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Forecasting South Korea's presidential election via multiparty dynamic Bayesian modeling

Seungwoo Kang, Hee-Seok Oh *

Department of Statistics, Seoul National University, Seoul 08826, South Korea

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ABSTRACT

Forecasting a presidential election's outcome is a long-standing topic in statistics and political science. However, a lack of historical data and a complex multiparty political system make it challenging to apply models developed so far to South Korea's presidential election. In addition, no suitable model has been proposed to address these issues, and there are no practical means by which to forecast presidential elections in South Korea. Here, we propose a flexible Bayesian framework for forecasting election outcomes at the provincial level by incorporating abundant pre-election polls into historical data. Hilbert spaces are employed to induce a multiparty forecast. Our framework provides numerous findings worth examining, such as long- and short-term opinion trends, the effect of fundamental conditions on vote share, and systematic bias in pre-election polls. The framework is applied to the 2022 South Korean presidential election, demonstrating that our framework is promising.

1. Introduction

The 20th presidential election of the Republic of Korea (South Korea), which ended on March 9th, 2022, was the tightest presidential election ever. The popular vote difference between the two major candidates, Suk-yeol Yoon and Jae-myung Lee, was 0.73%. Due to the narrow margin of popularity, a massive number of forecasts were issued during the campaign period—from politicians, political pundits, and pollsters—satisfying the nation's intense desire to know in advance who would win the election. During the campaign season of the 20th presidential election, an average of 3.5 national-level pre-election polls were released daily.

From the considerable supply of forecasts for the 2022 presidential election and all major elections worldwide, we can surmise that making prognoses on election outcomes *ex ante* fulfills people's curiosity. Election forecasts, however, are more than just entertaining guesses of who will win. For candidates and parties, they aid in diagnosing their pledges and strategies, they facilitate the design of campaign tactics, and they provide incentives to strengthen or weaken messages toward voters. For voters, forecasts assist them in making better decisions. For scholars, forecasts are tools for better understanding voting behavior and preferences, and they serve as guides for interpreting election results. As Lewis-Beck (2005) stated, election forecasts are 'intrinsically interesting in a healthy democracy.'

Efforts have been made to make these forecasts scientific, building theories upon a solid statistical footing. These scientific election-predicting methods can be classified into structural, aggregate, and synthetic models using the framework in Lewis-Beck and Stegmaier (2014). Structural models refer to forecasts based on classical regression techniques using historical patterns of election outcomes. Explanatory variables in these models are indicators known as *fundamentals*, which include macroeconomic indicators (e.g., election-year GDP, GNP, and unemployment rate) and political measures (e.g., president popularity and the number of consecutive terms).

* Corresponding author.
E-mail addresses: kangsw0401@snu.ac.kr (S. Kang), heeseok@stats.snu.ac.kr (H.-S. Oh).

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The response variable is, of course, the vote share of one candidate. These models are based on the firm election theory that voters are retrospective, rewarding or punishing incumbents by their past performance (Tufte, 1975). Structural models embrace early seminal works on election forecasting in political science (Abramowitz, 1988; Fair, 1978; Fiorina, 1978; Hibbs, 2000; Lewis-Beck & Rice, 1984). Aggregate models, as their name implies, aggregate several vote intention polls to track the underlying public opinion. Since polls are snapshots of the opinion at the polling time (which we refer to as nowcasts, not forecasts), the aggregate models estimate opinion trends across time until election day. A prominent example is RealClearPolitics, a political news website that combines polls to provide accurate estimates of public opinion. In the literature, Jackman (2005) and Pickup and Johnston (2007) are examples of aggregate models. For an overview of aggregate models, see, e.g., Blumenthal (2014), Pasek (2015). Finally, synthetic models integrate structural and aggregate methods, combining the power of solid election theory with the information provided by many polls. For instance, Brown and Chappell (1999), Lewis-Beck and Dassonneville (2015), Linzer (2013), Lock and Gelman (2010) are synthetic approaches to election forecasting. More recently, the website FiveThirtyEight (Silver, 2020) and the newspaper *The Economist* (Heidemanns et al., 2020; Morris, 2020) presented synthetic forecasting models of the 2020 U.S. presidential election.

Most presidential election models are examined outside of South Korea, and research on South Korean presidential election forecasting models is insufficient. There are three reasons for the lack of election prediction research in South Korea. Firstly, as Song (2019) pointed out, South Korea has a short history of democratic elections and hence limited historical data for model fitting and testing. Only eight presidential elections have been held in South Korea since its democratization (from the 13th (1987) to the 20th (2022) presidential elections) under the direct electoral system. It is challenging to construct structural models in this situation, where the degree of freedom is rapidly depleted due to many explanatory variables.

Secondly, South Korea's political system differs from that of the United States. For instance, the presidency of South Korea is under a single-term system, meaning that the running incumbent, considered as one of the most critical factors in forecasting the U.S. president (Abramowitz, 2021), is not a valid factor in South Korea. Similarly, variables deemed relevant in other nations' presidential election forecasting may not be significant in South Korea, so forecasters in South Korea should start from scratch. More importantly, because of the complexity and multiparty nature of South Korean presidential elections, models generated for two-party systems can only partially explain South Korean elections.

Thirdly, making forecasts is risky. As Gelman et al. (2020) wrote, 'one bad-luck election could make us look like fools.' Every election brings something new. Models that fit well with historical data may perform poorly in future elections. In South Korea where there are no players (forecasters) at the scene, there is a greater risk of announcing a fresh prognosis.

Given South Korea's harsh characteristics, this study aimed to provide a forecasting framework that meets all of these concerns. We propose, in particular, that South Korea's forecasting framework should satisfy the following three criteria:

(a) The framework should be capable of utilizing as much data as feasible. Economic and political indicators and an abundance of pre-election poll data released daily during the campaign should be used. This is to make accurate projections and better understand the presidential election by incorporating data. We should use a synthetic approach to predict elections.

(b) The framework should accommodate the unique aspects of the South Korean political system. For instance, the multiparty nature of the South Korean presidential election bears the possibility of the election being a tight race between more than two candidates. Therefore, it is desirable to use a general framework capable of handling such a situation. A multiparty model will help voters and candidates make decisions even if the race is two-party focused.

(c) In the words of Lewis-Beck and Tien (1996), the framework should be 'more than curve fitting' and seek to develop a 'good theory.' The framework should be able to make practical forecasts that are interpretable. However, just predicting the winner is insufficient; a framework should be established to provide interesting findings for a deeper comprehension of the election.

We now review the approaches for forecasting presidential elections in South Korea and argue that a new method is required. We classify the approaches using structural, aggregate, and synthetic models to facilitate the explanation.

One structural approach in South Korea is that of Song (2019), who built an *ex post* 2017 presidential election prediction model. However, their approach ignores South Korea's multiparty system, focusing solely on the vote share of the two major parties. This two-party approach is unavoidable in structural models, as Walther (2015) argued. As a result, this strategy fails to meet criteria (a) and (b).

Next, aggregate models include Park (2013) for South Korea's 2012 presidential election based on (Jackman, 2005). Following this work, news broadcasters MBC (Yeoron M; http://poll-mbc.co.kr) and SBS (Poliscore; https://poliscore.kr/) provided their aggregate models for the 2022 presidential election. These models, however, are not forecasts in the strictest sense. South Korea has a seven-day blackout period before election day, during which polls cannot be released. This means that the nowcast of public opinion ends a week before the election; the model cannot predict what will happen in the last week. Thus, no predictions can be made on election day. As a result, no aggregate models in South Korea meet criterion (c), which states that the framework should be able to provide forecasts.

Finally, to the best of our knowledge, there are no synthetic presidential election forecasting models in South Korea.

To that end, we adopt the dynamic Bayesian forecasting approach of Linzer (2013), designed for U.S. election forecasting, to sequentially update the projection of South Korea's 2022 presidential election from historical data using daily pre-election polls. We chose the approach of Linzer (2013) among the many synthetic models in the literature for two reasons.

(1) The reverse random walk idea used in Linzer (2013) links opinion trends (nowcasts) with forecasts on election day. This enables predictions even during South Korea's seven-day blackout period, a unique feature of South Korean elections that hinders forecasting for the aforementioned aggregate models. In addition, a forecast can be made on any date before the election in real time, i.e., dynamically, with the accuracy of the prognosis improving as election day approaches, which is impossible in synthetic models developed previously (Linzer, 2013).
(2) The model proposed by Linzer (2013) is the foundation for many state-of-the-art models. For instance, *The Economist* model for forecasting the 2020 U.S. presidential election is based on this study (Heidemanns et al., 2020). Also, extensions of this model to other countries with different political systems have been proposed by Walther (2015) and Stoetzer et al. (2019).

Although we rely on the work of Linzer (2013), we make some essential changes to suit the South Korean scenario. Our main contributions are summarized below. Firstly, the scarcity of historical data is considered in our model. In this high-dimensional scenario, where the number of variables may exceed the number of data points, we estimate coefficients using the Bayesian regression approach proposed by Lauderdale and Linzer (2015) with some modifications. This structural model's output generates a prior for a dynamic Bayesian model.

Secondly, a multiparty extension is proposed when updating the structural model with pre-election polls, suggesting a general methodology. We develop a multiparty dynamic Bayesian framework that predicts the vote share of the four major parties at the provincial level in real time in South Korea's 2022 presidential election. The most important contribution is the preservation of the interpretability of all model variables by using a Hilbert space structure of a simplex, commonly known as Aitchison geometry. We also added the effects of survey question-wording and survey method to our model while maintaining their interpretability.

Lastly, the proposed framework is the first attempt to meet all criteria (a)–(c) for forecasting election results in South Korea. Not only are forecasts of the final outcome made, but abundant findings are also produced, serving as a foundation for a 'good theory.' Specifically, the effect size of the fundamentals in South Korea is estimated, and long- and short-term trends in public opinion are tracked at the provincial and national levels. We believe that our pipeline contributes significantly to the literature on synthetic approaches to election forecasting and opens a wide range of possibilities for developing a sound election theory in South Korea.

The rest of the paper is organized as follows. We describe South Korea's political system and pre-election polling industry in Section 2. The proposed framework is described in Section 3. Section 4 presents the estimation results obtained by applying the proposed framework to South Korea's 2022 presidential election. This section shows and interprets forecasts as well as various findings from that election. Furthermore, a retrospective evaluation of our forecasts is provided. Finally, Section 5 outlines potential future topics and precautions, and offers conclusions.

All codes and data used for analysis are provided at https://github.com/seungwoo-stat/20th-PE-forecast.

2. Background

2.1. Presidential elections in South Korea

South Korea's single-term, five-year president is elected directly by a national popular vote. The candidate with the most valid votes at the national level is elected president. This differs from the U.S., which employs an indirect election system via the Electoral College group. Following the primaries for each political party, the election is run by one candidate from each political party or by independent candidates. In the 2022 presidential election, 14 candidates from 14 parties registered. Among the 14 candidates, four from the four major parties garnered significant attention: Jae-myung Lee of the liberal Democratic Party (the incumbent party in the 20th presidential election), Suk-yeol Yoon of the conservative People Power Party, Sang-jung Sim of the social-democratic Justice Party, and Cheol-soo Ahn of the conservative-liberal People Party.

Although the four candidates were the focus of attention, it was clear that the winner would be either Lee or Yoon. The Democratic Party and the People Power Party have been South Korea's two biggest political parties since South Korea's democracy (Choe, 2022). Hence, by focusing solely on the Democratic Party's two-major-party vote share (=vote share of the Democratic Party/(vote share of the Democratic Party + vote share of the People Power Party)), it will be possible to clarify the history of South Korean presidential elections.

The two-party vote share of the Democratic Party since the 14th presidential election is shown in Fig. 1 at the national and provincial levels. South Korea is divided into 17 first-tier administrative divisions: nine provinces, six metropolitan cities, one special city, and one special self-governing city, which are referred to as *provinces* for convenience. For historical reasons, we only use 15 of them in our analysis, merging two provinces with other provinces. Details on South Korea's provinces can be found in the supplementary material.

The figure shows that there are provinces with solid partisan leanings. For example, Gwangju, Jeollanam-do, and Jeollabuk-do offer strong Democratic Party support, with the Democratic Party's two-party vote share regularly reaching or exceeding 90% throughout the elections. Similarly, other provinces appear to have consistent support for the Democratic Party in all elections. Although

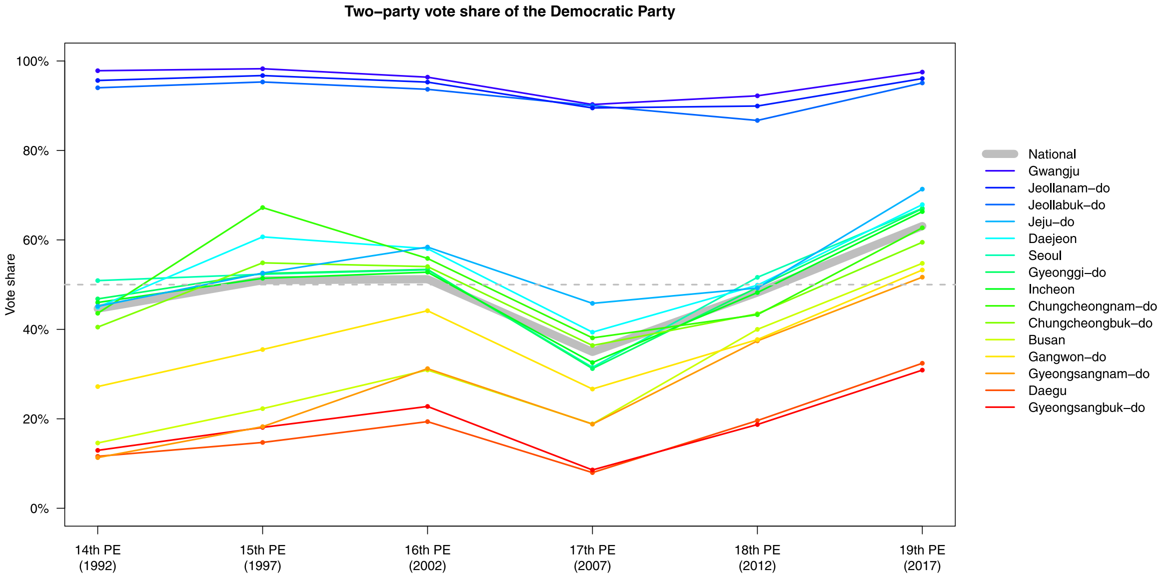

Fig. 1. Two-party vote share of the Democratic Party from the 14th presidential election to the 19th presidential election. National- and provincial-level results are shown with 16 different colored lines. Colors are assigned according to their ranks in the 19th presidential election.

the level of support varies over time, vote shares are rising and falling in tandem with an apparent *uniform swing* (Jackman, 2014) across provinces. These highly correlated national shifts are often called *parallel publics* (Gelman et al., 2020). For instance, in the 19th election, the vote share of the Democratic Party rose in all provinces compared to the 18th election, giving a landslide win for the Democratic Party.

It will be possible to identify which provinces' voting behavior is based on a high amount of regionalism and how long it has prevailed in South Korea if these uniform fluctuations can be eliminated. Hence, establishing a provincial-level forecasting model instead of directly projecting the national outcome will reveal the historical trends of each province and help forecast elections. These studies will also serve as a framework for studying elections at the provincial level, such as gubernatorial, local councilor, and general elections. Given the available data, it is, therefore, more valuable to analyze the South Korean presidential election at the provincial level.

Not only is analysis at this level interesting *per se*, but it is also necessary for an effective study. The data size increases by 15 times compared to only the national data, enabling a more precise fit with a higher degree of freedom. Nonetheless, separating the data into provinces is the only way to allow for a retrospective assessment of the forecast and the model. For instance, one can try working with a smaller unit by separating the data by age and gender, two factors known to affect the outcome of the South Korean election (Shin, 2001). Yet, the outcome of this analysis cannot be verified because the secret voting system conceals the vote totals for each of these factors. However, the source of the ballot is known, allowing us to confirm the provincial-level forecast. As a result, we conduct a provincial-level analysis, and the national-level projection can be provided by weighting and merging each provincial-level outcome.

2.2. Pre-election polls in South Korea

Pre-election polls or trial-heat polls capture the opinion of the public. That is, polls are not predictions of the election outcome but rather snapshots of voters' current preferences at the time of the survey (nowcasts), which fluctuate throughout the campaign depending on major events such as TV debates, scandals, and so on. Polls are also susceptible to sampling and non-sampling errors and can be inaccurate and misleading for forecasting purposes (Gelman & King, 1993; Jackman, 2005; Pasek, 2015; Smith, 1996). Nonetheless, because pre-election polls provide a direct estimate of voters' attitudes toward each candidate, the numerous poll results are of great interest to the general public and political scientists.

The polling industry in South Korea is massive. As of December 31st, 2021, 84 pollsters were registered in the National Election Survey Deliberation Commission (NESDC), a South Korean government agency that collects and examines all the election polls. The NESDC is notified of all polls encountered in news reports with details before their report; for instance, the purpose of the survey, sample size, survey method and coverage, and questionnaire of the poll are all uploaded to the NESDC website. Anyone can view the detailed documents of each pre-election poll on the NESDC website.

We examined two types of polls for our analysis from the NESDC: national- and provincial-level polls, in which the survey's population is set to all South Korean voters or to the voters in a specific province. Lower-level polls, such as sampling only voters from a particular second-tier or lower-level division, were not considered.

To our knowledge, all national-level presidential pre-election polls use a quota sampling method. That is, national opinion is sampled by assigning samples proportional to the voter population by age, gender, and residency (province clusters) to improve the survey accuracy. For example, if women aged 20 to 30 years living in Seoul constitute 5% of the voting population, samples are drawn and weighted so that this category constitutes 5% of the samples. Instead of using 15 provinces to represent residency, seven province clusters were used in general, obtained by merging provinces with similar partisan leanings. In this paper, these seven province clusters are denoted by SEO, SDG, CCG, HNG, DGG, BUG, and GJJ. Details on the province cluster formulation are in the supplementary material. For the survey's objectivity, the poll results for each of these quotas are also posted on the NESDC website. Thus, by looking at the poll results of each province cluster quota, we can obtain the opinions of the seven province clusters even if a national poll had been performed.

To forecast South Korea's 20th presidential election, we collected all provincial- and national-level poll results reported to the NESDC within 100 days of the election (from November 29th, 2021, to March 2nd, 2022). We chose 100 days before the election because the primaries for the four major parties ended in early November 2021. A margin of a few weeks was set to allow people to recognize the candidates of each party. Although the election was held on March 9th, 2022, all poll reports ended on March 2nd, 2022, because of South Korea's seven-day blackout before the election. We specifically collected pre-election polls that asked for vote intention over the phone. In summary, we gathered 307 national-level polls and 18 provincial-level polls. National polls have a sample size of at least 1000, while provincial polls have a sample size of at least 800. The total sample size was 387,108 from all of our 325 collected polls, which is approximately 0.9% of South Korea's voter population (18 years old or older).

The quality and design of the polls we collected varied greatly. While most polls were performed for two or three days, some were done for six days (Table 1). Except for the 30th and 31st of January 2022, the South Korean New Year holidays, polls were held every day in the previous 100 days until the 20th presidential election's blackout period.

Non-sampling errors in polls are known to occur due to a variety of factors depending on the survey's design, including the polling organization (referred to as *house effects*; Jackman, 2005), nonresponse (Walsh et al., 2009), question-wording (Pasek & Krosnick, 2010), polling method or survey mode (Pasek, 2015), and others. We concentrated on question-wording and polling method as examples of systematic errors. After inspecting the questionnaires of 325 polls, we classified wordings of vote intention questions into one of three categories (asked in Korean and translated by the authors): (1) Appropriate: 'Which candidate do you believe is the most *appropriate* for the presidency?' (2) Support: 'Which candidate do you *support*?' (3) Vote: 'Which candidate would you *vote* for?' These questions ask for a vote intention, but the implications are slightly different. Next, in our collection, the pollsters use two survey modes: an automated response system (ARS) by a machine, or a telephone interview conducted by a person. Table 2 counts the polls based on two sources of error: question-wording and polling method.

Table 1
Number of polls by their duration in days.

Days	1	2	3	4	5	6	Total
Count	31	195	73	14	2	10	325

Table 2
Number of polls by their question-wording and polling method. We categorized the poll based on the predominant method used when a single poll combined both ARS and interview methods.

Method	Question-wording			Total
	Appropriate	Support	Vote	
ARS	11	45	145	201
Interview	27	5	92	124
Total	38	50	237	325

We will see if these two factors cause a significant bias in South Korea's polling industry. Also, we will try to remove these biases by incorporating these effects into our proposed model for a more accurate forecast of the election outcome.

3. Proposed framework

3.1. Prior building: Fundamentals-based model

There is a large body of literature on predicting presidential election outcomes using political and economic factors, also referred to as *fundamental* conditions of the election. Most methods use regression-based models to identify the relationship between the fundamentals and elections. Long before the campaign season begins, the election outcome is predicted by fitting the statistical model to the historical data and plugging in the fundamental condition of the current election.

However, problems with this traditional method have been highlighted in the most recent decade (Lauderdale & Linzer, 2015; Silver, 2012). In summary, most fundamentals-based models in the literature lack accuracy and are overconfident. To use an adage, their model fit is 'too good to be true.' There are three main factors of the uncertainty overstatement argued by Lauderdale and Linzer (2015). First, for most fundamentals-based models in the United States, uncertainty in the model specification is not considered. It is unclear whether the selected covariates in the model, which is limited in number due to a small historical data set, are the most appropriate collection. Second, proper prediction intervals are not used. And third, the model is oversimplified. For example, national-level swings are ignored and the persistent, but the gradually changing partisan leanings of each state are not considered.

Lauderdale and Linzer (2015) used a Bayesian regression model to address these concerns. The model's hierarchical prior structure, in particular, regularizes regression coefficients, allowing it to contain more variables than the standard regression approach, without the danger of overfitting or cherry-picking predictors. Furthermore, posterior samples are used to compute proper credible intervals, and national-level swing terms and each state's partisan leaning terms are incorporated into the model with suitable stochastic processes.

It is worth noting that the 20th presidential election in South Korea occurred during the spread of the Omicron variant of COVID-19, putting political and economic situations in an extraordinary circumference. The impact of the global pandemic on election forecasting has been highlighted in several works, including the 2020 U.S. presidential election forecasting symposium (Dassonneville & Tien, 2021). For instance, Abramowitz (2021) abandoned economic variables in his model, while Lewis-Beck and Tien (2021) adjusted the outliers of their GNP variable. The coronavirus was observed to have an impact on the fundamental condition of the election. Thus, mitigating its effects on the election is critical. Fortunately, the model of Lauderdale and Linzer (2015), unaware of the global pandemic at the time of publication, can handle this issue because the coefficients are regularized, preventing the overestimation of a specific covariate effect and robustifying the forecast in the event of an outlying condition.

Hence, the model proposed by Lauderdale and Linzer (2015) addresses the issues raised by fundamentals-based regression models. As a result, while this approach widens the credible intervals of forecasts with less confident outcomes, we use it with a few modifications. We forecast 15 provincial-level vote outcomes, particularly the Democratic Party's two-major-party vote share in South Korea's 20th presidential election. The model's output will be a prior for a poll-based model that extends the two-party forecast to a multiparty forecast, as described in the following section. The Bayesian regression model for South Korea's 20th presidential election is summarized below.

We denote the two-party vote share of the Democratic Party at province p and election t as y_{pt} ($p = 1, \dots, 15$; $t = 14, \dots, 20$). Our primary goal is to estimate $y_{p,20}$ for each p, which is an unobserved value. We model the vote share with several additive terms as

$$y_{pt} = \alpha_{pt} + \sum_k \beta_k x_{kt} + \sum_l \gamma_l z_{lpt} + \delta_t + \epsilon_{pt},$$
$$(p = 1, \dots, 15;\ t = 15, \dots, 20). \tag{3.1}$$

The model divides the vote share analytically into two components: the long-term vote division expected from a specific province, and the short-term oscillation of a particular election. The former component is known as a *normal vote* or a *baseline vote* of a specific province (Converse, 1966), α_{pt}. The three terms on the right-hand side of (3.1) following α_{pt} express the latter component.

The normal vote is expressed using a dynamic linear model (West & Harrison, 1997). That is, the normal vote of one province remains stable over long periods but gradually shifts from its lagged vote share:

$$\alpha_{pt} \sim \mathcal{N}(\alpha_{p,t-1}, \sigma_\alpha^2), \quad \sigma_\alpha \sim \mathcal{N}_{1/2}(\sigma^2), \tag{3.2}$$

where $\mathcal{N}_{1/2}(\sigma^2)$ denotes the half-normal distribution (the distribution of $|X|$, where $X \sim \mathcal{N}(0, \sigma^2)$), indicating a parsimonious prior.

For normal vote interpretability, the effect is constrained so that α_{pt} expresses the party loyalty of province p in election t, assuming that election t is a tie between the Democratic Party and the People Power Party. Let w_{pt} be the proportion of valid votes in province p at election t among all valid votes in election t. That is, $\sum_p w_{pt} = 1$ for each t. Then, α_{pt} is constrained such that $\sum_p w_{pt}\alpha_{pt} = 0.5$ for all t. Thus, the initial value $\alpha_{p,14}$ in (3.2) can be given as

$$\alpha_{p,14} = y_{p,14} + 0.5 - \sum_{p'} w_{p',14} y_{p',14}. \tag{3.3}$$

Given these probability structures and the constraint, a normal vote in each province can be directly interpreted as long-term voting behavior after removing the election-specific forces.

Following, x_{kt} represents the kth national-level covariates in election t, and β_k is its coefficient. Similarly, z_{lpt} is the lth provincial-level covariate in province p for election t, and the coefficient to be estimated is γ_l. These variables incorporate fundamental conditions of the election into the model. To impose common hierarchical prior structures on these coefficients, covariates are standardized to zero mean and unit variance. Specifically, the coefficients are independently regularized with a hierarchical structure as

$$\beta_k \sim \text{Cauchy}(0, \sigma_\beta^2), \quad \sigma_\beta \sim \mathcal{N}_{1/2}(\sigma^2),$$
$$\gamma_l \sim \text{Cauchy}(0, \sigma_\gamma^2), \quad \sigma_\gamma \sim \mathcal{N}_{1/2}(\sigma^2),$$

to prevent overfitting with a large number of covariates. Due to a limited historical data set in South Korea, the posterior distributions of β_k and γ_l may not differ significantly from their priors when using a normal prior. It is implausible to find a regression coefficient posterior that indicates a significant effect. To alleviate the updating problem, a distribution with a thicker tail, the Cauchy distribution, is used. If more data are collected in the future, we may switch to the normal distribution. In addition, when the covariates are *performance variables*, i.e., variables deemed to influence the incumbent party, not the Democratic Party (e.g., GDP growth), covariates associated with election t are multiplied by -1 if the incumbent president in election t is the People Power Party to correct and reflect the effect of the covariates toward the incumbent party.

The next election-specific term in model (3.1) is δ_t, which denotes the national-level swing that is not explained by the covariates x_{kt}. Inclusion of this random effect in the model reflects the observation that vote shares across provinces are highly correlated (uniform swing; see Fig. 1) and prevents overconfident forecasts. We consider the following distribution $\delta_t \sim \mathcal{N}(0, \sigma_\delta^2)$, and $\sigma_\delta \sim \mathcal{N}_{1/2}(\sigma^2)$, independently.

Finally, ϵ_{pt} is an error term with a prior $\epsilon_{pt} \sim \mathcal{N}(0, \sigma_{\epsilon,t}^2)$, and $\sigma_{\epsilon,t} \sim \mathcal{N}_{1/2}(\sigma^2)$, independently distributed. The error term's dispersion varies in t, enabling election-specific variations.

Additional assumptions are required to sample posteriors of the variables in model (3.1). (i) Until the election is over, the proportion of valid votes in province p for the 20th election is unknown. So, we assume that it is equivalent to that of the 19th election, $w_{p,19} = w_{p,20}$. (ii)

We suppose that the dispersion of the error $\sigma_{\epsilon,20}$ for the 20th election equals $\sigma_{\epsilon,19}$ for the 19th election. (iii) We impose a uniform prior to the common hyperparameter σ. With these assumptions, we treat $y_{p,20}$ as a missing value and conduct Bayesian posterior sampling.

It is worth noting the limitations of the forecasting approach through the fundamentals-based model (3.1). Firstly, the additive structure implies that the estimated vote share can be greater than one or less than zero, which is a meaningless value. Secondly, the multiparty nature of the South Korean political system is ignored. Hence, the model only partially explains South Korean elections. Lastly, fundamentals-based forecasts that do not consider many poll data result in forecasts with poor accuracy (Silver, 2012).

As a solution to the first problem, generalized models, such as logistic regression models, can be used to constrain y_{pt} to remain within the [0, 1] interval. However, adopting a logistic model presents challenges. For example, when using a generalized model, we cannot simply interpret or constrain α_{pt}, as in Eqs. (3.2) or (3.3), because α_{pt} would be in a nonlinear relation with y_{pt}. Similarly, it is unclear how to handle the covariates and the swing term. Fortunately, all three issues are addressed in the next step while updating the forecast using poll data. As a result, we are not making additional efforts to complicate the model at this stage.

3.2. Updating: Multiparty dynamic Bayesian model

Up to this point, based on the fundamentals-based model, we forecast each province's two-major-party vote share. However, structural models that consider only historical voting patterns are incapable of dealing with the contextual information of the current election. Therefore, a synthetic model that combines all historical data with extensive pre-election polls should be preferred.

A synthetic model that deals with these polls was proposed by Linzer (2013), which uses a sequence of pre-election polls to update the structural model's forecast at any point. A dynamic Bayesian model is used to estimate the campaign period's opinion trend. As election day comes, the model's forecast becomes more accurate as campaign narratives are incorporated. Thus, the model is dynamic in that it regularly updates forecasts (Lewis-Beck & Stegmaier, 2014).

However, it is not straightforward to extend Linzer's model to account for the multiparty nature while keeping every variable in the model interpretable. A few solutions have been proposed, including (Stoetzer et al., 2019; Walther, 2015), who presented multiparty extensions of the model to forecast German and Swedish elections. Nonetheless, the model proposed by Walther (2015) is limited in that it estimates each party's support marginally, which is not an extension but a repeated application of the same model. Marginal estimation introduces a problem in which each candidate's support does not sum to one. Similarly, the approach of Stoetzer et al. (2019) is limited because the model's terms are challenging to interpret. For example, the house effect term in their model is defined on the vector space obtained after applying the additive logratio transformation (Aitchison, 1986) to a point on a simplex. In this transformed vector space, it is unclear what each value of the vector's component represents.

To this end, we aim to extend the dynamic Bayesian model to a multiparty setting where estimation is done jointly and all variables are interpretable. Specifically, we propose a multiparty dynamic Bayesian model that tracks daily multiparty public opinion trends at the provincial level using a simplex geometry and provides daily updated multiparty forecasts. Furthermore, our model estimates and incorporates the effects of various polling errors in an interpretable manner.

Let K be the number of pre-election polls, with each poll taking place in a specific province on a particular date. In our 20th presidential election poll data, polls estimate the opinion of seven province clusters ($p = 1, \ldots, 7$) from November 27th, 2021, until election day. Each date is represented by index $t = 1, \ldots, 103$. That is, $t = 1$ is November 27th, 2021, and $t = 103$ corresponds to the election day, March 9th, 2022. Suppose the kth poll surveys the preferences of n_k people. In that case, the number of respondents preferring each of the four candidates of the 20th presidential election can be denoted as a *response vector* $\mathbf{y}_k$ composed of nonnegative integers, and would be independently distributed as

$$\mathbf{y}_k = (y_{k1}, y_{k2}, y_{k3}, y_{k4})^\top \sim \text{Multinomial}(n_k, \widehat{\boldsymbol{\pi}}_k), \quad (k = 1, \ldots, K) \tag{3.4}$$

assuming a random sampling. Here, $\widehat{\boldsymbol{\pi}}_k$ is a probability vector denoting the opinion underlying the kth poll, i.e., the proportion of voters who would tell the kth poll their vote intention for each candidate. Although each poll typically surveys the proportion of undecided voters, as well as support for the other candidates, we only use the data for the four candidates, because our goal is to predict the four-party vote share. Eq. (3.4) incorporates the sampling error of each poll by modeling the survey result using the multinomial distribution. $\widehat{\boldsymbol{\pi}}_k$ would depend on several factors, including the province cluster and the poll date, as well as various systematic errors associated with the kth poll. So, the strategy is to factorize $\widehat{\boldsymbol{\pi}}_k$ to related terms and systematic errors in the poll.

To model $\widehat{\boldsymbol{\pi}}_k$, it is necessary to recognize that $\widehat{\boldsymbol{\pi}}_k$ is a point on a simplex, a space of vectors where its components are positive and sum to one. We refer to a point on a simplex as a *composition*. A simplex is not a vector space, since vector additions and scalar multiplications are not preserved in the space. Nevertheless, using Aitchison geometry, we can identify a simplex as a finite-dimensional Hilbert space. Appendix A summarizes the geometry and operations on this Hilbert space. We first exhibit the model and explain it in detail. This formulation, implemented via Aitchison geometry, is the central part of our work. Specifically, we factorize $\widehat{\boldsymbol{\pi}}_k$ using the following four configurations:

$$\begin{aligned}\widehat{\boldsymbol{\pi}}_k &= \boldsymbol{\pi}_k \oplus \boldsymbol{\eta}^{\text{wording}}_{i(k)} \oplus \boldsymbol{\eta}^{\text{method}}_{j(k)} \\ &= \boldsymbol{\xi}_{p(k),t(k)} \oplus \boldsymbol{\delta}^{\text{swing}}_{t(k)} \oplus \boldsymbol{\eta}^{\text{wording}}_{i(k)} \oplus \boldsymbol{\eta}^{\text{method}}_{j(k)},\end{aligned} \tag{3.5}$$

where each term is a point on a four-part simplex $\mathcal{S}^4$, and each part of the term represents one of the four candidates. In addition, $\oplus$ denotes the perturbation operation on a simplex, which is analogous to vector addition in Euclidean space.

Denoting the province cluster and the date of the kth poll as $p(k)$ and $t(k)$, respectively, $\boldsymbol{\pi}_k = \boldsymbol{\xi}_{p(k),t(k)} \oplus \boldsymbol{\delta}^{\text{swing}}_{t(k)}$ in Eq. (3.5) is a true opinion underlying in province cluster $p(k)$ at date $t(k)$. After removing the poll's systematic errors, the true preference of the voters would only depend on the region and the date when the poll was conducted. Thus, the election outcome of province cluster p would be $\boldsymbol{\xi}_{p,103} \oplus \boldsymbol{\delta}^{\text{swing}}_{103}$. Similarly, $\boldsymbol{\xi}_{pt} \oplus \boldsymbol{\delta}^{\text{swing}}_{t}$ for $t = 1, \ldots, 102$ would denote the daily voter preference trends throughout the election campaign period. The two terms composing the true opinion $\boldsymbol{\pi}_k$ are the national-level effect (swings) $\boldsymbol{\delta}^{\text{swing}}_{t(k)}$ and the provincial-level effect $\boldsymbol{\xi}_{p(k),t(k)}$. As Linzer (2013) argued, regional opinion patterns are often similar. As a result, the national-level swings represent a common deviation from long-term province-specific effects. By separating the terms, the accuracy of the model can be improved because national-level effects are estimated simultaneously across all province clusters, providing strength to province clusters that are not polled in the region on a specific date or when only a small number of samples are available.

For Bayesian analysis, a reverse random walk prior or a backward random walk prior is applied to these two effects, as in Heidemanns et al. (2020), Linzer (2013), Stoetzer et al. (2019). Specifically, on election day, the random walk begins with forecasts from the fundamentals-based model and walks through the beginning of the campaign. Thus, this stochastic process links the fundamentals-based forecast with the daily opinion trend, even during the blackout period. Thus,

$$\begin{aligned} \text{ilr}(\boldsymbol{\xi}_{pt})|\boldsymbol{\xi}_{p,t+1} &\sim \mathcal{N}_3(\text{ilr}(\boldsymbol{\xi}_{p,t+1}), \text{diag}(\sigma^2_{\xi 1}, \sigma^2_{\xi 2}, \sigma^2_{\xi 3})), \\ &\quad (t = 1, \ldots, 102) \qquad (3.6) \\ \sigma_{\xi l} &\sim \mathcal{N}_{1/2}(\sigma^2), \quad (l = 1, 2, 3) \\ \text{ilr}(\boldsymbol{\delta}^{\text{swing}}_t)|\boldsymbol{\delta}^{\text{swing}}_{t+1} &\sim \mathcal{N}_3\left(\text{ilr}\left(\boldsymbol{\delta}^{\text{swing}}_{t+1}\right), \text{diag}(\sigma^2_{s1}, \sigma^2_{s2}, \sigma^2_{s3})\right), \\ &\quad (t = 1, \ldots, 102) \qquad (3.7) \\ \sigma_{sl} &\sim \mathcal{N}_{1/2}(\sigma^2), \quad (l = 1, 2, 3) \end{aligned}$$

where ilr denotes the isometric logratio transformation that maps a point on a four-part simplex to three-dimensional Euclidean space (see Appendix A). As the name implies, ilr preserves the metric on the simplex to Euclidean space. Furthermore, since a simplex cannot directly use the normal distribution defined on a Euclidean space, this transformation makes it possible to use normal distributions for compositional data. Both (3.6) and (3.7) use a multivariate normal distribution with an independent variance structure. More complex structures, such as Lewandowski–Kurowicka–Joe priors (Lewandowski et al., 2009), can be used for covariance matrices. However, we used relatively simple prior structures in (3.6) and (3.7) to accelerate the convergence of posterior sampling.

For identifiability, we impose a restriction $\boldsymbol{\delta}^{\text{swing}}_{103} = (0.25, 0.25, 0.25, 0.25)^\top$ on the initial value of the random walk. This means that $\boldsymbol{\delta}^{\text{swing}}_{103}$ acts as a neutral element on a simplex, analogous to the zero vector in Euclidean space, so $\boldsymbol{\xi}_{p,103} \oplus \boldsymbol{\delta}^{\text{swing}}_{103} = \boldsymbol{\xi}_{p,103}$, and the true opinion of province cluster p on election day in each province should be set as the variable $\boldsymbol{\xi}_{p,103}$. As a result, we set $\boldsymbol{\xi}_{p,103}$ from the forecast obtained through the fundamentals-based model. Specifically, let y_p be a forecast of the Democratic Party's two-party vote share at province cluster p derived from the fundamentals-based model by weighting the forecast with $w_{p',19}$ to the forecasts of the provinces that belong to the province cluster p. To estimate each y_p, we use a normal distribution. In other words, if the fundamentals-based forecast of the province cluster p has posterior mean a_p and posterior sample standard deviation b_p, then we set $y_p \sim \mathcal{N}(a_p, b_p^2)$ for each p. Of course, any other distribution, such as a mixture of normals, can be used to denote the prior of y_p. Then, we set $\texttt{sim}_p \sim \text{Unif}[0, 0.05]$, $\texttt{ahn}_p \sim \text{Unif}[0, 0.15]$, $\texttt{lee}_p = (1 - \texttt{sim}_p - \texttt{ahn}_p) \times y_p$, $\texttt{yoon}_p = (1 - \texttt{sim}_p - \texttt{ahn}_p) \times (1 - y_p)$, and $\boldsymbol{\xi}_{p,103} = (\texttt{lee}_p, \texttt{yoon}_p, \texttt{sim}_p, \texttt{ahn}_p)^\top$, such that $\boldsymbol{\xi}_{p,103}$ is a composition. Candidate Sim and Ahn's weakly informative prior is set with our domain knowledge that they were also candidates in the 19th presidential election. Based on their vote share in the 19th election and pre-election polls early in the 20th presidential election campaign, we set them using a common uniform distribution in all province clusters with an appropriate interval.

The advantage of keeping $\boldsymbol{\xi}_{pt}$ and $\boldsymbol{\delta}^{\text{swing}}_t$ on a simplex is that we can directly interpret the effects. For instance, if the first component of both effects is greater (lesser) than 0.25, it indicates a lean toward (against) candidate Lee. Similarly, voters prefer Yoon over Ahn if the second component has a higher value than the fourth. The interpretation of the analysis results will be more apparent in Section 4.2.

Next, the last two terms of (3.5) denote the systematic errors resulting from the specification of the kth poll. We considered two effects: the effect of question-wording, and the effect of the survey method. Question-wording and survey method are grouped into three and two classes, respectively (see Section 2). We denote the wording of the kth poll as $i(k)$, and the method of the kth poll as $j(k)$.

Additionally, for the identifiability of the effects, we impose the constraint that polls are not biased on average:

$$\begin{aligned} \textstyle\bigoplus_{i=1}^{3} \boldsymbol{\eta}_i^{\text{wording}} &= \left(\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}\right)^\top, \\ \textstyle\bigoplus_{j=1}^{2} \boldsymbol{\eta}_j^{\text{method}} &= \left(\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}\right)^\top, \end{aligned} \qquad (3.8)$$

where $\bigoplus_{i=1}^{I} \boldsymbol{\eta}_i = \boldsymbol{\eta}_1 \oplus \cdots \oplus \boldsymbol{\eta}_I$. The condition in (3.8) is a naive constraint. If more information on the systematic errors is available, such as polls of the most recent presidential election and their meta-analysis outcomes, a constraint giving weights can be provided. For example, if we gain information that interviews are less biased than ARS methods in recent elections, we can provide constraints as weights, say $0.9 \otimes \boldsymbol{\eta}^{\text{method}}_{\text{interview}} \oplus 0.1 \otimes \boldsymbol{\eta}^{\text{method}}_{\text{ARS}} = (0.25, 0.25, 0.25, 0.25)^\top$, where $\otimes$ denotes the power transformation, an analog of scalar multiplication on a Euclidean space. However, it is unlikely and must be verified with the effort that the polls from the latest election have the same amount of bias as those from the current election. We thus use (3.8).

A prior is given to these systematic errors as well, using the ilr transformation:

$$\mathrm{ilr}(\boldsymbol{\eta}_i^{\mathrm{wording}}) \sim \mathcal{N}_3\left(\mathrm{ilr}\left(\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)^{\top}\right), \mathrm{diag}(\sigma_{\mathrm{w}1}^2,\sigma_{\mathrm{w}2}^2,\sigma_{\mathrm{w}3}^2)\right),$$
$$\mathrm{ilr}(\boldsymbol{\eta}_j^{\mathrm{method}}) \sim \mathcal{N}_3\left(\mathrm{ilr}\left(\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)^{\top}\right), \mathrm{diag}(\sigma_{\mathrm{m}1}^2,\sigma_{\mathrm{m}2}^2,\sigma_{\mathrm{m}3}^2)\right).$$

By inspecting the distribution of each component of the errors, it can be directly interpreted whether there is a significant bias in the question-wording and survey method of the poll. For example, suppose that the 95% credible interval of the first component of $\boldsymbol{\eta}_{\mathrm{ARS}}^{\mathrm{method}}$ is above 0.25. In that case, it can be concluded that the ARS survey overestimates Lee's support compared to the interview survey.

Assuming that σ, $\sigma_{\mathrm{w}l}$, and $\sigma_{\mathrm{m}l}$ ($l = 1, 2, 3$) independently follow a uniform distribution in addition to all of these model specifications, we estimate the posterior distribution of each variable. Then we can forecast the election outcome of the four candidates and daily trends in public opinion on any date during the campaign by supplying polls published up to that date and observing the posterior distribution of $\boldsymbol{\xi}_{pt} \oplus \boldsymbol{\delta}_t^{\mathrm{swing}}$. The forecast would only depend on the fundamentals-based model's outcome when no polls are supplied. As more polls are incorporated in the model, the accuracy and the precision of the forecast are enhanced, sometimes departing from the fundamentals-based model's forecast.

Although our model is depicted in the context of South Korea's 20th presidential election, the extension is simple. For example, in our analysis, the systematic errors are fixed (time-independent), but they can be modeled dynamically, as in (3.6) and (3.7). However, it should be noted that making a model complex requires sacrificing its efficiency. With many variables, the posterior sampling procedure may not converge well. We offer a forecasting framework, and it is up to the researcher to modify the model and interpret its outcome in the context of their election.

4. Estimation results

4.1. Fundamentals-based model

4.1.1. Variable selection

Selecting national- and provincial-level covariates is the first step in using the fundamentals-based model (3.1) to analyze the 20th presidential election. However, as pointed out in Section 1, research on South Korean fundamentals-based models is scarce. As a result, in addition to the studies of Song (2019) (a fundamentals-based model in South Korea) and Lauderdale and Linzer (2015) (a Bayesian regression model), we investigated all the forecasting models published in the *PS: Political Science & Politics* symposium for the 2020 U.S. presidential election, as well as widely cited works on presidential election forecasting, and adopted all of the available variables. The predictors used in each of the publications we examined are listed in the supplement.

Anyone who wants to investigate or improve our model should be able to accomplish the same end without much difficulty. We did not use variables that required additional research or effort. This ensures that the outcomes can be reproduced, which is one of the criteria for a forecasting model (Lewis-Beck, 2005). Refer to the supplement for an explanation of variables not considered in our analysis for the reproducibility of results.

Table 3 lists fundamental variables used in our model. Specifically, 12 national-level covariates and six provincial-level covariates are used. Predictors were obtained from the websites of the Korean Statistical Information Service (economic variables) and Gallup Korea (approval ratings). A detailed description of the variables used in our model is given in the supplement.

Some remarks on the variables are warranted. Firstly, when collecting predictors, it is assumed that the variables have the same effect on the two main parties. That is, the effect of economic variables, for example, does not depend on which party is in power, even if people may expect more from one party than from the other. As a result, for the performance variables (marked with 'a' in Table 3), we multiply −1 by the election in which the People Power Party is sitting. In addition, as stated in Section 3.1, all variables are standardized after dealing with the performance variables.

Secondly, for dummy variables such as `impeach`, `home`, and `homeground`, it should be acknowledged that there is a limit to coding various effects as 0 and ±1. Some events can have more impact on elections. For example, considering the covariate `homeground`, if the time difference between a candidate's employment as governor of a province and the presidential election increases, it may weaken the effect on the vote share. However, it is difficult to code various effects objectively. Consequently, our analysis should be interpreted as the *average* effect of each covariate.

4.1.2. Estimation

We used Rstan (Stan Development Team, 2022) to simulate the posterior distribution of parameters in model (3.1), with four chains and 10,000 iterations for each chain, including warm-up iterations. The trace plot and R-hat convergence diagnostics of each parameter were examined to confirm the convergence of the iterative procedure (Gelman et al., 2013).

Among the sampled parameters, we are most interested in the posterior distributions of α_{pt}, β_k, γ_l, and $y_{p,20}$. These parameters can be used to examine the long-term opinion trend captured by each province's normal vote, the effects of national-level predictors, the effects of provincial-level predictors, and the forecast of the 20th presidential election outcome (two-party vote share of the Democratic Party) in each province, respectively.

First, the left panel of Fig. 2 depicts the normal vote α_{pt}, with its posterior median value connected by a line for each of the 15 provinces. As stated above, normal votes can be understood as a baseline vote division expected from each province after removing election-specific forces.

Table 3
Covariates in the fundamentals-based model forecasting the 20th presidential election of South Korea. 2nd to 11th row: national-level predictors, 12th to 15th: provincial-level predictors.

Variable name	Explanation
`is_current`	Index of incumbency
`gdp4`, `gdp5`[a]	Yearly growth of real GDP in the 4th and 5th years
`gni4`, `gni5`[a]	Yearly growth of real GNI in the 4th and 5th years
`twice1`[a]	Incumbent party has been in the presidency for two consecutive terms
`twice2`[a]	Number of consecutive terms a party has been in the office
`approval`[a]	President's approval rating in 5/2' (4/4' for the 19th PE)
`net_approval`[a]	President's net approval rating
`D_primary`	Primary election results for the Democratic Party
`P_primary`	Primary election results for the People Power Party
`impeach`[a]	President impeachment by the National Assembly
`cpi4`, `cpi5`[a]	Yearly growth of CPI in the 4th and 5th years
`unemp4`, `unemp5`[a]	Unemployment rate in the 4th and 5th years
`home`	Home province of the candidate, with lagged adjustments
`homeground`	Candidate with experience as a provincial governor, with lagged adjustments

[a]Denotes a performance variable, which has associations with the incumbent party, not the Democratic Party. PE = presidential election, n/m' = nth year mth quarter of the president's term.

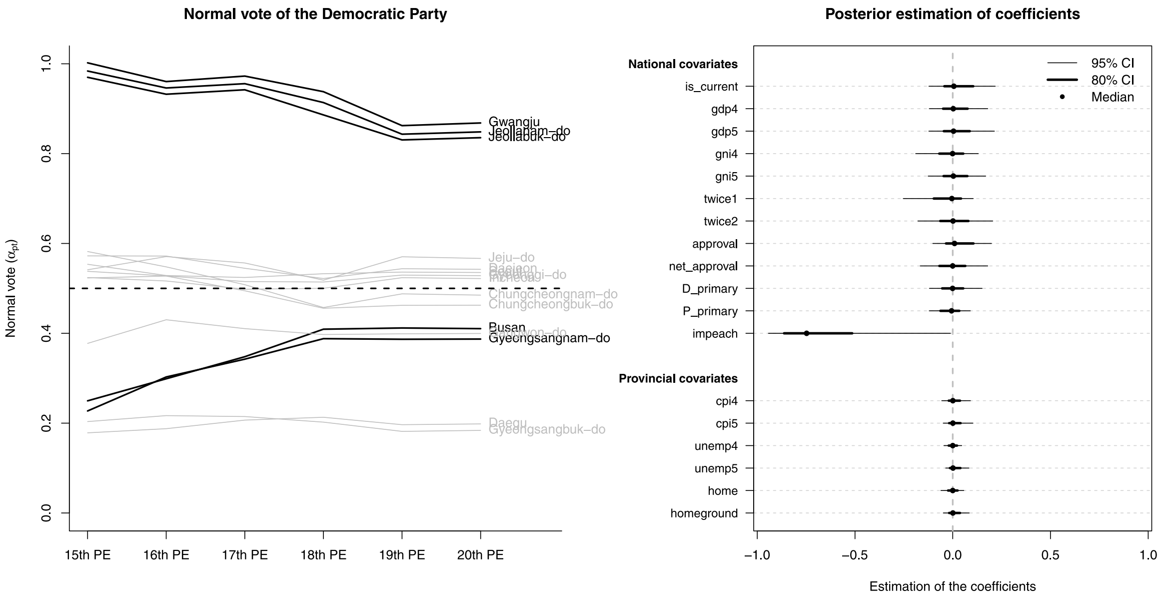

Fig. 2. (Left) Normal vote α_{pt} from the 15th to the 20th presidential elections for each province. The black provinces are those with the most changes from the 15th to the 19th presidential elections. The dotted horizontal line indicates a value 0.5. (Right) Posterior median and 95% and 80% credible intervals of each regression coefficient used in the fundamentals-based model.

From the 15th to 19th presidential elections, provinces where the change in the normal vote rate exceeds 0.1 are expressed by a black line and others by a gray line. Gwangju, Jeollanam-do, Jeollabuk-do, Busan, and Gyeongsangnam-do showed the most change, with α_{pt} decreasing in the former three provinces and α_{pt} increasing in the latter two provinces. In other provinces, support for the two major political parties has shown calm stability throughout the elections. These long-term trends reflect strong regionalism, which is a pervasive voting culture in South Korea (Shin, 2001).

It is important to note that normal votes should be interpreted as relative. We constrained normal votes to reflect an election between the two major parties that is perfectly balanced. Thus, it cannot be argued that Gwangju's baseline support for the Democratic Party is dwindling. The decrease in Gwangju's normal vote could be an artifact of the constraint. Instead, it must be stated that Gwangju's baseline loyalty to the Democratic Party is declining *in comparison to* the other provinces.

Second, the right panel of Fig. 2 shows the posterior distributions of the regression coefficients β_k and γ_l, along with their posterior medians and 95% and 80% credible intervals. The variable `impeach` appears to be the only significant variable in our analysis, as all 95% credible intervals of the other variables contain zero. Also, it seems that the provincial-level effects are more certain to be around zero than the national-level effects. Although we cannot say much about voting behaviors observed in South Korea's presidential election, the proposed model will be useful in developing election theories in the future. As more data become available in the future, the

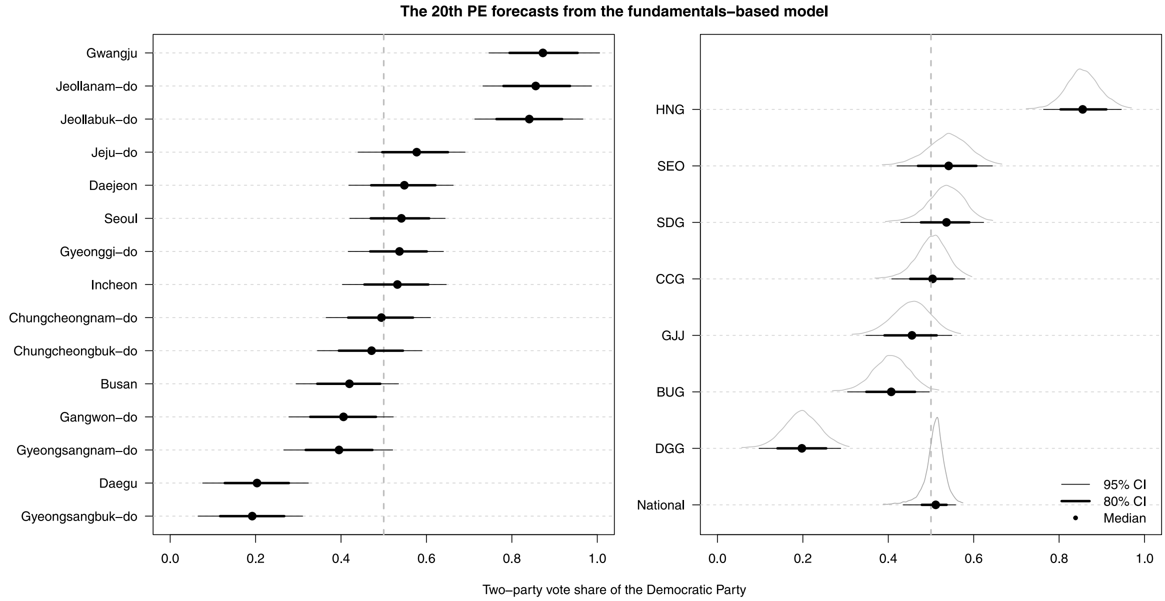

Fig. 3. (Left) Provincial-level forecast of the Democratic Party's two-party vote share in the 20th presidential election. Predictions are shown with their 95% and 80% credible intervals. Provinces are sorted by their posterior median values. (Right) Forecast of seven province clusters' results and national outcome with the kernel density estimate of each posterior distribution. Province clusters are sorted by their posterior median values. The dotted vertical line indicates a value of 0.5.

coefficient may deviate from zero, deriving a significant interpretation.

The negatively distributed `impeach` coefficient indicates that the president's impeachment in the National Assembly is negatively associated with the vote share of the incumbent candidate in the subsequent election, conditioned on the other variables. It is important to note that this is an association rather than a causal relationship. There may be latent variables that the model does not describe. Analyzing a causal relationship between the president's impeachment and the vote share in the next election is not a goal that our model can achieve. In addition, since each covariate was standardized and a common prior structure was imposed, careful progress should be made when interpreting the model's coefficients.

Finally, the left panel in Fig. 3 represents the forecasts of the 15 provincial-level election outcomes, expressed using the posterior median and credible intervals of $y_{p,20}$. It can be seen that some provinces are expected to have strong support for the Democratic Party or the People Power Party. In contrast, the credible interval for most provinces contains 0.5. Also, the forecast's credible intervals are too wide. Six of the 15 provinces have a longer 95% credible interval than the historical difference between the maximum and minimum two-party vote shares from the 15th to 19th presidential elections. This is due not only to the small size of the South Korean data set but also to the fact that fundamentals-based models cannot make sufficiently confident forecasts (Silver, 2012). In the following section, we show how using polls to update the result of the fundamental model makes our forecast more confident, with shorter credible interval lengths.

In the right panel of Fig. 3, the provincial-level forecasts are combined to create provincial cluster-level and national-level forecasts. The posterior distributions of $y_{p,20}$ are averaged with weights $w_{p,19}$, from the 19th presidential election. The posterior median of the national prediction indicates that the Democratic Party candidate, Lee, will win. Still, the national prediction's credible interval includes a substantively wide area below 0.5. We also plotted the kernel density estimate for each province cluster and national-level forecast. They are unimodal and not skewed. Consequently, it is not problematic to use normal approximation to these estimates and employ them as a prior distribution in the updating step.

4.2. Multiparty dynamic Bayesian model

4.2.1. Data preparation and model modification

The raw pre-election poll data from the NESDC cannot directly be used to update the forecast from the fundamentals-based model. Each survey data set spans several days and is expressed in percentages of vote intentions rather than the number of respondents supporting each candidate. The desired data format for the updating step is a vector of nonnegative integers corresponding to each candidate's vote intention (response vector) for a specific date. Therefore, some preprocessing is required before performing an iterative posterior sampling procedure.

To begin, we separated each national-level poll to obtain the vote intention percentages in each of the seven province clusters. This is a straightforward procedure because the NESDC website reports contain poll results for each province cluster for each national-level poll. Then, the sample size was multiplied by the vote intention percentages to determine the number of respondents who would vote for each candidate. However, since quota sampling is used and the reported outcome is a number obtained after weighting the quotas to match their proportion, multiplying a sample size by each percentage does not yield an integer value. We rounded the number to the nearest nonnegative integer to obtain a response vector. Finally, we separated the response vector by survey date because polls are conducted over multiple days (Table 1). The response vector was divided into integer vectors for each survey date, so adding each response vector equals the original vector. As a result, we have 5053 poll results ($K = 5053$), each of which is a response vector from a particular date. Instead of dividing the response into multiple days, the survey's start or end date can be used as a reference point.

Then, as in Linzer (2013), we modified our model of (3.5) to improve the convergence of the posterior sampling procedure. Specifically, instead of estimating $\boldsymbol{\xi}_{pt}$ in (3.5) at each t, three days are grouped and estimated at once. In particular, $\boldsymbol{\xi}_{p(k),w(t(k))}$ is used in place of $\boldsymbol{\xi}_{p(k),t(k)}$, where w is a function that maps three consecutive days to a single index. This modification is applied to the reverse random walk prior and improves the estimation process by reducing the number of parameters in the model. As argued by Linzer (2013), this simplification has little effect on the forecast outcome, because $\boldsymbol{\delta}_t^{\text{swing}}$ exhibits daily volatile oscillations yet $\boldsymbol{\xi}_{pt}$ changes gradually over time. Below, we show that this claim is also valid for the 20th presidential election.

4.2.2. Estimation

Similar to the fundamentals-based model, we used Rstan (Stan Development Team, 2022) for simulation, with eight chains and 5000 iterations for each chain, including warm-up iterations. Trace plot and R-hat diagnostics for each parameter were also examined.

We propose a way to generate summary statistics from the posterior samples before parameter estimation. In this model, many parameters are distributed on a simplex. For instance, $\boldsymbol{\xi}_{pw(t)}$, $\boldsymbol{\delta}_t^{\text{swing}}$, $\boldsymbol{\eta}_i^{\text{wording}}$, $\boldsymbol{\eta}_j^{\text{method}}$, and their perturbations are all distributed on a four-part simplex, implying that their posterior samples are points on the same simplex as well. Suppose that $\mathbf{y}_s \in \mathcal{S}^4$ $(s = 1, \ldots, S)$ is a posterior sample of any simplex-valued parameter. Assuming that $\mathbf{y}_s$ is a Euclidean vector, there is no guarantee that the estimator is on the simplex if a component-wise median or mean estimator of $\mathbf{y}_s$ is constructed. We consider a method used widely in non-Euclidean data analysis to solve the issue. A geometric median analogous to the median statistic is used:

$$\widehat{\mathbf{y}} = \operatorname*{argmin}_{\mathbf{z}\in\mathcal{S}^4} \sum_{s=1}^{S} d(\mathbf{y}_s, \mathbf{z}), \tag{4.1}$$

where $d(\cdot,\cdot)$ denotes a distance function on a simplex (see Appendix A). It is guaranteed that $\widehat{\mathbf{y}}$ is on a four-part simplex. Gradient descent-type algorithms can be used to calculate the geometric median (Cardot et al., 2013).

Similarly, we can use the Fréchet mean estimator, $\bar{\mathbf{y}} = \operatorname{argmin}_{\mathbf{z}\in\mathcal{S}^4} \sum_{s=1}^{S} [d(\mathbf{y}_s, \mathbf{z})]^2$, analogous to the mean estimator. The Fréchet mean on a simplex can directly be deduced to be equivalent to a component-wise geometric mean with a closure operation. We defer the proof to Appendix A.

As stated above, the parameter $\boldsymbol{\xi}_{pw(t)} \oplus \boldsymbol{\delta}_t^{\text{swing}}$ is defined as a true opinion underlying in province cluster p ($p = 1, \ldots, 7$) at date t ($t = 1, \ldots, 103$). When $t = 103$, it represents the true opinion on election day or a prediction of the election result. We only show the estimation results for the two clusters with the highest and lowest voter turnout, SDG and GJJ. The results of all province clusters are provided in the supplementary material.

Gyeonggi-do and Incheon comprise the SDG cluster, which accounts for about 30% of South Korean voters. Consequently, SDG is a crucial province cluster in choosing the winner. On the other hand, GJJ, a province cluster composed of Gangwon-do and Jeju-do, represents around 4% of the electorate in South Korea. The election outcome forecast from SDG and GJJ is shown in Fig. 4 at four different time points: 10, 7, and 4 weeks before the election (December 29th, 2021, January 19th, 2022, and February 9th, 2022), and one week before the election (March 2nd, 2022). Since pre-election polls are not released in South Korea during the final week before the election, the forecast made one week before the election is the last one. The geometric median in (4.1) is utilized to jointly estimate the median because $\boldsymbol{\xi}_{pw(t)} \oplus \boldsymbol{\delta}_t^{\text{swing}}$ is distributed on a four-part simplex. As a result, the median estimates on each date in Fig. 4 add up to one. Additionally, 95% credible intervals are estimated component-wise, as shown in the figure.

We observe the forecasts at each time point until the election and the short-term, daily opinion trends in Fig. 4. In contrast to what SDG consistently predicts, GJJ predicts that Lee would lose in that cluster. The outcomes from both clusters align with the fundamentals-based model in the right panel of Fig. 3. However, the updated forecasts are significantly more precise. For example, in the forecast made one week before the election, the length of the 95% credible intervals only goes up to 7% (or ±3.5%) for all candidates, as opposed to approximately 20% credible intervals lengths in the fundamentals-based model. The results are regularly updated from the fundamentals-based model, and credible intervals are getting narrower, indicating an accurate and precise prognosis.

The weighted average of the true opinions at the provincial level is then computed to produce national trends and forecasts. Since the exact number of voters in each province is unknown until the election, we used the weights from the 19th presidential election. As in Fig. 4, geometric medians and component-wise credible intervals are used. Forecasts fluctuate on a nationwide scale. Lee was predicted to win by a narrow margin in a forecast released 10 weeks before the election. However, from the seven-week prediction before the election, the model consistently predicts Yoon's victory, with credible intervals of the two major-party candidates overlapping less and less as time goes on. In the final prediction, we contend that

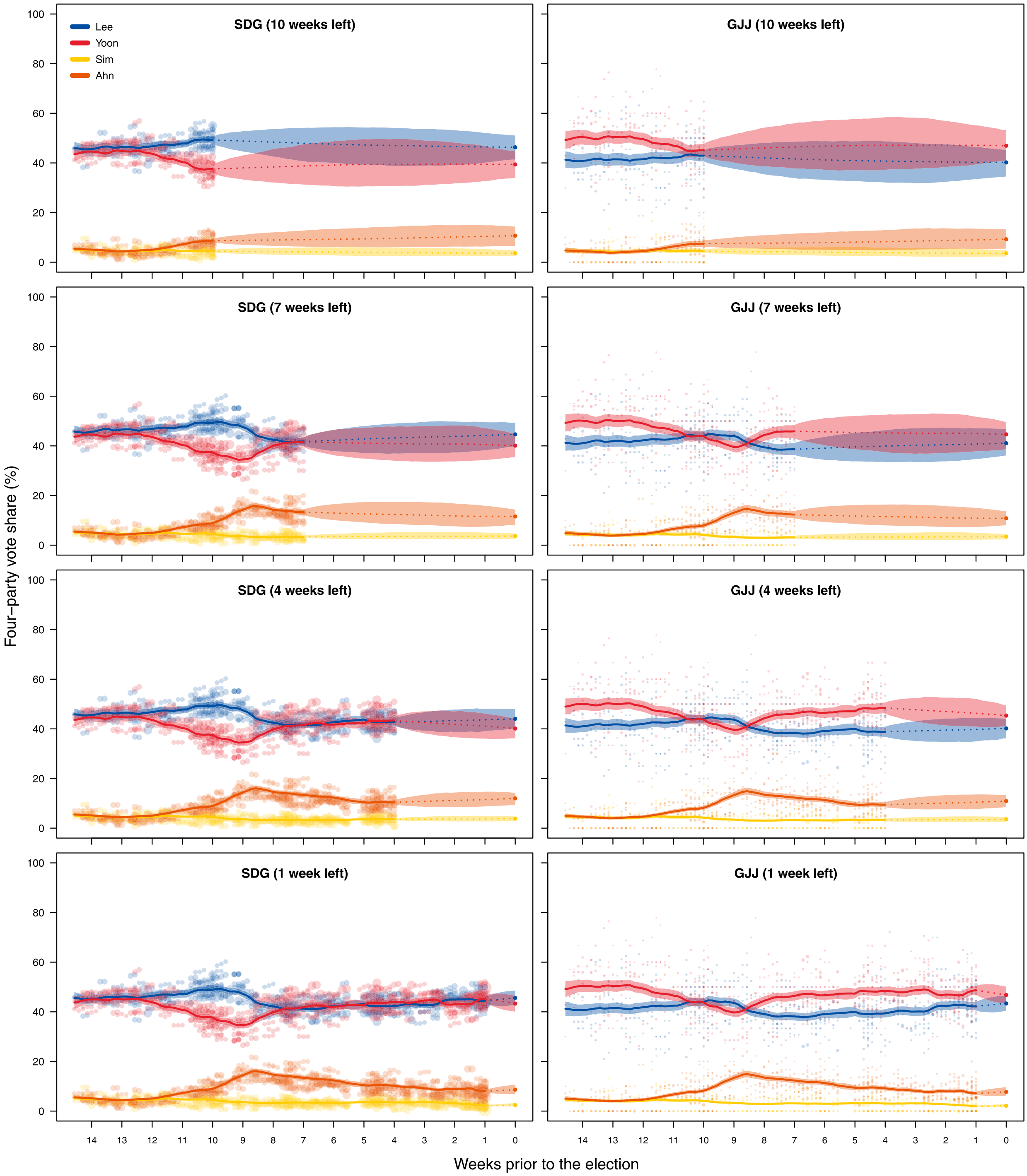

Fig. 4. Posterior medians (line) and 95% credible intervals (shaded area) of the true opinion trend in the province cluster SDG (left) and GJJ (right). Four time points are used to make a forecast. Points indicate pre-election poll results, whose size is proportionate to the sample size of that poll.

Yoon would receive 45.31% of the four-party vote, with a 95% credible interval (42.42%, 48.10%), and Lee would receive 43.80% with a 95% credible interval (41.44%, 46.28%). This result that Yoon would win the presidency reverses the forecast by the fundamentals-based model that Lee would win.

From Figs. 4 and 5, it is important to note that Lee and Yoon's trends are *not* symmetric. When one of the two major-party candidates sees a decrease in support, the other candidate does not see a corresponding increase in support. Since Sim's and Ahn's vote shares are also fluctuating, this phenomenon is natural. However, when

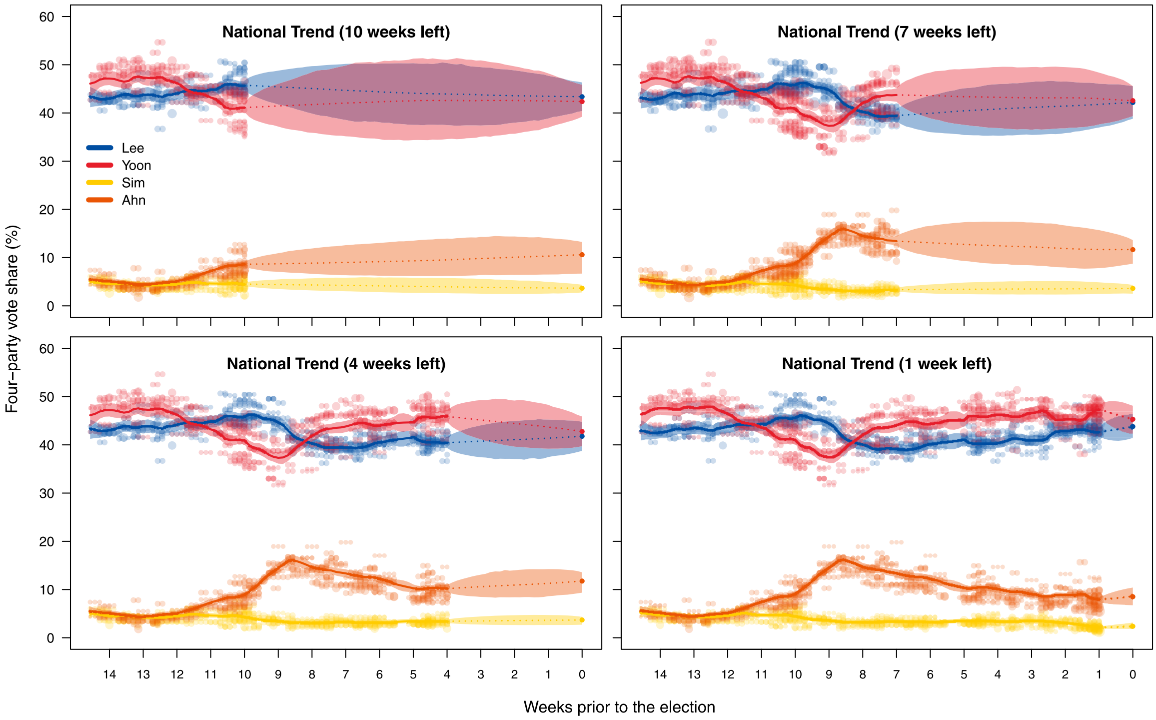

Fig. 5. Posterior medians (line) and 95% credible intervals (shaded area) of the national-level true opinion trend. A forecast is made using four time points. Points represent the results of pre-election polls, whose size is proportionate to the size of the poll's sample.

only two major parties are considered, as in two-party models, for example Linzer (2013), two parties are handled as if their vote shares are complementary. Therefore, compared to two-party models, our multiparty model has the advantage of being able to explain the election in greater detail.

In addition to the above-mentioned benefit of the multiparty model, we point out that tracking the popularity of the candidates from the non-two-major parties can help voters and candidates judge, even if one of the two major party candidates is a foregone conclusion, as in Fig. 5. The multiparty model can aid voters in casting strategic ballots. In other words, voters may elect a major-party candidate who can avert unfavorable election results rather than their preferred non-major party candidate. Candidates could withdraw from the campaign if their vote share forecasts dropped too low to keep their political position. Negotiations for unification amongst many candidates who share the same goal can be facilitated through such a framework.

We now look at the two factors that make up the true opinion. To see the national and provincial changes, we plot $\boldsymbol{\xi}_{pw(t)}$ and $\boldsymbol{\delta}_t^{\text{swing}}$ separately in Fig. 6. The geometric median and component-wise credible intervals are obtained using posterior samples. As shown in Figs. 4 and 5, two effects can be examined at any time point, but we only look at the effects on the last forecast on March 2nd, 2022, to evaluate the results following the inclusion of all pre-election polling data.

The province-specific effect $\boldsymbol{\xi}_{pw(t)}$ is consistent for all provinces, supporting the claim that grouping days by $w(t)$ will not significantly alter the forecast. On the other hand, national swings are volatile, changing daily. Accordingly, since most opinion shifts occur at the national level, opinion trends in each province cluster are highly correlated. For example, we observe that Ahn experienced a rise in popularity from 11 to 9 weeks before the election. We can claim that this fluctuation is common in all province clusters.

Finally, the biases arising from different survey wordings and methods can be visually analyzed in Fig. 7. Consolidating all the polls produced by March 2nd, 2022, the figure illustrates the effects of each systematic bias in pre-election polls. The interpretations are straightforward. It can be said that the effect is significant if a credible interval does not contain 0.25, which corresponds to zero for simplex-valued data.

In terms of the wording effect, it can be observed that the poll is more likely to favor Lee than Ahn when asked in terms of *votes*. In a similar vein, the *appropriate* wording is expected to increase support for Ahn and decrease support for the two major party candidates in relation to other wordings. The bias occurs more from the method, as no credible intervals include 0.25. Compared to the

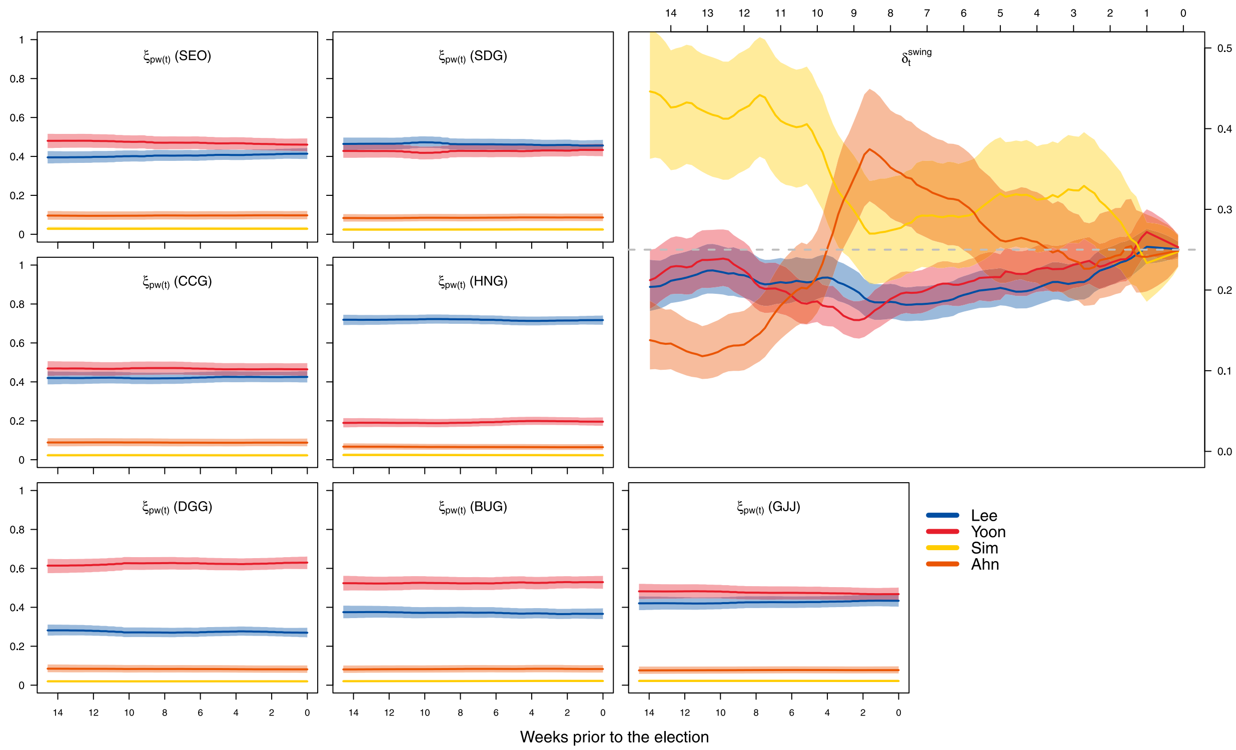

Fig. 6. (Small plots) Posterior medians (line) and 95% credible intervals (shaded area) of $\xi_{pw(t)}$ for each province cluster. (Large plot) Posterior medians (line) and 95% credible intervals (shaded area) of δ_t^{swing}. The dotted horizontal line indicates a value of 0.25.

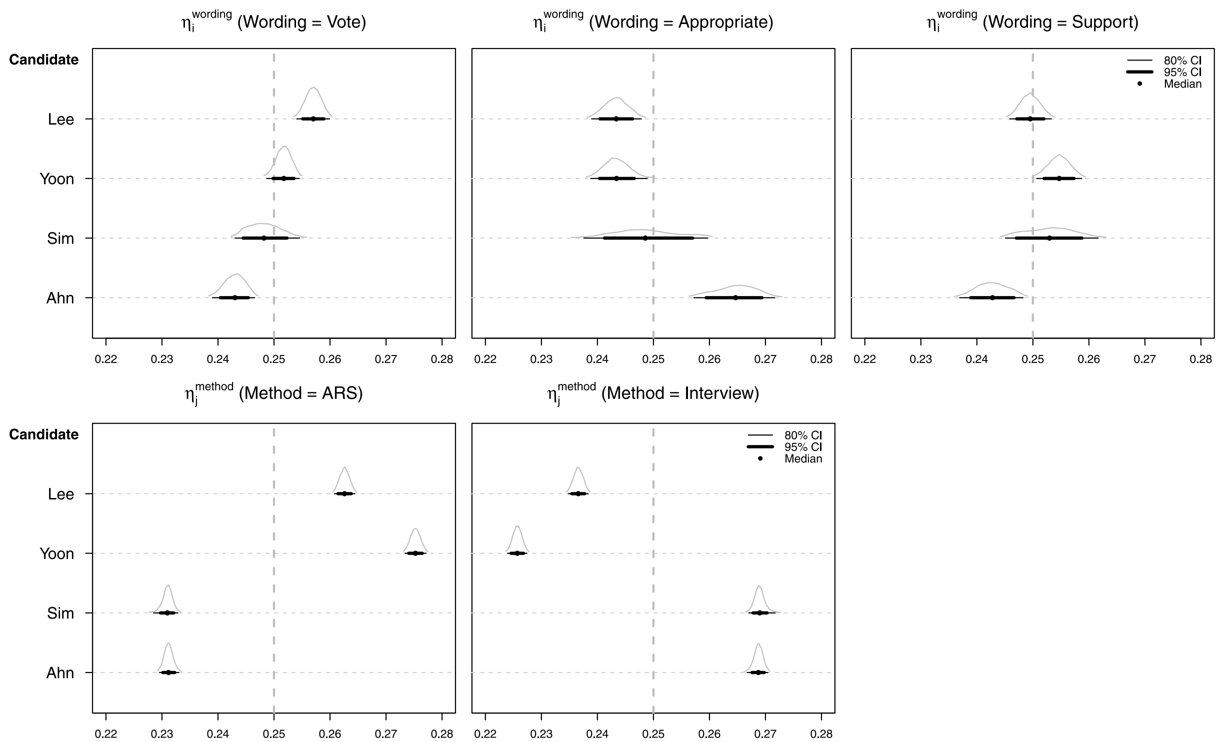

Fig. 7. Posterior medians, 95% and 80% credible intervals, and kernel density estimate (bell-shaped curve) of η_i^{wording} (above) and η_j^{method} (below) for each candidate.

interview mode, support is expected to be greater for Lee and Yoon and lower for Sim and Ahn when hiring the ARS mode. It should be emphasized that these effects must be viewed relatively. We used (3.8) to apply the restriction that polls are collectively unbiased. As a result, it is impossible to determine from these samples whether the polling industry is biased or which wording or methodology is closer to the actual view that underlies the polls. Only a relative evaluation of the bias is done.

4.3. Retrospective evaluation

Despite starting our analysis before the election, Yoon of the People Power Party was elected as South Korea's 20th president by a narrow margin with Lee at the time of writing this paper. Also, the four major parties took about 98.76% of the popular vote. This section conducts a retrospective analysis of our findings to determine whether our model can be used in the future.

In summary, the winner predictions for each province cluster are all correct. Furthermore, our model correctly predicts that Yoon will be elected president by a narrow margin at the national level. The supplementary material contains a detailed summary of our forecasts.

Our forecasts are a prediction of the four-major-party vote share. However, on March 3rd, 2022, the first day the poll was not announced, candidate Ahn dropped out. As a result, Ahn's vote share is structurally zero. According to our poll data ending on March 2nd, this event is unpredictable. Thus, his withdrawal is not reflected in our prediction. We considered two options to modify our forecasts. One: it computes the three-party vote share while ignoring Ahn's vote share. The vote share is renormalized with the three candidates who completed the race. The other: it uses polls based on the three candidates. Some pollsters asked for vote intentions under the assumption that Ahn had resigned. We can compute the three-party vote share using these conditioned polls. However, both ways are implausible. The first idea assumes that Ahn's votes will be distributed equally among the three candidates, which is unrealistic. The second is meaningless because voters will not consider Ahn's resignation. A conditional question only provokes reluctant respondents to speak about a situation they have never imagined. Thus, it is difficult to regard the conditioned polls as capturing the *true* opinion.

The moral of this unexpected event is that our model is only correct under the conditions we set. Our model made predictions under the condition that four candidates would run to the finish line and receive votes. Accordingly, the analysis and interpretation should be carried out carefully, keeping in mind our imposed conditions.

In conclusion, it is difficult to determine whether our forecast of the 20th presidential election is correct or accurate because one of our assumptions was wrong. Moreover, we cannot conduct a further meta-analysis of the pre-election polls. For example, we used the constraint (3.8), assuming that the polls were collectively unbiased. If Ahn had not resigned, this hypothesis could have been checked. We can compare polls using the ARS method and polls using the interview method alone to run the model and determine which method yields more accurate predictive results. Then we can make statistical claims about the methods' biases. Nonetheless, our study offers many valuable findings. Fundamentals-based model results and the long-term normal votes are worth investigating. Also, the short-term daily opinion trends and pre-election poll biases are still significant, as Ahn had his support as a presidential candidate until March 2nd, 2022. These findings aid in providing a deeper understanding of the election.

5. Conclusion

In this paper, we proposed a dynamic presidential election forecasting framework considering South Korea's insufficient historical data and multiparty system. The framework was applied to South Korea's 20th presidential election, held on March 9th, 2022. Along with predicting the four-major-party vote share in each province cluster, we examined the effects of fundamentals on vote share, bias in pre-election polls, and long- and short-term opinion trends.

Specifically, the effects of fundamentals were estimated using a fundamentals-based model based on the retrospective theory of voting. Due to the lack of historical data, the current model does not provide meaningful interpretations. In the future, however, relationships between the fundamentals and the election results will be available, with which we can build more concrete election theories for South Korea. Besides our approach, analyzing the causal relationship between the fundamentals and the election could be interesting. Further, bias in pre-election polls will be of great interest to pollsters, as it can be used to enhance the accuracy of their polls. It may be meaningful to evaluate whether the industry is biased, i.e., collectively biased, because we only examined the polls' bias relatively. Finally, long- and short-term trends in opinion provide a multiscale view of the South Korean presidential elections. We now have statistical evidence demonstrating that regionalism is prevalent in South Korea in the long and short term. In summary, we believe these findings provide valuable theories and lay groundwork for further studies and practical applications.

We spent much time making interpretations and emphasized a handful of cautions. We tried to avoid some interpretations in the analysis. For example, Bayesian forecasting models inherently provide a way to calculate the probability of winning. This is easily determined by counting the proportion of posterior samples in which one candidate receives a larger share of the vote than others. To acknowledge potential pitfalls in election forecasts, we did not compute these values. We were concerned that such probability-speaking may lead people to regard forecasting as a form of entertainment, leading the public to focus solely on the forecast *per se* rather than more important candidate scandals or election issues. We were also concerned that these forecasting interpretations would affect voting results if they were widely publicized before the election, focusing solely on entertainment. As noted by Gelman et al. (2020), Victor (2021), it takes time and effort to convey these probabilities to the general public. When making forecasts, we should remember that 'science is for explanation, not prediction' (Victor, 2021).

Our framework is now ready for evaluation. We do not claim that our model is flawless. In fact, no model is *true*, and election forecasting models always benefit from additional research. The next five years represent yet another opportunity.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Simplex geometry

A.1. Definitions

Here, we briefly review the Aitchison geometry on a simplex and the isometric logratio transformation (ilr) that maps points on the simplex to a Euclidean space. For more information, see, e.g., Aitchison (1986), Pawlowsky-Glahn et al. (2015).

Define a *D-part simplex* $\mathcal{S}^D = \{\mathbf{x} = (x_1, \ldots, x_D)^\top \in \mathbb{R}^D : \sum_{i=1}^D x_i = 1, x_j > 0, j = 1, \ldots, D\}$. Elements of $\mathcal{S}^D$ are called compositional data or compositions. Given a Euclidean vector $\mathbf{e} = (e_1, \ldots, e_D)^\top \in \mathbb{R}^D$ on a positive orthant, a *closure* operator $\mathcal{C}$ maps $\mathbf{e}$ to a unique composition, $\mathcal{C}(\mathbf{e}) = (e_1/\sum_{i=1}^D e_i, \ldots, e_D/\sum_{i=1}^D e_i) \in \mathcal{S}^D$.

Two operations are defined on a simplex $\mathcal{S}^D$ as an analog of a vector addition and a scalar multiplication on a vector space. *Perturbation* operation $\oplus$ is defined for two compositions $\mathbf{x}, \mathbf{y} \in \mathcal{S}^D$ as $\mathbf{x} \oplus \mathbf{y} = \mathcal{C}((x_1y_1, \ldots, x_Dy_D)^\top)$, and *power transformation* is defined as $\alpha \otimes \mathbf{x} = \mathcal{C}((x_1^\alpha, \ldots, x_D^\alpha)^\top)$, where $\alpha \in \mathbb{R}$. It can easily be seen that $\frac{1}{D}\mathbf{1}_D = \left(\frac{1}{D}, \ldots, \frac{1}{D}\right)^\top \in \mathcal{S}^D$ is a unique identity (neutral) element of perturbation, and 1 is an identity element of power transformation.

By defining an inner product on $\mathcal{S}^D$ as

$$\langle \mathbf{x}, \mathbf{y} \rangle = \frac{1}{D} \sum_{i<j} \log \frac{x_i}{x_j} \log \frac{y_i}{y_j}, \quad (\mathbf{x}, \mathbf{y} \in \mathcal{S}^D),$$

$\mathcal{S}^D$ turns into a complete inner product space, i.e., a Hilbert space of dimension $(D-1)$. In a standard way, the norm on $\mathcal{S}^D$ is defined as $\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle}$ and distance between two compositions $\mathbf{x}, \mathbf{y} \in \mathcal{S}^D$ as $d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} \oplus ((-1) \otimes \mathbf{y})\|$. The latter distance is known as the Aitchison distance and we shall call this Hilbert space geometry as Aitchison geometry.

Isometric logratio transformation (ilr) is an isometric map from a D-part simplex onto a $(D-1)$-dimensional Euclidean space $\mathbb{R}^{D-1}$ in the sense that $d(\mathbf{x}, \mathbf{y}) = \|\text{ilr}(\mathbf{x}) - \text{ilr}(\mathbf{y})\|_2$, where $\mathbf{x}, \mathbf{y} \in \mathcal{S}^D$ and $\|\cdot\|_2$ is the usual Euclidean norm. Any orthonormal basis on $\mathcal{S}^D$ can induce ilr. For example, considering the simplex $\mathcal{S}^4$, the explicit formulation of ilr and its inverse can be given as

$$\text{ilr}(\mathbf{x}) = \left(\sqrt{\frac{3}{4}} \log \frac{x_1}{\sqrt[3]{x_2x_3x_4}}, \sqrt{\frac{2}{3}} \log \frac{x_2}{\sqrt{x_3x_4}}, \sqrt{\frac{1}{2}} \log \frac{x_3}{x_4}\right)^\top, \quad (\mathbf{x} \in \mathcal{S}^4)$$

and

$$\begin{aligned}\text{ilr}^{-1}(\mathbf{e}) = \mathcal{C}\Bigg(\Bigg(&\exp\left\{\sqrt{\frac{3}{4}}e_1\right\}, \exp\left\{\sqrt{\frac{2}{3}}e_2 - \frac{1}{\sqrt{12}}e_1\right\},\\ &\times \exp\left\{\frac{1}{\sqrt{2}}e_3 - \frac{1}{\sqrt{12}}e_1 - \frac{1}{\sqrt{6}}e_2\right\},\\ &\exp\left\{-\frac{1}{\sqrt{12}}e_1 - \frac{1}{\sqrt{6}}e_2 - \frac{1}{\sqrt{2}}e_3\right\}\Bigg)^\top\Bigg).\\ &(\mathbf{e} \in \mathbb{R}^3)\end{aligned}$$

For more detailed explanations on isometric logratio transformations, see Egozcue et al. (2003).

A.2. Derivation of a geometric mean on a simplex

Note that the Fréchet mean of samples $\mathbf{y}_s \in \mathcal{S}^D$ for $s = 1, \ldots, S$ is defined as

$$\bar{\mathbf{y}} = \operatorname*{argmin}_{\mathbf{z} \in \mathcal{S}^D} \sum_{s=1}^S [d(\mathbf{y}_s, \mathbf{z})]^2,$$

where $d(\cdot, \cdot)$ denotes a distance function on a simplex defined above. Noting that ilr is an isometry, it holds that

$$\begin{aligned}\bar{\mathbf{y}} &= \operatorname*{argmin}_{\mathbf{z} \in \mathcal{S}^D} \sum_{s=1}^S \|\text{ilr}(\mathbf{y}_s) - \text{ilr}(\mathbf{z})\|_2^2\\ &= \text{ilr}^{-1}\left(\frac{1}{S}\sum_{s=1}^S \text{ilr}(\mathbf{y}_s)\right)\\ &= \text{ilr}^{-1}\left(\text{ilr}\left(\frac{1}{S} \otimes \left(\bigoplus_{s=1}^S \mathbf{y}_s\right)\right)\right)\\ &= \frac{1}{S} \otimes \left(\bigoplus_{s=1}^S \mathbf{y}_s\right),\end{aligned}$$

where $\|\cdot\|_2$ is the usual Euclidean norm. Hence, denoting $\mathbf{y}_s = (y_{s1}, \ldots, y_{sD})^\top$, we get

$$\bar{\mathbf{y}} = \mathcal{C}\left(\left(\prod_{s=1}^S (y_{s1})^{1/S}, \ldots, \prod_{s=1}^S (y_{sD})^{1/S}\right)^\top\right),$$

which is a component-wise geometric mean of $\mathbf{y}_s$ $(s = 1, \ldots, S)$ with a closure operator.

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ijforecast.2023.01.004. It includes information on South Korea's provinces, details on the fundamentals-based model's variable selection procedure, and a summary of the 2022 South Korean presidential election forecast and outcome.

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