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Essentials of Pension Economics

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ISBN 978-3-030-26495-6 ISBN 978-3-030-26496-3 (eBook)
https://doi.org/10.1007/978-3-030-26496-3

This Palgrave Pivot imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To my late father and mother

Preface

The idea of writing a book on *Pensions* was prompted by the urgent relevance of the issue. Pensions, whether paid by governments or by private companies, are the sole source of income for millions of people around the world. By 2050, two billion elderly people will have to be ensured some form of income. At the same time, the prospect facing the young generations is of a gloomy future, leaving scant room for savings to support their parents, their older fellow citizens or themselves when old, the actual direction their savings take depending on the social arrangements affecting them. This is why pensions have as much to do with social policy as with economics, the essential role of the latter being to investigate and reveal the, possibly hidden, trade-offs of alternative courses of action.

Despite its relevance, the economic dimension of the pension issue finds little, if any, room among the subjects taught worldwide in university economics, management and even political science programmes. This compendious volume should ideally act as an incentive for the many professors of micro-, macro- and public economics to try to include a short course on *Pensions* in their undergraduate or graduate syllabuses, as I have done in the past fifteen years, albeit struggling with occasional notes, material or research articles that may well advance the frontiers of economic research but are hardly likely to fire the interest of students, professionals and policymakers.

This is why I have agreed with the publisher's project to produce a short handbook for the Pivot series focusing on the essential toolbox of pension economics. While the brevity of the volume may make it approachable, it has an inevitable side effect. Many colleagues will be surprised to find a number of topics missing from the table of contents, such as utility maximization or behavioural theory in retirement decisions, the role of pension design in possibly fostering economic growth and many other fundamental issues that should be covered by a longer book. I must also apologize to the many scholars and experts in the field who may search in vain for their important works in the list of references. This is another drawback of writing a succinct textbook. On the other hand, if the aim of attracting the interest of students, professional experts and policymakers to the *essentials* of such a relevant topic should be achieved, then the many opportunity costs and indeed the effort involved in producing such a succinct volume will be amply repaid. Once provided with the toolkit to understand the pros and cons of different pension designs, interested readers will have plenty of opportunities to access the many papers and books focusing on the nuances and more advanced topics of pension economics.

Bearing the entire responsibility for any shortcomings to be found in this book, I wish to thank for their invaluable comments and suggestions, but also for the many private and public conversations that have furthered my understanding of pension systems, Mirko Bevilacqua, Robert Holzmann, Edward Palmer and Ole Settergren. My thanks, are also due to Graham Sells for his patient English language editing. Let me finally express my gratitude and admiration to my mentor Sandro Gronchi, who introduced and accompanied me through the intricate issue of pensions after we met, in 1997, at Sapienza University of Rome where I was assistant professor and he full professor of economics. Sandro's approach to *pensions* from the standpoint of a brilliant economist and intellectual who understands the need to combine theoretical rigour and practical relevance eventually moulded my desire to devote theoretical research to the many problems connected with pension provision, solution of which can significantly increase the well-being of so many human beings all around the world.

Rome, Italy — Sergio Nisticò

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CHAPTER 1

Introduction

Abstract In most cases, reaching old age implies losing the human capital out of which labour services can be given in exchange for income. Saving during adulthood is therefore necessary to accumulate other forms of capital, real or financial, whose services the 'old' can give in exchange for the goods and services being produced by the 'young'. Mandatory pension plans represent an efficient response to myopic behaviour leading individuals to save too little for their old age. Moreover, even forward-looking individuals face difficulties in drawing up an efficient saving plan for retirement, since they would be inclined to protect themselves from the risk of having no income in the event of surviving well beyond the average age of death.

Keywords Human capital · Retirement savings · Old age · Mandatory pension plans

In a pure market economy hypothetically populated by selfish individuals, access to the goods and services supplied entails drawing on wages derived from the sale of labour services, or rents or interest thanks to ownership of valuable assets, or alternatively using the proceeds from selling those assets. In any case, owning one or the other form of capital, human, real or financial, is a necessary condition to satisfy one's needs and wants. Such a condition is necessary but not sufficient, given that the mere possession of either form of capital does not ensure that there

S. Nisticò, *Essentials of Pension Economics*,
https://doi.org/10.1007/978-3-030-26496-3_1

is a demand for it. The typical case is when, in a downturn, workers' human capital and firms' physical assets remain unemployed, thus depriving their owners of any source of income. Besides these, there are cases in which individuals lack any form of capital irrespective of market failures. Take the case of people lacking financial or real assets whose health or physical conditions are so poor as to reduce the 'marketability' of their human capital, thus requiring some sort of solidarity, in the past possibly provided within the family or by charitable organizations. It is, therefore, hardly surprising that modern market economies have developed specific institutions designed to replace those occasional and unsystematic forms with a State-managed system of solidarity. It was in particular with the development of the *welfare state* that many countries around the world decided to create a series of programmes to ensure a minimum level of well-being for all the members of society. Free access to public schooling and health care ensures that human capital is fairly distributed among individuals regardless of 'initial conditions'. Moreover, the welfare state provides individuals with monetary benefits whenever they are deprived of their usual sources of income. Disability and unemployment allowances are typical forms of subsidies protecting individuals when accidents and adversities reduce the marketability of their human capital during working age. Moreover, the marketability of human capital progressively decreases with age.

In fact, reaching old age has its blessings but in most cases it also eventually leaves individuals with no marketable human capital and, hence, with no labour services to sell. In principle, rational and forward-looking individuals could voluntarily 'save for retirement', i.e. accumulate assets during working age, such that during retirement the absence of proceeds from the worn-out human capital can be offset by those from physical or financial capital. However, individuals are not immune to myopic behaviour, leading to saving too little for retirement. Conversely, even forward-looking individuals face difficulties in drawing up an efficient saving plan for retirement. In fact, since most individuals are risk-averse, they would be inclined to protect themselves from the longevity risk, i.e. from the risk of having no income in the event of surviving well above the average age of death. On the other hand, by definition, only a part, let's say a half, of the members of each birth cohort can be of above-average age at death, the remaining half being destined to be of below-average age at death. As a consequence, if all individuals are risk-averse, given the uncertainty surrounding individual life expectancy, many of them will have *ex post* saved too much, thus (regrettably) leaving assets as bequests on their demise. One of the efficient solutions to this problem is risk-pooling through insurance against the longevity risk. The higher the number of individuals who join the insurance pool, the easier will it be to match the required annual savings rate (the premium) with the average age at death, thus avoiding saving too much. In fact, myopic behaviour and risk-pooling account for the widespread public, compulsory old-age insurance plans paying a pension, i.e. a lifelong annuity to all individuals reaching retirement age.

This book deals with the economic problems involved in the organization of a pension system. Chapter 2 spells out the basic concepts and the meaning of the various expressions used in the jargon of pension economics. Chapter 3 is devoted to a short and fairly elementary technical digression. Chapter 4 describes the essential properties of alternative pension designs in highly simplified economies where individuals are assumed to live two periods only, one as active workers and one as retired. Chapter 5 analyses the redistributions implied by different pension designs in the context of real lifetime spans while Chapter 6 addresses population ageing and how the different types of pension plans can meet the challenge. Chapter 7 concludes.

CHAPTER 2

The Basic Concepts

Abstract Consumption smoothing throughout the lifecycle and poverty preventing are two fundamental goals of pension systems. The former has traditionally been played by Bismarckian, earnings-related pensions, the latter by Beveridgean, flat-rate pensions. Alternatively, pensions can be computed through personal accounts recording either monetary contributions paid in the system or 'points' earned in one way or another before retirement. Fully funded systems always have enough assets invested in the financial markets to extinguish all their liabilities towards workers and retirees. Pay-as-you-go (PAYG) systems have no assets and extinguish their liabilities by resorting to the contribution revenues. Under changing economic and demographic conditions, defined benefit systems ensure solvency by adjusting the contribution rate charged on active workers, whereas defined contribution systems automatically adjust expenditures to the varying contribution revenues.

Keywords Consumption smoothing and poverty preventing · Bismarckian earnings-related and Beveridgean flat-rate benefits · Pay-as-you-go and funded pension systems · Defined benefit and defined contribution pension systems · Personal accounts

The idea of a compulsory old-age insurance plan managing the longevity risk had already emerged with the enlightened thinkers of the late eighteenth century. According to Condorcet the Government should

S. Nisticò, *Essentials of Pension Economics*,
https://doi.org/10.1007/978-3-030-26496-3_2

secure "to him who attains old age a support, arising from his savings, but augmented by those of other persons, who, making a similar addition to a common stock, may happen to die before they shall have the occasion to recur to it" (Condorcet 1795, p. 331). It took around a century for Condorcet's idea to be put in practice, given that the first public, compulsory pension system based on insurance principles was set up in 1889 when the Prussian Chancellor Otto von Bismarck extended the compulsory insurance for work accidents and sickness, introduced a few years before, to invalidity and old age. In the Bismarckian model both pensions and contributions—the latter paid by both workers and employers—are based on individual earnings. The Bismarckian earnings-related principles began to be adopted by other European countries in the early twentieth century and in the United States after the Great Depression with the Social Security Act of 1935, one of the most significant elements of Roosevelt's New Deal. A few years later, a different approach, based on flat-rate benefits and contributions emerged in the UK with the well-known Government Report on *Social Insurance and Allied Services* conceived and drafted by the economist and civil servant William Beveridge (1942). Most of the ideas included in the Beveridge Report were to be implemented a few years later with the 1946 National Insurance Act introducing the Basic State Pension in the UK.

2.1 The Different Aims of a Public Pension System

Bismarckian earnings-related pension systems provide individuals with a tool for *consumption smoothing* throughout the lifecycle (see Focus 2.1). On the other hand, Beveridgean flat-rate pensions aim to provide only a minimum pension, a safety net *preventing poverty* during old age. Moreover, pension systems may aim to achieve *redistributions* during old age in favour of 'deserving' groups of the population (Barr 2006). In the past, protected categories have included the armed forces, civil servants, and women or workers employed in particular industries. Besides these redistributions that are transparently declared, it is not infrequent for redistributions to be generated opaquely by the complexity of the pension system.

Universal protection against poverty during old age as well as compulsory insurance against the longevity risk so as to achieve consumption smoothing raises the issue of how much *paternalism* and *protection* is optimum for society. Shouldn't individuals be left with freedom to choose whether and how much they want to save for old age? Isn't excessive and widespread protection an incentive for individuals to behave as free-riders, leaving society with the burden of their improvident conduct? In his report, Beveridge warned that "the State in organising security should not stifle incentive, opportunity, responsibility; in establishing a national minimum, it should leave room and encouragement for voluntary action by each individual to provide more than that minimum" (Beveridge 1942, pp. 6–7). Beveridge favoured universal flat-rate cash payments 'in return for' a flat-rate contribution, opposing means-testing precisely because it clashes with the contributory principle. In fact, "payment of a substantial part of the cost of benefit as a contribution irrespective of the means of the contributor is the firm basis of a claim to benefit irrespective of means" (ibid., p. 12). The Beveridge Report sketched an overall social insurance system covering several risks together. The idea of a comprehensive insurance system has inspired many social security plans around the world that generally provide an overall insurance for old age, disability and survivor (OASDI). It is worth mentioning, however, that the inclusion of a lifelong pension to be paid to the surviving spouse of the insured, now widespread around the world, was explicitly rejected by Beveridge, whose proposal was not to grant permanent pensions to "widows of the working age without dependent children" but rather to provide them only with "a temporary benefit at a higher rate than unemployment or disability benefit, followed by training benefit when necessary" (ibid., p. 11).

In what follows we will disregard survivors and disability insurance and concentrate on the economic properties of alternative retirement plans. In particular, we will focus on the conditions for old-age pension systems to be financially solvent and the extent to which they achieve redistributions both among individuals with different career patterns belonging to the same cohort (or generation), and among individuals with similar career patterns belonging to different cohorts. In the jargon of pension economics, the issue of redistributions is often referred to with the highly debatable word 'fairness', preceded by the label intra-generational when assessing redistributions among members of the same birth cohort or intergenerational when assessing how the system performs through time toward different cohorts.

2.2 Computing Old-Age Benefits

When setting up a public, compulsory pension system, policymakers need, among other things, to define a rule to determine the amount of the annuity to be paid to retirees, the choice depending on the specific aim they want to achieve.

Typical Beveridgean, poverty-preventing pension benefits amount to the ‘subsistence’ wage or to a fraction of average earnings in the economy.

On the other hand, a typical Bismarckian pension annuity aims to ensure consumption smoothing. It is essentially earnings-related, i.e. a share of the individual’s reference earnings, computed as the average of a number of either end-of-career or best earnings, while the share is generally a multiple of workers’ years of service. The multiple, say 2%, is called the accrual rate, representing the percentage of reference earnings paid as a pension for each year of service. While last earnings rules are meant to ensure some sort of continuity in the standard of living when moving from work to retirement, best earnings rules protect the workers whose earnings tend to decline in the last years. Given their scope, after being first awarded to any new retiring worker, earnings-related pensions should be adjusted or indexed, annually, according to growth in average earnings, whereas flat-rate pensions are generally indexed according to the cost of living.

A variant of earnings-related rules can be found in the ‘point systems’, which assign to each insured worker a certain number of points for each year of work. Points can be accumulated in proportion either to individual earnings (as in the German system) or to contributions (as in the French version). At retirement, the first pension is calculated through multiplication of accumulated points by the value of a point, arbitrarily chosen each year by the policymaker or by a board of actuaries. The yearly percentage change of the point value also determines the rate at which pensions already being paid are indexed through time.

An alternative way of computing a pension benefit is to use personal accounts, i.e. to open an account in the name of each insured worker recording all their old-age contributions plus interest. Credits can be earned also for non-marketable deserving activities, such as child-rearing or community service. At retirement, the balance of the account is converted into a pension through division of the balance by a ‘divisor’ reflecting retiring workers’ life expectancy. Pensions are then indexed taking into account the interests maturing on the account after retirement. The essential property of personal accounts is that they can avoid any type of explicit redistributions by crediting a uniform annual rate of interest to all accounts, thus limiting redistributions to those, inevitable in old-age insurance, from the shorter-lived to the longer-lived. In fact, while the 'withdrawals' of the former will fall short of their contribution balance, those of the latter will exceed their balance.

2.3 Measuring Pension Adequacy

Assessment of the adequacy of pensions awarded to retiring workers should also be made in the light of the aim that the pension system seeks to achieve. One and the same average pension disbursed by a pension system can be considered either generous, if its aim is merely to prevent poverty among the elderly, or poor if disbursed by systems whose aim is to achieve consumption smoothing, especially in countries where average earnings are high. Moreover, seemingly low pensions could turn out to be fairly generous in a country where the elderly have access to various forms of support such as free health services, old-age care and housing allowances.

Still, pension adequacy needs to be measured in one way or another and identifying a reliable form of measurement is no easy task, especially if the idea is also to compare pension systems of different countries. This is why the benefits provided in any country by its public pension system should, first of all, be related to workers' earnings in the same country, thus separating the merits of the pension systems from those of the economy as a whole. A typical measure of adequacy from the viewpoint of consumption smoothing is the replacement rate, i.e. the ratio of the pension to some specification of workers' earnings. If the pension is related to the last annual earnings, the replacement rate measures the percentage of pre-retirement consumption opportunities that the pension ensures for retiring workers.

On the other hand, by taking the ratio of the economy-wide average pension to the economy-wide average earnings, generally referred to as the pension level, it is possible to assess the extent to which a pension system protects the elderly from the risk of poverty. According to the International Labour Office (ILO), this is achieved in countries where the public pension system ensures a pension level of at least 45%, where the denominator is the wage of an ordinary adult male worker.

Data on the average pension disbursement might hide significant differences among individual retirees. Such is the case of pensions awarded in the same year to retirees belonging to the same birth cohort, but also of retirees with the same career pattern belonging to different cohorts.

Given that earnings profiles differ among occupations in the same countries and for similar occupations across different countries, comparing pensions to the very last earnings overestimates the replacement rate for career profiles with earnings that decline in the last years of activity, and vice versa. Many institutions, e.g. the OECD (2017, Ch. 4), compute 'theoretical' replacement rates for hypothetical workers distinguished according to average lifetime earnings, career length and earnings growth, the benchmark being a hypothetical worker starting work at 20 years of age with a full contribution record (e.g. 40 or 45 years) and average earnings throughout the career. Relating pensions to the average of some last earnings, e.g. 5 or 10, significantly reduces the dispersion of individual replacement rates awarded by any pension system.

Computing replacement rates through division of pensions and earnings gross of taxes implies a distortion whenever the tax structure on pension benefits differs from that on workers' incomes. Accordingly, we should distinguish between gross and net replacement rates. Net replacement rates are typically higher than the corresponding gross rates given that pensions are generally lower than the last years' earnings and most countries have progressive taxation which reduces the tax burden after retirement. The difference between net and gross replacement rates increases with the level of the contribution rate given that retirees are exempted from paying old-age contributions.

2.4 Financing Pension Systems

Universal protection from the (longevity) risk of poverty during old age is costly. Both Bismarckian and Beveridgean systems are generally designed according to the above-mentioned insurance principles, i.e. they collect contributions, the premiums to finance lifelong pensions awarded to those who outlive retirement age. Generally, old-age insurance premiums (contributions) are charged to both workers and employers, though in varying proportions around the world (see Focus 2.1). Premiums are generally collected in percentages of workers' earnings (the self-employed are required to pay their share as both employee and employer), possibly up to a specified ceiling. The more the system is geared to protecting from the risk of poverty rather than ensuring consumption smoothing, the lower is the ceiling above which contributions are not due. In fact, low contribution ceilings significantly reduce the replacement rates ensured by the public pension scheme, requiring either the presence of a compulsory workplace plan or workers to contribute to supplementary, private old-age insurance.

The logic behind charging workers and employers a contribution rate to finance pension expenditure is that old-age insurance should be considered part of the labour costs. Quite often, however, the State also contributes to the pension scheme, thus placing a part of the burden on general tax revenue, which derives also from factors of production other than labour. It is worth stressing that, in a market economy, the entire burden of contributions may fall on workers despite the formal participation imposed on employers (Nisticò 2013). In fact, many authoritative studies confirm that employers are generally able to shift the entire burden onto the workers, i.e. to reduce salaries in proportion to the overall payroll taxes charged on firms.

The extent to which a pension system can provide generous pensions at a low contribution rate, whether paid by the workers, the employers, or drawn from the general tax revenue, depends on both demographic and economic factors. Demography matters because its trend determines the number of retirees (beneficiaries) relative to the number of adults (potential contributors) in each period of time. Also, from the viewpoint of each birth cohort, the longer the period of eldership relative to that of adulthood, the higher the ratio between required savings and benefits. Adulthood and eldership are obviously relative concepts, since the age at which individuals are considered adult or elderly varies both cross-sectionally, i.e. among different countries in the same period of time, and longitudinally, i.e. over different historical periods in the same country. Nowadays, in most OECD countries all people aged 15–65 are considered part of the adult population, while the over 65-year-olds are considered elderly, despite the fact that many people retire over the age of 65 while labour market entry is postponed well beyond 15 years of age in most industrialized countries. The ratio of elderly to adults is referred to as the old-age dependency ratio, and it is increasing in most OECD countries. The main determinants of the trend in the old-age dependency ratio are the fall in the fertility rate, roughly definable as the number of births per woman, and the increase in survival rates at late ages, also determining life expectancy at retirement, i.e. the expected duration of benefits. In many less developed countries with very high fertility rates and low life expectancy, both the probability of individuals reaching retirement age and the expected duration of the benefits in favour of the few reaching retirement are low. In the richer countries, where fertility rates are very low and life expectancy high, it is quite the opposite. Both life expectancy and health conditions are improving in all countries, which implies that the dividing line between adulthood and eldership is also shifting, thus leading to a generalized increase in the age at which people have the right to retire.

On the other hand, fundamental economic factors, such as average earnings and employment rates, measuring the employed share of working-age population, also matter in that they determine the evolution of the contribution base. In fact, in countries with high employment rates and productivity growth, the burden of old-age pensions can, *ceteris paribus*, be borne by a high percentage of adults whose average earnings are substantially growing over time. This is true also from the viewpoint of any single birth cohort of individuals; the higher the average earnings and periods of contributions of cohort members during adulthood turn out to be, the lower will be the ratio between the yearly contribution rate and the value of pensions during eldership. As will be clear below, when members of each birth cohort save for their own retirement without relying on other cohorts' contributions, the fundamental economic factor determining the cost of acquiring a pension during retirement is the rate of return with which the market rewards savings.

The question of whether old-age insurance is managed by requiring each birth cohort to finance its own prospective benefits or all cohorts of active workers to share the burden of benefits to be paid to the cohorts of retirees is of the utmost importance in pension economics, pointing to the fundamental distinction between funded and pay-as-you-go (PAYG) systems. A funded pension system is organized in such a way that the contributions paid by each birth cohort during working age are accumulated in a fund, out of which pensions will be paid when the cohort members reach retirement. If promised pensions and charged contributions are computed according to rigorous actuarial methods, the system can be said to be fully funded, meaning that each cohort's share of the fund will be exhausted on the death of the 'last' cohort member. Conversely, in pay-as-you-go systems the State enforces a sort of intergenerational agreement requiring all active workers belonging to different birth cohorts to accept that, year by year, their contributions pay for the pensions to be disbursed to existing retirees, also belonging to different birth cohorts. The proposed agreement is to be renewed in the future, thus ensuring that pensions to present workers will be paid out of contributions charged on future active cohorts. Whereas fully funded systems can be terminated in any moment in time by using the funds to extinguish the existing obligations, those relying on pay-as-you-go financing cannot, since they need to renew the agreement with current and future generations of workers indefinitely to meet their obligations towards 'past' cohorts of workers. It is intuitive that pay-as-you-go financing, requiring an authority to enforce renewal of the intergenerational agreement, is exclusive of public pension systems. On the other hand, funding is the only possibility for private systems to guarantee contributors the property rights on their future pensions. However, from the viewpoint of active workers, benefits are only expected rather than guaranteed, considering the risk that "the pension might not actually be paid at all on account of the pension scheme becoming insolvent" (Blake 2006, p. 2). Public, pay-as-you-go systems incur the political risk of policymakers refraining from making the necessary adjustments when they clash with electoral consensus. Funded schemes, whether private or public, are exposed to the financial risk that the accumulated funds will be depleted due to mismanagement or major financial shocks.

Focus 2.1: How Much Should We Save for Retirement?

As mentioned in the Introduction, rational and forward-looking individuals could be left with the freedom to choose how much to save for their retirement. Disregarding inherited and bequeathed assets for the sake of simplicity, according to the life-cycle theory (LCT) individuals plan to distribute their lifetime earnings over the various periods of life to achieve 'consumption smoothing'. In order to do so, the savings rate is positive (wealth is accumulated) before retirement whereas it is negative (wealth is depleted) afterwards. The LCT was developed by Franco Modigliani together with his graduate student William Brumberg. The collaboration between the two produced two fundamental papers, the first of which appeared in the early 1950s (Modigliani and Brumberg 1954) while the second remained an unpublished manuscript because of Brumberg's premature death until it was included (Modigliani and Brumberg 1980) in the second volume of Modigliani's collected papers.

In fact, public pension systems can be considered the instruments through which individuals accept being 'lashed to the mast', i.e. to have some compulsory saving imposed on them, aware that their weak willpower could lead to the irrational behaviour of consuming too much during adulthood and too little during eldership. However, the question arises as to how much individuals should be forced to save. If the public pension scheme were to absorb all savings dictated by LCT behaviour, an individual with no assets, expecting her income to grow in line with the interest rate, should save every year a fraction of her income equal to the expected duration of retirement relative to the overall consumption period (including both earning and retirement spans). In fact, assuming that the interest rate is zero and that "our hypothetical prototype plans to consume his income at an even rate throughout the balance of his life" (Modigliani and Brumberg 1954, p. 397), if he starts his career expecting to earn 60,000£/year for 40 years and to live for 20 years after retirement, as the present trend in life expectancy in OECD countries suggests, he should save one third, 20/(40+20), of his income for 40 years to be able to consume 40,000£/year throughout the 60 years of his life. However, consumption smoothing should not be interpreted strictly, as in the last example, and the LCT recognizes that different individuals tend to have different preferences about the specific shape of the planned smoothing, such that a friend of our prototype might prefer to consume more during adulthood than during eldership. This is why many public pension systems impose a much lower contribution rate than the one (33%) opted for by the LCT prototype and leave individuals the task of topping up savings for retirement according to their preferences, a sort of compromise between freedom and paternalism. Moreover, contributions to old-age insurance are generally charged only on the share of earnings below a pre-set ceiling, thus showing that poverty prevention is often the main goal of public pension plans rather than pure consumption smoothing. Indirect evidence of the soundness of the LCT derives from the observation that recourse to voluntary contributions to private pension plans is the norm in countries with a low ceiling and a low rate of contribution to the public, compulsory pension system. Such is the case, among others, of the United States, where the contribution rate for old-age insurance is 10.6%, and of the UK, where the flat-rate State pension is only £165/week and, as of 2019, the minimum contribution rate to a workplace pension scheme with automatic registration is 8%. Conversely, subscription to supplementary pension plans is negligible in countries with a very high rate of contribution to the public scheme, such as Italy, where the 33% compulsory contribution rate entirely absorbs the LCT theoretical figure. Mandatory contribution rates in most European countries and Japan are around 20% of earnings (OECD 2017).

In fact, many public pension systems started in the early 1900s as funded plans but had to switch to pay-as-you-go financing as the only feasible way to guarantee retirees the promised annuity when, after the Second World War, depletion of the pension fund became evident. Switching from funding to pay-as-you-go financing is always an option, whereas the opposite is no easy task given that it means requiring active cohorts to accumulate their contributions in a fund, thus depriving existing retirees of their promised pensions. The notorious instance of shift from pay-as-you-go to funding was in Chile where, in 1981, the State budget was drawn on to keep paying existing pensions while new workers were required to contribute to privately managed pension funds.

2.5 The Financial Indicators of a Pension System

Just as bank deposits represent liabilities, i.e. money that banks owe to depositors, so old-age contributions also represent liabilities, i.e. money that the pension system owes to insured workers. Unlike the case with bank depositors, who can claim their money back at any time, the credit of insured workers becomes liquid only when and if they reach retirement age. Moreover, except for the rare cases of lump sum benefits, pension systems extinguish liabilities by awarding a lifetime annuity. At any point in time, the liabilities of a pension system measure the value of the benefits to be disbursed in the future to both active and retired workers.

In order to assess the system's liabilities towards the latter, their pensions already being paid, life expectancy determined by survival rates at the various ages after retirement, as reckoned by mortality tables produced by National Statistical Offices, is the crucial element to be considered for measurement. In turn, liabilities towards active workers also depend on the rules determining the credit attributed to existing insured workers for each unit of money they paid in as contributions and/or for each year of service. Given that many public pension systems tend to generate redistributions, liabilities may correspond only loosely to paid-in contributions and may even arise also regardless of contributions, as for instance when individuals earn pension credits for deserving activities such as periods of child or elderly care.

On the other side of the balance sheet, pension systems should record their assets. The stock of assets at the moment of evaluation is the result of the past differences between the two flows of revenues and expenses. The flow of interest earned through fund management constitutes an additional component of the system's revenues to be added to the contribution flow. Whenever the flow of revenues exceeds the flow of pension disbursement and management costs, assets are accumulated (invested) in a fund.

The extent to which assets match liabilities determines the system's degree of funding, measured as the ratio of assets to liabilities. A system can be said to be fully funded if, at the moment of evaluation, its assets are exactly equal to its liabilities, the degree of funding thus equalling one. At the other extreme, the degree of funding is zero for systems with no assets. Systems with no assets have to rely entirely on PAYG financing to meet their obligations towards retirees. On the other hand, PAYG systems can be said to have some form of assets, i.e. claims on future workers' contributions (see Focus 2.2), the main difference with funded assets being that they are not invested in the financial markets. This is why funded systems can be referred to with the label 'financial' as opposed to PAYG systems, which can be labelled as 'non-financial' (Gora and Palmer 2004).

As mentioned above, many public pension systems started with the ambition to remain fully funded but had to switch to PAYG financing when their assets were depleted, and their funding dropped to very low levels or even to zero. Conversely, after the Second World War, given the ample growth in earnings and high ratio of contributors to retirees, for several years the contribution flow of many public systems exceeded their pension expenditure, thus leading to increased funding.

The degree of funding may show a cyclical pattern, increasing during periods of positive employment growth and temporary high ratio of contributors to retirees and decreasing a few decades later when the high number of contributors, especially when survival rates are on the increase, translates into a high number of retirees. Thus, a positive but cyclical degree of funding does not alter the PAYG financing structure of public systems, since the reserves behave as a buffer fund expanding and contracting according to the cyclical pattern of expenditure and revenues. At present, the contraction of buffer funds, where they still exist, is further exacerbated by the fall in the fertility rate registered in the past few decades in most OECD countries, which reduces the contribution revenue for any given employment rate. Buffer funds should never fall below a critical threshold since they ensure the system's solvency even when the system's revenues fall short of expenses. The ability of the reserves to ensure solvency is measured by the fund ratio, resulting from dividing reserves by the system's expenses, showing for how many years the system could disburse benefits in the absence of revenues. For instance, the US social security Board of Trustees considers a 100% fund ratio as the benchmark to test the financial adequacy of its trust fund (see Focus 2.2).

Finally, it is worth mentioning that, regardless of the system's solvency, pension expenditure in any given country can be measured against the country's Gross Domestic Product (GDP). The ratio of pension expenditures to GDP shows the extent to which the needs of the retirees are being satisfied against the needs of the younger generations and against other possible uses of the resources allocated to their financing (Barr and Diamond 2006).

It should be clear from the above that the flows of revenues and expenditure in a given year, or the stocks of assets and liabilities in a moment of time, do not tell us very much about the solvency of a pension system, whose liabilities might come due at a much later date than the moment of evaluation. This is why the financial position of pension systems is monitored by means of actuarial balances whose ultimate aim is to assess whether current rules need to be changed in order to ensure solvency over a reasonably long period of time (see Focus 2.2). By contemplating the 'likely' evolution of the structure of the insured population, of their earnings during adulthood and of their pensions during eldership over a long enough period of time (75 years in the United States), actuarial balances assess whether there will be a moment in time in which, given the expenditure generated by the need to redeem the liabilities that reach maturity year by year, the system's assets will be exhausted and the contribution revenue will fall short of pension expenditure. Whenever this situation crops up, prompt adjustment of the system's rules is called for, aiming either to increase the contribution rate or to reduce the liabilities that will reach maturity over the horizon covered by the actuarial balance. The typical case of need for such adjustment is that of having, one way or another, to reduce the liabilities towards the workers belonging to the populous baby-boom cohorts of the 1950s and 1960s, whose contributions have significantly exceeded pension expenditure for many years, but are now going to claim their pensions precisely when the number of contributors is particularly low, due to the fall in fertility rates registered in many countries as of the early 1980s.

Workers' and retirees' claims on present and future governments, if not backed by corresponding assets or buffer funds, are often referred to as the governments' 'implicit pension debt', where the epithet 'implicit' evidences that the creditors are not bondholders of the 'explicit' government debt. The other side of the coin of the implicit debt is the 'pension wealth' of insured workers and retirees. As will emerge clearly from the analyses carried out in Sects. 4.2 and 4.5, the financial sustainability of the implicit debt, as in the case of governments' outstanding debt, does not depend on the ability of the system to instantly redeem it, i.e. to be fully funded, but rather on the sustainability of the interest rate that the system pays on its liabilities. In fact, regardless of the social importance or possible redistributive aims of pension systems, and regardless of whether the system relies on funding or PAYG financing, to assess their financial solvency, overall pensions should be seen as paying back to insured workers their 'pension wealth' i.e. their claims on the system's resources according to the current rules. Since the workers' pension wealth can be split into two components, the contributions paid and the interest matured, it is important to understand that the system's rules imply the system's interest rate and that the rate is the crucial variable determining the system's long-run solvency. The more generous the pensions awarded are relative to the contributions paid in by workers, the higher will be the system's interest rate, and vice versa.

Focus 2.2: Assessing the Solvency of a PAYG Pension System Through 'Actuarial Balance' or Double-Entry Bookkeeping

The 1935 Social Security Act that established compulsory old-age insurance in United States also established the Social Security Board in charge of reporting to Congress on, among other things, the financial operations of the old-age and survivor insurance (OASI) Trust Fund. In its annual report, the Board first provides an update of the relevant figures, including the number of beneficiaries and contributors, the stock of reserve at the beginning and at the end of the calendar year and the cash flows responsible for any variation in the stock, income from contributions and interest, on the one hand, and benefit expenditures on the other. The report goes on to provide both short-term and long-term projections—the former covering a 10-year horizon, the latter extending over a 75-year period—of the relevant figures under current legislation. Both projections are run for three different scenarios, the intermediate one reflecting the Trustees' best estimate while the low-cost and high-cost scenarios are produced to cope with the uncertainty surrounding both demographic and economic variables. Projecting the system's income and expenditures over such a long period of time entails making assumptions about the determinants of their future development, i.e. about the number of retirees and the amount of their benefits, on the one hand, and the number of workers covered and their earnings on the other In fact, compiling an actuarial balance means making assumptions for the whole projection period on the trends in fundamental demographic factors—the fertility rate, the change in survival rates at all possible ages, net immigrations—and economic indicators—productivity and average growth of earnings, consumer price index, unemployment rate and the interest rate on the trust fund assets. These assumptions differ for the three alternative scenarios. According to the results of the projections, the Board formulates recommendations to lawmakers to adopt the necessary remedies for the old-age insurance to be able to protect also future generations of workers. The short-term findings contained in the last report (Board of Trustees 2018) show that, under the intermediate assumptions, for the first time since 1982, in 2018 expenditure will exceed total income, including interest on the fund assets, thus leading to a decline in the system's reserves. Despite the negative net inflows, the fund ratio will remain above 100% within the short-term horizon. However, the long-term projections are less comforting. Costs are projected to exceed income for the entire period, leading to a depletion of the OASI fund in 2034, calling for either an increase in the contribution rate or a substantial reduction of the benefits, or indeed some combination of the two. The corrections, the Board adds, should be adopted sooner rather than later in order to distribute the burden of adjustment over a higher number of generations since deferral would imply more drastic corrections imposed on fewer birth cohorts. The fact that, so far, the US Congress has not taken any significant correcting measure is a typical instance of political risk.

A wholly different approach to assessing and securing financial solvency for the system has been legislated by the Swedish Parliament in 2000. The mechanism relies on double-entry bookkeeping presenting a yearly income statement and balance sheet. The income statement reports all flows—contributions plus interests on the fund assets, on the one hand, and pension disbursements plus administrative costs on the other—affecting the value of the fund assets, as well as the flows affecting the system's liabilities, such as newly awarded pension credits to workers plus indexation of existing pensions, on the one hand, and pension disbursement on the other. The balance sheet shows the resulting stocks of assets and liabilities at the end of the year, which are thus assessed "mainly on the basis of events and transactions that are verifiable at the time of valuation" (Pensionsmyndigheten 2018). The Swedish accounting system is particularly innovative in that, despite relying essentially on PAYG financing, it shows not only the fund assets but also the contribution assets (CAs), defined as the pension liabilities that can be sustained by the current contribution flow, were such a flow to remain constant in the future, together with the age-structure of the insured population. The idea of the Swedish reformers was that there must exist CAs for PAYG systems given that, when designed to be solvent, they can rely on contributions to extinguish their liabilities 'as they go'. To transform the flow of contributions into a stock value, it is multiplied by the number of years it takes for a unit of such a flow to be reimbursed. The longer the time span, called turnover duration (TD), turns out to be, the higher will be the level of existing liabilities that can be sustained by the current contribution flow. By repeating measurement of CAs annually it is possible to take into account historical changes in the fundamental demographic and economic factors such as fertility, age-related net migration and mortality, contribution base and age-related average incomes, while avoiding projections. The CAs are then added to the fund assets, and whenever the sum of the two falls below liabilities, an automatic balance mechanism is triggered that reduces the growth rate of liabilities to ensure long-run balance. The automatic nature of the adjustments protects the Swedish system from political risk.

Thus, besides using actuarial balances, the financial solvency of pension systems can be deduced 'logically' from economic analysis by identifying the sustainable interest rate for each pension system according to its degree of funding. If the system's rules are such that the system's actual rate on existing liabilities exceeds its sustainable level, the liabilities will grow faster than they 'should', thus leading to a fall in the system's degree of funding, eventually reaching the point where it becomes zero, while the contribution revenue will not cover pension disbursement. Conversely, the system's degree of funding rises whenever the interest rate on existing liabilities falls short of the sustainable rate.

2.6 Ensuring Solvency Under Uncertainty

Given the frequent mismatch between the trends in revenues and expenses, pension systems should specify the correction mechanisms to be implemented whenever economic or demographic factors threaten their solvency. Due to these adjustments, both workers and retirees are exposed to the risk that the cost-benefit ratio of participating in the system be, *ex post*, different from what it was *ex ante*. Borrowing from the vocabulary typical of occupational pension plans, generally supervised by Government agencies that ensure that their liabilities are fully backed by a reserve fund, the pension jargon distinguishes between defined benefit (DB) and defined contribution (DC) schemes.

In DB plans, the employer is bound by contract to pay each insured worker reaching retirement age an annuity computed according to a specified rule, e.g. a flat-rate or earnings-related pension. The plan also specifies the indexation rule of the benefit, which is typically adjusted either according to the cost of living or to average earnings growth. Whether or not workers are also required to contribute to the plan, according to the DB legal arrangement it is the employer who, as sponsor of the plan, bears the risk that additional financial resources be needed to meet the outstanding obligations, the sources of risk for the employer being both economic, e.g. the return on the invested funds, and demographic, e.g. increases in life expectancy at retirement. To the extent that any worsening of a firm's financial position ultimately affects its employees through lower wage growth, a DB plan can be said to protect retirees and shift the burden of adjustment onto the active workers. Conversely, if there is an upturn in their financial position, DB plans do not distribute to retirees any portion of the gains that, by enhancing the firm's financial position, will in one way or another benefit current workers.

In DC plans, the employer is committed to paying all workers a fixed contribution, defined either as a percentage of their earnings or as a fixed sum, while the amount of the pension is not guaranteed but depends on the development of the financial and demographic indicators of the plan. In fact, whether or not workers are also required to pay a fixed contribution to the plan, according to the DC legal arrangement retirees bear the burden of any adjustment needed to respond to economic and/or demographic threats. This is why DC plans generally use personal accounts, ensuring that benefits reflect the contributions paid in and life expectancy of retiring workers. Neither the sponsor nor active workers are required to offset negative shocks affecting the financial position of the plan. Conversely, retirees will enjoy more generous pensions whenever the plan's financial position improves.

The DB–DC distinction has also been extended to public pension systems, despite the obvious fact that adjustments of either the contribution burden on active workers or the generosity of pensions in favour of retirees can be legislated by policymakers at any time. In fact, the DB–DC distinction, even if not as binding as it is in private pension provisions, is still used for public pension systems according as to whether or not their rules specify the amount of the pension benefit. Hence, the DB plans are committed to paying at retirement a pension specified either as a flat-rate amount or in terms of workers' earnings and years of service, whereas DC plans do not commit to such a promise and look to personal accounts to compute the pension annuity. As emerges clearly from the analysis carried out in the next chapters, DC systems using personal accounts can, if well designed, be solvent whatever the chosen contribution rate. On the other hand, in DB plans, adjustments of the contribution rate are required for the contribution revenue to suffice to honour the commitment with retiring workers (in the case of PAYG systems) or for the fund's assets not to fall short of liabilities (in the case of fully funded systems).

Given the strains deriving from demography and the slowing growth of earnings, starting from the late 1980s almost all DB public pay-as-you-go systems, after exhausting the opportunities to raise the contribution rate to continue paying pensions in accordance with the announced formulas, have initiated a period of socially costly parametric reforms aiming to curb pension expenditure. Typical parametric reforms are reduction of accrual rates, extension of reference earnings to career-long average and raising normal retirement age, which cuts expenditure on newly awarded pensions while curbing the indexation rate reduces expenditure on pensions already in payment. This is why the DB nature of most public pension systems should now be interpreted in the narrow sense of mere adoption of well-defined, but possibly temporary, benefit rules.

A sort of continuous discretional 'fine-tuning' of both contributions and pensions, thus escaping the DC–DB distinction, has been adopted by the French and German points systems which revise both the point value and the contribution rate yearly, in an attempt to distribute the burden of the necessary adjustments fairly between active workers and retirees. However, unless designed to mimic personal accounts, point systems cannot avoid opaque and regressive redistributions (Gurtovaya and Nisticò 2019).

To avoid continually retracting unsustainable promises and to remedy the regressive redistributions implied by DB, earnings-related rules (Gronchi 1995), in the early 1990s Italy and Sweden decided to implant the logic of personal accounts, hitherto considered exclusive of fully funded schemes, into the body of PAYG financing. Latvia, Poland and, more recently, Norway followed suit. The outcome of the implant is now known as the Notional or Non-Financial Defined Contribution (NDC) scheme. The emergence of the new scheme extended the spectrum of policy alternatives, since it is now clear that achieving the goals of fairness and solvency typical of personal accounts does not mean embarking on the costly and much-debated transition from PAYG to funding. This is why the NDC solution is attracting the interest of international institutions, such as the OECD and the World Bank, as well as policymakers, guided by the "desire to change the pension system and not merely to adjust parameters and rules" (Whitehouse 2012, p. 85) of the existing DB earnings-related schemes. In fact, with approximately six-year intervals as of 2003, three International Conferences have been devoted to the topic of NDC pensions. The first two were jointly organized by the World Bank and Sweden while Italy joined the other two sponsors for a third Conference held in the Autumn of 2017. With the participation of major scholars in the field and policymakers, the focus was on whether, and if so to what extent, the NDC model can be replicated in other countries and how its design and implementation can be improved. The three conferences gave birth to a total of five volumes containing almost 80 chapters authored by about 150 scholars and institutional policy experts (Holzmann and Palmer 2006; Holzmann et al. 2012, 2019), constituting an indispensable source of knowledge, inspiration and tools for readers interested in going beyond the essentials of the NDC system to be found in this book.

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CHAPTER 3

The Little Mathematics You Need

Abstract Growth rates and factors measure the change of a variable through time. The growth factor is computed by dividing the value of a variable at the end of a period by the value it had at the beginning of the period. Knowing the growth factor and the initial value of a variable, we can identify its future value. The present value of a variable equals its value at the end of the period divided by its growth factor. The growth rate of a financial capital during a period measures its Internal Rate of Return (IRR). When financial capital evolves through several periods because of sequential cash flows of opposite sign, its IRR can be computed only by trial and error.

Keywords Growth rates and factors · Present values and future values · Discounting · Internal rate of return

This chapter is a sort of technical interlude. It is based on elementary mathematics, necessary for an understanding of the functioning of alternative pension systems. We recommend it in particular to readers who tend to fight shy of mathematics. We are confident that they, but also the more skilled readers, will appreciate our effort at simplification.

S. Nisticò, *Essentials of Pension Economics*,
https://doi.org/10.1007/978-3-030-26496-3_3

3.1 Understanding Changes: Growth or Discount Factors and Rates

One of the main features of pension economics lies in focusing on 'changes' of different variables—e.g. contribution revenues, pension expenditures, employed or retired workers, average or total pensions and earnings—in a time interval of any given length. We refer to the interval as a 'period', or a 'year', and we assume that all relevant transactions, such as paying contributions or receiving pensions, or changes of status, such as being employed or retiring, take place in 'one shot' at the beginning of each period. With this assumption, we can avoid the complexities implied in measuring changes in continuous time consisting of infinitesimally small units.

Let us start by defining the following concepts:

x_t the value that variable x takes on at the beginning of period t;

x_{t+n}, the value that variable x takes on at the beginning of period $t+n$;

$x_{t+n} - x_t$, the change, or growth, of variable x during the time interval between the beginning of period t and the beginning of period $t+n$;

g_t^x, the growth rate of the variable x in period t, i.e. the change in each unit comprised in x during the interval between the beginning and the end of period t;

$1 + g_t^x$, the growth factor of the variable x in period t, i.e. the value that each unit comprised in x at the beginning of period t takes on at the end of the same period t.

The definitions given above imply the following relations among the values that the variable x takes on at the beginning of two adjacent periods, t and $t+1$:

$$A1.\, x_{t+1} = x_t \cdot \left(1 + g_t^x\right),$$

$$A2.\, x_t = \frac{x_{t+1}}{(1 + g_t^x)},$$

$$A3.\, \left(1 + g_t^x\right) = \frac{x_{t+1}}{x_t},$$

the latter of which implies

$$A4.\, g_t^x = \frac{x_{t+1}}{x_t} - 1.$$

Formulas $A1 - A4$ are the basic tools to analyse changes in economic variables.

*A*1 determines the *future value* of variable x at the beginning of period $t+1$ making use of the information on the value of the same variable x at the beginning of period t and its growth rate in the interval between the beginning of period t and that of period $t+1$. Let us suppose, for instance, that a retired worker receives a pension of £24,800 in year 2019 and that pensions are adjusted yearly according to the last available figures on the inflation rate, amounting to, say, 2.5%, or 0.025 in decimal figures, then the pension that the retiree will receive in 2020 equals its value in 2019 multiplied by its growth factor, as follows:

$$\text{£}24{,}800 \cdot (1+0.025) = \text{£}25{,}420.$$

*A*2 determines the *present*, or *discounted value* that variable x takes on at the beginning of period t using the information on the value of the same variable x at the beginning of period $t+1$ and of its growth rate during period t.

Suppose, for instance, that average earnings in 2019 amount to £40,600/year and that their growth rate between 2018 and 2019 has been 1.5%. With this information we can assess average earnings in 2018 by *discounting* the 2019 figure as follows:

$$\frac{\text{£}40{,}600}{1+0.015} = \text{£}40{,}000.$$

Note that the expression of the growth factor $\left(1+g_t^x\right)$ is referred to as the *discount factor* when used (at the denominator) to compute the present, or discounted, value as in the previous example.

Finally, with *A*3 and *A*4 we can determine the growth factor and growth rate of variable x during period t with the help of the information on the values that x takes on at the beginning of periods t and $t+1$. Supposing, for instance, that the contribution revenue of a pension system amounts to £156 billion in 2019 and had been £150 billion in 2018, we can compute its growth factor as:

$$\frac{\text{£}156{,}000{,}000{,}000}{\text{£}150{,}000{,}000{,}000} = 1.04$$

and its growth rate as:

$$\frac{\text{£}156{,}000{,}000{,}000}{\text{£}150{,}000{,}000{,}000} - 1 = 0.04 = 4\%.$$

Note that when using the latter formula with reference to the values that the financial capital takes on at the beginning of periods t and $t+1$, the resulting growth rate is generally referred to as the Internal Rate of Return (IRR) on the capital invested in period t (for one period).

The basic tools $A1 - A3$ can easily be extended to analysis of the change between two non-adjacent periods as follows:

$$B1.\, x_{t+n} = x_t \cdot \left[\left(1 + g_t^x\right) \cdot \left(1 + g_{t+1}^x\right) \cdots \left(1 + g_{t+n-1}^x\right)\right],$$

$$B2.\, x_t = \frac{x_{t+n}}{\left[(1 + g_t^x) \cdot \left(1 + g_{t+1}^x\right) \cdots \left(1 + g_{t+n-1}^x\right)\right]},$$

$$B3.\, \left[\left(1 + g_t^x\right) \cdot \left(1 + g_{t+1}^x\right) \cdots \left(1 + g_{t+n-1}^x\right)\right] = \frac{x_{t+n}}{x_t}.$$

where the expression in square brackets represents the overall, or compound, growth factor of the variable x between t and $t + n$ with $n > 1$.

$B1$ determines the *future value* of variable x at the beginning of period $t + n$ using the information on the value of the same variable x at the beginning of period t and of its growth rates in all n periods included in the interval between the beginning of periods t and $t + n$. Suppose, for instance, that a worker is hired in 2019 by a company for a salary amounting to £58,000/year for the first year and then growing by 5% (0.05), 7.5% (0.075) and 10% (0.1), respectively, in the three years to follow. The salary that the worker will be paid in 2022 corresponds to its value in 2019 multiplied by its growth factors in 2020, 2021 and 2022, as follows:

$$£58{,}000 \cdot [(1 + 0.05) \cdot (1 + 0.075) \cdot (1 + 0.1)] = £72{,}014.25.$$

$B2$ determines the *present*, or *discounted value* that the variable x takes on at the beginning of period t relying on the information on the value of the same variable x at the beginning of period $t + n$ and of its growth rates in all n periods included in the interval between t and $t + n$.

Suppose, for instance, that the number of workers contributing to a pension system in 2019 amounts to 353,430 and that its growth rates in the three preceding periods had been 2.0% (0.02), 10% (0.1) and 5.0% (0.05), respectively. We can infer the number of contributors in 2016 by *discounting* the 2019 figure as follows:

$$\frac{353{,}430}{[(1 + 0.02) \cdot (1 + 0.1) \cdot (1 + 0.05)]} = 300{,}000.$$

Finally, $B3$ determines the overall growth factor of variable x in the interval between t and $t + n$ relying on the information on the values that x takes on at the beginning of periods t and $t + n$. Supposing, for instance, that the expenditure of a pension system amounts to £286 billion in 2019 and had been £220 billion in 2016, we can compute its overall growth factor in the 2016–2019 interval as follows:

$$\frac{£286{,}000{,}000{,}000}{£220{,}000{,}000{,}000} = 1.3$$

showing that each monetary unity included in the value of pension expenditure in 2016 has grown to 1.3 in 2019 at an overall growth rate of 30% (0.3). Note that nothing can be inferred about the single growth factors of expenditures in the years included in the interval if we rely solely on the information on its values at the two extremes of the interval. For these factors to be computable, we should know not only the extreme values but also the 'intermediate' values taken on by expenditure in 2017 and 2018, which would enable us to use *A*3 for the pairs of adjacent years 2016–2017, 2017–2018 and 2018–2019. However, we can still determine the 'average' growth factor in the interval 2016–2019 by recalling the notion of geometric mean, i.e. by extracting the n^{th} root of both sides of the third solution of *B*3, as follows:

$$\sqrt[n]{(1+g_t^x)\cdot\left(1+g_{t+1}^x\right)\cdot\cdots\cdot\left(1+g_{t+n-1}^x\right)} = \sqrt[n]{\frac{x_{t+n}}{x_t}},$$

where the expression on the right-hand side equals the (geometric) average of the growth factors (on the left-hand side) of variable x in the n intervals comprised between t and $t+n$ and

$$\sqrt[n]{(1+g_t^x)\cdot\left(1+g_{t+1}^x\right)\cdot\cdots\cdot\left(1+g_{t+n-1}^x\right)} - 1 = \sqrt[n]{\frac{x_{t+n}}{x_t}} - 1$$

is the corresponding average growth rate, i.e. the *hypothetical* constant annual growth rate of variable x in the n intervals comprised between t and $t+n$ that would produce the same compound growth factor computed according to *B*3. With reference to the example above, the average growth factor of pension expenditure amounting to £220 billion in 2016 and reaching the level of £286 billion in 2019 is

$$\sqrt[3]{\frac{£286{,}000{,}000{,}000}{£220{,}000{,}000{,}000}} = 1.09139$$

while the corresponding average growth rate is

$$\sqrt[3]{\frac{£286{,}000{,}000{,}000}{£220{,}000{,}000{,}000}} - 1 = 0.09139 = 9.139\%.$$

In order to verify the soundness of the above result, we can apply *B*2 to compute the future value in 2019 of £220 billion in 2016, assuming a constant growth rate of 9.139% per year during the three years to follow:

$$£220{,}000{,}000{,}000 \cdot (1.09139) \cdot (1.09139) \cdot (1.09139) =$$
$$£220{,}000{,}000{,}000 \cdot (1.09139)^3 = £286{,}000{,}000{,}000,$$

which confirms that 1.09139 is the average of the three, unknown, growth factors that have produced the overall growth factor of 1.3 in the 2016–2019 interval. Note, finally, that also

Focus 3.1: Computing Changes in the Cost of Living Using the Consumer Price Index

Many public pension systems adjust pensions from year to year according to the change in the cost of living. Note that, by applying the notion of geometric mean, we can compute the average increase in the cost of living within a series of adjacent periods of time if we know the values that the consumer price index (CPI) takes on at the beginning and end of the selected time span. In fact, in any given year the value of the CPI measures the cost of a given basket of goods and services whose cost is 'set' at 100 in a base year, arbitrarily chosen, all annual values of the CPI relating to 100 as the annual monetary costs registered for the given basket relate to the monetary cost that the same basket had in base year. Let's consider, for instance, the values of the CPI in two non-adjacent years, e.g. 172.192 as of January 2000 and 195.267 as of January 2005 computed by the U.S. Bureau of Labor Statistics to measure the cost of living for All Urban Consumers in United States. By using the following formula:

$$\sqrt[5]{\frac{195.267}{172.192}} - 1 \cong 0.025 \cong 2.5\%$$

we arrive at the average percentage increase in the cost of living in United States in the five-year period between January, 2000 and January, 2005. The reader can verify that the same result would have been obtained by using the 'intermediate' values of the index as of January 2001, 2002, 2003 and 2004, amounting to, respectively, 177.042, 179.867, 184.000, and 188.908, to compute all yearly growth factors of the CPI for the pairs of adjacent years 2000–2001, 2001–2002, 2002–2003, 2003–2004 and 2004–2005 and then averaging them by using the formula of the geometric mean. Obviously, this latter, more painstaking method, would have provided more information, showing not only the average increase in the cost of living in United States in the 2000–2005 period but also the varying values of the yearly percentage changes within the same period.

*B*3, like *A*3, when used with reference to financial values, computes the IRR on the capital invested in period *t* for *n* periods.

Given that most of our analysis will refer to a hypothetical steady state characterized by constant growth rates of the relevant variables, it is worth recalling the form that formulas $B1 - B3$ take on under the constant growth assumption:

$$x_{t+n} = x_t \cdot (1 + g_x)^n,$$

$$x_t = \frac{x_{t+n}}{(1 + g_x)^n},$$

$$(1 + g_x) = \sqrt[n]{\frac{x_{t+n}}{x_t}}$$

which implies:

$$g_x = \sqrt[n]{\frac{x_{t+n}}{x_t}} - 1,$$

where the notation g_x for the growth rate of variable x is now 'freed' from the temporal index, given that it is assumed to be constant over time.

In economics, several variables are 'compounded', i.e. are the product of two or more separate variables, each growing at a different rate. Take the case of a firm's total revenues changing through time according to the changes in the unit price of its output and the changes in the number of units sold to its customers. Two typical, compound variables to be analysed in pension economics are contribution revenues and pension expenditures. The former are accounted for by the number of active workers contributing to the system and the average contribution paid by each worker, the latter by the average pension disbursed and the number of retirees.

Let us assume that the variable s is the product of variables y and z such that:

$$s_t = y_t \cdot z_t$$
$$s_{t+n} = y_{t+n} \cdot z_{t+n}.$$

Assuming, moreover, that variables y and z grow at constant rates, g_y and g_z, respectively, the value that variable s takes on in period $t + n$ can be expressed as:

$$s_{t+n} = y_t \cdot \left(1 + g_y\right)^n \cdot z_t \cdot (1 + g_z)^n = s_t \cdot \left(1 + g_y\right)^n \cdot (1 + g_z)^n,$$

showing that the expression

$$\left(1 + g_y\right) \cdot (1 + g_z)$$

is the growth factor of variable s, product of variables y and z, while

$$g_s = \left(1 + g_y\right) \cdot (1 + g_z) - 1$$

is the growth rate of the same variable s. The same result can be reached recalling that

$$g_s = \sqrt[n]{\frac{s_{t+n}}{s_t}} - 1 = \sqrt[n]{\frac{y_t \cdot \left(1 + g_y\right)^n \cdot z_t \cdot (1 + g_z)^n}{y_t \cdot z_t}} - 1,$$

which clearly simplifies to the same expression of g_s identified above.

Note that the following expression:

$$g'_s = g_y + g_z$$

is an approximation of the correct value of the compound growth rate g_s only when g_y and g_z are quite small.

An example can help to understand the logic of compound growth factors. Suppose, for instance, that a worker has just signed a two-year contract according to which the company will pay a yearly salary of £86,000 for the first year and a higher salary for the second year covering the increase in cost of living plus another 20% increase to augment the worker's purchasing power. What will the worker's salary be in the second year if the inflation rate is 5%? The correct answer is:

$$£86{,}000 \cdot (1.05) \cdot (1.2) = £108{,}360$$

corresponding to a salary growth rate of:

$$(1.05) \cdot (1.2) - 1 = 0.26 = 26\%.$$

Note that computing the second year's salary according to the simple sum (25%) of the two growth rates of inflation (5%) and of the agreed upon real salary increase (20%) wouldn't have fulfilled the terms of the contract. In fact, given that a hypothetical bundle of goods and services costing £86,000 in the first year, cost £86,000 · (1.05) = £90,300 in the second year, it is easy to verify that raising the salary up to only £86,000 · (1.25) = £107,500 would not allow the worker to buy 1.2 units of the same bundle of goods and services, as the contract dictates, but only = £107,500/£90,300 ≅ 1.19 units.

Note, moreover, that 'decomposing' growth factors follow the same logic as composing them. In fact, given that:

$$1 + g_s = \left(1 + g_y\right) \cdot (1 + g_z),$$

computing one of the unknown components, e.g. $1 + g_y$, requires dividing the compound growth factor by the known component as follows:

$$1 + g_y = \frac{(1 + g_s)}{(1 + g_z)},$$

such that

$$g_y = \frac{(1 + g_s)}{(1 + g_z)} - 1.$$

In fact, given that the growth rate of salaries can be decomposed into two components, one covering the cost of living and the other corresponding to the *real* growth rate, we can verify the error implied in using the simple sum of the growth rates by computing, through 'decomposition', the percentage change in the purchasing power of a worker's salary that was raised by 25% when the inflation rate was 5% as follows:

$$\frac{(1 + 0.25)}{(1 + 0.05)} - 1 \cong 1.19.$$

3.2 More on the IRR

As mentioned above, $A3$ enables us to compute the rate of return internal (implicit) in any pair of capital values measured at the beginning of two adjacent periods as follows:

$$\mathrm{IRR}_{t\to t+1} = \frac{K_{t+1}}{K_t} - 1$$

where K_t denotes the value of the capital invested at the beginning of period t and K_{t+1} the value of the capital one period later. Similarly, when comparing the values that the capital takes on at the beginning of two non-adjacent periods, we can compute the IRR resorting to $B3$ as follows:

$$\mathrm{IRR}_{t\to t+n} = \sqrt[n]{\frac{K_{t+n}}{K_t}} - 1,$$

where the notations are self-explanatory.

We should now ask how to compute the rate of return internal to several cash flows some of which disbursed and others earned, such as the contributions paid in and the pensions received by a worker participating in a pension system. Note that this is exactly the same problem that firms have when evaluating the rate of return expected from an investment project that implies a series of initial costs and a series of future net revenues. Just as firms need to compare the expected rate of return of an investment with the market interest rate, so it is for workers seeking to assess the advantage of an old-age insurance plan against the possibly available alternative of investing their savings, e.g. in financial markets.

In order to understand the maths involved, we should preliminarily understand the logic of our problem. In fact, it is important to recall that we are talking about a multiperiod savings plan that specifies the cash flows due by the two parties in each period but does not declare (make explicit) the, supposedly constant, rate of interest remunerating the worker's capital (pension wealth) before the plan is terminated. In fact, the outstanding debt of the plan, on which interest is due, increases during the accumulation phase and decreases during the disbursement phase. When the disbursement phase is terminated with the last disbursement, the contract between the parties is terminated and the plan will have repaid the entire debt. Imagine, for the sake of simplicity, a savings plan with a two-period accumulation phase and a two-period disbursement phase. At the beginning of periods 1 and 2, the worker pays contributions, denoted as C_1 and C_2, respectively, while at the beginning of periods 3 and 4 the plan pays the worker pensions, denoted as P_1 and P_2, respectively. The worker's capital will evolve throughout the four periods, according to the cash flows and according to the rate of interest, which we denote as g_k, as follows:

$K_1 = C_1$ at the beginning of period 1;
$K_2 = C_1 \cdot (1 + g_k) + C_2$ at the beginning of period 2, soon after the worker pays the second contribution;
$K_3 = C_1 \cdot (1 + g_k)^2 + C_2 \cdot (1 + g_k) - P_1$ at the beginning of period 3, soon after the plan pays the worker the first pension;
$K_4 = C_1 \cdot (1 + g_k)^3 + C_2 \cdot (1 + g_k)^2 - P_1 \cdot (1 + g_k) - P_2$ at the beginning of period 4, soon after the plan pays the worker the second and last pension.

On looking at the expression of K_4 and recalling that with the last pension (P_2) the plan must fully redeem its debts such that the worker's outstanding wealth is $K_4 = 0$, we find that the rate of return implicit in the four cash flows must be a value of g_k ensuring that:

$$C_1 \cdot (1 + g_k)^3 + C_2 \cdot (1 + g_k)^2 - P_1 \cdot (1 + g_k) - P_2 = 0,$$

which is equivalent to saying the rate of return implicit in the four cash flows is a value of g_k ensuring that the overall future values of benefits repay the overall future values of contributions, both computed in period 4. Note, however, that reference to period 4 is not a necessary condition. In fact, we can see our story also from the viewpoint of another period, e.g. period 1, which can easily be done by dividing all the terms of the above equation by $(1 + g_k)^3$ as follows:

$$C_1 + \frac{C_2}{(1 + g_k)} - \frac{P_1}{(1 + g_k)^2} - \frac{P_2}{(1 + g_k)^3} = 0.$$

According to the above condition, we can also define the internal rate of return as a value of g_k ensuring that the present or discounted values of contributions pay the discounted values of benefits, both computed at the beginning of period 1.

The above equation is 3rd-degree and therefore has three solutions for its unknown, the growth factor of the outstanding capital

$(1 + g_k)$. However, according to Descartes's rule, the number of positive solutions equals the number of reversal of signs of the polynomial's coefficients. This rule is particularly helpful for pension plans with a contribution phase followed by a disbursement phase that, necessarily, exhibit only one reversal of sign, thus ensuring that there is only one economically meaningful solution of $(1 + g_k)$. In fact, the negative values of $(1 + g_k)$ should be discarded in that they imply $g_k < -1$, whereas $g_k = -1 = -100\%$, i.e. not being paid any pension during retirement, is the worst outcome that we would contemplate for workers. We will not enter into discussion of the highly complex formula solving our 3rd-degree equation given that in reality the number of yearly contributions and pensions is so high that the equation to be solved can be of a much higher degree (e.g. 59th for a worker contributing 40 years and drawing pensions for 20 years) for which no solving formulas exist. In fact, for these equations the positive solution for the discount factor can be identified only by trial and error.

In order to understand how to use the trial-and-error method to identify a solution for the rate of return implicit in a multiperiod contribution phase followed by a multiperiod disbursement phase, it is convenient to work on the following four-period example characterized by two periods of accumulation and two periods of disbursement. Let's assume that the plan is characterized by the following four cash flows:

$C_1 =$ £20,000, the contribution due at the beginning of period 1;
$C_2 =$ £26,000, the contribution due at the beginning of period 2;
$P_1 =$ £30,000, the pension to be paid at the beginning of period 3;
$P_2 =$ £36,000, the pension to be paid at the beginning of period 4.

The evolution of the worker's pension wealth clearly depends on the succession of the cash flows and on the rate of interest g_k, which is our 'unknown'. The four cash flows are shown along the main diagonal of Table 3.1 whereas their evolution through time can be followed in each row. The values that the worker's pension wealth takes on at the beginning of each period, soon after each cash flow is paid or disbursed, are shown in the bottom, shaded row of the Table, by adding up (vertically) the values taken in each year by each cash flow, including their 'capitalization' at the rate of interest g_k. Note that pensions appear in the bottom row with a ‘minus’ sign since they reduce the workers’ pension wealth.

Recalling that the last cash flow $P_2 = £36,000$ extinguishes the obligation of the plan towards the worker, such that the worker’s pension wealth in period 4 (displayed in the fourth cell of the Table’s bottom row) must be zero, the IRR of the plan is shown by the (only) positive solution of the following equation in $(1 + g_k)$, where the left-hand side computes the worker’s net pension wealth at the end of the third year, i.e. just before the retiree receives the second, and last, pension (on the right-hand side):

$$\underbrace{\left\{\underbrace{\left[\underbrace{£20{,}000\cdot(1+g_k)}_{\text{Pension wealth after 1 year}}+£26{,}000\right]\cdot(1+g_k)-£30{,}000}_{\text{pension wealth after 2 years}}\right\}\cdot(1+g_k)}_{\text{pension wealth after 3 years}}=£36{,}000,$$

which can be written as:

$$£20{,}000\cdot(1+g_k)^3+£26{,}000\cdot(1+g_k)^2-£30{,}000\cdot(1+g_k)=£36{,}000.$$

Let’s now start our trial-and-error procedure to identify a value of $g_k \geq -1$ that verifies the above equality. Our first attempt can reasonably be $g_k = 0.15$, given that the first pension exceeds the second contribution by roughly 15% while the second pension may extinguish the obligation generated by the first contribution at a 15% compound interest rate. The

Table 3.1 The evolution of a worker’s pension wealth according to four known cash flows and the unknown rate of interest

$t = 1$	$t = 2$	$t = 3$	$t = 4$
$C_1 = £20{,}000$	$£20{,}000\cdot(1+g_k)$	$£20{,}000\cdot(1+g_k)^2$	$£20{,}000\cdot(1+g_k)^3$
	$C_2 = £26{,}000$	$£26{,}000\cdot(1+g_k)$	$£26{,}000\cdot(1+g_k)^2$
		$P_1 = £30{,}000$	$£30{,}000\cdot(1+g_k)$
			$P_2 = £36{,}000$
$K_1 = £20{,}000$	$K_2 = £20{,}000\cdot(1+g_k)+£26{,}000$	$K_3 = £20{,}000\cdot(1+g_k)^2+£26{,}000\cdot(1+g_k)-£30{,}000$	$K_4 = £20{,}000\cdot(1+g_k)^3+£26{,}000\cdot(1+g_k)^2-£30{,}000\cdot(1+g_k)-£36{,}000$

negative outcome of this first guess is illustrated in Table 3.2, showing that, if the interest rate implicit in the plan were 0.15, the worker's pension wealth would amount to:

£49,000 at the beginning of period 2, equal to the first contribution gross of interests plus the second contribution;
£26,350 at the beginning of period 3, equal to the values of the two contributions gross of interests and net of the first pension;
£−5697 at the beginning of period 4, equal to the values of the two contributions gross of interests matured for one extra period diminished by the first pension received at the beginning of period 3 gross of the 'lost' interest on it and by the second pension.

This latter circumstance, that the worker's pension wealth computed at a 15% interest rate is not zero after the payment of the second pension, shows that there is something wrong with our guess. In fact, it shows that if the plan was actually meant to reward the worker's contributions with an interest rate of 15%, after paying the first pension of £30,000, the obligation towards the worker could have been extinguished with the payment of a second pension lower than £36,000 by £5697, which is the 'excess' payment shown in the fourth cell in the bottom row of Table 3.2.

The circumstance that the plan promises a second pension more generous than that implied by our guess that the IRR of the plan is 15% suggests that the actual IRR is $g_k > 0.15$ and that we should go on in

Table 3.2 The evolution of a worker's pension wealth according to four given cash flows and the assumed rate of interest of 15%

$t = 1$	$t = 2$	$t = 3$	$t = 4$
$C_1 = £20{,}000$	$£20{,}000 \cdot (1 + 0.15) = £23{,}000$	$£20{,}000 \cdot (1 + 0.15)^2 = £26{,}450$	$£20{,}000 \cdot (1 + 0.15)^3 = 30{,}417.5$
	$C_2 = £26{,}000$	$£26{,}000 \cdot (1 + 0.15) = £29{,}900$	$£26{,}000 \cdot (1 + 0.15)^2 = £34{,}385$
		$P_1 = £30{,}000$	$£30{,}000 \cdot (1 + 0.15) = £34{,}500$
			$P_2 = £36{,}000$
$K_1 = £20{,}000$	$K_2 = 23{,}000 + £26{,}000 = £49{,}000$	$K_3 = £26{,}450 + £29{,}900 - £30{,}000 = £26{,}350$	$K_4 = £30{,}417.5 + £34{,}385 - £34{,}500 - £36{,}000 = £ - 5{,}697.$

our trial and error, trying a higher value, e.g. $g_k = 0.25$. This is done in Table 3.3, which shows, however, that also this second guess is wrong.

In fact, if the interest rate implicit in the plan were 0.25, the worker's pension wealth would amount to:

£51,000 at the beginning of period 2;
£33,750 at the beginning of period 3;
£6187.5 at the beginning of period 4.

This latter circumstance, that the worker's pension wealth computed at a 25% interest rate is still positive after payment of the second pension, shows that the actual IRR of the plan is lower than 25%. In fact, if the plan was actually meant to reward the worker's contributions with an interest rate of 25%, after paying the first pension of £30,000, the obligation towards the worker had to be extinguished by paying a second pension in excess of £36,000 by £6187.5, which is the 'deficiency' payment shown in the fourth cell in the bottom row of Table 3.3.

Therefore, given the two contributions of $C_1 = £20{,}000$ and $C_2 = £26{,}000$, by paying two pensions of $P_1 = £30{,}000$ and $P_2 = £36{,}000$ the plan is implicitly rewarding the worker's pension wealth with an annual interest rate $0.15 < g_k < 0.25$. As is shown in Table 3.4, $g_k = 0.20$ is the IRR of the plan. In fact, at the interest rate of 20%, the worker's pension wealth would amount to:

Table 3.3 The evolution of a worker's pension wealth according to four given cash flows and the assumed rate of interest of 25%

$t = 1$	$t = 2$	$t = 3$	$t = 4$
$C_1 = £20{,}000$	$£20{,}000 \cdot (1 + 0.25) = £25{,}000$	$£20{,}000 \cdot (1 + 0.25)^2 = £31{,}250$	$£20{,}000 \cdot (1 + 0.25)^3 = 39{,}062.5$
	$C_2 = £26{,}000$	$£26{,}000 \cdot (1 + 0.25) = £32{,}500$	$£26{,}000 \cdot (1 + 0.25)^2 = £40{,}625$
		$P_1 = £30{,}000$	$£30{,}000 \cdot (1 + 0.25) = £37{,}500$
			$P_2 = £36{,}000$
$K_1 = £20{,}000$	$K_2 = 25{,}000 + £26{,}000 = £51{,}000$	$K_3 = £31{,}250 + £32{,}500 - £30{,}000 = £33{,}750$	$K_4 = £39{,}062.5 + £40{,}625 - £37{,}500 - £36{,}000 = £6{,}187.5$

£50,000 at the beginning of period 2;
£30,000 at the beginning of period 3;
£0 at the beginning of period 4.

The circumstance that the last year's pension wealth computed at a 20% interest rate is precisely exhausted ($K_4 = 0$) when the retired worker receives the second and last pension of £36,000 shows that the actual IRR of the plan is precisely 20%. In other words, our trial-and-error procedure has shown that the four cash flows characterizing the plan *imply* that the worker's pension wealth (as resulting from the succession of the cash flows) is yearly credited with a 20% interest rate until it is finally depleted with withdrawal of the last pension. On looking at the last column of Table 3.4, it should be clear that this amounts to saying that the IRR is the specific interest rate, 20% in our example, at which the pair of benefits repay the pair of contributions, both computed in $t = 4$ (£34,560 + £37,440 = £36,000 + £36,000). The same equality can be verified by computing the present value of contributions and benefits in any other period, e.g. period 1, at the rate of 20%. In fact, given that the present values in $t = 1$ of C_2 (paid in $t = 2$), P_1 (received in $t = 3$) and P_2 (received in $t = 4$) amount, respectively, to £26,000/1.2 = £21,666 , £30,000/1.2^2 = £20,833 and £36,000/1.2^3 = £20,833, the circumstance that by applying a 20% discount rate, the sum of the discounted values of the two contributions computed in $t = 1$ (£20,000 + £21,666 = £41,666) equals the sum of the discounted values of the two benefits (£20,833 + £20,833 = £41,666) shows that the pair of contributions, gross of interest maturing at the rate of 20%, is precisely paying for the pair of benefits promised by the plan. Note, finally, how our first trial,

Table 3.4 The evolution of a worker's pension wealth according to four given cash flows and the assumed rate of interest of 20%

$t = 1$	$t = 2$	$t = 3$	$t = 4$
C_1 = £20,000	£20,000 · (1 + 0.20) = £24,000	£20,000 · (1 + 0.20)^2 = £28,800	£20,000 · (1 + 0.20)^3 = 34,560
	C_2 = £26,000	£26,000 · (1 + 0.20) = £31,200	£26,000 · (1 + 0.20)^2 = £37,440
		P_1 = £30,000	£30,000 · (1 + 0.20) = £36,000
			P_2 = £36,000
K_1 = £20,000	K_2 = 24,000 + £26,000 = £50,000	K_3 = £28,800 + £31,200 − £30,000 = £30,000	K_4 = £34,560 + £37,440 − £36,000 − £36,000 = £0

with the rate of 15%, should be interpreted. The value of benefits computed in $t = 4$ exceeds the value of contributions, showing that the pair of benefits eventually repay more than the contributions gross of interest at 15%. From a different viewpoint, the 15% assumption shows that the value of contributions computed in $t = 1$ is lower than that of benefits, implying that if the rate of interest were 'only' 15% they would not suffice to pay for the pair of benefits promised by the plan. The opposite holds in our second trial with the wrong guess of a 25% interest rate.

Focus 3.2: Computing the IRR with a Spreadsheet

Do we need to go through a long and tiresome iteration based on trial-and-error, such as summarized in our Tables 3.2, 3.3 and 3.4, whenever we need to compute the IRR of a pension plan? A few decades ago, the answer would have been 'yes', whether we computed with pencil and paper or with a pocket calculator. Nowadays, with the development of computing technologies, we can unhesitatingly answer 'no', given that a simple spreadsheet can do the job for us. With reference to the same example of a four-year pension plan summarized in Table 3.4, the steps to follow are shown in the three images presented in Fig. 3.1.

The first step consists in inserting the cash flows, respecting the sequence corresponding to their actual occurrence through time, in a number of adjacent cells equal to the number of cash flows. For the spreadsheet to perform the calculation, it is necessary to differentiate the sign of inflows from that of outflows. Note that we have decided to write the values of the cash flows in a vertical sequence of cells and, contrary to our procedure in the three Tables, to label the contributions as negative cash flows and the pensions as positive ones. In fact, the opposite choices would have

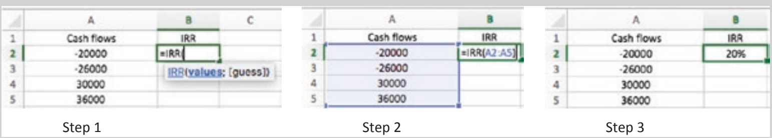

Fig. 3.1 The three steps to compute the IRR of a pension plan with a spreadsheet

been equivalent for the spreadsheet to provide the correct solution. We then have to choose any free cell in the spreadsheet to insert the following command: = IRR followed by an open parenthesis. The second step consists in indicating after the open parenthesis the sequence of cells in which we have inserted the cash flows, of which the spreadsheet is asked to compute the IRR, and closing the parenthesis. As shown in the image presented in the centre of the Figure, this can easily be done by selecting the relevant cells or by writing, within the open parenthesis the first and the last cell separated by a colon. The last step consists in pressing the 'enter' key on your keyboard and reading the solution that appears immediately after. Does the speed with which we see the answer mean that the spreadsheet has a formula to perform its computation? The answer is obviously 'no' and the speed is simply explained with the enormous number of trials that the spreadsheet is able to perform in a unit of time. Note that when the sequence of cash flows is particularly complex, the spreadsheet could ask for our help in choosing the first 'guess', as shown in the image on the left.

CHAPTER 4

Four Archetypal Pension Systems

Abstract The basic properties of pension systems are analysed by means of a 2-overlapping-generations model. Non-financial, pay-as-you-go (PAYG) systems are either 'defined benefit' (NDB), disbursing earnings-related pensions or defined contribution (NDC), computing pensions according to the logic of personal accounts. The former apply the equilibrium contribution rate to adapt contribution revenues to changes in the economic and demographic factors affecting pension expenditures; the latter adapt expenditures to revenues by crediting to personal accounts the sustainable rate of return equal to the growth rate of aggregate earnings. Financial, fully funded systems can also ensure that assets match liabilities, thus maintaining their 'unit' degree of funding by either adjusting the contribution rate (FDB) or by crediting the market interest rate to the personal accounts (FDC).

Keywords Financial defined benefit (FDB) and defined contribution (FDC) pension systems · Non-financial defined benefit (NDB) and defined contribution (NDC) pension systems · Sustainable rate of return · Contribution revenues and pension expenditures · Earnings growth rate

With this chapter, we come to the core of the book. The aim here is to explain the main forces at work in a pension system according to its financing and adjusting method, i.e. according as to whether it is funded

S. Nisticò, *Essentials of Pension Economics*,
https://doi.org/10.1007/978-3-030-26496-3_4

or based on pure pay-as-you-go (PAYG) financing and whether it ensures solvency on the basis of the defined benefit (DB) or defined contribution (DC) mechanisms. We simplify the analysis assuming that DB schemes compute pensions according to an earnings-related formula whereas DC schemes use personal accounts. To highlight the fact that funded systems invest their assets in financial markets whereas PAYG systems lack any financial assets, we will use the labels 'financial' as synonym of funded and 'non-financial' as synonym of PAYG, thus borrowing from Gora and Palmer (2004) the use of the following four acronyms:

NDB for systems adopting PAYG financing while adjusting the contribution rate charged to workers to ensure solvency;
FDC for funded systems relying on the automatic adjustments of PAs to remain fully funded;
FDB for funded systems adjusting the contribution rate to remain fully funded;
NDC for systems adopting PAYG financing while relying on the automatic adjustments of PAs to ensure solvency.

4.1 A Two-Overlapping-Generation Model: Logic and Assumptions

In reality, the functioning of a pension system in a given calendar year involves transactions of individuals belonging to several different birth cohorts or generations. For instance, assuming that by law individuals under the age of 16 years cannot work, retirement is allowed as of 65 years and compulsory at 70 years and that 110 years is the maximum observed age, in any calendar year the revenues of a pension system will derive from the contributions paid by 50–55 different generations of workers while expenditures will consist of the pensions disbursed to 40–45 different generations of retirees. The size of each of these overlapping generations of workers and retirees will differ according to the varying trends in fertility and employment rates as well as survival rates at the various ages that have occurred for each generation in the 95 years prior to that of observation. Moreover, employment rates and average earnings of each cohort of workers depend on the choices of the workers and firms, but also of the policymakers.

Given the complexity of the economic and demographic reality with which pension systems interact, we will have to apply 'Ockham's Razor', so named after the English philosopher William of Ockham (1287–1324), a principle of scientific investigation according to which we should avoid complexities that are not necessary to highlight the *relevant* features of the reality we seek to explain. The importance of this principle can readily be grasped by imagining that you have a perfect reproduction, i.e. a 1:1 scale road map, of the landscape you want to explore without getting lost. Reality already lies all around you, and looking at a perfect reproduction of it to settle your doubts would probably just increase your chance of getting lost. A simplified reproduction, such as a 1:1,000,000 map, is probably what you need. On the other hand, good researchers should refrain from excessive simplification, since relying on an over-simplified map would also increase the chance of getting lost. In the attempt to find a good compromise between the two opposite risks, we proceed in two steps. In the first step, we use a two-overlapping-generation model (OLG), i.e. highly stylized representation of a pension system in which individuals participate for one period only as workers and one period only as retirees. In the first period of their life, they pay contributions in proportion to their earnings whereas in the second they receive a pension that can be either earnings-related or based on the logic of 'personal accounts'. The circumstance that individuals' lives extend to two periods only explains why only two generations, the active and the retired, overlap in each period. Moreover, we assume that, in any period, all workers earn the same income and that all relevant variables grow through time at constant rates. This first step brings out the main properties of the different pension systems according to their financing and adjustment mechanisms. Finally, we assume that the burden of ensuring solvency is entirely borne by the workers out of their contributions, despite the fact that, in reality, pension system financing comes, at least in part, from employers and/or from the general tax revenue. The second step consists in 'zooming' on more realistic individual careers representing several years of participation in the system as both worker and retiree. With this second step we can address the fundamental issue of the redistributions among individuals according to the length and pattern of their careers, possibly embedded in the technicalities of the alternative pension designs.

The four archetypal systems on which we focus in this section are displayed in the 2 × 2 matrix presented in Table 4.1. Note that, as mentioned above, we assume that DB systems disburse earnings-related pensions while DC systems compute pensions according to the value of the contributions workers paid in their personal accounts.

Table 4.1 Four archetypal pension systems distinguished according to adjusting and financing method

Adjusting method / *Financing method*	*Defined benefit (earnings-related pensions)*	*Defined contribution (personal-accounts pensions)*
Pay-as-you-go (non-financial)	NDB	NDC
Funding (financial)	FDB	FDC

We will start with the two systems on the main diagonal of the matrix, which are the most common and widespread around the world. In fact, combining PAYG financing with DB earnings-related pensions is typical of compulsory public schemes, whereas the combination of funding with DC-personal accounts is typical of supplementary pension plans. We will then move on to the systems on the counter diagonal. While FDB supplementary pension plans are progressively disappearing from the scene, as mentioned above, NDC systems are becoming a reference point for many policymakers unwilling to legislate continuous increases in the contribution rate to pay public pensions under the current economic and demographic scenario.

We will analyse our archetypal pension systems across three time periods, $t-1$, t and $t+1$, that we may also refer to as 'years', by using the following notations:

w earnings per worker in year t;
L number of workers employed in year t;
g_w constant growth rate of workers' income;
g_L constant growth rate of the number of workers;
c contribution rate, i.e. the contribution per unit of income;
a the accrual rate applied when the pension is earnings-related;
r the market rate of interest, i.e. the rate of return on the assets invested by funded systems;
π_{NDC} the rate of interest credited to personal accounts by NDC systems;
π_{FDC} the rate of interest credited to personal accounts by FDC systems.

There is an important point that needs to be made clear: in our simplified two-OLG settings, mortality patterns are deterministic rather than probabilistic. In particular, individuals survive two periods only with certainty, one period as workers and one period as retirees. Consistently with this assumption, all workers in any given period, will survive and claim for a pension in the following period. As to the amount of the pension, DB earnings-related systems, whether PAYG or funded, compute it through multiplication of the given accrual rate by the individual's income earned in the preceding period, when active. On the other hand, DC-personal accounts systems, whether PAYG or funded, compute the pension as a lump sum amounting to the value of the contributions paid in the system when active, gross of interests matured for one year.

The following properties of our model will recur in analysis of all archetypal pension systems:

contribution revenues in any year are the product of three factors, the contribution rate, earnings per worker and the number of workers employed;

pension expenditures in any year are the product of two factors, the pension and the number of retirees;

earnings per worker in years $t-1$ and $t+1$ are derived from earnings per worker in year t by computing, respectively, the discounted value and the future value of w at its growth rate g_W;

the number of workers in years $t-1$ and $t+1$ is derived from the number of workers in year t by computing, respectively, the discounted value and the future value of L at its growth rate g_L;

the number of retirees in years $t-1$, t and $t+1$ simply replicates the number of workers in the preceding years $t-2$, $t-1$ and t, respectively.

4.2 NDB Systems

The sequence of contribution revenues and pension expenditures in an NDB system, hypothesized to have started in year $t-1$, is summarized in Table 4.2.

Expenditures in year $t-1$, shown in the cell at the intersection of the left column and bottom row of Table 4.2, represent the 'pension gift' to the cohort of workers active in year $t-2$ who will be paid the established earnings-related pension despite having paid no contributions themselves. Given that the system relies on PAYG financing, its degree of funding, hereafter denoted as k, will have to be zero throughout the three periods under observation. This is first of all achieved by ensuring

Table 4.2 Evolution of an NDB system

	$t-1$	t	$t+1$
Contribution Revenues	$c\cdot\frac{w}{1+g_w}\cdot\frac{L}{1+g_L}$	$c\cdot w\cdot L$	$c\cdot w\cdot(1+g_w)\cdot L\cdot(1+g_L)$
Pension Expenditures	$a\cdot\frac{w}{(1+g_w)^2}\cdot\frac{L}{(1+g_L)^2}$	$a\cdot\frac{w}{1+g_w}\cdot\frac{L}{1+g_L}$	$a\cdot w\cdot L$

that the system does not accumulate reserves at the outset, i.e. by ensuring that the contribution revenue exactly matches the pension expenditures of year $t-1$. Given its DB nature, the contribution rate will, therefore, have to be set at the level c^* that ensures:

$$c^*\cdot\frac{w}{1+g_w}\cdot\frac{L}{1+g_L}=a\cdot\frac{w}{(1+g_w)^2}\cdot\frac{L}{(1+g_L)^2},$$

from which:

$$c^*=\frac{a}{(1+g_w)\cdot(1+g_L)}.$$

We may call expression c^* the equilibrium contribution rate. In fact, it will be readily seen that if we substitute it in the corresponding cells of the top row, the contribution revenues will exactly match pension expenditures also in the years t and $t+1$, thus ensuring that $k=0$ in all periods.

Note that the value of the equilibrium contribution rate is a function of both the generosity of pensions, as reflected by the value of the accrual rate a, and of the expected dynamics of aggregate earnings (the contribution base), as reflected by the growth factors of employment and individual earnings. In particular, the more (less) generous accrual rate a turns out to be, the higher (lower) will be, ceteris paribus, the equilibrium contribution rate. Conversely, the higher (lower) the growth rates of employment and earnings prove, the lower (higher) will be, *ceteris paribus*, the equilibrium contribution rate. This latter circumstance can readily be appreciated on noting that contribution revenues are a function of current employment and earnings, whereas pension expenditures depend on their past values. Therefore, pensions to be disbursed in proportion to any given value of a can be financed at a lower contribution rate in economies characterized by high values of g_w and g_L, where current aggregate earnings are high relative to their past values, than in economies with low values of g_w and g_L.

By charging the equilibrium contribution rate, the system will be solvent, i.e. in any period its contribution revenues will suffice to pay pensions according to the established accrual rate, thus extinguishing its current liabilities while leaving no assets. On the other hand, by charging contributions to extinguish 'old' liabilities, 'new' liabilities arise to be extinguished with the future revenues deriving from the contributions charged to future cohorts of active workers according to the equilibrium contribution rate. This ability to be at the same time indebted, destitute of assets and solvent is the essence of pension systems relying on PAYG financing.

Having identified the conditions for a DB system to be solvent while maintaining its PAYG financing through time, it is worth considering what rate of interest the system ultimately pays when 'borrowing' from each active cohort the contributions needed to extinguish outstanding liabilities towards the retiring cohort. In other words, we want to find out what rate of return a pure NDB system ensures, implicitly, when disbursing pensions to the retired cohorts 'in exchange for' the contributions the same cohort paid when active. Given our simplifying assumptions on the life duration of each cohort, we can start from formula *A*3 in Chapter 3, i.e. from the formula computing the internal rate of return (IRR) for two adjacent cash flows, represented, in this case, by the contributions paid and the pensions received. With reference to the cohort contributing in year t the overall amount $c \cdot w \cdot L$, while receiving in year $t + 1$ pensions amounting to $a \cdot w \cdot L$, we get:

$$\mathrm{IRR}_{t \to t+1} = \frac{a \cdot w \cdot L}{c \cdot w \cdot L} - 1 = \frac{a}{c} - 1,$$

which shows that, given the generosity of a DB scheme as reflected by the value of the accrual rate (a), the lower (higher) the contribution rate the system charges on active workers, the higher (lower) will be the IRR the system grants to contributors. At the same time, we want to identify the specific IRR implicitly awarded by a pure NDB system that charges the equilibrium contribution rate, i.e. by a system whose revenues are constantly equal to expenditures such that $k = 0$ throughout the observation period, as required by PAYG financing. This is done by substituting the expression of c^* for c in the formula of the IRR:

$$\mathrm{IRR}^*_{t \to t+1} = \frac{a}{\frac{a}{(1+g_w)\cdot(1+g_L)}} - 1 = (1 + g_w) \cdot (1 + g_L) - 1.$$

It can readily be seen that the same result can be obtained by computing the IRR of other cohorts, e.g. that contributing in year $t - 1$. Therefore, according to the above expression, the following statement holds:

Statement 1
The IRR awarded to any cohort of workers by a PAYG–DB (NDB) system that charges the equilibrium contribution rate equals the growth rate of aggregate earnings obtained by compounding the growth rates of individual earnings and employment.

Statement 1, generally known as the Samuelson–Aaron theorem, is of the utmost importance in pension economics. It reveals the interest rate 'hidden' within the technicalities of sustainable, earnings-related DB formulas, showing also that it is not necessary to accumulate and invest in the financial markets for retirement savings to be rewarded with a positive interest rate. Given the simplifying assumptions of our model, it needs emphasizing that the growth rate of aggregate earnings measures the rate of return for each cohort as a whole, while the individual IRRs tend to be higher or lower than the Samuelson–Aaron rate according to the duration and dynamics of the career (see Focus 5.1).

Focus 4.1: More on the Samuelson–Aaron Theorem

The Samuelson–Aaron theorem dates back around sixty years, to the publication of Paul Samuelson's "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money" (Samuelson 1958). In his article, Samuelson argued that in a sort of primitive economy, with non-negative steady growth and constant age structure of population, where "all ice melted, and so did all chocolates … [such that] workers could not carry goods over into their retirement years", i.e. a world in which "trade with Mother Nature current consumption goods in return for future consumption goods" implies a −1 rate of interest (ibid., p. 468), setting up a PAYG social security arrangement is welfare improving. In fact, in Samuelson's hypothetical economy, the 'biological' interest rate, equal to the (supposedly non-negative) growth rate of the population (g_L) and coinciding with the growth rate of aggregate earnings given the assumption of constant productivity ($g_w = 0$), emerges as one of the mathematical solutions clearing the market for savings. This solution exists despite the fact that the working of free exchanges in the savings market would unavoidably end up with "a negative market interest rate, rather than with the biological … interest rate corresponding to the social optimum" (Samuelson 1958, p. 78). On the other hand, any social security programme forcing equality between supply (contributions) and demand (pensions) for savings in a PAYG setting, rewards compulsory contributions with the 'biological' interest rate, thus allowing attainment of the social optimum. In order to prove his *paradox*, Samuelson used both a 2-OLG model like ours, and a 3-OLG model wherein individual life lasts three periods deterministically, two of which, at age 1 and 2, being devoted to production, while the third, at age 3, is spent in retirement.

A few years later, Henry Aaron (1966) extended Samuelson's *paradox* to a world in which also labour productivity grows steadily and the market interest rate can be non-negative, showing that a PAYG social security arrangement is still welfare-improving provided that the market interest rate is lower than the growth rate of taxable earnings, obtained by compounding the growth rate of individual earnings and the growth rate of the insured population. The method applied by Aaron to prove the theorem is quite similar to ours, though without the simplifying assumption of 2 OLGs only, and with the provision of a defined pension benefit amounting to the current average wage for all retirees rather than a fraction of individual 'last earnings'. In fact, the first step in Aaron's proof is identification of the equilibrium contribution rate allowing the contributions charged on active cohorts to pay the defined benefit to all retirees. In the second step, Aaron computes the present value of pensions and contributions of a cohort of contributors, showing that the former is higher than the latter when discounting them at the market interest rate, as in our trial-and-error example in Sect. 3.2, when the discount rate (15%) is lower than the IRR (20%). The final step confirms that the equality between the present values of the cohort's pensions and contributions holds when the two are discounted at the growth rate of aggregate earnings. The assumption of steady growth in both earnings and employed population ensures that what is true for one cohort is also true for any other cohort of contributors. Recently, Gronchi and Nisticò (2008) elaborated a 4 OLGs model, allowing for different career patterns within each cohort of contributors, to prove that the assumption of steady growth is not necessary for the theorem to hold. On the other hand, Settergren and Mikula (2006, pp. 123–125) have shown that out of a steady state and allowing for changes in mortality patterns, the Samuelson–Aaron IRR is only one of the two components of the IRR deriving from PAYG financing, the additional one being captured by changes in expected turnover duration (see Focus 2.2).

4.3 FDC Systems

For a pension system to be fully funded, its assets must constantly equal its liabilities, such that $k = 1$ in any period of time. This is why funded systems do not disburse pensions at the outset, in year $t-1$. In fact, we will assume that the entire contribution revenues charged on the first cohort of insured workers is accumulated and invested in a fund yielding the market interest rate r. In year t, the interest earned on the fund will constitute a second inflow for the system, alongside the contribution revenues. This is why in Table 4.3, showing the evolution of our simplified FDC scheme, there are two additional rows showing, respectively, the value of the assets accumulated in the fund and the interest matured on them. For the reasons explained above, neither interest nor pension expenditures appear in year $t-1$, when the system comes underway. Note that, to simplify perusal of the table, we use the notations $\mathbf{CR}, \mathbf{PE}$

Table 4.3 Evolution of a FDC system

	t − 1	t	t + 1
Contribution Revenues	$c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L}$	$c \cdot w \cdot L$	$c \cdot w \cdot (1+g_w) \cdot L \cdot (1+g_L)$
Funded Assets	$c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L}$	$FA_{t-1} \cdot (1+r) + CR_t - PE_t$	$FA_t \cdot (1+r) + CR_{t+1} - PE_{t+1}$
Interests	–	$FA_{t-1} \cdot r$	$FA_t \cdot r$
Pension Expenditures	–	$c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L} \cdot (1+\pi_{FDC})$	$c \cdot w \cdot L \cdot (1+\pi_{FDC})$

and FA for the values of contribution revenues, pension expenditures and funded assets as of year t. And note, moreover, that assets in years t and $t+1$ result from the value of previous-year assets gross of interest, plus net revenues of the same year (CR − PE).

Given its DC nature, the system will choose and fix the contribution rate c while the ensuing contribution flow in $t-1$ will be accumulated as system assets. Moreover, our DC system computes pensions according to the balance of personal accounts. The problem is to identify the rate of return π_{FDC} to be credited to all accounts for the system to be fully funded, i.e. for its liabilities to match its assets exactly, such that $k=1$. Since the assets are invested in a fund yielding the market interest rate, the liabilities in year $t-1$ are none other than the present value, discounted at the market interest rate, of the pensions to be disbursed in year t. Therefore, for assets to match liabilities the following condition must hold:

$$c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L} = \frac{c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L} \cdot (1+\pi_{\text{FDC}})}{(1+r)},$$

which implies

$$\pi_{\text{FDC}}^{\wedge} = r.$$

In fact, it can be readily seen that by crediting to all personal accounts a rate of return equal to the market interest rate, pension expenditure in year t, (PE_t) amounts to:

$$c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L} \cdot (1+r),$$

which implies

$$\text{FA}_{t-1} \cdot (1+r) = \text{PE}_t$$

such that the assets, gross of interest, will suffice to pay pensions in year t, or in other words extinguish outstanding liabilities. In fact, when $\pi_{\text{FDC}} = r$, the expression of FA_t simplifies to CR_t thus showing that the new contribution revenues $c \cdot w \cdot L$ can be entirely accumulated and invested in the fund to match exactly the present value of newly arising liabilities $c \cdot w \cdot L \cdot (1+\pi_{\text{FDC}})/(1+r)$, i.e. the present value of the pensions to be disbursed in year $t+1$ if $\pi_{\text{FDC}} = r$. Therefore, the following statement holds:

Statement 2
The rate of return to be credited to all personal accounts for a DC system to be fully funded through time equals the market interest rate.

According to Statement 2, by crediting the market interest rate to all personal accounts, the system's liabilities towards each individual worker can be extinguished by liquidating the corresponding assets constituted by the contributions paid in by the same individual worker, gross of interest matured. Therefore, in our 2-OLG model, at the turn of each period the fund will simultaneously be diminished by the old assets gross of interest and refilled with the new contribution revenues, which are $(1 + g_w) \cdot (1 + g_L)$ times higher than those of the previous year. Therefore, the following statement also holds:

Statement 3
In an FDC system, funded assets increase yearly at the growth rate of aggregate earnings resulting from compounding the growth rates of individual earnings and employment.

Here it needs to be pointed out that the manager of an FDC system does not need to actually sell the specific assets bought with the contributions of each active cohort to pay the pensions of its members when they reach retirement age. In fact, despite the fact that in a personal-accounts scheme, each cohort member acquires claims (property rights) amounting exactly to the value of the contributions paid in, there is no reason why the system should not use the liquidity constituted by the contributions paid by active cohorts to extinguish the liabilities towards retiring cohorts. On the other hand, the contribution flow of any year might exceed or fall short of pension expenditures, thus requiring, respectively, the difference to be invested in, or disinvested from, the fund. In fact, it is more appropriate to picture our system manager as someone yearly performing the following operations: collecting contributions from the active-cohort and interest on the accumulated assets, disbursing pensions to the retiring-cohort and, finally, investing or disinvesting according to the sign of net inflows, i.e. according to whether the two inflows net of the outflow are positive or negative. Perusing the following expression of the system's net inflows in year t:

$$c \cdot w \cdot L + r \cdot c \cdot \frac{w}{1 + g_w} \cdot \frac{L}{1 + g_L} - c \cdot \frac{w}{1 + g_w} \cdot \frac{L}{1 + g_L} \cdot (1 + \pi_{\mathrm{FDC}}),$$

we can try to identify some general rule on how contribution revenues (the first addend) compare to pension expenditures (the third addend) in an FDC scheme. In order to answer this question, let us first of all assume that the market interest rate coincides with the growth rate of overall earnings, such that $\pi_{FDC} = r = (1 + g_w) \cdot (1 + g_L) - 1$. It is readily seen that under this assumption pension expenditures become:

$$c \cdot \frac{w}{1 + g_w} \cdot \frac{L}{1 + g_L} \cdot (1 + g_w) \cdot (1 + g_L) = c \cdot w \cdot L,$$

which cancels out exactly with the contribution revenues of the same year t, while interest can be entirely reinvested in the fund. Conversely, if $\pi_{FDC} = r > (1 + g_w) \cdot (1 + g_L) - 1$, pension expenditures exceed contribution revenues, such that the share of interest generated by the positive difference $r - \left[(1 + g_w) \cdot (1 + g_L) - 1\right]$ will have to be used to fill the gap between pensions and contributions and only the remaining share, generated by $(1 + g_w) \cdot (1 + g_L) - 1$, can be reinvested in the fund. Finally, if $\pi_{FDC} = r < (1 + g_w) \cdot (1 + g_L) - 1$, contribution revenues exceed pension expenditures such that the difference between the two can be reinvested in the fund together with interests. In all cases, the fund will grow in accordance with Statement 3.

4.4 FDB Systems

Table 4.4 shows the evolution of an FDB scheme. As in the case of an FDC, we assume that the system accumulates and invests the entire contribution revenues of year $t - 1$ in a fund. However, in contrast with an FDC, pensions are defined in terms of an accrual rate and it is the task of the contribution rate to ensure that the system's funded assets match liabilities

Table 4.4 Evolution of a FDB system

	$t-1$	t	$t+1$
Contribution Revenues	$c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L}$	$c \cdot w \cdot L$	$c \cdot w \cdot (1+g_w) \cdot L \cdot (1+g_L)$
Funded Assets	$c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L}$	$FA_{t-1} \cdot (1+r) + CR_t - PE_t$	$FA_t \cdot (1+r) + CR_{t+1} - PE_{t+1}$
Interests	–	$FA_{t-1} \cdot r$	$FA_t \cdot r$
Pension Expenditures	–	$a \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L}$	$a \cdot w \cdot L$

throughout the observation period. The problem is therefore to identify the value of c that enables the system to be fully funded, given the commitment towards the workers to pay a pension equal to a share a of their earnings.

Recalling that the liabilities in year $t-1$ simply amount to the present value of the pensions to be disbursed in year t, for assets to match liabilities the following condition must hold:

$$c \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L} = \frac{a \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L}}{(1+r)},$$

which implies

$$c^{\wedge} = \frac{a}{1+r}.$$

The expression of $c^{\wedge}$ can be referred to as the funding contribution rate, since by substituting it in all the cells in the top row, the funded assets will exactly match the system's liabilities also in the years t and $t+1$, thus ensuring that $k=1$ in all periods. In fact, after substitution, the expression of assets in year t becomes:

$$\mathrm{FA}_t = a \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L} + \frac{a}{1+r} \cdot w \cdot L - a \cdot \frac{w}{1+g_w} \cdot \frac{L}{1+g_L},$$

which simplifies to

$$\frac{a}{1+r} \cdot w \cdot L,$$

amounting exactly to the present value of pensions to be disbursed in year $t+1$. In other words, by charging the contribution rate $c^{\wedge}$ the system ensures that each cohort accumulates exactly the assets needed to extinguish liabilities towards its members. Note that the value of the funding contribution rate is a function of both the value of the accrual rate a, as in NDB systems, and of the market interest rate. This latter circumstance can readily be appreciated on noting that pensions to be disbursed in proportion to any given value of a can be financed at a lower (higher) contribution rate in economies characterized by higher (lower) values of the yield of accumulated assets. Having identified the conditions for a DB system to remain fully funded through time, it is worth verifying what rate of interest the system ultimately pays when 'borrowing' from each active cohort the contributions needed to extinguish outstanding liabilities towards its members. Again, we can start from formula *A*3 in Chapter 3, i.e. from the following formula computing the IRR implicitly awarded to the cohort contributing in year t:

$$\mathrm{IRR}_{t \to t+1} = \frac{a \cdot w \cdot L}{c \cdot w \cdot L} - 1 = \frac{a}{c} - 1,$$

and then, by substituting the expression of $c^{\wedge}$ for c in the formula of the IRR:

$$\mathrm{IRR}^{\wedge}_{t \to t+1} = \frac{a \cdot w \cdot L}{\frac{a}{1+r} \cdot w \cdot L} - 1 = r.$$

Clearly, the same result can be obtained by computing the IRR of other cohorts. Therefore, according to the above expression, the following statement holds:

Statement 4

The IRR awarded to any cohort of workers by an FDB system that charges the funding contribution rate equals the market interest rate.

Also in this case, it is worth emphasizing that the market interest rate measures the rate of return implicitly awarded to each cohort as a whole, while the individual IRRs tend to be higher or lower than it according to the duration and dynamics of the career (see Focus 5.1). On the other hand, by charging the funding contribution rate, an FDB system behaves, as a whole, precisely like an FDC, such that Statement 3 still holds and the contribution revenues in each year equal, exceed or fall short of pension expenditures according as to whether the growth rate of overall earnings equal, exceed or fall short of the market interest rate.

4.5 NDC Systems

Table 4.5 shows the evolution of an NDC system. Like their FDC 'relatives', NDC systems pay pensions according to the value of personal accounts and do not rely on adjustments of the contribution rate to ensure constancy in their degree of funding, which will have to be $k = 0$ throughout the three periods under observation, as in the case of NDB systems, their other 'relatives'. It is worth recalling that personal accounts in NDC systems are 'virtual', meaning that PAYG financing imposes that workers' contributions be 'directly' disbursed as pensions to the retirees of the time. Therefore, no actual assets are accumulated in

Table 4.5 Evolution of an NDC system

	$t-1$	t	$t+1$
Contribution Revenues	$c\cdot\frac{w}{1+g_w}\cdot\frac{L}{1+g_L}$	$c\cdot w\cdot L$	$c\cdot w\cdot(1+g_w)\cdot L\cdot(1+g_L)$
Pension Expenditures	$c\cdot\frac{w}{(1+g_w)^2}\cdot\frac{L}{(1+g_L)^2}\cdot(1+\pi_{NDC})$	$c\cdot\frac{w}{1+g_w}\cdot\frac{L}{1+g_L}\cdot(1+\pi_{NDC})$	$c\cdot w\cdot L\cdot(1+\pi_{NDC})$

the accounts, though the system manager keeps record of the individual account balances.

If the system is not to accumulate reserves at the outset, after setting the contribution rate, as required by the DC nature of the system, the 'pension gift' to the cohort of workers active in year $t-2$ must equal the ensuing contribution revenues in year $t-1$. This is done by identifying the system's rate of return π_{NDC} to be credited to all personal accounts for the following equality between revenues and expenditures to hold:

$$c\cdot\frac{w}{1+g_w}\cdot\frac{L}{1+g_L}=c\cdot\frac{w}{(1+g_w)^2}\cdot\frac{L}{(1+g_L)^2}\cdot(1+\pi_{\text{NDC}}),$$

which implies

$$1=\frac{(1+\pi_{\text{NDC}})}{(1+g_w)\cdot(1+g_L)}$$

and, hence:

$$\pi^*_{\text{NDC}}=(1+g_w)\cdot(1+g_L)-1.$$

In fact, it can be readily seen that by crediting to all personal accounts a rate of return equal to the growth rate of aggregate earnings, pension expenditures equal contribution revenues also in years t and $t+1$, such that the following statement holds:

Statement 5

The rate of return to be credited to all personal accounts for a DC system to be solvent while relying on pure PAYG financing through time equals the growth rate of aggregate earnings obtained by compounding the growth rates of individual earnings and employment.

The reasons why in our simplified economy the revenues of an NDC system suffice to pay pensions to current retirees if, and only if, the personal accounts are credited with a rate of return equal to the growth rate of aggregate earnings can be summarized as follows:

with PAYG financing, current contribution revenues are the only resources available to extinguish liabilities towards the workers who paid contributions one year before;
with a DC setting, where the contribution rate is fixed, such that the dynamics of the system's revenues depend solely on the dynamics of the contribution base, the growth rate of aggregate earnings is precisely the rate at which contribution revenues grow through time;
when pensions are computed according to the balance of personal accounts, the claims of current retirees in any year amount to the value of the contributions they paid as workers one year before, gross of interest computed at the rate π_{NDC};
if, and only if, $\pi_{\text{NDC}} = (1 + g_w) \cdot (1 + g_L) - 1$, in any year current contributions, amounting to past contributions times $(1 + g_w) \cdot (1 + g_L)$, suffice to pay pensions amounting to the same value as past contributions times $(1 + \pi_{\text{NDC}})$.

The following differences between Statement 5 and Statement 1 also need to be emphasized.

with NDB systems, the (implicit) rate of return of the contributions paid by any cohort of workers equals the growth rate of aggregate earnings if and only if the contribution rate is set at its equilibrium level;
NDC systems are in equilibrium *for whatever contribution rate*, if and only if the rate of return of the contributions equals the growth rate of aggregate earnings;
NDC systems *transparently* credit to all personal accounts the sustainable rate of return equal to the growth rate of aggregate earnings whereas the rate of return awarded by sustainable NDB systems is hidden within the technicalities of the specific pension formula;
by explicitly crediting the sustainable rate of return to all individual accounts, NDC systems preclude significant redistributions among the members of each cohort, whereas DB schemes cannot avoid non-negligible differences in the individual rates of return.

Focus 4.2: The Sustainable Rate of Return in a Generic PA–DC System

We have shown that crediting the market rate of interest matured on the fund's assets to all personal accounts ensures that FDC systems remain fully funded through time, whereas for NDC systems to be solvent without accumulating reserves the rate of return to be credited is the growth rate of total earnings. We are now in a position to identify the general expression of the sustainable rate of return valid for any pension scheme with any degree of funding $0 \leq k \leq 1$. In order to do so, let us imagine that any system must specify and declare its desired degree of funding such that the problem for the system's manager is to identify a rule for the rate of return to be credited to all personal accounts for the growth of liabilities and of funded assets to constantly match each other, thus allowing the scheme to maintain its actual degree of funding constant through time. The same rule will also reveal the rate of return implicitly paid by any DB scheme with constant degree of funding. We will use the following simplifying notations:

$$W_{t-1} = \frac{w}{(1+g_w)} \cdot \frac{L}{(1+g_L)};$$

$$\dot{w} = (1+g_w) \cdot (1+g_L) - 1.$$

Let us start by noting that in a DC scheme that relies on personal accounts, the liabilities, hereafter denoted as LIA, towards contributors at the beginning of any year when interest has yet to be credited to their accounts, coincide with the contributions the same workers paid into the system. Note, moreover, that the value of the funded assets at time $t-1$ is the share k of the contribution revenues not 'donated' to the first cohort of retirees. We will assume that this share k is precisely our generic DC system's desired degree of funding:

$$\frac{\mathrm{FA}_{t-1}}{\mathrm{LIA}_{t-1}} = \frac{k \cdot c \cdot W_{t-1}}{c \cdot W_{t-1}} = k.$$

Therefore, for this degree to remain equal to k at time t, the rate of return π_{GDC} credited to the account balances of a generic PA-DC system must ensure that

Simplifying and reordering:

and hence: $$\frac{(1+\pi_{GDC})}{(1+\dot{w})} = \frac{k \cdot (1+r) + (1+\dot{w})}{(1+\dot{w})} - k$$

$$\overset{\circ}{\pi}_{GDC} = \dot{w} + k \cdot (r - \dot{w}) = k \cdot r + (1-k) \cdot \dot{w}.$$

The two expressions of the sustainable rate of return of a generic DC system, after the first and second equal sign respectively, although identical from a mathematical viewpoint, express two different but equally important viewpoints. According to the first, the 'benchmark' rate of return is the growth rate of aggregate earnings crediting of which ensures equality between revenues and expenditures in any year, thus allowing interest to be reinvested in the fund. However, if the market interest rate were higher (lower) than the growth rate of aggregate earnings, crediting the benchmark only would imply the fund growing more (less) than liabilities, thus leading to an increase (a decrease) of k. Therefore, for assets to keep pace with liabilities, the benchmark rate must be corrected with the interest deriving from the difference, positive or negative, between the two rates, obviously proportionate to the ratio k of assets to liabilities. According to the second expression, $\overset{\circ}{\pi}_{GDC}$ is the weighted average of $\overset{\wedge}{\pi}_{FDC}$ and $\overset{*}{\pi}_{NDC}$, the weights being the desired degree of funding k and its complement, respectively. Note that

$$\overset{\circ}{\pi}_{GDC} = \overset{\wedge}{\pi}_{FDC} \quad \text{if } k = 1$$

and

$$\overset{\circ}{\pi}_{GDC} = \overset{*}{\pi}_{NDC} \quad \text{if } k = 0.$$

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CHAPTER 5

Pension Benefits in a More Realistic Setting

Abstract Earnings-related pensions are computed according to the years of contributions, a certain number of last (or best), earnings and an accrual rate, i.e. the percentage of valorized earnings awarded as pension for each year of service. Indexation of existing pensions can be anchored either to wage or to CPI growth. Earnings-related pensions tend to award higher internal rate of returns (IRRs) to short and dynamic careers than to long and flat ones. Intra-generational unfairness is mitigated when applying lower accrual rates to higher income brackets, as in the U.S. system. Personal Accounts ensure fairness by computing the first annuity dividing the account balance at retirement by a 'divisor' reflecting retiring workers' life expectancy, while anchoring indexation to the system's rate of return 'net' of the frontloading factor.

Keywords Fairness and intra-generational redistributions · CPI and wage indexation · Account balance and divisors · Frontloading rate · Personal accounts indexation rule

Real-life careers and retirement spans last more than one period. This is why, having shown the fundamental properties of the four archetypal systems, we will now relax the 2-OLG assumption and focus on the properties of alternative pension formulas in a realistic context in which individuals work for n years and spend their time in retirement for m years, earning their income when active and receiving a pension when

S. Nisticò, *Essentials of Pension Economics*,
https://doi.org/10.1007/978-3-030-26496-3_5

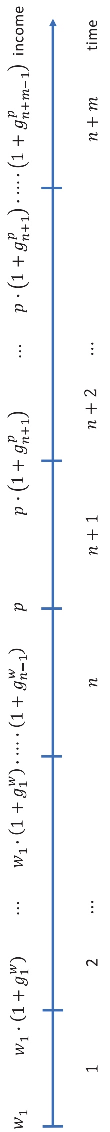

Fig. 5.1 Realistic working and retirement time spans

retired, as shown in Fig. 5.1, where *p* denotes the first pension annuity and g^p_{n+i} its indexation rate after the *i*th year of retirement.

We will focus in particular on earnings-related and personal-account formulas, typical of defined benefit (DB) and defined contribution (DC) systems respectively, emphasizing their relative performance both in terms of adequacy of the resulting annuities and in terms of possible disparities in treatment between different individuals according to their career patterns. Note that many pension systems set a ceiling on individual earnings used to calculate both mandatory contributions and pension benefits. The average ceiling in OECD countries is around 2.3 times the average economy-wide earnings (OECD 2017, p. 90). The lower the ceiling to pensionable earnings proves, the lower will be, *ceteris paribus*, the replacement rate that the system ensures for retiring workers. On the other hand, low ceilings make it more expedient for workers to subscribe to supplementary pension plans with voluntary contributions.

5.1 The Earnings-Related Pension

The majority of OECD countries compute public pensions according to some measures of individual earnings. As mentioned in the introductory chapter, the factors influencing the earnings-related pension are:

- the number of contributory years;
- reference earnings, hereafter denoted as RE, i.e. the specific set of workers' annual pensionable earnings counting for the computation;
- the accrual rate, i.e. the percentage of reference earnings, counting as pension for each year of work.

The typical earnings-related pension formula is:

$$p = n \cdot a \cdot \mathrm{RE},$$

where RE is generally the following average of a predefined number *r* of last earnings

$$\mathrm{RE} = \left[\sum_{i=n-r+1}^{n} w_i \cdot \prod_{j=i}^{n} \left(1 + \gamma_j\right) \right] / r,$$

where

$$\sum_{i=n-r+1}^{n} w_i = w_{n-r+1} + w_{n-r+2} + \cdots + w_n,$$

is the sum of the last r yearly earnings and

$$\prod_{j=i}^{n} (1 + \gamma_j),$$

are the compound growth factors, i.e. the indexes, to be used for valorization of past earnings of each year i where the valorisation rate γ typically reproduces the growth rate of the CPI or of the economy-wide average earnings. For instance, the valorized earnings of the second-to-last year of the career amount to

$$w_{n-1} \cdot (1 + \gamma_{n-1}) \cdot (1 + \gamma_n),$$

whereas those of the third-to-last year amount to

$$w_{n-2} \cdot (1 + \gamma_{n-2}) \cdot (1 + \gamma_{n-1}) \cdot (1 + \gamma_n),$$

and so on, showing that the ‘older’ earnings to be valorised are, the more growth factors will have to be included in the index. The higher (lower) the rate at which past earnings are valorised the higher (lower), ceteris paribus is the system’s rate of return on workers’ contributions.

Note that the value of r can range from 1 to n, the extremes of the range showing the opposite cases, respectively, of pensions depending exclusively on the very last earnings and pensions depending on all earnings. Since the reforms came underway at the end of the twentieth century, many OECD countries have now adopted the all earnings rule (OECD 2017), whereas last earnings formula had been the norm hitherto (Disney 1999).

The main reason underlying the move from the last earnings towards all-earnings formulas is the unjustified premium that the former grant to short and fast-rising careers relative to long and flat careers. However, it can be proved that the all-earnings version can only reduce but not eradicate the unfairness inherent in the typical earnings-related formula (see Focus 5.1). In fact, a different variant of earnings-related formula addresses the unfairness issue by differentiating the accrual rate according to the earnings level. The following earnings-related formula, allowing for higher accrual rates on lower earnings brackets, is in effect in the United States:

$$p = a_1 \cdot \min(\mathrm{RE}, B_1) + \begin{cases} 0 \text{ if } \mathrm{RE} < B_1 \\ a_2 \cdot [\min(\mathrm{RE}, B_2) - B_1] \end{cases} + \begin{cases} 0 \text{ if } \mathrm{RE} < B_2 \\ a_3 \cdot (\mathrm{RE} - B_2) \end{cases},$$

where a_1, a_2 and a_3 are the three accrual rates, RE is the average of the highest 35 annual earnings valorised according to economy-wide average earnings growth, and B_1 and B_2 ($B_1 < B_2$) are the 'bend points' defining the three annual earnings brackets. The alleged progressiveness of the system derives from the values of the accrual rates. These are $a_1 = 0.9$, $a_2 = 0.32$ and $a_3 = 0.15$, while the values of the two bend points, annually recomputed according to the average wage growth, are $B_1 = \$11,112$ and $B_2 = \$66{,}996$ for those reaching the age of 62 (the lowest admitted retirement age) in 2019. Note that, despite the apparent absence of the number n of contributing years—which on the other hand finds a place in the typical earnings-related formula—the US formula penalizes short careers in that US social security, when computing RE, uses as many zeros as the difference between 35 and the retiring worker's length of career. Despite the fact that the sharp difference between the three accrual rates grants much higher replacement rates to low-earning than to high-earning workers, it has been shown that the US formula has poor redistributive properties in terms of individual IRRs in particular vis-à-vis certain, not infrequent, career patterns (Nisticò and Bevilacqua 2018).

It is also to be emphasized that pure and simple earnings-related formulas provide scant incentive to postpone retirement beyond the standard, or normal, retirement age given that the modest increase in pension gained with an additional year of work does not compensate for the lower number of expected annual benefits implied by postponing retirement. This is why, in conjunction with the ongoing adverse demographic scenario, the normal retirement age is being raised in almost all OECD countries, in many cases with an automatic link to the increases in life expectancy at retirement. On the other hand, the effective age at which workers claim their earnings-related pension is significantly lower than the normal age, due to the widespread use of early retirement (OECD 2017, p. 126). In fact, correcting the pension resulting from the formula according to the difference between the effective and the normal retirement age is becoming the norm. In the United States, where the standard retirement age is being raised to 67, the pension is cut by 6.6% for each year of early retirement and increased by 8% for each year of postponement relative to 67. Those figures are often referred to as actuarial adjustments, given that they are based on the effect of different retirement patterns on both revenues and expenditures considering the probability of surviving at the different ages (see Focus 5.2).

Focus 5.1: Assessing the Possible Unfairness of the Earnings-Related Formulas

Assessing the extent to which pension formulas transparently redistribute in favour of some specific career patterns, or opaquely redistribute in favour of others, can be done by computing individual IRRs, e.g. with a spreadsheet as in Focus 3.2, for different series of inflows and outflows, reflecting contributions and benefits paid into and received from the pension system by some typical categories of workers. For instance, a broad stream of literature, e.g. Caldwell et al. (1999), Gustman and Steinmeier (2001), Coronado et al. (2002), and Liebman (2002), has focused on the regressive redistributions generated by the correlation between lifetime income and life expectancy that makes the payback period in favour of low-income workers systematically shorter than the average. It is important to emphasize that proof of these regressive redistributions is quite robust only considering redistribution from low income females to high income females and from low income males to high income males. In fact, considering—the more relevant—total insurance collective, redistribution from low-income to high-income contributors is far less clear, given that the generally lower incomes and longer life of females compensate for the income and life expectancy correlation within sexes. Before then, Gronchi (1995) had calculated the impact of a shift from the 'last 5' to the all-earnings formula introduced in Italy in 1992 on the real IRRs of thirty typical careers of employees distinguished according to average real wage growth and length. The study, showing that moving from the 'last 5' to the all-earnings rule could reduce but could not eliminate the unfairness typical of last-earnings formulas, convinced the Italian Unions to support the new, radical shift to the NDC scheme that was introduced by the Italian Parliament in 1995. In fact, the tables contained in Fig. 5.2 showed Italian policymakers that:

(i) with both the last-earnings and the all-earnings formula and for both men and women, the IRR decreases with the length and increases with the speed of the career, thus revealing the unfairness of the system; (ii) the all-earnings formula produces lower IRRs for all careers and also a lesser dispersion (as measured by the standard deviation), albeit still significant; (iii) with the move to the all-earnings formula, legislated together with the increase in the contribution rate (from 28.4 to 33%) and the gradual increase in the standard retirement age (from 55 to 60 for women and from 60 to 65 for men), the real IRR of all 30 careers remained well above the growth rate of real aggregate earnings expected from then on (1.5–2.0%), thus showing that the reform could not ensure solvency for the Italian system.

Individual IRRs in Italy before 1992's reform

MEN

Working years	real wage growth 0,5%	1,0%	1,5%	2,0%	2,5%	3,0%	aver.	st. dev.
20	4,4%	4,6%	4,8%	5,0%	5,2%	5,4%	4,9%	0,07
25	4,2%	4,4%	4,7%	4,9%	5,1%	5,3%	4,8%	0,08
30	4,0%	4,3%	4,5%	4,8%	5,1%	5,3%	4,7%	0,09
35	3,9%	4,2%	4,4%	4,7%	5,0%	5,3%	4,6%	0,10
40	3,8%	4,1%	4,4%	4,7%	5,0%	5,3%	4,5%	0,11
aver.	4,1%	4,3%	4,6%	4,8%	5,1%	5,3%	4,7%	
st. dev.	0,05	0,04	0,03	0,02	0,01	0,01		0,10

WOMEN

Working years	real wage growth 0,5%	1,0%	1,5%	2,0%	2,5%	3,0%	aver.	st. dev.
20	5,0%	5,2%	5,4%	5,6%	5,8%	6,0%	5,5%	0,06
25	4,8%	5,0%	5,2%	5,4%	5,6%	5,9%	5,3%	0,07
30	4,5%	4,8%	5,0%	5,3%	5,5%	5,8%	5,2%	0,08
35	4,4%	4,6%	4,9%	5,2%	5,5%	5,7%	5,0%	0,09
40	4,2%	4,5%	4,8%	5,1%	5,4%	5,7%	4,9%	0,10
aver.	4,6%	4,8%	5,1%	5,3%	5,6%	5,8%	5,2%	
st. dev.	0,06	0,05	0,04	0,03	0,02	0,02		0,09

Individual IRRs in Italy after the 1992 reform

MEN

working years	real wage growth 0,5%	1,0%	1,5%	2,0%	2,5%	3,0%	aver.	st. dev.
20	3,6%	3,6%	3,8%	3,9%	4,0%	4,2%	3,9%	0,05
25	3,3%	3,5%	3,6%	3,8%	3,9%	4,0%	3,7%	0,06
30	3,2%	3,3%	3,5%	3,6%	3,7%	3,8%	3,5%	0,06
35	3,0%	3,2%	3,3%	3,4%	3,5%	3,6%	3,4%	0,06
40	2,9%	3,1%	3,2%	3,4%	3,5%	3,6%	3,3%	0,06
aver.	3,2%	3,4%	3,5%	3,6%	3,7%	3,8%	3,5%	
st. dev.	0,08	0,05	0,05	0,06	0,06	0,06		0,08

WOMEN

working years	real wage growth 0,5%	1,0%	1,5%	2,0%	2,5%	3,0%	aver.	st. dev.
20	4,4%	4,5%	4,6%	4,7%	4,8%	5,0%	4,7%	0,04
25	4,1%	4,2%	4,3%	4,5%	4,6%	4,7%	4,4%	0,05
30	3,8%	4,0%	4,2%	4,3%	4,4%	4,5%	4,2%	0,05
35	3,7%	3,8%	4,0%	4,1%	4,2%	4,3%	4,0%	0,05
40	3,5%	3,7%	3,8%	4,0%	4,1%	4,2%	3,9%	0,06
aver.	3,9%	4,1%	4,2%	4,3%	4,4%	4,5%	4,2%	
st. dev.	0,08	0,06	0,06	0,07	0,06	0,07		0,08

Fig. 5.2 The effects on individual real IRRs of moving from the 'last 5' to the all-earnings rule

The same finding of the unfairness of the last earnings rule can be reached 'theoretically' without resorting to simulations. In fact, by using the above-mentioned 4-OLG model allowing for different career patterns within each cohort of contributors, Gronchi and Nisticò (2008) proved that within a DB setting based on last earnings, the growth rate of aggregate earnings constitutes merely a sort of average of the sustainable individual IRRs, with flat and long careers implying a lower IRR than short and dynamic ones.

5.1.1 *Setting the Indexation Rule of Earnings-Related Pensions*

It is part of the DB logic that after awarding the first pension annuity according to the chosen formula, a rule must be announced as to how the annuity changes through time, i.e. as to the value that the growth rate of pensions g_t^p takes in any calendar year t. For those public pension systems whose main goal is to prevent poverty among the elderly, after awarding the first pension, possibly above the poverty line, subsequent annuities should maintain the pension's purchasing power unaltered. This is achieved by enforcing the following indexation rule:

$$g_t^p = g_t^{\mathrm{CPI}},$$

where

$$g_t^{\mathrm{CPI}} = \frac{\mathrm{CPI}_{t+1}}{\mathrm{CPI}_t} - 1,$$

is the growth rate of the CPI, i.e. the growth rate of the cost of living, during period t and

$$p_{t+1} = p_t \cdot \frac{\mathrm{CPI}_{t+1}}{\mathrm{CPI}_t},$$

computes the level of the annuity to be paid at the beginning of any year $t + 1$ given the pension paid at the beginning of t.

If pensions are uprated according to changes in CPI, their real value, i.e. their purchasing power, remains constant through time. In fact, recalling that the growth factor of nominal pensions can be decomposed into two components, the growth factor of the cost of living and the growth factor of pensions' purchasing power (see Sect. 3.1), the equality $g_t^p = g_t^{\mathrm{CPI}}$ implies that:

$$g_t^{rp} = \frac{1 + g_t^p}{1 + g_t^{\mathrm{CPI}}} - 1 = 0,$$

where g_t^{rp} denotes the growth rate of the pension in real terms during period t. On the other hand, the growth rate of workers' nominal earnings before retirement tends to exceed that of the cost of living, which implies:

$$g_t^{rw} = \frac{1 + g_t^w}{1 + g_t^{CPI}} - 1 > 0,$$

where g_t^{rw} denotes the growth rate of real earnings during period t computed by deducting the growth rate of CPI from the growth rate of nominal earnings during the same period. In fact, uprating pensions in payment according to CPI growth alone is not consistent with compulsory old-age insurance designed to support individuals' consumption smoothing. Exacerbation of the negative shock for retirees implied by the replacement rate being less than unit is avoided by uprating nominal pensions according to the growth rate of average nominal earnings, i.e. by setting the following indexation rule:

$$g_t^p = g_t^w,$$

which implies:

$$g_t^{rp} = g_t^{rw}.$$

Uprating pensions according to average nominal earnings growth is generally referred to as wage indexation, despite the fact that wages represent only a share of economy-wide earnings, including for instance also those of the self-employed, and that the different forms of earnings do not necessarily grow through time at the same rate.

With the worsening of the economic and demographic scenario, many public pension systems designed to ensure consumption smoothing, and thus initially enforcing wage indexation, moved to CPI indexation, which is now the norm, although some DB systems rely on a mix between the two. Unfortunately, reducing pension expenditures by fixing annuities in real terms can be very harmful for both retirees and workers close to retirement who have scant chances of increasing their savings for retirement. The graphs presented in Fig. 5.3 show the case of a hypothetical worker whose earnings, both gross and net of a 20% contribution rate (assumed to be entirely paid by workers), grow steadily at 2% in real terms for 40 years, starting from the normalized initial level of 100 and 80, respectively and then retiring with the promised first pension equal to 50% of final gross earnings, such that the net replacement rate, i.e. the ratio between the first pension and the last earnings net of pension contributions, amounts to 62.5%. The graph shows that the pension, on which contributions are not due, equals the real earnings, net of pension contributions that the worker had received around 23 years before retiring and that CPI (flat real) indexation implies that

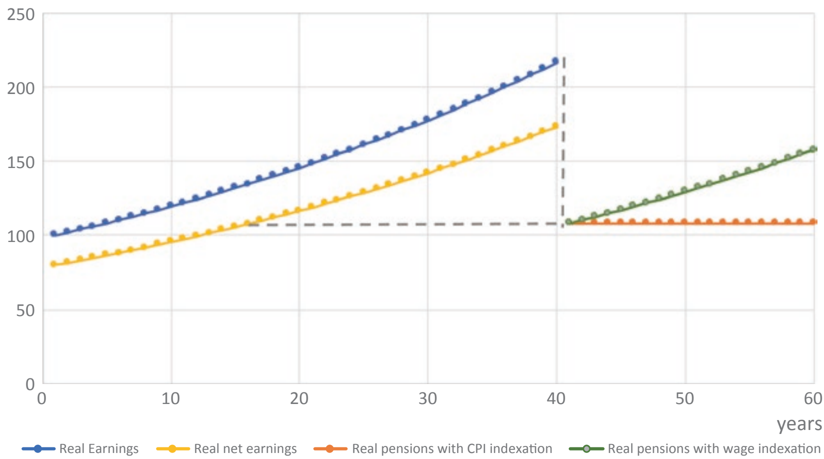

Fig. 5.3 Moving from work to retirement with CPI or with Wage indexation

the retired worker should adapt to such a remote standard of living for the rest of his/her life. Conversely, with wage indexation, retirees can more easily accept the new, lower standard of living given the prospect of seeing their pensions increasing in real terms at the same rate as they were used to with their earnings when working. On the other hand, CPI indexation is psychologically more acceptable by workers whose end-of-career earnings decline in real terms.

5.1.2 The Problem of 'Vintage Pensions'

The problems with mere CPI indexation are not limited to the difficulty for individuals to accept that their own income stops increasing in real terms after retirement. Comparison of one's experience as a pensioner with that as a worker could be considered a relatively minor issue if individuals were informed in advance of the prospect of their public pension merely maintaining their purchasing power unaltered through time. In fact, CPI indexation raises a much more serious issue of intergenerational fairness, i.e. a problem of disparity between 'similar' retirees belonging to different birth cohorts. Suppose, for instance, that Ms. Smith retires in 2019 after *n* years of work with the following last earnings pension:

$$p_{\text{Smith, 2019}} = n \cdot a \cdot w_{\text{Smith, 2018}}.$$

One year later, also Ms. Jones retires after n years of work, $n-2$ years of which spent together with her older friend Ms. Smith, performing the same tasks for the same company with the same annual earnings. Therefore, Ms. Jones's pension will be:

$$p_{\text{Jones, 2020}} = n \cdot a \cdot w_{\text{Jones, 2019}}.$$

On the other hand, since all earnings grow yearly at the rate, supposedly constant, g_w, the following relation holds between Ms. Jones and Ms. Smith's last earnings:

$$w_{\text{Jones, 2019}} = w_{\text{Smith, 2018}} \cdot (1 + g_w),$$

such that:

$$p_{\text{Jones, 2020}} = p_{\text{Smith, 2019}} \cdot (1 + g_w).$$

There is nothing wrong with Ms. Jones's first pension, drawn in 2020, being $(1+g_w)$ times higher than Ms. Smith's first pension, drawn in 2019 as it was for the first, the second up to the nth salary earned by the younger friend one year later than the older one. In fact, precisely because of this circumstance, when Ms. Jones was hired and joined Ms. Smith in performing the same task as hers the two friends were earning exactly the same salary and such equality held in each of the $n-2$ years of common work. It is evident that for the two friends, accustomed for $n-2$ years to earn the same salary for the same service every year, to experience the same equality during retirement, Ms. Smith's pension drawn in 2019 should be indexed according to the growth rate of earnings such that:

$$p_{\text{Smith, 2020}} = p_{\text{Smith, 2019}} \cdot (1 + g_w) = p_{\text{Jones, 2020}}.$$

If, on the other hand, Ms. Smith's pension were indexed to CPI growth, we would have:

$$p_{\text{Smith, 2020}} = p_{\text{Smith, 2019}} \cdot (1 + g_{CPI}) = p_{\text{Jones, 2020}} \cdot \frac{1 + g_{CPI}}{1 + g_w},$$

implying that Ms. Smith's second pension annuity in 2020 will amount to around 98% of the first pension awarded to her younger friend in the same year if real earnings grow at 2% per year. In other words, the further problem generated by CPI indexation of pensions in payment is that it generates vintage pensions, i.e. a marked disparity among pension annuities awarded, *ceteris paribus*, in different years. Consider that if the earnings growth factor exceeds that of the CPI by 2%, the pensions awarded in year $t - 10$ will amount to:

$$\left(\frac{1 + g_{\mathrm{CPI}}}{1 + g_w}\right)^{10} = \left(\frac{1}{1.02}\right)^{10} = 82\%,$$

of those awarded to similar workers in year t. Given the ongoing increase in life expectancy, it is not infrequent that 20 or even 30 cohorts of similar retired workers draw their pensions in the same calendar year, with the consequence that the 20-year-old and 30-year-old CPI-adjusted annuities amount to just 67 and 55%, respectively, of those newly awarded in the current year.

5.2 The Technicalities of the PA–DC Scheme

Personal accounts work as a savings deposit. During the active period, paid in contributions increase the balance of the deposit together with the accrued interests. After retirement, while interest still accrues on the account, withdrawal of pension annuities reduces the balance. The logic of personal accounts is that expected pension annuities must exhaust the deposit balance, i.e. that the balance of the personal account drops to zero after the last expected withdrawal for the 'representative' member of each cohort whose actual life duration coincides with life expectancy. This one-to-one correspondence between expected pensions and contributions is generally referred to as 'fairness', in the sense of the absence of any redistributions apart from those from the shorter-lived to the longer-lived, implicit in any old-age insurance plan. As we have seen in Sects. 4.3 and 4.5, personal accounts can also ensure automatic solvency for any contribution rate provided that the rate of return to be credited to all accounts is appropriately chosen in accordance with the system's degree of funding, equalling the market interest rate or the growth rate of aggregate earnings for funded or PAYG schemes, respectively. It should be clear that personal accounts also allow for retirement age flexibility within an interval chosen by the policymaker (Gronchi et al. 2019). The workers who choose to end the accumulation phase and enter into the disbursement phase earlier than the others accept, *ceteris paribus*, that their account balance at retirement is lower and their life expectancy is higher, such that annual withdrawals, i.e. pension annuities, will be lower than those afforded by a higher retirement age. Figure 5.4 compares the evolution

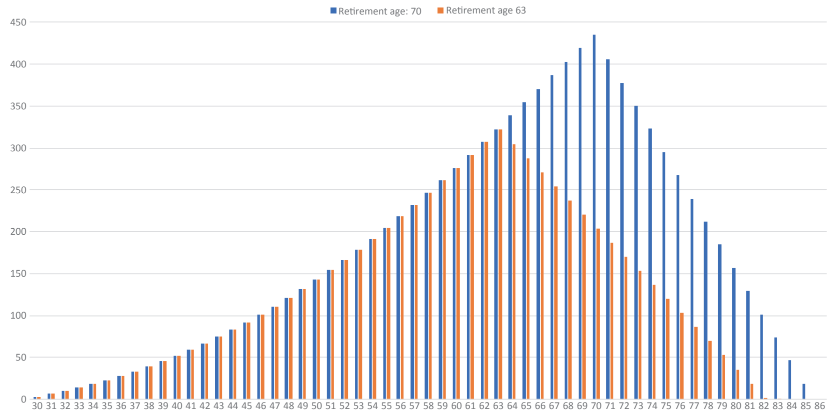

Fig. 5.4 The evolution of the personal account (thousands of £.) according to retirement age

of a hypothetical personal account under the two alternative choices of the individual retiring at the age of 70 or at the age of 63. Comparison shows that retiring at 70, after contributing for 40 years, significantly increases the worker's account balance and shortens the value *m* of the expected number of annuities (life expectancy at retirement) from around 20 to 16, thus allowing for much higher annual pensions, as shown by the steeper disbursement phase at 70. Note that surviving at 70 implies an overall higher number of expected life years than those foreseen at 63 (see Focus 5.2).

5.2.1 Spreading the Account Balance Throughout Life Expectancy at Retirement: Computing and Indexing the Pension Annuity

We should now ask how systems based on personal accounts compute and then index the first pension annuity to ensure that the account balance is exhausted by the *m* withdrawals expected at retirement, taking into consideration that interest continues to accrue on the account also after retirement. Let us suppose that Ms. Brown retires on 1 January 2020, when her account balance reaches the value of £100,000 and her life expectancy is 10 years. To grasp the logic of personal accounts it will help, provisionally, to imagine the balance split into 10 equivalent parts, amounting to £10,000, designated to finance the 10 expected annuities. This even balance spreading is shown in column 3 of Table 5.1. The first

Table 5.1 The working of personal accounts after retirement—standard profile

Date	*Annuity number*	*Balance spreading*	*Sustainable interest rate*	*Matured interest (compound)*	*Pension annuities*	*Resulting indexation*
1.1.2020	1	£10,000	–	–	£10,000	–
1.1.2021	2	£10,000	10%	£1000	£11,000	10%
1.1.2022	3	£10,000	5%	£1550	£11,550	5%
1.1.2023	4	£10,000	5%	£2127.5	£12,127.5	5%
1.1.2024	5	£10,000	10%	£3340.25	£13.340.25	10%
1.1.2025	6	£10,000	10%	£4674.275	£14,674.275	10%
1.1.2026	7	£10,000	12%	£6435.188	£16,435.188	12%
1.1.2027	8	£10,000	8%	£7750	£17,750	8%
1.1.2028	9	£10,000	10%	£9525	£19,525	10%
1.1.2029	10	£10,000	8%	£11,087	£21,087	8%
1	2	3	4	5	6	7

slice of the balance is immediately withdrawn as the first pension annuity whereas the remaining 9 mature interest at the sustainable interest rate until they are eventually withdrawn. Therefore, the first pension is:

$$p = \frac{£100{,}000}{10} = £10{,}000,$$

where the denominator is the balance divisor reflecting Ms. Brown's life expectancy at retirement or, more precisely, the life expectancy of the cohort Ms. Brown belongs to (see Focus 5.2).

Given that the sustainable interest rate is pegged to a changing economic variable, be it the market interest rate, the growth rate of aggregate earnings or any combination of the two, its value changes unpredictably through time, as shown in column 4. Column 5 shows the overall interest maturing on the corresponding slices of the balance before they are withdrawn as pension annuities. For instance, the £2127.5 compound interest matured on the fourth slice of the account when it is withdrawn as the £12,127.5 fourth pension annuity is computed as follows:

$$£10{,}000 \cdot 0.1 + £11{,}000 \cdot 0.05 + £11{,}550 \cdot 0.05 = £2127.5.$$

Column 7 shows the indexation rate resulting from the value of the annuities, shown in column 6. For instance, the indexation rate in 2024, is computed as follows:

$$\frac{£13{,}340.25}{£12{,}127.5} - 1 = 10\%.$$

Note that the indexation rate is 'endogenous', meaning that it results from the balance spreading and from the corresponding annual interest rate. In fact, the even balance spreading shown in Table 5.1 generates an indexation rate exactly equal to the sustainable interest rate, as emerges on comparing columns 4 and 7. In order to understand why this is so, consider the circumstance that each slice differs from the one that became an annuity one year earlier precisely on account of the extra interest matured during the last year.

However, balance spreading doesn't need to be even, since Ms. Brown might prefer heavier withdrawals from her balances in the first years of retirement.

This is achieved with the spreading shown in Table 5.2, where each slice is 1.5% higher than the following one, implying that all slices, denoted as s_i, are linked to the first as follows:

$$s_i = \frac{s_1}{(1 + 1.5\%)^{i-1}} \quad i = 2, 3, \cdots, 10.$$

Table 5.2 The working of personal accounts after retirement—frontloaded profile

Date	*Annuity number*	*Balance spreading*	*Sustainable interest rate*	*Matured interest (compound)*	*Pension annuities*	*Resulting indexation*
1.1.2020	1	£10,683	–	–	£10,683	–
1.1.2021	2	£10,525	10%	£1052.5	£11,577	8.37%
1.1.2022	3	£10,370	5%	£1607.35	£11,977.35	3.45%
1.1.2023	4	£10,216	5%	£2173.45	£12,389.45	3.45%
1.1.2024	5	£10,066	10%	£3362.3	£13,428.3	8.37%
1.1.2025	6	£9917	10%	£4635.48	£14,552.48	8.37%
1.1.2026	7	£9770	12%	£6287.18	£16,057.18	10.34%
1.1.2027	8	£9626	8%	£7460.15	£17,086.15	6.4%
1.1.2028	9	£9484	10%	£9033.51	£18,517.51	8.37%
1.1.2029	10	£9343	8%	£10,358.59	£19,701.59	6.4%
1	2	3	4	5	6	7

For the deposit to be exhausted with the last withdrawal, it will suffice to impose the following condition that the sum of all slices equals the account balance at retirement:

$$s_1 \cdot \sum_{i=1}^{10} \frac{1}{(1+1.5\%)^{i-1}} = £100{,}000,$$

which can be solved for s_1 thus allowing for determination of the first pension (which coincides with the first slice) as follows:

$$p = s_1 = \frac{£100{,}000}{\sum_{i=1}^{10} \frac{1}{(1+1.5\%)^{i-1}}} = \frac{£100{,}000}{1 + \frac{1}{1.015} + \frac{1}{1.015^2} + \cdots + \frac{1}{1.015^9}} = £10{,}683,$$

which accounts for the value of the first pension (as well as of the first slice) shown in the first row of Table 5.2. Note that the balance divisor still reflects life expectancy, since it is the sum of 10 terms mirroring the 10 expected annuities to be paid to Ms. Brown. On the other hand, the divisor is now lower than 10 because of the uneven spreading that allows Ms. Brown to be paid higher pensions in the earlier years of retirement at the cost, however, of lower pensions in later years. In this sense, the disbursement profile shown in Table 5.2 is referred to as 'frontloaded', due to the first five slices being higher and the last five being lower than £10,000, while 1.5% is, in this case, the frontloading rate. It is no surprise, therefore, that applying the same annual interest factors to the two different sets of slices, we get the two different disbursement profiles shown in Tables 5.1 and 5.2.

In fact, we need to recall that the deposit exhaustion constraint implies that the pensions following the first are obtained by adding interest to the saved slices. For instance, the value that the third slice will have when it is withdrawn is:

$$s_3 = \frac{s_1}{(1+1.5\%)^2} \cdot (1+10\%) \cdot (1+5\%),$$

while the value of the fourth will be:

$$s_4 = \frac{s_1}{(1+1.5\%)^3} \cdot (1+10\%) \cdot (1+5\%) \cdot (1+5\%),$$

which explains the values for the third and the fourth pensions awarded to Ms. Brown in years 2022 and 2023, respectively.

Having clarified the logic of the one-to-one correspondence that ensures fairness of PA, we should now ask what is the indexation rule that a PA system should announce 'in advance' to be sure that the whole disbursement phase is consistent with the first pension and the future interest that will accrue to Ms. Brown's account. On comparing the formula of the fourth with that of the third pension (slice) we can see that the resulting indexation rate when Ms. Brown is awarded her fourth pension is:

$$g_{p,4} = \frac{s_4}{s_3} - 1 = \frac{s_1 \cdot \frac{(1+10\%)\cdot(1+5\%)\cdot(1+5\%)}{(1+1.5\%)^3}}{s_1 \cdot \frac{(1+10\%)\cdot(1+5\%)}{(1+1.5\%)^2}} = \frac{1+5\%}{1+1.5\%} - 1 = 3.45\%,$$

which confirms the value of the indexation rate shown in Table 5.2 for 2023 and clarifies the rule to be announced, i.e. that the indexation rate is each year equal to the 'difference' between the sustainable interest rate and the chosen frontloading rate.

In general, the two requisites for a generic (see Focus 4.2) PA pension scheme to ensure sustainability and fairness can be generalized as follows:

1. First pension rule:

$$p = \frac{AB_R}{\sum_{i=1}^{m} (1+\delta)^{1-i}},$$

where

$$AB_R = c \cdot \sum_{i=1}^{n} w_i \cdot \prod_{j=i}^{n} \left(1 + \overset{\circ}{\pi}_{GDC,j}\right),$$

denotes the personal account balance at retirement and δ the 'frontloading rate' established by the policymaker. Note that the balance divisor increases with life expectancy and decreases with the frontloading rate.

2. Indexation rule of all pensions to be disbursed at the beginning of calendar year $t+1$:

$$g_t^p = \frac{1 + \overset{\circ}{\pi}_{GDC,t}}{1+\delta} - 1,$$

where $\pi^{\circ}_{\mathrm{GDC},t}$ denotes the value that the sustainable interest rate takes on in calendar year t (see Focus 4.2).

The generality of the two rules can readily be grasped given that for $\delta = 0$ the denominator of the first pension rule amounts to life expectancy at retirement while the expression of the indexation rule equals the system interest rate, as it does for the standard profile described in Table 5.1. On the other hand, the divisor of the account balance at retirement increases with life expectancy m, which in turn decreases as retirement age increases, as shown in Fig. 5.4. Therefore, PA systems ensure fairness at whatever retirement age, since workers who choose to retire at younger ages with a higher divisor 'pay' for their longer benefits in terms of smaller annuities. Nevertheless, some upper limit should exist above which employers can impose retirement on their employees who might overestimate their working capacity, while a lower one is also advisable because excessively young retirement ages, implying low pensions, increase the risk of poverty among the elderly. This is why PA systems generally announce an interval of admitted retirement ages together with the corresponding divisors.

5.2.2 The Trade-Off Between the Frontloading and Indexation Rates

According to the first rule, more frontloaded pension profiles can be generated by raising the value of δ, which increases, *ceteris paribus*, the generosity of the first pension. This is very tempting for pension systems aiming at fostering the appeal of their plan, be it public or private. Nevertheless, one should resist such a temptation since the indexation rate resulting from the second rule would risk becoming very low or even negative whenever the value of the sustainable interest rate did not exceed the chosen frontloading rate. The consequences of high frontloaded rates are shown in Fig. 5.5, where the standard and the frontloaded profiles presented in Tables 5.1 and 5.2 have been recomputed under the assumption of a sustainable, real interest rate of 1.5%, which implies a flat real annuity for the 1.5% frontloaded profile and a 1.5% real indexation for the standard profile.

Note that the low indexation rate generated by high frontloading rates gives rise to the same problem as the vintage pensions discussed in Sect. 5.1.2.

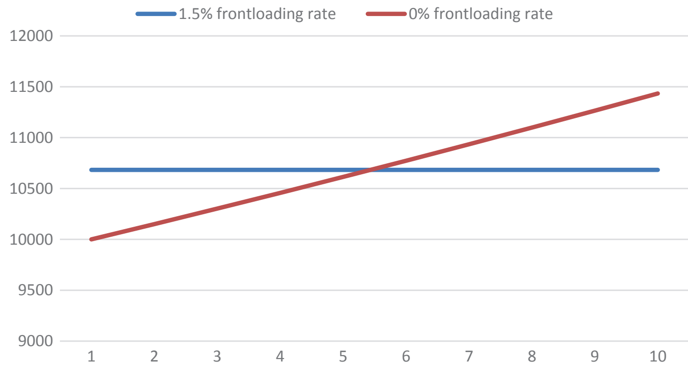

Fig. 5.5 The standard and frontloaded annuities in real terms when the sustainable interest rate adjusted for inflation is 1.5%

Whereas vintage pensions are tolerable in private, voluntary pension plans, the same cannot be said of public compulsory plans where they lead to social turbulence and, possibly, to periodic equalizations that compromise both the fairness and the sustainability of the system. In fact, the 1.5 and 1.6%, chosen for δ by Italy and Sweden, respectively, are definitely too high considering the low growth expected worldwide in the near future. Norway, deterred by the Swedish 2010 and 2011 negative indexations, seems to have understood the risk, and has set the frontloading rate at a much more convenient value of 0.75%.

5.2.3 *Assessing the Adequacy of PA Pensions*

The appeal of the DB earnings-related formula lies in its ability to provide workers with an intelligible expectation of the replacement rate of their first pension after n years of work. On the other hand, it is in the nature of PA–DC systems not to make any promise to workers but that their stream of pension annuities will correspond to their lifelong contributions, gross of the sustainable interest. Given the technicalities involved in computing the account balance at retirement and the annuities, significant information support is necessary for PA systems to ensure transparency, i.e. to let workers see what they will get in exchange for their contributions. This is why PA systems generally provide workers with a periodical statement showing their present account balance, the relative weights of paid in contributions and accrued interest and an estimate of the pension that can be expected in one or more well specified career scenarios and according to the various retirement ages included in the interval defined by the system. Whereas this good practice is widespread among supplementary or voluntary PA–FDC systems, Sweden is the only country we are aware of that provides this fundamental informative support to workers registered in a compulsory NDC scheme by means of the so-called Orange Envelope sent out yearly by the Swedish Pension Agency to all insured workers.

On the other hand, a rough estimate of the expected replacement rate is at hand also for NDC public systems crediting to all account balances an interest rate (π) equal to the growth rate of aggregate earnings. In fact, assuming that the earnings of a typical worker grow through time precisely at the growth rate of aggregate earnings, then the following equality holds between the future values at retirement of all yearly contributions:

$$c \cdot w_1 \cdot \prod_{j=1}^{n} \left(1 + \overset{\circ}{\pi}_{\mathrm{GDC},j}\right) = c \cdot w_2 \cdot \prod_{j=2}^{n} \left(1 + \overset{\circ}{\pi}_{\mathrm{GDC},j}\right) = \cdots = c \cdot w_n \cdot \left(1 + \overset{\circ}{\pi}_{\mathrm{GDC},n}\right),$$

such that the account balance at retirement results from multiplying the future value of the last contribution by the number of contribution years, while the first pension is

$$p = \frac{n \cdot c \cdot w_n \cdot (1 + \pi_n)}{d},$$

where d denotes the balance divisor substantially equal to life expectancy at retirement. If we divide both sides of the above expression by the last wage w_n, we can see that the gross replacement rate to be expected by this typical worker is

$$\frac{p}{w_n} = c \cdot \frac{n}{d} \cdot (1 + \pi_n),$$

whereas the replacement rate net of contributions is:

$$\frac{p}{w_n \cdot (1 - c)} = c \cdot \frac{1}{1 - c} \cdot \frac{n}{d} \cdot (1 + \pi_n),$$

which shows that the main determinants of the replacement rate are the contribution rate (c) and the ratio between active and expected retirement years (n/d), which favours long and dense careers, while the impact of the last interest factor ($1 + \pi_n$) is negligible (Nisticò and Bevilacqua 2013). In fact, whereas the contribution rate plays no role for the long-run solvency of a PA system, its value is fundamental in determining the adequacy of the pension. For instance, if our typical worker plans to retire after 40 years of work when his/her life expectancy is around 20 years ($n/d = 2$), the expected gross replacement rate will be around twice the contribution rate, and so only 30% if the contribution rate is 15% or 40% if the contribution rate is 20% of gross earnings. Note that the corresponding net figures are higher: 35 and 50%, respectively. It is worth emphasizing that flat-career workers whose earnings growth rate falls short of the sustainable rate of return should expect higher replacement rates, given that their end-of-career earnings are lower than those of our typical worker. The reverse applies to fast-rising careers. This property of PA systems of awarding uniform rates of return while differentiating replacement rates in favour of long and flat careers stands out in contrast with the outcome of systems based on DB-last earnings rules that award substantially uniform replacement rates while differentiating IRRs in favour of short and fast-rising careers (see Focus 5.1).

5.3 The Functioning of PA Schemes When Reality Parts Company with Models

In our 2-overlapping-generation model described in Sect. 4.1, we assumed that the number of workers of age 1 is year by year $1 + g_L$ times greater than the previous year. Moreover, we assumed constant mortality, i.e. death rates constantly equal to 0% between age 1 and age 2 and 100% at age 2. These two assumptions imply that the number of retirees is also year by year $1 + g_L$ times lower than the number of workers. However, fertility and employment rates change through time irregularly and death rates are showing a continuous decline, in particular at high ages. This latter pattern implies, among other things, that the younger cohorts' life expectancy at retirement is higher than that of the older cohorts. Moreover, it is worth recalling that with a PA scheme all the insured workers who die before retirement leave their account balances as a sort of inheritance that the pension system should allocate in one way or another, e.g. to surviving relatives, to surviving members of the same cohort of the deceased, as in the Swedish NDC scheme, or as a reserve fund to offset other possible causes of imbalances. In what follows, we will try to understand how such features of reality affect our previous conclusions on the financial solvency and fairness of PA systems.

5.3.1 Changes in Employment Growth

Non-steady employment growth affects PA systems differently, according as to whether they are funded or PAYG financed. In fact, changes in employment growth do not affect, *ceteris paribus*, the solvency of funded PA systems, given their particular feature of not having to borrow from active cohorts to be able to pay pensions to the retired. In fact, even a substantial fall in the number of contributors does not affect the solvency of an FDC system provided that it computes its liabilities according to the sustainable rate of return on contributions (the market interest rate), such that the accumulated funds will always suffice to extinguish liabilities, more populated retiring cohorts having accumulated more funds than less populated ones.

The same cannot be said of the NDC systems, which could have a short-run liquidity problem whenever the members of the current cohorts of contributors fall short of its expected, steady-state level. However, provided that changes in the employment growth rate are cyclical, this circumstance does not impede the long-run sustainability of NDC systems, given that the less populated cohorts of current contributors will then become less populated cohorts of retirees precisely when the new, more populated cohorts of contributors enter on the scene. Therefore, cyclical changes in employment growth produce temporary unbalances which are positive (surpluses) when the employment growth rate increases and negative (deficits) when it decreases (Valdés-Prieto 2000; Gronchi and Nisticò 2008). This is why a capable enough buffer fund, 'inflating' during the surplus phase and then 'deflating', is a fundamental complement to NDC pension systems. An additional problem implied by non-steady employment growth is that the system interest rate during phases of expansion is higher than during recessions. This may cause disparities in the individual IRRs both between and within cohorts. To stabilize the system's rate, the Swedish NDC scheme chose not to distribute the volatile 'employment dividend' and to anchor the system interest rate to the more stable average wage growth, while ensuring long-run sustainability with temporary adjustments of the system interest rate through an automatic balance mechanism (Settergren 2003; Settergren and Mikula 2006) that is triggered whenever the liabilities exceed the sum of contribution assets gross of the buffer fund, as explained in Focus 2.2. Note, however, that triggering the balance mechanism cuts the system's interest rate, which reintroduces volatility and, hence, some sort of intergenerational unfairness. Note also that the same problem arises in FDC schemes given the volatility of the market interest rate, showing a sort of unavoidable trade-off between solvency and intergenerational fairness.

5.3.2 Changes in Longevity

Increasing longevity bears important consequences for all pension plans. In DB schemes, an increase of the contribution rate and/or of retirement age must be legislated. In PA systems, whether funded or PAYG, the effect is different. According to the current trend in longevity, individuals born in a given year live longer than those born in previous ones. Therefore, for one-to-one correspondence between contributions and pension annuities to hold for all birth cohorts, divisors at all admitted retirement ages should increase by year of birth (i.e. by cohort). In particular, the set of updated divisors should be assigned to a cohort when it reaches the lower bound of the retirement age interval and be based on the cohort's specific life expectancies at ages included in the interval which, unfortunately, can be ascertained only after the last cohort members, if any, have reached the maximum age. Therefore, at the time when the cohort starts retiring such data can only be estimated according to either one of the following two approaches.

Focus 5.2: Estimating Life Expectancy Through Period and Cohort Life Tables

Life tables follow a hypothetical cohort from its birth to its complete extinction applying some death rates, observed or theoretically estimated, at each possible age. Death rates are computed as the ratio of the deceased in any given year to those exposed to the risk in the same year, while survival rates are their complements. The enclosed Fig. 5.6 shows the well-known Lexis diagram (Lexis 1875), which can be fruitfully used to explain the logic of estimating death and survival rates, and hence life expectancy at any possible age, of individuals belonging to a specific cohort born in any calendar year. Calendar time is measured on the horizontal axis of the diagram, whereas age is measured on the vertical axis. Each individual life is represented by a 45° upward sloping straight line crossing the horizontal line drawn in correspondence to any age one year after having crossed the line of the previous age. Broken lines represented with a round cap show a decease, while the location on the diagram of the round cap indicates the time and the corresponding age of the deceased. For instance, the members of the cohort born in 1956 reaching 62 in 2018 are exposed to the risk of dying before reaching 63 either in 2018 or in 2019. If they survive at 63, then they will be exposed to the risk of dying before reaching 64 (either in 2019 or in 2020) and so on up to the maximum observed age, say 110. Therefore, assuming that this 1956 cohort will start retiring at 63 in 2019, its life expectancy at 63 cannot be ascertained in 2018, when its divisors should be announced, as is shown by the question marks inserted along the diagonal lane showing the path to be followed by the cohort. Those rates could be ascertained only by following its vicissitudes after 2018 up to its complete extinction (in 2066 provided that some of its members reach the maximum age of 110). *Cohort* life tables adopt sophisticated techniques to project the death and survival rates of the retiring cohort, i.e. to predict how the lifelines of any given cohort will evolve in the future. On the other hand, *period* life tables provide a possible solution that does not require debatable projections, in that they rely exclusively on the data observed 'vertically', in 2018. As shown by the vertical bar of the diagram, those exposed to the risk of dying at 63 in 2018 can be computed by counting: (i) the lines of the members of the 1955 cohort crossing the 63-year horizontal line between 1.1.18 and 1.1.19 and (ii) the lines of the 1954 cohort crossing the 63-year horizontal line after 1 January 2017 and the 1.1.18 vertical line after turning 63, thus showing that they survived at 63 and were still alive on 1 January 2018. It is then sufficient to count the deceased (the round caps) among the exposed to compute the most updated measure of the death rate between 63 and 64. Note, however, that collecting the data necessary to compute death rates at higher ages it is necessary to observe the older cohorts.

For instance, the death rates at the age of 100 that can be observed in 2018 involve the cohorts born in 1918 and 1919 whose survival rates fall far below the survival rates at the same age that will be shown in 2056 by our 1956 retiring cohort. This is why divisors computed according to *period* life tables can be referred to as backward-looking, in contrast with the forward-looking ones computed according to *cohort* life tables.

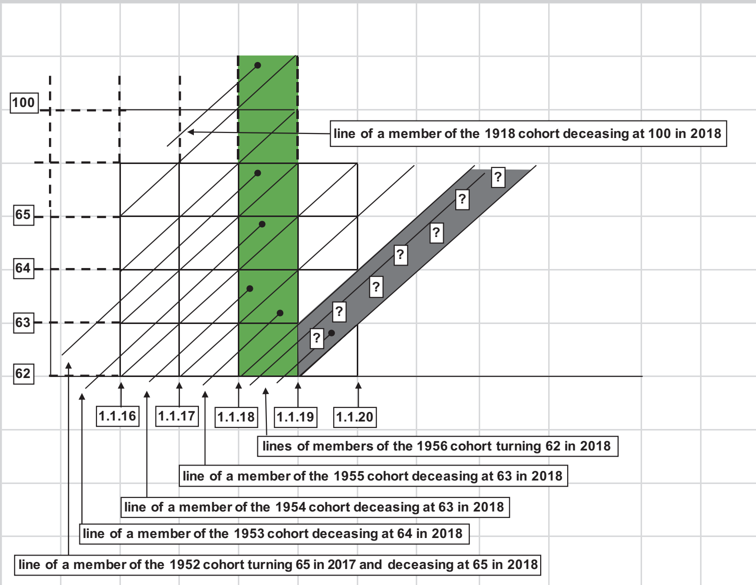

Fig. 5.6 Lexis diagram

The first approach consists in projecting *cohort* life tables, i.e. tables specifically projected for the cohort itself. With 'perfect foresight', this choice would produce exact forward-looking divisors, ensuring perfect fairness.

The second choice is to admit that perfect foresight is a chimera and simply derive cohort residual lives from the most updated usual *period* life tables, based on observation of the previous cohorts. This second choice produces obsolete, backward-looking divisors that are lower than they should be, thus threatening the solvency of PA systems (see Focus 5.2).

Divisor obsolescence jeopardizes both the fairness and the sustainability of PA systems. In fact, money withdrawn from personal accounts tends to exceed money previously deposited plus interest matured, while yearly pension expenditures tend to exceed the corresponding contribution revenues. Note that perfect foresight does not prevent forward-looking divisors from producing imbalances. In fact, it has been proven that they produce 'hypersustainability', i.e. each year pension expenditures tend to be lower than contribution revenues (Gronchi and Gismondi 2008). Moreover, the lack of perfect foresight makes forward-looking divisors scarcely viable from a socio-political point of view. In fact, workers would have to accept having their pensions computed on the basis of possibly erroneous forecasted mortality tables. Therefore, whereas forward-looking divisors are more congenial to privately managed complementary FDC systems, backward-looking divisors appear to be inevitable for public NDC systems. On the other hand, even the most widespread model to project cohort life tables, the Lee-Carter model (Lee and Carter 1992), has, so far, systematically underestimated longevity (Alho et al. 2013).

PA systems can minimize divisor obsolescence, thus fostering both fairness and sustainability by updating life tables annually and setting the minimum retirement age as high as possible, given that obsolescence is higher at low retirement ages (Gronchi et al. 2019). Note, incidentally, that lump sum payments of the entire account balance at retirement rather than disbursing annuities would solve the problems posed by increasing longevity. However, this is not a practical solution given that it implies shifting the burden on individuals of 'sensibly' spreading the account balance at retirement on residual life expectancy, with the consequence that the poverty rate among, possibly myopic, retirees would certainly increase. Moreover, the efficiency gains implied by the insurance against longevity risk would be lost.

5.3.3 The Problems Posed by Heterogeneous Mortality

A final observation concerns the 'insurance nature' of the annuities disbursed by PA systems which, inevitably, exhaust the contributions (gross of interest) only of the 'representative' cohort member whose residual life at retirement is exactly m: *ex post*, the longer-lived will have withdrawn more than they paid in, whereas the shorter-lived will have withdrawn only a part of their contribution balance. However, the equivalence of benefits and contribution would still be guaranteed *ex ante* if all individuals (regardless of social class, work environment, gender, region of residence, etc.) had the same probability of falling into the two subsets of the shorter-lived and longer-lived, implying that any subset of workers has the same life table.

Unfortunately, there is no reason why this should be the case. *Ceteris paribus*, the rich are longer-lived than the poor, women longer-lived than men, married people longer-lived than single, rural residents longer-lived than city dwellers. The consequences of heterogeneous mortality both on the uniformity of individual IRRs and on sustainability could be sterilized in various ways, e.g. by diversifying the balance divisors or the contribution rates by homogeneous social groups; however, diversification would be technically difficult (Holzmann et al. 2019), while its social acceptability could not be taken for granted.

Actually, when removing the *ceteris paribus* clause, the contrasting correlations generate quite a blurred picture. Suffice it to consider how the correlation between longevity and income could evaporate insofar as women, who have lower earnings, are longer-lived.

The correlation between life expectancy and income, and thus between the duration of the pension and the size of the notional capital, also undermines the sustainability of PA systems, given that the extra benefits to be paid to the longer-lived exceed those left by those who pre-decease them.

Focus 5.3: Computing Divisors

The one-to-one correspondence between pensions and contributions implies that, at retirement, after *n* years of contributions, the first pension is computed to achieve exhaustion of the deposit at a point corresponding precisely to the retiring worker's life expectancy, i.e. after the *m*th pension annuity is withdrawn. However, since life expectancy at retirement is computed by taking into account the probabilities that retiring workers will survive at each possible age after retirement up to the maximum observed age (e.g. 110 years), a more rigorous PA formula should include all potential benefits up to the maximum age weighted with their respective probabilities. Taking into account that each annuity following the first (withdrawn at the beginning of year $n+1$) is equal to the previous one times the indexation factor, denoting with h the maximum number of retiring years and with π_j the rate of return that the system credits to the account in each year $n+j$ ($j = 1, 2, \cdots, h-1$), the probabilistic deposit exhaustion constraint of a member of birth cohort B retiring at age x takes the following form:

$$\mathrm{AB}_{R,B,x} = p + \frac{\varphi_{B,x+2} \cdot p \cdot \left(1 + g_{p,n+1}\right)}{(1 + \pi_{n+1})} + \frac{\varphi_{B,x+3} \cdot p \cdot \left(1 + g_{p,n+1}\right) \cdot \left(1 + g_{p,n+2}\right)}{(1 + \pi_{n+1}) \cdot (1 + \pi_{n+2})} + \cdots + \frac{\varphi_{B,x+h} \cdot p \cdot \left(1 + g_{p,n+1}\right) \ldots \left(1 + g_{p,n+h-1}\right)}{(1 + \pi_{n+1}) \ldots (1 + \pi_{n+h-1})},$$

where $\varphi_{B,x+i}$ ($i = 1, 2, \cdots, h$) is the probability that the individual belonging to birth cohort B reaches age $x + i$ conditional on being alive at age $x + i - 1$. Given the frontloading rate (δ) chosen by the system and recalling the first pension and indexation rules of PA schemes, the first pension becomes:

$$p = \frac{\mathrm{AB}_{R,B,x}}{1 + \varphi_{B,x+1} \cdot \frac{1}{1+\delta} + \varphi_{B,x+2} \cdot \frac{1}{(1+\delta)^2} + \cdots + \varphi_{B,x+h} \cdot \frac{1}{(1+\delta)^{h-1}}},$$

where the denominator is the divisor at age x to be announced to the members of cohort B when they reach the lower limit of the interval. Note that the divisor can easily be computed for all possible retirement ages on the basis of the chosen frontloading rate and of all survival rates recorded in the last updated life table. The Swedish NDC system, where the lower limit of the interval of admitted retirement ages is 61, has a sophisticated mechanism by which it first announces provisional divisors to the cohort that just turned 60 and then updates the divisors before the cohort turns 65, eventually recomputing the pension of those who have chosen to retire between 61 and 65 with the 'obsolete' divisors. The enclosed Table 5.3 shows the series of definitive divisors attributed, year by year, to each birth cohort involved in the NDC system, applied in Sweden as of 1998 and computed with the rigorous, probabilistic method indicated above.

Table 5.3 Confirmed annuity divisors for the Swedish NDC scheme (Pensionsmyndigheten 2018)

Age / *Cohort*	*61*	*62*	*63*	*64*	*65*	*66*	*67*	*68*	*69*	*70*
1938	17.87	17.29	16.71	16.13	15.56	14.99	14.42	13.84	13.27	12.71
1939	17.94	17.36	16.78	16.19	15.62	15.04	14.47	13.89	13.32	12.76
1940	18.02	17.44	16.86	16.27	15.69	15.11	14.54	13.96	13.39	12.82
1941	18.14	17.56	16.98	16.39	15.81	15.23	14.65	14.08	13.50	12.94
1942	18.23	17.65	17.06	16.48	15.89	15.31	14.74	14.16	13.59	13.02
1943	18.33	17.75	17.16	16.58	15.99	15.41	14.84	14.26	13.68	13.11
1944	18.44	17.86	17.28	16.70	16.11	15.54	14.96	14.38	13.80	13.23
1945	18.55	17.96	17.38	16.80	16.22	15.64	15.07	14.48	13.91	13.33
1946	18.64	18.05	17.47	16.89	16.31	15.73	15.16	14.57	13.99	13.41
1947	18.73	18.15	17.56	16.98	16.40	15.83	15.24	14.66	14.07	13.49
1948	18.83	18.24	17.66	17.07	16.49	15.91	15.33	14.74	14.16	13.58
1949	18.89	18.31	17.72	17.13	16.55	15.97	15.38	14.79	14.21	13.63
1950	18.98	18.39	17.80	17.21	16.63	16.05	15.46	14.87	14.28	13.70
1951	19.06	18.48	17.89	17.30	16.71	16.13	15.54	14.95	14.37	13.78
1952	19.14	18.55	17.96	17.37	16.78	16.20	15.61	15.02	14.43	13.85
1953	19.20	18.62	18.03	17.44	16.85	16.26	15.68	15.09	14.50	13.91

5.3.4 Coordinating PA with Disability and Survivor Plans

Personal accounts are essentially designed to manage old-age insurance. They are hardly compatible with the strong heritage of public pension systems charging a single contribution rate and awarding old-age, survivor and disability pensions. As to disability, the public PA systems need to split the system into different plans. Disability will have to be financed through a different contribution rate or from general tax revenues while disability allowances, possibly means-tested, are awarded to citizens regardless of their being workers or not. Following the Swedish model, such a plan should be charged to pay contributions on allowances (as if they were salaries) to the old-age plan. The contributions, in part paid also by the disabled, would be credited to old-age personal accounts and contribute to old-age pensions.

Survivors' benefits could, in principle, be included in the old-age PA system by increasing divisors to take into account the further annuities that are expected on the basis of survivors' ages and of the probability that the deceased has a survivor. However, redistributive flows would occur from single to married retirees and from similar-age to distant-age couples, with negative effects on fairness. As an alternative, as is common practice in FDC systems, one could admit the choice between 'one-head' divisors and 'two-head' divisors, the latter based on survivors' ages. Note, however, that despite complying with the fairness objective of PA systems, such a choice would jeopardize sustainability given both the moral hazard problems and the relevant database needed for correctly computing 'general' two-head divisors (Gronchi and Nisticò 2006). In this case, too, the Swedish choice to adopt a fiscalized survivors' programme paying means-tested, and possibly temporary, allowances, is convincing.

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CHAPTER 6

The Challenges for Public Pension Schemes

Abstract With population ageing, life expectancy at retirement increases and the ratio of contributors to retirees decreases. To ensure solvency, defined benefit (DB) systems undergo continuous parametric reforms, including raising retirement age, whereas personal accounts, defined contribution (PA-DC) systems are endowed with automatic adjustments ensuring solvency in both their funded and pay-as-you-go (PAYG) versions, albeit at the cost of jeopardizing pension adequacy. Within financial defined contribution (FDC) and non-financial defined contribution (NDC) systems, retiring workers are given the free choice of either accepting lower benefits or maintaining benefit adequacy by postponing retirement. Moreover, following the logic of personal accounts workers can see contributions as mandatory savings rather than taxes on their labour services. The extent to which a costly transition from PAYG to fully funded systems can counteract the effects of population ageing is debatable.

Keywords Population ageing · Parametric reforms · Political risk and automatic adjustments · Retirement age and pension adequacy · Pension contributions and taxes · Transition from PAYG to funding

The age structure of a country is a fundamental determinant of the cost of pension provisions. It can be represented with piled-up horizontal bars, each showing the size, or the percentage, of the population aged 0–4, 5–9, 10–14 and so on up to the maximum age. During the last

S. Nisticò, *Essentials of Pension Economics*,
https://doi.org/10.1007/978-3-030-26496-3_6

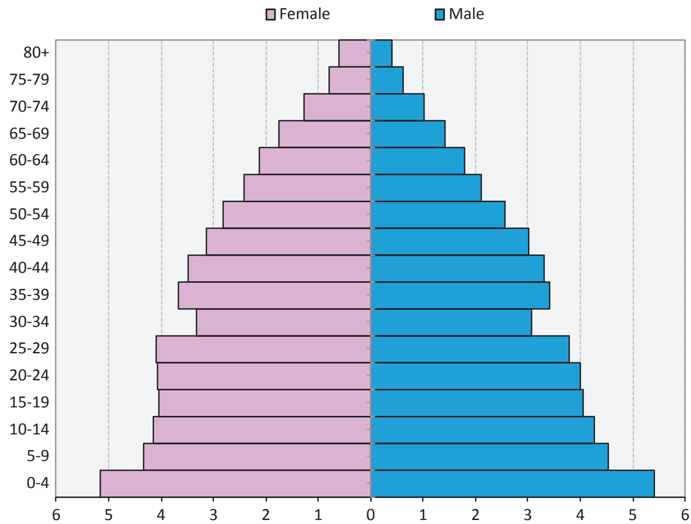

Fig. 6.1 Age structure in percentage of total population for high-income countries in 1950. Author's elaboration based on United Nations (2017) data

century, when the majority of compulsory pension systems was designed and implemented in what are now known as high-income countries, the age structure was represented by the typical 'population pyramid', the bars progressively shrinking at higher ages, as represented in Fig. 6.1, which shows the age structure of the population of high-income countries in 1950. Note that the exception of the bar representing the cohorts aged 30–34 in 1950 depends on the lower number of births during the First World War.

As pointed out in Chapter 2, because of the simultaneous decline in the fertility rate and in death rates at all ages, the population age structure is changing, giving rise to what is known as population ageing. For high-income countries, it is rapidly taking a shape closer to a classical modern skyscraper than an ancient pyramid, as shown by the actual age structure in 2018 and by the projections for 2050 reported in Fig. 6.2.

Despite lagging behind the high-income countries, low-income countries are also facing population ageing, given that fertility and death rates are declining worldwide (Department of Economic and Social Affairs of the United Nations Secretariat, Population Division 2017, p. 2). Note

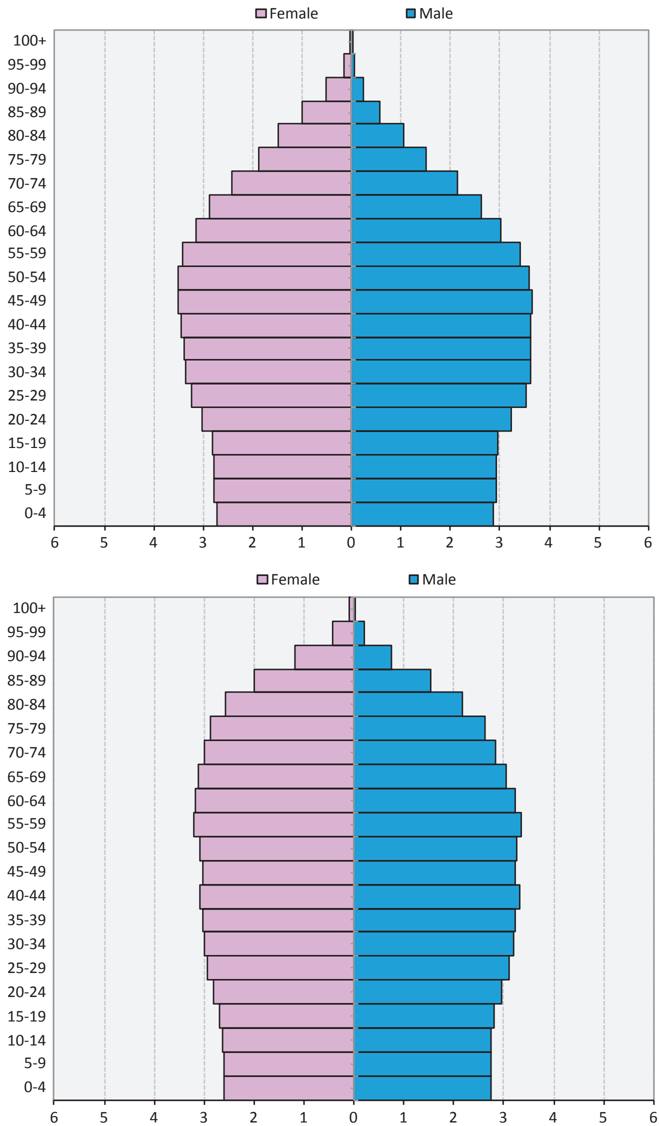

Fig. 6.2 Age structure in percentage of total population for high-income countries in 2018 (above) and in 2050 (below). Author's elaboration based on United Nations (2017) data

that not only is the age structure changing in shape but it is also increasing in height, which raises the question of whether the ongoing-pattern of increasing life expectancy will go on indefinitely. In this respect, scholars engaged in research on longevity can be divided into two groups, those who don't see any reason why the surprisingly linear (constant) increase of 3 months per year in longevity should drop in the future, e.g. Oeppen and Vaupel (2002), and those who think that "the human body is now running up against inherent limits that the genetically fixed attributes of our biology impose" (Olshansky 2018, p. 195). Predictions of a slow-down in longevity increase are based on the fact that much of the past gains in expected years of life resulted from the survival rates at young ages, now substantially exhausted, whereas the still ongoing increase in survival rates at higher ages inevitably produces lower gains in expected years of life.

One of the most striking effects of the present demographic trend is the progressive increase in the old-age dependency ratio, i.e. the ratio of potential retirees to potential contributors. In fact, whereas the elderly in high-income countries, defined as people aged over 64, accounted for just 12.3% of the number of adults aged 15–64 in 1950, they now represent 27.5% and will be around 46.5% of their 'active' counterparts in 2050. Note that for some countries, such as Italy, the old-age dependency ratio is expected to settle at over 60% in the coming decades (United Nations 2017).

6.1 The Threat to Solvency and Adequacy

The current demographic scenario has serious implications for both the solvency of the systems and the adequacy of pension provisions, although they differ for pay-as-you-go (PAYG) and funded systems.

6.1.1 The Simple 'Algebra' of Pay-as-You-Go Financing

In a solvent PAYG system, the value of pensions that can be disbursed to retirees is inevitably linked, in each calendar year, to the amount of contributions paid by employed workers. In fact, for contribution revenues to be able to finance pension expenditures, in each calendar year the following equality must hold:

$$\bar{p} \cdot R = c \cdot \bar{w} \cdot L,$$

where $\bar{p}$ denotes the financially sustainable average pension, R the number of retirees, $\bar{w}$ average earnings, while L again denotes the number of workers. If a PAYG system shares the DB philosophy, shifting the burden of any needed adjustments onto the contributions to be paid by the workers, the above equation can show the effect of the worsening demographic scenario on the contribution burden. In fact, by solving it for the contribution rate:

$$c^* = \frac{\bar{p}}{\bar{w}} \cdot \frac{R}{L},$$

we can compute the impact on the equilibrium contribution rate of keeping the pension level ($\bar{p}/\bar{w}$) at a targeted level given the ongoing increase of the old-age dependency ratio (R/L). In fact, supposing that retirement is compulsory at 65 and that all persons aged over 65 receive a pension regardless of whether or how much they have contributed to the system when active, the ratio of retirees to workers on the right-hand side of the equation above can be expressed as:

$$\frac{R}{L} = \frac{P\left(65^+\right)}{e \cdot P(15 - 65)},$$

where $P\left(65^+\right)$ and $P(15 - 65)$ denote population above 65 and between 15 and 64, respectively, while e denotes the employment rate. Taking into account that, at present, the average employment rate in OECD countries falls short of 70% (OECD 2019), the envisaged increase in the old-age dependency ratio to around 0.5 implies that the ratio of retirees to workers will amount to over 70%, showing that the target of a 45% pension level implies an equilibrium contribution rate to the old-age pension scheme rising to over 32%. On the other hand, with the exception of Italy, PAYG public systems generally charge a contribution rate much lower than that and are reluctant to raise it given the possible negative effect on employment. A typical case is to be seen in United States, where the envisaged depletion of the trust fund calls for an increase in the contribution rate well above the current 10.6%, but there is still no political agreement as to whether this measure should be implemented. In some cases, the solution envisaged is to shift the burden of guaranteeing all retirees a minimum pension, regardless of their contribution record, onto the general tax revenues, thus reducing the burden of overall pension expenditure on workers and employers. In order to prevent pension expenditures from driving up either overall taxation (which ultimately also falls on the workers and employers) or the public debt, the standard, minimum retirement age is being raised over 65 to increase the number of actives and reduce the number of retirees. For instance, if the legal retirement age were to be raised to 70, the redefined old-age dependency ratio in 2050 would amount to around 35% rather than 50%, and the ratio of retirees to workers would be:

$$\frac{R}{L} = \frac{P(70^{+})}{e \cdot P(15-69)} \cong 0.5,$$

thus considerably reducing the burden of the ongoing demographic scenario on the equilibrium contribution rate. However, taking into account the slowness of the political processes when the discretionary measures to be agreed upon and adopted will hurt voters in one way or another—either the workers or the retirees—PAYG–DB systems are exposed to the political risk of indefinitely postponing the required adjustments.

For non-financial defined contribution (NDC) systems, the implications of the constraints imposed by PAYG financing are different, given that solvency cannot be ensured by raising the contribution rate, nor is there a legal retirement age to be raised, the free choice of retirement age within a pre-set interval being one of the essential properties of these systems. In fact, once appropriately regulated, NDC systems are endowed with automatic mechanisms that can ensure long-run solvency without any further political intervention. A better understanding of the different mechanisms governing PAYG–PA systems can be gained by solving the equality between contribution revenues and pension expenditures for the pension level as follows:

$$\frac{\bar{p}}{\bar{w}} = c \cdot \frac{L}{R},$$

which shows that the fall in the ratio of contributors to retirees on the right-hand side implies, ceteris paribus, an inevitable fall in the pension level. This is why some form of automatic increase of the lower bound of the retirement age interval would certainly foster pension adequacy of NDC pensions. On the other hand, given that the workers can choose their own retirement age, the demographic effect of the increase in the dependency ratio can be offset by workers voluntarily retiring over 65. Note that if workers chose to retire at 70, the ratio of active workers to retirees on the right-hand side would come to 2, thus showing that if the average age at retirement reaches 70, the pension level ensured by NDC systems by the middle of this century will be around twice the chosen contribution rate.

6.1.2 Funded Systems

The current demographic scenario also has severe implications for funded systems despite the fact that in these systems birth cohorts are financially independent of one another. In this case, too, the implications for the DB earnings-related version differ from those of the more common, DC-PA version. As to the former, it is obvious that increasing life expectancy at retirement raises the liabilities of the system towards both workers and retirees, thus requiring higher assets and, hence, higher contribution rates and/or longer contribution periods to keep the system fully funded, while maintaining the existing benefit generosity. As for the latter version, the automatic adjustment of the benefits to life expectancy, together with crediting the sustainable rate of return to all accounts, ensures that assets keep pace with liabilities for any given contribution rate. On the other hand, the automatic adjustments raise the fundamental issue of pension adequacy, as in NDC systems. In fact, as shown in Sect. 5.2.3, if the market interest rate does not deviate significantly from the growth rate of earnings, the replacement rate that workers can expect from a PA system, whether funded or PAYG financed, amounts essentially to the product of two factors, namely the contribution rate and the ratio of contributing to retirement years. It is therefore evident that the higher the number of years that retiring workers expect to live, the lower will be the replacement rate ensured by PA systems for any given contribution rate. Note that with unisex life expectancy at 65 likely to rise to around 24 years (United Nations 2017) in the next few decades, the replacement rate to be expected by the 'average' worker retiring at 65 with a 20% contribution rate and a contribution record of 35 years is around a mere 29%. Therefore, also in funded PA systems workers are induced to voluntarily postpone retirement age if they wish to offset the negative effect of increasing life expectancy on the replacement rate. Let us add that although increasing the contribution rate is not an option to ensure long-run solvency for PA systems, it is in fact an option for solvent systems, both in their funded and PAYG versions, to foster pension adequacy.

At this point we need to take a brief look at how the automatic adjustment mechanisms of public NDC systems affect workers and retirees. Let us note, first of all, that it is in fact possible to conceive other DC systems wherein pension expenditures would still be *endogenously* determined according to the exogenous dynamics of the revenues, e.g. by disbursing to each retiree, regardless of his/her contribution record, the same pension amounting, in any given year, to the ratio between the contribution revenues and the number of retirees. Such a DC system, despite possibly appearing 'just' according to same equity principle, would be unfair according to our notion of fairness in the sense of uniformity of individual internal rate of returns (IRRs).

Conversely, PA systems react to any shock, be it of demographic or economic nature, by adapting pension expenditures to the contribution revenues, while ensuring intra-generational fairness. On the other hand, because of their DC nature, i.e. of relieving the contribution rate of the burden of ensuring financial stability, they are generally considered unfair towards the cohorts of retirees required to accept lower pensions while exempting active workers from sharing the burden of the needed adjustment. However, given that the most important adjustment mechanism in PA systems is the continuous steering of the rate of return credited to personal accounts, which of course affects the indexation rate of pensions in payment and hence undoubtedly harms retirees, closer examination of how NDC systems react to the worsening of their financial position shows that the active generations of workers do in fact pay a part of the bill. In reality, their account balances and hence their future pensions are de facto being reduced relative to what they would have been if the credited rate of return had not been lowered. It will then be their choice whether or not to offset the reduction of their pension wealth by postponing their retirement age. This is why continuous transparency and information is a fundamental ingredient of PA systems, as explained in Sect. 5.2.3. Moreover, given that bad health and heavy jobs may compromise this kind of free choice, sickness and disability insurance, as well as improvement in the quality of the workplace are necessary ingredients for the free choice to be effective.

6.2 An Alternative to Automatic Adjustments: The Point Systems

In Sect. 6.1.1 we saw how DB and DC systems deal with the constraints imposed by PAYG financing, with the former adjusting the contribution rate while the latter leave retiring workers the option between retirement age or amount of pension. On the other hand, also many DB systems are introducing a change in retirement age and a reduction in pension benefits, with the consequence that the DB–DC distinction is to some extent blurring in the case of public PAYG systems. In fact, a mix of all forms of adjustment can be employed, discretionarily rather than automatically, with the ultimate aim of enforcing some equitable distribution of the gains or losses generated by PAYG financing. This is precisely the strategy of the 'point systems' (PSs), such as the statutory supplementary schemes introduced in France in the aftermath of the Second World War and in Germany in 1992. Actually, the logic underlying PS is very similar to that of PA, the main differences being that points rather than 'money' accumulate on workers' accounts and that the number of accumulated points is transformed into a pension annuity according to the value that a 'managing board' attributes to the same points in the year of retirement. The same value of the point is then used to uprate pensions in disbursement in any given year, such that the yearly change in the point value determines the indexation rate of existing pensions.

6.2.1 The German Point System

In the German version of the PS, the points acquisition rule is earnings-related as in the Bismarckian tradition. More precisely, the pension points (*PP*) credited in any given year to each worker are computed according to the ratio between his/her earnings and the average earnings in the same year. For instance, the pension points earned by a hypothetical worker *i* in year *t* are:

$$PP_{i,t} = \frac{w_{i,t}}{\bar{w}_t},$$

whereas the pension to be awarded in the same year *t* to an individual *r* retiring after *n* years of work is:

$$p_{r,t} = \sum_{j=1}^{n} PP_{t-j,r} \cdot AF_r \cdot PPV_t,$$

where PPV_t denotes the value of one point in calendar year *t* and AF_r denotes the adjustment factor 'correcting' the pension according to a possible discrepancy between the statutory and retirement age of the individual *r*. As mentioned above, pensions are adjusted according to the annual percentage change in *PPV*, whose 'first' value was set in 1991, so as to match the pension entitlement an average-income worker earned by contributing one year according to the previous rules (Queisser 1996). It is with the yearly adjustments of *PPV* that the German system ensures its solvency. In fact, it was precisely by introducing a sustainability factor that links the growth factor of *PPV* (positively) to the growth factor of aggregate earnings, as imposed by the sustainability conditions in a steady state with constant age structure of population, but also (negatively) to the growth factors of the contribution rate and of the ratio of retirees to contributors, that the German system aimed to achieve a burden-sharing mechanism, transferring also to retirees a part of the cost falling on active workers to meet current obligations. A parallel aim of the process is to whittle down the PAYG public pillar in favour of occupational and private, supplementary funded systems (Börsch-Supan and Wilke 2004, p. 29). Recurrent changes in the sustainability factors need to be, and actually have been, made in order to ensure that the development of employment growth and life expectancy does not lead the contribution rate to exceed, and the net replacement rate to fall short of, some pre-set targets. The result is a sort of hybrid system of DB and DC ensuring financial solvency through continuous adjustments of both the contribution rate and the number and generosity of pension annuities, with the sustainability factor incorporating "a self-stabilizing feedback mechanism into the system similar to the notional rate-of-return mechanism in NDC systems" (Börsch-Supan and Wilke 2006, p. 603). On the other hand, the simplicity and transparency of the NDC automatic mechanisms stand out in contrast with the complexity and the residual intra-generational unfairness of the German Point system (Gurtovaya and Nisticò 2018, pp. 380–381).

6.2.2 The French Point System

Despite being 'supplementary' to the first PAYG pillar, the French version of the PS charges contributions and disburses pensions to around 40 million individuals. Formulated in the middle of the last century, it can be considered an embryonic form of the NDC scheme (Palmer 2006, p. 17). On the other hand, the discretionary elements in the French PS are even more pronounced than those embedded in the German one, thus marking a profound difference with the logic of PAs. In fact, the French version of the PS distinguishes between the 'buy' and the 'sell' value of the points, both set annually through an agreement signed by the representatives of the employers' and employees' organizations. A further, fundamental discretionary element of the French PS is what is known as the 'calling rate', i.e. the ratio between the effective contribution rate (EPR) paid by workers and the point acquisition rate (PAR), i.e. the contribution rate ensuring the right to acquire pension points. Higher values of the PAR are applied to higher income brackets identified by bend points. No contributions are due above a pre-set income ceiling. The key role of the calling rate should be evident, given that by setting it above unit, the system raises the contributions accruing to the system without raising liabilities, i.e. without adding points to workers' accounts. In fact, the number of pension points earned in any year is given by the ratio of the individual contributions ensuring the right to acquire pension points and the purchase cost of one pension point (APV) while, similarly to the logic of the German PS, the pension to be awarded in year t to an individual r retiring after n years of work at the statutory pension age is:

$$p_{r,t} = \sum_{j=1}^{n} PP_{t-j,r} \cdot CPV_t,$$

where CPV_t is the sell value of the points, whose yearly percentage change governs the indexation rate of pensions in payment, as for the German PS. Note that actuarial adjustments according to the contributions record are applied to those drawing their pension before the statutory pension age, which is being gradually raised from 65 to 67.

Summarizing, the French PS can discretionarily manoeuvre its four levers to make burden-sharing adjustments in response to the ongoing problems of higher dependency ratios and low economic growth (Gurtovaya and Nisticò 2019, p. 123). The levers produce:

- a lower growth rate of *CPV*, which harms retirees by reducing current pension expenditure;
- a higher value of APV, which reduces pension liabilities towards workers;
- a higher value of the calling rate, which harms workers by increasing the contribution revenue without increasing their pension credits;
- a higher value of PAR, which increases contribution revenue together with pension liabilities.

6.3 Intergenerational Fairness

In Chapter 5, we explained how pension formulas can generate intra-generational unfairness, i.e. discrepancies among the IRRs of individuals belonging to the same birth-cohort according to their career pattern. We also explained that PA systems are referred to as actuarially *fair* in that they substantially ensure equality of IRRs to all members of any given birth cohort. The aim of the German and French PS is to be 'fairer' to different generations in that they discretionarily distribute the burden on active and retirees alike. This ambition calls for a discussion of the fundamental issue of intergenerational fairness, by which we mean uniformity of IRRs awarded to different birth cohorts.

In fact, in Chapter 4 we saw that financially sustainable pension systems must, implicitly or explicitly, adjust the rate of return rewarding compulsory old-age contributions according to the growth rate of aggregate earnings, if PAYG financed, or to the market interest rate, if fully funded, and both variables show great volatility over the decades. Suffice it to recall the striking differences in GDP and income growth rates before and after the early 70s and in real interest rates during the period spanning from the mid-80s to the mid-2000s and after (World Bank 2019). Thus, the birth cohorts that have the chance to reach the peak of their pension wealth during a long period of high rates of returns will eventually benefit, *ceteris paribus*, from a much higher IRR than the birth cohorts whose compulsory savings accumulate during periods of very low, or even negative, rates of return awarded by the pension systems to ensure its financial stability. It was precisely with the aim of reducing intergenerational unfairness that many public, earnings-related PAYG systems have wisely accumulated a buffer fund during the periods of very high growth in earnings (essentially after the Second World War and until the 1970s), i.e. they charged the (at that time) high number of contributors a contribution rate higher than that sufficient to raise the revenues needed to disburse the defined pension benefit to the (at that time) low number of retirees. If buffer funds are in place, NDB schemes can avoid raising the contribution rate as soon as the contribution revenues fall short of pension expenditure, thus avoiding the negative impact on the IRR awarded to the present cohorts of contributors. As to NDC schemes, with buffer funds the rate of return credited to personal accounts can be kept far more stable than would otherwise be necessary to ensure yearly equality between revenues and expenditures, as is the case in Sweden (see Sect. 5.3.1). Those buffer funds are now being exhausted because of the ongoing increase in the ratio of retirees to contributors, given that a high percentage of the many past contributors are now retiring (with a very high life expectancy), precisely when the number of present contributors is particularly low. On the other hand, the ongoing demographic scenario leaves scant room for the prospect of young cohorts now entering the labour market having their contributions rewarded with the same IRR as earned by the recently retired cohorts. The problems of intergenerational fairness are clearly exacerbated when, because of the above-mentioned political risk, public pension systems avoid taking the necessary measures to ensure financial stability in good time. Given that, sooner or later, the adjustments will have to be enforced, the longer they are delayed the greater will be the difference between the IRR awarded to the 'protected' cohorts and that of the 'sacrificed' ones.

As from the early 1990s, a number of scholars (Auerbach et al. 1991, 1994) have been pointing out the burden for younger and future generations of ensuring that the Government complies with its intertemporal budget constraint, i.e. that it does not "default on its liabilities in essence satisfying the constraint through a tax on its creditors" (Auerbach et al. 1994, p. 75). Essentially, this exercise of generational accounting computes the present value of taxes, net of benefits, that will have to be charged to existing and future generations for the ongoing fiscal policy to be sustainable. Despite the fact that the basic approach of generational accounting is essentially prospective, i.e. it focuses only on the *remaining* lifetime taxes and benefits of existing and future cohorts—thus being unsuitable for intergenerational comparison, in that it overstates the net benefits for older generations whose remaining tax burden is negligible but had borne a significant tax burden in the past—it can be extended so as to take into account also the past net taxes of existing generations (ibid., pp. 85–87), thus becoming suitable to assess the degree of intergenerational fairness of alternative pension policies. On the other hand, as shown in Focus 5.1, a more transparent methodology could rely on computing the IRRs of typical workers belonging to different cohorts according to the historical and projected evolution of benefits, contribution rates, retirement age and life expectancies, as in Disney (2004, pp. 286–291).

Although generational accounting is open to criticism in several respects (Haveman 1994), in particular the need to make arbitrary assumptions about the discount rate used to compute the present value of net taxes and the failure to take into account the benefits for each generation of Government expenditure, when applied to pension design, the approach has the merit of emphasizing the political risk implied by deferring the necessary parametric adjustments to pension systems that have turned out to be unsustainable in the ongoing demographic and economic scenario.

6.4 Different Views on Pension Reforms

In fact, introducing a PA system, endowed with automatic adjustments not needing any further parametric reform, can be seen as a decision of the legislative body to be 'lashed to the mast' (Brooks and Weaver 2006), just as Homer's mythical Odysseus told his crewmen to lash him to the mast of the ship so that he could hear the alluring voices of the Sirens without the risk of running after them.

Moreover, when pensions are disbursed according to PAs, given their fairness, individuals perceive contributions not as a tax but as compulsory savings (Disney 2004). Therefore, the existence of a PA public pension scheme may not necessarily distort individual savings decision and labour market participation. Individuals could merely complement the compulsory contribution rate to the public pension scheme with their voluntary savings; the higher the former, the lower the latter. This is true in particular if the PA public scheme is fully funded, thus remunerating individual savings with the market interest rate, which is the rate of return available to voluntary savers (Blake 2006, p. 106). This is why many economists around the world have traditionally advocated reducing the generosity of public, mandatory PAYG systems and going back to fully funded, PA schemes as a way to avoid the tax, a distorting component of contributions to public pension systems, pointing out in particular the positive labour-force participation incentive, especially among the elderly, in reducing the generosity of public pensions (Gruber and Wise 1999, 2004).

However, the emergence of the idea that PAs could also be 'implanted' into the body of PAYG financing and the ensuing implementation of NDC schemes in Italy, Latvia, Sweden and Poland in the 1990s, later followed by Norway, has shown that fairness rather than funding is the necessary condition to avoid the tax elements in pension contributions. So much was in fact admitted by Feldstein, one of the most convinced advocates of funding (Feldstein 1998, p. 8), who recalled that the proposal to adopt PAs in a PAYG settings dates back to a proposal made in the United States by James Buchanan at the end of the 1960s (Buchanan 1968). On the other hand, Feldstein points out that the average rate of return on the assets accumulated in a funded pension system (the sustainable rate of return of funded systems), be it public or privately managed, can be much higher than the growth rate of earnings (the sustainable rate of return of PAYG systems), thus implicitly imposing a tax on mandatory contributions. For any system, this tax can be computed, at any specified date, on the basis of the difference between the present values of benefits discounted at the growth rate of earnings and of the same benefits discounted at the higher rate awarded by funded systems.

The major thread running through Feldstein's argument is that many public PAYG systems have kept the rates of return above their sustainable level in order to increase their political appeal, thus imposing an ever-increasing contribution burden on the younger generations of workers (Feldstein 1998, p. 5). Of course, raising the contribution burden cannot go on indefinitely, which brings us to the inevitable reduction of the generosity of benefits. In this respect, NDC systems, being endowed with mechanisms ensuring sustainability for a given contribution rate, also escape this part of Feldstein's criticism. On the other hand, the possible systematic differences in the rate of returns awarded by sustainable PAYG and funded systems merit discussion. In fact, a slightly different definition of fairness considers only funded PA systems fully fair in that they ensure a rate of return equal to the market interest rate, whereas, according to this definition, PAYG–PA systems are quasi-actuarially fair (Lindbeck 2006; Lindbeck and Persson 2003).

According to the arguments mentioned above, funded systems should be 'superior' to unfunded systems since they ensure a rate of return higher than that ensured by PAYG systems. What they fail to take into account is the fact that this was not the case in the past, when the very high rates of productivity and employment growth allowed PAYG financing to ensure higher returns than funded plans and there is no reason why the same situation should not arise again in the future, in particular when the high administrative costs, which significantly reduce the individual account balances, are deducted from the gross returns of invested funds (Diamond 1996, pp. 76–78). Moreover, we should recall that many public pension systems started as fully funded, while shifting to

PAYG financing was necessary because of the depletion of the funds or because of the need to use the funds to support other programmes of public interest, as in the case of U.S. Social Security (Musgrave 1981, p. 97). In other words, PAYG financing should not be considered an 'original sin' for which some remedy should be sought. In fact, PAYG financing not only allowed the first cohorts of retirees to receive a pension without having contributed in the past but also, in many other cases, it allowed all those cohorts that had contributed to a funded system—and whose financial assets were, for whatever reason, depleted—to secure a pension. Similarly, many pension systems aiming to be fully funded, in particular in their DB-earnings-related version, didn't adjust the system's parameters to compensate for the lower-than-expected returns on their assets. The consequent fall in the system's degree of funding, bringing an increasing burden to bear on the sponsor, called upon to fill the gap between assets and liabilities, eventually led to the closure of many plans, or to their transformation into a PA scheme.

Moreover, the option of going back to fund the system's liabilities should not be discussed through abstract comparison of the rates of return of the two financing methods. More appropriately, we should take into account the costs of the transition implied by the choice to make the present generations of workers accumulate their contributions in a fund, which deprives the PAYG system of the resources needed to extinguish the debt towards the retired and retiring workers. If the option of making current generations bear the burden of the transition by paying twice, i.e. for their own funded pension and for the PAYG pension of the retired, is rejected, then the only remedy is to borrow the necessary money at the current interest rate, thus imposing on future generations the burden of the interest on the required loans. However, such a burden would clearly offset the potential benefits that policymakers promised to those generations when advocating the transition (Orszag and Stiglitz 2001, pp. 24–25). In fact, "completion of the phase-out of a pay-as-you-go defined benefit system does not necessarily mark a stage with higher capital, since completion of a debt-financed transition leaves a higher level of explicit debt, roughly offsetting the accumulation in the new funded system" (Diamond 1996, p. 79). Alternatively, as was the case with Chile, which went through the shift from PAYG to funding in 1981, the presence of a budget surplus can in fact do the job, but this solution is now unlikely given the present state of public finances in many countries potentially interested in the transition. Another issue to be considered is whether it is true that embarking on the costly transition towards a fully funded system would actually benefit the economy by fostering national savings, and also whether any such hypothetical increase in savings would lead to higher investments and greater economic growth (Cesaratto 2005, 2006).

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CHAPTER 7

Concluding Remarks

Abstract The debate on pensions is inevitably influenced by differences in political viewpoints as to the extent to which the State should replace the market when the latter fails to ensure the well-being to a large portion of population. However, knowledge of the fundamentals of pension economics can help to purge the debate from preconceived disagreement, as can be seen in the case of the growing consensus as to the need to amend the perverse and regressive redistributions of many DB earnings—related pension systems. Hopefully this book provides the patient readers with the tools needed to weigh up the pros and cons of alternative policy proposals, being aware that there is no magic wand at hand to solve the complex problems threatening the future of public pension systems.

Keywords Fundamentals of pension economics · The pension debate · The future of public pension systems

Political stances and values inevitably influence the debate on economic policy proposals, the recurrent issue being whether the redistributions generated by government intervention do ultimately satisfy efficiency requirements and bring some advantage to the potential beneficiaries. It is possible that public pension systems have been excessively burdened with redistributive goals, as well as the task of favouring early retirement to the advantage of companies leaving obsolete industries, restructuring

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https://doi.org/10.1007/978-3-030-26496-3_7

or intending to introduce labour-saving innovations. While few scholars would object that a poverty-preventing first pillar *à la* Beveridge financed by general tax revenue is needed, the debate being rather about its level and whether it should be universal or means-tested, disagreement waxes strong over the need for a robust public system supporting individuals in consumption smoothing while abstaining from any kind of redistribution. On the other hand, substantial agreement could be reached on the reasons why a collective, mandatory pension system is welfare-enhancing, given the myopia of individuals and the problem they would face in managing the longevity risk. Quite certainly there would also be widespread consensus on the need to avoid the perverse and often regressive redistributions of many DB earnings—related pension systems, even in their funded version. On all other policy options, some disagreement among scholars is inevitable, and indeed open-minded disagreement is generally preferable to a priori consensus. There are different options as to how to set up a well-designed pension system and, ultimately, the chimera of a perfect pension system should be obvious, leaving scope to work out what is the best design given the specific cultural, political and social conditions in each country. Workers and retirees benefit more from the good management of an imperfectly designed pension system than from the bad management of a perfectly designed one.

In fact, there is no magic wand to solve the complex problems that slower economic growth and ageing hold for pension systems. The present and expected well-being of millions of people around the world is at stake—people who might start to feel the same sense of insecurity experienced by past generations before the organisation of robust welfare states ensured the opposite sense of *social security*. Reaching this extraordinary goal, made possible thanks to the contribution of brilliant intellectuals and enlightened policymakers, was certainly facilitated by the demographic/employment dividend that allowed a small contribution of the active population to ensure generous pensions for the elderly. Now that slower economic growth and ageing are becoming the norm for at least several decades to come, we should all, whatever our age, agree to revise our expectations and accept some adjustments in our healthier and longer lives. Hopefully this book will have provided the patient readers with the tools needed to weigh up the pros and cons of alternative policy proposals, with the awareness that one of them, i.e. getting rid of public pension systems altogether, would leave all the problems there, to be managed individually, by the family, or by charitable organisations.

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