bound the losses in a precise way. Given the coarseness and precision of the data this seems to us good enough for the practical purposes of a stress test.

3. Applying systemic loss evaluation to public EBA data

We now analyze a dataset published by the European Banking Authority (EBA). The dataset is from the 2016 EBA stress testing exercise. It contains exposure as well as impairment data and was the basis of the pan-European bank solvency stress test of 2016. This dataset allows for comparing the risk assessment that would result from the standard EBA methodology and from our loss evaluation method, which takes potential fire sale effects into account.

From this analysis we can see how our ideas can be applied to a practical stress testing situation, and it simultaneously gives us some

[6] Compare to Proposition 3.1. and its proof in Banerjee and Feinstein (2021). By our assumptions on $\theta(\delta)$ and φ our map fulfills the assumptions needed for $\varphi \circ q$.

[7] We refer readers interested in the details for the special case of our model to a working paper version Breuer et al. (2021) and the GitHub archive for this paper.

Table A.4
Balance sheet of bank b at $t = 0$.

Assets	Liabilities
Central banks and central governments: Loans	
Central banks and central governments: Bonds	
Institutions	
Corporates	
Retail	Debt
Equity	
Other noncredit obligations	
Residual position	Core tier 1 equity

first ideas of the quantitative importance of such an extended evaluation of potential losses. It gives us only a first idea because the data allow us to consider only a limited set of marketable securities which are held on the bank balance sheet. We cannot look beyond government bonds and at balance sheet items. There is nothing, however, in our framework that would exclude in principle a wider consideration if data were available. It also gives us only a first idea because the loss evaluation is derived from an assumption about bank behavior which is not yet empirically validated. We are confident, however, that our framework is flexible enough to accommodate more elaborate and more realistic behavioral models of the fire sale process.

3.1. Organizing the EBA data into stylized bank balance sheets

We now give a brief high-level description of our data. A more detailed description is given in the appendix. Readers interested in every detail of the data compilation can consult our GitHub repository cited in the introduction.

In the annual transparency exercise, EBA discloses detailed bank-by-bank data for given reference dates, usually June and December. Information is published for a wide set of banks across 26 countries at the highest level of consolidation in the European Union (EU-27) and the European Economic Area (EEA) as well as for some banks from the UK. The data are made available on the EBA website and provide disclosure on banks' assets and liabilities, capital positions, risk exposure amounts, leverage exposures and asset quality as well as information on sovereign exposures.

Every two years the EBA also conducts a bank solvency stress test for the largest banks in the EU and EEA. The sample of banks is smaller than in the transparency exercise. The selection threshold is at a value of total assets larger than 30 billion euro.

Under some assumptions on the aggregation of data detailed in the appendix and by using our theoretical computational framework of fire-sale equilibrium, we construct stylized balance sheets for each bank at $t = 0$, $t = 1$ and $t = 1 + \tau$.

The stylized balance sheet we get in this way for each bank for the 2016 data is given in Table A.4.

This scheme uses the asset classification of the reporting standard according to the internal rating-based approach (IRB) at the highest level of consolidation.

We need to explain a few features of this scheme in more detail: Not all banks report to the EBA according to the IRB standard. Some banks report assets partially also according to the standard approach (STA). To organize the data into a unique scheme like in Table A.4 we have to make an assumption about how we map STA into IRB classifications, where necessary. The detailed assumption how we do this is described in the appendix.

Observe that for the position "central banks and central government" we split the position into loans and (sovereign) bonds. Thus when we bring our model to the EBA data, sovereign bonds are the only on balance sheet marketable assets for which we have exposure information. The price impact effects of distressed fire sales can thus only be partially described given our data.

Table A.5
Summary statistics of the data from the 2016 EBA stress testing exercise. There are 51 banks in the sample. The table shows the minimum value the 25% quantile, the median, the 75% quantile, the mean and the standard deviation for total assets, the ratio of equity to unweighted total assets (CET1 ratio), the ratio of (unweighted) total assets over core tier 1 equity (Leverage ratio) as well as the share of the total value of sovereign bond exposures in total assets. All figures are in billion euro.

Statistics	Total assets	CET1 ratio	Leverage ratio	Bond ratio
Min	33.70	0.02	7.70	0.01
Q25	154.08	0.04	17.55	0.05
Median	234.57	0.05	20.47	0.07
Q75	744.83	0.06	23.31	0.11
Max	2218.57	0.13	47.35	0.30
Mean	526.53	0.05	20.87	0.08
StDev	548.06	0.02	6.55	0.06

Finally observe that in our organization of the data we use an asset class called "residual position". This position is constructed as the difference between the sum of the value of all asset positions reported as exposed to credit risk in the stress test and the value of total assets reported by banks in their annual reports. That such a gap can be not only negative (total value of EBA assets smaller than total assets) but also positive (total assets smaller than total value of EBA assets) is a consequence of the regulatory reporting framework. A more detailed analysis of these (sometimes substantial) gaps is given in the appendix.

3.2. What do the data look like? Some summary statistics

We now give a brief descriptive overview of our data. Table A.5 displays some descriptive statistics for the distribution of total assets, the (unweighted) ratio of Core Tier 1 equity over total assets (CET1), the leverage ratio λ as well as the share of the value of sovereign bonds in the value of total assets of the 2016 EBA stress test exposure data. We can see that all of the 51 banks in this sample have total assets of at least 30 billion euro. The average capital ratio equals about 5% with a standard deviation of 2 percentage points. The equity base, if computed without the usual Basel II risk weighting is relatively small. The leverage ratio shows the same information (just expressed as the inverse of the tier 1 capital ratio). We display it here because it is a critical ratio in our behavioral model. From the table we see that even without any shock or stress there is at least one bank with a leverage ratio way above the critical threshold, which we have set for this analysis $\lambda^* = 33$.[8] The average value of sovereign bond holdings in this sample is about 8%.[9]

The key variable in our analysis is leverage. Leverage is not only critical for the overall resilience to shocks – what is usually studied in traditional stress testing – but it is also critical for potential fire sales and thus loss amplification processes. Fig. A.1 shows that already without any stress, in both samples there are banks which already exceed the threshold of $\lambda^* = 33$ even without any shock.[10]

3.3. Applying the model

The precise shape of the price impact function φ is a question actively discussed in the market microstructure literature. For a recent overview see, for example, Bouchaud (2010). There is an ongoing discussion about what price impact functions look like empirically.

[8] The Basel 3 framework sets the minimum required leverage at 3% which is why we (defining leverage as exposure/capital in contrast to Basel which uses capital/exposure) set λ^* at this particular value.

[9] This corresponds to estimates given in the literature. For example Gennaioli et al. (2018) report a figure of 9% in a sample of 191 big banks around the globe.

[10] The Basel 3 framework sets the minimum required leverage at 3% which is why we (defining leverage as exposure/capital in contrast to Basel which uses capital/exposure) set λ^* at this particular value.

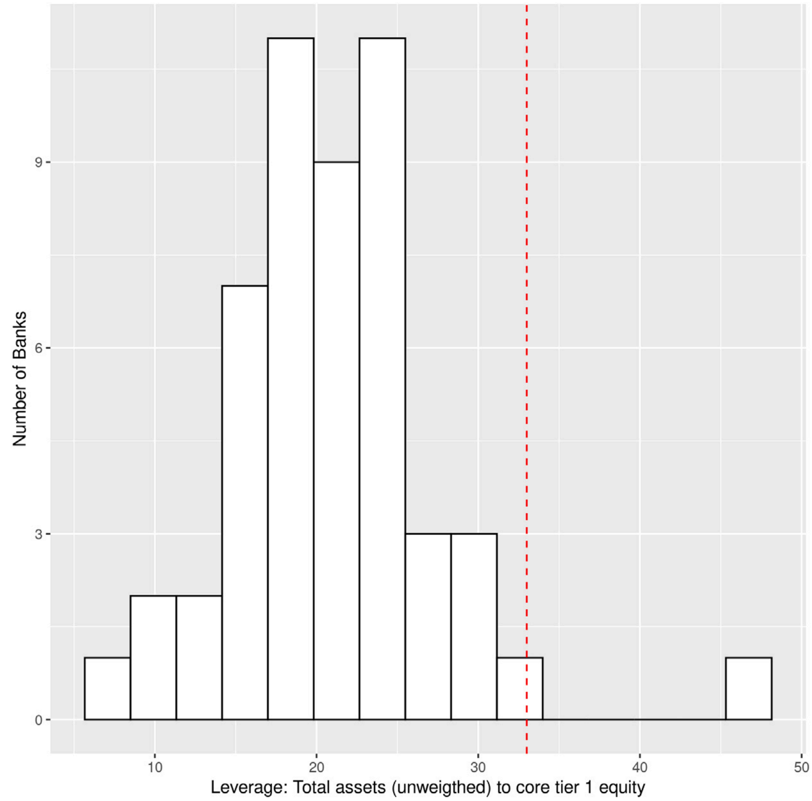

Fig. A.1. Histogram of leverage in the 2016 EBA stress testing banking sample. The dashed (red) line is the critical leverage threshold above which in our model behavioral reactions are taken into account in the evaluation of losses. In this particular figure the critical threshold is set at $\lambda^* = 33$.

Recent work by Kyle and Obizhaeva (2016) provide new evidence based on market micro structure invariance principles. They show that market microstructure invariance implies a transaction cost model where the percentage costs of trading of a particular asset is proportional to the product of volatility, two invariant constants and a general invariant price impact function.

The shape of the function can be determined only empirically. Following the empirical literature on price impact we assume that this function is a square root function (see Bouchaud, 2010).

Using the square root specification we get:

$$\varphi_i(q_i) = \sigma_i \kappa \sqrt{\frac{q_i}{ADV_i}} \tag{14}$$

where q_i is the aggregate volume in value terms (say euro or US dollar) of security i sold in the market, σ_i is the volatility of (daily) prices of security i, κ is a constant of order unity independent of the asset class and ADV_i is the average daily volume (turnover) of security i.

Our Eq. (14) is consistent with Eq. (18) in Kyle and Obizhaeva (2016) under the assumption that one of their constants is zero. This last condition implies that spread costs are ignored. Unobservable quantities in their model are absorbed in our constant κ. Following the terminology of Kyle and Obizhaeva (2016), the quantity q_i is the "aggregate bet". In the invariance equations the original expressions contain bet volatility and bet volume. Both of these quantities are defined for the business time τ, in their terminology. Expressing these two quantities in terms of observable variables, daily returns volatility and average daily volume, bet volatility scales as $\sqrt{\tau}$ while bet volume scales as τ. As a result the τ dependence of price impact cancels out in the square root case.[11]

3.4. The EBA 2016 bank solvency stress test

Let us now study our first case, the EBA 2016 stress test. We want to go through the following thought experience: let us look at the published stress test data first and examine the assessment which resulted from this analysis. We then ask the hypothetical question: what would our assessment have looked like if we had factored in the potential fire sale effects as captured by our framework. Comparing these two cases helps us understand how and to what extent both approaches differ.

The sample of banks which participated in the 2016 EBA stress test consisted of 51 banks from 15 EU or EEA countries, 37 were located in countries participating in the single supervisory mechanism (SSM) and 14 banks were based in non-SSM countries (Denmark, Hungary, Norway, Poland, Sweden and the UK). The scenario considered in the stress test assumed a deviation of EU GDP from its baseline level by 3.1% in 2016, 6.3% in 2017 and 7.1% in 2018. It furthermore considered a shock in residential and commercial real estate prices as well as for foreign exchange rates in Central and Eastern Europe under the adverse scenario. The assumption on the advanced economies, including Japan and the US was that cumulative GDP growth would be between 2.5% and 4.6% lower than under the baseline scenario in 2018. For the main emerging economies the stress test assumed total GDP between 4.5% and 9.7% below the baseline projections in 2018, with a stronger

[11] Cont and Schaanning (2017) use different impact functions based on exponential functions rather than the square root function. In their specification, therefore, τ has to be specified to pin down the price impact. We stick to the square root function because the literature presents some evidence that this function is actually often observed in the context of price impact events (see Bouchaud, 2010). Note that our results do not depend on the exact form of the impact function but only on Assumption 3. The impact functions used in Cont and Schaanning (2017) do – for instance – fulfill Assumption 3 and could therefore be used as well.

impact for Brazil, Russia and Türkiye. Finally the stress test defined an adverse scenario for a number of key prices such as long-term interest rates, FX rates, stock prices, inflation and swap rates. These scenarios are processed by the participating banks to "translate" them using their own analytical frameworks into impairments according to the EBA methodology (European Banking Authority, 2016a). The results of the stress test are reported in European Banking Authority (2016b).

Note that we do not exactly reproduce the EBA stress test here, since we do not implement the full EBA methodology for this analysis. We do, for instance, not consider risk weighting, we do not consider (exogenous) market risk and operational risks and we do not model income flows but confine ourselves to balance sheets only. The reason why we take so many bold shortcuts here is to focus on the key question of this section: how does a loss assessment based on the EBA data differ between an approach where we use impairments only and an approach where we factor in additional losses from fire sales. While the EBA stress test makes a stress assessment focused on 8 different metrics,[12] we focus for our purposes on leverage, equity losses and the number and size of affected institutions.

Results under the assumption of no fire sales. Let us first look at the pure credit risk losses implied by the EBA data under the adverse scenario in terms of the leverage ratio λ. This plot may be compared with Figure 13 on page 23 in European Banking Authority (2016b). The comparison shows that the leverage numbers look very similar, despite the fact that we do not reproduce the EBA stress test exactly.[13]

From Fig. A.2 we can see that under the adverse scenario the median leverage as well as the share of banks with significantly higher leverage increases compared to the initial position. The median leverage increases at all horizons above the 75% quantile in the initial state. There are also a number of banks, which would not survive the stress test without help from outside. They are not able to maintain a leverage ratio below the critical boundary of $\lambda^* = 33$. We display this fact graphically in a bar chart in Fig. A.3. The names of the affected institutions as well as their rank among the 51 banks in terms of total assets are written into the chart for the initial position as well as for the adverse scenario at all horizons.

As we can see from Fig. A.3, there is one bank which is above the threshold already in the initial state. Under the one year ahead adverse scenario there are five additional banks exceeding the threshold. If we go two years ahead, an additional bank is going to join the club. Finally in 2018 at the three-year horizon we have again only seven banks above the threshold.

Note that the banks who get in trouble in the stress scenario are very large in terms of total assets. These seven or eight banks made up about 6% of total assets in 2016 and 2018. The eight banks in 2017 made up a share of 8% of the total assets of all banks participating in the stress test. In terms of the GDP of the 19 Euro area countries (EA-19) the total assets of distressed banks make up a share of 56%, 72% and 52%. The institutions which come into trouble are thus really huge and certainly too big to rescue for the national states in which they are residing.

In terms of losses in core tier 1 equity relative to the initial position, we can say that in aggregate terms the loss of tier 1 equity in the stress scenarios would be about 16% in 2016, 17% in 2018 and 16% relative to the aggregate tier 1 equity position at the initial date.

Table A.6
Values of the fixed point δ^*. The rows display the different asset classes of marketable securities, the columns display the value of δ^* for the years 2016, 2017 and 2018 in the adverse EBA scenario. The parameter κ is set to a value of 5 in this computation. For the given data the fixed points are unique. We thus only report one value for each security class.

Bond	δ^*_{2016}	δ^*_{2017}	δ^*_{2018}
DE	0.0192	0.0213	0.0198
ES	0.0021	0.0021	0.0021
FR	0.0327	0.0363	0.0338
GB	0.0416	0.0439	0.0434
IT	0.0438	0.0558	0.0477
JP	0.0023	0.0027	0.0025
US	0.0169	0.0187	0.0176
Rest_of_the_world	0.0055	0.0063	0.0057

Table A.7
Maximum price impact — the impact which would result if all banks sold their entire bond portfolio - and the relative impact in the EBA stress scenario compared to the maximum impact for all adverse scenarios.

Bond	$\delta_{\max}$	$\delta^*_{2016}/\delta_{\max}$	$\delta^*_{2017}/\delta_{\max}$	$\delta^*_{2018}/\delta_{\max}$
DE	0.05	0.40	0.44	0.41
ES	0.06	0.03	0.03	0.03
FR	0.07	0.48	0.54	0.50
GB	0.06	0.75	0.79	0.78
IT	0.11	0.41	0.52	0.44
JP	0.00	0.77	0.92	0.84
US	0.03	0.56	0.62	0.58
Rest_of_the_world	0.01	0.39	0.45	0.41

Results under the assumption of fire sales. Now let us compare these numbers under the assumption that we also factor in potential fire sales of sovereign bonds. Note that when computing the price impact, according to our impact equation (14), all parameters, except the parameter κ, are pinned down by data. From κ we only know that it is empirically "of order unity", which allows for quite a wide range of values. If we had a time series of observed impact events, we could estimate the value of this parameter. Here we can only make assumptions, which are more of less arbitrary. The order unity constraint is, for example, compatible with values between 1 and 9 but not with 20 or 50. For our simulation we set $\kappa = 5$.

When we consider the potential for fire sales we have to compute a fire sale equilibrium for all stress test horizons. Given our data, it turns out that the fixed points are unique. The values of the discount at the fire sale equilibrium are given in Table A.6.

Is this impact large or small? We can get a feeling for the order of magnitude by bench-marking the price impact against a hypothetical maximum impact which could occur here if all banks sold their *entire* sovereign bond portfolio. The result of such a hypothetical sovereign bond "meltdown-situation" is shown in Table A.7.

From the table we can see that the impact in the stress scenario is about half of the impact of a situation in which every bank would sell its entire sovereign bond portfolio. This means that the price impact in a stress scenario can be significant.

In terms of banks, which exceed the threshold of critical leverage $\lambda^* = 33$ under such an evaluation of losses we see that we can observe a "systemic effect". The fire-sale effects push banks beyond the critical threshold which would have stayed below the threshold in the EBA scenario. We have now two additional banks, which get into trouble, as a result of the fire sale "dynamics": Banco Popolare - Società Cooperativa and BNP Paribas.[14] One of them is huge: In terms of total assets BNP Paribas is the second largest bank in the sample. Banco Popolare - Società Cooperativa is only the 40th largest bank. Their combined total assets amount to roughly 7% of the entire total

[12] Transitional CET1 capital ratio, fully loaded CET1 capital ratio, transitional leverage ratio, transitional CET1 capital, cumulative credit risk losses (impairment or reversal of impairment on financial assets not measured at fair value through profit or loss), cumulative gains or losses arising from operational risk, cumulative market risk losses including CCR, cumulative profit or loss for the year. See European Banking Authority (2016a) for definitions and details.

[13] Note that in Fig. 13 in EBA the leverage ratio is represented as CET1 over total assets whereas we have defined it reciprocally as total assets over CET1.

[14] PNB Paribas is in trouble in the fire sales scenario in all adverse scenarios, while it is not above the threshold in the adverse EBA scenario 2016 and 2018.

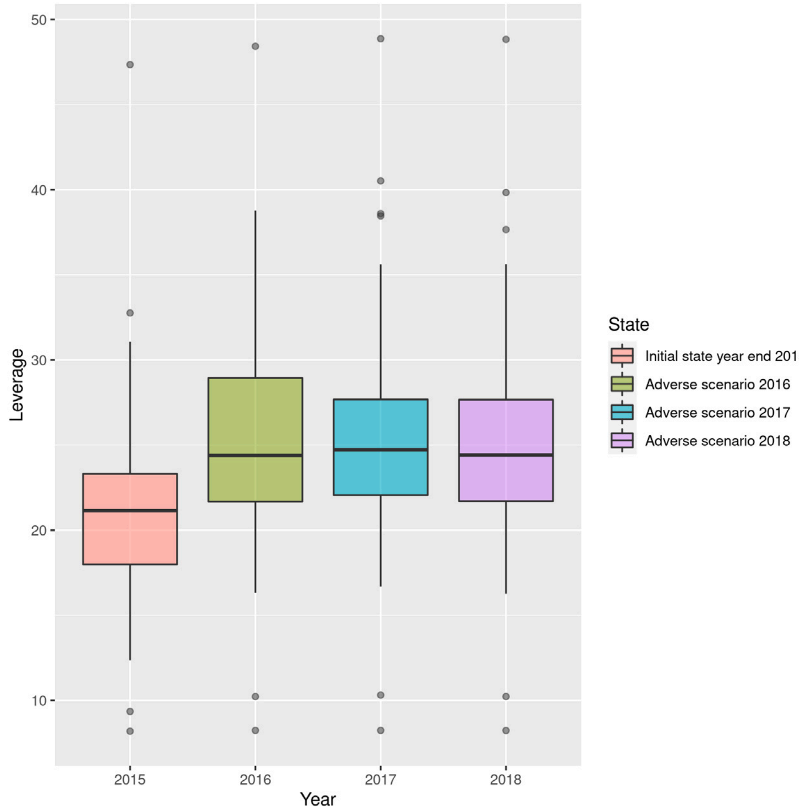

Fig. A.2. Distribution of leverage λ in the EBA stress test. The left most boxplot shows the initial state of the banking system at year end 2015. This corresponds to $t = 0$ in our model. The next box plots show the leverage distribution under the adverse EBA scenario at different time horizons, 2016 (one year ahead), 2017 (two years ahead) and 2018 (three years ahead). In terms of our model, these horizons would correspond all to different assumptions about $t = 1$. There are a few banks which exceed the critical leverage threshold of $\lambda^* = 33$ under the adverse scenario.

assets of all banks combined or about 17% of EA-19 GDP. This means that the factoring in of fire-sale losses reveals indeed a huge amount of *additional* losses which will be concealed in the traditional EBA approach.

We observe that we cannot gauge the entire potential of fire sales, given our data. Of the 9 banks which are at or above the threshold of $\lambda^* = 33$, N.V. Bank Nederlandse Gemeenten, Lloyds Banking Group Plc, Deutsche Bank AG, Banca Monte dei Paschi di Siena S.p.A. and Société Générale S.A. sell their *entire* bond portfolio but still are unable to restore a stable capital structure. They are not able to restore even the critical leverage constraint. If we had more marketable assets in our data, the fire sales of significant institutions would affect other asset classes and would be bigger. It cannot be excluded that we might even run into a major systemic crisis.

Cont and Schaanning (2017) discuss for the EBA 2016 data at which threshold of losses fire selling of marketable assets might cascade into a fully-fledged systemic crisis. We refer the interested readers for details of such a threshold analysis to their paper.

When we look at losses in aggregate core tier 1 equity, taking fire sales into account, the loss amounts to 19%, 21% and 19% in the adverse scenario at the different horizons of 2016, 2017 and 2018. This is significantly more than the 16%, 17% and 16% we observed for the stress scenarios not taking into account potential fire sales effects. A more detailed picture can be given from looking at the distribution of Tier 1 equity losses in the entire sample of banks under the assumption of no fire sales compared to the case with fire sales, shown in Fig. A.4. We see that in the case where we evaluate losses taking into account potential fire sales the box-plot is stretched in the upper quartiles of the distribution as well as shifted upward. This means that the entire distribution shifts and the losses become more severe.

The overall conclusion from the analysis of the 2016 EBA stress test is that whether we factor in potential fire-sale processes or not can

Table A.8
Query scheme for the IRB exposures from the file `TRA_CR.csv`.

Variable	Value	Meaning
Period	201512	December 31 2015
Portfolio	3, 4	Foundation IRB (F-IRB), Advances IRB (A-IRB)
Item	1690201	Exposure values (IRB)
Scenario	1	Actual Figures
Status	1, 2	Non defaulted assets, defaulted assets
Exposure	1100, 2000, 3000, 4000, 6100, 6300	Central banks and government, Institutions Corporates, Retail Equity, Other
Perf_status	0	No breakdown by performance status

make a significant difference. We are not in a position, given our data, to pin down more precisely when this difference will be most relevant. We have no precise data on the value of the parameter κ; we have no precise and full picture of marketable securities which can become part of a fire sale but only a small though significant subset: sovereign bonds. We also do not have an empirically validated theory of bank behavior in distress.

We think, however, that our model does not preclude the closure of these gaps in data and modeling in principle. Our results indicate that the significance of indirect losses would be even more pronounced when these gaps are closed. We therefore think that loss evaluations should take potential fire sales effects into account to get a more comprehensive picture of the potential fall outs from financial distress.

4. Conclusions

When considering potential impacts of financial distress in a banking stress test, taking into account fire-selling effects in the evaluation

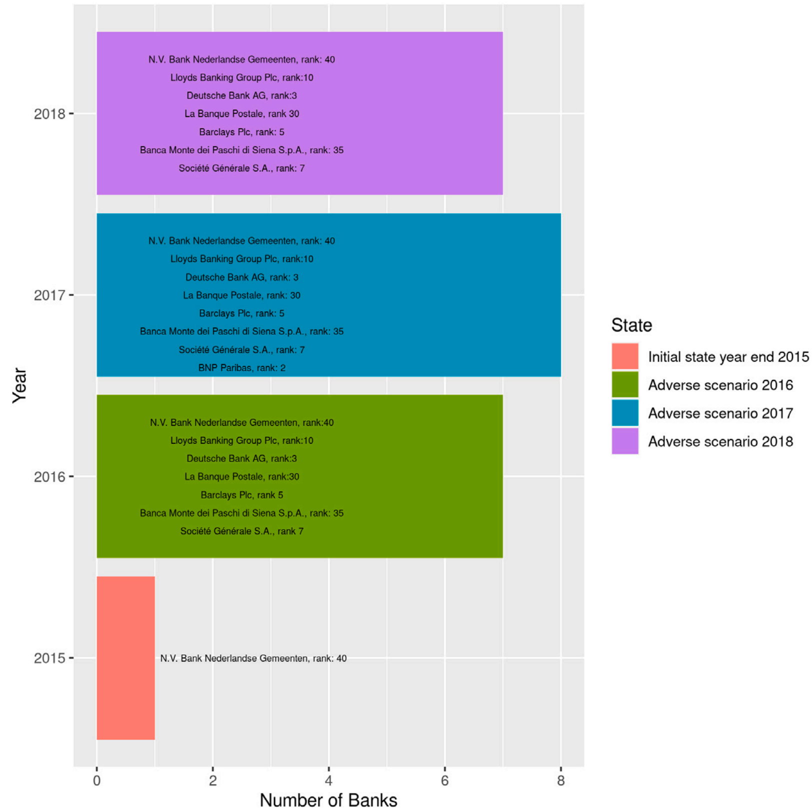

Fig. A.3. Number of banks with a leverage λ above the threshold $\lambda^* = 33$ in the initial state at year end 2015, in the adverse scenario in 2016, 2017 and 2018. The names of the banks as well as their size rank among the 51 banks in terms of total assets are given as annotations right of the bar (initial state year-end 2015) or in the bars (Adverse scenario 2016, 2017, 2018).

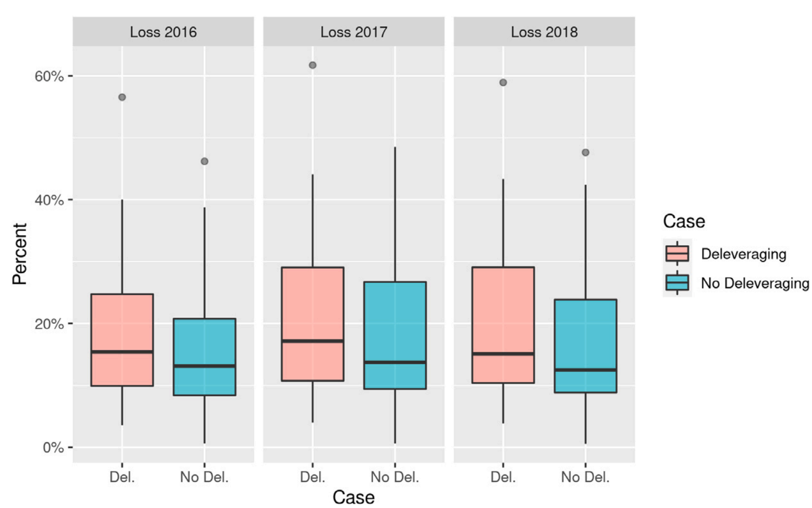

Fig. A.4. The Figure shows three comparative boxplots of the distribution of CET1 losses relative to the initial position for the adverse scenario in 2016, 2017 and 2018. The left box in each of the three plots (in red) gives a plot of the distribution of these losses for the case where fire sales are factored into the evaluation of losses. The right box plot in each of the three plots (the blue box) shows the distribution when potential fire sales are ignored, as in the EBA stress test. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

of losses is very important. Ignoring these effects might lead to a too benign assessment of risk while ignoring important and quantitatively significant indirect loss potentials. We show for the 2016 EBA stress test that a stress test ignoring these effects would overlook important and quantitatively significant losses. Tests would consider institutions resilient which are actually fragile.

A key message of our paper is that we offer a fire-sale framework to stress testing practitioners which is practical, simple and rigorous and can be integrated with the standard stress test very easily. The framework uses all familiar concepts of stress testing as practiced today and allows for an add on of a fire sale analysis. It is our hope that the results of our paper will encourage more stress testing practitioners to

Table A.9
Query scheme for the IRB impairments from the file `TRA_CR.csv`.

Variable	Value	Meaning
Period	201612, 201712, 201812	December 31 2016, 2017, 2018
Portfolio	2	IRB
Item	1690205	Impairment rate (IRB)
Scenario	2,3	Baseline, Adverse
Status	0	No break down by status
Exposure	1100, 2000,	Central banks and government, Institutions
	3000, 4000,	Corporates, Retail
	6100, 6300	Equity, Other
Perf_status	0	No-breakdown by performance status

Table A.10
Query scheme for the STA exposures from the file `TRA_CR.csv`.

Variable	Value	Meaning
Period	201512	December 31 2015
Portfolio	1	Standard Approach (STA)
Item	1690301	Exposure values (STA)
Scenario	1	Actual Figures
Status	1, 2	Non defaulted assets, defaulted assets
Exposure	1100, 1200	Central banks and government, regional government
	1300, 1400	Public sector entities, multilateral development banks
	1500, 1600	International organizations, central banks
	1700, 2000	General governments, institutions
	3000, 4000	Corporates, retail
	5000, 6400	Secured by mortgages, Items with particularly high risk
	6500, 6600	Covered bonds, Claims on inst. and corp. with a ST credit assessment
	6700, 6100	Collective investments undertakings (CIU), equity
	6200, 6300	Securitization, Other non-credit obligation assets
Perf_status	0	No-breakdown by performance status

Table A.11
Query scheme for the STA impairments from the file `TRA_CR.csv`.

Variable	Value	Meaning
Period	201612, 201712, 201812	December 31 2016, 2017, 2018
Portfolio	1	Standard Approach (STA)
Item	1690305	Impairment rate (STA)
Scenario	2, 3	Baseline scenario, adverse scenario
Status	0	No break down by status
Exposure	1100, 1200	Central banks and government, regional government
	1300, 1400	Public sector entities, multilateral development banks
	1500, 1600	International organizations, central banks
	1700, 2000	General governments, institutions
	3000, 4000	Corporates, retail
	5000, 6400	Secured by mortgages, Items with particularly high risk
	6500, 6600	Covered bonds, Cl. on inst. and corp. with a ST c.a.
	6700, 6100	Collective investments undertakings (CIU), equity
	6200, 6300	Securitization, Other noncredit obligation assets
Perf_status	0	No-breakdown by performance status

Table A.12
Query scheme for the equity and leverage ratio figures in `TRA_OTH.csv`.

Variable	Value	Meaning
Period	201512	December 31 2015
Item	1690106	Common tier 1 equity
Scenario	1	Actual figures

Table A.13
Query scheme for sovereign bond figures from the file `TRA_SOV.csv` AFS means available for sale, FVO means fair value through profit and loss, and HFT means held fro trading.

Variable	Value	Meaning
Period	201512	December 31 2015
Item	1690503, 1690506	
	1690507, 1690508	Net direct exposures AFS, FVO, HFT
SOV_maturity	8	All maturities

include fire sale analysis in their toolkit and thereby help us to collectively improve and increase our knowledge about this key amplification mechanism of financial distress.

Appendix. Data: Sources and compilation

A.1. EBA - Exposures and impairment data

The exposure data are composed from raw data provided via the webpage of the European Banking Authority.[15]

[15] https://eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2016. Readers who are interested in a line by line documentation of how the exposure data are constructed precisely are welcome to study the R-scripts `make_balance_sheets_2016.R` which is contained in the `data-raw` subfolder in the GitHub repository https://github.com/Martin-Summer-1090/syslosseval which hosts the code used for all data compilations and

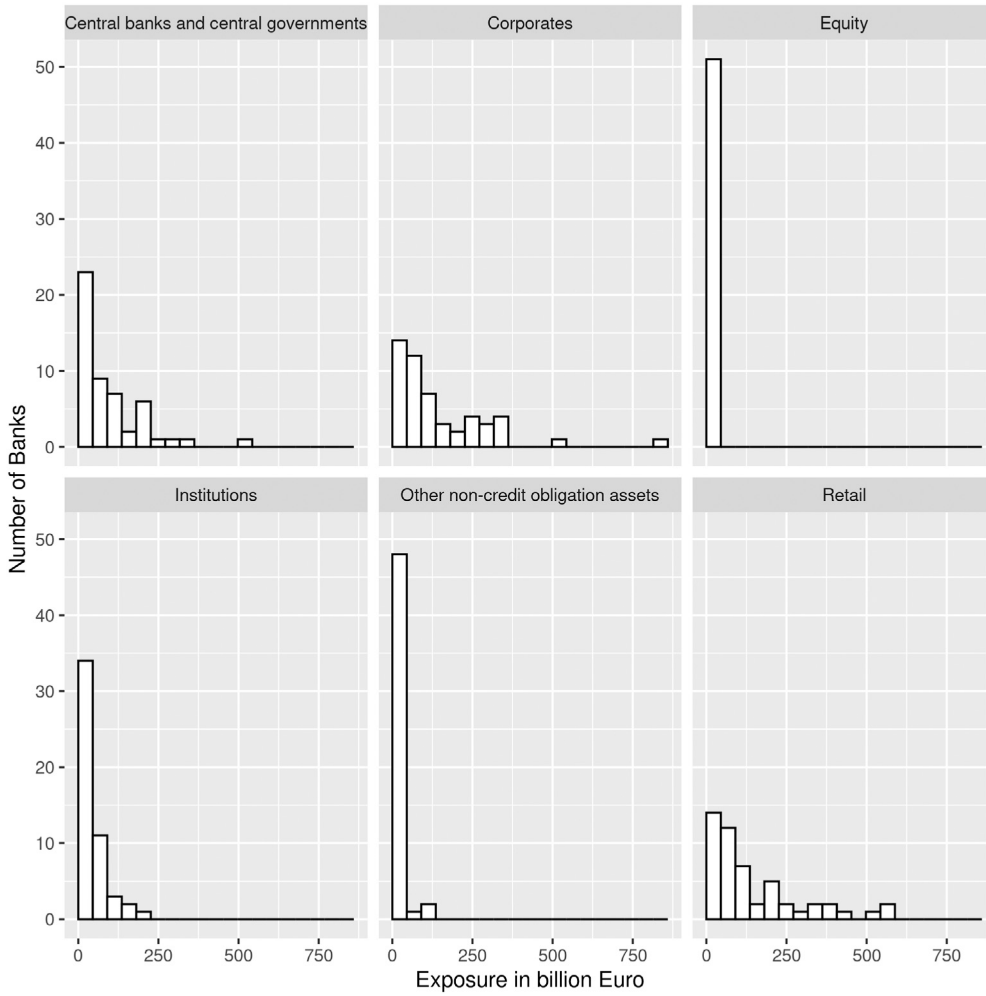

Fig. A.5. Histograms of exposure size in the different IRB exposure categories for the cross section of banks in the EBA 2016 stress test.

Exposure and impairment data. We first retrieve the IRB credit risk exposures from the file TRA_CR.csv and filter the data-file according to Table A.8.

The exposure values for F-IRB and A-IRB positions as well as for defaulted and non-defaulted assets are added up for each bank and each country to which the banks are exposed for each of the different exposures.[16]

The impairment data, which report impairment rates,[17] we retrieve from the file TRA_CR.csv as in Table A.9.

These impairment rates are reported for one year, two year and three years into the future for a baseline as well as for an adverse scenario.

The next step is to retrieve all the exposures reported according to the STA approach. Here the query scheme is as in Table A.10

For the impairment data on the STA positions we use the query described in Table A.11.

As with the IRB case we organize these data in the same format in one long-format data table with the same variables.

Data on bank equity. We also retrieve data which are independent of the accounting framework (IRB, STA) and which are stored in the data file TRA_OTH.csv on the EBA website. These data are the common tier 1 equity, tier 1 equity and the leverage ratio. The data are retrieved using the following query summarized in Table A.12.

From Tables A.8 and A.10 it can be seen that the IRB and STA data do not use the same classification of assets. To organize the data in a coherent and uniform balance sheet we have to make some assumptions. We map the STA positions to the IRB scheme. We make our assumption on the mapping precise here:

Assumption 3. Our mapping uses the following rules:

1. Exposures 1100, 1200, 1300, 1400, 1500, 1600 and 1700 STA are mapped into Exposure 1100 IRB and then added with the IRB values into an overall position for central banks and central government.
2. Exposure 2000 in IRB and Exposure 2000 STA are added to one position Institutions.

computations used in this paper. We describe the filters used for the 2016 data here in detail.

[16] This aggregation step is necessary because the EBA data leave the respective field for the aggregate IRB exposure empty in the raw data file.

[17] The impairment rate is a ratio of the impairment flow which contains the probability of default as well as the loss given default, and the exposure. The EBA file contains only the ratio but not the nominator and the denominator of this ratio separately.

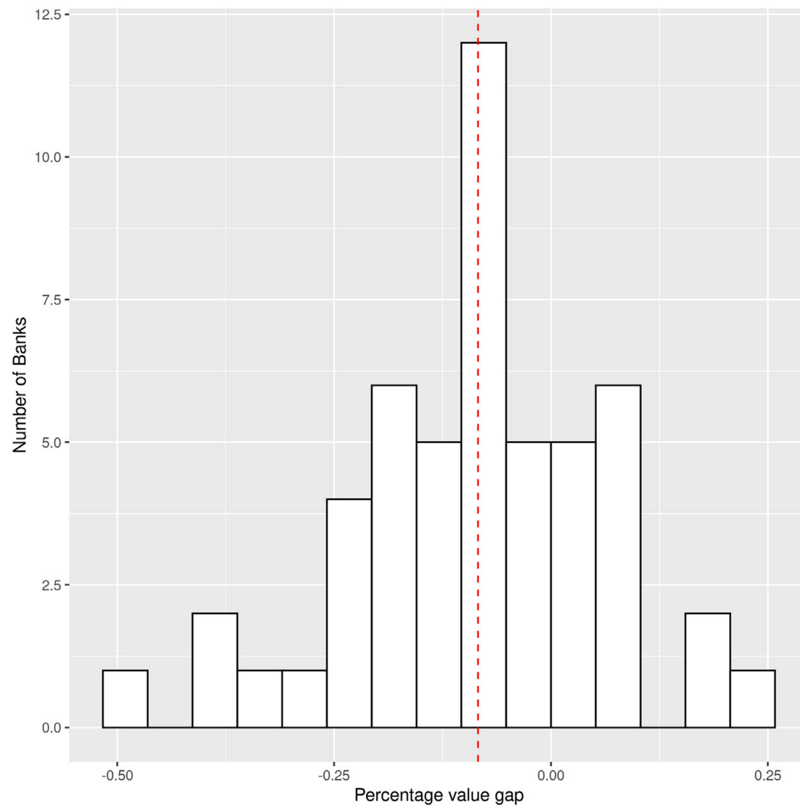

Fig. A.6. Histogram of the value gap between the total value of reported EBA exposures and total assets as reported in the bank balance sheet as a percentage of reported total assets.

3. Exposure 3000 in IRB and Exposure 3000 STA are added to one position Corporates.
4. Exposure 4000 in IRB and Exposure 4000 and 5000 in STA are added to one position Retail.
5. Exposure 6100 in IRB and Exposure 6100 in STA are added to one position Equity.
6. Exposure 6200 in IRB and Exposure 6200 in STA are added to one position Securitization.
7. Exposure 6300 in IRB and Exposure 6300 in STA are added to one position Other.

When we have to add up impairment rates across STA categories we use the exposure weighted averages across the subcategories for aggregation of impairment rates.

The biggest exposures are held against governments, corporates and households. The positions which are classified as equity and other obligations are significantly smaller. Exposures toward institutions are in between. A more detailed picture of the exposure distribution across banks is given in Fig. A.5.

A.2. Total assets and residual position

The EBA exposures reports positions which are subject to credit risk according to the supervisory rules. Thus when we add up the assets of each bank, reported in the `TRA_CR.csv` file, we will not get the total assets of the bank but most of the times less than that and in rare cases more than that. These gaps can be quantitatively substantial. The reported sum of assets subject to credit risk is smaller than the total assets reported in the published balance sheet of a bank, if the regulatory reporting framework allows the bank to exclude certain exposures from reporting because they have no credit risk (according to the reporting requirements). Sometimes an actual exposure is considered as not revealing the actual risk and the regulatory framework forces banks to apply certain multipliers to these positions. In that case the total value of exposures with credit risk may even exceed the value of total assets reported in the balance-sheet.

To deal with this we introduce the residual position as an additional artificial asset class, if the value of the total EBA exposures is less than the total assets reported in the balance sheet. In the case the EBA position is larger we take this value as the total asset figure. Unfortunately these residuals can be fairly large and can go in either direction. They also show no clear systematic pattern over time. In the 2016 sample the negative gaps dominate the value gap. We cannot fully clarify these discrepancies which must have its deeper roots in the financial regulatory reporting framework. To get a rough quantitative impression about the magnitude of these discrepancies we show a histogram displaying the distribution of the gap between the total value of reported EBA assets and the value of total assets as reported in the balance sheet as a percentage of total assets (see Fig. A.6).

A.3. Attributing sovereign bond exposures

The EBA data contain information about the exposures of each bank in government bonds. This information is stored in the `TRA_SOV.csv` file on the EBA website. Sovereign exposures contain subcategories of securities available for sale (AFS), positions designated at fair value through profit and loss (FVO) and securities held for trading (HTF). This allows a split of the overall position into loans and securities. This allows the application of our framework to a limited but very important security class held on the bank balance sheet.

The precise query for these data is given in Table A.13.

We subtract the sum of the exposure values of 1690503, 1690506, 1690507 and 1690508 from the total position 1100 governments and central banks. This difference is the value we attribute as a sovereign bond position for each of the 51 banks in our sample. Though the order of magnitude of sovereign bond exposures to the total assets of a bank look right on average there are some problems we cannot

Table A.14
Query scheme for sovereign bond figures from the file `TRA_SOV.csv` AFS means available for sale, FVO means fair value through profit and loss, and HFT means held for trading.

Variable	Value	Meaning
Period	201912	December 31 2019
Item	2020811	Total carrying amount of non-der. financial assets
SOV_maturity	8	All maturities
Accounting Portfolio	0	No breakdown by accounting portfolio

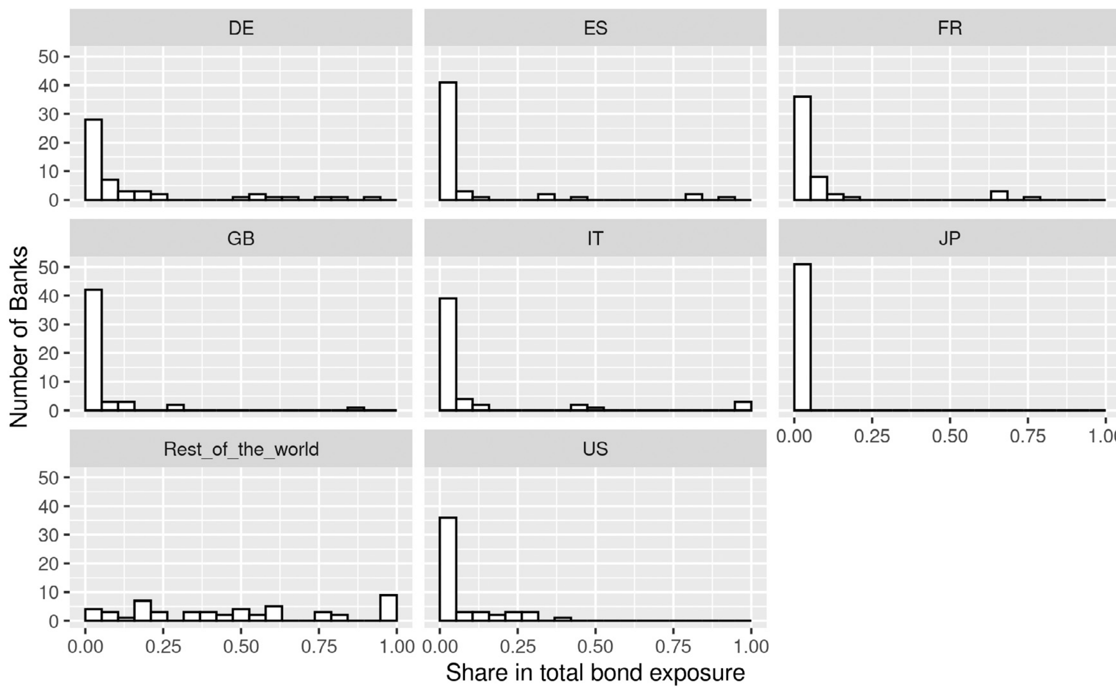

Fig. A.7. Histograms of exposure size in the different IRB exposure categories in the cross section of banks in the EBA 2016 stress test.

Table A.15
Geographical distribution of sovereign bonds across regions for the aggregate banking system in 2016.

Country	Share
DE	0.11
ES	0.08
FR	0.09
GB	0.10
IT	0.09
JP	0.01
US	0.11
Rest of the world	41.00

fully explain. It is for instance not always the case that the sum of 1690503, 1690506, 1690507 and 1690508 is strictly smaller than the total position 1100. If 1100 reports the entire exposure to central banks and central governments including all loans and securities this should in theory be the case. In the case where this sum exceeds the value reported under 1100 we assume that the entire exposure is held in government bonds. (See Table A.14.)

Table A.15 shows the geographical distribution of sovereign bond exposures. About half of the exposure is in countries for which we can access public data on average daily volume and the volatility of sovereign bond prices.

We finally show a histogram displaying the distribution of bond exposures in the cross section of banks toward every of the individual regions in Fig. A.7.

A.4. Market data for bonds and the residual position

Bond prices. Bond prices are retrieved from http://us.spindices.com. Table A.16 gives the data we get from this site. We retrieve the data for Germany, Spain, Great Britain, France, Italy, Japan and the United States. All other countries are aggregated in a position rest of the world.

We show the time series of the indices in Fig. A.8.

Average daily volume for sovereign bonds. We collect the data for average daily volumes from various public sources from the internet. This collection process is rather messy, because the data are only partially available. They are stored in different formats and are often only available as graphics. We give a table describing the sources for our volumes data, for the countries we can actually use in our analysis.

To compute an average daily volume figure for the rest of the world we use an idea from Cont and Schaanning (2017). They observe a high correlation between the nominal debt outstanding and the average daily volume. Figures about the nominal debt outstanding can be retrieved from the BIS international debt statistics (https://www.bis.org/statistics/secstats.htm). Denote the nominal debt outstanding in country i by N_i and using the ADV data we have, following Cont and Schaanning (2017), we run the regression:

$$\log ADV_i = c_1 \log(N_i) + c_0 + \epsilon_i$$

Then we use the values of the estimated parameters c_1 and c_0 and the relation to assign an expected average daily volume for the rest of the world by adding all nominal values outstanding except for the countries where we have direct observations.

Table A.16
Description and sources of sovereign bond indices used in the paper.

Country	Index
Germany	Germany Sovereign Bond Index
Spain	Spain Sovereign Bond Index
France	France Sovereign Bond Index
Great Britain	U.K. Gilt Bond Index
Italy	Italy Sovereign Bond Index
Japan	Japan Sovereign Bond Index
United States	U.S. Treasury Bond Index
Rest of the World	S&P Global Developed Aggregate Ex-Collateralized Bond Index

Table A.17
Sources of average daily volumes data of sovereign bonds.

Country	Link
Germany	https://www.deutsche-finanzagentur.de/en/institutional-investors/secondary-market/
Spain	https://www.tesoro.es/sites/default/files/estadisticas/15I.xlsx
France	https://www.afme.eu/reports/data/details//Government-Bond-Data-Report-Q2-2019
Great Britain	https://www.dmo.gov.uk/data/gilt-market/turnover-data/
Italy	https://infostat.bancaditalia.it/
Japan	https://asianbondsonline.adb.org/data-portal/
US	https://www.sifma.org/resources/research/us-treasury-trading-volume/
Rest of the world	Computed

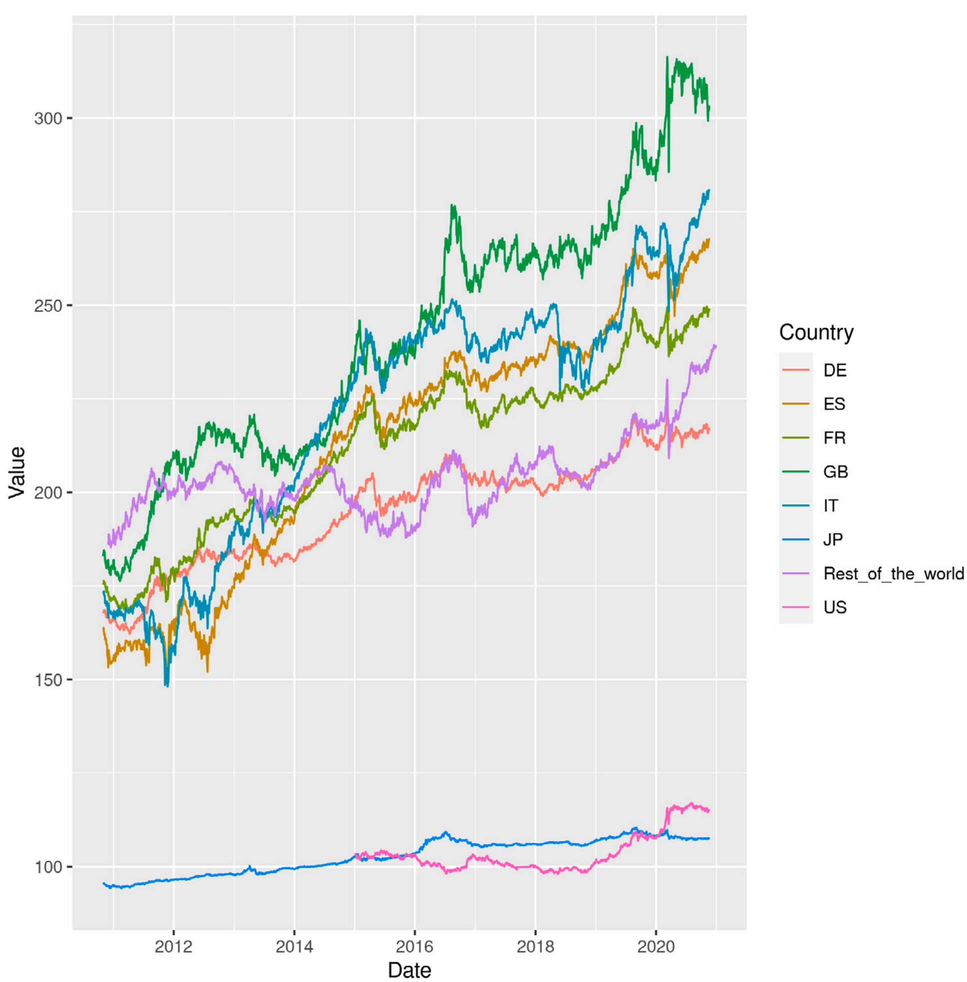

Fig. A.8. Time series of the different bond indices of Fig. A.8. The graph shows that the bonds of GB and Italy are the most volatile while US, JP and DE show the least volatility.

Table A.18
Average daily volumes of different sovereign bond classes for the year 2016. The individual country values are from public data sources listed in Table A.17. The figure for the rest of the world is based on an estimation.

Country	Volume	Unit	Currency
DE	17039.68	Million	Euro
ES	8288.12	Million	Euro
FR	8500.00	Million	Euro
IT	5164.63	Million	Euro
JP	36736.51	Million	Euro
GB	34853.66	Million	Euro
US	467657.66	Million	Euro
Rest_of_the_world	97852.20	Million	Euro

Readers who are interested in all details are referred to the R-script `make_price_volume_data.R` in the folder `data-raw` in the GitHub repository for the `syslosseval` package. Here we show the numbers in Table A.18.

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