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Non-Gaussian models for CoVaR estimation

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ABSTRACT

In this paper we show how to obtain estimates of CoVaR based on models that take into consideration some stylized facts about multivariate financial time series of equity log returns: heavy tails, negative skew, asymmetric dependence, and volatility clustering. While the volatility clustering effect is captured by AR-GARCH dynamics of the Glosten-Jagannathan-Runkle (GJR) type, the other stylized facts are explained by non-Gaussian multivariate models and copula functions. We compare the different models in the period from January 2007 to March 2020. Our empirical study conducted on a sample of listed banks in the euro area confirms that, in measuring CoVaR, it is important to capture the time-varying dynamics of the volatility. Additionally, a correct assessment of the heaviness of the tails and of the dependence structure is needed in the evaluation of this systemic risk measure.

1. Introduction

The contribution of financial institutions to systemic risks is a widely debated topic in the literature. Relevant articles include (Adrian & Brunnermeier, 2016; Billio et al., 2012; Bonaccolto et al., 2019; Girardi & Ergün, 2013), and (Liu et al., 2021).

In Adrian and Brunnermeier (2016) the distress of a financial institution j is defined as the event $\{y^j = \mathrm{VaR}_\alpha^j\}$, where y^j is the random variable representing the log returns of the institution and VaR_α^j the corresponding value-at-risk (VaR) at tail level α. Here we consider the conditional value-at-risk ($\mathrm{CoVaR}^{\leq}$) measure, that is the systemic risk measure where the conditioning event is the distress represented through the inequality $\{y^j \leq \mathrm{VaR}_\alpha^j\}$. This allows us to have a systemic risk measure which can be backtested (see Girardi & Ergün, 2013, and Banulescu et al., 2021), by using the standard tests developed for VaR. Conversely, the original $\mathrm{CoVaR}^{=}$ of Adrian and Brunnermeier (2016) is simple to estimate, but not so simple to backtest, because the tests to backtest the VaR cannot be applied. As shown in Bianchi and Sorrentino (2020), if the model for stock log returns is based on the multivariate normal random variable, a close formula for $\mathrm{CoVaR}^{=}$ is available.

Assuming that the $\mathrm{CoVaR}^{\leq}$ is a proper systemic risk measure, we explore to which extent the model assumptions on the univariate log returns and on the dependence structure affect the estimates of this risk measure. Then we conduct a backtesting analysis to obtain a robust model comparison.

It is widely known that the size of daily log returns varies over time and that if the volatility is high, it tends to remain high, and if it is low, it tends to remain low. This means that volatility moves in clusters and for this reason it is necessary to capture such observed behavior (see Rachev et al., 2011). Additionally, the CoVaR estimation is by definition a multi-dimensional problem. The multivariate normal model is usually applied in practical applications for finance, mainly because both the theoretical and practical complexity of a model increases if one moves from a normal to a non-normal framework.

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However, multivariate normal distribution has two main drawbacks: (1) its marginal distributions are normal, therefore it does not capture empirically observed skewness and excess-kurtosis; (2) its dependence structure is symmetric, therefore it does not capture asymmetry of dependence during extreme market movements and the dependence of tail events. For the reasons above, in this work we implement multivariate non-normal models with volatility clustering for CoVaR estimation.

We assume an AR-GARCH model for the univariate time series (see Glosten et al., 1993) and then we analyze different dependence structures. Among possible multivariate parametric models applied to finance, we select the multivariate normal tempered stable (MNTS) model, the multivariate generalized hyperbolic (MGH) model, and four copula functions: normal, t, BB1 and BB7, as described in De Luca and Rivieccio (2012), and Jaworski (2017). Both non-normal multivariate distributions and copula functions are widely known in the literature.
Kurosaki and Kim (2013b) developed a model based on MNTS distribution to estimate the CoVaR, and subsequently (Kurosaki & Kim, 2013a) and Biglova et al. (2014) studied a mean-CoAVaR strategy applied to portfolio optimization to mitigate the potential loss arising from systemic risk. Bianchi et al. (2016) and Bianchi and Tassinari (2020) analyzed both MNTS and MGH models applied to risk assessment and portfolio optimizations (see also Fallahgoul & Loeper, 2021; Shao et al., 2015, and Bianchi et al., 2019). Copula functions have largely been used in financial econometrics application, since the seminal work of Jondeau and Rockinger (2006) introducing the copula-GARCH model. For comparison purposes, we also analyze the constant and the dynamic conditional correlation (CCC and DCC) model with multivariate normal innovations (see Bauwens et al., 2006).

Our empirical study confirms that, even if in most cases the goodness-of-fit tests reject the normal distributional assumption, there are no significant differences in terms of VaR performance between Gaussian and non-Gaussian models, if the time-varying dynamics of the volatility is captured and the tail level is not extreme (i.e. $\alpha = 0.05$). Conversely, there are differences in terms of CoVaR performance. In estimating the CoVaR$^{\leq}$, which focuses on the tails of a bivariate random variable, a correct assessment of the heaviness of the tails and of the dependence structure is important, particularly during market turmoil, when the differences between Gaussian and non-Gaussian models become significant. However, the practical complexity of a model and the computational time needed for its estimation are considerable in a non-Gaussian framework. Additionally, ad-hoc algorithms should be used in order to implement these models in an efficient way. A key question is whether adding this complexity is worth the improvement in results obtained. Our empirical study shows that this complexity is necessary if, in estimating CoVaR, one wants to capture some stylized facts about multivariate financial log returns.

The remainder of the paper is organized as follows. In Section 2 we define the univariate model, the dependence structure, the ΔCoVaR$^{=}$ and the ΔCoVaR$^{\leq}$ and we show the necessary formulas to compute these risk measures. In Section 3 we describe how the estimation and risk measure evaluation procedures have been implemented. After having described in Section 4 the market data considered in this study, the main empirical results are discussed in Section 5. In Section 6 we compare the different distributional assumptions through a backtesting exercise. In Section 7 we compare the ΔCoVaR with the score defined by the Financial Stability Board (FSB) for global systemically important banks' (GSIBs) bucket allocation and we introduce a score adjusted for the information coming from the stock markets. Section 8 concludes.

2. Methodology

For each institution j, the random variable y_t^j represents the log returns of the market value of equity S_t^j, that is $y_t^j = \log(S_t^j/S_{t-1}^j)$. Superscript *sys* denotes the entire financial system, i.e. the capitalization-weighted portfolio of all financial institutions in the selected sample or an index that is representative of the stock market and frequently used by financial professionals (e.g. the S&P 500 index or the Euro Stoxx 50 index).

At time t, given the VaR with tail level α of the institution j ($\mathrm{VaR}^j_{\alpha,t}$), for a given tail level β, the CoVaR$^{=}$ of the system conditional on institution j being in distress (i.e. market returns of bank j are equal to its VaR$_\alpha$) is defined as (see Adrian & Brunnermeier, 2016)

$$P\left(y_t^{sys} \leq \mathrm{CoVaR}^{=j}_{\beta,\alpha,t} \mid y_t^j = \mathrm{VaR}^j_{\alpha,t}\right) = \beta. \tag{2.1}$$

A tail level α equal to 1% (2.5% or 5%) denotes a distress state of the world. Then the institution j contribution to systemic risk (i.e. the ΔCoVaR$^{=}$) is defined as the difference between the CoVaR conditional on the institution being under distress and the CoVaR in the median state (with $\alpha = 0.5$), that is

$$\Delta\mathrm{CoVaR}^{=j}_{\beta,\alpha,t} = \mathrm{CoVaR}^{=j}_{\beta,\alpha,t} - \mathrm{CoVaR}^{=j}_{\beta,0.5,t}.$$

From equations (17) and (18) in Adrian and Brunnermeier (2011), if one assumes a multivariate GARCH model with normal innovations, the ΔCoVaR$^{=}$ can be written as a function of the volatility of the system and the correlation between the institution j and the system.

Similarly, for a given tail level β, the CoVaR$^{\leq}$ of the system conditional on institution j being in distress (i.e. market returns of bank j are less or equal to its VaR$_\alpha$) is equal to

$$P\left(y_t^{sys} \leq \mathrm{CoVaR}^{\leq j}_{\beta,\alpha,t} \mid y_t^j \leq \mathrm{VaR}^j_{\alpha,t}\right) = \beta. \tag{2.2}$$

Under the definition in Eq. (2.2), it makes no sense to set $\alpha = 0.5$ to define the median state, as done under the definition in Eq. (2.1). For this reason it is necessary to introduce a different benchmark state to define the ΔCoVaR$^{\leq}$. Then by following (Girardi & Ergün, 2013), we define the benchmark state as the event $\{\mu_t^j - \sigma_t^j \leq y_t^j \leq \mu_t^j + \sigma_t^j\}$ with probability of occurrence p_t^j, where μ_t^j and σ_t^j are, respectively, the conditional mean and the conditional standard deviation of institution j. We refer to this event as the *one-sigma* event, in which the log returns of the institution j are far no more than one standard deviation (e.g. one-sigma) from the mean. Thus, as in Eq. (2.2), we define

$$P\left(y_t^{sys} \leq \mathrm{CoVaR}^{\leq j}_{\beta,\sigma,t} \mid \mu_t^j - \sigma_t^j \leq y_t^j \leq \mu_t^j + \sigma_t^j\right) = \beta. \tag{2.3}$$

Under the definition in Eq. (2.2), the institution j contribution to systemic risk is defined by

$$\Delta\mathrm{CoVaR}^{\leq j}_{\beta,\alpha,t} = \mathrm{CoVaR}^{\leq j}_{\beta,\alpha,t} - \mathrm{CoVaR}^{\leq j}_{\beta,\sigma,t}. \tag{2.4}$$

$\Delta\mathrm{CoVaR}^{\leq j}_{\beta,\alpha,t}$ captures the negative externality that institution j imposes on the system. It should be noted that while the first term of the right-hand side of Eq. (2.4) represents the tail event (there is α), the second term represents the *one-sigma* event (there is σ).

Under our framework, the measurement of systemic risk is divided into three steps: (1) the estimate of the univariate models on the time series of log returns; (2) the estimate of the multivariate model on the data extracted from the first step; (3) the computation of the risk measure (VaR) and of the systemic risk measure (CoVaR).

We assume for univariate log return processes an AR(1)-GARCH(1,1) model with Glosten-Jagannathan-Runkle (GJR) dynamics for the volatility, that is

$$\begin{aligned} y_t &= a y_{t-1} + \sigma_t \varepsilon_t + c \\ \sigma_t^2 &= \xi_0 + \xi_1 \left(|\sigma_{t-1}\varepsilon_{t-1}| - \gamma\left(\sigma_{t-1}\varepsilon_{t-1}\right)\right)^2 + \eta_1 \sigma_{t-1}^2 \end{aligned} \tag{2.5}$$

where ε_t is a collection of independent and identically distributed random variables with zero mean and unit variance. As observed in Kim et al. (2011), the following equality holds

$$\mathrm{VaR}^{y}_{\alpha,t+1} = a y_t + \sigma_{t+1}(\mathrm{VaR}^{\varepsilon}_{\alpha,t+1}) + c, \tag{2.6}$$

which in practice means that it is possible to compute the VaR of y on the basis of the quantile of a standardized random variable ε.

After having estimated for each bank and for the system the univariate discrete-time dynamic volatility model defined in Eq. (2.5), we extract the innovations and estimate different multivariate models (normal, MNTS and MGH) or dependence structures (copula).

On the basis of the univariate and multivariate estimates, it is possible to evaluate the CoVaR for each institution j and time t. Then Eq. (2.2) can be written as

$$\frac{P\left(y_t^{sys} \leq \mathrm{CoVaR}^{\leq j}_{\beta,\alpha,t}, y_t^j \leq \mathrm{VaR}^j_{\alpha,t}\right)}{P\left(y_t^j \leq \mathrm{VaR}^j_{\alpha,t}\right)} = \beta \tag{2.7}$$

and by the definition of VaR it follows that

$$P\left(y_t^{sys} \leq \mathrm{CoVaR}^{\leq j}_{\beta,\alpha,t}, y_t^j \leq \mathrm{VaR}^j_{\alpha,t}\right) = \alpha\beta.$$

In the multivariate cases, given the density f_t^j of the bivariate random variable defined by y_t^{sys} and y_t^j, then the following equalities can be considered

$$\begin{aligned} &\int_{-\infty}^{CoVaR^{\leq j}_{\beta,\alpha,t}} \int_{-\infty}^{VaR^j_{\alpha,t}} f_t^j(x,y)dxdy = \alpha\beta, \\ &\int_{-\infty}^{CoVaR^{\leq j}_{\beta,\sigma,t}} \int_{\mu_t^j-\sigma_t^j}^{\mu_t^j+\sigma_t^j} f_t^j(x,y)dxdy = p_t^j\beta, \end{aligned} \tag{2.8}$$

to obtain an estimate of $\Delta\mathrm{CoVaR}^{\leq}$. It should be noted that, as in Eq. (2.6), the CoVaR can be computed by considering the density of the bivariate random variable defined by the innovations obtained through the AR-GARCH filtering (i.e. ε_t^{sys} and ε_t^j) and then by performing a location and scale transformation based on the estimated AR-GARCH parameters. As observed by Banulescu et al. (2021), it is interesting to highlight that by slightly modifying the integral in Eq. (2.8) it is possible to obtain an estimate of the marginal expected shortfall.

In the copula function cases, as shown in Bernard et al. (2012), if one considers the dependence structure between y_t^{sys} and y_t^j, then the following equality can be considered

$$P(y_t^{sys} \leq x, y_t^j \leq y) = C\left(F_{y_t^{sys}}(x), F_{y_t^j}(y)\right)$$

and Eq. (2.7) can be rewritten as

$$C\left(F_{y_t^{sys}}(CoVaR^{\leq j}_{\beta,\alpha,t}), F_{y_t^j}(VaR^j_{\alpha,t})\right) = \alpha\beta \tag{2.9}$$

where C is a given copula function. Similarly, it is not difficult to see that the following equality holds

$$\begin{aligned} &C\left(F_{y_t^{sys}}(CoVaR^{\leq j}_{\beta,\sigma,t}), F_{y_t^j}(\mu_t^j + \sigma_t^j)\right) \\ &\quad - C\left(F_{y_t^{sys}}(CoVaR^{\leq j}_{\beta,\sigma,t}), F_{y_t^j}(\mu_t^j - \sigma_t^j)\right) = p_t^j\beta. \end{aligned}$$

In Section 5 the CoVaR$^{\leq}$ based on these multivariate models is compared to the CoVaR$^{=}$ based on the multivariate normal GARCH model as defined by Adrian and Brunnermeier (2016) and empirically studied by Bianchi and Sorrentino (2020).

3. Implementation

As already observed, the practical complexity of a model increases as soon as one moves from a Gaussian to a non-Gaussian framework. For this reason, in this section we describe in detail how we implemented our estimation procedures.

As observed in Section 2, the systemic risk measure estimation is divided into three steps. In the first step, we estimate a univariate AR-GARCH model on the time series of log returns. In the second step, we estimate the dependence structure by applying different multivariate approaches. In the third step, we compute the VaR at the given tail level α and then, by setting α (or σ) and β, we compute the CoVaR. In Fig. 1 we report the main steps of our workflow.

We compare the non-normal models with the constant and dynamic conditional correlation models with multivariate normal innovations and we refer to them as *CCC* and *DCC*.

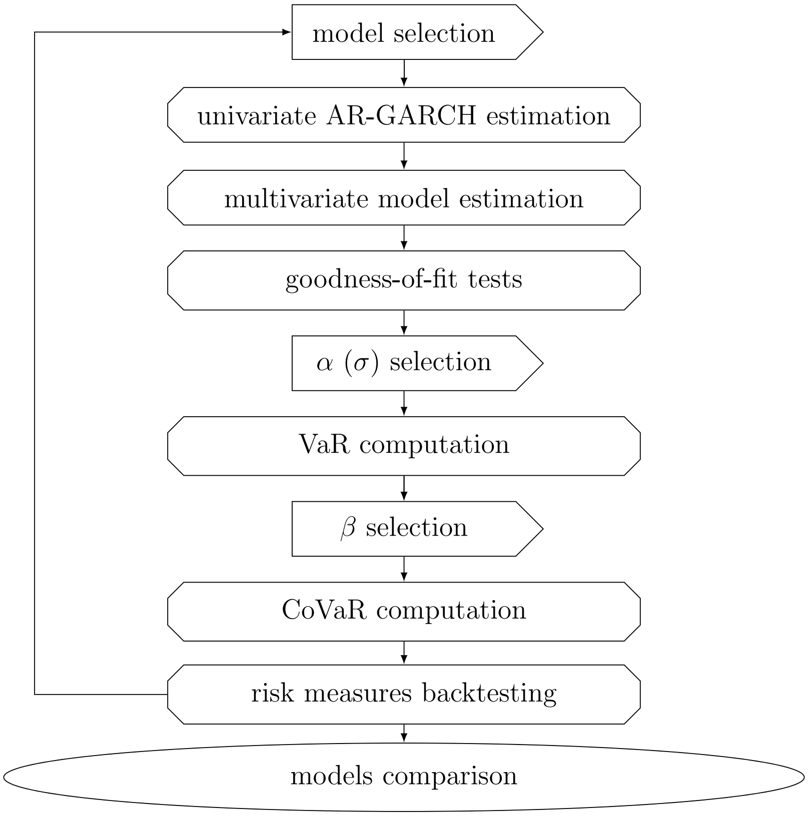

Fig. 1. Flowchart representing the steps of our analysis.

3.1. Model estimation

The first step is the estimation of the univariate autoregressive GARCH model with GJR dynamics for both system and banks log returns. This step is performed through the *garchFit* function of the package *fGarch* of R. While for the multivariate models we consider a normal distributional assumption to extract the innovations, for the copula model we consider the skew-t distributional assumption. The former approach can be viewed as a quasi-maximum likelihood estimation (QMLE) approach (see Goode et al., 2015). Additionally, at each estimation step we verify if the autoregressive component is statistically significant: if it is not, we estimate the model without the autoregressive component. The DCC model allows one to assume the same AR-GARCH structure of Eq. (2.5) for the marginal components and can be estimated through the *dccfit* function of the package *rmgarch* of R.

After having estimated the univariate dynamic volatility models, we extract the innovations and estimate the multivariate models and the copula ones. We consider the multivariate normal tempered stable (MNTS) model, the multivariate generalized hyperbolic (MGH) model, as described in Bianchi et al. (2019), and the best copula function in terms of AIC among normal, t, BB1 and BB7 copulas, as described in De Luca and Rivieccio (2012).

The estimation of both the MNTS and MGH is time-consuming from a computational point of view and for this reason we decided to run as few estimation procedures as possible, thus instead of estimating a bivariate model we perform a 13-dimensional estimation. Conversely, the estimation of copula functions in large dimensions can be problematic from a numerical error perspective, thus instead of estimating a 13-dimensional model we perform bivariate estimations. Thus, for the MNTS and MGH models we estimate the 13-dimensional model by using an ad-hoc procedure implemented in R considering an expectation–maximization maximum-likelihood approach (see Bianchi et al., 2016). The copulas are instead estimated on bivariate time series (i.e. for each j, the couple ε^{sys} and ε^{j} is considered) through the *BiCopSelect* function of the *VineCopula* package of R. For the CCC model, the correlation matrix is the only parameter to estimate.

There are remarkable differences in terms of computational time between the two approaches. While for a one-step estimation of the four competitor bivariate copulas 0.9 s are needed, for a one-step estimation of the multivariate MNTS (MGH) model the computational time is around 150 (10) s. For each estimation day, in the copula case we have 12 independent bivariate estimations and in the multivariate case we have a single multivariate estimation. This means that the computing time of the copula and MGH model is similar, but the computing time of the MNTS model is 15 times larger. By considering that the overall computing time is around 24 h for the faster models, to deal with the MNTS estimation problem we relied on an efficient R code making use of the packages *foreach* and *doParallel* and ran it on a multi-core platform (a Linux based system with Intel processors).

3.2. Risk measures estimation

After the two estimation steps described in Section 3.1, we forecast the one-day ahead volatility obtained from the estimated AR-GARCH parameters. In the DCC case we also forecast the one-day ahead correlation obtained from the dynamic model parameters. While in the MNTS (MGH) model ε^{sys} and ε^{j} are assumed NTS (GH) distributed, in the copula model they are assumed skew-*t* distributed (see Fernandez & Steel, 1998). To evaluate the univariate VaR for each single bank *j*, while in the NTS (GH) case one needs to numerically invert the cumulative distribution function obtained by means of the fast Fourier transform algorithm (see Bianchi et al., 2019), in the skew-*t* case the VaR can be directly obtained through the *qsstd* function of the *fGarch* package of R. In the CCC case, the evaluation of the VaR is straightforward, since the *qnorm* function of the *stats* package of R can be used.

While the procedure to obtain VaR estimates is well-known in the related literature, the estimation of the CoVaR$^{\leq}$ is more challenging. A closed formula for CoVaR is available under normal distributional assumptions (see Bernard et al., 2012); for non-normal models, as shown in Bernard and Czado (2015) and Girardi and Ergün (2013), numerical procedures are needed to find the CoVaR$^{\leq}$ satisfying Eq. (2.8) or (2.9).

In the copula case, by following the approach described in Bernard et al. (2012), it is possible to obtain a CoVaR estimate by means of the inversion of the copula function in Eq. (2.9). After having estimated the bivariate copula function, the CoVaR estimate is obtained through the one-dimensional root finding algorithm implemented in the *uniroot* function of the *stats* package of R (a classical algorithm in numerical analysis). The procedure is implemented by also considering the *BiCopCDF* function of the *VineCopula* package of R. This function evaluates the cumulative distribution function of the parametric bivariate copula of Eq. (2.9).

In the multivariate case, the integrals in Eq. (2.8) are evaluated by the numerical integration algorithm implemented in the *quadrature* function of the *mvQuad* package of R. Thus, the CoVaR estimate is obtained through the one-dimensional root finding algorithm implemented in the *uniroot* function of the *stats* package of R.

Except for the correlation matrix, the CoVaR estimation procedure for both CCC and DCC models is identical: while the latter model considers the historical correlation, the former considers the one-day ahead forecast obtained through the *dccforecast* function of the package *rmgarch* of R.

While for the copula model the computing time for VaR and CoVaR is instantaneous and the overall computing time is a few minutes, this is not the case for multivariate models. In contrast to the copula case, the time needed to estimate the CoVaR for the MNTS and the MGH model for a given day is around eight minutes. For this reason, also in this case we relied on a multi-core implementation. However, the VaR estimation algorithm is 60 times faster than the CoVaR one.

As far as CoVaR estimation is concerned, the main bottleneck is the numerical evaluation of the double integral in the MNTS case. A large number of points are needed for the bidimensional grid in order to avoid numerical errors in the evaluation of this integral. In order to obtain an efficient algorithm, the grid is rescaled on the basis of the domain of integration. We consider the trapezoidal rule and a grid with 10,000 points (100 points for each one-dimensional side of the grid). By considering that the integral is focused on the tails in the third quadrant of the bidimensional space, the number of points is large enough. As a robustness check, we increase the number of points from 10,000 to 90,000 (from 100 to 300 points for each one-dimensional side of the grid) and evaluate the CoVaR with both α and β equal to 0.05 at three randomly selected days in the sample period analyzed in this study. For both the MNTS and the MGH model, the absolute percentage difference between the two estimates is on average around 1%, but with a computational time that is almost ten times slower (more than one hour) if one considers the grid with 90,000 points. However, if the same empirical exercise is conducted on a day when CoVaR reaches a very large value, the absolute percentage difference dramatically decreases.

It should be noted that the evaluation of the density function of a bivariate MNTS random variable is based on both a numerical integration and the FFT algorithm (see Bianchi et al., 2016 for the computational aspects related to the evaluation of this density function).

4. Data

Before starting with the empirical analysis, in this section we provide a description of the data used in this study. We obtained from Refinitiv the daily dividend adjusted closing prices from January 2002 to March 2020 for a sample of listed banks in the euro area belonging to the main or to the additional GSIB assessment sample for a total of 12 banks. These banks are: Deutsche Bank (D:DBK), Commerzbank (D:CBK), Unicredit (I:UCG), Intesa (I:ISP), BNP Paribas (F:BNP), Société Générale (F:SGE), Crédit Agricole (F:CRDA), BBVA (E:BBVA), Santander (E:SAN), Banco de Sabadell (E:BSAB), KBC Group (B:KC) and ING Bank (H:INGA). We refer to these European banks as GSIBs, even if for some of these banks there are no additional capital requirements. In this study the system is the Euro Stoxx 50 index. The time period in this analysis includes the high volatility period after the Lehman Brothers filed for Chapter 11 bankruptcy protection (September 15 2008), the eurozone sovereign debt crisis, during which, in November 2011, the spread between the 10-year Italian BTP and the German Bund with the same maturity exceeded 500 basis points, the turmoil after the Italian political elections in 2018 and the recent financial market crash at the end of the first quarter of 2020.

The CoVaR (ΔCoVaR) is estimated on the basis of the time series from the previous five years. For example, the CoVaR (ΔCoVaR) for December 6 2007 is estimated

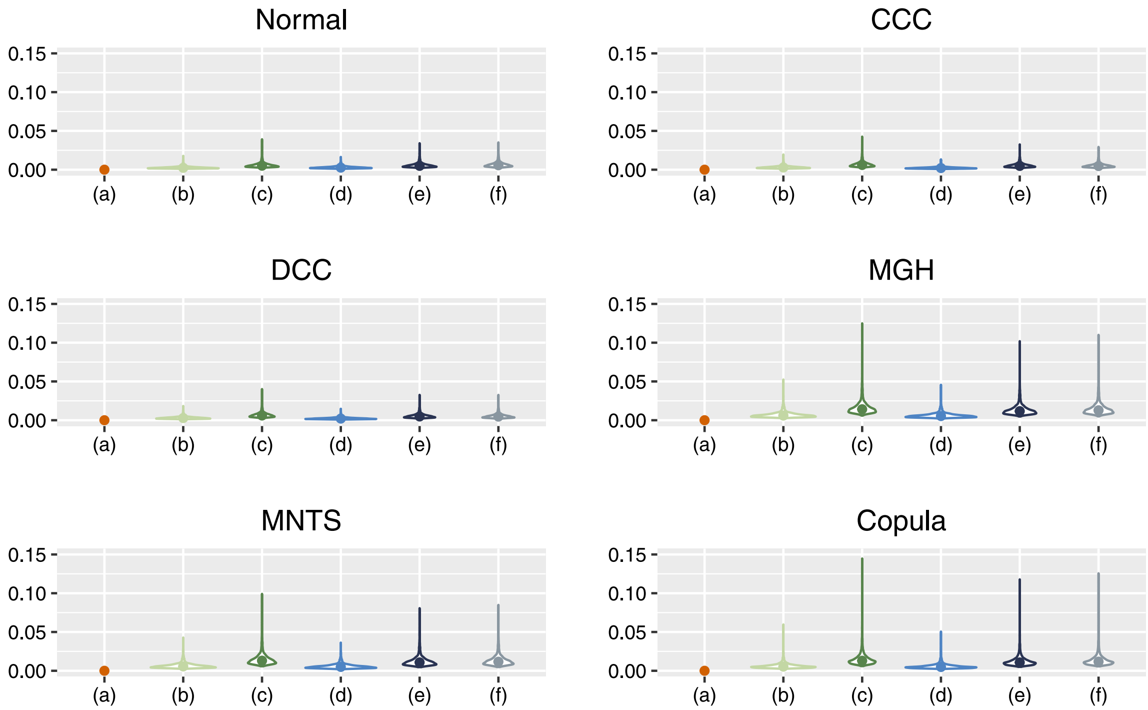

Fig. 2. We report the time series (median values across all banks) from January 2 2007 to March 30 2020 of the $\text{CoVaR}^{=}$ based on the multivariate normal model (*normal*) and the $\text{CoVaR}^{\leq}$ based on the CCC, the DCC, the MGH, the MNTS, and the copula model for various levels of α and β, that is (a) $\alpha = \beta = 0.05$, (b) $\alpha = 0.05$ and $\beta = 0.025$, (c) $\alpha = 0.05$ and $\beta = 0.01$, (d) $\alpha = 0.025$ and $\beta = 0.05$, (e) $\alpha = \beta = 0.025$, and (f) $\alpha = 0.01$ and $\beta = 0.05$. All values are changed in sign. The differences with respect to case (a) are reported. The dot inside the violin plot represents the median value.

from the data for the period from December 6 2002 to December 5 2007. For each bank and each model we consider 3,389 estimations from January 2 2007 to March 30 2020.

5. Empirical results

In this section we compare the three non-normal models with the CCC and the DCC model. We estimate the $\text{CoVaR}^{\leq}$ under these five multivariate assumptions. Additionally, we estimate the $\text{CoVaR}^{=}$ under the multivariate normal distributional assumption, that is, the systemic risk measure originally proposed by Adrian and Brunnermeier (2016). We refer to it as the *normal* model.

As far as the copula model is concerned, on the basis of the AIC criterion the t-copula is selected in most of the cases (around 88% over the 40,668 bivariate estimations), in very few cases the normal copula (0.5%), and in all other cases the BB1 (around 12%). As observed above, it should be noted that only 0.9 s are needed to perform a one-step estimation of four bivariate copulas.

For each model and each bank we conduct the Kolmogorov–Smirnov (KS) and the Anderson–Darling (AD) test over the entire estimation window from January 2 2007 to March 30 2020 for a total of 44,057 KS (AD) tests for each model (3,389 for each bank). As far as the KS test is concerned, while the normal model has a rejection rate equal to 51.5%, for non-normal models the null hypothesis is rejected in 6.7% of the cases, mainly caused by one bank with a rejection rate equal to 82% in the MNTS (MGH) case and to 3% in the copula case. The overall performance of the three non-normal models is comparable, with the exception of that bank. As far as the AD test is concerned, while the normal model has a rejection rate equal to 57.9%, for all non-normal models the null hypothesis is almost never rejected.

In Figs. 2 and 3 we show the behavior of the CoVaR, then in Figs. 4 and 5 we depict the dynamics of the ΔCoVaR with α and β equal to 0.05. In order to show the differences between models and between tail levels (α and β), we report in Figs. 2 and 3 the median time series computed across all banks from January 2 2007 to March 30 2020 of the $\text{CoVaR}^{=}$ based on the multivariate normal model (i.e. the *normal* model) and the $\text{CoVaR}^{\leq}$ based on the CCC, the DCC, the MGH, the MNTS, and the copula model for various levels of α and β. While in Fig. 2, for each model, we report the differences with respect to the case in which α and β are both equal to 0.05, in Fig. 3 for each couple of α and β analyzed in this work, we report the differences with respect to the $\text{CoVaR}^{=}$ (i.e. the normal model). Fig. 2 shows that the CoVaR of a given parametric model depends strongly on the selected tail levels, particularly in the non-Gaussian cases and in stressed market scenarios. Fig. 3 depicts significant differences between Gaussian and non-Gaussian models for all tail levels analyzed in this work. Under the multivariate normal distributional assumption, the overall difference between normal, CCC, and DCC models does not seem material.

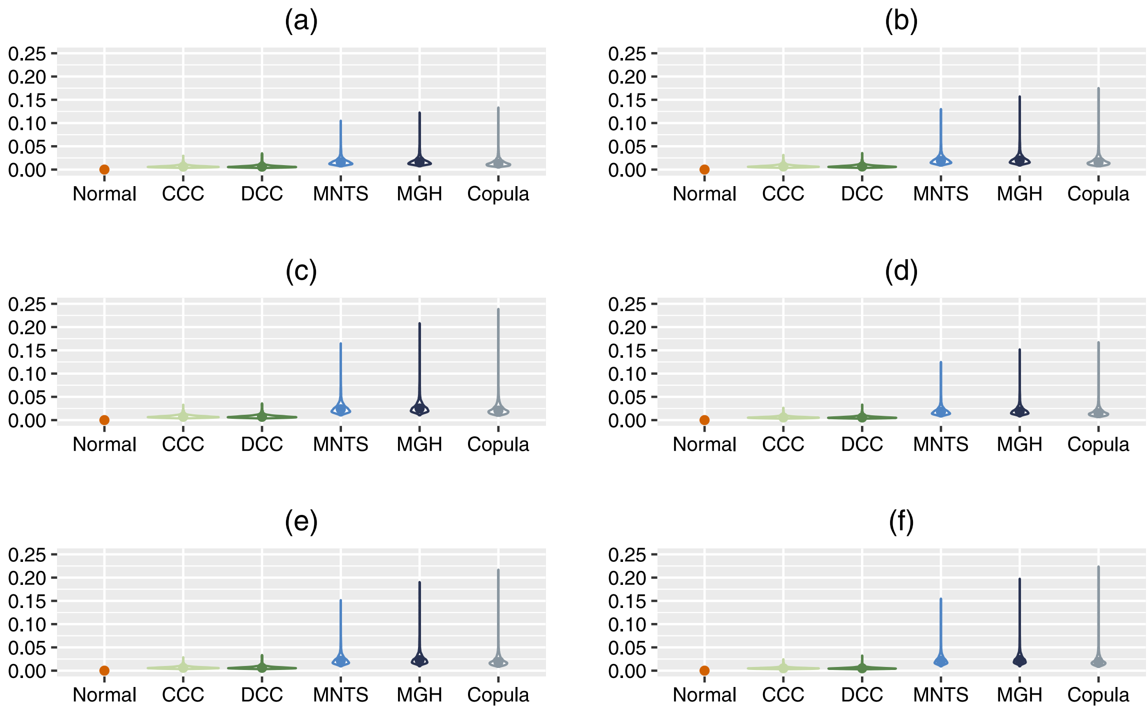

Fig. 3. We report the time series (median values across all banks) from January 2 2007 to March 30 2020 of the CoVaR$^{=}$ based on the multivariate normal model (*normal*) and the CoVaR$^{\leq}$ based on the CCC, the DCC, the MGH, the MNTS, and the copula model for various level of α and β, that is (a) $\alpha = \beta = 0.05$, (b) $\alpha = 0.05$ and $\beta = 0.025$, (c) $\alpha = 0.05$ and $\beta = 0.01$, (d) $\alpha = 0.025$ and $\beta = 0.05$, (e) $\alpha = \beta = 0.025$, and (f) $\alpha = 0.01$ and $\beta = 0.05$. All values are changed in sign. The differences with respect to the normal model are reported. The dot inside the violin plot represents the median value.

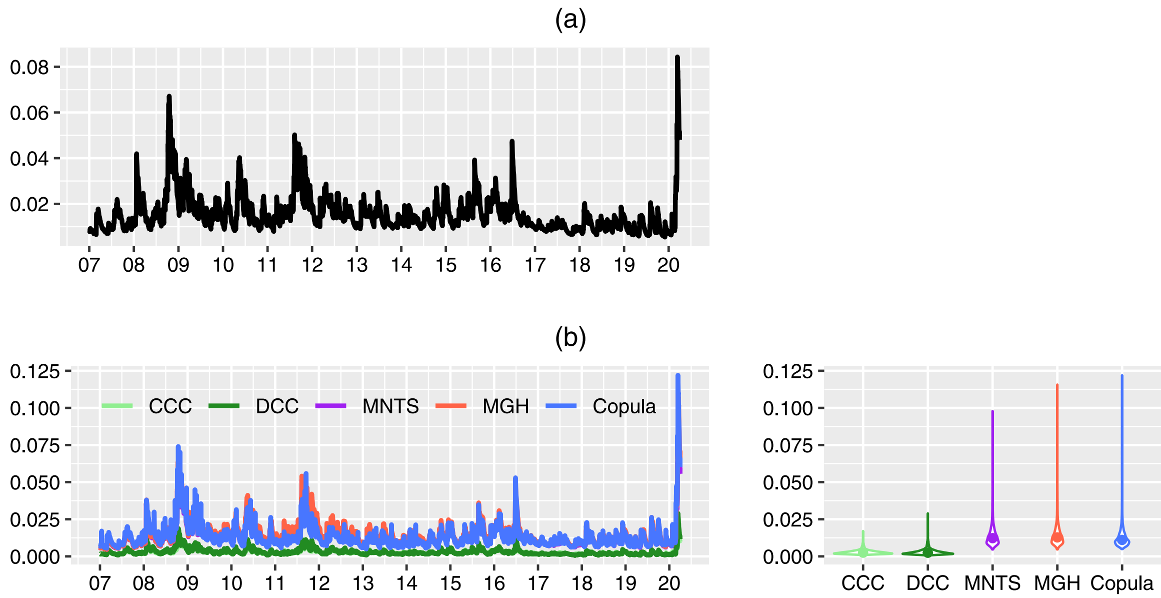

Fig. 4. In panel (a) we report the time series (median values across all banks) from January 2 2007 to March 30 2020 of the ΔCoVaR$^{=}$ based on the multivariate normal GARCH model with GJR dynamics. In panel (b), for each model we report for the same time window the time series and the violin plot of the differences between the ΔCoVaR$^{\leq}$ and the above ΔCoVaR$^{=}$. In all cases we consider α and β equal to 0.05. All values are changed in sign. The dot inside the violin plot represents the median value.

In Fig. 4 we report the median time series computed across all banks of the ΔCoVaR$^{=}$ based on the multivariate normal model. We consider α and β equal to 0.05 in Eq. (2.4). Then we compare the ΔCoVaR$^{=}$ to the ΔCoVaR$^{\leq}$ evaluated under different distributional assumptions. In panel (b) we report the differences between

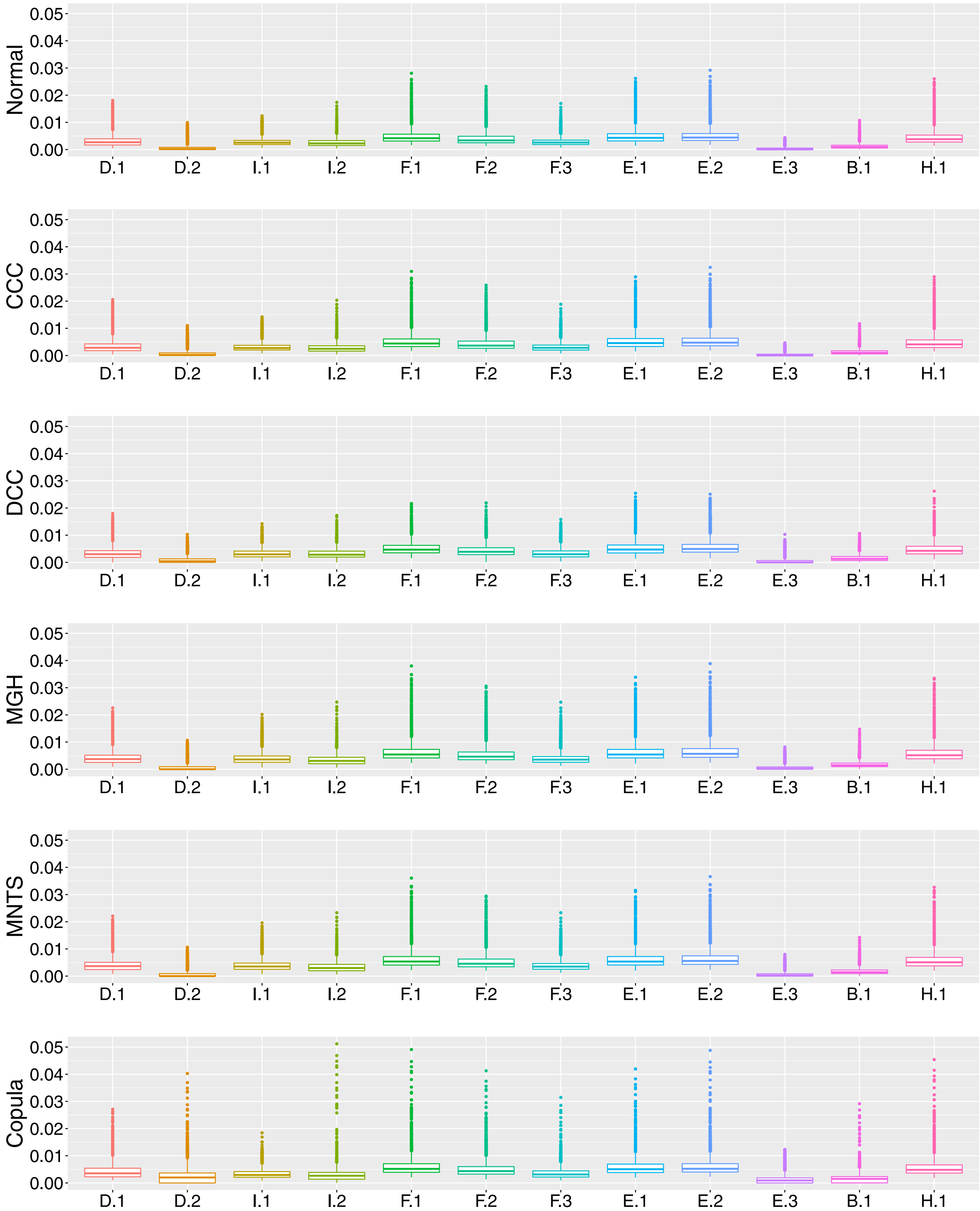

Fig. 5. For each model and each institution we report the box plot of the difference with respect to the minimum computed across all institutions from January 2 2007 to March 30 2020. We consider the $\text{CoVaR}^{=}$ based on the multivariate normal model (*normal*) and the $\text{CoVaR}^{\leq}$ based on the CCC, the DCC, the MGH, the MNTS, and the copula model. In all cases we consider α and β equal to 0.05. All values are changed in sign.

the $\Delta\text{CoVaR}^{\leq}$ and the $\Delta\text{CoVaR}^{=}$. As already observed for the CoVaR, in the CCC and the DCC case in panel (b) it appears that there are no remarkable differences between $\Delta\text{CoVaR}^{=}$ and $\Delta\text{CoVaR}^{\leq}$. This means that, at least for the sample considered in this study, if one considers the normal distributional assumption, the two systemic risk measures $\text{CoVaR}^{\leq}$ and $\text{CoVaR}^{=}$ are more or less equivalent, even if the former is slightly higher during market turmoil. The difference increases in non-normal cases, it is relevant, and it varies over time. As shown in the violin plot on the right side of panel (b), it ranges from a median value of 0.25% in the CCC and case to 1.28% in the MGH case. The maximum distance is reached in March 2020 and it ranges from 1.7% in the CCC case to 12.1% in the copula case. We recall that the violin plot is a method of drawing numeric data and combining a box plot with a kernel density plot. The dot inside the violin plot represents the median value. As already observed for the CoVaR, as expected, also for the ΔCoVaR there are significant differences between Gaussian and non-Gaussian models and these differences increase during market turmoil.

It should be noted that, given a model, the variability across banks of the $\Delta\text{CoVaR}^{\leq}$ is low in absolute value but it is not negligible, at least for the sample of banks considered in this empirical exercise. This variation is low in absolute value because we are analyzing only the main large European banks. The same argument also holds for the $\Delta\text{CoVaR}^{=}$. In order to obtain a more meaningful visual comparison of different institutions, in Fig. 5 for each model and each institution we report the box plot of the difference with respect to the minimum ΔCoVaR computed across all institutions. Here we are considering α and β equal to 0.05. In the non-normal cases the difference is slightly larger. It should be noted that, under this different perspective, the variability across banks becomes more evident. This difference, which we refer to as ΔCoVaR^{*}, will be considered in Section 7.

6. Backtesting CoVaR

The CoVaR defined in Eq. (2.2) can be backtested in two-steps (see Girardi & Ergün, 2013). In both steps it is possible to follow the approach proposed in Christoffersen (2010). First we conduct a preliminary VaR backtest by considering the entire observation window and defining a first hit sequence. This hit sequence is 1 if the loss of the financial institution on that day is larger than its predicted VaR level, and zero otherwise. Then we define a subset of observations on the basis on the distress of the financial institution j (i.e. $y_t^j \leq \text{VaR}_{\alpha,t}^j$). Thus, by looking at this subset, we can backtest the CoVaR. In more detail, we compare the CoVaR forecast with the ex-post loss of the financial system and define a second hit sequence which is 1 if the loss of the financial system on that day is larger than its predicted CoVaR level, and zero otherwise.

For evaluating the accuracy of forecasted VaR and CoVaR for the models analyzed in this paper, we perform the likelihood ratio (LR) tests proposed by Christoffersen (1998, 2010). The LR tests use the number of violations (i.e. the hit sequences defined above), where violations occur when the actual loss exceeds the estimated VaR (CoVaR). The LR test consists of three parts: (1) the LR test of unconditional coverage (LR_{uc}), which is the same as the proportion of failures test by Kupiec (1995), (2) the LR test of independence (LR_{ind}), and (3) the joint test of coverage and independence (LR_{cc}). In Fig. 6, for both VaR and CoVaR for all analyzed models and various values of α and β, we report the number of banks (over a total of 12 banks) whose p-values of the LR_{uc} and the LR_{cc} are less than 0.05. We do not report the p-values for the LR test of independence.

In order to backtest the CoVaR, a large number of observations are needed. Here we are considering 3,389 observations; this means that there are enough observations to backtest almost all tail levels α, but it is not possible to backtest all meaningful tail levels β. For example, even if we are considering more than 18 years of daily data, it is not possible to study the CoVaR for α and β equal to 0.01, because in this case the theoretical number of exceedances is less than one.

We further consider the dynamic quantile (dq) test proposed by Engle and Manganelli (2004), that can be viewed as a more general formulation of the tests proposed above and two loss functions, that is the magnitude loss (LM) and the asymmetric magnitude loss (LA) function as defined by Amendola and Candila (2016). The farther the actual log return is from the expected one, the larger is the value of these loss functions. The LA function is built to penalize more heavily models with a higher number of violations with respect to the number of expected violations. The two functions are

$$LM = \begin{cases} 1 + (y_t - \text{VaR}_t)^2 & \text{if} \quad y_t < \text{VaR}_t \\ 0 & \text{if} \quad y_t \geq \text{VaR}_t \end{cases}$$

and

$$LA = \begin{cases} P(1 + |y_t - \text{VaR}_t|) & \text{if} \quad y_t < \text{VaR}_t \\ |y_t - \text{VaR}_t| & \text{if} \quad y_t \geq \text{VaR}_t \end{cases}$$

with $P = \exp((\hat{\alpha} - \alpha)/\alpha)$ if $\hat{\alpha} > \alpha$ and $P = 1$ in the opposite case, where $\hat{\alpha}$ is the empirical coverage.

In Tables 1 to 3 we report, for each bank and for all α and β considered in this study, the p-values of the three tests (uc, cc and dq) and the values of the two loss functions (LM and LA). To highlight the model with the worst performance, the p-values lower than 0.05 and the maximum value by row of the loss functions are in bold in the tables. Since the CCC and the DCC model show a similar performance, in Tables 1–3 we report only the DCC model.

As far as the VaR backtesting is concerned, the non-normal models (MNTS, MGH, and copula) largely outperform the DCC (CCC) model, at least for the tail probability levels that are usually of interest (i.e. for α equal to 0.01 and 0.025). However, for α equal to 0.05, the DCC (CCC) model shows a better performance than the copula model, even if in the copula model case the null hypothesis is rejected in one case only. This in practice means that capturing the volatility clustering effect is of paramount importance in evaluating risk measures, at least for the dataset analyzed in this study. In contrast, in the CoVaR backtesting, the DCC (CCC) model is always rejected and the non-normal model always presents a satisfactory performance. By looking at the LA values the MGH model seems slightly better in comparison with its competitor models.

As far as the VaR estimates are concerned, the results in Tables 1 to 3 show that the LM function is smaller for

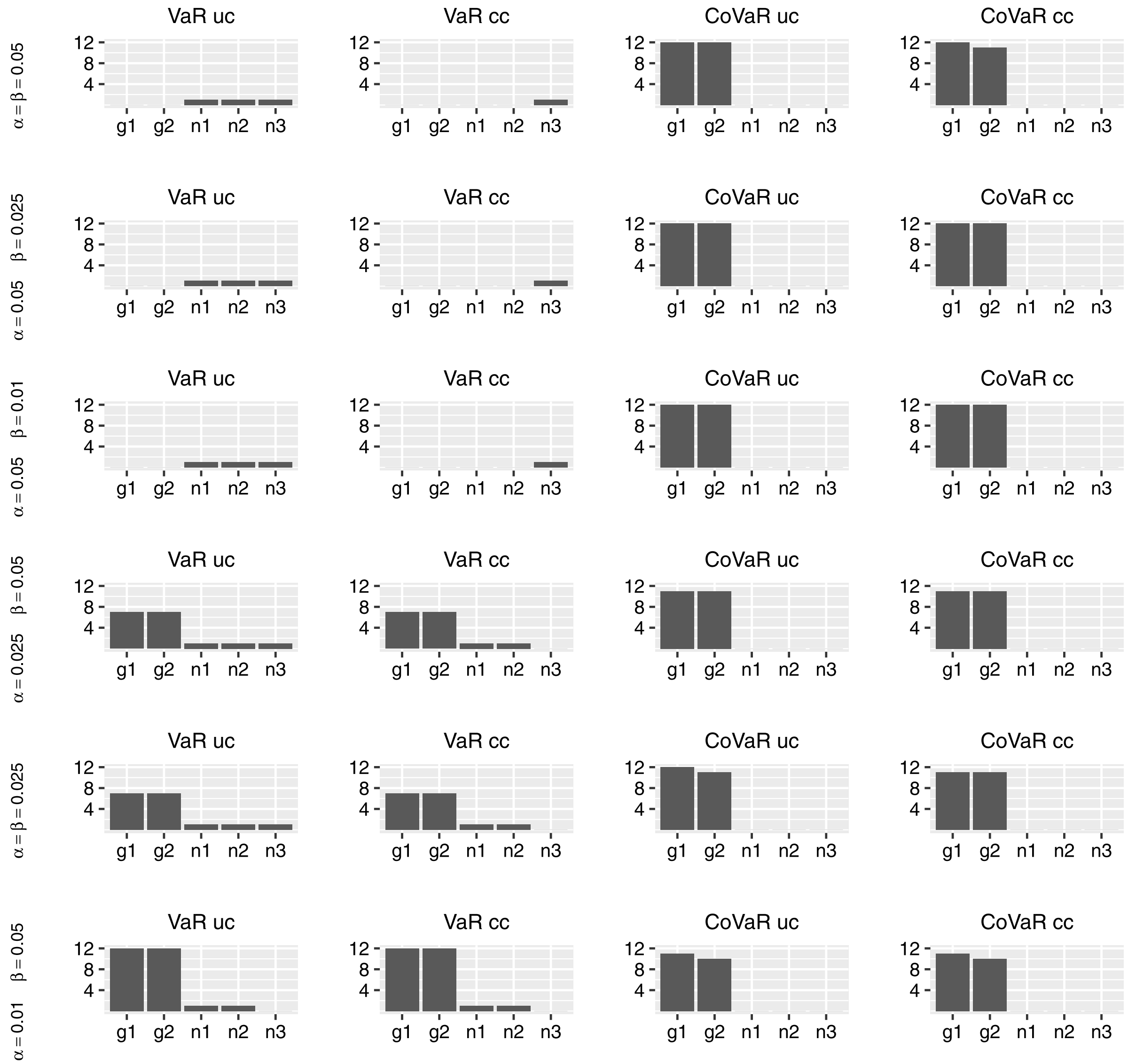

Fig. 6. Number of rejections (p-values less than 0.05) of the LR test of unconditional coverage (uc) and coverage and independence (cc) for both VaR and CoVaR for all analyzed models: CCC (g1), DCC (g2), MNTS (n1), MGH (n2), copula (n3). The number of banks is equal to 12.

the MGH and the MNTS models with respect to the copula model in 12 cases out of 12 when $\alpha = 0.05$, in nine cases out of 12 when $\alpha = 0.025$, and in eight cases out of 12 when $\alpha = 0.01$. Similar findings hold for the LA function, with the exception of the $\alpha = 0.01$ case. However, it should be noted that, on average, the DCC (CCC) model also shows satisfactory results in terms of loss functions.

As far as the CoVaR estimates are concerned, the results in Tables 1 to 3 show that the LM function is smaller for the MGH and the MNTS models with respect to the copula model in 23 cases out of 36 when $\alpha = 0.05$, in 16 cases out of 24 when $\alpha = 0.025$, and in four cases out of 12 when $\alpha = 0.01$. Similar findings hold for the LA function. However, it should be noted that while, on average, there are no remarkable differences between the three non-normal models in terms of loss functions, the performance of the DCC (CCC) model is not satisfactory.

At least for the time series and the values of α and β analyzed in this study, the non-normal multivariate models (i.e. the MGH and the MNTS model) slightly outperform the copula from both a VaR and a CoVaR perspective. The DCC (CCC) approach is satisfactory only in the VaR case with $\alpha = 0.05$, but its performance is not good enough in the CoVaR backtesting for all tail levels α and β considered in this study.

7. GSIBs indicators

In this section, we compare the ranking provided by the ΔCoVaR with the ranking identified by the FSB for

Table 1
P-values of the LR test of unconditional coverage (uc), coverage and independence (cc) and dynamic quantile tests (dq) for both VaR and CoVaR for all analyzed models and banks. The values of the magnitude (LM) and of the asymmetric loss (LA) functions are also reported. We consider $\alpha = 0.05$ and (a) $\beta = 0.05$, (b) $\beta = 0.025$, (c) $\beta = 0.01$. The *p*-values lower than 0.05 and the maximum value by row of the loss functions are in bold.

		DCC					MGH					MNTS					Copula				
		uc	cc	dq	LM	LA	uc	cc	dq	LM	LA	uc	cc	dq	LM	LA	uc	cc	dq	LM	LA
D.1	VaR	0.45	0.66	0.08	**0.05**	**0.09**	0.84	0.97	0.25	0.05	0.09	0.85	0.98	0.15	0.05	0.09	0.72	0.73	0.06	0.05	0.09
	CoVaR (a)	**0.00**	**0.01**	**0.02**	**0.11**	**0.13**	0.56	0.66	0.93	0.04	0.07	0.62	0.68	0.93	0.04	0.07	0.66	0.52	0.88	0.06	0.08
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.08**	**0.11**	0.88	0.92	1.00	0.02	0.06	0.69	0.82	1.00	0.03	0.07	0.24	0.39	0.80	0.04	0.07
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.06**	**0.15**	0.37	0.65	1.00	0.02	0.06	0.35	0.63	1.00	0.02	0.06	0.38	0.66	1.00	0.02	0.06
D.2	VaR	0.73	0.36	0.58	0.05	0.09	0.41	0.16	0.08	0.05	0.10	0.50	0.16	0.07	0.05	0.10	0.23	0.46	0.27	**0.05**	**0.10**
	CoVaR (a)	**0.00**	**0.00**	**0.00**	**0.13**	**0.15**	0.51	0.42	0.54	0.06	0.09	0.31	0.27	0.26	0.07	0.10	0.93	0.66	0.88	0.05	0.08
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.08**	**0.12**	0.81	0.87	1.00	0.03	0.06	0.80	0.86	1.00	0.03	0.06	0.15	0.26	0.61	0.04	0.08
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.07**	**0.52**	0.41	0.69	1.00	0.02	0.06	0.40	0.68	1.00	0.02	0.06	0.44	0.71	1.00	0.02	0.06
I.1	VaR	0.17	0.39	0.65	0.06	0.10	0.41	0.70	0.84	0.05	0.10	0.45	0.75	0.87	0.05	0.10	0.08	0.20	0.41	**0.06**	**0.10**
	CoVaR (a)	**0.01**	**0.03**	0.23	**0.10**	**0.12**	0.14	0.29	0.87	0.03	0.06	0.14	0.29	0.87	0.03	0.06	0.90	0.57	0.74	0.05	0.08
	CoVaR (b)	**0.00**	**0.01**	**0.03**	**0.07**	**0.10**	0.45	0.71	0.98	0.02	0.05	0.82	0.89	1.00	0.02	0.06	0.34	0.49	0.92	0.04	0.07
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.05**	**0.10**	0.41	0.68	1.00	0.02	0.06	0.41	0.67	1.00	0.02	0.06	0.47	0.73	1.00	0.02	0.06
I.2	VaR	0.66	0.69	0.53	0.05	0.09	0.91	0.88	0.56	0.05	0.09	0.91	0.88	0.56	0.05	0.09	0.37	0.42	0.38	**0.05**	**0.09**
	CoVaR (a)	**0.00**	**0.01**	**0.03**	**0.11**	**0.13**	0.58	0.45	0.63	0.06	0.09	0.58	0.45	0.64	0.06	0.09	0.34	0.27	0.49	0.07	0.09
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.10**	**0.13**	0.92	0.90	1.00	0.02	0.06	0.92	0.90	1.00	0.02	0.06	0.13	0.23	0.31	0.04	0.08
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.07**	**0.56**	0.36	0.62	1.00	0.02	0.06	0.36	0.62	1.00	0.02	0.06	0.42	0.68	1.00	0.02	0.06
F.1	VaR	0.66	0.55	0.91	0.05	**0.09**	0.84	0.89	0.99	0.05	0.09	0.97	0.86	0.99	0.05	0.09	0.66	0.38	0.87	**0.05**	0.09
	CoVaR (a)	**0.00**	**0.00**	**0.01**	**0.12**	**0.14**	0.83	0.66	0.91	0.05	0.08	0.87	0.66	0.91	0.05	0.08	0.45	0.70	0.71	0.06	0.09
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.10**	**0.13**	0.50	0.76	0.99	0.02	0.05	0.91	0.90	1.00	0.02	0.06	0.12	0.20	0.52	0.05	0.08
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.07**	**0.53**	0.37	0.64	1.00	0.02	0.06	0.36	0.62	1.00	0.02	0.06	0.39	0.65	1.00	0.02	0.06
F.2	VaR	0.41	0.05	0.21	0.05	0.09	0.45	0.06	0.29	0.05	0.09	0.41	0.05	0.25	0.05	0.09	0.73	0.12	0.47	**0.05**	**0.09**
	CoVaR (a)	**0.00**	**0.00**	**0.00**	**0.13**	**0.15**	0.71	0.68	0.93	0.04	0.07	0.72	0.68	0.93	0.04	0.07	0.79	0.57	0.92	0.05	0.08
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.09**	**0.13**	0.60	0.82	0.99	0.02	0.05	0.99	0.90	1.00	0.03	0.06	0.19	0.31	0.66	0.04	0.08
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.07**	**0.37**	0.32	0.58	1.00	0.02	0.06	0.32	0.57	1.00	0.02	0.06	0.34	0.60	1.00	0.02	0.06
F.3	VaR	0.85	0.26	**0.03**	0.05	0.09	0.84	0.35	**0.04**	0.05	0.09	0.97	0.30	**0.04**	0.05	0.09	0.55	0.25	0.15	**0.05**	**0.09**
	CoVaR (a)	**0.00**	**0.00**	**0.00**	**0.13**	**0.15**	0.56	0.63	0.89	0.04	0.07	0.85	0.59	0.85	0.05	0.08	0.30	0.24	0.53	0.07	0.09
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.10**	**0.15**	0.88	0.90	0.99	0.02	0.06	0.71	0.80	0.97	0.03	0.07	0.25	0.39	0.69	0.04	0.07
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.08**	**0.77**	0.37	0.64	0.99	0.02	0.07	0.36	0.62	0.99	0.02	0.06	0.40	0.66	0.99	0.02	0.06
E.1	VaR	0.13	0.31	0.11	0.06	0.09	0.33	0.51	0.21	0.05	0.09	0.33	0.51	0.20	0.05	0.09	0.06	0.16	**0.03**	**0.06**	**0.09**
	CoVaR (a)	**0.00**	**0.00**	**0.00**	**0.12**	**0.14**	0.46	0.57	0.94	0.04	0.07	0.46	0.57	0.94	0.04	0.06	0.92	0.58	0.93	0.05	0.08
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.09**	**0.12**	0.43	0.70	0.98	0.02	0.05	0.79	0.88	1.00	0.02	0.05	0.18	0.29	0.72	0.04	0.07
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.07**	**0.32**	0.42	0.69	1.00	0.02	0.06	0.42	0.69	1.00	0.02	0.06	0.48	0.74	1.00	0.02	0.05
E.2	VaR	0.07	0.12	0.10	**0.06**	**0.09**	0.20	0.23	0.22	0.05	0.09	0.33	0.38	0.29	0.05	0.09	0.11	0.19	0.17	0.06	0.09
	CoVaR (a)	**0.00**	**0.00**	**0.01**	**0.11**	**0.13**	0.65	0.63	0.94	0.04	0.07	0.70	0.64	0.94	0.04	0.07	0.87	0.56	0.90	0.05	0.08
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.09**	**0.12**	0.41	0.68	0.98	0.02	0.05	0.79	0.88	1.00	0.02	0.05	0.17	0.27	0.71	0.04	0.07
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.07**	**0.29**	0.44	0.71	1.00	0.02	0.06	0.42	0.69	1.00	0.02	0.06	0.46	0.72	1.00	0.02	0.05
E.3	VaR	0.15	0.25	**0.02**	0.06	0.09	**0.04**	0.11	**0.01**	0.06	0.09	**0.04**	0.11	**0.01**	0.06	0.09	**0.01**	**0.02**	**0.00**	**0.06**	**0.09**
	CoVaR (a)	**0.02**	0.06	0.06	**0.09**	**0.11**	0.33	0.48	0.94	0.04	0.07	0.54	0.59	0.96	0.04	0.07	0.29	0.44	0.94	0.03	0.06
	CoVaR (b)	**0.00**	**0.02**	**0.00**	**0.06**	**0.09**	0.67	0.84	0.99	0.02	0.06	0.67	0.84	0.99	0.02	0.06	0.98	0.88	1.00	0.02	0.06
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.04**	**0.08**	0.49	0.75	1.00	0.02	0.06	0.49	0.75	1.00	0.02	0.06	0.52	0.78	1.00	0.01	0.06
B.1	VaR	0.55	0.25	**0.03**	0.05	0.10	0.61	0.39	**0.04**	0.05	0.10	0.66	0.38	**0.04**	0.05	0.10	0.26	0.42	0.12	**0.05**	**0.10**
	CoVaR (a)	**0.00**	**0.00**	**0.00**	**0.13**	**0.16**	0.68	0.79	0.84	0.06	0.09	0.67	0.79	0.84	0.06	0.09	0.13	0.22	0.16	0.08	0.11
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.10**	**0.14**	0.78	0.83	1.00	0.03	0.07	0.77	0.83	1.00	0.03	0.07	0.53	0.67	0.97	0.03	0.07
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.07**	**0.33**	0.39	0.66	1.00	0.02	0.07	0.39	0.65	1.00	0.02	0.06	0.43	0.70	1.00	0.02	0.06
H.1	VaR	0.78	0.23	0.68	0.05	0.09	0.56	0.27	0.66	0.05	0.09	0.36	0.28	0.58	0.05	0.09	0.66	0.14	0.33	**0.05**	**0.09**
	CoVaR (a)	**0.00**	**0.00**	**0.00**	**0.13**	**0.15**	0.43	0.58	0.96	0.04	0.06	0.47	0.61	0.96	0.04	0.06	0.67	0.50	0.91	0.06	0.08
	CoVaR (b)	**0.00**	**0.00**	**0.00**	**0.10**	**0.15**	0.25	0.51	0.99	0.01	0.05	0.61	0.83	1.00	0.02	0.05	0.24	0.38	0.80	0.04	0.07
	CoVaR (c)	**0.00**	**0.00**	**0.00**	**0.08**	**0.57**	0.77	0.94	1.00	0.01	0.06	0.75	0.93	1.00	0.01	0.05	0.39	0.65	1.00	0.02	0.06

Table 2
P-values of the LR test of unconditional coverage (uc), coverage and independence (cc) and dynamic quantile tests (dq) for both VaR and CoVaR for all analyzed models and banks. The values of the magnitude (LM) and of the asymmetric loss (LA) functions are also reported. We consider $\alpha = 0.025$ and (d) $\beta = 0.05$, (e) $\beta = 0.025$. The *p*-values lower than 0.05 and the maximum value by row of the loss functions are in bold.

		DCC					MGH					MNTS					Copula				
		uc	cc	dq	LM	LA	uc	cc	dq	LM	LA	uc	cc	dq	LM	LA	uc	cc	dq	LM	LA
D.1	VaR	**0.01**	**0.01**	**0.04**	**0.03**	**0.08**	0.27	0.49	0.73	0.03	0.08	0.27	0.49	0.73	0.03	0.08	0.27	0.49	0.51	0.03	0.08
	CoVaR (d)	**0.01**	**0.05**	**0.01**	**0.11**	**0.13**	0.91	0.79	0.99	0.05	0.08	0.91	0.79	0.99	0.05	0.08	0.32	0.38	0.74	0.07	0.10
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.10**	**0.14**	0.69	0.87	1.00	0.03	0.07	0.69	0.87	1.00	0.03	0.07	0.69	0.87	1.00	0.03	0.07
D.2	VaR	0.64	0.82	0.51	**0.03**	0.08	0.53	0.81	0.69	0.02	0.08	0.60	0.87	0.70	0.02	0.08	0.89	0.86	0.37	0.03	**0.08**
	CoVaR (d)	**0.00**	**0.00**	**0.00**	**0.15**	**0.18**	0.32	0.41	0.85	0.08	0.11	0.34	0.42	0.86	0.08	0.11	0.10	0.13	0.32	0.09	0.12
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.12**	**0.23**	0.49	0.73	1.00	0.04	0.08	0.50	0.74	1.00	0.04	0.07	0.25	0.45	0.98	0.05	0.08
I.1	VaR	0.08	0.19	0.35	**0.03**	**0.09**	0.28	0.16	0.16	0.02	0.08	0.28	0.16	0.16	0.02	0.08	0.68	0.38	0.51	0.02	0.08
	CoVaR (d)	**0.01**	**0.02**	**0.00**	**0.12**	**0.14**	0.90	0.84	0.99	0.05	0.08	0.90	0.84	1.00	0.05	0.08	0.35	0.43	0.75	0.07	0.10
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.09**	**0.12**	0.44	0.69	1.00	0.04	0.08	0.44	0.69	1.00	0.04	0.08	0.52	0.75	1.00	0.04	0.07
I.2	VaR	**0.01**	**0.01**	0.14	**0.03**	**0.08**	0.22	0.22	0.64	0.03	0.08	0.27	0.26	0.69	0.03	0.08	0.06	0.07	0.27	0.03	0.08
	CoVaR (d)	**0.00**	**0.00**	**0.01**	**0.14**	**0.16**	0.70	0.81	0.99	0.04	0.07	0.91	0.79	0.99	0.05	0.08	0.41	0.46	0.80	0.07	0.10
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.10**	**0.14**	0.71	0.87	1.00	0.03	0.07	0.69	0.87	1.00	0.03	0.07	0.78	0.91	1.00	0.03	0.07
F.1	VaR	**0.01**	**0.01**	**0.03**	**0.03**	**0.08**	0.85	0.19	0.52	0.02	0.07	0.77	0.17	0.47	0.02	0.07	0.64	0.26	0.59	0.03	0.07
	CoVaR (d)	**0.00**	**0.00**	**0.00**	**0.17**	**0.20**	0.54	0.74	1.00	0.04	0.07	0.56	0.75	1.00	0.04	0.07	0.25	0.28	0.34	0.08	0.11
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.11**	**0.17**	0.96	0.95	1.00	0.02	0.07	0.97	0.95	1.00	0.02	0.06	0.88	0.94	1.00	0.02	0.06
F.2	VaR	0.22	0.21	0.21	**0.03**	**0.08**	0.60	0.34	0.60	0.02	0.08	0.53	0.30	0.55	0.02	0.08	0.97	0.50	0.51	0.03	0.08
	CoVaR (d)	**0.00**	**0.00**	**0.00**	**0.17**	**0.20**	0.59	0.77	1.00	0.04	0.07	0.98	0.81	0.97	0.05	0.08	0.41	0.45	0.41	0.07	0.10
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.10**	**0.15**	1.00	0.95	1.00	0.03	0.07	0.99	0.95	1.00	0.03	0.07	0.93	0.95	1.00	0.02	0.06
F.3	VaR	0.37	0.46	0.45	**0.03**	0.08	0.89	0.52	0.27	0.03	0.08	0.94	0.48	0.44	0.02	0.08	0.43	0.69	0.58	0.03	**0.08**
	CoVaR (d)	**0.00**	**0.00**	**0.00**	**0.16**	**0.19**	0.74	0.69	0.99	0.06	0.09	0.70	0.67	0.99	0.06	0.09	0.28	0.31	0.60	0.08	0.10
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.12**	**0.20**	0.58	0.77	0.98	0.03	0.07	0.55	0.75	0.98	0.04	0.07	0.65	0.82	0.99	0.03	0.07
E.1	VaR	**0.00**	**0.00**	**0.00**	**0.04**	**0.08**	0.31	0.56	0.19	0.03	0.07	0.31	0.56	0.61	0.03	0.07	**0.02**	0.06	0.13	0.03	0.07
	CoVaR (d)	**0.00**	**0.01**	**0.02**	**0.12**	**0.14**	0.73	0.79	0.99	0.04	0.07	0.73	0.79	0.99	0.04	0.07	0.76	0.66	0.96	0.06	0.09
	CoVaR (e)	**0.00**	**0.01**	**0.01**	**0.07**	**0.10**	0.68	0.83	1.00	0.03	0.07	0.68	0.83	1.00	0.03	0.07	0.83	0.89	1.00	0.03	0.06
E.2	VaR	**0.00**	**0.01**	**0.00**	**0.03**	**0.07**	0.27	0.26	0.14	0.03	0.07	0.31	0.31	0.16	0.03	0.07	0.06	0.07	0.12	0.03	0.07
	CoVaR (d)	**0.00**	**0.00**	**0.00**	**0.15**	**0.17**	0.72	0.78	0.99	0.04	0.07	0.73	0.79	0.99	0.04	0.07	0.41	0.42	0.77	0.07	0.09
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.10**	**0.13**	0.69	0.84	1.00	0.03	0.07	0.68	0.83	1.00	0.03	0.06	0.78	0.88	1.00	0.03	0.06
E.3	VaR	**0.00**	**0.00**	**0.00**	**0.03**	**0.08**	**0.00**	**0.01**	**0.00**	0.03	0.07	**0.01**	**0.02**	**0.00**	0.03	0.07	0.05	0.09	**0.00**	0.03	0.07
	CoVaR (d)	0.11	0.26	**0.03**	**0.09**	**0.11**	0.45	0.68	0.98	0.04	0.07	0.48	0.69	0.99	0.04	0.07	0.59	0.77	0.99	0.04	0.07
	CoVaR (e)	0.11	0.21	0.43	**0.05**	0.08	0.92	0.94	1.00	0.03	0.07	0.89	0.94	1.00	0.03	0.06	0.40	0.63	1.00	0.04	**0.08**
B.1	VaR	**0.02**	**0.01**	**0.00**	**0.03**	**0.09**	0.19	0.09	**0.00**	0.03	0.09	0.15	0.09	**0.00**	0.03	0.09	0.31	0.43	**0.04**	0.03	0.08
	CoVaR (d)	**0.00**	**0.00**	**0.00**	**0.14**	**0.17**	0.68	0.77	0.99	0.04	0.08	0.67	0.77	0.99	0.04	0.07	0.89	0.75	0.94	0.05	0.08
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.09**	**0.13**	0.78	0.92	1.00	0.02	0.06	0.76	0.92	1.00	0.02	0.06	0.81	0.93	1.00	0.02	0.06
H.1	VaR	0.15	0.29	0.28	**0.03**	**0.08**	0.68	0.38	0.64	0.02	0.08	0.68	0.38	0.64	0.02	0.08	0.89	0.24	0.27	0.03	0.08
	CoVaR (d)	**0.00**	**0.00**	**0.00**	**0.15**	**0.18**	0.25	0.49	0.99	0.02	0.08	0.25	0.49	0.99	0.02	0.05	0.74	0.69	0.91	0.06	0.09
	CoVaR (e)	**0.00**	**0.00**	**0.00**	**0.10**	**0.15**	0.99	0.95	1.00	0.02	0.06	0.99	0.95	1.00	0.02	0.06	0.92	0.95	1.00	0.02	0.06

Table 3
P-values of the LR test of unconditional coverage (uc), coverage and independence (cc) and dynamic quantile tests (dq) for both VaR and CoVaR for all analyzed models and banks. The values of the magnitude (LM) and of the asymmetric loss (LA) functions are also reported. We consider $\alpha = 0.01$ and (f) $\beta = 0.05$. The p-values lower than 0.05 and the maximum value by row of the loss functions are in bold.

		DCC					MGH					MNTS					Copula				
		uc	cc	dq	LM	LA	uc	cc	dq	LM	LA	uc	cc	dq	LM	LA	uc	cc	dq	LM	LA
D.1	VaR	**0.00**	**0.00**	**0.00**	**0.02**	0.07	0.23	0.40	0.66	0.01	**0.07**	0.23	0.40	0.66	0.01	0.07	0.49	0.51	0.61	0.01	0.07
	CoVaR (f)	**0.01**	**0.02**	**0.01**	**0.14**	**0.17**	0.52	0.69	0.52	0.07	0.12	0.52	0.69	0.52	0.07	0.11	0.45	0.63	1.00	0.08	0.11
D.2	VaR	**0.00**	**0.00**	**0.00**	**0.02**	0.08	0.13	0.18	0.57	0.01	0.08	0.13	0.18	0.56	0.01	0.08	0.39	0.44	0.67	0.01	**0.08**
	CoVaR (f)	**0.00**	**0.00**	**0.00**	**0.18**	**0.22**	0.57	0.73	0.58	0.07	0.10	0.57	0.73	0.58	0.07	0.10	0.18	0.29	**0.01**	0.10	0.14
I.1	VaR	**0.00**	**0.00**	**0.00**	**0.02**	0.08	0.11	0.23	0.43	0.01	0.08	0.16	0.30	0.44	0.01	**0.08**	0.49	0.61	0.58	0.01	0.08
	CoVaR (f)	**0.01**	**0.01**	**0.00**	**0.15**	**0.18**	0.53	0.68	0.98	0.08	0.12	0.56	0.71	0.98	0.08	0.11	0.69	0.80	0.99	0.07	0.10
I.2	VaR	**0.00**	**0.00**	**0.00**	**0.02**	0.08	0.30	0.37	**0.01**	0.01	0.08	0.30	0.37	**0.01**	0.01	0.08	0.07	0.10	**0.00**	0.01	**0.08**
	CoVaR (f)	**0.00**	**0.00**	**0.00**	**0.17**	**0.21**	1.00	0.90	0.99	0.05	0.08	1.00	0.90	0.99	0.05	0.08	0.62	0.77	0.53	0.07	0.10
F.1	VaR	**0.00**	**0.00**	**0.00**	**0.02**	**0.07**	0.74	0.11	**0.00**	0.01	0.07	0.74	0.11	**0.00**	0.01	0.07	0.60	0.16	**0.01**	0.01	0.07
	CoVaR (f)	**0.00**	**0.01**	**0.00**	**0.16**	**0.19**	0.75	0.83	0.97	0.06	0.10	0.75	0.83	0.97	0.06	0.10	0.91	0.88	1.00	0.05	0.09
F.2	VaR	**0.01**	**0.02**	**0.02**	**0.01**	0.08	0.39	0.54	0.85	0.01	0.08	0.49	0.61	0.84	0.01	**0.08**	0.74	0.70	0.82	0.01	0.08
	CoVaR (f)	**0.00**	**0.01**	**0.00**	**0.16**	**0.20**	0.69	0.89	1.00	0.03	0.08	0.66	0.87	1.00	0.03	0.07	0.60	0.84	1.00	0.03	0.07
F.3	VaR	**0.00**	**0.00**	**0.00**	**0.02**	**0.07**	0.85	0.66	0.08	0.01	0.07	0.85	0.66	0.08	0.01	0.07	0.60	0.63	0.68	0.01	0.07
	CoVaR (f)	**0.00**	**0.00**	**0.00**	**0.20**	**0.26**	0.38	0.56	0.45	0.09	0.12	0.38	0.56	0.45	0.09	0.12	0.42	0.61	0.90	0.08	0.11
E.1	VaR	**0.00**	**0.00**	**0.00**	**0.02**	**0.07**	0.98	0.71	0.86	0.01	0.06	0.85	0.68	0.84	0.01	0.06	0.18	0.24	0.56	0.01	0.06
	CoVaR (f)	**0.02**	**0.02**	**0.00**	**0.13**	**0.15**	0.35	0.53	0.99	0.09	0.12	0.38	0.56	0.99	0.09	0.12	0.55	0.66	0.53	0.07	0.10
E.2	VaR	**0.00**	**0.00**	**0.00**	**0.02**	**0.07**	0.39	0.54	**0.04**	0.01	0.06	0.39	0.54	**0.04**	0.01	0.06	0.49	0.51	0.15	0.01	0.07
	CoVaR (f)	**0.00**	**0.00**	**0.00**	**0.17**	**0.21**	0.25	0.40	0.51	0.10	0.14	0.25	0.40	0.52	0.10	0.14	0.45	0.63	0.53	0.08	0.11
E.3	VaR	**0.00**	**0.00**	**0.00**	**0.02**	**0.07**	**0.00**	**0.01**	**0.00**	0.02	0.07	**0.00**	**0.01**	**0.00**	0.02	0.07	0.72	0.64	**0.04**	0.01	0.07
	CoVaR (f)	0.23	0.30	**0.00**	**0.08**	0.11	0.65	0.87	0.99	0.04	0.08	0.65	0.87	0.99	0.04	0.07	0.40	0.58	0.10	0.08	**0.12**
B.1	VaR	**0.00**	**0.00**	**0.00**	**0.02**	0.08	0.23	0.40	0.19	0.01	0.08	0.23	0.40	0.19	0.01	**0.08**	0.98	0.65	0.71	0.01	0.08
	CoVaR (f)	0.08	0.20	0.05	**0.11**	**0.13**	0.97	0.90	0.98	0.05	0.09	0.97	0.90	0.98	0.05	0.09	0.55	0.81	1.00	0.03	0.07
H.1	VaR	**0.00**	**0.00**	**0.00**	**0.02**	0.07	0.49	0.43	0.49	0.01	**0.08**	0.49	0.43	0.49	0.01	0.08	0.72	0.66	0.68	0.01	0.07
	CoVaR (f)	**0.01**	**0.01**	**0.00**	**0.15**	**0.17**	0.69	0.80	0.99	0.07	0.10	0.69	0.80	1.00	0.07	0.10	0.88	0.88	1.00	0.06	0.09

the GSIBs as defined in the (Financial Stability Board, 2011) policy document and in its yearly updates. The FSB approach relies on firm-specific information on size, interconnectedness, substitutability, complexity, and cross-jurisdictional activity and it considers annual accounting and other data provided to regulators by financial institutions (see also the Basel Committee on Banking Supervision, 2011 and 2013). Even if we have a small sample, we try to empirically assess the relation between these categories and the ΔCoVaR.

More precisely, the GSIB framework analyzes bank activities over 12 indicators. Each bank indicator is compared with the aggregate indicators of all banks in the specified sample (see Basel Committee on Banking Supervision, 2014). These indicators are grouped into five categories (i.e. size, interconnectedness, substitutability, complexity, and cross-jurisdictional activity). To calculate the score needed to determine the additional requirement, for each given indicator, the bank reported value for that indicator is divided by the corresponding sample total, and the resulting value is then expressed in basis points (bps). The final score is obtained as the weighted average of the 12 indicators or as the simple average of the five category scores (the average is rounded to the nearest whole basis point). Both specific bank and aggregated indicators are available on the FSB website.[2] The score represents bank activities as a percentage of the sample total and is used to determine the bank additional requirement. A higher score results in a higher requirement. On the basis of the indicators obtained from the FSB website we compute the score for a subsample of listed European banks for which these indicators are available. These scores are not equal to the official ones, because here we are considering only a subsample of banks. Here the denominators (i.e. the sample totals) needed to compute the scores are defined by considering only a subsample of listed European banks.

As observed in Section 5, the variability of the ΔCoVaR across institutions is low in absolute value, and it decreases further when annual averages are computed. However, even if one looks at the annual maximum values, the variability across institutions is still low in absolute value. Additionally, it should be noted that while, if one looks at daily estimates, the non-normal ΔCoVaR is able to capture tail events, this seems not to be necessarily the case if one looks at annual averages. For these reasons, in this section we define a score on the basis of the ΔCoVaR* computed in Section 5.

By following the same approach implemented for the evaluation of the GSIBs indicators, for each model we compute, as a first attempt, the annual average of the ΔCoVaR and rescale it in order to obtain the corresponding score expressed in basis points. We consider α and β equal to 0.05. However, there are no remarkable differences in the ΔCoVaR scores, both across institutions and across models, if one applies the GSIB methodology to the ΔCoVaR values. For this reason, we compute the score on the basis of the annual average of the ΔCoVaR difference with respect to the minimum computed across all institutions (i.e. the annual average of the ΔCoVaR* defined in Section 5). That is, on each given estimation day, we evaluate this minimum ΔCoVaR of the sample and then we compute the difference between each bank estimate and the minimum. Thus, we evaluate the annual average of this difference. This indicator is the annual average of daily ΔCoVaR* shown in Fig. 5, rescaled so that the sum across all banks is equal to 10,000 basis points (i.e. 100%).

Both indicators and scores are reported in Table 5, where the sum of each column is by construction equal to 10,000 basis points (i.e. 100%). The banks are ordered from the most to the least systemic according to the GSIBs score. By looking at the values and at the colours of these scores, it appears evident that the variability across banks of the score defined on the basis of the ΔCoVaR* is high. As already observed, this variability comes from the results shown in Fig. 5, where the time series of the

2 See https://www.bis.org/bcbs/gsib/.

Table 4
Minimum distance weights and estimated ARPE by considering the GSIB scores over the years from 2013 to 2019. The weights are obtained by minimizing Eq. (7.1).

Model	Size	Inte	Subs	Comp	Cja	ARPE
Normal	0.7512	0.0829	0.0000	0.0000	0.1659	0.4117
DCC	0.6187	0.1534	0.0000	0.0000	0.2279	0.4071
MGH	0.8365	0.0737	0.0000	0.0000	0.0898	0.4188
MNTS	0.7970	0.0946	0.0000	0.0000	0.1084	0.4185
Copula	0.8900	0.0947	0.0000	0.0000	0.0153	0.3708

differences with respect to the minima computed across all institutions are reported.

In Table 5 we report the average indicators and scores estimated on the observations from 2013 to 2019. For each bank and each year, we computed the average indicators on daily ΔCoVaR* estimates, then we evaluated the average of these annual indicators from 2013 to 2019. There is a large difference between the GSIBs score and the score based on the ΔCoVaR*, particularly for some banks (see also Jokivuolle et al., 2018).

Thus we define the average relative percentage error (ARPE) with respect to the GSIBs score as

$$ARPE = \frac{1}{nobs}\sum_{j,t}\frac{|score_{j,t}^{GSIBs} - score_{j,t}^{\Delta CoVaR^*}|}{score_{j,t}^{GSIBs}} \tag{7.1}$$

where $nobs$ is the number of observations and $score_{j,t}^{GSIBs}$ and $score_{j,t}^{\Delta CoVaR^*}$ represent the GSIBs score and the score computed on the basis of the ΔCoVaR* estimates. The ARPE from 2013 to 2019 across the ten banks in the sample is around 60%, indicating that the GSIBs score contains different information with respect to the ΔCoVaR* indicator. As observed above, the GSIBs score is computed as

$$score_{j,t}^{\mathrm{GSIBs}} = \sum_i \omega_i Category_i$$

where i goes from 1 to 5 (size, interconnectedness, substitutability, complexity and cross-jurisdictional activity) and the weights ω_i are all equal to 0.2.

Since the ARPE value is high, we try to modify the weights ω_i of the five categories in order to minimize the percentage difference in Eq. (7.1), and we refer to them as *minimum distance weights*. This allows one to also understand which of the categories defined above has a stronger relation with the ΔCoVaR* indicator and to define an indicator that takes into account not only the firm-specific information coming from the five categories but also the information coming from financial markets. By considering these new weights (see Table 4), the ARPE with respect to the GSIBs score decreases to around 40%. These new weights are computed by considering the annual values of indicators and scores (not the average ones) for a total of 94 observations (seven for each bank). As shown in Table 4, for all models, substitutability (Subs) and complexity (Comp) have minimum distance weights equal to zero. The size (Size) weight ranges from 61.8% for the DCC model to 89% for the copula model, the interconnectedness (Inte) weight ranges from 7.4% for the MGH model to 15.3% for the DCC model, and the cross-jurisdictional activity (Cja) weight ranges from 1.5% for the copula model to 22.8% for the DCC model. It seems that the ΔCoVaR is correlated only with size, interconnectedness and cross-jurisdictional activity. Conversely, it is not correlated with the other two categories (i.e. substitutability and complexity). The variation of the indicators over time and across all banks is explained by only three of the five categories. The market-based indicator seems to capture neither substitutability nor complexity. As expected, size is a major driver, mainly because larger banks have a greater impact on the system (i.e. the Euro Stoxx 50 index).

Once we have these new weights, we can compute the corresponding score, which is a sort of adjusted GSIBs score, as reported in Table 5. This adjusted score can be view as a GSIBs score adjusted for the information coming from the stock market. It is a market-based systemic risk indicator as close as possible to the GSIBs score. In Table 5 we report the adjusted score obtained under the MNTS distributional assumption. Without replacing a subjective assessment of financial stability risks or questioning the GSIBs identification and buffer calibration process, our findings suggest including this market-based information in a toolbox to investigate the system risk contribution of a bank.

8. Conclusions

In this paper we compute the CoVaR and the ΔCoVaR under different distributional assumptions and different CoVaR definitions (i.e. CoVaR$^{=}$ and CoVaR$^{\leq}$). In order to backtest the CoVaR$^{\leq}$, we conduct an empirical test over more than 18 years of daily stock log returns. We estimate three different models allowing for volatility clustering, heavy tails, and non-linear dependence. These multivariate non-normal models outperform the normal one in terms of log returns fitting and CoVaR backtesting for all tail levels analyzed in this study. The backtesting exercise shows that the performance of the MGH (MNTS) model is slightly better compared to the copula model; however, this latter approach is very promising from a computational point of view. Our empirical study confirms that, in measuring CoVaR, it is important to capture the time-varying dynamic of the volatility. Additionally, a correct assessment of the heaviness of the tails and of the dependence structure is needed in the estimation of this systemic risk measure. Finally, we compare the score obtained through the ΔCoVaR measure with the GSIBs score defined by the FSB and introduce a score adjusted for the information coming from the stock markets.

Table 5
Average indicators and scores over the years from 2013 to 2019. For each bank and each year, we computed the average indicators on daily ΔCoVaR* estimates, then we evaluated the average of these annual indicators from 2013 to 2019. The adjusted score is computed on the basis of the weights minimizing the average relative percentage error between the GSIBs and the ΔCoVaR* score in the MNTS case. For the ΔCoVaR*, we consider α and β equal to 0.05.

ticker	ΔCoVaR* indicators					GSIBs indicators and scores						
	Normal	DCC	MGH	MNTS	Copula	Size	Inte	Subs	Comp	Cja	Score	Ad.score
D.1	702	686	799	792	783	1221	1309	2845	3074	1348	1959	1244
F.1	1410	1386	1367	1371	1335	1664	1435	2177	2171	1872	1864	1667
F.2	1197	1195	1201	1205	1153	1066	1166	1344	1385	867	1165	1052
F.3	817	838	847	848	820	1351	1199	1157	893	687	1057	1258
E.2	1450	1450	1402	1404	1332	1246	1084	489	430	1730	996	1288
I.1	809	834	791	794	752	849	1061	465	524	940	768	880
H.1	1216	1208	1243	1244	1235	943	870	420	359	1224	763	969
D.2	88	95	52	50	414	465	645	485	554	348	499	468
E.1	1322	1312	1301	1301	1249	627	496	329	264	701	483	623
I.2	990	996	996	991	928	569	736	288	346	283	444	551
total	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	

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