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On the role of fundamentals, private signals, and beauty contests to predict exchange rates[☆]

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ABSTRACT

This paper proposes a model where heterogeneous agents formulate their predictions of exchange rates based on a Bayesian learning process and higher-order beliefs where fundamentals and private information are used. We exploit survey data on professional forecasts to estimate the model through a Bayesian approach. Our analysis shows that higher-order beliefs are crucial, as they improve the ability to make predictions of exchange rates due to the possible coordination among agents. Moreover, public information plays the most critical role in determining individual predictions. Although the precision of the private signal is higher than the public one, information publicly revealed does exert a disproportionate influence, and differences in the estimated signals determine the equilibrium strategy of each agent as a combination of personal beliefs and higher-order expectations.

1. Introduction

Should we believe that the agent's expectations only depend on each individual's available information? If so, could such expectations drive the equilibrium outcome? Can other investors' choices matter in determining such expectations?

Systematic biases in individual expectations have received attention since Lucas (1972). Recent evidence has tried to better comprehend the limitations agents face in the acquisition process of information, as made by Sims (2003) and Woodford (2002), amongst others. This aspect is now explored and even associated with the accumulated evidence that a price system based on competitive markets can aggregate information dispersed in the economy. Along this line of reasoning, public disclosures of private information might facilitate information aggregation and dissemination through strategic interactions among agents. A consensus forecast is often considered an aggregator of heterogeneous private information and represents a coordination device that might help forecasters to make inferences about others' beliefs. It is therefore of interest to define a modeling strategy that accounts for it. More specifically, the impact of private and public information is usually evaluated by looking at a coordination game. The net effect of their combination is ambiguous. On one side, people interact strategically with complementary or substitutable actions. On the other side, public disclosure of information through higher-order beliefs

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may lead to unintended consequences regarding overreaction in the market. The strategic interactions may even have some extreme consequences, due to potential herding behavior. Unlike private information, public information is common knowledge among forecasters through the mechanism of higher-order beliefs. Whenever large fractions of individuals follow this behavior, public information is crucial to coordinate their expectations while reducing the role of private sources, independently of the precision of that information.

Following this line of research, we propose an explicit coordination model à la Morris and Shin (2002) that aims at disentangling the effect of private and public information in the expectation formation mechanism. Indeed, the extent to which different information sources are relevant in evaluating the economic aggregates is still unexplored in the literature. To the best of our knowledge, only a few contributions have tried to estimate the importance of higher-order expectations in making predictions. We extend the analysis by proposing a structural model to evaluate whether agents pay attention to other market participants' actions. We focus on the exchange rate market, where agents form their expectations by combining attention to economic fundamentals and personal knowledge. One of the main difficulties in this framework is the observability of private information, hardly revealed by the simple observation of prices. In the exchange rate literature, it is associated with order flow (Evans & Lyons, 2002; Menkhoff et al., 2016), which tends to be accessible only to a select group of large dealing banks. An alternative option adopted in the literature is to use survey data on individual forecasts, which appears to be a reliable tool to retrieve private information for econometric models. We mostly use the Foreign Exchange Consensus Survey data, as in Jongen et al. (2012), by looking at the EUR/USD currency pair to estimate private signals and evaluate the role of fundamentals and private information in determining expectations of exchange rates. We thus estimate a Bayesian learning process with private and public information and observe how the precision of signals guides the equilibrium strategy. Empirical results are obtained by casting our model in a state-space form that allows for Bayesian inference.

The analysis yields two main results. First, we find that higher-order beliefs account for about 86% of the prediction value. Neglecting this factor leads to poor inference about fundamental dynamics, forcing agents to rely primarily on private information, which is quite heterogeneous, and thus to make lousy predictions. Second, public information plays the most critical role in determining individual expectations, by more than 80% compared to private information. The importance of public information drops dramatically without higher-order expectations. Although the precision of the private signal is higher than that of the public signal, publicly disclosed information exerts a disproportionate influence. Our main conclusion is that higher-order beliefs help agents to retrieve more precise information about macroeconomic fundamentals and thus allow forecasters to improve their ability to predict exchange rates, even in the short run. Our framework and the related results appear to be robust to alternative extensions that we briefly offer in the appendix, which discusses the issue of bounded rationality and so forth.

The remainder of this paper is as follows. Section 2 encompasses the relevant literature in the field, while Section 3 describes survey data on individual exchange rate forecasts. Section 4 introduces the theoretical setup. Section 5 presents our empirical results, and some comments are given in Section 6 about the posterior estimates and policy implications. Concluding remarks are in Section 7.

2. Related literature

We are not the first to analyze a model in which a coordination game discloses the importance of higher-order beliefs among agents. In particular, our analysis speaks directly to several strands of literature involving, on one side, the use of relevant (or not) information in the determination of forecasting expectations and price determination and, on the other side, the role that public information may have on aggregate outcomes while considering the importance of survey heterogeneity for empirical estimations. This section is helpful, first, to understand how our contribution is placed in the literature and, second, to highlight the pros and cons of our approach compared to previous papers.

Starting from Keynes (1936)'s beauty contest game, it was asked whether the role of extrinsic information – interpreted as random uninformative events, i.e., sunspots – may influence agents' behavior in coordination games. In their seminal work, Cass and Shell (1983) explored the influence of such extrinsic information on economic activities. They showed that in the presence of multiple equilibria, agents condition their decisions on publicly observable, although intrinsically uninformative, signals. The interesting aspect is that, due to their public nature, such irrelevant information may not be focal for the determination of agents' beliefs and, as a consequence, on which equilibrium agents coordinate. Under a lab design, Duffy and Fisher (2005) were the first to provide experimental evidence for the occurrence of sunspots. They investigated whether news on price – not connected with market fundamentals – may generate sunspots, and discovered that whenever people do not share a common view on the sunspot and the related environment, such a public announcement does not play any role. The conditions for a coordination problem emerge under some training phases of the subjects. Siebert and Yang (2021) analyzed the conditions when a sunspot event may dissuade people from choosing a coordination strategy under the evaluation of their changes in risk and payoff. Their claim was that a sunspot can be less relevant if the process of convergence to it is less risky than a potential convergence to the payoff-dominant equilibrium and not so much riskier than the one related to the risk-dominant equilibrium. Our approach is complementary, as we allow for a coordination game generated by public information – not random sunspots – in higher-order beliefs, while estimating the importance of private components in the determination of exchange rates through a structural model.

The second strand of literature – much more related to our paper – investigates the strategic interactions among agents in the future realization of price under the complementarity and substitutability assumptions and the potential convergence to the well-defined rational expectations equilibrium (REE). Based on a forecasting learning process à la Hommes et al. (2005), Bao et al. (2012) proposed an experimental design where agents forecast the asset price, knowing that their forecasts and the ones of the other agents matter in the price determination. Although agents are uncertain about the exact relationship between their forecasts and the price, they showed that whenever individuals have an incentive to make opposite forecasting predictions compared to what the others do – so the actions are strategic substitutes – price forecasts and the aggregation of the real prices converge to the REE.[1] Instead, in the case of strategic complementarity, they do not follow the REE with large variations for both forecasting and the resulting price. However, the impact of strategic substitutability or complementarity is not well defined but depends on the heterogeneity of individuals. For instance, Haltiwanger and Waldman (1985, 1989) looked at an environment with both naïve and sophisticated agents. The authors argued that under strategic complementarity, agents have much more incentive to mimic the others' actions in higher-order beliefs, while the opposite is true for strategic substitutes. Along this line of reasoning, Colasante et al. (2018, 2019) extended the analysis on both strategic complementarity and substitutability while eliciting short- and long-run expectations about the evolution of market price. In their design, agents receive incentives to submit their predictions at the beginning of each period and revise their beliefs under a new set of information. On one side, they showed that in the short run, there is less coordination in the market with strategic substitutability, due to the opposite incentives that agents have in predicting low or high prices. However, their price convergence to the REE is faster, since expectations are, on average, close to the fundamental value. On the other side, strategic complementarity implies oscillatory dynamics of prices diverging in the long run with a lower rate of convergence, although the coordination among agents is easier to make.

The third strand of literature we refer to focuses on the role that public and private information plays in price determination when there is strategic interaction among agents. In particular, when public information is released, it may have an unexpected consequence, as it may reduce the production of private information from agents (as a crowding-out effect), or alternatively, the weight assigned to the public sources increases independently of their precision. Our model shows that this is possible in the exchange rate market, and the reason is motivated by the direct influence that public information has on the higher-order belief process of agents as a coordination device along the lines of Morris and Shin (2002). This framework was recently extended by Coibion et al. (2021), studying the macroeconomic expectations of managers. They looked at a unique survey measuring not only the expectations of each agent about fundamentals but also what in higher-order beliefs they think other managers expect for inflation.[2] They showed new evidence on the learning process through a series of information treatments for modeling new dimensions of a noisy-information game. However, overweighting effects can appear even without an explicit incentive to coordinate as in Ruiz-Buforn et al. (2021). They proposed an experiment to understand whether the bounded rationality of agents determines this effect through the commonality component (higher-order beliefs), showing that even a small common noise among agents may improve the aggregation of information, causing an overreaction of the market to public disclosure. This is similar to our point, although our contribution looks at the importance of commonality while including the evaluation of the coordination game in a strategic complementarity framework. Our coordination game has a similar structure to that found in the literature concerned with information sharing (e.g., Angeletos & Pavan, 2004, 2007; Vives, 1997). This choice represents a simple and flexible tool to link the theoretical model to its empirical counterpart and allows us to get estimates of the parameters in the individual utility function. The impact of public disclosure can be motivated even by the continuous process of beliefs revisions. Broer and Kohlhas (2022) confirmed individual overreaction in forecasts of GDP and inflation and explained this result as mainly due to the overconfidence which causes agents to over-revise their forecasting process, misperceiving others' responses to information.

Our analysis resembles some existing work in the literature, although in a different environment. In this respect, survey forecasts are pivotal in making inferences under heterogeneity. For example, Lahiri and Sheng (2008, 2010) followed the same idea of evaluating forecasters' rationality and optimal use and reaction to information. In particular, considering inflation and GDP predictions, they proposed a heterogeneous setting by assuming that agents differ because of their prior beliefs and different abilities to interpret new public signals. This second factor is crucial for addressing the disagreements observed in survey data forecasting. Following their intuition, our approach confirms the role of public information (with the overweighting effect shown in the literature) with the difference that, in our case, all agents share the same prior beliefs on the target variable. Heterogeneity is introduced by explicitly including individual private signals in the agent's information set.[3] The capacity of investors to predict asset prices based on past observations and

[1] See also Bao et al. (2013), where the authors designed an experiment considering the possibility that agents correctly forecast future prices on average and always solve optimization tasks based on the available information. In this case, including both dimensions in the analysis, the authors showed that all treatments converge to the REE.

[2] Interestingly, exploiting a daily online U.S. survey, Coibion et al. (2022) recently investigated whether a new monetary policy introduced by the Federal Reserve in 2020 impacts the inflation expectations of households. They confirmed that the public announcement of the policy went largely unnoticed by the population, and even those who heard the news did not include it in their expectations.

[3] See Lui et al. (2011) for evaluating the ability of forecasting of employers based on qualitative expectational data.

current information on interest rates and dividends was investigated by Colasante et al. (2017) in an experimental setup. Interestingly, they found that agents are coordinated, meaning they are able to infer the predictions of other participants. Rangvid et al. (2013) confirmed this finding by observing that forecasters consider other agents' expectations, or at least their average (consensus) while forming their own.

Finally, we were inspired by the contributions made on the expectations and learning-to-forecast literature on macroeconomic theory modeling the behavior of exchange rates. Building upon work by Bacchetta and Van Wincoop (2004, 2006), who explored the implications of heterogeneity in expectations, we differentiate by proposing an expectation formation process on the current exchange rate and then derive one-step-ahead expectations through the Kalman filter, as made by Coibion and Gorodnichenko (2012), on forecasting in the inflation case. Bacchetta and Van Wincoop (2004) instead developed a framework where agents observe current exchange movements that are basically inconsistent with their future expectations. Searching for an explanation for this inconsistency, a weight higher than average is assigned to some fundamentals chosen as so-called scapegoats. Fratzscher et al. (2015) developed an empirical test of this theory using a proxy of scapegoat effects from a survey from Consensus Economics regarding the weights assigned by panelists to several macroeconomic fundamentals. The authors found that including these expectations improved the explanatory power of the fundamentals.[4] Bacchetta and Van Wincoop (2006) focused instead on order flows and introduced a possible explanation for the empirical results verified by Evans and Lyons (2002), Payne (2003), and Froot and Ramadorai (2005).[5]

3. Foreign exchange survey data

We mainly consider data on individual expectations obtained from the *Foreign Exchange Consensus Forecasts* survey by Consensus Economics of London. We also examine more recent data sampled at a quarterly frequency from Bloomberg, spanning from 2012:Q4 to 2022:Q1. Empirical evidence based on this second dataset is reported in Appendix E.

In the Consensus Economics survey, panelists are asked to forecast spot rates at different maturities on the second Monday of each month. The sample is composed of almost 250 panelists spread worldwide, and 40 of them are personally identifiable by their names. Some panelists systematically provide predictions in each publication, while others appear with a lower frequency. There are also cases in which the panelist is included yet the corresponding forecast does not appear.

Although the survey refers to different currencies, we focus on EUR/USD one-month-ahead forecasts from January 2006 to June 2012.[6] Our analysis is conducted by taking into account individual forecasts, i.e., forecasts reported by personally identifiable panel members in the publication. The average expectation of all members is also published and indicated by *Consensus Forecasts*.

Some issues with this dataset involve missing data, as panelists do not always provide their forecasts for the next month. For the EUR/USD exchange rate, we thus built our dataset as follows: first, we collect all individual forecasts from 2006 to 2012, and second, we record only predictions from panelists with a response rate higher than 40 percent, thus selecting 15 time series of individual expectations. This results in an unbalanced panel based on 15 time series, nine of which hold fewer than five missing data. On average, for each month there are more than 13 individual forecasts available, with at least 11 individual predictions observed, thus guaranteeing a sufficient cross-sectional heterogeneity among forecasters and a rather stable cross-sectional dimension over time. Over the whole sample, the response rate by the panelists is larger than 90%. Fig. 1 shows that the average one-month-ahead forecasts approximate fairly well with the actual exchange rates. As stressed by Jongen et al. (2012), expectations are dispersed, confirming heterogeneity among panelists. In particular, the lower panel of Fig. 1 shows that the dispersion is relatively moderate from January 2006 to September 2007. Then it increases until reaching a peak in January 2009 and finally declines and stabilizes from November 2009 to June 2012. Regarding the representativeness of panelists, it is worth noting that some of them represent major dealing banks. More specifically, eight of them belong to the top 10 currency traders of the forex market with more than 70% of the cumulative market share. Furthermore, some of the institutions selected provide trading platforms for smaller banks. This procedure is called white labeling and is highly efficient for market dynamics, although it induces a concentration of information. In this way, white-label banks can directly observe small banks' trading flows and extract from these data possibly relevant information at lower costs (King et al., 2012). Some of the information described above is summarized in Table 1.

A preliminary analysis aimed to assess whether professional forecasters look at the beliefs of others, based on Rangvid et al. (2013), who examined the regression

$$\hat{\mathbb{E}}_t^i[s_{t+1}] = \alpha_i + \theta_i \mathrm{E}[\bar{\mathbb{E}}[s_{t+1}]] + \gamma_i \hat{\mathbb{E}}_{t-1}^i[s_t] + \epsilon_{i,t} \quad (1)$$

where $\hat{\mathbb{E}}_t^i[s_{t+1}]$ is the exchange rate forecast for the forecaster i at time t, $\mathrm{E}[\bar{\mathbb{E}}_t[s_{t+1}]]$ denotes the expected

[4] In the short run, the heterogeneity in the individual evaluation may lead to overrating the random macroeconomic fundamentals.

[5] Evans and Lyons (2002) exploited data about bilateral transactions among FX dealers via Reuters Dealing 2000–1 electronic trading system. They followed Meese and Rogoff (1983)'s methodology to investigate the out-of-sample forecasting ability of their linear model. To evaluate possible feedback effects of exchange rates on order flow, Payne (2003) and Froot and Ramadorai (2005) elaborated on a *VAR* model based on the size of transactions and interest rate differentials. Their methodology allowed for a more precise estimation of the information provided by the order flow and for estimations of the long- and short-run effects of international flows on exchange rates and their relation to fundamentals.

[6] We also considered other exchange rates, such as USD/JPY with the same frequency and timespan and GBP/USD, where we collected expectations on a two-monthly frequency. The results are available upon request.

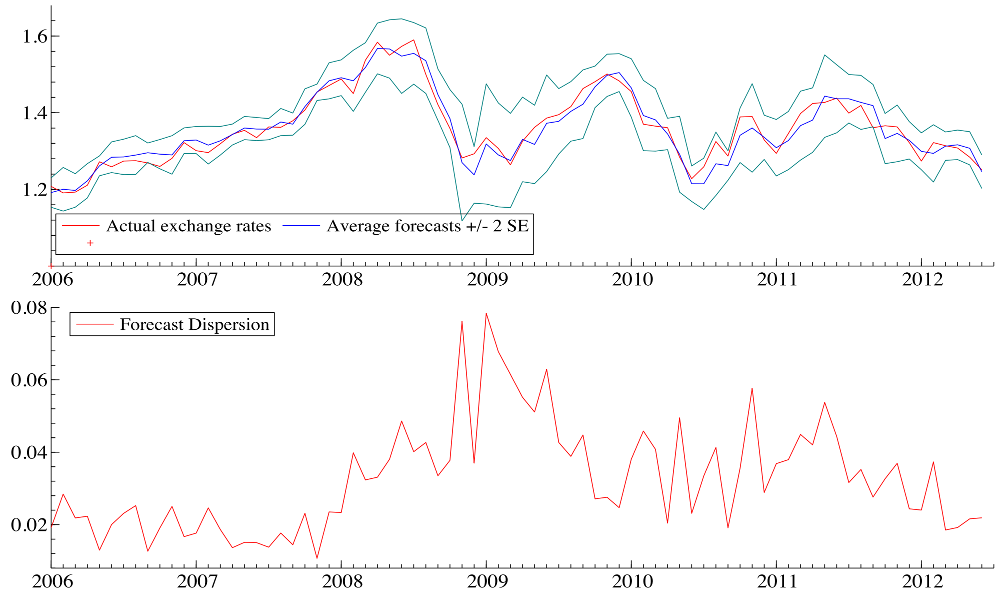


Fig. 1. EUR/USD. Upper panel: Actual vs. average forecasts together with a 95% confidence interval from January 2006 to June 2012. Lower panel: Estimated dispersion over time.

Table 1
EUR/USD. Predictions of individual forecasters. Column 2: number of missing forecasts; Column 3: companies with proprietary trading platforms (implementing white labeling); Column 4: The company has been in the top 10 currency traders list; Column 5: ADF test for the forecast error (*p*-values in parentheses); Column 6: "yes" if the rational expectation hypothesis is confirmed and "no" otherwise.

	No. missing	Own platform	Top 10	ADF test	RE?
Bank of Tokio – Mitsubishi	1			−3.392 (0.011)	yes
Barclays Capital	31	yes	yes	−1.654 (0.455)	no
BNP Paribas	7		yes	−3.669 (0.005)	yes
BoA – Merrill Lynch	4		yes	−3.256 (0.017)	yes
Citigroup	45	yes	yes	−1.700 (0.431)	no
Commerzbank	4			−4.186 (0.000)	yes
Deutsche Bank Research	18	yes	yes	−1.850 (0.356)	no
General Motors	0			−3.472 (0.009)	yes
HSBC	1	yes	yes	−2.686 (0.077)	no
IHS Global Insight	0			−3.226 (0.019)	yes
J.P. Morgan	10	yes	yes	−2.041 (0.269)	no
Oxford Economics	7			−3.394 (0.011)	yes
Royal Bank of Canada	0			−2.594 (0.094)	no
UBS	3	yes	yes	−1.764 (0.398)	no
WestLB	2			−3.427 (0.010)	yes

Table 2
EUR/USD: Random parameter panel model.

	Consensus data (2006–2012)		Bloomberg data (2013–2021)	
	Estimate	95% Conf. Int.	Estimate	95% Conf. Int.
Consensus	.4626	[.254, .670]	.2428	[112, .373]
$\mathbb{E}^i_{t-1}[s_t]$	.4561	[.265, .647]	.6099	[.487, .732]
const.	.1107	[.052, .169]	.1687	[.130, .206]
Number of forecasters	15		43	
Obs. per forecaster	63.8		35.8	

consensus forecast at t, whereas lagged individual expectations are used as control variates. As a proxy for the expected consensus, we used the lagged observed consensus, as in Rangvid et al. (2013). To allow for heterogeneity across forecasters, a random coefficient model was used, and its estimates are reported in Table 2.

These results suggest a strong and significant effect of the consensus on individual expectations, which is also interpreted as herding in Rangvid et al. (2013).[7] Using an extended version of the consensus dataset, Frenkel et al. (2020) obtained different results, examining the conditional probability that individual forecasts overshoot/undershoot actual exchange rates given that the forecast exceeds the consensus forecast, and finding anti-herding behavior. This evidence is likely due to the different definitions of herding used in the two tests. Nevertheless, it highlights the importance of investigating how information about higher-order beliefs should be taken into account when considering exchange rate models.

As a further preliminary exercise, we investigate whether this survey on expectations is consistent with the rational expectation hypothesis (REH). The test implemented is based on Liu and Maddala (1992) and checks whether the forecasting errors $s_{t+1} - \hat{\mathbb{E}}_t^i[s_{t+1}]$ are covariance stationary. In case they are, we cannot discard the rational expectation hypothesis.[8] The results of this test are reported in the last two columns of Table 1, where we indicate by "no" the rejection of the REH. Overall, the REH is not always supported by the data and therefore is questionable, even at this short forecasting horizon, thus suggesting that a different specification for subjective expectations might be suitable in this case.

4. Forming expectations of exchange rates

Our analysis is based on a model for exchange rates in which a number of heterogeneous agents form their expectations by combining different sources of information. Following Engel and West (2005), we assume that exchange rates are modeled by fundamentals and future expectations of currencies. As mentioned in the previous section, here, we deviate from the rational expectation paradigm which assumes that agents form their expectations about future prices based on econometric forecasts. Therefore, in line with Bacchetta and Van Wincoop (2006), we assume that exchange rates are determined by aggregate expectations as follows:

$$s_t = \lambda \bar{\mathbb{E}}_t[s_{t+1}] + (1-\lambda) f_t - \lambda \psi_t \tag{2}$$

where $\bar{\mathbb{E}}_t[s_{t+1}]$ are average expectations about future exchange rates, λ is a parameter, and ψ_t is the liquidity premium, whereas the fundamental f_t is a combination of macroeconomic variables.

We postulate that agents adopt a Bayesian rule to process information about future exchange rates, and that they account for strategic interactions, since we assume that they have an incentive not to deviate from the average market forecast.

We first tackle the problem agents face when gathering information to predict exchange rates, and then we model the predictions in the case of strategic interactions.

Consider an economy populated by a finite series of predictors, $i = 1, \ldots, n$. In period t, each agent i observes noisy private and public signals about the exchange rate s_t. In our Bayesian setup, prior knowledge about s_t is summarized by a random walk, i.e., a priori:

$$s_t \sim N(s_{t-1}, \sigma_\gamma^2). \tag{3}$$

Furthermore, each agent receives a common public signal about the fundamental f_t as a function of the exchange rate:

$$f_t = s_t + \eta_t \qquad \eta_t \sim N(0, \sigma_\eta^2) \tag{4}$$

and a private personal signal:

$$x_{it} = s_t + \epsilon_{it} \qquad \epsilon_{it} \sim N(0, \sigma_\epsilon^2) \tag{5}$$

So while information about the fundamental f_t is common knowledge among agents, the private signal x_{it} is specific to agent i and not observed by others. In the following, we parameterize distributions in terms of precisions instead of variances; that is, $\rho_s = \sigma_\gamma^{-2}$, $\rho_f = \sigma_\eta^{-2}$ and $\rho_x = \sigma_\epsilon^{-2}$. The posterior knowledge about exchange rates for each agent is thus Gaussian with precision $\rho_{\tilde{y}} + \rho_x$, where $\rho_{\tilde{y}} = \rho_s + \rho_f$ and mean $\mathbb{E}_t^i[s_t|f_t; x_{it}] = \alpha_{x_i} x_{it} + \alpha_{\tilde{y}} \tilde{y}_t$, where $\tilde{y} = \left(\rho_s s_{t-1} + \rho_f f_t\right) / \left(\rho_s + \rho_f\right)$, $\alpha_{\tilde{y}} = \rho_{\tilde{y}} / \left(\rho_{\tilde{y}} + \rho_x\right)$ and $\alpha_{x_i} = \rho_x / \left(\rho_{\tilde{y}} + \rho_x\right)$.

Based on public and private signals at t, the expected prediction of s_{t+1} is thus obtained through the one-step-ahead prediction of the Kalman recursions based on Eq. (3), i.e., $\mathbb{E}_t^i[s_{t+1}] = \mathbb{E}[s_t|x_{it}, f_t, s_{t-1}]$. Thus, by applying the Kalman filter recursions, we find that the best guess about s_{t+1} based uniquely on the subjective knowledge is $\mathbb{E}_t^i[s_t|x_{it}; f_t]$.

Note that agents exploit just lagged information about exchange rates, as is common in the adaptive expectation literature, such as, De Grauwe and Markiewicz (2013) and Lansing and Ma (2017).[9] This mechanism is consistent with the timeline of events described in Elias (2016), which can be summarized as follows:

- At the end of period $t-1$, the exchange rate s_{t-1} is realized.
- At the beginning of t, the value of f_t is realized.
- During t, agents make their predictions for s_{t+1} based on s_{t-1} and f_t.
- At the end of period t the exchange rate s_t is realized.

This timing convention is natural in our setup, since, in the *Consensus* survey, we know when it is published, i.e., the second Monday of each month, but we do not know the exact time the panelists formed their expectations. We thus assume that all agents made their predictions before the end of t.[10]

[7] A broad definition of herding is that the expected consensus affects individual forecasters. Significant coefficients, albeit with different signs, can be obtained by adding lagged exchange rates in Eq. (1). In addition, simulated data from the estimated model in Section 6 yield robust results compared to the Rangvid et al. (2013) test.

[8] Here, we test for unit roots on the forecasting errors through a standard ADF test without trend and with at most four lags.

[9] For asset pricing, a similar hypothesis was used in Hommes et al. (2005) and Branch and Evans (2011).

[10] Furthermore, as stressed by Menkhoff et al. (2016), amongst others, the currency market is mainly developed as a decentralized over-the-counter market where participants trade with one another

When forming predictions, agents do not just combine public and individual signals. They also pay attention to actions made by other market participants. To introduce strategic interactions, following Morris and Shin (2002), we suppose that each agent builds an individual forecast by minimizing the following expected loss function:

$$\mathbb{E}_t\left[-(1-\delta)(e_{it}-s_{t+1})^2-\delta(e_{it}-\bar{e}_t)^2\right] \tag{6}$$

where $e_{it} = \mathbb{E}^i_t[s_{t+1}]$ is the individual expectation of subject i at time t, and $\bar{e}_t$ is the average forecast or consensus. The parameter $\delta \in (0,1)$ is a scalar that describes the intensity of the coordination motive, i.e. the importance that agent i attaches to the expectations of other market predictors. The first component, sometimes indicated as forecasting cost, is a quadratic loss in the distance between the expectation and the actual exchange rate, while the second component is a quadratic loss in the distance between the expectation and the consensus. As stressed by Rangvid et al. (2013), agents might be interested in not deviating from the consensus, whether to preserve their reputation, to take advantage of other's private information, or because they believe other agents look at the consensus itself, namely, the beauty contest motive. This is the factor related to higher-order beliefs, whose weight is expressed by the parameter δ. To keep the algebra simple, we assume that the best prediction of each agent is based on current information, that is, x_{it} and f_t, even though a generalization that also includes past information can be easily derived, as in Coibion and Gorodnichenko (2012).

Following Marinovic et al. (2011), and assuming that each agent adopts a linear strategy, namely, $e_{it} = \varphi_x x_{it} + \varphi_{\tilde{y}}\tilde{y}_t$, it can be proved (see Appendix A for technical details) that optimal individual predictions must be

$$\mathbb{E}^i_t(s_{t+1}) = \varphi_x x_{it} + \varphi_{\tilde{y}}\tilde{y}_t, \tag{7}$$

where

$$\varphi_x = \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x+\rho_{\tilde{y}}} \quad \text{and} \quad \varphi_{\tilde{y}} = \frac{\rho_{\tilde{y}}}{(1-\varrho)\rho_x+\rho_{\tilde{y}}} \tag{8}$$

while $\varrho = \frac{\delta n-\delta}{\eta-\delta}$. The weight of the beauty contest, δ, identifies the importance of the predictors' expectations. When $\delta = 0$, the predictor's optimal choice coincides with her personal expectation, and public signals increase in relevance as δ gets larger. Furthermore, the sensitivity of the predictor's expectations to the exchange rate depends on the quality of private and public signals in terms of precision, and agents assign lower weights to the private signal, while public sources act as a coordinating mechanism for predictions of others' actions when δ is larger.

By just averaging with respect to the individual forecasts, the consensus is

$$\bar{\mathbb{E}}_t[s_{t+1}] = \varphi_x\bar{x}_t + \varphi_{\tilde{y}}\tilde{y}_t, \tag{9}$$

through various communication modes at a non-visible standard rate at any point in time. Also, as suggested by Hommes et al. (2005), in models with heterogeneous agents, asset prices realized at t depend on aggregate expectations of future prices. Since agents at t do not observe other agents' predictions, they are unable to evaluate the current price s_t precisely, and therefore they exploit information up to time $t-1$.

and can be used in Eq. (2) to obtain

$$s_t = \beta_1 f_t + \beta_2\bar{x}_t + \beta_3 s_{t-1} + \beta_4\psi_t, \tag{10}$$

where $\beta_1 = (1-\lambda+\lambda\tau_2\varphi_{\tilde{y}})$, $\beta_2 = \lambda\varphi_x$, $\beta_3 = \lambda\tau_1\varphi_{\tilde{y}}$, $\beta_4 = -\lambda$, $\tau_1 = \frac{\rho_s}{\rho_s+\rho_f}$, and $\tau_2 = \frac{\rho_f}{\rho_s+\rho_f}$. Note that in this way, exchange rates explicitly depend on public and private information.

So far, we have assumed that agents use a prediction rule that may be misspecified and disconnected from the actual data generation process. This systematic misperception of reality could be the driving force behind the empirical results presented in this paper. However, in Appendix B, we propose a further extension of our model by considering boundedly rational agents à la (Hommes & Zhu, 2014; Lansing & Ma, 2017), where individual forecasts are statistically consistent with observations of the true exchange rate process. We find that the main results of the paper hold even in this case. We also study the case of rational expectations as a limit case, again in Appendix B.

5. Empirical model

5.1. Methods and data

We consider a state-space model to closely mimic the theoretical framework described in Section 4. Our empirical model reads as follows:

$$\begin{aligned}
s_t &= \beta_1 f_t + \beta_2\bar{x}_t + \beta_3 s_{t-1} + \beta_4\psi_t + \epsilon_{s,t} && (11)\\
f_t &= \alpha_0 + \alpha_1 f_{1,t} + \alpha_2 f_{2,t} + \epsilon_{f,t} && (12)\\
f_{1,t} &= \phi_{01} + \phi_{11} f_{1,t-1} + \phi_{12} f_{2,t-1} + \epsilon_{f_1,t} && (13)\\
f_{2,t} &= \phi_{02} + \phi_{21} f_{1,t-1} + \phi_{22} f_{2,t-1} + \epsilon_{f_2,t} && (14)\\
\tilde{y}_t &= \frac{\rho_f}{\rho_f+\rho_s} f_t + \left(1-\frac{\rho_f}{\rho_f+\rho_s}\right) s_{t-1} && (15)\\
\psi_t &= \rho_\psi\psi_{t-1} + \epsilon_{\psi,t} && (16)\\
\mathbb{E}^i_t[s_{t+1}] &= \varphi_{\tilde{y}}\tilde{y}_t + \varphi_x x_{it}, \qquad i=1,\dots,N && (17)\\
x_{it} &= x_{it-1} + \epsilon_{x_i,t}, \qquad i=1,\dots,N && (18)
\end{aligned}$$

Then, Eq. (11) is the empirical counterpart of Eq. (10) that describes the dynamics of exchange rates as a function of private information, i.e., $\bar{x}_t$, economic fundamentals f_t, and past exchange rates, where β_i denotes explicit functions of the parameters. Instead, the parameters defining the expectation formation mechanism are λ, δ, ρ_f, ρ_s, and ρ_x, while the others are called reduced-form parameters.

To be consistent with the theoretical model of Section 4, Eq. (4) would imply an ARIMA (0, 1, 1) representation for the fundamentals f_t with MA parameter $\theta_f = \left(\sqrt{q_f^2+4q_f}-2-q_f\right)/2$ and Gaussian noise, $\epsilon_{f,t}$, with variance $\sigma^2_f = -\sigma^2_\eta/\theta_f$, where $q_f = \sigma^2_\gamma/\sigma^2_\eta$ is the signal-to-noise ratio.

We choose to keep the empirical model as flexible and simple as possible to parsimoniously describe time series features of real data and to avoid potential identification issues linked to the estimation procedure. For this reason, we consider a VAR(1) model with unconstrained parameters for the fundamentals, to account for stationary or unit roots.[11]

The identity in Eq. (15) is the dynamics of the public information defined as a convex combination of past exchange rates and fundamentals, as derived in Section 4. In turn, Eq. (17) identifies the mechanism that forms individual expectations as a mixed effect of private and public information, weighted respectively by φ_x and $\varphi_{\tilde{y}}$ defined in Eq. (8). The liquidity premium ψ_t is an autoregressive process, even though we set $\rho_\psi = 0$ to be consistent with the theoretical setup of Bacchetta and Van Wincoop (2006). We assume that x_{it} in Eq. (18) are non-observable random walks, to model the expected high persistence in the subjective private information.[12] The shocks $\epsilon_t = \left(\epsilon_{s,t}, \epsilon_{f,t}, \epsilon_{f_1,t}, \epsilon_{f_2,t}, \epsilon_{\psi,t}, \epsilon_{x_i,t}\right)$, $i = 1, \dots, N$ are all Gaussian with mean zero and standard deviation, respectively, $\sigma_s, \sigma_f, \sigma_{f_1}, \sigma_{f_2}, \sigma_\psi$ and σ_{x_i}, whereas N is the number of informed agents that make predictions of exchange rates.

This potentially unobservable system can be compactly rewritten as

$$\Gamma_0 \mathbf{x}_t = c_x + \Gamma_1 \mathbf{x}_{t-1} + \Gamma_\epsilon \boldsymbol{\epsilon}_t \tag{19}$$

where $\mathbf{x}_t = \left(s_t, f_t, f_{1,t}, f_{2,t}, \tilde{y}_t, \psi_t, \mathbb{E}_t^i, x_{it}\right)$, $i = 1, \dots, N$, while Γ_0, Γ_1, and Γ_ϵ are parameters matrices. Pre-multiplying Eq. (19) with Γ_0^{-1} gives

$$\mathbf{x}_t = \Theta_c + \Theta_x \mathbf{x}_{t-1} + \Theta_\epsilon \boldsymbol{\epsilon}_t. \tag{20}$$

To take the model to the data, we consider as observables current exchange rates, $\hat{s}_t$, expectations $\hat{\mathbb{E}}_t^i[s_{t+1}]$ from the *Consensus* survey and the two fundamentals $\hat{f}_{i,t}$, $i = 1, 2$, that is, $\hat{\mathbf{y}}_t = (\hat{s}_t, \hat{f}_{j,t}, \hat{\mathbb{E}}_t^i[s_{t+1}])$, with $j = 1, 2$ and $i = 1, \dots, N$.[13] We selected observable expectations $\hat{\mathbb{E}}_t^i[s_{t+1}]$ for $N = 15$ institutions which represent the most influential companies providing predictions for exchange rates in the whole market, as stressed in Section 3.

We base our analysis on Taylor-rule fundamentals, by considering as f_t a linear combination of inflation rate and output gap differentials, namely, $(\pi_t - \pi_t^*)$ and $(y_t - y_t^*)$. Data on macroeconomic fundamentals are obtained from the Federal Reserve Economic Data (FRED) database. In particular, we use monthly data on consumer price indexes and industrial production indexes. A measure of the output gap is computed by removing the trend from the logarithm of the industrial production index through the Hodrick–Prescott filter, as suggested by Lansing and Ma (2017).[14] Finally, exchange rates are observed the business day before the survey was conducted, as suggested in Fratzscher et al. (2015). Observables are thus linked to our model as

$$\hat{\mathbf{y}}_t = S\mathbf{x}_t \tag{21}$$

where S is a selection matrix. Note that Eqs. (20) and (21) are respectively the transition and the measurement dynamics of a linear and Gaussian state-space system.

Finally, to deal with non-stationarity on data, we represent the model in differences

$$\Delta\hat{\mathbf{y}}_t \equiv \begin{bmatrix} \Delta\hat{s}_t \\ \Delta\hat{f}_{1,t} \\ \Delta\hat{f}_{2,t} \\ \Delta\hat{\mathbb{E}}_t^i[s_{t+1}] \end{bmatrix} = \begin{bmatrix} \hat{s}_t - \hat{s}_{t-1} \\ \hat{f}_{1,t} - \hat{f}_{1,t-1} \\ \hat{f}_{2,t} - \hat{f}_{2,t-1} \\ \hat{\mathbb{E}}_t^i[s_{t+1}] - \hat{\mathbb{E}}_{t-1}^i[s_t] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{\gamma}_i \end{bmatrix}, \quad i = 1, \dots, N \tag{22}$$

in which we also added some Gaussian measurement errors $\tilde{\gamma}_i$ with standard deviation σ_{E_i}. Lagged variables are managed by defining $\tilde{\boldsymbol{x}}_t = (\boldsymbol{x}_t, \boldsymbol{x}_{t-1})$, ultimately obtaining the reduced form

$$\begin{aligned} \Delta\hat{\mathbf{y}}_t &= \tilde{S}\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{\gamma}}_t \\ \tilde{\boldsymbol{x}}_t &= \tilde{\Theta}_c + \tilde{\Theta}_x \tilde{\boldsymbol{x}}_{t-1} + \tilde{\Theta}_\epsilon \tilde{\boldsymbol{\epsilon}}_t. \end{aligned} \tag{23}$$

5.2. Prior distributions and inferential methods

To make inferences regarding the parameters, we recur to Bayesian estimation methods, and in particular, to Markov chain Monte Carlo (MCMC) algorithms.

The model is characterized by two types of parameters. The first, called structural parameters and indicated by $\boldsymbol{\theta}$, refer to the ones necessary to define the theoretical model in Eqs. (2)–(6). They are, amongst others, λ, δ, ρ_{x_i}, ρ_f, and ρ_s, the parameters describing the dynamics of fundamentals and private information, together with the measurement errors. The others are functions of the structural ones, and are, for instance, β_i, $i = 1, \dots, 4$, $\varphi_{\tilde{y}}$, and φ_x. These can be derived as a by-product of the structural parameters. Through MCMC, structural parameters are sequentially simulated from their posterior distribution and then posterior draws for β_j, $j = 1, \dots, 4$, φ_x, and $\varphi_{\tilde{y}}$ are computed. As standard practice for DSGE models (An & Schorfheide, 2007), we update parameters through a random walk Metropolis–Hastings algorithm and then, for each draw, we compute the likelihood and acceptance probabilities using the state-space representation of Eq. (23).

Our primary interest is to capture the effect of higher-order beliefs, identified by δ, to summarize the relevance that each agent assigns to the expectations of other market participants. Our second goal is to measure the role of private and public information, to determine the actual expectation. We need to explore the coefficients φ_x and $\varphi_{\tilde{y}}$ obtained from Eq. (8).

11 Note that some estimates based on the ARIMA specification are qualitatively similar and are reported in Appendix D. In Appendix B, we also derive a theoretical model consistent with the stationary hypothesis of f_t.

12 As for the fundamentals, Eq. (5) implies that private information follows an ARIMA (0, 1, 1) process. As above, we prefer to simplify their dynamics by assuming a slightly different parameterization.

13 We use the symbol ˆ to distinguish between observed and theoretical variables.

14 Note that other choices are possible. For instance, empirical results based on money supplies and output differentials are similar and are available upon request.

Table 3
Posterior computation (MCMC) – Structural parameters.

	Posterior distribution			Prior information		
	Parameter value	Mean	95% Cred. Int.	Mean	S.D.	Type
$p(\beta_1 \vert \hat{\mathbf{y}})$	$\beta_1 = 1 - \lambda + \lambda\varphi_{\tilde{y}} \frac{\rho_f}{\rho_f+\rho_s}$	0.4420	[0.380,0.498]			
$p(\beta_2 \vert \hat{\mathbf{y}})$	$\beta_2 = \lambda\varphi_x$	0.1680	[0.106,0.223]			
$p(\beta_3 \vert \hat{\mathbf{y}})$	$\beta_3 = \lambda\varphi_{\tilde{y}} \frac{\rho_s}{\rho_f+\rho_s}$	0.3900	[0.335,0.465]			
$p(\beta_4 \vert \hat{\mathbf{y}})$	$\beta_4 = -\lambda$	−0.8323	[−0.890,−0.773]			
$p(\alpha_1 \vert \hat{\mathbf{y}})$		1.3597	[0.758,1.959]	0	1	Normal
$p(\alpha_2 \vert \hat{\mathbf{y}})$		−1.1759	[−2.013,−0.260]	0	1	Normal
$p(\phi_{01} \vert \hat{\mathbf{y}})$		−0.0109	[−0.189,0.177]	0	1	Normal
$p(\phi_{02} \vert \hat{\mathbf{y}})$		−0.0174	[−0.231,0.163]	0	1	Normal
$p(\phi_{11} \vert \hat{\mathbf{y}})$		0.1002	[−0.061,0.252]	0	1	Normal
$p(\phi_{12} \vert \hat{\mathbf{y}})$		0.0357	[−0.035,0.096]	0	1	Normal
$p(\phi_{21} \vert \hat{\mathbf{y}})$		0.0328	[−0.150,0.236]	0	1	Normal
$p(\phi_{22} \vert \hat{\mathbf{y}})$		0.5270	[0.405,0.636]	0	1	Normal
$p(\rho_f \vert \hat{\mathbf{y}})$		0.8634	[0.678,1.047]	1	0.1	Gamma
$p(\rho_s \vert \hat{\mathbf{y}})$		1.2237	[1.040,1.423]	1	0.1	Gamma
$p(\rho_x \vert \hat{\mathbf{y}})$		3.6449	[2.989,4.320]	4	0.4	Gamma
$p(\varphi_{\tilde{y}} \vert \hat{\mathbf{y}})$	$\varphi_{\tilde{y}} = \frac{\rho_f+\rho_s}{(1-\varrho)\rho_x+(\rho_s+\rho_f)}$	0.7982	[0.735,0.870]			
$p(\varphi_x \vert \hat{\mathbf{y}})$	$\varphi_x = \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x+(\rho_s+\rho_f)}$	0.2018	[0.130,0.265]			
$p(\sigma_s \vert \hat{\mathbf{y}})$		1.1249	[0.766,1.489]	0.6	0.16	Inv. Gamma
$p(\sigma_f \vert \hat{\mathbf{y}})$		10.6740	[8.606,13.412]	0.6	0.16	Inv. Gamma
$p(\sigma_{f1} \vert \hat{\mathbf{y}})$		0.5118	[0.439,0.607]	0.6	0.16	Inv. Gamma
$p(\sigma_{f2} \vert \hat{\mathbf{y}})$		1.1481	[0.982,1.350]	0.6	0.16	Inv. Gamma
$p(\sigma_\psi \vert \hat{\mathbf{y}})$		0.7742	[0.397,1.277]	0.6	0.16	Inv. Gamma
$p(\lambda \vert \hat{\mathbf{y}})$		0.8323	[0.773,0.890]	0.5	0.1	Beta
$p(\delta \vert \hat{\mathbf{y}})$		0.8608	[0.779,0.920]	0.5	0.1	Beta

In the theoretical model of Section 4, we show that the coefficient φ_x measures the relevance of private information in the formation process of expectations, while, $\varphi_{\tilde{y}}$ indicates the importance of public information. They both depend on δ, which is positively correlated to the public signal compared to the private one.

Our prior choices regarding the parameters are summarized in Table 3.[15] Overall, we consider prior densities that match the domain of the parameters. We select a prior distribution for δ with average 0.5 (and standard deviation 0.1), consequently assigning equal weight to the two incentives present in the decision-making function of our predictors (Eq. (6)).

A priori, we assume that public and private information play the same role when agents form their expectations, without forcing the model to privilege certain sources of information. This guess is consistent with the hypothesis that φ_x and $\varphi_{\tilde{y}}$ are, on average, a priori equal. Since these weights depend on the precision coefficients ρ_f, ρ_s, ρ_x, and δ, we found prior distributions for them that, at least on average, give $E[\varphi_x] = E[\varphi_{\tilde{y}}] = 0.5$. To obtain this result, we set the prior distributions for ρ_f and ρ_s as gamma with mean 1 and standard deviation 0.1, whereas ρ_x is still gamma, but with the larger expected value, namely, 4 and standard deviation 0.4.[16] The discount factor λ is a beta variable with mean 0.5 and a standard deviation of 0.1. Furthermore, we assume a weakly informative prior for both α_1 and α_2 that is Gaussian with mean 0 and with rather large variance with respect to the mean, i.e., 1. Finally, the standard deviations of the shocks, including standard deviations of the measurement errors, are dispersed. Their standard deviations, in particular, are quite large with respect to the corresponding expected values. They are inverse-gamma variables with a mean of 0.6 and a standard deviation of 0.16.

6. Posterior estimates

All computations are based on software written in Ox′ 8.0 (Doornik, 2007) combined with the state-space library SsfPack (Koopman et al., 1999). Posterior estimates were obtained by running 100,000 iterations of the MCMC algorithm with a burn-in of 50,000, which is a sufficient number of iterations to remove dependence on initial conditions. As standard practice in macro-econometrics, initial conditions were obtained by maximizing the posterior mode for the parameters. The main results are summarized in Table 3, which includes posterior estimates of the structural relevant parameters, namely posterior averages and credibility intervals, together with the moments of the priors. Other parameter estimates are reported in Appendix C.

Overall, priors and posteriors differ considerably, thus suggesting that the contribution of the data/likelihood is substantial, and that the relevance of the prior assumptions does not drive the posterior results.[17]

The first result relates to the estimate of δ, where the posterior average confirms the critical role of the beauty contest mechanism in the predictor's evaluation process. Individuals assign a higher weight than expected (86%) to

15 Prior choices for measurement errors and the precision of private information are summarized in Appendix C.

16 An extensive sensitivity analysis suggested that posterior estimates of φ_x and $\varphi_{\tilde{y}}$ are robust with respect to this choice.

17 This evidence is also supported by a battery of robustness checks available upon request.

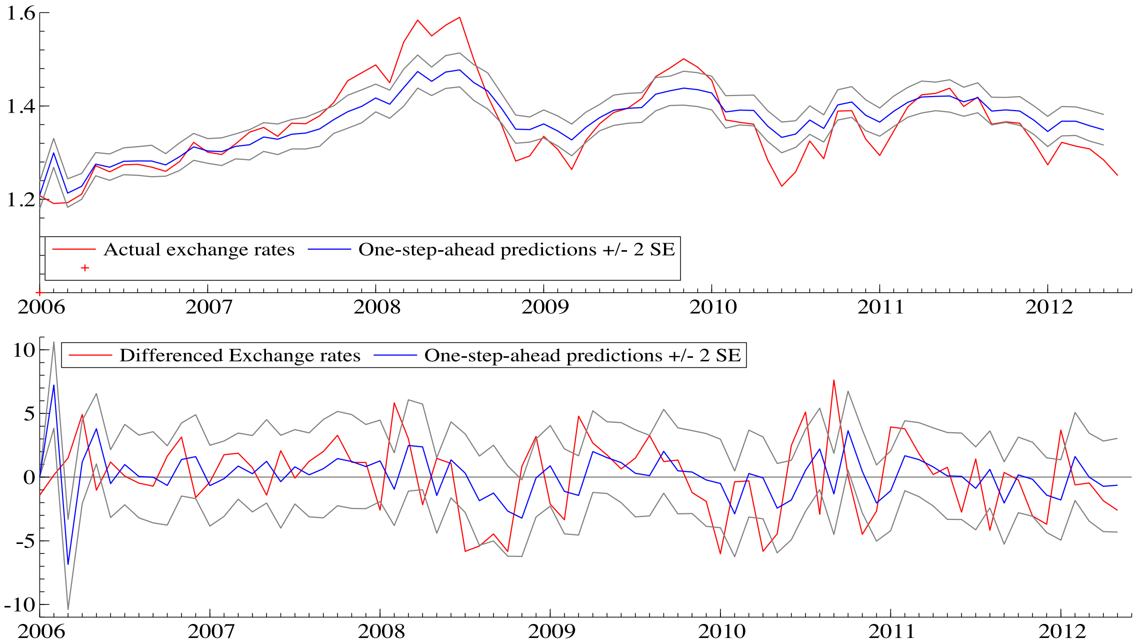


Fig. 2. Upper panel: Actual exchange rates (red line) vs. predicted exchange rates (blue line) together with 95% credibility bands. Lower panel: Actual exchange rates returns (red line) vs. predicted exchange rates returns (blue line) together with 95% credibility bands. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

deviations from the consensus, and limited weight (14%) to their individual ability to predict exchange rates based on their subjective information.

A second important outcome pertains the roles public and private information play in determining individual forecasts. Our analysis is based on the coefficients φ_x and $\varphi_{\tilde{y}}$. The results suggest that predictions rely about 80% on public information, whereas private information accounts for just 20%.

These two findings are consistent with previous theoretical results on higher-order beliefs. The combination of higher-order beliefs and the information structure ensures relatively rational behavior in the decision-making process. When agents give more importance to the consensus expectations than to their assessment, they implicitly reduce the relevance of their private signal. Although private signals are more accurate than public signals, since ρ_x is larger than ρ_f and ρ_s, the role of higher-order beliefs is largely confirmed. These results are substantially confirmed even in cases where agents are boundedly rational, as stressed in Appendix B.

Further analysis focuses on the model capacity to deliver short-run predictions, as displayed in Fig. 2, in which one-step-ahead forecasts for exchange rates and their returns are computed.

In this case, our model predicts actual data reasonably well, although it slightly underpredicts actual exchange rates when individual forecast dispersion is high.[18] To be more precise on this point, we evaluated the out-of-sample performance of the model by comparing its forecasting performance to the standard random walk benchmark. To do this, we re-estimated the model using the first 40 observations and then evaluated the one-step-ahead predictions for the remaining observations. Instead of computing the standard Diebold and Mariano test, whose asymptotic distribution has been shown to be non-Gaussian when the nested model is a martingale difference, we consider the adapted version proposed in Clark and West (2006, 2007). Therefore, in line with Molodtsova and Papell (2009), we implemented the adjusted version of the test that accounts for this imprecision. The test statistic is 0.826, with a p-value of 0.409, suggesting that the model is a slight improvement over the random walk, although the difference is not statistically relevant.

To emphasize the relevance of higher-order beliefs, we also estimated the model by setting $\delta = 0$, thus making consensus irrelevant. For this case, the estimates are reported in Table F.9 in Appendix F. Under this assumption, empirical results change dramatically. First, the estimated weight assigned to the private signal is much larger compared to the public one, i.e., 60% vs. 40%. Second, the estimate of σ_f is about three-times larger than the estimate based on the complete model, moving from 10 to 24.

18 As one reviewer correctly pointed out, consensus dynamics predicts exchange rates quite well, better than either our model or the random walk benchmark. This may seem counterintuitive. However, the main role of observed expectations and their measurement errors in our model is to extract information about private signals, since other proxies are not easily accessible. This is necessary to test our proposed expectation formation mechanism on real data. In fact, survey data are not used as predictors of exchange rates. Therefore, our forecasting mechanism does not fully benefit from the observed consensus.

These findings suggest that consensus defines a mechanism that helps agents to retrieve information on public signals, thus delivering more precise and reliable expectations on fundamentals. Neglecting consensus leads to a poor understanding of the fundamental dynamics, forcing agents to rely primarily on private information that is rather heterogeneous, as noted by looking at the observable expectations in Section 3. Consequently, we also observe that the weight β_1 assigned to fundamentals in the exchange rates equation changes substantially, moving from .44 in the complete model to .24 in the no-consensus model. Our understanding is that imprecise knowledge about f_t is related to a substantial inability to form expectations on it, and for this reason agents are unable to assign the correct weight to macroeconomic information. Therefore, our results appear consistent with the scapegoat hypothesis of Bacchetta and Van Wincoop (2004). When $\delta = 0$, predictions tend to be poor, displaying flat dynamics around the average exchange rates (see the right panel of Fig. F.5 in Appendix F).

To assess the role of private signals, as a further exercise, we consider the case of $\rho_x = 0$, thus assuming an irrelevant role for private information. The results are reported in Table F.9 and Fig. F.5 (left panel). In this case, we observe that the empirical results do not substantially deviate from the complete model. This outcome suggests that the main factor that helps describe exchange rates is the consensus mechanism, which allows agents to gather information on fundamentals that are the most relevant indicators to explain exchange rate dynamics.

Therefore, public information, following the beauty contest analogy, acts as a coordinating mechanism. It is a central result, firstly, proposed by Morris and Shin (2002). We intentionally integrated it into our framework to test its presence and intensity in the context of the exchange rate market. Furthermore, it delivers a complementary result to the empirical test of the scapegoat model posited by Bacchetta and Van Wincoop (2004) and implemented by Fratzscher et al. (2015), who found that using survey predictions of fundamentals as proxies for scapegoat effects improves the ability to explain exchange rate movements. Public information is, therefore, able to capture movements in actual exchange rate dynamics. These findings stress the links between higher-order beliefs and the dominant impact of public information. On a rational level, predictors look for information on fundamentals. However, they end up attributing excess weight to public information, which is not informative, at least in the short run. This is likely due, on one side, to the presence of higher-order beliefs, and, on the other side, to the uncertainty entailed by the heterogeneity of expectations. The fundamental is, therefore, transformed into a scapegoat in the event of uncertainty regarding structural parameters. In particular, the higher value of the conditional variance of the fundamental, σ_f (around 11), suggests that the short-term uncertainty proposed by Fratzscher et al. (2015) is congruent with uncertainty stemming from changes in fundamentals, specifically generating the scapegoat effect discussed by Bacchetta and Van Wincoop (2004).

This analysis suggests that fundamentals are reasonable predictors even in the short run. However, their relevance is emphasized if agents perceive the dynamics of the fundamental precisely, that is, in case σ_f tends to be small. Private information is not relevant for forecasting purposes, even though its represents a sizable ingredient in defining the expectation formation mechanism, since $\varphi_x > .20$.

We also compare our results with a naive rational expectation model with no private signals that closely mimics the dynamics defined in Eq. (2). In particular, we consider

$$s_t = \lambda \mathbb{E}_t[s_{t+1}] + (1-\lambda) f_t - \lambda \psi_t + \epsilon_t, \tag{24}$$

in which f_t is described in Eq. (12), while ψ_t is an i.i.d. sequence. Rational expectations are defined such that $\mathbb{E}_t[s_{t+1}] = s_t + \eta_t$, where η_t is a Gaussian shock with zero mean and constant variance. We estimated the exchange rate dynamics according to the rational expectations model using MCMC. The parameter estimates are reported in Appendix F. Furthermore, for each posterior draw of the parameters, we solved the rational expectation system with Sims (2002) using the Ox package LiRE (Mavroeidis & Zwols, 2007). An analytical solution is provided in Appendix B. Then, for each parameter, we simulated the one-step-ahead prediction produced by the rational expectation model. Fig. 3 compares the predictions and the average trajectory from the survey.[19] Fig. 3 shows that the predictions differ significantly from the observed expectations and in fact, the prediction's performance is similar to the no-consensus case i.e., $\delta = 0$ (see also Table F.9). Interestingly, the estimate of σ_f is large even in this case, thus suggesting some agent's difficulty making inferences based on public information. This feature appears to be the main cause for the poor performance of these two models in terms of forecasting. Our conclusion on this point is that whenever agents are unable to derive a precise inference on the public information, their predictions will perform poorly. This points to the important role that higher-order beliefs plays in our structural estimates.

Finally, to study the dynamic interactions between variables expressed in levels, it is useful to analyze how exchange rates react to structural shocks. This assessment is illustrated in Fig. 4. We show the impulse response functions (IRFs) for exchange rates resulting from positive economic shocks related to exchange rates, fundamentals, liquidity, and private information. For each shock, the impulse response functions are shown along with the 95% credible intervals.

In the graphs shown in Fig. 4, the red line relies on the IRF, while the grey band is the 95% credible interval for the IRF. The top-left panel shows how an exchange rate shock affects the overall dynamics of the exchange rate at short horizons, fading away in a few months. The same happens for the shock to the linear combination of the two fundamentals (top-right panel), which also affects the dynamics of the rate. Such a result is consistent with the standard monetary model with flexible prices, where

19 Here, rational expectations and predictions were computed as the average trajectory compared to the posterior draws based on the parameters of the model.

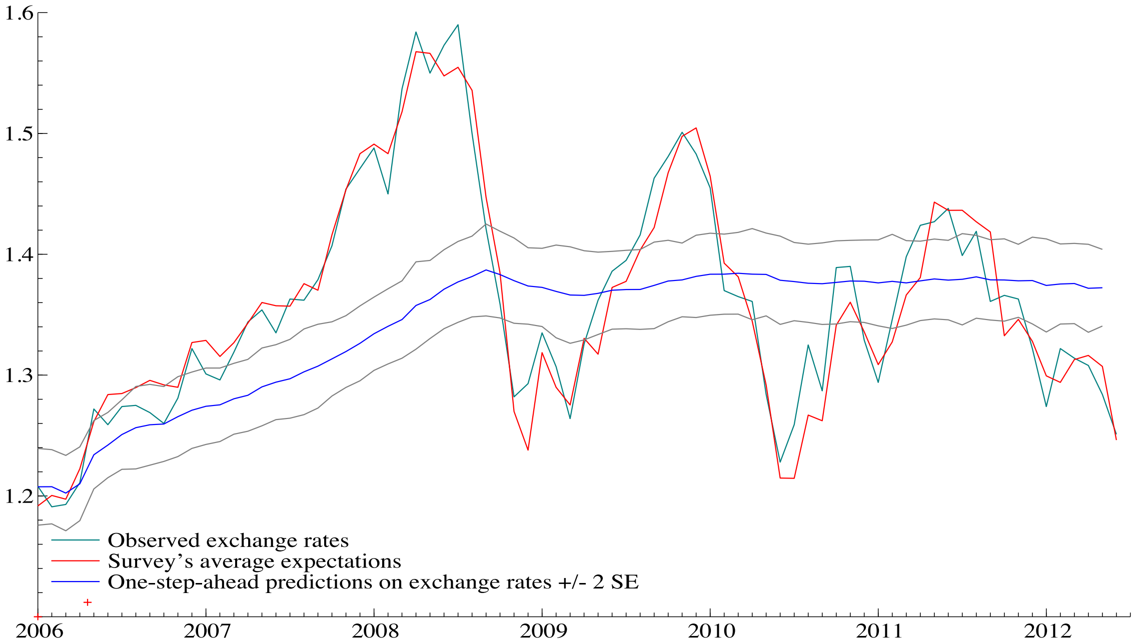


Fig. 3. Actual exchange rates (green line) and average from the survey (red line), together with one-step-ahead predictions and their approximated confidence bands (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

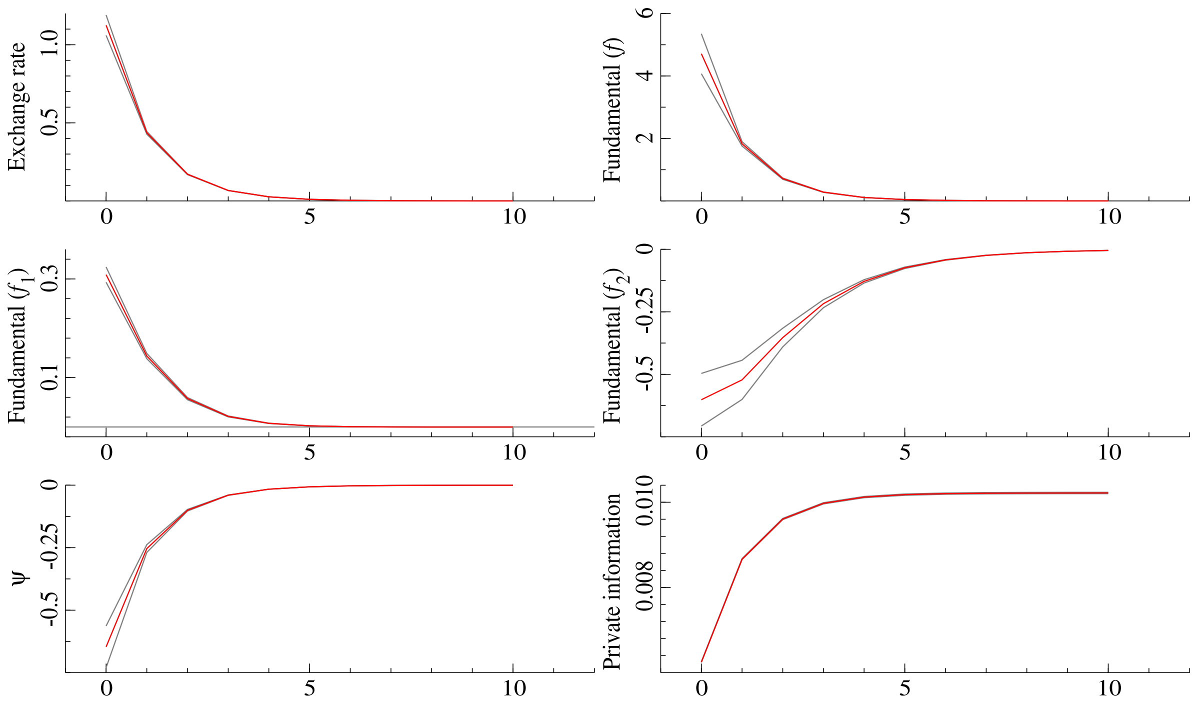


Fig. 4. Impulse response functions and 95% credible intervals. Each panel represents how different shocks affect the exchange rate dynamics. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

an anticipated monetary shock only temporarily affects the exchange rate. The bottom-left panel shows how a shock to liquidity has a substantial effect on exchange rate dynamics, in line with (Evans & Lyons, 2002), who showed the importance of order flow in the determination of the exchange rate. However, this effect is absorbed with time as a result of the decision to model liquidity as white noise. The final graph of Fig. 4 illustrates the increased role of private information, particularly in the short run, although the results are rather small in magnitude.

7. Concluding remarks

This paper showed the role of higher-order beliefs in a coordination game among investors to predict the value of the exchange rate. In particular, we looked at an exchange rate market, investigating how the impact of higher-order beliefs may significantly vary based on the level of information that investors may consider. Morris and Shin (2002) theoretically found that higher-order beliefs lead agents in a market to abandon highly informative but not commonly known private signals in favor of focal public ones. We extended this framework to a dynamic context to observe whether higher-order beliefs (and, consequently, public information) have a crucial role when the optimization process relies on both individual evaluation and coordination among agents. Our model assumes that agents have quadratic-payoff functions. They wish to match their action to the fundamental and do it together, i.e., coordinate with others' actions. The primary justification for this structure was to focus the analysis on two novel aspects. First, we focused on the possibility of perfectly estimating the impact of private and public sources of information. Thus, we were able to quantify the effect of each signal and understand the weight that each agent devotes to others' opinions. Second, we observed the impact of evaluating macroeconomic news on agents' behavior.

The results showed that higher-order beliefs predominantly matter in predicting the exchange rate. Our test thus confirms the importance of public information, as suggested by several contributions in the literature. The take-home message, in particular, is that the importance of higher-order beliefs turns upon the relevance of the public signal as the key factor in the expectation formation mechanism. Private information is also important, and explains some heterogeneity among agents, even though it does not substantially help investors to improve their predictions, at least in the short run. In a nutshell, if public signals are combined with high precision, then forecasts are also accurate, but this happens only when agents account for consensus when forming their predictions. Therefore, consensus appears to be the mechanism that helps agents to retrieve high-quality information from the market.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Davide Raggi reports financial support was provided by Einaudi Insttute for Economics and Finance.

Appendix A. Optimal forecasting strategy

In this section we describe the details that allow to build an optimal forecasting strategy for each agent i.

We denote the individual expectation of subject i at time t, i.e., $e_{it} = \mathbb{E}_t^i[s_{t+1}]$, while $\bar{e}_t \equiv \int_j e_{jt}(.)dj$ and $\sigma_e^2 \equiv \int_j [e_{it}(.) - \bar{e}_t(.)]^2 dj$ are respectively the average or consensus, and the dispersion of investors' expected evaluations in the economy. If we do not consider strategic interactions, we know that the posterior knowledge about exchange rates for each agent i is Gaussian with precision $\rho_{\tilde{y}} + \rho_x$, where $\rho_{\tilde{y}} = \rho_s + \rho_f$ and mean $\mathbb{E}_t^i[s_t|f_t; x_{it}] = \alpha_{x_i} x_{it} + \alpha_{\tilde{y}}\tilde{y}_t$, where $\tilde{y} = \left(\rho_s s_{t-1} + \rho_f f_t\right) / \left(\rho_s + \rho_f\right)$, $\alpha_{\tilde{y}} = \rho_{\tilde{y}} / \left(\rho_{\tilde{y}} + \rho_x\right)$ and $\alpha_{x_i} = \rho_x / \left(\rho_{\tilde{y}} + \rho_x\right)$.

The preferences of agents are explicitly characterized by a concave increasing function $U(e_{it}, \bar{e}_t, \sigma_e^2, s_{t+1})$ as in Angeletos and Pavan (2007). We assume that the dispersion σ_e has only a second-order non-strategic effect, i.e., $U_{e\sigma} = U_{s\sigma} = 0$, while $U_\sigma(e_{it}, \bar{e}_t, 0, s_{t+1}) = 0$, $\forall e_{it}; \bar{e}_t; s_{t+1}$. Under perfect information about the exchange rate s_{t+1}, due to symmetry ($e_{it}(.) = \bar{e}_t(.) = s_{t+1}$, $\forall i$), the best response is given by the unique equilibrium characteristics where the predictors' choice exactly coincides with their expectation. In the case of imperfect information, by contrast, optimality is required for any $(x_{it}; f_t)$ in the predictor's choice. For a finite number of investors, the individual's expected utility assumes the following form:

$$U(e_{it}, \bar{e}_t, \sigma_e^2, s_{t+1}) = -(1-\delta)(e_{it} - s_{t+1})^2 - \delta(e_{it} - \bar{e}_t)^2 \quad (A.1)$$

Maximizing the expected utility for e_{it}, we obtain that:

$$e_{it}(x_{it}; f_t; \rho_{\tilde{y}}; \rho_x) = (1-\delta)\mathbb{E}_t^i[s_{t+1}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] + \delta\mathbb{E}_t^i[\bar{e}_t|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] \quad (A.2)$$

which can be rewritten as:

$$e_{it}(x_i; f; \rho_{\tilde{y}}; \rho_x) = (1-\delta)\mathbb{E}_t^i[s_{t+1}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] + \delta\frac{e_{it}}{n} + \delta\frac{n-1}{n}\mathbb{E}_t^i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] \quad (A.3)$$

where $\mathbb{E}_t^i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] = \mathbb{E}_t^i[(\frac{e_{1t}+\cdots+e_{i-1t}+e_{i+1t}+\cdots e_{nt}}{n})| x_{it}; f_t; \rho_{\tilde{y}}; \rho_x]$. In the unique equilibrium with heterogeneous information, each individual $i \neq j$ at time t follows a linear strategy, as follows:

$$e_{it}(x_{it}; f_t; \rho_{\tilde{y}}; \rho_x) = \varphi_x x_{it} + \varphi_{\tilde{y}}\tilde{y}_t \quad (A.4)$$

According to this strategy, the predictor's expectation about the other $(n-1)$ agents is linear in $(f_t; s_{t+1})$ and is given by:

$$\mathbb{E}_t^i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] = \varphi_x\mathbb{E}_t^i[x_{-it}] + \varphi_{\tilde{y}}\tilde{y}_t$$

Then, according to Eq. (5), $\mathbb{E}_t^i[x_{-it}] = \mathbb{E}_t^i[s_t + \epsilon_{-it}]$. Moreover since $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ and using it in Eq. (3), $\mathbb{E}_t^i[s_t + \epsilon_{-it}] = \mathbb{E}_t^i[s_{t+1}]$. Therefore,

$$\mathbb{E}_t^i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] = \varphi_x\mathbb{E}_t^i[s_{t+1}] + \varphi_{\tilde{y}}\tilde{y}_t \quad (A.5)$$

Plugging Eqs. (A.5) and (A.4) into Eq. (A.3),

$$e_{it} = (1-\delta)\mathbb{E}_t^i[s_{t+1}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] + \delta\frac{e_{it}}{n} + \delta\frac{n-1}{n}\mathbb{E}_t^i[\varphi_x s_{t+1} + \varphi_{\tilde{y}}\tilde{y}_t|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] \quad (A.6)$$

Rearranging it, we obtain:

$$e_{it} = (1-\varrho+\varrho\varphi_x)\left[\frac{\rho_x}{\rho_{\tilde{y}}+\rho_x}x_{it} + \frac{\rho_{\tilde{y}}}{\rho_{\tilde{y}}+\rho_x}\tilde{y}_t\right] + \varrho\varphi_{\tilde{y}}\tilde{y}_t \quad (A.7)$$

where $\varrho = \frac{n\delta-\delta}{n-\delta}$. According to Eq. (A.4), the coefficients $(\varphi_x; \varphi_{\tilde{y}})$ for the optimal linear strategy must therefore satisfy:

$$\varphi_x = \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x + \rho_{\tilde{y}}} \quad \text{and} \quad \varphi_{\tilde{y}} = \frac{\rho_{\tilde{y}}}{(1-\varrho)\rho_x + \rho_{\tilde{y}}} \tag{A.8}$$

as the unique solution of the system. Therefore, the optimal solution of the learning game relies on the individual expectation about the next-period exchange:

$$\mathbb{E}_t^i(s_{t+1}) = \varphi_x x_{it} + \varphi_{\tilde{y}}\tilde{y}_t, . \tag{A.9}$$

Appendix B. The case of rational and boundedly rational agents

In this section, we study how the misperception of the actual law of motion (the ALM, or true process) of the exchange rates affects the empirical results. In fact, one of the main issues researchers face when dealing with heterogeneous or adaptive expectations is to find conditions that make their forecasts about a relevant variable free from systematic forecasting errors. One way to carry market information into the forecasting problem is to assume that individual forecasting rules (the perceived law of motion, or PLM) and market dynamics (the ALM) share at least the same statistical moments.

To shed some light on this point, we follow Hommes and Zhu (2014) and Lansing and Ma (2017) by assuming that investors fail to recognize precisely the true exchange rate process (ALM), by applying a parsimonious forecasting rule to predict exchange rates (PLM). However, they can observe actual exchange rates, where the ALM is the data generating process. In this respect, we assume that agents are *boundedly rational*. Hommes and Zhu (2014) suggest that individual forecasts must be statistically coherent with observational evidence. The intuition is that informed rational (or nearly rational) agents take advantage of the ALM by looking at market equilibrium data on exchange rates, and they exploit such information to compute their forecasts.

To account for this feature, we slightly modify our theoretical model, following the setup proposed by Lansing and Ma (2017), in particular regarding the way information on fundamentals enter the model, even though the role of higher-order beliefs and private information remains unchanged. As in our original contribution, we assume that exchange rates are described by

$$s_t = \lambda\bar{\mathbb{E}}_t[s_{t+1}] + (1-\lambda)f_t - \lambda\psi_t \tag{B.1}$$

where f_t is the process of fundamentals. Differently with respect to Eq. (4), we assume that f_t is a stationary autoregressive process ($|\rho| < 1$):

$$f_t = \rho f_{t-1} + u_t, \qquad WN(0, \sigma_u^2) \tag{B.2}$$

where u_t represents innovations that are white noise with variance σ_u^2.

In this version of the model, the law of motion agents perceive (that corresponds to the prior information perceived by agents in Eq. (3) is

$$s_t = s_{t-1} + \alpha u_t + \gamma_t, \qquad \gamma_t \sim \mathcal{N}(0, \rho_s^{-1}) \tag{B.3}$$

This basically represents the random walk prior, in which agents also react to innovations on the fundamentals through the parameter α. As in the main contribution, we assume that private information is described by

$$x_{it} = s_t + \epsilon_{it}, \qquad \epsilon_{it} \sim \mathcal{N}(0, \rho_x^{-1}) \tag{B.4}$$

Through Eqs. (B.3)–(B.4), and similarly with the results of the paper, it can easily be proved that individual non-strategic expectations for each agent i are

$$\begin{aligned} E_t^i[s_{t+1}|s_{t-1}, f_t, x_{it}] &= \frac{\rho_s}{\rho_s+\rho_x}(s_{t-1}+\alpha u_t) + \frac{\rho_x}{\rho_s+\rho_x}x_{it} \\ &= \alpha_y\tilde{y}_t + \alpha_x x_{it}, \end{aligned} \tag{B.5}$$

When agents consider strategic interactions through the usual loss function in Eq. (6), then

$$\begin{aligned} E_t^i[s_{t+1}] &= \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x+\rho_s}x_{it} + \frac{\rho_s}{(1-\varrho)\rho_x+\rho_s}\tilde{y}_t \\ &= \varphi_{\tilde{y}}\tilde{y}_t + \varphi_x x_{it} \end{aligned} \tag{B.6}$$

where $\varrho = \frac{\delta n - n}{n-\delta}$. If we plug these individual expectations into Eq. (2), we get

$$\begin{aligned} s_t &= \lambda\bar{E}[s_{t+1}] + (1-\lambda)f_t - \lambda\psi_t \\ &= \lambda\varphi_x\bar{x}_t + \lambda\varphi_{\tilde{y}}(s_{t-1}+\alpha u_t) + (1-\lambda)f_t - \lambda\psi_t \\ &= \lambda\varphi_x\bar{x}_t + \lambda\varphi_{\tilde{y}}s_{t-1} + \alpha\lambda\varphi_{\tilde{y}}u_t + (1-\lambda)f_t - \lambda\psi_t \\ &= \beta_1 f_t + \beta_2\bar{x}_t + \beta_3 s_{t-1} + \beta_4\psi_t + \beta_5 f_{t-1} \end{aligned} \tag{B.7}$$

where $\beta_1 = \lambda\varphi_{\tilde{y}}\alpha + 1 - \lambda$, $\beta_2 = \lambda\varphi_x$, $\beta_3 = \lambda\varphi_{\tilde{y}}$, $\beta_4 = -\lambda$ and $\beta_5 = \lambda\varphi_{\tilde{y}}\alpha\rho$. Eq. (B.7) is the actual law of motion for the exchange rates (ALM).

It is worth noting that agents need to pin down the parameter α to actually implement their forecasts. Since α is a regression coefficient in the PLM, it is statistically identified by

$$\alpha = \frac{cov(\Delta s_t, u_t)}{\sigma_u^2}.$$

Following Lansing and Ma (2017), α can be calibrated by taking advantage of the moments obtained from the true process that delivers the observed (by the agents) exchange rates. This calibration allows agents to link their perceived law of motion, which in principle can be misspecified, with the ALM that delivers actual observations on exchange rates. To do that, it is convenient to rewrite the ALM as

$$\Delta s_t = \lambda\varphi_x\bar{x}_t + (\lambda\varphi_{\tilde{y}}-1)s_{t-1} + \alpha\lambda\varphi_{\tilde{y}}u_t + (1-\lambda)f_t - \lambda\psi_t. \tag{B.8}$$

In terms of the ALM, $cov(\Delta s_t, u_t)$ is

$$\begin{aligned} cov(\Delta s_t, u_t) = {}& \lambda\varphi_x cov(\bar{x}_t, u_t) + (\lambda\varphi_{\tilde{y}} - 1)cov(s_{t-1}, u_t) + \\ & + \alpha\lambda\varphi_{\tilde{y}}\sigma_u^2 + (1-\lambda)cov(f_t, u_t) - \lambda cov(\psi_t, u_t) \end{aligned} \tag{B.9}$$

If we assume that u_t is uncorrelated with ψ_t and $\bar{x}_t$, and we realize that $cov(s_{t-1}, u_t) = 0$ and $cov(f_t, u_t) = \sigma_u^2$, we obtain that when agents are boundedly rational, it must be that

$$\begin{aligned} \alpha &= \frac{cov(\Delta s_t, u_t)}{\sigma_u^2} = \alpha\lambda\varphi_{\tilde{y}} + (1-\lambda) \\ &\Rightarrow \alpha = \frac{1-\lambda}{1-\lambda\varphi_{\tilde{y}}} \end{aligned} \tag{B.10}$$

Table B.4
EUR/USD: Posterior computation (MCMC) – Parameter estimates in the case of bounded rationality and without the consistency hypothesis. Here, * refers to a prior used just for the data-driven model. f is a combination of output gaps, short-term interest rates (lagged), and inflation.

	Data driven		Bounded rationality		Prior information		
	Mean	95% Cred. Int.	Mean	95% Cred. Int.	Mean	S.D.	Type
$p(\beta_1\|\hat{\mathbf{y}})$	0.6923	[0.6317, 0.7574]	0.4395	[0.3411, 0.5394]			
$p(\beta_2\|\hat{\mathbf{y}})$	0.2715	[0.2014, 0.3347]	0.2187	[0.1302, 0.3039]			
$p(\beta_3\|\hat{\mathbf{y}})$	0.5381	[0.4182, 0.6687]	0.6073	[0.4051, 0.7686]			
$p(\beta_4\|\hat{\mathbf{y}})$	−0.8096	[−0.8743,−0.7259]	−0.8261	[−0.9054,−0.6966]			
$p(\beta_5\|\hat{\mathbf{y}})$	−0.4682	[−0.5880,−0.3487]	−0.2504	[−0.3402, −0.1792]			
$p(\rho\|\hat{\mathbf{y}})$	0.9320	[0.8978, 0.9521]	0.9428	[0.8820, 0.9743]	0.5	0.2	Beta
$p(\alpha\|\hat{\mathbf{y}})$	0.9529	[0.7529, 1.1492]	0.4395	[0.3411, 0.5394]	1	0.1	Normal*
$p(\rho_s\|\hat{\mathbf{y}})$	1.1212	[0.7643, 1.5240]	1.2309	[0.8487, 1.7399]	1	0.2	Gamma
$p(\rho_x\|\hat{\mathbf{y}})$	2.8829	[1.9214, 4.0567]	2.5157	[1.4484, 4.0751]	4	1.0	Gamma
$p(\varphi_{\tilde{y}}\|\hat{\mathbf{y}})$	0.6624	[0.5600, 0.7676]	0.7320	[0.5759, 0.8529]			
$p(\varphi_x\|\hat{\mathbf{y}})$	0.3376	[0.2324, 0.4400]	0.2680	[0.1471, 0.4241]			
$p(\sigma_s\|\hat{\mathbf{y}})$	3.8813	[3.2188, 4.7284]	3.9022	[3.1251, 5.2817]	0.6	0.2	Inv. Gamma
$p(\sigma_u\|\hat{\mathbf{y}})$	0.3487	[0.2999, 0.4091]	0.3474	[0.3022, 0.3998]	0.6	0.2	Inv. Gamma
$p(\sigma_\psi\|\hat{\mathbf{y}})$	1.0718	[0.5381, 1.8110]	0.6653	[0.3830, 1.1311]	0.6	0.2	Inv. Gamma
$p(\lambda\|\hat{\mathbf{y}})$	0.8096	[0.7259, 0.8743]	0.8261	[0.6966, 0.9054]	0.5	0.1	Beta
$p(\delta\|\hat{\mathbf{y}})$	0.7181	[0.6574, 0.7726]	0.6534	[0.5237, 0.7786]	0.5	0.1	Beta

As a limit case, we also consider rational expectations, starting from Eqs. (B.1)–(B.2). Following Hommes and Zhu (2014), we find by repeated forward substitutions that the solution is

$$s_t^* = \frac{1-\lambda}{1-\lambda\rho} f_t - \lambda\psi_t. \tag{B.11}$$

This result implies that under *full* rationality, the only information agents need is knowledge of the fundamentals f_t, while private information is irrelevant. Furthermore, there is no coordination mechanism in this setting. Of course, this model is not nested in ours, but it is empirically equivalent to the case where $\delta = 0$ in our model, as emphasized in Section 6 and Appendix F.

In the following, we report the estimates of the model in case α is a free parameter (the case of no bounded rationality), versus the case in which α is calibrated with Eq. (B.10). The empirical model is thus

$$s_t = \beta_1 f_t + \beta_2 \bar{x}_t + \beta_3 s_{t-1} + \beta_4 \psi_t + \beta_5 f_{t-1} + \epsilon_{s,t} \tag{B.12}$$
$$f_t = \rho f_{t-1} + u_t \tag{B.13}$$
$$\tilde{y}_t = \alpha f_t + s_{t-1} - \alpha\rho f_{t-1} \tag{B.14}$$
$$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi,t} \tag{B.15}$$
$$\mathbb{E}_t^i[s_{t+1}] = \varphi_{\tilde{y}} \tilde{y}_t + \varphi_x x_{it}, \qquad i = 1,\dots,N \tag{B.16}$$
$$x_{it} = x_{it-1} + \epsilon_{x_i,t}, \qquad i = 1,\dots,N \tag{B.17}$$

To estimate this model, we define the fundamental process as in Lansing and Ma (2017), even though different definitions of f_t provide equivalent results. Here, f_t reflects the use of Taylor-rule fundamentals in the specification of the exchange rate model and is described as follows:

$$f_t = -b\theta(i_{t-1} - i_{t-1}^*) - b(1-\theta)[g_\pi(\pi_t - \pi_t^*) + g_y(y_t - y_t^*)] \tag{B.18}$$

in which π_t, y_t, and i_{t-1} are, respectively, the inflation rate, output gap, and past short-term interest rates from the home country, whereas the starred variables represent foreign variables. Furthermore, following Lansing and Ma (2017), the parameters of this combination are calibrated as $g_\pi = 1.5$, $g_y = .5$, $g_s = 0.1$, $\theta = 0.8$, and $b = \frac{1}{1+(1-\theta)g_s} \approx 0.98$.

The column labeled "Data driven" in Table B.4 summarizes the result obtained in case α is estimated as a free parameter, thus assuming that agents do not at all consider the actual law of motion while building their expectations. These results are in line with the main results obtained in the original manuscript. The column labeled "Bounded rationality" displays estimates in which α is not estimated, but is computed with Eq. (B.10). Our finding is that the estimate of δ does not change much in the two cases, moving from .72 to .65, thus suggesting that the knowledge (or partial knowledge) of the true model by the forecasters does not alter their consideration of the consensus factor. Furthermore, the weights assigned to public and private information are similar to the ones reported in the paper.

Appendix C. Other parameter estimates

In this section we report the results related to other parameters that are not relevant in our analysis. In particular, Table C.5 summarizes the posterior output of the precision of the private signal for the 15 forecasters considered with Eq. (18), whereas Table C.6 relates to the standard errors of the measurement errors of Eq. (22).

Appendix D. Different specification of the empirical model

Here, we report our estimates of the structural parameters in case we assume the dynamics of fundamentals and private information fully consistent with the expectation formation mechanism described in Eqs. (4) and (5). It is in fact easy to prove that Eq. (4), together with the random walk hypothesis for exchange rates of Eq. (3), corresponds to a standard ARIMA (0, 1, 1) process with MA parameter $\theta_f = \left(\sqrt{q_f^2 + 4q_f} - 2 - q_f\right)/2$ and Gaussian noise, $\epsilon_{f,t}$, with variance $\sigma_f^2 = -\sigma_\eta^2/\theta_f$, where $q_f = \sigma_\gamma^2/\sigma_\eta^2$

Table C.5
EUR/USD: Posterior computation (MCMC) – Other parameters.

	Posterior distribution		Prior information		
	Mean	95% Cred. Int.	Mean	S.E.	Type
$p(\sigma_{x_1}\|\hat{\boldsymbol{y}})$	0.5589	[0.377,0.825]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_2}\|\hat{\boldsymbol{y}})$	0.5753	[0.379,0.830]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_3}\|\hat{\boldsymbol{y}})$	0.5862	[0.391,0.912]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_4}\|\hat{\boldsymbol{y}})$	0.5688	[0.368,0.871]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_5}\|\hat{\boldsymbol{y}})$	0.5264	[0.352,0.891]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_6}\|\hat{\boldsymbol{y}})$	0.6048	[0.411,0.984]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_7}\|\hat{\boldsymbol{y}})$	0.6576	[0.419,0.931]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_8}\|\hat{\boldsymbol{y}})$	0.5955	[0.391,0.946]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_9}\|\hat{\boldsymbol{y}})$	0.6603	[0.394,1.187]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{10}}\|\hat{\boldsymbol{y}})$	0.5711	[0.401,0.829]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{11}}\|\hat{\boldsymbol{y}})$	0.5579	[0.375,0.886]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{12}}\|\hat{\boldsymbol{y}})$	0.6306	[0.417,0.985]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{13}}\|\hat{\boldsymbol{y}})$	0.5469	[0.367,0.784]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{14}}\|\hat{\boldsymbol{y}})$	0.5943	[0.385,1.055]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{15}}\|\hat{\boldsymbol{y}})$	0.8072	[0.443,1.338]	0.6	0.16	Inv. Gamma

Table C.6
EUR/USD: Posterior computation (MCMC) – Measurement errors.

	Posterior distribution		Prior information		
	Mean	95% Cred. Int.	Mean	S.E.	Type
$p(\sigma_{E_1}\|\hat{\boldsymbol{y}})$	1.4172	[1.170,1.726]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_2}\|\hat{\boldsymbol{y}})$	1.8006	[1.444,2.306]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_3}\|\hat{\boldsymbol{y}})$	2.6439	[2.256,3.217]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_4}\|\hat{\boldsymbol{y}})$	3.8281	[3.212,4.606]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_5}\|\hat{\boldsymbol{y}})$	0.8853	[0.573,1.431]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_6}\|\hat{\boldsymbol{y}})$	3.2557	[2.792,3.866]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_7}\|\hat{\boldsymbol{y}})$	2.8402	[2.380,3.543]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_8}\|\hat{\boldsymbol{y}})$	1.5519	[1.300,1.841]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_9}\|\hat{\boldsymbol{y}})$	2.6605	[2.307,3.073]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{10}}\|\hat{\boldsymbol{y}})$	1.9824	[1.654,2.360]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{11}}\|\hat{\boldsymbol{y}})$	2.9641	[2.487,3.541]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{12}}\|\hat{\boldsymbol{y}})$	2.9884	[2.480,3.571]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{13}}\|\hat{\boldsymbol{y}})$	2.5116	[2.158,2.986]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{14}}\|\hat{\boldsymbol{y}})$	3.0270	[2.527,3.584]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{15}}\|\hat{\boldsymbol{y}})$	1.6876	[1.402,1.980]	0.6	0.16	Inv. Gamma

is the signal-to-noise ratio. Similar results can be obtained for private signals. Under this assumption, the transition equation of the state-space representation is

$$s_t = \beta_1 f_t + \beta_2 \bar{x}_t + \beta_3 s_{t-1} + \beta_4 \psi_t + \epsilon_{s,t} \tag{D.1}$$

$$f_t = f_{t-1} + \theta_f \epsilon_{f,t-1} + \epsilon_{f,t} \tag{D.2}$$

$$\tilde{y}_t = \frac{\rho_f}{\rho_f + \rho_s} f_t + \left(1 - \frac{\rho_f}{\rho_f + \rho_s}\right) s_{t-1} \tag{D.3}$$

$$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi,t} \tag{D.4}$$

$$x_{it} = x_{it-1} + \theta_x \epsilon_{x_i,t-1} + \epsilon_{x_i,t}, \qquad i = 1, \ldots, N \tag{D.5}$$

The measurement equations are thus

$$\mathbb{E}_t^i[s_{t+1}] = \varphi_{\tilde{y}} \tilde{y}_t + \varphi_x x_{it} + \epsilon_{e_i,t}, \; \epsilon_{e_i,t} \sim \mathcal{N}(0, \sigma_{E_i}^2) \; i = 1, \ldots, N \tag{D.6}$$

$$f_{j,t} = \alpha_j f_t + \epsilon_{j,t}, \quad \epsilon_{j,t} \sim \mathcal{N}(0, \sigma_{f_j}^2) \; j = 1, \ldots, K \tag{D.7}$$

$$\hat{s}_t = s_t \tag{D.8}$$

that in compact form read as

$$\hat{\boldsymbol{y}}_t = S\mathbf{x}_t + \boldsymbol{\epsilon_{y,t}}, \tag{D.9}$$

where the observables are $\hat{\boldsymbol{y}}_t = \left(\mathbb{E}_t^i[s_{t+1}], f_{j,t}, \hat{s}_t\right)$, $i = 1, \ldots, N$, $j = 1, \ldots, K$, S is a matrix of coefficients, and $\boldsymbol{\epsilon_{y,t}}$ is the vector of measurement errors.

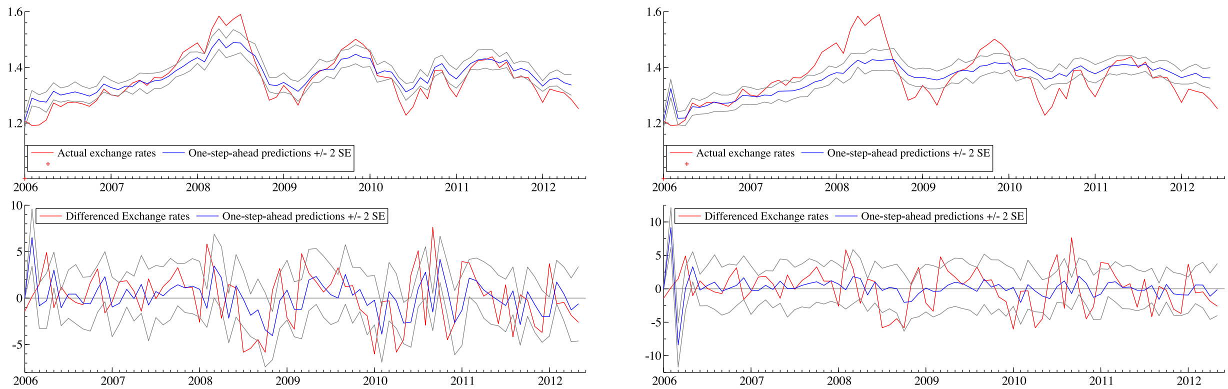
Table D.7
EUR/USD: Posterior computation (MCMC) – Structural parameters with ARIMA specification.

	Mean	95% Cred. Int.	Prior
$p(\beta_1\|\hat{\boldsymbol{y}})$	−0.6885	[−0.760, −0.598]	
$p(\beta_2\|\hat{\boldsymbol{y}})$	0.0460	[0.037, 0.057]	
$p(\beta_3\|\hat{\boldsymbol{y}})$	0.7540	[0.704, 0.800]	
$p(\beta_4\|\hat{\boldsymbol{y}})$	−0.8884	[−0.960, −0.781]	
$p(\rho_f\|\hat{\boldsymbol{y}})$	1.0995	[0.884, 1.378]	$\mathcal{G}(1, 0.1)$
$p(\rho_s\|\hat{\boldsymbol{y}})$	3.5322	[3.162, 3.912]	$\mathcal{G}(1, 0.1)$
$p(\rho_x\|\hat{\boldsymbol{y}})$	2.5928	[2.299, 2.909]	$\mathcal{G}(4, 0.2)$
$p(\lambda\|\hat{\boldsymbol{y}})$	0.8884	[0.781, 0.960]	$\mathcal{B}(0.95, 0.025)$
$p(\delta\|\hat{\boldsymbol{y}})$	0.9081	[0.885, 0.926]	$\mathcal{B}(0.5, 0.025)$
$p(\varphi_{\tilde{y}}\|\hat{\boldsymbol{y}})$	0.9482	[0.937, 0.957]	
$p(\varphi_x\|\hat{\boldsymbol{y}})$	0.0518	[0.043, 0.063]	
$p(\sigma_s\|\hat{\boldsymbol{y}})$	0.2121	[0.177, 0.258]	$\mathcal{IG}(0.6, 0.2)$
$p(\sigma_\phi\|\hat{\boldsymbol{y}})$	0.2361	[0.194, 0.287]	$\mathcal{IG}(0.6, 0.2)$
$p(\theta_f\|\hat{\boldsymbol{y}})$	−0.5769	[−0.615, −0.534]	
$p(\sigma_f\|\hat{\boldsymbol{y}})$	1.2610	[1.164, 1.361]	
$p(\theta_x\|\hat{\boldsymbol{y}})$	−0.4350	[−0.461, −0.409]	
$p(\sigma_x\|\hat{\boldsymbol{y}})$	0.9429	[0.903, 0.984]	

It is worth noting that f_t is a linear combination of observable macroeconomic indicators, namely $f_{j,t}$, $j = 1, \ldots, K$. Such indicators allow us to measure the public signal f_t summarizing the economic dynamics in the

Table E.8
Posterior computation (MCMC) – Structural parameters.

	Posterior distribution			Prior information		
	Parameter value	Mean	95% Cred. Int.	Mean	S.D.	Type
$p(\beta_1\|\hat{\mathbf{y}})$	$\beta_1 = 1-\lambda+\lambda\varphi_{\tilde{y}}\frac{\rho_f}{\rho_f+\rho_s}$	0.3574	[0.311,0.436]			
$p(\beta_2\|\hat{\mathbf{y}})$	$\beta_2 = \lambda\varphi_x$	0.3389	[0.251,0.410]			
$p(\beta_3\|\hat{\mathbf{y}})$	$\beta_3 = \lambda\varphi_{\tilde{y}}\frac{\rho_s}{\rho_f+\rho_s}$	0.3037	[0.243,0.373]			
$p(\beta_4\|\hat{\mathbf{y}})$	$\beta_4 = -\lambda$	−0.9385	[−0.962,−0.909]			
$p(\alpha_1\|\hat{\mathbf{y}})$		0.0985	[−0.251,0.481]	0	1	Normal
$p(\alpha_2\|\hat{\mathbf{y}})$		1.6269	[0.959,2.365]	0	1	Normal
$p(\phi_{01}\|\hat{\mathbf{y}})$		0.0045	[−0.176,0.195]	0	1	Normal
$p(\phi_{02}\|\hat{\mathbf{y}})$		0.0197	[−0.170,0.213]	0	1	Normal
$p(\phi_{11}\|\hat{\mathbf{y}})$		−0.0603	[−0.221,0.106]	0	1	Normal
$p(\phi_{12}\|\hat{\mathbf{y}})$		0.0302	[−0.181,0.208]	0	1	Normal
$p(\phi_{21}\|\hat{\mathbf{y}})$		−0.0079	[−0.092,0.087]	0	1	Normal
$p(\phi_{22}\|\hat{\mathbf{y}})$		0.1132	[−0.073,0.330]	0	1	Normal
$p(\rho_f\|\hat{\mathbf{y}})$		1.0115	[0.823,1.242]	1	0.1	Gamma
$p(\rho_s\|\hat{\mathbf{y}})$		1.0344	[0.892,1.197]	1	0.1	Gamma
$p(\rho_x\|\hat{\mathbf{y}})$		3.7461	[2.964,4.455]	4	0.4	Gamma
$p(\varphi_{\tilde{y}}\|\hat{\mathbf{y}})$	$\varphi_{\tilde{y}} = \frac{\rho_f+\rho_s}{(1-\varrho)\rho_x+(\rho_s+\rho_f)}$	0.6387	[0.559,0.734]			
$p(\varphi_x\|\hat{\mathbf{y}})$	$\varphi_x = \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x+(\rho_s+\rho_f)}$	0.3613	[0.266,0.441]			
$p(\sigma_s\|\hat{\mathbf{y}})$		0.9923	[0.778,1.299]	0.6	0.16	Inv. Gamma
$p(\sigma_f\|\hat{\mathbf{y}})$		20.7374	[14.482,28.266]	0.6	0.16	Inv. Gamma
$p(\sigma_{f1}\|\hat{\mathbf{y}})$		0.7283	[0.592,0.898]	0.6	0.16	Inv. Gamma
$p(\sigma_{f2}\|\hat{\mathbf{y}})$		0.2640	[0.215,0.325]	0.6	0.16	Inv. Gamma
$p(\sigma_\psi\|\hat{\mathbf{y}})$		0.6740	[0.455,1.009]	0.6	0.16	Inv. Gamma
$p(\lambda\|\hat{\mathbf{y}})$		0.9385	[0.909,0.962]	0.5	0.1	Beta
$p(\delta\|\hat{\mathbf{y}})$		0.7019	[0.606, 0.792]	0.5	0.1	Beta



(a) $\rho_x = 0$ (b) $\delta = 0$

Fig. F.5. One-step-ahead predictions of exchange rates when private information is irrelevant (left panel) vs. no consensus (right panel).

market. We use a dynamic factor model (see Forni et al., 2000; Stock & Watson, 2011 for a survey of these methods) to define a link between a potential non-observed factor f_t and real data. A similar approach is proposed by Boivin and Giannoni (2006) for DSGE model estimation. This strategy has the advantage that we do not need to explicitly describe the dynamics of each macroeconomic factor $f_{j,t}$, but just the aggregate one as a public signal, thus keeping our empirical model fully consistent with the theoretical framework. The results are displayed in Table D.7.

As for the previous results, public information appears dominant with respect to private information. Furthermore, estimates on δ confirm the relevance of the beauty contest mechanism, as stressed in Section 6.

Appendix E. Bloomberg survey data

The Bloomberg database consists of a survey of one-step-ahead forecasts for about 150 banks or financial institutions, 43 of them without missing observations, made from 2012:Q4 to 2022:Q1 for a total of 37 periods. The panelists partially overlap the ones included in the Consensus database. From the overall dataset, we selected 15 banks, to keep the number of panelists consistent with the one used in the main text. We considered forecasts from the Australia & New Zealand Banking Group, Barclays, BNP Paribas, Citigroup, Commerzbank, Danske Bank, DZ Bank, ING Financial Markets, JPMorgan Chase, Morgan Stanley, Nomura Bank International, RBC Capital Markets, Sumitomo Mitsui Trust Bank, DNB, and Societé Generale.

Table F.9
EUR/USD: Posterior computation (MCMC) – Structural parameters in case private information is irrelevant, with no consensus, and in the case of rational expectations.

	Case 1: $\rho_x = 0$		Case 2: $\delta = 0$		Case 3: Rational expectations		Prior information		
	Mean	95% Cred. Int.	Mean	95% Cred. Int.	Mean	95% Cred. Int.	Mean	S.D.	Type
$p(\beta_1\|\hat{\mathbf{y}})$	0.5187	[0.446,0.596]	0.2472	[0.200,0.289]					
$p(\beta_2\|\hat{\mathbf{y}})$	0.0000		0.5667	[0.499,0.631]					
$p(\beta_3\|\hat{\mathbf{y}})$	0.4813	[0.404, 0.553]	0.1861	[0.155,0.224]					
$p(\beta_4\|\hat{\mathbf{y}})$	−0.8005	[−0.865, −0.723]	−0.9331	[−0.955,−0.909]					
$p(\alpha_1\|\hat{\mathbf{y}})$	0.7670	[−0.635, 2.243]	0.6633	[0.343,0.965]	0.0085	[−0.183, 0.198]	0	1	Normal
$p(\alpha_2\|\hat{\mathbf{y}})$	−1.4201	[−2.467,−0.410]	−2.0779	[−2.596,−1.578]	−0.0740	[−0.263, 0.125]	0	1	Normal
$p(\phi_{01}\|\hat{\mathbf{y}})$	−0.0099	[−0.231,0.176]	−0.0188	[−0.204,0.191]			0	1	Normal
$p(\phi_{02}\|\hat{\mathbf{y}})$	−0.0115	[−0.168,0.181]	0.0154	[−0.176,0.203]			0	1	Normal
$p(\phi_{11}\|\hat{\mathbf{y}})$	0.1003	[−0.050,0.274]	0.0832	[−0.041,0.230]			0	1	Normal
$p(\phi_{12}\|\hat{\mathbf{y}})$	0.0298	[−0.037,0.104]	0.0392	[−0.018,0.106]			0	1	Normal
$p(\phi_{21}\|\hat{\mathbf{y}})$	0.0231	[−0.176,0.202]	−0.0378	[−0.264,0.113]			0	1	Normal
$p(\phi_{22}\|\hat{\mathbf{y}})$	0.5302	[0.395,0.647]	0.5360	[0.387,0.686]			0	1	Normal
$p(\rho_f\|\hat{\mathbf{y}})$	0.8016	[0.641,0.959]	1.0707	[0.851,1.276]			1	0.1	Gamma
$p(\rho_s\|\hat{\mathbf{y}})$	1.2052	[1.023,1.409]	1.1060	[0.885,1.347]			1	0.1	Gamma
$p(\rho_x\|\hat{\mathbf{y}})$	0.0000		3.3701	[2.820,3.940]			4	0.4	Gamma
$p(\varphi_{\bar{y}}\|\hat{\mathbf{y}})$	1.0000		0.3931	[0.335,0.459]					
$p(\varphi_x\|\hat{\mathbf{y}})$	0.0000		0.6069	[0.541,0.665]					
$p(\sigma_s\|\hat{\mathbf{y}})$	1.0077	[0.574,1.412]	1.1336	[0.829,1.466]	1.0524	[0.435, 2.548]	0.6	0.16	Inv. Gamma
$p(\sigma_f\|\hat{\mathbf{y}})$	7.9546	[6.344,9.801]	24.2984	[19.370,32.697]	48.0409	[11.728, 141.525]	0.6	0.16	Inv. Gamma
$p(\sigma_{f1}\|\hat{\mathbf{y}})$	0.4891	[0.424,0.589]	0.5078	[0.435,0.596]	0.6710	[0.577, 0.783]	0.6	0.16	Inv. Gamma
$p(\sigma_{f2}\|\hat{\mathbf{y}})$	1.1511	[0.988,1.362]	1.1837	[1.024,1.420]	1.1619	[1.000, 1.352]	0.6	0.16	Inv. Gamma
$p(\sigma_\psi\|\hat{\mathbf{y}})$	0.7710	[0.379,1.562]	1.1837	[0.657,0.933]	1.1961	[0.492, 3.427]	0.6	0.16	Inv. Gamma
$p(\lambda\|\hat{\mathbf{y}})$	0.8005	[0.723,0.865]	0.9331	[0.909,0.955]	0.8029	[0.447, 0.956]	0.5	0.1	Beta
$p(\delta\|\hat{\mathbf{y}})$	0.4584	[0.273,0.662]	0.0000				0.5	0.1	Beta

On the basis of these new data, we estimated the model on Eqs. (11)–(18). The results are reported in Table E.8.

These results support the empirical evidence provided in Section 6. In particular, the estimate of δ is still large, albeit smaller than the previous estimates, moving from $\approx .82$ to $\approx .70$. These estimates confirm the relevance of public information in the expectation formation mechanism, with an assigned weight of about 64%, compared to the .76% from the Consensus database.

Appendix F. Rational expectation vs. no consensus or private information

In the following, we report estimates of the models in which private information or consensus is not considered: $\rho_x = 0$ or $\delta = 0$, respectively. As a comparison, estimates from the rational expectation model are reported, namely

$$s_t = \lambda \mathbb{E}_t[s_{t+1}] + (1-\lambda) f_t - \lambda \psi_t + \epsilon_t, \tag{F.1}$$

in which f_t is described in Eq. (12). As stressed above, rational expectations are defined so that $\mathbb{E}_t[s_{t+1}] = s_t + \eta_t$, where η_t is a Gaussian shock.

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